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VOL. 149.—PART I.

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MDCCCLIX.

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THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the Council-books and Journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume; the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgement of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

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PHILOSOPHICAL TRANSACTIONS.

I. *Researches on the Foraminifera.*

By WILLIAM B. CARPENTER, *M.D.*, *F.R.S.*, *F.G.S.*, *&c.*

Received June 17,—Read June 17, 1858.

PART III.

ON THE GENERA PENEROPLIS, OPERCULINA, AND AMPHISTEGINA.

119. IN my preceding memoirs, I have shown that two very dissimilar types of structure present themselves among Foraminifera; one characterized by its simplicity, the other by its complexity. In the former, of which *Orbitolites*, *Orbiculina*, and *Alveolina* are typical examples, the calcareous skeleton does not present any definite indications of organization, but seems to have been formed by the simple calcification of a portion of the homogeneous sarcode-body of the animal; that sarcode-body is but very imperfectly divided into segments, the communications between the cavities occupied by these segments being very free and irregular; the form of the segments themselves, and the mode of their connexion, are alike inconstant; and even the plan of growth, on which the character of the organism as a whole depends, though preserving a general uniformity, is by no means invariably maintained. In the latter, to which *Cycloclypeus* and *Heterostegina* belong, we find the calcareous skeleton presenting a very definite and elaborate organization; the several segments of the body are so completely separated from each other, that they remain connected only by delicate threads of sarcode; each segment thus isolated has its own proper calcareous envelope, which seems to be moulded (as it were) upon it, and this envelope or shell is perforated with minute parallel tubuli, closely resembling those of dentine except in the absence of bifurcation or ramification; the partition-walls between adjacent segments are consequently double, and are strengthened by an intermediate calcareous deposit, which is traversed by a system of inosculating passages that seems properly to belong to it. The form of the segments, their mode of communication, and consequently the general plan of growth, have a very considerable degree of constancy; and altogether the tendency is strongly manifested in

this type, to the greater individualization of the parts of the composite body, which in the preceding must be looked upon rather as constituting one aggregate whole.

120. I purpose in the present memoir to carry on this contrast, by presenting a detailed comparison of the structure of two generic forms, which, whilst they so far agree in general plan of growth as not only to have been ranked by M. D'ORBIGNY side by side in his order *Hélicostègues*, but to have been placed by other systematists in close apposition, differ in the most marked manner as to all the particulars just enumerated. Both these types are of peculiar interest:—the first, *Peneroplis*, on account of the very wide range of variation it presents, which has led to the establishment of three *genera*, apparently distinguishable by well-marked differences in conformation, upon what I feel satisfied will prove to be but individual modifications of one and the same *specific type*;—the second, *Operculina*, as being the nearest existing representative of *Nummulites*, and consequently as affording not merely the key to the elucidation of the structure, but also the basis for the determination of the value of the reputed species, of that genus, by the study of the range of variation which it presents; this range being, though more restricted than in the preceding case, still quite sufficient to justify a large multiplication of species, in the estimation of those who do not practise that extended method of comparative inquiry, on the importance of which I have dwelt in a former Memoir (¶ 74). With the latter of these genera, as also with *Nummulites*, I shall prove that the genus *Amphistegina* is closely allied; although M. D'ORBIGNY, misled by the marked want of symmetry and by the alternation in the disposition of the chambers, which are exhibited by certain forms of that type, has placed it in a different order, *Entomostègues*. For I shall have to show that a gradational variety in this respect, ending in complete symmetry, may coincide with such a uniformity in general structure, that even a very decided departure from symmetry must be regarded as a character of little value in classification, compared with agreement in the organization of the shell and in those peculiarities in the conformation of the animal which are indicated by it; and further, that a most marked difference in degree of organization exists between two species of *Amphistegina*, which so closely resemble each other externally that the young of one may easily be mistaken for the adult of the other.

Genus PENEROPLIS.

121. *History.*—The genus *Peneroplis* was first instituted by MONTFORT* to distinguish a peculiar type of minute polythalamous shells, which had been previously described and figured by FICHTEL and MOLL†, and had been ranged by them with numerous others under the comprehensive designation *Nautilus*; and MONTFORT correctly indicates its distinctive character, as "bouche de toute la longueur de la base, et percée sérialement par une file des pores," though he seems to have very erroneously interpreted the signi-

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* BUFFON de SONNINI, 1802–1805; Mollusques, tome iv. p. 1. pl. 42; and Conchyliologie Systématique, 1808, p. 258.

† Testacea Microscopica, Vindob. 1798–1803, p. 91. tab. 16. figs. *a*, *i*.

fication of those pores. For he goes on to state, "Cette coquille est encore pellucide, et permet de lire au travers de son têt la série et la disposition des nombreuses cellules de chaque concameration; ces cellules deviennent plus grandes à mesure que la coquille prend plus d'accroissement. Il est probable que leur nombre repond à celui des animaux qui les habitent, et qui les constituent simultanément pour former un nouveau rang." This subdivision of the principal chambers into cells, each occupied by a separate animal, exists only in the imagination of MONTFORT, who seems to have been misled by the peculiar markings of the surface of the shell, which, as will be presently seen, are merely superficial. LAMARCK, not adopting MONTFORT's genus, referred the *Nautilus planatus* of FICHTEL and MOLL to the genus *Cristellaria*, with which it has no relationship whatever; and having copied their figures into the 'Encyclopédie Méthodique[*],' he designated one set of forms as *C. planata*, and another as *C. dilatata*, subsequently reuniting them, however, under a new specific name, *C. squammula*[†]. The genus *Peneroplis* has been recognized by BLAINVILLE[‡] and by EHRENBERG[§]; and D'ORBIGNY has applied this name, in his various writings on Foraminifera, to the form described by FICHTEL and MOLL, whilst he has created a new generic term, *Dendritina*, for a series of closely allied forms, in which there is a single large dendritic aperture instead of a linear series of separate apertures. The distinctive characters of these two genera, as last given by him[‖], are as follows:—"*Peneroplis*; coquille nautiloïde comprimée, pourvue de nombreuses ouvertures sur une seule ligne à la dernière loge seulement. Cavité simple:—*Dendritina*; ce sont des *Peneroplis* dont les ouvertures anastomosées forment une dendrite." In addition to these, I shall cite his definition of another genus, established by LAMARCK in 1801, which, as I shall presently show, consists, like *Dendritina*, of mere varieties of *Peneroplis*:—"*Spirolina*; coquille nautiloïde dans le jeune âge, projetée en crosse dans l'âge adulte."

122. The genus *Peneroplis* is very widely diffused through warmer latitudes; indeed few collections of Foraminifera from sands or dredgings taken from the Mediterranean, the Ægean Archipelago, the Red Sea, the East or West Indies, the Philippine seas, or the shores of Australia or the Polynesian Islands, will fail to present numerous examples of it. A few specimens have been found on British coasts; but it is surmised by Professor WILLIAMSON that these have been brought by the Gulf-stream from the West Indian Seas. "Amongst the multitude of West Indian seeds and other light objects thus thrown upon our north-western shores, it was to be expected that some of the tropical Foraminifera would be entangled; and such may have been the case with the species under consideration[¶]." The finest examples I have met with were contained in

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[*] Tab. 467, figs. 1, *a*, *b*, *c*, 2; *a*, *b*, *c*.

[†] Animaux sans Vertèbres, tom. vii. p. 607.

[‡] Malacologie, p. 372.

[§] "Tabellarische Charakteristik der Bryozoën-Classe und sämmtlicher Familien und Gattungen der Polythalamien," in Berlin Transactions, 1838.

[‖] Cours Élémentaire de Paléontologie, 1849, tom. ii. p. 198.

[¶] On the Recent Foraminifera of Great Britain, published by the Ray Society, 1858, p. 46.

Mr. Cuming's Philippine Collection; whilst the most numerous were kindly furnished me by Mr. J. Gwyn Jefferys from his dredgings in the Gulf of Genoa. We shall hereafter find that, as in the case of *Orbitolites*, particular varieties of conformation prevail, though by no means exclusively, in particular localities.

123. *External Characters.*—The ordinary form of the shell of *Peneroplis* (of which an ideal representation is given in Plate I. fig. 2) is an extremely flat spire of about two turns and a half, opening out rapidly in its last half turn (Plate II. fig. 1). In the young shell, each whorl is merely adherent to the preceding, so that the first-formed portion is not concealed by the subsequent growth; but it very commonly happens that the last whorl spreads itself out to such a degree as partially to invest the preceding (Plate II. figs. 1, 3), the extension towards the centre, however, seldom reaching so far as to conceal the original umbilicus. Although this lateral extension is sometimes confined to the inner margin of the spire, as is seen in fig. 1, yet it often occurs along the outer margin also, as is seen in fig. 3; being usually limited, however, to the last four or five chambers. But sometimes, instead of opening out and partially investing the previous whorls, the spire is prolonged in a straight line, and several of its successive chambers present little or no progressive increase in size (Plate II. fig. 5); though even in this variety the last four or five chambers are often seen to spread themselves out rather suddenly, so as to extend along the inner margin of the straight portion. Between these different extremes of conformation, every intermediate gradation presents itself.

124. The surface of the shell is very strongly marked by depressed bands, which indicate the place of the septa between the chambers; and between these septal bands the walls of the chambers rise in flattened arches. In a direction transverse to the septal bands, we almost uniformly observe a strongly-marked striation; the striæ running parallel at tolerably regular intervals, which average about $\frac{1}{1400}$th of an inch, from one septal band to another. This striation, which imparts a very characteristic physiognomy to these minute shells, seems due to a sort of plication or ridge-and-furrow arrangement of the shelly wall (Plate II. fig. 20), which may not improbably have the effect of imparting to it increased strength. The plication generally disappears at the junction of the walls of the chambers with the septa; and consequently we do not usually see it at the mouth when the shell is viewed endways. Sometimes, however, it is continued on to the septum itself; and its character is then extremely well displayed, as in Plate II. fig. 15, which, however, represents not the typical form, but one of the varieties to be hereafter noticed. On the prominences of the plicæ, there are frequently to be seen rows of extremely minute puncta (fig. 20); these, however, are not the apertures of passages through the shell, as might not unnaturally be supposed; but are, like the punctations of *Orbiculina* (¶ 88), mere depressions of its surface,—as I have ascertained by the careful examination of very thin sections. It is remarkable that the plication of the shell is sometimes wanting, though the punctations may still present themselves in rows, as shown in fig. 23; whilst in other cases, not only are the plicæ deficient, but the punctations are distributed uniformly over the entire surface, as shown in fig. 24. That

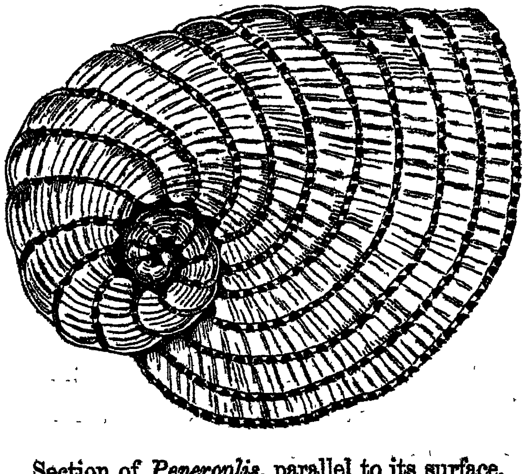
these variations are not indicative of any specific difference, is at once proved by the fact that the shells which exhibit them in one part, present the ordinary character of surface in another. In Plate II. fig. 22 is shown a case in which similar punctations present themselves on the septum closing the mouth of the shell of a *Dendritina*, which, as will presently appear, is only a modified *Peneroplis*.

125. The septum which closes the mouth of the shell is perforated by numerous isolated pores, arranged in a single linear series (Plate I. fig. 2, Plate II. fig. 1); the number of these pores depends upon the length of the septal plane (a very convenient term which I adopt from Professor WILLIAMSON), and thus it usually increases with the age of the individual, each chamber opening externally by a larger number of pores than did that which preceded it. The typical form of these pores seems to be circular, though they are apt to present various departures from that shape; they usually lie in a sort of furrow, formed by the projection of the lateral borders of the mouth somewhat beyond the septum; and, as in *Orbitolites* and *Orbiculina*, each one is surrounded by a prominent annulus of shell.

126. The texture of the shell very closely resembles that of *Orbitolites* and *Orbiculina*, but is somewhat more porcellanous. As in those genera, the shell presents an opaque white hue, when it is viewed by light reflected from its surface; whilst thin sections examined by transmitted light are of a brownish yellow or dark amber colour. Its substance is apparently quite homogeneous, no other trace of structure presenting itself than the plications and punctations already referred to; and its texture is not nearly so firm as that of those Foraminiferous shells which possess a minutely-tubular organization.

127. On examining a thin section of a typical *Peneroplis*, taken through the median plane between the lateral surfaces (Fig. I.), the central chamber is seen to have the globose form which has been shown to characterize the primordial segment of the Foraminifera already described; from this first chamber *a single passage* leads to the second,

Fig. I.



Section of *Peneroplis*, parallel to its surface.

which communicates in like manner by a single passage with the third, as does the third with the fourth; the fourth chamber, however, communicates by *two passages* with the fifth, as does the fifth with the sixth, and the sixth with the seventh. In the septum between the seventh and eighth, with which the second whorl may be considered as commencing, there are *three* apertures; and this number continues for the four consecutive partitions which divide the chambers forming the next. half-convolution. Then, however, commences a very remarkable increase; for whilst in each of the next two partitions there are *four* passages, the numbers in the four succeeding partitions which divide the chambers completing the second turn are respectively 6, 9, 11, and 14; whilst in the last eight partitions which divide the chambers of the outer half-whorl; the numbers of the apertures are respectively 14, 20, 26, 28, 30, 35, 44, and 48. The average distance of the apertures from each other remains nearly the same throughout; so that their number pretty closely corresponds with, and may be taken to represent, the length of the septal plane which they traverse in each case; and it is not a little remarkable, that whilst this number should only increase from 1 to 4 in the first convolution and a half, it should so rapidly augment from 4 to 48 in the last half-convolution.

128. As I have not been fortunate enough to obtain any other than dried specimens of this organism, I have not had the opportunity of examining the structure and arrangement of its. soft parts. Such an opportunity, however, has presented itself to Professor EHRENBERG, who collected living specimens in his expedition to the Red Sea; and by treating these with dilute acid, he has freed the animal body from its enclosing shell, precisely as I was able to do in the case of *Orbitolites*. The figure* which he has given of the animal corresponds in every important particular with what an examination of the shell would lead me to expect; for it represents a series of segments of a generally-homogeneous substance, corresponding in form, dimensions, and connexions with the successive chambers of Fig. I. The only departure that I can discover from what I should myself have anticipated, lies in this—that the successive segments are connected along the inner margin of the convolutions by a band much broader and thicker than the threads which pass between other parts of the segments; so that this band would seem to establish a principal connexion, to which the other threads might be considered as secondary. Now after a very careful examination both of the septal planes of numerous specimens, and of sections taken in the direction of Fig. I, I feel myself justified in the positive assertion that no such principal aperture exists at the inner margin of each septum, as would be required to give passage to such a band as is figured by Professor EHRENBERG. Consequently I can only account for this feature in his delineation of the animal (the idea of a difference in the conformation of the shell being negatived by the precise correspondence between his figures of it and my own, as well as by my familiarity with the Red Sea type of *Peneroplis*), by supposing that, like some of his other figures, it rather represents his *idea* of the structure of the animal, than what he actually saw in

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* Abhandl. der Konigl. Akad. der Wissenschaft. zu Berlin, 1838. Physik, taf. 2, fig. 1.

its body, this principal band being apparently regarded by him as an intestinal canal, by which he supposed all the segments to be connected together.—Professor Ehrenberg's figure represents a marking-out of spherules in the midst of the otherwise homogeneous substance of the body, closely corresponding to those which I have described in the animal of *Orbitolites* (¶ 43); and it also shows the skeletons of various Diatomaceæ imbedded in the substance of the four outer segments, as I have myself found them in *Orbitolites* (¶ 34). Consequently I can have no hesitation in the belief that the general structure and mode of life of these two forms are analogous; and that there does not exist in *Peneroplis*, any more than in *Orbitolites*, that hypothetical organization which Professor Ehrenberg has attributed to both.

129. *Monstrosity.*—The only example of monstrosity which I have met with in this type, is obviously analogous to the "monstrosities by excess" which I have described as not unfrequent in *Orbitolites*; consisting (at least apparently) in the formation of a secondary body by outgrowth or gemmation from the first. This secondary body does not seem, however, to have its origin in the primordial segment, to which similar outgrowths may always be traced in *Orbitolites* (¶ 62).

130. *Affinities.*—As regards its general organization and plan of growth, it seems clear that *Peneroplis* bears a very near approximation to that form of *Orbiculina* in which the later increase takes place like the earlier upon the spiral type (¶ 87), and in which there is but a single plane of chambers with a single row of marginal pores (¶ 90): the only essential difference, in fact, consisting in this, that each successive increment is formed in the case of *Peneroplis* by a continuous segment of sarcode, occupying an undivided chamber of the shell; whilst in the case of *Orbiculina* each segment is subdivided into sub-segments strung like the beads of a necklace on a continuous stolon of sarcode, the chamber being correspondingly subdivided by transverse partitions having a continuous passage through all. The very close alliance of the two forms is shown by the fact, that specimens are occasionally to be met with in collections of *Orbiculinæ*, which, while presenting the general physiognomy of that genus, exhibit the deficiency of transverse partitions which is the distinguishing character of *Peneroplis*; so as to render it uncertain whether such specimens should be considered as *Orbiculinæ* in which (as certainly happens occasionally in *Orbitolites*, ¶ 61) the usual partitions are deficient, or whether they truly belong to *Peneroplis*, their resemblance to *Orbiculina* being superficial only, and their presence among specimens of that genus merely accidental. In either case it is obvious that the affinity between these two types is very close; and in fact young *Orbiculinæ* frequently present such a strong external resemblance to *Peneroplis*, that their true nature cannot be determined without such an examination of their internal structure as will serve to disclose the transverse partitioning of their chambers. It may be fairly inquired whether the peculiar striation of the surface of *Peneroplis*, which so strongly suggests the idea of internal partitioning as to have led Montfort to assert its existence, is not really a rudiment of that structure.

131. *Varieties.*—The relationship just indicated being borne in mind, it will scarcely

excite surprise to find that *Peneroplis*, like *Orbiculina*, presents very considerable diversities of conformation. The first and simplest departure from what has been described as its typical character, consists in a duplication of the series of pores in each septum (Plate II. fig. 7 *a*), the spire being at the same time less compressed, so that the septal plane is wider in proportion to its length. Now it is not a little remarkable that this is almost uniformly the case with specimens furnished by particular localities, whilst those obtained from others not very remote exhibit almost as uniformly the extremest elongation and narrowing of the septal plane with only a single row of apertures; and hence it might not unreasonably be maintained that this difference should be accounted of specific value. In reply to this, however, there is not only the analogy of *Orbitolites* and *Orbiculina*, in which an indefinite multiplication in the rows of marginal pores may take place during the growth of the individual, but also the fact that in *Peneroplis* the two forms cannot be distinguished at an early age, either by the shape of the shell or by the disposition of the pores, which are often arranged neither in a single nor in a double row, but on a sort of mixture of both plans, as shown in Plate II. figs. 9 and 10; whilst among the more advanced examples of each type, it is not at all uncommon to meet with individuals which present a combination of the characters of both, the septal plane having a single row of pores in one part of its length with a double row in another. Sometimes, moreover, in one of the less compressed forms of the shell, although there is but a single row of pores, it is obvious from the elongated shape of these that they indicate a tendency to duplication (fig. 8). Moreover we find in the variety with a complete double row the same disposition as in the ordinary *Peneroplis* to the substitution of the rectilineal for the spiral mode of growth, as we see in fig. 7. Hence I consider that it may be unhesitatingly asserted that the duplication of the row of pores, and the increased turgidity of the spire which it accompanies, are but features of individual variation, and cannot be admitted to rank as specific differences. And in this view I am glad to find myself borne out by Professor WILLIAMSON, who defines *Peneroplis*[*], not (like M. D'ORBIGNY) as having only a single row of apertures, but as having "septal orifices scattered over the long narrow septal plane;" whilst his figure of *Peneroplis planatus* shows these orifices arranged for the most part in a double row, one portion having them even more multiplied.

132. What is the relationship to the typical *Peneroplis*, however, of that group of forms to which D'ORBIGNY has given the generic designation *Dendritina*, is a question of more difficulty. These are characterized, as we have seen, by the possession of a single large aperture sending out dendritic ramifications in each septum (Plate I. fig. 1, Plate II. figs. 12, 13); but this is by no means the whole of their differentiation. For the spire, instead of being compressed, is very turgid; and its successive whorls not merely surround but also invest those which have preceded them; so that what may be appropriately termed *alar prolongations* (*al*, figs. 12 *a*, 12 *b*, Plate II.) of the chambers of even the last whorl often extend nearly to the umbilicus. The geographical distribution

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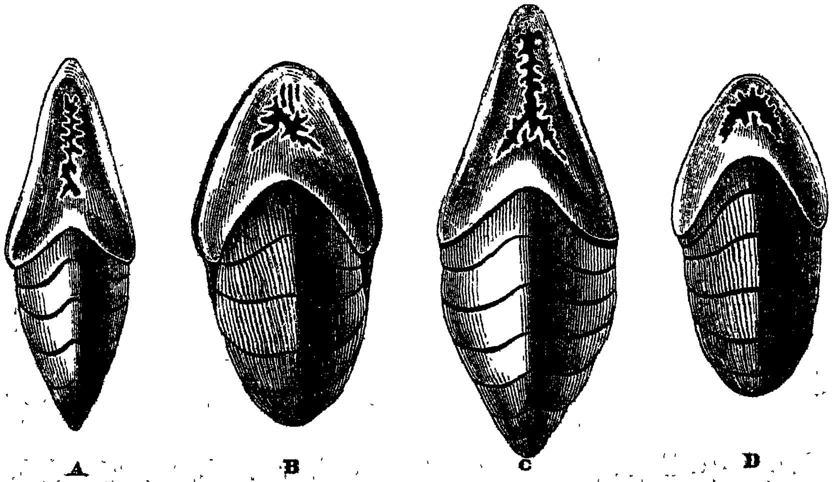
[*] The Recent Foraminifera of Great Britain, p. 44.

of *Dendritina*, moreover, is peculiar; for, so far as I am aware, this type is restricted to the tropical ocean. I have not met with it in dredgings from any part of the Mediterranean or the Red Sea, where *Peneroplis* abounds; while the largest specimens I have seen are those contained in Mr. Cuming's Philippine explorations, some of these measuring ·078 inch in diameter, and ·030 in thickness or breadth. To such as content themselves with glancing at strongly-marked examples of this type, the propriety of its generic, or at any rate of its specific separation from *Peneroplis*, would seem indubitable. Nevertheless I think that I shall be able to show adequate grounds for the belief, that the two forms cannot be separated by any definite line of demarcation; and that they must therefore be ranked as belonging not merely to the same genus, but even to the same species.

133. In the first place I would refer to the fact that the peculiar plication and punctation of the surface of the shell, which are such marked features in the physiognomy of *Peneroplis*, are repeated in *Dendritina* (Plate II. figs. 21, 22) in a manner so precisely similar, as strongly to impress every one who has his attention directed to the aspects of these two forms respectively, with the idea of their very close relationship.

134. Secondly, the differences of general configuration between *Peneroplis* and *Dendritina* are differences of *degree*, and present themselves in very variable amount in different individuals. For, starting from those forms of *Peneroplis* in which the spire is least compressed, the transition is easy to those *Dendritinæ* whose spire is least turgid, and whose septal plane is not broader in proportion to its length than it often is in *Peneroplis*. From the most compressed forms of *Dendritina* to those which have the most turgid spires and the widest septal planes, the gradation is insensible, scarcely any two individuals according in their proportions; thus we find that whilst the septal plane is sagittate in some (Fig. II. A, C), it tends to become reniform in others (B, D), the margin of the spire, which is almost carinated in the first case, becoming obtuse and even cresentic in the second. Again, the extent of the investment of the earlier whorls

Fig. II.



Front views of four specimens of *Dendritina*.

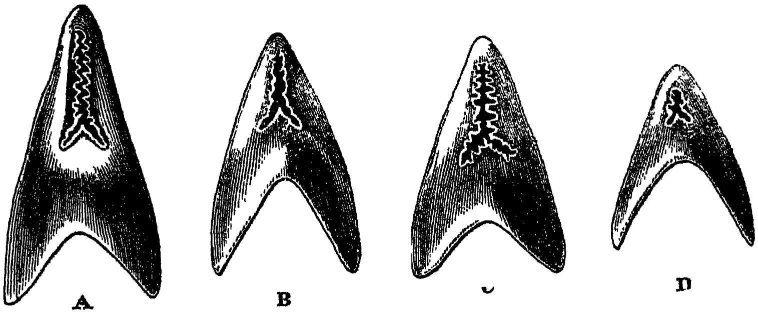
by the later, varies as much as the degree of turgidity; for whilst in some of the most compressed forms, as in *Peneroplis*, each whorl does little more than apply itself to the margin of the preceding (Fig. III. A, C), the more turgid the spire becomes, the more completely (generally speaking) does it embrace the preceding, the alar prolongations of the chambers thus coming to bear a large proportion to their principal cavity (Fig. III. B, D).

135. Thirdly, we observe in *Dendritina* precisely the same tendency to rectilineal extension in the later period of growth, as in *Peneroplis*; for although a distinct generic term *Spirolina* has been given to the form presenting this modification, I think it must be apparent to every one that the example delineated in Plate II. fig. 11, bears just the same relation to the typical *Dendritina*, that those shown in figs. 5, 7 bear to the typical *Peneroplis*. The transition from *Dendritina* to *Spirolina* is well seen in figs. 12, 13; in the first of which we see the last whorl apparently about to disengage itself from the earlier ones, whilst in the second that disengagement has been completed. I cannot conceive that any one could refuse to regard either of these specimens as a *Dendritina*; and yet it is obvious that their continued increase upon the plan which has already manifested itself in the formation of the later chambers, would convert them into *Spirolinæ*. Such continued increase has obviously taken place in the specimen represented in fig. 11, which, up to the time of the substitution of the rectilineal for the spiral mode of growth, has all the appearance of an ordinary *Dendritina*.

136. Still it may be said that, notwithstanding all these points of resemblance, the difference between *Peneroplis* and *Dendritina* is clearly marked out by the difference in the modes of communication between the chambers of the two types; and that such a difference is sufficiently important to constitute a valid generic character. I freely admit that this would be the case, if the difference were constantly to present itself between all the individuals of the same type, as it does between their characteristic examples (Plate I. figs. 1, 2); but the fact is far otherwise. We have seen that among those which would be unhesitatingly ranked under the designation *Peneroplis*, there is not only a tendency to multiplication of the rows of separate pores, but also an occasional fusion of two or more pores, so as to form a single large pore of irregular shape. On the other hand, among the unquestioned *Dendritinæ*, we observe not merely that the form of the single large dendritic aperture is extremely variable, but that it is frequently so simple as to suggest the idea of having been formed by the coalescence of a linear series of pores. The most characteristic forms of the dendritic aperture that I have met with, are shown in Plate II. figs. 12 *a*, 13 *a*; two examples of a remarkable departure from this have just been seen in Fig. II. B, D, where the proportions of the aperture are altogether reversed, its breadth being much greater than its length, and its central part being enlarged at the expense of its ramifications; while through such an aperture as that represented in Plate II. fig. 22, we are conducted back to the ordinary type, though its form is much simplified. On the other hand, in Fig. II. A, C, we have marked examples of a narrowing and elongation of the aperture, with such a reduction

of its dendritic ramifications, that it comes to present little more than a linear fissure. From a comparison of these cases, it will be seen that the form of the aperture bears a pretty constant relation to that of the septal plane; the broadest apertures presenting themselves in the individuals which have the most turgid spire, and the narrowest in those whose spire is most compressed; whilst the proportionate development of the two principal alar prolongations seems related to the degree of that alar extension of the chambers over the whorl they enclose, of which I have already spoken. But the most satisfactory proof of the wide extent of range of variation in the form of the aperture in *Dendritina*, is afforded by a comparative examination of the apertures connecting different chambers of the same individual. Thus in the interior of the very shell that presented the peculiarly characteristic example, Plate II. fig. 12 *a*, we find a form, *b*, closely corresponding to that shown in Fig. II. C; and in four septa of the inner part of another shell, we have the simple forms of aperture represented in Fig. III. A, B, C, D.

Fig. III.



Septal planes and apertures from different parts of the same specimen of *Dendritina*.

137. But further, not only do we thus meet with examples of each type which present more or less of approximation towards the other, but we also not unfrequently encounter individuals in which the characters of the two types are so blended that it is difficult, if not impossible, to say to which they should be referred. Thus in Plate II. fig. 8, we have a young specimen with a linear series of apertures, the marked elongation of which shows each to be formed by the fusion of two, whilst there is an additional pair precisely in the situation of the alar prolongations in fig. 12 *b*; and it is obvious that a longitudinal coalescence of these pores would produce an aperture exactly resembling that of fig. 12 *b*. Again, in another young specimen shown in Plate II. fig. 16, a partial fusion of the separate pores into a single dendritic aperture has actually taken place (fig. 16 *a*), whilst another broad aperture is seen just below this. In fig. 10 we see numerous pores, some small and rounded, others large and irregular, each of the latter being obviously formed by the coalescence of two or three of the separate pores; and it is evident that a closer approximation of the whole would produce a single large dendritic aperture. Other varieties of the same kind are presented in figs. 9 and 14. But a yet more remarkable example is shown in fig. 15; for here the coalescence has pro-

ceeded so far as to produce a number of separate branching apertures, and nothing is wanting but the removal of the line of shell which passes down the middle of the septum, to unite these into the most characteristic form of the single ramifying orifice. This individual, like the one represented in fig. 6, was already beginning to assume the *Spirolina* form, the rounded shape of the mouth showing that the spire has detached itself completely from the previously-formed convolutions: in figs. 4 and 11 *a* are shown the mouths of more advanced examples of the same type, which present such a combination of the large dendritic aperture of *Dendritina* with the isolated pores of *Peneroplis*, as to complete (in my opinion) the proof that no valid distinction can be drawn between these two types, either from the number, the isolation, the position, or the shape of the apertures in the septa. This conclusion will probably become less surprising than it may at first sight appear to such as are accustomed to the study of the more constant characters presented by the shells of Mollusca, when it is borne in mind that the apertures seem to have no other function than to give passage to threads of sarcode; and that it is the general character of the Rhizopod type, for such extensions of the substance of the body to be entirely destitute of constancy either as to position or number.

138. If the foregoing conclusion be admitted, it follows that not merely must the genera *Dendritina* and *Spirolina* be relinquished, but that both these forms must be regarded as mere varieties of *Peneroplis planatus*. From the circumstance that the *Dendritina*-type is found only in tropical seas, and that it attains a far larger size than the ordinary *Peneroplis*-type, whilst the latter seems to be almost starved out (as it were) in the Mediterranean,—presenting itself there under its humblest form, and scarcely extending itself at all into northern seas,—I am disposed to think that temperature has a considerable influence in producing these varieties. I should be far, however, from attributing the entire result to this agency alone, since we find *Peneroplis* presenting itself abundantly in tropical seas, along with *Dendritina*. But as it is in the Mediterranean forms that we meet with the extreme of flattening and with the most uniform singleness of the row of pores, whilst it is among the tropical forms that we find *Peneroplis* not merely attaining its greatest dimensions, but also presenting the closest approximation to *Dendritina* in the turgidity of its spire and in the arrangement of its septal apertures, it seems obvious that the development of this type of structure is favoured by a constantly high temperature; and further, that *Dendritina* may be regarded as the highest form, towards which *Peneroplis* tends in proportion as it is subjected to that influence.—As I have not had the opportunity of examining the fossil forms of this type, I am as yet unable to say how far the foregoing conclusions will be borne out by the phenomena which they present.

Genus OPERCULINA.

139. The collection of Mr. CUMING contains a large number of Nautiloid Foraminifera, differing considerably from one another not only in size and proportions, but also

in general aspect and particular features, yet having so many structural characters in common as to be obviously referrible to the same generic type. This type I shall designate, provisionally at least, as *Operculina*; since, although it does not correspond with the definition of this genus as given by its founder M. D'ORBIGNY, it is obviously related most closely to the fossil forms which have been described under that designation by M. D'ORBIGNY himself, as well as by MM. D'ARCHIAC and HAIME, and to the recent type described as *Operculina* by Mr. CARTER.

140. *History.*—The genus *Operculina* was created by M. D'ORBIGNY in 1825, for the reception of a group of Foraminifera bearing a strong general resemblance to *Nummulites*, but considered by him as differing from it in the extreme compression of its discoidal shell, in the smaller number of its whorls, in the non-investment of the earlier whorls by the later, so that the former remain apparent on both sides through the whole of life, and also in the form of the aperture, which is a transverse slit in *Nummulites*, but triangular in *Operculina*. The following is the definition of this genus given by him in his 'Foraminifères Fossiles de Vienne' (p. 117):—"Coquille libre, équilaterale, ovale ou discoidale, très comprimée, formée d'une spire non embrassante, régulière, également apparent de chaque côté, à tours contigus, et croisant très rapidement. Loges nombreuses, étroites, la dernière formant saillie de toute la largeur de la spire, à tous les âges percée d'une ouverture visible triangulaire, contre le retour de la spire." In his 'Cours Élémentaire de Paléontologie' (tom. ii. p. 197), he defines the genus by the following characters alone:—"Coquille comprimée à tours apparents, pourvue d'une ouverture triangulaire contre le retour de la spire." I think it necessary also to quote from the same work M. D'ORBIGNY'S definitions of three other genera nearly allied to *Operculina*; since, as will presently appear, the characters which he assigns to them are based on an entirely wrong idea of the distinctive features presented by their respective types.—*Nonionina.* "Coquille nautiloïde, pourvue d'une seule ouverture en fente transversale, non masquée, placée contre le retour de la spire."—*Nummulites.* "Coquille lenticulaire à tours embrassants, pourvue d'une ouverture en fente transversale, souvent masquée, contre le retour de la spire."—*Assilina.* "Ce sont des *Nummulites* dont les tours de la spire, ne rejoignant pas le centre, sont tous plus ou moins apparents." In the admirable Monograph of MM. D'ARCHIAC and HAIME[*], the genus *Nummulites* is defined as follows:—"Coquille libre, symmétrique, orbiculaire, plane, discoïde, lenticulaire ou subglobuleuse, composée d'une lame calcaire pliée et enroulée suivant un même plan, criblée de pores ou canaux de trois grandeurs différentes, à tours plus ou moins serrés, plus ou moins embrassants, quelquefois simplement juxtaposés. Dans le jeune âge, le dernier est saillant; plus tard, les tours se rapprochent de telle sorte qu'à l'état adulte le dernier paraît se souder tout à fait à l'avant dernier. Canal spiral divisé par des cloisons transverses, plus ou moins nombreuses, percées à leur base, contre le retour de la spire, d'une ouverture linéaire transverse. Ces derniers caractères semblent s'être atténués vers la fin de la vie de l'animal" (p. 70). The genus *Nonionina* is said by these

[*] Description des Animaux Fossiles du Groupe Nummulitique de l'Inde. Paris, 1853.

authors to differ from *Nummulites* in respect of its flatter form, the comparative shortness of its spire, the smaller number of chambers, and the non-embrace of its convolutions; as also in absence of that narrowing of the later convolutions which leads to the entire closure of the spire in *Nummulites*. The genus *Operculina* is said (*loc. cit.*) to be distinguished from *Nummulites* only by the depression of its form, the small number of its convolutions, and the rapid increase in breadth of the last whorl, which seems constantly to remain open. The genus *Assilina*, having no sufficiently distinctive characters, must (according to these able investigators) be merged in *Nummulites*.—It is obvious, from these remarks, that MM. d'Archiac and Haime had fully recognized the two fundamental errors of M. d'Orbigny's definition of *Operculina*; viz. his statement that the convolutions are non-embracing, and his description of the form of the aperture as triangular. And although Mr. Carter[*] does not allude to the discrepancy between the structure of the recent type described by him and the generic definition of M. d'Orbigny, yet it is evident that in assigning to it the designation *Operculina* he was guided rather by its general resemblance to the fossils on which that generic name had been conferred, than by the conformity of its structure to M. d'Orbigny's definition. The errors of that definition are easily accounted for; since it is only by such an examination of thin transverse sections as M. d'Orbigny (I have reason to believe) never made, that the fact of the earlier whorls being really embraced by the later, notwithstanding the apparent freedom of the latter, can be substantiated; and the determination of the form of the aperture is very likely to be erroneously made, when the examination is limited to fossil specimens, in which it is frequently but very indistinctly traceable.

141. The minute structure of this type has been already investigated by two excellent observers. Under the designation of "an undescribed species of *Nonionina*," Professor Williamson[†] has given an account of the structure of a shell abounding in the Manilla sand, which, not merely from his figures and descriptions, but from a comparison of the specimens which he has kindly enabled me to make, I know to be one of the smaller forms of the Philippine *Operculina*. Mr. H. J. Carter[‡] has still more minutely described the organization of an *Operculina* which he obtained in great abundance on the south-east coast of Arabia, the shells coming up attached to the grease of the sounding lead from sandy bottoms of between ten and twenty fathoms depth; and of the identity of this with a larger form of Philippine *Operculina* I am enabled to speak, from comparison of specimens which Mr. Carter has obligingly transmitted to me. I find the description of each to be in the main correct so far as it goes, but to be defective in some essential particulars. Thus Professor Williamson does not notice the canal-system except in the marginal cord; and Mr. Carter, though he has more fully described the canal-system, has not only missed one important part of it, but has misapprehended the structure of the marginal cord. The investigations of each, moreover, have been limited

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[*] Annals and Magazine of Natural History, Sept. 1852.

[†] Transactions of Microscopical Society, First Series, vol. iii. p. 112.

[‡] Annals and Magazine of Natural History, Sept. 1852.

to one particular form of this protean type; and neither seems in the least degree aware of its extraordinary tendency to variation. The rich store of material placed at my disposal by Mr. CUMING has not only enabled me to prosecute my inquiries into the minute structure of this organism upon specimens of unparalleled size and degree of development, but has also enabled me to bring together a great body of data for comparison as to the extent of variation which it may undergo, alike in external conformation and in internal organization. And as the information hence obtained has a most important bearing upon the study of the closely-allied genus *Nummulites*, I venture to think that, notwithstanding all which has been done by Professor WILLIAMSON and Mr. CARTER, it is desirable that I should give a detailed account of my own investigations into the structure of *Operculina*, as well as a summary of the results of my comparison of its multiform varieties. It will be advantageous in the first place to examine into what appears to be the characteristic structure of the type, and then to inquire into the degree of modification to which this may be subject on each point.

142. *External characters.*—A large proportion of Mr. CUMING's specimens accord in their general aspect with the one represented in Plate III. fig. 7, which is closely conformable to Mr. CARTER's type of *Operculina Arabica*, except in its somewhat larger dimensions. It is a compressed spiral of about ·25 inch in diameter, and about ·015 inch in thickness, consisting of between three and four convolutions gradually increasing in breadth; these are in general nearly flat, but are sometimes a little arched between their inner and outer margins, sometimes depressed so as to present a slight concavity, especially near the outer margin of the last whorl. The chambers are about seventy-five in number, commencing from a primordial spheroidal cell and progressively increasing in dimensions with the widening of the spire; their septa have for the most part a radial direction, but they bend backwards near the outer margin of each whorl; and they are marked externally by bands which are distinguished by their semitransparent aspect from the dull brownish hue of the general surface. These septal bands are commonly on the same plane with the intervening portions of the shell; but sometimes the walls of the chambers are a little arched between the septa that bound them, so that the septal bands are slightly furrowed; whilst the walls of the chambers are sometimes a little depressed, so that the septal bands are prominent; and such varieties may present themselves in different parts of one and the same shell. Not unfrequently the whole surface is seen to be marked by very minute punctations; but more commonly they are larger and fewer in number, and are often arranged in pretty regular lines parallel to the septal bands,—one, two, or three rows of such punctations being seen on the wall of each chamber (Plate V. fig. 7); these punctations, when sufficiently magnified, are found to be spots of semitransparent shell-substance resembling that of the septal bands; and, as in the case of these, their surface is sometimes on the same plane with that of the general surface of the shell, sometimes a little elevated so as to form papillæ, and sometimes a little depressed into minute fossæ (fig. 8). These punctations often present themselves abundantly on some parts of the surface, whilst they are entirely absent from others. Sometimes, instead of a limited number of comparatively large and

regularly arranged punctations, we find a vast number of very minute papillæ, scattered without order over the entire surface of the chambers (fig. 6); these, again, may be absent from some parts of a shell, over other portions of which they are abundantly distributed. There is commonly a large semitransparent tubercle at the umbilicus, and smaller tubercles are often seen along the septal bands, especially of the earlier whorls: the umbilical and the septal tubercles are well seen in Plate III. fig. 6, and (less characteristically) in several other figures of the same Plate; and the septal tubercles are shown on a much larger scale in Plate V. fig. 11. Not unfrequently, however, the umbilicus is depressed, instead of being elevated into a tubercle; and the moniliform tubercles along the septal bands are wanting.

143. The departures from this typical form, however, are very wide. A glance at any considerable aggregation of specimens reveals to us an extraordinary variety of size and shape; and our attention is specially attracted by a series of which an example is represented in Plate III. fig. 9, which are not merely distinguished by a size greatly above the average—their long diameter reaching nearly ·4 inch,—but also by the extraordinary flattening of the later convolutions, and the rapidity with which the spire opens out. The approximation between the two lateral walls of the chambers is here so close, that not only are the septal bands rendered very prominent by the depression of the outer surface between them, but even the outer marginal band stands up as a ridge, from which the walls of the chambers slope down. In fact, an examination of this form leaves the observer impressed with surprise that any room can be left for the animal, the segments of which must be extraordinarily attenuated, losing in thickness what they gain in area. Now a careful comparison of this form with the ordinary type, not only makes it obvious that the former differs from the latter in no other particular than this attenuation, which (as already pointed out in the cases of *Heterostegina* and *Peneroplis*) is a common feature of the later growths in Foraminifera, but also that the attenuation takes place in such different degrees in different individuals, that any attempt to use it as a differential character is completely baffled by the continuous gradation of forms that is presented, between the one which has been assumed as the typical, and such as most widely depart from it in this particular.

144. The collection of Mr. CUMING also includes, however, a considerable proportion of comparatively small specimens, ranging from ·08 to ·20 inch in diameter, which present a very different configuration. In the smallest of these, represented in Plate III. fig. 1, the spire, instead of being flat or even somewhat hollowed, is arched from its inner to its outer margin, so that the breadth of the septal plane is equal to its length, or nearly so; and the like is seen in the somewhat larger specimens of which fig. 2 is an example. This variety of conformation, however, being limited to shells whose earlier period of growth is evinced by their smaller number of convolutions and of chambers, and being seen, moreover, in the earlier whorls of those which afterwards present the greatest flattening, may safely, I think, be regarded as a character of age. It will be recollected that the young of the flattened *Peneroplis* more resembles *Dendritina* in the comparative turgidity of its spire; but I do not find that *Operculina* ever continues long

to increase upon such a plan; for the compression of the spire always shows itself in the third whorl, if it has not previously done so, and is accompanied with a corresponding augmentation of its breadth, so that the septal plane becomes narrowed and elongated, as is seen in comparing fig. 4 with fig. 3. That no basis for specific distinction is afforded by the most marked differences in the form of the spire, as shown in the proportions of the septal plane (A, B, C, D), will be made evident by the examination of such a transverse section as is shown in Fig. V. (p. 21); in which it will be observed that differences equal to those presented by the most compressed and the most turgid forms of the spire, are exhibited by the successive convolutions of one and the same individual.

145. Another marked feature of difference among the specimens of this collection, is the depression or elevation of the central region relatively to the peripheral. In what I have assumed as the typical form, the central region presents the same general level with the rest, though the umbilicus itself is often marked by a prominent tubercle. In such forms as are represented in Plate III. figs. 1 and 2, the umbilical region is rather depressed than elevated; and this depression is often observed in older specimens whose early growth has taken place on this type. But there is a group of specimens, of which three successive ages are represented in Plate III. figs. 3, 4, 5, 8, that have the whole central region so exceedingly prominent as to form a cone, whose apex is marked either by one large tubercle, or by a cluster of smaller ones; and this conformation gives so peculiar a physiognomy to the shells which present it, that few systematists would hesitate in placing them apart as specifically different from the rest. On a careful comparison of a large number of individuals, however, it becomes apparent that this difference, like the preceding, is gradational; every degree of prominence being traceable from the individuals which have the umbilicus marked only by a tubercle, as in fig. 7, through those in which the region generally is slightly elevated, as in fig. 8, to those in which it presents the most marked projection, as in figs. 3–5. We shall presently find (¶ 153) that this difference depends mainly on the degree in which the investing layer, prolonged from the later convolutions over the surface of the earlier, is separated from that surface by the extension of the alar prolongations of the chambers of the investing whorls; as to which point there is a most remarkable diversity, not only among different individuals, but between the several convolutions of the same individual.

146. A fourth very obvious character of differentiation among the individuals of this collection, consists in the presence or absence of tubercles on the septal bands. In what I have described as the typical form (Plate III. fig. 7), there are no considerable prominences over the greater part of the surface; the septal bands are generally smooth and continuous; and it is only in the central region that we observe any departure from this uniformity, the umbilicus being occupied by a small tubercle, and the smooth septal bands being replaced in the first whorl by moniliform rows of little tubercles. In other instances, however, we find not only the central tubercle, but the rows of tubercles marking the septa, much larger and more prominent; and this marking-out of the septal bands by elevated tubercles is not limited to the first whorl, but extends to the second, and even to the third, as is well shown in fig. 6. The specimens whose central region is

very prominent (Plate III. figs. 3, 4, 5), usually show this feature in the most decided manner; but it is occasionally presented also in no less a degree by those whose umbilicus is flat or even depressed. The most remarkable departure from the ordinary type is seen in a small group of specimens in which the tubercles are not only extremely large and prominent (Plate IV. fig. 6), but are distinguished by their opaque whiteness (Plate V. fig. 12), which contrasts strongly with the semi-transparency ordinarily characterizing these prominences. But even this character has no diagnostic value; for a specimen which has the tubercles opaque in one part may have them semi-transparent in another; and as to the size of the tubercles, their degree of prominence, and the proportion of the entire spire over the septa of which they occur, there is every degree of variety. In one of my largest and flattest specimens, the central region is strongly tuberculated, but the tubercles almost suddenly cease when the spire begins to open out, and the septal bands are thenceforth as smooth as in the ordinary type.

147. A less obvious but still very decided feature of individual difference, consists in the presence or absence of papillary elevations *between* the septal bands. I have already spoken (¶ 142) of the frequent existence of symmetrically arranged spots, sometimes slightly depressed, but more commonly elevated, that are distinguished from the rest by the semitransparency of their shell; these spots are sometimes considerably enlarged, and their elevation increased, so that they become prominent papillæ, closely resembling the tubercles upon the septal bands. Their size and disposition vary considerably. Sometimes they are small, numerous, and scattered without any definite arrangement over the entire surface of the wall of the chamber, whilst in other cases they are considerably larger, and form single, double, or even triple rows between the septal bands (Plate V. fig. 10). Another remarkable variety of external aspect is produced by the elevation of the general surface into rounded eminences closely abutting on one another (like the pustules of the skin in a case of confluent small-pox), and distinguished from the preceding by the absence of any peculiarity in the texture of the shell. That these and other analogous variations of surface-marking have no value as differential characters, is at once demonstrated, not merely by their gradational approximation in different individuals, but by the fact that they are presented in very different degrees on different parts of the surface of the very same shell; the chambers of one part of the spire being strongly marked by certain of these peculiarities, whilst those of another may only present indications of them, and those of a third may be perfectly smooth.

148. The collection of Mr. Cuming, however, contains a group of forms which are at once distinguished from the rest by their general physiognomy (Plate III. figs. 11, 12), and which, when their characters are examined in detail, appear to be separated from them by well-marked differences. Their aspect is much more lustrous, and their hue much whiter, varied, however, by a tinge of green diffused in irregular patches; the spire does not in the largest specimens make above three turns, and it begins to open out sooner than in the type already described; the septa are usually considerably more convex anteriorly, and are also rather more distant from each other, so that the interval between them is greater in proportion to the breadth of the spire; the shape of the

chambers being thus modified, and the number of chambers in each whorl being diminished; the septal bands, not merely of the earlier whorls, but even of the last-formed portion of the shell, are raised into prominent tubercles; and multiple rows of large tubercles are seen between the septal bands. It is to this multiplication of smooth tubercles composed of a variety of shell-substance which reflects light much more strongly than the rest, that the more glistening aspect of this type is chiefly due; and I have not met in it with any instances of that general brownish coloration of the surface which is the ordinary characteristic of the other. The most constant and remarkable distinctive feature of this type, however, is the presence of a large hemispherical cluster of semitransparent tubercles in the centre of the spire (Plate III. fig. 10).

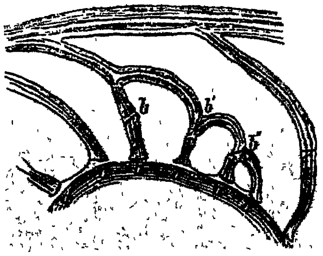
149. Having had no difficulty in setting apart a large number of specimens agreeing very closely with each other, and differing from the rest, in all the foregoing characters, I should have arrived at the unhesitating conclusion that this type deserves to take rank as a species of *Operculina* distinct from the preceding, were it not for the circumstance that every here and there I met with an example in which the differential characters were less strongly marked than usual. Thus in some individuals which preserve the general proportions of the spire, the green coloration is wanting, the large central cluster of tubercles is replaced by a single tubercle of extraordinary size, the septal bands of the neighbourhood are not more tuberculated than in many examples of the ordinary type, and the rows of tubercles over the chambers are either wanting altogether, or are not more prominent than in many individuals of the preceding type. This evidence of the negative value to be attached to the number or prominence of the tubercles as a specific character, is confirmed by a curious fact of an opposite nature; namely, that in certain individuals we find them developed to an extraordinary and obviously abnormal degree (Plate V. fig. 9).—From the difficulty of deciding to which type particular specimens are to be referred, I had been almost led to adopt the conclusion of the specific identity of this with the ordinary form; when Dr. Gould, of Boston (N. E.), kindly placed in my hands some *Operculinæ*, which had been collected on the coast of Japan by the recent American expedition to that country. These specimens combined in so remarkable a manner the most distinctive features of the two types, —namely, the general form and proportions of the one, with the umbilical hemispheric cluster of tubercles and the general abundance of tubercular elevations characteristic of the other,—as to remove all doubt from my mind with regard to their specific identity.

150. *Internal Structure.*—The study of the internal organization of *Operculina* may be prosecuted in two modes;—by the examination, under sufficient magnifying powers, of thin transparent sections taken in different directions;—and by viewing under a low power, as opaque objects, fragments obtained by breaking the shell, especially (as Mr. Carter was the first to suggest) after these have been allowed to absorb carmine or indigo by being placed upon water in which either of these colours has been rubbed up. By the former method alone can certain minutiæ of structure be detected; but the latter is extremely serviceable (as I found in my study of *Nummulites*, op. cit.) in enabling the observer to trace out the relations of various parts, which sections exhibit to him

disconnected from one another. In Plate I. fig. 3, an attempt is made to bring together in an ideal representation the most important features of the internal structure of *Operculina*, as disclosed by these two methods of examination; the warranty for all its details being supplied by the delineations of the separate parts that are given in other figures, which are all taken from preparations in my possession.

151. Owing to the circumstance that large specimens of *Operculina* are rarely if ever quite flat, it is next to impossible to make a section through the median plane that shall traverse all the whorls to the central cell; a sufficiently near approach to this, however, has been made in the section represented in Plate VI. fig. 3, and Plate IV. fig. 10, to bring into view the general disposition of the chambers. This is much less regular than would be supposed from a superficial examination of the exterior. For while there is a certain general average in the proportion which their long diameter (that is, the breadth of the whorl) bears to the short (or distance between the septa), which may be stated as about 4½ to 1, this is by no means constantly maintained, the long diameter being sometimes as much as 7 times, and sometimes no more than 2½ times, the short. The ordinary course of the septa, too, is often strangely departed from, as is seen in Plate VI. fig. 3. There are few individuals which do not present—besides abnormal sinuosities, greater or less in degree—very marked irregularities in the conformation of the chambers, analogous to those which I have described in *Nummulites**. Frequently a septum, instead of passing continuously from the inner to the outer margin of the whorl, stops short without reaching the latter, and bends backwards to join the last-formed septum; and sometimes a second septum unites itself to the first in the same manner, as shown at *a*, Plate VI. fig. 3. The abortion is often still more marked; thus at *b* we see three septa thus interrupted, of which two do not traverse half the distance, and the third not a quarter, thus dividing the space between two complete septa into four small chambers along the inner margin, and one large irregular chamber extending to the outer. Another case of the same kind is seen in an earlier part of the same whorl; and in Fig. IV. an additional illustration on a larger scale is given of the like irregularity, for the purpose of showing the arrangement of the communicating passages. Generally speaking, the more nearly we approach the centre of the spire, the more regularly do we find the septa disposed, until we come into close proximity with the central cell.

Fig. IV.



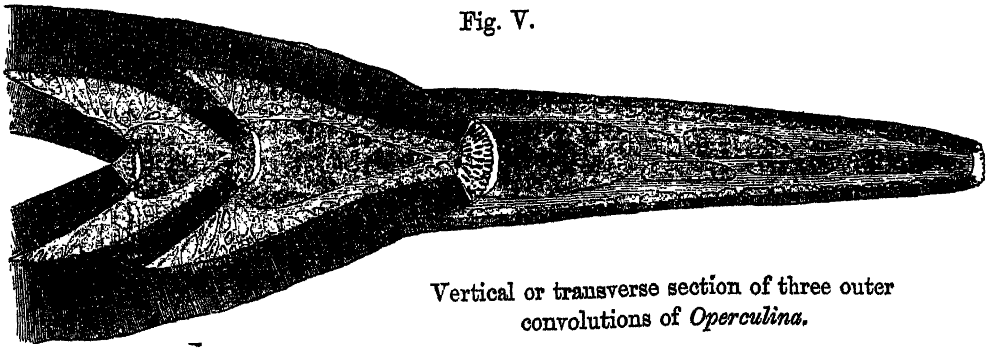
Irregular disposition of septa in *Operculina*:—*a*, *a*, *a*, normal apertures at inner margin of spire; *b*, *b′*, *b″*, apertures of communication between abnormally divided chambers.

* Quarterly Journal of the Geological Society, vol. vi. 1850, p. 23.

In Plate IV. fig. 5 we have an illustration of what may be considered the normal mode of commencement of the spire; from which it will be seen that it originates, as in Foraminifera generally, in a spheroidal cell, from which others are successively developed around it, the earlier chambers having no very definite shape, but those which succeed them gradually coming to assume the characteristic form and proportions. The like is shown in vertical section in Fig. VII. B.

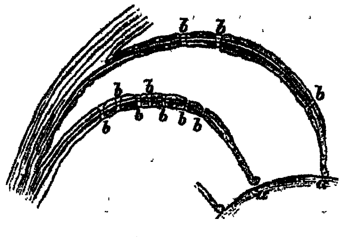
152. Each chamber communicates with the neighbouring chamber on either side, by a long narrow crescentic fissure left by the non-adhesion of the septum to the outer margin of the preceding whorl (Plate I. fig. 3). This fissure (Figs. IV., VI. *a*, *a*, *a*) is frequently not to be seen in such a section as is represented in Plate VI. fig. 3, owing to the circumstance that this does not happen to pass through the plane in which it lies; and it is best brought into view either by making thin transverse sections (as Fig. V.), or by breaking a specimen transversely and examining its fractured edges, by which such views will be obtained as are presented in Fig. VIII. (p. 26) A, B, C, D. Besides this principal aperture, we observe in the septa, especially of the larger chambers, a variable number of secondary pores (Plate I. fig. 3 *e*), generally circular, and of comparatively small size; these are dispersed without any regularity, as is shown in Figs. V., VI., VIII. They may or may not be brought into view in a horizontal section, according as its plane does or does not happen to pass through them; but when they are thus traversed (Fig. VI. *b*, *b*, *b*), it is seen that these secondary pores, like the principal aperture, establish a direct communication between adjacent chambers*. In the irregular formation repre-

Fig. V.



Vertical or transverse section of three outer convolutions of *Operculina*.

Fig. VI.



Section of two septa of *Operculina* through the median plane, showing the secondary pores, *b*, *b*, *b*, in addition to the principal orifices, *a*, *a*, at the inner margin of the spire.

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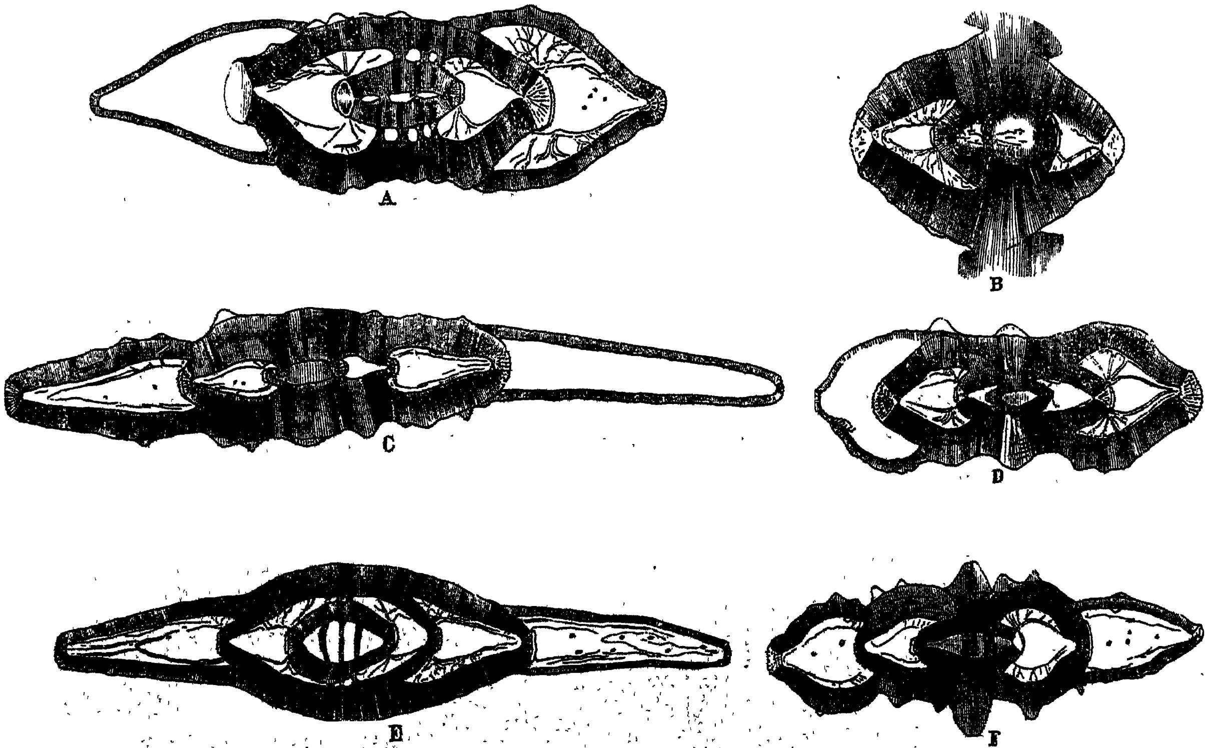
* These secondary pores were first observed by me in *Nummulites lævigata* (*op. cit.* p. 24); I then believed, however, that they do not pass through both layers of the septa, but only establish a communica-

sented in Fig. IV., these secondary pores (b', b'', b''') present a disposition which would seem to indicate that they are scarcely less important for the maintenance of the communication between the chambers, than is the principal aperture itself; and it is curious to observe how exactly they repeat at b' and b'' that mode of communication—between *two* segments on the inner side and *one* on the outer—which is so characteristic of *Orbitolites* (¶ 17) and *Orbiculina*. Each septum, as in *Cycloclypeus* and *Nummulites*, is composed of two layers (Plate I. fig. 3 *d*, *d*), each being the proper wall of one of the chambers which it separates; these layers are commonly in close apposition with each other; but they separate at certain spots, to give passage to the very curious system of interseptal canals to be presently described.

153. A comparison of numerous vertical sections brings into view a most remarkable variety in the form and disposition of the chambers, not merely in different individuals, but in different parts of the same individual. Commencing with one of the flattest variety, Fig. VII. c, we observe that the breadth of the septal plane is very small in comparison with its length; and that although the spiral lamina of each whorl except the last obviously extends itself over that of the preceding whorls, the cavity of the cham-

Fig. VII.



Vertical or transverse sections of six specimens of *Operculina*, exhibiting marked variations in the form and proportions of the convolutions.

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tion between the chambers and the interseptal passages. In this idea I am now satisfied, by the examination of *Operculina*, that I was in error; and I am disposed to regard these secondary pores of *Nummulites* and *Operculina* as representing the multiple communications between the chambers of *Peneroplis* (¶ 127).

bers is not so extended, those of each whorl being bounded internally by the external margin of the preceding. Passing from this, however, to forms which are slightly elevated in the centre (Fig. VII. E), we find not only that the breadth of the septal plane is increased, but that the cavity of each chamber is extended into two *alæ*, which are more or less prolonged over the enclosed whorls; so that the spiral lamina of the investing whorl is kept by their interposition from coalescing with that which it embraces, except in the central region. The degree of this extension, however, varies in different convolutions; the alæ being usually smaller in the chambers of each consecutive whorl, and being absent altogether from those of the last,—a transition well shown in Fig. V., which also exhibits the marked variation in the general proportions of the chambers that may present itself between the inner and the outer whorls. In forms which are distinguished by a yet more turgid spire, the proportions of the chambers are very considerably modified; but among these there is a very considerable difference in the degree in which the chambers of the later whorls are prolonged over the earlier; thus in Fig. VII. A, whose centre is on a level with the rest of the spire, we see the alæ prolonged so as quite to reach that region; whilst in D and F, which have the central region depressed, though the general proportions of the chambers correspond with those of the preceding, the alæ are so little extended that the spiral laminæ of the successive whorls coalesce not only at the centre, but at some distance around it. After an attentive examination and comparison of a great number of vertical sections, I feel justified in affirming that no importance can be attached either to the form and proportions of the chambers, whether shown in the relative length and breadth of the septal plane, or in the degree in which their alæ are prolonged over the enclosed whorls, as furnishing characters of specific difference; seeing that it is not only found to vary gradationally when a sufficiently large series of specimens is compared, but that it is often equally inconstant in different parts of the same individual.

154. The spiral lamina, in typical specimens, is much thicker in the inner than in the outer convolutions, that of the last whorl being always comparatively thin (Fig. V.); and a careful examination of transparent vertical sections makes it apparent, that the greater thickness of the spiral lamina of the earlier whorls is partly due to the prolongation of that of the later whorls over them, and to its coalescence with them. I have not been able to satisfy myself that the spiral lamina of the last whorl is thus extended over the preceding whorls; and it has rather appeared to me to be merely applied to the margin of that which it surrounds. The difficulty of certainly determining this point, chiefly arises from the circumstance that the spiral lamina of the last whorl is reduced in thickness in proportion to its extension, as if in consequence of a limitation of the amount of calcareous matter employed in its formation. The spiral lamina is made up of a variable number of lamellæ of a minutely tubular substance (Plate IV. fig. 9), so exactly corresponding with that which I have already described in *Nummulites* (*loc. cit.*) and in *Cycloclypeus*, that any detailed description of it is unnecessary. I have to add, however, that in examining extremely thin transverse sections of this tubular shell-substance with a sufficiently high magnifying power, I have been able to perceive that the

orifices of the tubuli are separated by a very delicate areolation (fig. 9*b*), as if the whole substance were composed of an aggregation of prisms with a tubulus in the centre of each. And this circumstance appears to me to afford the explanation of an observation which I have elsewhere recorded in regard to certain fossil specimens of *Nummulites* (*loc. cit.*),—namely, that where the characteristic tubular structure is wanting, an appearance of prismatic structure is occasionally presented*; for it is not by any means difficult to conceive that the same molecular change which obliterates the one feature, should bring the other into increased distinctness. The size of the tubuli of *Operculina* is much greater at and near the inner surface of the walls of the chambers, than it is at a little distance from them, as will be seen by comparing figs. 1 and 3 of Plate IV. with figs. 2 and 4; so that when a thin lamina from the innermost part of the wall is examined under a high magnifying power, the tubes are seen almost to fill the areolæ, as is shown in Plate IV. fig. 9 *a*, which is drawn on the same scale as *b*.

155. As is *Nummulites* and *Cycloclypeus*, the shell-substance over the septa is not penetrated by tubuli, and is consequently far more transparent than the rest; and it is in consequence of this difference in texture, that the septal bands are so strongly marked on the external surface, as well as in sections parallel to it (Plate IV. figs. 10, 11; Plate VI. figs. 1, 2). The same is true, also, of the substance of the tubercles, whether occurring as elevations of the septal bands, or upon intermediate parts of the spiral lamina. In the latter case, the tubercles are seated upon inverted cones of the like substance, which do not usually reach down to the inner lamina, so that the internal surface presents the orifices of the tubuli regularly disposed over the interior of the chambers (as seen in Plate IV. figs. 1, 3), even where sections passing at a little distance from these show rounded or elongated spots of transparent substance (figs. 2, 4), around which the tubuli are crowded together, as if they had been displaced by its interposition. Such spots occasionally present themselves in the ordinary type of *Operculina*, as seen in Plate IV. fig. 10; but it is of course in sections of the strongly-tuberculated variety that we find them most marked, and this especially in the investing layers of the central region, where, from the size and approximation of the tubercles, the spaces between them are rendered almost opaque by the crowding together of the tubuli (Plate IV. fig. 11). The tubuli are generally seen, moreover, to be somewhat more crowded in the neighbourhood of the septal bands.—The greatest local development of the non-tubular substance is seen in those varieties which have a large elevated tubercle in the centre (Plate III. figs. 3-5); a section of such a variety has been shown in Fig. VII. D, where it is seen that this tubercle springs from the spiral lamina of the

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* As MM. D'ARCHIAC and HAIME, in referring to my statement as to the appearance of prismatic structure in *Nummulites*, would appear to entertain some doubt as to the correctness of my observation (see p. 61 of their Monograph), I think it right not merely to say that fig. 12, Pl. IV. of my Memoir on Nummulites is a most exact representation of a section still in my possession, but also to mention that the vertical fracture of well-preserved specimens of *N. lævigata* often presents an edge so strongly resembling that of the prismatic shell-substance of *Pinna* when fractured in the same direction (though on a much smaller scale), as to have led experienced microscopic observers to a belief in their similarity, before my discovery of the minutely tubular structure of *Nummulites*.

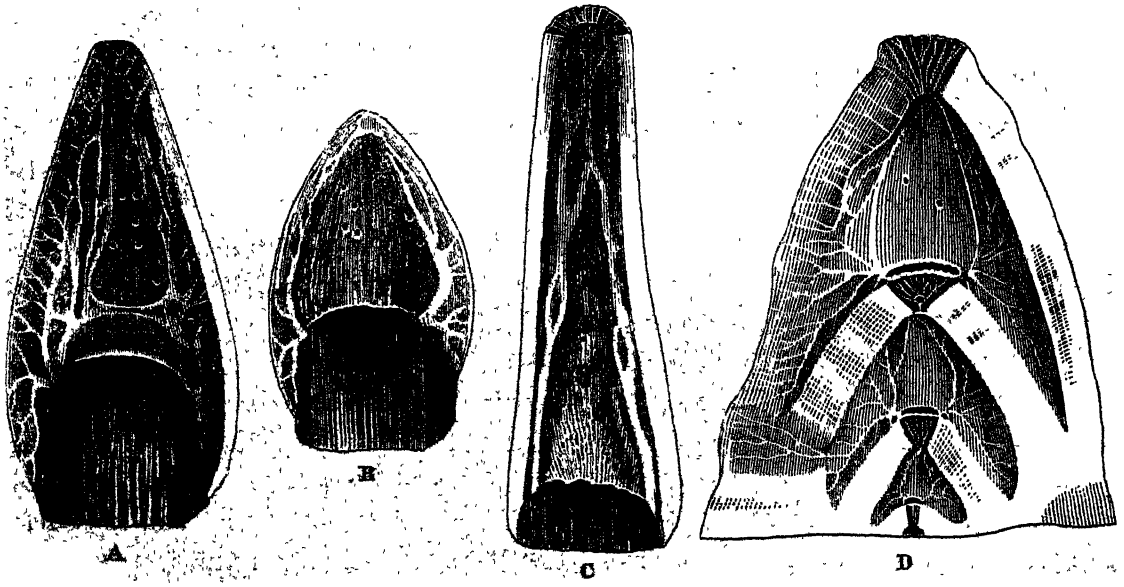
innermost convolution, but that it increases in diameter as it is built upon (so to speak) by each of the whorls by which this is subsequently invested.

156. The texture of the shell-substance which bounds the chambers at the outer margin of each whorl, however, presents a marked departure from that of the spiral lamina; a difference analogous to that which was first indicated by me in *Nummulites* (*loc. cit.*), and which has been found by MM. D'Archiac and Haime to present itself so constantly in that genus, that they give to this marginal portion the special designation "bourrelet." Mr. Carter has already recognized in *Operculina* the speciality in the structure of this part of the shell; but he has fallen into what I believe to be an error in his interpretation of that structure, describing it as made up of an aggregation of fusiform spicules, and hence designating it as "the spicular cord." Now I am very familiar with the appearance which has led him to take this view of its nature, since it is one which is commonly presented by thin sections that pass either through the median plane or parallel to it, as shown in Plate IV. fig. 13; but this is only one out of many aspects which are presented by sections taken in various directions; and a comparison of the whole seems to me to lead to quite a different conclusion,—namely, that the supposed spicular composition of this "marginal cord" (as it may be appropriately termed) is due to the peculiar manner in which the homogeneous substance of which it is composed is traversed by the set of canals that are correctly described by Mr. Carter as forming the "marginal plexus." For if the section should happen to traverse a portion of this plexus in which the canals form a tolerably regular network of elongated meshes in one plane, the appearance delineated in fig. 13 is presented; but just as often a less complete layer of the network is traversed by the section, and the form of the meshes is very inconstant, so that the appearance presented is rather that shown in fig. 14. Moreover the appearances exhibited by transverse sections of this marginal cord, highly magnified (figs. 15, 16), do not at all confirm the idea of a spicular arrangement, but are altogether conformable to the view I have expressed; the most complete confirmation to which is afforded by tangential sections, of which a very successful example,—giving a beautiful view, not only of the canal-system of the "marginal cord," but also of its communications with that of the septa, and of its relations with the two principal spiral canals to be presently described,—is represented in Plate IV. fig. 12. In this it is clearly seen how irregular is the disposition of the inosculating passages, whilst there is not a trace of the spicular structure, which ought, if it existed, to be as well brought into view in a section taken in this direction, as it is in the one shown in fig. 13, which is taken in a plane at right angles to it. The "marginal cord" is traversed on its external surface by longitudinal furrows, which are sometimes very shallow (fig. 15), but sometimes dip down deeply like those between the convolutions of the brain, as shown in fig. 16. These furrows usually form a kind of network with fusiform interstices (Plate V. fig. 5), whilst in other cases they run parallel to each other and inosculate more rarely (fig. 4). From the correspondence between their arrangement and that of the passages in the interior, I am inclined to think that they belong to the same system with these, being, in fact, canals not covered in by shell-substance, that communicate

with the plexus within by inosculating branches, whose apertures may sometimes be detected in the bottom of the furrows. This furrowed surface of the marginal band, and the existence upon it of apertures communicating with a set of canals in its substance, were specially pointed out by me in *Nummulites* (*loc. cit.*), although the changes produced by fossilization prevent the full extent of the "marginal plexus" from being traced out in that genus. The general relation of the marginal cord (*a a*) to the two spiral laminæ (*b*, *b*) with which it unites itself to form the boundary of the chambers, as seen in Plate IV. figs. 15, 16, is precisely the same in *Operculina* as in *Nummulites*.

157. The "marginal plexus" of canals communicates freely with the system of "interseptal canals," which are disposed between the two layers of the septa, as I have described in *Nummulites* and in *Cycloclypeus*, and the existence of which in *Operculina* has been already indicated by Mr. CARTER. The distribution of these is well seen, not only in vertical sections which have happened to traverse the septa, as in Figs. V., VII., and in Plate IV. fig. 8, but also in specimens laid open by fracture in the same direction, especially after the canals have been more distinctly marked out by the imbibition of a colouring liquid (Fig. VIII., and fig. 2, Plate V.). It will be seen from these delineations, all of which are faithfully copied from specimens in my possession, that the distribution of the interseptal canals, whilst presenting a certain general uniformity of plan, is by no means constant in detail. In Figs. VII. A, and VIII. D, we see two principal trunks passing from the two angles of the fissure at the interior edge of each septum, towards the marginal band at its exterior (as in fig. 2 *d*, Plate VI.), and sending out branches which ramify over the part of the septum that joins the spiral lamina (Fig. VIII. D), leaving the intervening portion untraversed; and this is the only arrangement which has been described by Mr. CARTER. But on looking at the other figures, it

Fig. VIII.



Septal planes of four specimens of *Operculina*, showing varieties in the disposition of the interseptal canals.

will be seen that much variety presents itself; thus in Fig. VIII. A (as in fig. 2, Plate V.) we observe two or more such trunks proceeding from each angle of the fissure, those of the same side frequently inosculating with each other; in Fig. VIII. C (as in fig. 8, Plate IV.) the two principal trunks on the opposite sides inosculate as they approach one another in their course towards the outer margin of the septum; and in Fig. V. (page 21) this inosculation is seen to take place much nearer the inner margin of the septum of the last whorl, so that the greater part of the septal plane is occupied by the plexus formed by their interlacement. The interseptal canals of one septum are occasionally, if not invariably, connected with those of another by loops formed between the ramifications of those canals which extend along the alar prolongations of the septa towards the centre of the shell; such inosculating loops are seen at the centre of fig. 6, Plate VI. Moreover, branches of the same system are generally found to penetrate the non-tubular shell-substance whenever it accumulates in any quantity; these branches are at once distinguished from the ordinary tubuli by their much larger size, their diameter being commonly about $\frac{1}{5000}$th of an inch.

158. The branches of the interseptal canals which proceed towards the spiral laminæ are sometimes continued onwards in the non-tubular shell-substance that constitutes the septal bands; and they are also frequently seen to diverge from the septa over the walls of the chambers,—still, however, being usually invested by a layer of transparent shell-substance, as is well shown at *e*, fig. 2, Plate VI. This arrangement, again, corresponds with what I have described and figured in *Nummulites*; but I have not been able to detect in *Operculina* the apertures by which in *Nummulites* these canals communicate with the cavity of the chamber, a circumstance which may perhaps be attributed to the relatively smaller size of these passages in *Operculina*, and the consequent facility with which their apertures may be confounded with those of the ordinary tubuli. In fig. 2, Plate VI. is represented an appearance of areolation I have several times met with in the walls of the chambers,—though usually less conspicuously than in this instance,—which has made me suspect that the ramifications of the interseptal canals sometimes (if not as a regular rule) form a network in the spiral lamina over the entire surface of the chambers.

159. The interseptal system of each whorl is connected with the marginal plexus of the preceding by a very remarkable arrangement, which seems to have altogether escaped Mr. Carter's notice. At each junction of the marginal cord with the tubular substance of the spiral lamina, the orifice of a large canal (Plate IV. fig. 15 *c*, *c*) may be seen in a vertical section which divides this canal transversely; and either of these canals may be clearly traced, in sections parallel to the surface, which have happened to pass through its plane, as at *d*, *d*, fig. 10, Plate IV., *b*, *b*, fig. 1, Plate VI., and very conspicuously in fig. 5, Plate IV., in which, by a fortunate accident, one of the spiral canals has been laid open in its entire course through the inner whorls, and can be traced nearly to the central cell. In the tangential section, fig. 7, Plate IV., the two canals can be seen running on the outside of what may be called the *nucleus*, consisting of the central cell and of the chambers immediately surrounding it. The mode in which

the interseptal passages branch off from these two spiral canals, is well seen in fig. 1, Plate VI., and also in fig. 10, Plate IV.; and this arrangement is shown in fig. 5, Plate IV. to extend to the septa between the chambers of the innermost convolution. The relations of the different parts of the system are brought extremely well into view in fig. 12, Plate IV., which represents a tangential section of a young shell, the rapid curve of whose spire causes the planes of even the contiguous septa to vary greatly in their direction. The spiral canals, though only running along the angles of the marginal cord, pretty obviously communicate with the plexus of passages which it contains; and thus the interseptal system of one whorl is brought into direct connexion with that of the preceding. It is to be remembered, however, that independently of such connexion, the spiral mode of growth of itself brings about a continuity of the canal-system throughout, by means of the marginal plexus; the consecutive whorls not being added one to another like the successive annuli of *Cycloclypeus*, each of which is (so to speak) *closed* or complete in itself; but being formed by the prolongation of the spiral lamina and of the marginal cord, which may be considered as always *open* to indefinite extension.

160. In regard to the uses of this canal-system, I have not at present anything to add to what I have already stated when describing its distribution in *Cycloclypeus* (¶ 107): I shall have the opportunity, however, of discussing them more fully in a subsequent Memoir upon forms in which this system is still more remarkably developed.

161. I have attempted to determine by a re-examination of my preparations of *Nummulites*, with the additional light afforded by the structure of *Operculina*, whether an arrangement of the canal-system prevails in the former at all comparable to that which I have shown to exist in the latter. In those which have undergone the ordinary fossilizing process, it is difficult to discover more than vague indications of its existence. But the peculiar process of silicification of *Foraminifera*, to which attention has been recently drawn by Professor EHRENBERG [*], has afforded an unexpected verification of the anticipations which the similarity of these two genera in other points of structure had led me to form as to this. For by the infiltration of silex tinged with silicate of iron into the cavities of Foraminifera, whilst their calcareous shells have undergone decomposition and removal, a most perfect series of *casts* are presented, not merely of the chambers, but of the canal-system where this exists. And in his figure of such a cast of *Nummulites striata* (taf. 5. fig. 2), Professor EHRENBERG has given a most beautiful representation of the canal-system of the marginal cord, and of its communications with the interseptal canals, so closely corresponding with that which, long before the publication of his memoir, I had worked out in *Operculina*, that no doubt can fairly remain as to the essential similarity between these two types in this important feature of their organization [†].

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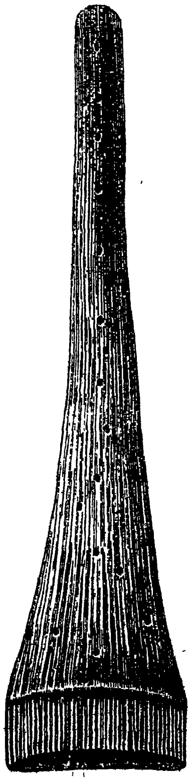
[*] Über den Grünsand und seine Erläuterung des organischen Lebens. Aus den Abhaldlungen der Königl. Akademie der Wissenschaften zu Berlin, 1855.

[†] I may here remark upon the confirmation afforded by other parts of this most valuable memoir to the results of my researches upon *Orbitolites* and *Cycloclypeus*; the various figures given by Professor EHRENBERG

162. Hence it would appear that there is no other essential difference between *Operculina* and *Nummulites**, than that which consists in the closing in of the spire, affirmed by MM. D'ARCHIAC and HAIME to be a constant character of the latter, whilst in the former the spire seems ordinarily to open out so long as it continues to increase. But even this is not an invariable difference; for I have met with several specimens of *Operculina*, in which the spire closes in by a somewhat abrupt inflexion of the marginal cord, so as to produce a rapid diminution in the size of the last four or five chambers, ending (as it would appear) in a complete cessation of growth. Examples of this kind are represented in fig. 8, Plate III. and fig. 3, Plate V.; and in Fig. IX. is shown the closing septum of one of these specimens upon a more enlarged scale. Looking, however, to the circumstance that these specimens bear but a very small proportion to those which exhibit no such tendency, and also to the fact that some of the specimens in which this closing-in is seen are very far from having attained their full growth, I am disposed to regard it as an abnormal, or at least as only an occasional occurrence in *Operculina*; and I must confess that, notwithstanding the positive assertion of MM. D'ARCHIAC and HAIME, I still entertain doubts as to whether it is to be accounted as a uniform characteristic of *Nummulites*. At any rate, the occasional occurrence of this condition in *Operculina* deprives the character of that constancy which is requisite to make it good for generic differentiation; and if *Operculina* and *Nummulites* are to be retained as separate genera, I cannot perceive by what features they are to be distinguished, save by the marked compression in form, the limited number of convolutions, and the external display of the whole spire, which are the obvious though not very important characteristics of the former.

Fig. IX.



Front view of the septal plane closing in the last chamber of *Operculina*.

163. It only now remains to speak of certain appearances indicative of *reparation after injuries*, which throw some light upon the physiology of this type of organization. Specimens are not unfrequently met with, exhibiting such irregularities as are delineated in fig. 1, Plate V.; and we can scarcely, I think, be wrong in concluding that these irregularities have commonly been produced by fracture. A portion of the outer part of the spire being broken off, the wound heals by the formation of new shell at the margin; but in its further progress the spire often shows the effect

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in his Fourth Plate, of the chambers and passages in different species of an organism which he designates *Orbitoides*, being in the closest conformity with my representations of the corresponding parts of these two genera, between which the form described by Professor EHRENBERG seems to be a connecting link.

* I have already (¶ 104, *note*, Philosophical Transactions, 1856, p. 558) corrected the mistake into which I fell, when treating of the structure of *Nummulites*, in regarding the non-tubular *columns* of the fossil shell as having been *passages* subsequently filled up.

of the injury, in a narrowing of that part of the convolution which succeeds it, the new chambers being formed as it were on the contracted basis to which their predecessors have been reduced. A very remarkable feature in all these reparations is *the continuation of the marginal cord along the fractured edge.* It might have been supposed that the wound would have been healed with some rude exudation, which would have given rise to an amorphous shell-structure; but instead of this, we find the reparation chiefly effected by a new production (probably by extension from the old) of that portion which gives the most evidence in its canalicular structure of being intimately connected with the vital changes taking place in the organism. This phenomenon is beautifully displayed in the specimen represented in fig. 4, Plate VI., which exhibits another remarkable feature, for which it is difficult to account,—a reversal in the direction of growth which has taken place after the fracture. Although the specimen is unfortunately not complete, there can be (I think) no reasonable doubt that its convolutions must have taken the course indicated by the dotted lines; and it would seem that after the first two convolutions had been formed, the spire had been broken across, almost through its centre. It will be at once seen, by comparing the direction of the septa of the second convolution with that of all those which surround it, that the latter have been reversed; and although the part in which the reversal actually took place is unfortunately wanting, yet there can be little doubt that it was made as indicated in the figure (*a*). But the most curious feature in the specimen is this,—that the extension (*ac*) of the marginal cord which has closed in the fractured portion has obviously proceeded not from the later (*b*), but from the earlier (*c*) of the two fractured extremities of the second convolution; and it is in this *backward* growth that the reversal would appear to have originated. A somewhat similar retrograde increase has been already noticed among the modes in which *Orbitolites* is occasionally repaired (¶ 39); and it obviously shows that every portion of the organism is equally capable of extending itself when left free to do so. It is worthy of remark, that the growth of this specimen subsequently to the injury, has taken place more after the plan of *Nummulites* than on the ordinary plan of *Operculina*; the convolutions being more numerous than usual, and their rate of increase slow. There is evidence, however, in the presence of a small fragment of the final whorl, that it underwent the thinning-out which is characteristic of its type. Notwithstanding the great number of specimens which I have examined, I have not met with one that presented any such departure from the normal type of growth as would deserve to be termed a *monstrosity*; so that it would seem as if such aberrations were more frequent in the *cyclical* than in the *helical* type.

Genus Amphistegina.

164. *History.*—This genus was first constituted by M. d'Orbigny in 1825, for the reception of a type of Foraminiferous shells which does not seem to have been previously noticed by those who have given their attention to this group, in consequence, it may be, of its limitation to the seas of warm latitudes. Notwithstanding the very close relationship which, as I shall presently show, it bears to *Nummulites* and other Nautiloid

forms of the order *Hélicostègues*, it was considered by M. D'ORBIGNY to depart from them in a character which he regarded of such fundamental importance as to serve as the basis of a distinct Order, that of *Entomostègues*; of which the following is his most recent definition[*]:—" Animal composé de segments alternes, formant une spirale. Coquille composée de loges empilées ou superposées sur deux axes alternant entre elles, et s'enroulant en spirale." The mode of increase of these shells, he elsewhere says[†], presents a singular mixture of that of the *Enallostègues* with alternating chambers, and of the spiral involution of the *Hélicostègues*. I have already had occasion (¶ 111) to point out, that in placing *Heterostegina* in this order, M. D'ORBIGNY has misconceived the structure of that genus; and I shall now have occasion to show that he has fallen into an equally grave error in regard to *Amphistegina*. His definitions of that genus in the 'Foraminifères Fossiles de Vienne' and in the 'Cours Élémentaire de Paléontologie,' are by no means accordant with each other: I take the last as the most authoritative.—" Coquille déprimée a spire embrassante, pourvue de loges alternes d'une coté et non de l'autre, séparées, intérieurement, par des cloisons longitudinales[‡]."

165. The only inquiry yet made, so far as I am aware, into the minute structure of the shells of this type, is that of Professor W. C. WILLIAMSON, in the memoir[§] to which I have already had such frequent occasion to refer. His investigations were made on small specimens of the *Amphistegina gibbosa*, a species which seems to be pretty generally diffused through the tropical ocean, and which has been recently dredged up in great abundance by Mr. MᶜANDREW in the neighbourhood of Teneriffe. Professor WILLIAMSON showed by means of horizontal and vertical sections that *Amphistegina* has the general structure of *Nummulites*, but with this marked difference (as he considered), that the shell is *inequilateral*; the spire making its convolutions obliquely instead of revolving in the same plane (in other words, being *turbinoid* instead of *nautiloid*), and the alar prolongations of the chambers being much larger, and extending further, on one side than on the other. He did not detect any indications of a canal-system in this shell; nor does he mention that division of the septa into two laminæ, one belonging to each of the contiguous chambers, which usually goes along with the other characters of this type.

166. *Organization.*—My own inquiries have been chiefly made upon a set of specimens contained in Mr. CUMING'S Philippine Collection, which present this type in a condition of far higher development than it has been elsewhere seen to attain; but I have also had the advantage of examining large numbers of specimens from New Holland, Teneriffe, and various parts of the Indian Ocean, as well as (through the kindness of Mr. W. K. PARKER) numerous fossil specimens from tertiary deposits in different parts of the globe. Of these last, nearly all seem to belong to the same species, *A. gibbosa*, as that described by Professor WILLIAMSON; and it is remarkable, that whilst the average dia-

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[*] Cours Élémentaire de Paléontologie et de Géologie, tom. ii. p. 201.

[†] Foraminifères Fossiles de Vienne, p. 199.

[‡] *Op. cit.* p. 201.

[§] "On the Minute Structure of the Calcareous Shells of some Recent Species of Foraminifera," in Transactions of Microscopical Society, First Series, vol. iii. p. 105.

meter of the recent specimens does not elsewhere exceed ·065 inch, it attains in some of the Philippine examples as much as ·115 inch, thus equalling that of the largest fossil specimens which have come under my notice. The general correctness of Professor WILLIAMSON's description of this species I can fully confirm; but the comparison of a large number of sections leads me to the unhesitating conclusion that the inequilaterality which has been considered the distinctive character of this type is so inconstant, as to forbid our assigning any value to it as a *generic*, still less as an *ordinal* character. It is often very strongly marked in the fossil *Amphisteginæ* of the Vienna and St. Domingo tertiaries; but it is as frequently absent altogether, the shell being perfectly equilateral; and between these two extremes of form we find every intermediate gradation. So it frequently presents itself among the small specimens of *Amphistegina gibbosa* which are common in tropical seas, and I have met with specimens exhibiting as great a want of symmetry as the one of which Professor WILLIAMSON has delineated a section; yet on the whole, I should say, that in the great bulk of the smaller specimens which have passed under my notice, the want of symmetry is but slight; whilst perfect equilaterality both of external form and of internal arrangement not unfrequently occurs; and in those large specimens from the Philippines which present this type in its highest development, there is little or no departure from exact symmetry.—But further, Mr. CUMING's collection furnishes numerous examples of a remarkable species whose general conformity of structure to *Amphistegina gibbosa* seems to leave no doubt of its near relationship to it (the resemblance of its younger specimens to that specific type being so close as to have led me in the first instance to a belief in the specific identity of the two), and yet the most exact symmetry here presents itself as the rule, departures from it being exceptional, and never proceeding to any considerable extent. It may be considered, therefore, as satisfactorily established, that want of lateral symmetry is not to be held as an essential character of the shells of this genus, although more frequently occurring in it than in *Nummulites* or *Operculina*, to which, as I shall now show, this type bears an extremely close relationship.

167. The species to which I have just referred being hitherto undescribed, I shall designate it *A. Cumingii*. In its young state it is lenticular in form (Plate V. figs. 13, 14), and is only distinguishable from the symmetrical variety of *A. gibbosa* by the smaller number and greater angular distance of the septal bands which radiate from the centre and suddenly turn backwards as they approach the margin. (The radiating portions of these septal bands, as will presently appear, mark the direction of the portions of the septa that intervene between the alar prolongations of the chambers; whilst the recurrent portions mark the direction of the portions of septa that separate the principal cavities of the chambers.) As the *A. Cumingii* advances in life, however, a marked change presents itself in its form, corresponding to that which I have shown to occur in *Heterostegina*, *Peneroplis*, and *Operculina*; for the spire, after about its fourth turn, begins to flatten itself out, and separates itself henceforth from the central portion of the shell, to which it affords a very partial investment (figs. 15–17), the spiral lamina

being still continued over it, but the cavity of the chambers being limited to the peripheral portion of the whorl. Thus the form of the shell is so completely altered, that it would not be recognized as the same if the successive stages of this alteration could not be traced, especially as the characteristic radiating lines are no longer to be seen on the prominent centre. Its diameter undergoes an extraordinary increase; the specimen from which fig. 17 was taken measuring no less than ·30 inch by ·25 inch.

168. The organization of the interior of the shell, as displayed by sections passing either through the median plane or parallel to it (Plate VI. fig. 6), and by sections taken perpendicularly to the median plane, or vertically (Plate VI. fig. 5), is closely conformable to that of *Operculina*; the chief difference arising out of the extension of the alar prolongations of the chambers of each successive whorl, and of their dividing septa, over the entire surface of the penultimate whorl, except at the centre of the spire, which is commonly occupied by a solid pillar of non-tubular substance, having the general shape of a double cone, whose apex is in the primordial cell, and whose base is on the most prominent part of each surface. As it is very seldom that the successive whorls lie precisely on the same level, a section which traverses the outer convolutions in the median plane will pass above or below the plane of the inner whorls; and it will thus, in traversing the surfaces of these, bring into view the centripetal prolongations of the septa intervening between the alar prolongations of the chambers of the whorls which invest them, as is shown in the central portions of fig. 6. These centripetal prolongations are composed of non-tubular shell-substance, and the portion of the spiral lamina above them is also non-tubular; and thus it is that their position is marked on the exterior by radiating bands, which never rise in *A. Cumingii* into tubercles or ridges, though they sometimes become thus elevated in *A. gibbosa*. These septal bands are, generally speaking, those of the last-formed whorl alone, those of all the preceding whorls being concealed by the investment they have received; but in unsymmetrical specimens (as often happens in *A. gibbosa*, and occasionally in *A. Cumingii*), the alar prolongations do not extend to the centre of the flatter side; so that the septal bands which radiate from the centre are those of the penultimate whorl, and are interrupted as they approach the margin by a new series belonging to the last-formed convolution. The reason of the obliteration of the septal bands on the prominent portion of the shell in advanced specimens of *A. Cumingii* is at once explained when we look at fig. 5; for as the last whorl extends itself peripherally, the alar prolongations of its chambers and therefore of its septa are withdrawn from the central region, and thus no new septal bands will be formed; whilst the continuation of the thick spiral lamina over the whole surface of the penultimate whorl prevents *its* septal bands from displaying themselves. In this respect there is a marked contrast between *A. Cumingii* and *A. gibbosa*; for on the surface of the oldest specimens of the latter we see not only the numerous septal bands of the outer whorl, but indications of those of the penultimate whorl, which show themselves as imperfect or broken lines between the complete radiating bands. This is probably due to that elevation of the septal bands of which mention has

already been made, which causes them to unite at intervals with the spiral lamina that is subsequently formed over them; so that they are marked on its external surface by the non-tubularity of its substance in those parts which immediately overlie them.

169. Allowance being made for the differences arising out of this diversity in their plan of conformation, we find an extraordinary resemblance in the general organization of *Amphistegina Cumingii* to that of the *Operculina* already described, as will be apparent from a comparison of figs. 5, 6, Plate VI. to figs. 8, 10, Plate IV. of *Operculina*. Hence it will be quite unnecessary to enter into any minute description of its structure. In regard to the mode of communication between the chambers,—by one principal orifice in the form of a narrow slit along the inner margin of each septum, with a variable number of secondary apertures irregularly disposed in different parts of the septal plane,—there is an absolute identity with *Operculina*; as there is also in the distribution of the interseptal passages between the two layers which each septum here unmistakeably exhibits. The marginal cord, likewise, exhibits the same peculiarity of organization as in *Operculina*; being channeled out by freely inosculating passages which communicate with those of the interseptal system (Plate VI. fig. 6). This marginal system of passages is even more developed than in *Operculina*; and a study of its distribution will, I think, remove all doubt as to the correctness of my interpretation of the appearances often presented by sections of the marginal cord in that genus (¶ 156). There is this difference, however, in the relation of the interseptal to the marginal system of canals in these two types,—that the interseptal canals of *Amphistegina* appear to take their origin directly from some of the large superficial passages of the marginal cord on which the septum abuts, instead of branching from a pair of regular spiral canals as in *Operculina*, no distinct evidence of such canals having here presented itself to me.

170. The relationship of *Amphistegina Cumingii* to those forms of *Nummulites* in which the alar prolongations of the chambers of each whorl are continued, with centripetal prolongations of the septa, over the whole surface of the penultimate whorl, is obviously extremely close; in fact, I can discern no distinguishing character between these two types, except that which is afforded by the tendency of the last turn of the spire of *Amphistegina* to open out, whilst that of *Nummulites* closes in. The double-coned axis of non-tubular substance would seem to be a distinctive character of *Amphistegina*; but I am not inclined to lay much stress upon it, as it presents itself in very different proportions in different individuals.

171. Whilst the type which I have now been describing is thus closely related to the highest because most specialized forms of Foraminiferous organization, it is singular that the smaller species to which I have referred under the name of *A. gibbosa* should present such a marked inferiority of conformation. Its spiral lamina is as minutely tubular as that of its congener; but I have not been able to detect the least trace of a system of interseptal canals; and it is especially to be noticed, that the marginal cord, which exhibits such a high development of the canal-system in *A. Cumingii*, consists in *A. gibbosa* of homogeneous non-tubular shell-substance, without any vestige of canals;

though traversed here and there by a stray tubulus resembling the straight parallel tubuli of the spiral lamina. I am inclined to attribute this difference in part to the smaller scale of the last-named species, which prevents any part of its shell from being far removed from the contact of sarcode; the absence of a canal-system being analogous, under this point of view, to the absence of Haversian canals in laminæ of bone which are thin enough to draw their nourishment directly from the nearest vascular surface. The smaller species may thus be considered as—so to speak—a degraded form of the larger; the other differences in its structure being of very subordinate value. These differences, however, serve for its recognition by external characters; for whilst the number of radiating septal bands to be seen in *A. Cumingii* is never much greater than twenty, and in young specimens does not reach half that amount, that of the radiating septal bands in *A. gibbosa* commonly exceeds thirty; besides which, the backward turn which these bands take, when they have passed the margin of the penultimate whorl, is far more striking in the last-named species than in that which I have been specially describing. The granular character of the surface in the neighbourhood of the mouth, which seems due (as Professor WILLIAMSON has pointed out) to a secondary deposit of minute papillæ of non-tubular shell-substance, is another distinguishing feature of *A. gibbosa.* To this species I am disposed to refer all the three fossil forms of *Amphistegina* that are described by M. D'ORBIGNY* under the names of *A. Hauerina*, *A. mamillata*, and *A. rugosa*, since these differ no more from each other than do the recent examples of the species,—their chief distinctions being based on the degree of their departure from bilateral symmetry, and on the limitation of the alar prolongations on the flatter side to the marginal portion of the included whorl; characters, which I have shown to possess no constancy, and to be therefore quite valueless for systematic purposes. I have examined several large *Amphisteginæ* from the miocene of St. Domingo, which seem to conform to the same type; but in consequence of the alterations brought about by fossilization, I have not been able to determine satisfactorily whether the apparent absence of a canal-system indicates that it had no existence in the recent organism; so that the identification of the species must rest on the less satisfactory characters furnished by the number and direction of the septal bands.

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* Foraminifères Fossiles de Vienne, pp. 207–209.

Explanation of the Plates.

PLATE I.

Fig. 1. Ideal figure of *Dendritina*, partly laid open, so as to show the arrangement of its chambers, the nature of their communications by a single fissure of irregular form (simpler in the earlier whorls, but more complex in the later), the overlapping of the earlier whorls by the alar prolongations of the chambers of the later, and the manner in which the last turn of the spire tends to detach itself from the earlier convolutions.

Fig. 2. Ideal figure of *Peneroplis*, partly laid open, so as to show the arrangement of its chambers, the nature of their communication by isolated pores, which are arranged in a single row in the later whorls, but frequently form a double row in the earlier, the rapid increase in the breadth of the spire coincident with its very extraordinary flattening in the last convolution, and the very slight degree in which the earlier whorls are covered in by the later.

Fig. 3. Ideal figure of *Operculina*, laid open to show the details of its structure:—*a a a*, marginal band (the "spicular cord" of Mr. Carter, the "bourrelet" of MM. d'Archiac and Haime), divided transversely at a', so as to show the orifices of its canals, the distribution of which is seen at $a''a''$ in a tangential section of the band, and at $a'''a'''$ in a section through the median plane; *b*, *b*, *b*, external surface of the chambers, marked out by the septal bands; *c*, *c*, *c*, interior of the chambers of the outer whorl, the alar prolongations of which extend, as shown at c', c', over the surface of the interior whorl, reaching nearly to the centre of the spire; the chambers are separated by the septa *d*, *d*, *d*, formed by the union of two layers of shell, one belonging to each chamber, and having spaces between them in which lie the interseptal canals, whose smaller branches are seen irregularly divided in the septa d', d', whilst in the septum d'' one of the principal trunks is laid open through its whole length; at the approach of each septum to the marginal cord of the preceding whorl is seen the fissure which forms the principal connexion between the chambers; at *e*, *e* are seen the secondary orifices of communication, and in the same septa is shown the distribution of the interseptal system of canals, branching from *f*, *f*, the two spiral canals; and at *g*, *g* are seen the conoidal columns of translucent shell-substance, forming tubercular projections on the surface, which frequently overlie the septa, and are penetrated by branches of the interseptal system of vessels.

PLATE II.

Fig. 1. Side view of a characteristic specimen of *Peneroplis*:—1 *a*, front view, or septal plane, of the same, showing its single row of isolated pores:—40 diam.

Fig. 2. Front view of a young specimen, intermediate in its characters between *Peneroplis* and *Dendritina*:—40 diam.

Fig. 3. Side view of a specimen of *Peneroplis*, of which the later chambers are widely extending themselves laterally:—40 diam.

Fig. 4. Septal plane of cylindrical prolongation of *Spirolina* (see fig. 11), showing transition between the isolated pores of *Peneroplis* and the coalesced fissures of *Dendritina*:—40 diam.

Fig. 5. Side view of a specimen of *Peneroplis*, of which the later chambers are extending themselves longitudinally:—40 diam.

Fig. 6. Front view of a young specimen, intermediate in its characters between *Peneroplis* and *Dendritina*:—40 diam.

Fig. 7. Side view of a specimen of *Peneroplis*, of which the later chambers are extending themselves longitudinally, and of which the spire is more turgid, and the pores arranged in a double row, as shown at 7 *a*:—40 diam.

Figs. 8, 9, 10. Front views of young specimens of *Peneroplis*, showing various departures from the normal type in the form of the septal plane and the disposition of the apertures:—40 diam.

Fig. 11. Side view of *Spirolina*, which bears the like relation to the *Dendritina*-type shown in figs. 12 and 13, that the specimens represented in figs. 5 and 7 bear to the ordinary *Peneroplis*-type; the septal plane of its cylindrical extension, with apertures of intermediate form between those of *Peneroplis* and of *Dendritina*, is shown at 11 *a*:—40 diam.

Fig. 12. Side view of a typical specimen of *Dendritina*:—at 12 *a* is shown the septal plane and single large dendritic orifice of its last chamber, and at 12 *b* the same, from the preceding whorl; *al*, *al*, alar prolongations of the chambers, which bear a much larger proportion to the principal cavity in the middle than in the later part of the growth:—40 diam.

Fig. 13. Side view of a specimen of *Dendritina* tending towards the *Spirolina*-type, the last turn of the spire having already detached itself, and the last chamber having completely lost its alar prolongations, as seen at 13 *a*:—40 diam.

Fig. 14. Front view of a young specimen of *Peneroplis*, presenting a very marked departure from the ordinary type in the form of its septal plane and the disposition of its apertures:—40 diam.

Fig. 15. Septal plane of a specimen resembling in general form that represented in fig. 13, but showing a want of coalescence of the fissures of which the dendritic orifice is made up; the ridge-and-furrow arrangement of the walls of

the chambers is here prolonged in an unusual manner over the borders of the septal plane:—100 diam.

Figs. 16, 17, 18, 19. Side and front views of young specimens of *Peneroplis*, showing various departures from the normal type in the form of the septal plane and in the disposition of the apertures:—40 diam. (See also figs. 2, 6, 8, 9, 10, 14.)

Fig. 20. Portion of the ordinary surface of *Peneroplis* more highly magnified, showing its ridge-and-furrow plication, and its rows of minute punctations:—100 diam.

Fig. 21. Portion of the ordinary surface of *Dendritina*, showing precisely similar characters:—100 diam.

Fig. 22. Septal plane of a specimen of *Dendritina*, showing the punctations continued over its surface:—100 diam.

Fig. 23. Portion of the surface of a specimen of *Peneroplis*, on which the plications had become obsolete, but the punctations are still arranged in rows corresponding to them in distance:—100 diam.

Fig. 24. Portion of the surface of a specimen of *Peneroplis*, over which the punctations are scattered without definite arrangement:—100 diam.

PLATE III.

Figs. 1–9. Side and front views of specimens of *Operculina* at various ages, showing differences in the form and proportions of the spire, in the surface-markings, and in the number and arrangement of the tubercles marking the septal bands:—10 diam.

Fig. 10. Hemispherical cluster of tubercles occupying the umbilicus of the variety of *Operculina* represented in figs. 11, 12:—25 diam.

Figs. 11, 12. Side and front views of young and mature specimens of peculiar variety of *Operculina*, characterized by the rapid opening-out of the spire, and by the extraordinary development of tubercles, not only along, but also between the septal bands:—10 diam.

PLATE IV.

Fig. 1. Part of the internal surface of one of the chambers of an *Operculina* that presented large tubercles on its exterior, showing the uniform distribution of the tubuli except along the septal bands, and the comparatively large size of their orifices:—120 diam.

Fig. 2. Section through the wall of a contiguous chamber of the same, showing the partial deficiency of tubuli in certain spots, with their crowding together in

the neighbourhood of these, and the diminution in the size of the tubuli as they pass towards the outer surface:—120 diam.

Fig. 3. Part of the internal surface from another *Operculina*, presenting smaller and more numerous tubercles on its exterior:—120 diam.

Fig. 4. Section through the wall of a contiguous chamber of the same:—120 diam.

Fig. 5. Section through the median plane of the earlier convolutions of an *Operculina*, showing the spheroidal form of the primordial chamber, the irregular forms of the chambers that immediately succeed it, and one of the spiral canals, which may be traced in the marginal cord almost to the centre of the spire:—75 diam.

Fig. 6. Portion of the external surface of an *Operculina*, of which the tubercles were particularly prominent:—75 diam.

Fig. 7. Vertical section of the inner convolutions of an *Operculina*, which has happened to pass close to the primordial chamber and the one that springs from it, and which shows at *a* the two spiral canals running along their exterior:—100 diam.

Fig. 8. Vertical section of a typical *Operculina*, passing through nearly the centre of the spire; showing at *a*, *a* the spiral lamina of the second whorl, separated from that of the whorl which it invests by the alar prolongations of the chambers of the latter, which extend nearly to the centre of the spire; *b*, the spiral lamina of the last convolution, thinned away in proportion to its extension, and coalescing with that of the preceding convolution near the margin of the latter, the alar prolongations of the outer chambers being comparatively small; *c*, septum between contiguous chambers, traversed by the interseptal system of canals; *d*, *d*, marginal cord divided transversely (see figs. 15, 16); *e*, *e*, fissure of communication between contiguous chambers; *f*, *f*, spiral canals divided transversely:—40 diam.

Fig. 9. Very thin section of a lamina of the tubular shell-substance of *Operculina*, magnified 120 diameters, cutting the tubuli transversely, and showing a delicate areolation between them; *b*, prismatic appearance of a portion of the same, magnified 250 diameters; *a*, another portion, magnified 250 diameters, showing the comparatively large size of its tubuli close to the internal surface.

Fig. 10. Portion of a section of a typical *Operculina* (see Plate VI. fig. 3), taken through the median plane:—*a*, *a*, marginal cord traversed by canal-system; *b*, *b*, walls of the chambers, composed of minutely-tubular substance; *c*, septal bands with their lateral branches, composed of transparent non-tubular shell-substance; *d*, *d*, one of the spiral canals:—50 diam.

Fig. 11. Portion of a similar section of a tuberculated specimen of *Operculina*, showing the clear spaces left by the absence of tubuli in the spots beneath the tubercles, and the semi-opacity produced by the crowding together of the tubuli elsewhere:—50 diam.

Fig. 12. Portion of a vertical section passing tangentially along the exterior of the innermost convolution of a young *Operculina*, and affording a most advantageous display of its canal-system:—*a a*, marginal cord of the inner convolution; *b b*, *b b*, spiral lamina of the outer convolution; *c c*, *c c*, spiral canals; *d d*, *d d*, origins of the interseptal canals from the spiral canals; *e*, *e*, *e*, projection of the septal bands of non-tubular shell-substance into ridges on the surface:—120 diam.

Fig. 13. Portion of the marginal cord, *a*, *a*, of *Operculina*, as seen in a section passing through the median plane, showing the fusiform spaces formed by the inosculation of the canals:—120 diam.

Fig. 14. Another portion, showing more elongated and less regular spaces:—120 diam.

Fig. 15. Transverse section of the marginal cord, *aa*, as seen in a vertical section of *Operculina* (fig. 8), showing its slightly furrowed external surface, the canal-system which traverses its interior, its junction with the ordinary tubular substance, *b b*, of the spiral lamina, and the spiral canals, *c c*:—120 diam.

Fig. 16. A similar section from another specimen, showing deep furrows upon the external surface of the marginal cord:—120 diam.

PLATE V.

Fig. 1. Side view of an *Operculina*, of which a portion of the last whorl has apparently been broken off, and the injury repaired by the continuation of the marginal cord along the line of fracture:—10 diam.

Fig. 2. Portion of the outermost and penultimate convolutions of *Operculina*, laid open by vertical fracture:—*a*, external surface of marginal cord of the convolution inclosed by them; a', a', transverse sections of the marginal cord in the outer convolutions; *b b*, the two marginal canals, whose transverse sections, $b' b'$, are seen in the penultimate convolution; *c*, septum between the contiguous chambers of the penultimate convolution; c', the same in the outer convolution; *d d*, origins of interseptal system of canals; *e*, *e*, fissures of communication between contiguous chambers; *f*, secondary pores; *g g*, thick spiral lamina of tubular substance, formed by the coalescence of that $g' g'$ of the last whorl over that of the penultimate whorl:—75 diam.

Fig. 3. Side view of an *Operculina*, of which the last whorl has closed in by a gradual diminution in the size of its chambers:—10 diam.

Figs. 4, 5. External surface of marginal cord, *a a*, showing two modes of disposition of its furrows, in which are seen the orifices of some of the canals; at $a'a'$ the cord is seen in transverse section; and at *b b* is shown its junction with the spiral lamina:—75 diam.

Figs. 6 to 8, 10 to 12. Varieties in the superficial characters of *Operculina*:—40 diam.

Fig. 9. Young specimen of the tuberculated variety of *Operculina*, showing an extraordinary development of the central clusters of tubercles:—10 diam.

Figs. 13 to 17. Successive stages of growth of *Amphistegina Cumingii*:—10 diam.

PLATE VI.

Fig. 1. Section of a portion of the outer whorl of *Operculina*, parallel to the surface, showing a marked irregularity in the disposition of the septa:—*a a*, marginal cord; *b b* one of the spiral canals; *c*, *c*, septal bands of transparent shell-substance; *d*, tubular shell-substance of spiral lamina forming the walls of the chambers; *e*, *e*, prolongations of interseptal canals in spiral lamina:—100 diam.

Fig. 2. Similar section from another specimen, showing an appearance of reticulation in the spiral lamina:—*a a*, marginal cord; *c*, septal band; *d*, one of the interseptal canals communicating with canal-system of marginal cord; *e*, prolongation of interseptal canals in spiral lamina:—100 diam.

Fig. 3. Section of *Operculina*, traversing its median plane except in its central portion, and showing, especially at *a* and *b*, great irregularities in the disposition of its chambers:—12 diam.

Fig. 4. Portion of a specimen of *Operculina*, which seems to have undergone fracture at an early age, and to have thenceforth grown in a reversed direction, as will be seen on comparing the direction of the septa in the whorls *a* and *b*; the marginal cord appears to have been continued from *c* to *a* along the line of fracture, and thence to have grown onwards to *b*:—50 diam.

Fig. 5. Vertical section of *Amphistegina Cumingii*, showing its conformity in all essential respects to the type of *Operculina*, and the differences produced by the smaller size of its chambers in proportion to the thickness of its spiral lamina:—50 diam.

Fig. 6. Portion of section of *Amphistegina Cumingii*, traversing its outer whorls in the median plane, but passing along the surface of its inner convolutions, so as to bring into view the alar prolongations of the septa in their passage towards the centre, and the occasional inosculation of their canal-systems by loops in that region:—50 diam.

II. *Description of some Remains of a Gigantic Land-Lizard* (Megalania prisca*, Owen) *from Australia. By Professor* Owen, *F.R.S. &c.*

Received May 13,—Read June 17, 1858.

In a collection of fossil bones from pleistocene (?) deposits forming the bed of a tributary of the Condamine River, west of Moreton Bay, Australia, recently purchased by the British Museum, were the three vertebræ which form the subject of the following description.

They formed part of a collection composed chiefly of remains of *Diprotodon*, *Nototherium* and other large extinct Marsupials, and presented the same colour, specific gravity and partially fossilized state; but their anatomical characters were very different, and demonstrated these vertebræ to belong to a large reptile of the Lacertian order.

These characters are, the well-turned hemispheric anterior cup (Plates VII. and VIII., *c*) and posterior ball (ib. *b*) for the articulation of the body of the vertebra with those of contiguous vertebræ, the tubercle (ib. *d*) at the fore-part of each side of the base of the neural arch for the articulation of the rib, and the contracted area of the neural canal (Plate VII., fig. 4, *n*): the latter character is extreme, and denotes the small proportional size of the spinal chord in correlation with the habits and powers of a sluggish cold-blooded quadruped.

The neural arch has coalesced with the centrum; the cup and ball are both placed obliquely (Plate VIII. fig. 2), the latter (ib. *b*) looking upward and backward; both these characters concur with the single costal tubercle on each side, in determining the lacertian character of the vertebræ in question.

The best preserved of the three vertebræ (Plate VII. figs. 1, 2, 3 & 4) wants the right posterior zygapophysis; the articular surface of the costal tubercle (fig. 1, *d*) is also abraded. The following are dimensions of this vertebra:—

	Inches.	Lines.
Length of centrum	3	3
Length of non-articular lower surface of centrum	2	0
Breadth of centrum, behind the ball	1	11
Vertical diameter of centrum, behind the ball	1	4
Vertical diameter of cup	1	9
Transverse diameter of cup	2	5
Breadth of neural arch above the costal tubercles	4	7

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* Μέγας, great, and ἠλαίνω, to roam about; in reference to the terrestrial nature of the great Saurian, which was noticed under the above name in the author's 'Lectures on Fossil Reptilia,' delivered in the Theatre of the Museum of Practical Geology, Jermyn Street, 'Synopsis,' March 1858.

	Inches.	Lines.
Vertical diameter from highest part of neural arch	3	4
Transverse diameter of anterior outlet of neural canal	0	9
Transverse diameter of posterior outlet of neural canal	0	4
Vertical diameter of anterior outlet of neural canal	0	3
Vertical diameter of posterior outlet of neural canal	0	5
Longest diameter of anterior zygapophysis	1	6
Vertical diameter of costal tubercle	1	0
Transverse diameter of costal tubercle	0	7
Antero-posterior extent of base of neurapophysis	1	10

The anterior articular concavity of the centrum, as shown by the above admeasurements, is a transverse ellipse, very obliquely placed, looking downward and forward: this aspect is shown in the section of the smaller vertebra (Plate VIII. fig. 2, *c*). The articular surface is smooth; in the larger vertebra (Plate VII. fig. 4, *c*) it presents a shallow transversely lengthened pit at the centre.

The under surface of the body (*ib.* fig. 3) is flat from before backward, broad, and slightly convex transversely; it is a little roughened, with two very shallow depressions anteriorly, as if for aponeurotic attachments, but shows not a trace of hypapophysis. The hind articular convexity corresponds in size and shape with the anterior cup, and has an opposite aspect; the plane of the base of the ball (*ib.* fig. 1, *b*) forms an angle of 45° with that of the lower surface of the centrum; the lateral margins of the ball project a little beyond the narrow constriction which there divides it from the rest of the centrum.

In front of this constriction the sides of the body rise, expanding, with an outer convexity, as they advance to the costal tubercle, which is situated upon the fore-part of the base of the neurapophysis. Some wrinkled and vascular impressions beneath the tubercle indicate the place of the obliterated suture between the neural arch and centrum.

The base of the costal tubercle (*ib.* fig. 1, *d*) is a vertical ellipse, of the dimensions above given. On the right side its convexity is shown, the articular surface not being abraded beyond the thin crust of bone which formed it: on the left side the abrasion exposes the coarser cancellous texture of the bone beneath. Above the tubercle, a ridge or angle formed by the meeting of the anterior and lateral surfaces of the neurapophysis extends to the outer end of the anterior zygapophysis. The articular surface of this process is flat, and looks upward and inward. The broad upper surface of the neural arch between the processes is rather concave, traversed by a low median ridge (Plate VII. fig. 2, *n s*). On each side of this, at the fore-part of the arch, is a small tubercle; behind this the surface is roughened by small shallow pits arranged in progressively lengthening curved lines, indicative of a place of ligamentous attachment; between this surface and the zygapophysis is a broad and very shallow smooth depression. The posterior half of the upper surface of the neural arch is convex, and extends outward at its back part to form the posterior zygapophysis (ib. *z'*), the articular surface of which looks downward and outward.

The fracture which has broken away the right posterior zygapophysis extends a little to the left of the median line, and there may, therefore, have been a neural spine from the posterior third of the arch; but it must have been thin, and was probably, if it existed, low; it is represented along the rest of the arch by the ridge (*n s*) above mentioned, which nowhere rises above two lines in height. Thus the breadth of the neural arch exceeds its length, and much exceeds its height.

The extreme contraction of the neural canal (*ib.* fig. 4, *n*) forms a striking feature in this vertebra, especially in regard to the vertical diameter of the anterior outlet: in the posterior outlet this diameter is the longest. The anterior outlet is unsymmetrical, the right nerve or vessel from the chord indenting more deeply the vertebra, as it leaves the canal. The posterior outlet shows the inward projection of a ridge from the middle of each side of the canal; external to this outlet a low ridge rises vertically from the margin of the articular ball to the lower end of the surface of the zygapophysis.

The free surface of the vertebra is in general smooth, or with shallow linear markings and impressions, as above described.

This vertebra belongs to the dorsal series.

The second vertebra (Plate VIII. figs. 1 & 2), which is somewhat smaller, appears, by the greater extent of the costal tubercle (fig. 1, *d*), and by the longitudinal depression on each side of the mid-part of the under surface, to have come from the cervical region. The left side of this vertebra was so much mutilated that I had it ground down to a flat surface, such as would have been left by a vertical longitudinal section of the bone. This exposed the shape of the neural canal (fig. 2, *n*), which, from its shallow anterior outlet (ib. *a*), deepens to the middle of the vertebra, by the sinking of the floor of the canal into the substance of the centrum, whence it contracts a little towards the posterior outlet. The ridge indicated at the side of the canal in the preceding vertebra is here seen to commence anteriorly from the upper part of the canal, and describing a curve similar to, but not quite parallel with, that of the floor, to terminate behind near the middle of the canal. The large vascular canals and coarse cancellous texture of the substance of both the centrum and the neural arch are also shown by this section. There was no neural spine on this vertebra, but only the low median ridge corresponding to, but less developed than, that partially shown in the foregoing vertebra.

The subtriangular surface formed by the small shallow impressions in curvilinear lines is also present on each side of the fore-part of the median ridge of the neural arch in this vertebra, anterior to which is a tubercular rudiment of a 'zygosphene[*].'

The costal tubercle (*ib.* fig. 1, *d*) commences about three lines from the lateral border of the anterior cup *c*, and extends to near the anterior zygapophysis, *z'*. The length of the tubercle is 2 inches 3 lines; its greatest breadth is 9 lines.

The shape and aspects of the articular surfaces, both on the centrum and neural arch, are the same as in the former vertebra. The general configuration of the vertebræ is likewise closely similar.

[*] This term is defined in the works cited in the note, p. 46.

The length of the centrum is 3 inches; the vertical diameter of the vertebra is 2 inches 10 lines. The greatest vertical diameter of the neural canal is 6 lines.

The third vertebra of the present gigantic Lizard consists of a mutilated centrum only (Plate VIII. figs. 3 & 4): it shows the median tract defined by the lateral grooves (fig. 3, *v*) on the under surface, the grooves being broader and deeper and more decidedly bent outward at their fore-part than in the foregoing and somewhat smaller vertebra; the median tract is simply convex from side to side, and is straight lengthwise; it does not project at any part below the level of the under surface; on the contrary, the convex outer sides of the grooves project below its level. The part of the neural canal (fig. 4), preserved on the upper broken surface, shows the transverse expansion at its fore-part, and a low median ridge from the floor of that part of the canal.

The mid-tract of the lower surface of the centrum (*ib.* fig. 3), though, in the cervical and anterior dorsal vertebræ, defined by the lateral grooves, *v*, is not produced at any part as a hypapophysis, but resembles, in this respect, the under surface of the hinder neck-vertebræ in the Monitors (*Varanus*, *Hydrosaurus*, fig. 3 *a*).

The small vertical diameter of the centrum, in proportion to its breadth, and the oblique position of the terminal cup and ball, are well displayed in the present mutilated specimen, which appears to have been rolled and water-worn.

In comparing the vertebræ of *Megalania* with those of existing Lizards, the biconcave vertebræ of the Geckos, and the complex procœlian vertebræ, with zygosphene and zygantrum, of the Iguanians[*], are at once set aside; as are also the compressed and carinate or subcarinate cervical and dorsal vertebræ of the *Rhynchocephalus* (*Hatteria*, Gray) of New Zealand. Among the less modified vertebræ of other Lacertians, those of the Australian Monitors and Lace-lizards (*Hydrosaurus*, Wagler) make the nearest approach to the vertebræ of *Megalania*.

They present the same oblique position of the cup and ball (Plate VIII. figs. 1 *a*, 2 *a*), flatness and breadth of the under surface of the centrum (*ib.* fig. 3 *a*), constriction at the base of the ball and lateral expansion thence forwards to the costal tubercle; the same relative size and aspect of zygapophyses; the same curvilinear pittings and fine wrinklings affecting the otherwise smooth and compact outer surface of the bone; the same contrast between the vertical and transverse diameters of the two outlets of the neural canal; and the same lateral and infero-median ridges in that canal. The chief distinctions are, the much more contracted area of the neural canal, and the minor development of the neural spine, in *Megalania* (compare fig. 4 with 4 *a*); also the shortness of the vertebræ in proportion to their breadth, in the large fossil Lizard.

In *Hydrosaurus varius* the ridge representing the neural spine begins, as in *Megalania*, at the fore-part of the neural arch, but is relatively higher, although low and equal: this makes it probable that the homologous ridge in the vertebra (Plate VIII. fig. 4, *n s*) of *Megalania* is the sole representative of the neural spine: in *Hydrosaurus giganteus* the ridge rises to the proportions of a spinous process. But, apart from these generic or subgeneric differences, the correspondence is complete between the extinct and above-cited existing Monitor-lizards of Australia.

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[*] Owen, 'Catalogue of the Osteological Series in the Museum of the Royal College of Surgeons,' 4to, vol. i. 1853, p. 145, no. 668; and 'Principes d'Ostéologie Comparée,' 8vo, Paris, 1855, pl. 13 *a*, fig. 11.

The three or four vertebræ at the base of the neck, supporting free ribs, in *Hydrosaurus*, exhibit the same modification of the costal tubercle as in the second vertebra of *Megalania* (Plate VIII. fig. 1, *d*): those vertebræ, also, show the median tract defined by the two side-grooves on the under surface. The vertebræ in advance of these, in *Hydrosaurus*, have the median tract produced into a hypapophysis.

Vertebræ of *Megalania* will probably be discovered showing this character; and, indeed, from the correspondences already determined, it may be inferred that the same local modifications of the vertebral column prevailed in the bony structure of *Megalania* as in that of the existing Australian Monitors.

Among these land-lizards, it is interesting to find that one species, which attains the length of upwards of 6 feet, has been dignified with the specific name of *giganteus*, on account of this unusually large size compared with the generality of existing *Lacertilia*.

Whether among the vast and unexplored wildernesses of the Australian continent any living representative of the more truly gigantic *Megalania* still lingers, may be a question worth the attention of travellers. But, most probably, like the gigantic Marsupials, *Diprotodon* and *Nototherium*, with whose fossil remains those of *Megalania* were associated in the tertiary deposits now cut through by the Condamine and its tributaries, the gigantic land-lizard has long been extinct.

The procœlian type of vertebra, or that in which the articular cup is in front and the ball behind, characteristic of the fossils above described, was first introduced in the Reptilian class, according to present knowledge, during the liassic period; but, from that to the upper oolite, it was manifested only by Pterodactyles. The earliest examples of procœlian vertebræ in Lacertian reptiles date from the Wealden period; in Crocodilian reptiles the procœlian type first appears at the cretaceous period *.

The only known procœlian lizards comparable in size with the large Australian one, represented by the three above-described vertebræ, are the *Mosasaurus* and *Leiodon* of the Greensand and Chalk strata.

The inferior depth of the cup and production of the ball, the vertical position of both at right angles with the axis of the centrum, and many minor modifications of the vertebræ of those large Lacertians of the cretaceous period, show great differences between them and the fossil procœlian vertebræ of the freshwater tertiary deposits of Australia. The Mosasauroids were, in fact, a family of marine lizards so distinct from existing species, as to form the type of a suborder.

The fossil remains of European procœlian lizards, which, like the large vertebræ under description, belong to the type of true land-lizards, all belong to species of the ordinary average size. The miocene and pliocene formations of the continent, and especially of France, have furnished several genera and species *.

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* In the Greensand of New Jersey, North America (Proceedings of the Geological Society, January 31, 1849); in the Upper Greensand near Cambridge, England; in the 'Calcaire pisolithique' of Mont Aimé, Departement de la Marne; and in the upper cretaceous stone at Maestricht.

The chief peculiarity of the Australian fossil lizard is its great size; the vertebræ rival in bulk those of the largest living Crocodiles. The 'Monitor de la Nouvelle-Hollande' (*Varanus*, MERREM, *Hydrosaurus*, WAGLER) has thirty vertebræ between the skull and sacrum. Taking the average length of the vertebra at 3 inches, this would give 7 feet 6 inches as the length of the trunk of the *Megalania*; and were the proportions of the head and tail like those of the great Lace-lizard of Australia (*Hydrosaurus giganteus*, GRAY), the total length of the *Megalania* would be about 20 feet.

On the very probable hypothesis that the jaws and teeth of *Megalania* were of the same type as those of the *Hydrosaurus*, it must have been carnivorous, and, by its bulk and strength, very formidable.

DESCRIPTION OF THE PLATES.

PLATE VII.

Fig. 1. Side view of a dorsal vertebra, *Megalania prisca*.
Fig. 1 *a*. Side view of a dorsal vertebra, *Hydrosaurus giganteus*.
Fig. 2. Upper view of a dorsal vertebra, *Megalania prisca*.
Fig. 2 *a*. Upper view of a dorsal vertebra, *Hydrosaurus giganteus*.
Fig. 3. Under view of a dorsal vertebra, *Megalania prisca*.
Fig. 3 *a*. Under view of a dorsal vertebra, *Hydrosaurus giganteus*.
Fig. 4. Front view of a dorsal vertebra, *Megalania prisca*.
Fig. 4 *a*. Front view of a dorsal vertebra, *Hydrosaurus giganteus*.

PLATE VIII.

Fig. 1. Side view of a cervical vertebra, *Megalania prisca*.
Fig. 1 *a*. Side view of a cervical vertebra, *Hydrosaurus giganteus*.
Fig. 2. Vertical section of a cervical vertebra, *Megalania prisca*.
Fig. 2 *a*. Vertical section of a cervical vertebra, *Hydrosaurus giganteus*.
Fig. 3. Under view of an anterior dorsal vertebra, *Megalania prisca*.
Fig. 3 *a*. Under view of an anterior dorsal vertebra, *Hydrosaurus giganteus*.
Fig. 4. Horizontal section of an anterior dorsal vertebra, *Megalania prisca*.
Fig. 4 *a*. Horizontal section of an anterior dorsal vertebra, *Hydrosaurus giganteus*.

[All the figures are of the natural size, and have been drawn on the stone without reversing.]

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* See LARTET, 'Notice sur la Colline de Sansans,' 8vo, 1851, p. 39; GERVAIS, 'Paléontologie Française,' pp. 258, 259.

III. *On some Remarkable Relations which obtain among the Roots of the Four Squares into which a Number may be divided, as compared with the corresponding Roots of certain other Numbers. By the Right Hon. Sir* Frederick Pollock, *F.R.S., Lord Chief Baron.*

Received April 26,—Read May 20, 1858.

The property of numbers, which is the subject of this paper, first presented itself to my attention in the case of the odd squares 1, 9, 25, 49, &c. $(2n\mp1)^2$; any two adjoining odd squares may be divided (each of them) into 4 square numbers, whose roots will have this remarkable relation to each other: two of them will be identically the same; and as to the other two, one of them will be 2 less, and the other will be 2 more than the roots of the preceding or subsequent odd square; for example, 25 and 49 may be divided into squares, the roots of which being placed below, will appear thus:—

	25	**49**	so	**49**	**81**
	−2, 1, 4, 2	−4, 1, 4, 4		0, 2, 3, 6	−2, 2, 3, 8
or thus	0, 0, 3, 4	−2, 0, 3, 6.			

In comparing the roots of the adjoining odd squares, 2 roots (placed in the middle) are the same; of the others, one is 2 more, the other 2 less than the corresponding roots of the other.

The following Table presents the result of a comparison of the roots of all odd squares up to $27^2=729$:—

1	**9**	**121**	**169**
0, 0, 1, 0	−2, 0, 1, 2	−2, 2, 7, 8	−4, 2, 7, 10
9	**25**	**169**	**225**
0, 1, 2, 2	−2, 1, 2, 4	0, 3, 4, 12	−2, 3, 4, 14
25	**49**	−2, 4, 7, 10	−4, 4, 7, 12
−2, 1, 4, 2	−4, 1, 4, 4	−4, 5, 8, 8	−6, 5, 8, 10
0, 0, 3, 4	−2, 0, 3, 6	−6, 4, 9, 6	−8, 4, 9, 8
49	**81**	**225**	**289**
0, 2, 3, 6	−2, 2, 3, 8	−4, 3, 10, 10	−6, 3, 10, 12
81	**121**	−6, 5, 10, 8	−8, 5, 10, 10
0, 1, 4, 8	−2, 1, 4, 10	**289**	**361**
−2, 4, 5, 6	−4, 4, 5, 8	−2, 5, 8, 14	−4, 5, 8, 16
−4, 0, 7, 4	−6, 0, 7, 6	−6, 3, 12, 10	−8, 3, 12, 12

361
0, 1, 6, 18
−2, 1, 10, 16
−4, 7, 10, 14
−6, 9, 10, 12

441
−2, 1, 6, 20
−4, 1, 10, 18
−6, 7, 10, 16
−8, 9, 10, 14

441
−8, 8, 13, 12
−10, 4, 15, 10

529
−10, 8, 13, 14
−12, 4, 15, 12

529
−8, 10, 13, 14
−2, 5, 10, 20

625
10, 10, 13, 16
−4, 5, 10, 22

625
−2, 4, 11, 22
&c.

729
4, 4, 11, 24
&c.

[*It is to be remembered, that throughout this paper the root of any square may be assumed to be positive or negative at pleasure, and* 0 *is considered as an even square.*]

This relation among the roots belongs also to the numbers resulting from the addition of any (the same) even number to the adjoining odd squares, and within certain limits (to be noticed presently) to the numbers resulting from the subtraction of the same even number: thus,

49−4=45
−2, 3, 4, 4
0, 0, 3, 6

49−2=47
−3, 2, 5, 3

49
0, 2, 3, 6

49+2=51
−1, 3, 4, 5

49+4=53
0, 1, 4, 6
&c.

81−4=77
−4, 3, 4, 6
−2, 0, 3, 8

81−2=79
−5, 2, 5, 5

81
−2, 2, 3, 8

81+2=83
−3, 3, 4, 7

81+4=85
−2, 1, 4, 8
&c.

The addition may proceed indefinitely, but the subtraction has this limit: on deducting 30 from 49, and also from 81, the numbers become 19 and 51, which are terms in the series 1, 3, 9, 19, 33, 51, &c. $(2n^2+1)$; and any 2 alternate terms of that series will become by continued addition adjoining odd squares.

Any 2 alternate terms of the series may be represented by $2.(n-1)^2+1$, and $2.(n+1)^2+1$, or $2n^2-4n+3$, and $2n^2+4n+3$; add $2n^2-2$ to each, and they become $(2n-1)^2$, and $(2n+1)^2$, that is, adjoining odd squares.

If, instead of the odd squares, a series of the even squares +1 be assumed, the adjoining terms will have similar properties; thus,

4+1=5
0, 0, 2, 1
+1, 0, 0, 2

16+1=17
−2, 0, 2, 3
−1, 0, 0, 4

$16+1=17$
$+1, 0, 0, 4$

$36+1=37$
$-1, 2, 4, 4$
&c.

$36+1=37$
$-1, 0, 0, 6$

$64+1=65$
$-3, 2, 4, 6$
&c.

And each of these may in like manner be increased, or (subject to a similar limit) be diminished; these, however, are derived from the alternate terms of another series, 1, 5, 13, 25, &c. $(2n^2+2n+1)$, $(n-2)^2+(n-1)^2$, and $n^2+(n+1)^2$ will represent any 2 alternate terms of the series; and if to $2n^2-6n+5$, and also to $2n^2+2n+1$ there be added $2n^2-2n$, they become $4n^2-8n+5$, and $4n^2+1$, or $(2n-2)^2+1$, and $(2n)^2+1$, or, adjoining even squares $+1$.

The various examples of these or similar relations among the roots of the 4 squares into which numbers may be divided are endless; the increase and decrease of the variable roots is not always by 2; it may be by any other number. But instead of multiplying examples, it will be better to enter on the proof of what has been already stated, which will furnish the means of investigating every instance that can be produced. The proof depends upon,—

1st, a general property of all odd numbers which (as far as I am aware) has not hitherto been noticed; and

2ndly, a general theorem relating to odd numbers in arithmetical progression.

The property of odd numbers is this:—Every odd number may be divided into 4 squares, in such manner that 2 of the roots will be equal, 2 of them will differ by 1, 2 of them will differ by 2, 2 of them by 3, and so on, as far as the number is capable (from its magnitude) of having roots large enough to form the difference required. The difference may be algebraical, and result from one of the roots being considered as a negative quantity. For example, there is only one mode of dividing the number 23 (a number of the form $(8n+7)$) into 4 squares; these must be 1, 4, 9, 9, and their roots ± 1, ± 2, ± 3, ± 3:—

3 and 3 are equal, difference 0,
3 and 2 differ by 1,
3 and 1 differ by 2,
2 and -1 differ by 3.
-3 and 3 differ by 6,
-2 and 3 differ by 5,
-1 and 3 differ by 4,

The differences of the roots may therefore be 0, 1, 2, 3, 4, 5 or 6; a greater difference than 6 is (in the number 23) impossible; the least numbers that would make a difference of 7 would be 3 and 4, and the sum of their squares would be 25.

The same form or mode of dividing a number into 4 squares does not always furnish every possible difference (as in the case of 23); thus 13 in one form has the roots 0, 0, 2, 3; these furnish as differences 0, 1, 2, 3, 5. To obtain the difference 4, the other form of dividing 13 into 4 squares (viz. 1^2, 2^2, 2^2, 2^2) must be resorted to, and then -2, $+2$ differ by 4: so 39 has two modes of dividing into 4 squares; in one the roots are

$^{1}77$ is $+1,6,6,2$, and therefore $^{9}117$ is $-3,6,6,6$. Obviously it is easy to throw the proof into a general algebraic form; the general property of odd numbers above stated may therefore be considered as established.

The theorem alluded to is this:—If any odd number of odd numbers, beginning with 13 (or any other number), be in arithmetical progression with a common difference of 4 [or any other number] (for the purpose of this example let the series be 13, 17, 21, 25, 29, 33, 37, with a common difference of 4), then if the common difference (4) be assumed as the index of the difference of roots to the middle term in the series, and the higher terms beyond the middle have as indices of the differences of their roots (4+1), (4+2), (4+3), &c. in succession, and the lower terms have as indices (4−1), (4−2), (4−3), &c., the series with the indices will be $^{1}13$, $^{2}17$, $^{3}21$, $^{4}25$, $^{5}29$, $^{6}33$, $^{7}37$; then if the terms less than the middle term be divided into four squares with exterior roots, having the differences indicated by the respective indices [which may be done by the property of odd numbers just proved], thus,

$^{1}13$	$^{2}17$	$^{3}21$	$^{4}25$
1, 2, 2, 2	0, 2, 3, 2	−1, 0, 4, 2	−2, 1, 4, 2
2, 0, 0, 3		+1, 0, 2, 4	0, 0, 3, 4

then the terms greater than the middle term will have this relation to the terms less than the middle term, the two terms next to the middle term will have their exterior roots,—one, less by 1, the other, greater by 1, than those of the other. The two terms next but one will have their exterior roots,—the one, less by 2, the other, greater by 2, and so on, increasing as the pairs of terms become more distant from the centre; and all the pairs of terms (equidistant from the middle term) will respectively have the same middle roots; thus

$^{4}25$	$^{5}29$	$^{6}33$	$^{7}37$
−2, 1, 4, 2	0, 0, 2, 5	−2, 2, 3, 4	−2, 2, 2, 5
			−1, 0, 0, 6

Comparing them, the result is as stated above; the difference between the exterior roots of the respective pairs is half the difference between the respective indices.

The algebraic proof of this theorem is very easy. Let there be an arithmetic series with n as a middle term and p as a common difference; then the series with the indices of the differences of roots will be

$$^{p-2}(n-2p),\quad ^{p-1}(n-p),\quad ^{p}n,\quad ^{p+1}(n+p),\quad ^{p+2}(n+2p),\ \&c.;$$

and any two terms equidistant from the middle term may be represented by $(n-mp)$, $(n+mp)$, and their indices will be $(p-m)$, $(p+m)$; then if $^{p-m}(n-mp)$ have its middle roots r, s, and the first of its exterior roots a, the other exterior root must be $a+p-m$ in order that the difference of the roots may correspond with the index and equal $(p-m)$; then the corresponding roots of $n+mp$ will be, according to the theorem, $(a-m), r, s, a+p$. If the first set of roots be squared, the sum is

$$a^2+r^2+s^2+(a+p-m)^2=a^2+r^2+s^2+(a+p)^2+m^2-2am-2pm;$$

if the second set of roots be squared, the sum is $a^2-2am+m^2+r^2+s^2+(a+p)^2$, which is $2pm$ more than the former; if, therefore, the first sum of squares equals $n-mp$, the second will equal $n+mp$; therefore the pair of terms that are at the distance (m) from the centre will have their middle roots the same, and their exterior roots one less by m, the other greater by m than those of the other.

In this proof p, the common difference, may be odd or even, and n, the middle term, may be odd or even; thus

$$\begin{array}{ccccc} {}^{2}\mathbf{2} & {}^{3}\mathbf{6} & {}^{4}\mathbf{10} & {}^{5}\mathbf{14} & {}^{6}\mathbf{18} \\ -1,0,0,1 & -1,0,1,2 & -2,1,1,2 & -2,0,1,3 & -3,0,0,3 \end{array}$$

is a series composed of even numbers, all of which obey the theorem; but frequently an even number is not so divisible as to form the required difference. To form every difference is a property which belongs universally to odd numbers only, not to even numbers; the common difference may be only 1; the numbers from 25 to 41 are all (both odd and even) divisible into four squares, whose roots conform to the theorem, 33 being the middle term.

$$\begin{array}{ccccccccc} {}^{-7}\mathbf{25} & {}^{-6}\mathbf{26} & {}^{-5}\mathbf{27} & {}^{-4}\mathbf{28} & {}^{-3}\mathbf{29} & {}^{-2}\mathbf{30} & {}^{-1}\mathbf{31} & {}^{0}\mathbf{32} & \\ 4,0,0-3 & 5,0,0,-1 & 5,1,1,0 & 2,2,4,-2 & 5,0,0,2 & 3,2,4,1 & 3,3,3,2 & 0,4,4,0 & \\ & & & & & & & & {}^{1}\mathbf{33} \\ & & & & & & & & 0,4,4,1 \\ {}^{9}\mathbf{41} & {}^{8}\mathbf{40} & {}^{7}\mathbf{39} & {}^{6}\mathbf{38} & {}^{5}\mathbf{37} & {}^{4}\mathbf{36} & {}^{3}\mathbf{35} & {}^{2}\mathbf{34} & \\ -4,0,0,5 & -2,0,0,6 & -1,1,1,6 & -3,2,4,3 & +1,0,0,6 & 0,2,4,4 & 1,3,3,4 & -1,4,4,1 & \end{array}$$

To apply this theorem as a proof of the matters stated in the beginning of the paper, all the examples, whether of the odd squares, or of the even squares +1, or those numbers increased or decreased, may be made terms in an arithmetic series; $4n^2-4n+1$ and $4n^2+4n+1$ (which represent any 2 adjoining odd squares) have a difference of $8n$, which is divisible by 4, and therefore they may form terms in an arithmetical series; thus

$$4n^2-4n+1,\quad 4n^2-2n+1,\quad 4n^2+1,\quad 4n^2+2n+1,\quad 4n^2+4n+1,$$

the common difference being $2n$, and the odd squares will be two places from the middle term; their exterior roots will therefore be greater and less by 2. So any 2 adjoining odd squares, $4n^2$ and $4n^2+8n+4$, differ by $8n+4$, which is divisible by 4; and the adjoining even squares, +1, may in like manner be made terms in an arithmetic series.

I propose now to apply the theorem generally to other instances of odd numbers having 2 roots equal and the other 2 roots differing by any number whatever, the one root being greater, the other less by that number; for example, the alternate odd squares may be divided into 4 squares in such manner that 2 roots of the one may equal 2 roots of the other, and the differences of the remaining roots will be 4:

$$\begin{array}{cc} {}^{1}\mathbf{9} & {}^{9}\mathbf{49} \\ 0,2,2,1 & -4,2,2,5 \\ +1,0,2,2 & -3,0,2,6 \end{array}$$

$^{3}25$	$^{11}81$
0, 0, 4, 3	−4, 0, 4, 7
−1, 2, 4, 2	−5, 2, 4, 6

so the odd squares taken every third term, as

9	81
+2, 0, 1, 2	−4, 0, 1, 8
25	**121**
+2, 1, 2, 4	−4, 1, 2, 10

present a difference of 6 for the exterior roots.

The theorem affords a solution of all these and every other instance. For in the case of any arithmetical progression of odd numbers having an odd number of terms, the terms of which have been indexed as directed (by making the common difference the index of the middle term, &c.), it will be found that all the pairs of terms equidistant from the middle term are derived from 2 terms of one, or other, of the 2 series above-mentioned by adding the same even number to both; if the indices be even, they are derived from terms in the series 1, 3, 9, 19, &c.; if the indices be odd, the terms are derived from the series 1, 5, 13, 25, &c., and the terms in the series may immediately be found, as they are the terms having the same indices as the pair of terms in the arithmetical progression. Let

$$^{0}5,\ ^{1}9,\ ^{2}13,\ ^{3}17,\ ^{4}21,\ ^{5}25,\ ^{6}29,\ ^{7}33,\ ^{8}37$$

be an arithmetic series (with a common difference 4 and 21 the middle term) indexed according to the theorem; place under each term that term in either of the two series which has the index of the term in the arithmetical series; thus

$^{0}5$	$^{1}9$	$^{2}13$	$^{3}17$	$^{4}21$	$^{5}25$	$^{6}29$	$^{7}33$	$^{8}37$
$^{0}1$	$^{1}1$	$^{2}3$	$^{3}5$	$^{4}9$	$^{5}13$	$^{6}19$	$^{7}25$	$^{8}33$
4	8	10	12	12	12	10	8	4

Deducting the one from the other, it is obvious that the terms equidistant from 21 are derived from terms of the two series, by adding to them the same number; if the terms in the 2 series be adjacent, the difference of the exterior roots will be 1, if alternate, 2, and so on; if the terms in the 2 series be m places distant, the differences of the exterior roots will be m: this may be shown generally in an algebraic form, thus.

Let the middle term of an arithmetic series be n and the common difference be an even number $2m$, the series with its indices will be

$$^{2m-3}(n-6m),\ ^{2m-2}(n-4m),\ ^{2m-1}(n-2m),\ ^{2m}n,\ ^{2m+1}(n+2m),\ ^{2m+2}(n+4m),\ ^{2m+3}(n+6m),\ \&c.$$

and the terms of the 2 series to be placed under each term will be

$$\&c.\ (2m^2-4m+3),\ (2m^2-2m+1),\ (2m^2+1),\ (2m^2+2m+1),\ (2m^2+4m+3),\ \&c.;$$

deducting the one from the other, the remainders will be

$$n-(2m^2+5),\ \big(n-(2m^2+3)\big),\ \big(n-(2m^2+1)\big),\ n-(2m^2+1),\ n-(2m^2+1),\ n-(2m^2+3),\ n-(2m^2+5),$$

where obviously the remainders equidistant from the middle term are equal to one another.

Now the difference between the adjoining terms of either of the 2 series will always be divisible by 2, the difference between the alternate terms will be divisible by 4, the difference between 2 terms that are m apart will be divisible by $2m$.

For the nth term of the series 1, 3, 9, 19, &c. is $2n^2+1$, and the $(n+m)$th term is $2n^2+4nm+2m^2+1$, and their difference, $4nm+2m^2$, is divisible by $2m$. So in the other series (1, 5, 13, 25, &c.) the nth term is $2n^2+2n+1$, and the $(n+m)$th term is

$$(n+m)^2+(n+m+1)^2=2n^2+2n+1+2m^2+4mn+2m;$$

and their difference, $2m^2+4mn+2m$, is also divisible by m. An arithmetical series may therefore always be formed, which will give the required difference of the roots according to the distance of the pair of terms from the middle term.

The general result therefore is, that if any 2 odd numbers be assumed, they will either have this relation to each other of the roots of the 4 square numbers into which they may be divided, or a third odd number may be found, which will connect them together by having that relation to each.

If the 2 odd numbers be the result of an addition of the same even number to any 2 terms of either of the two series, they will have this relation of the roots, and the difference of the exterior roots will depend upon the distance of the 2 terms from each other; and conversely, if any 2 odd numbers have this relation of the roots, they are derived from 2 terms of the same series by the addition of the same even number to both; but if the 2 odd numbers have not this relation of their roots to each other, then a third odd number may be found having that relation to each of them.

Assume any 2 odd numbers as 13 and 105, deduct 12 from each of them so as to reduce the smaller to 1, and the other to 93, the next term in either series less than 93 is 85, 85+8=93. Select that term in the series to which 85 belongs, which by the addition of 8 becomes a term in the other series, this is 25, 25+8=33, and 33 is the number which connects 1 with 93; for

$^{0}\mathbf{1}$	and	$^{8}\mathbf{33}$
0, 0, 1, 0		−4, 0, 1, 4

have the relation which appears from their roots, and

$^{7}\mathbf{33}$	and	$^{13}\mathbf{93}$
−3, 2, 2, 4		−6, 2, 2, 7
−2, 0, 2, 5		−5, 0, 2, 8

have a similar relation in two ways; then add 12 to 1, to 33, and to 93, and 13, 45 and 105 will have this relation among their roots:

$^{0}\mathbf{13}$	$^{8}\mathbf{45}$	$^{7}\mathbf{45}$	$^{13}\mathbf{105}$
+2, 1, 2, 2	−2, 1, 2, 6	−3, 2, 4, 4	−6, 2, 4, 7
0, 2, 3, 0	−4, 2, 3, 4	−2, 0, 4, 5	−5, 0, 4, 8
		−1, 2, 2, 6	−4, 2, 2, 9

It is obvious, that what is done with these numbers may be done with any other odd numbers, and it would be superfluous to give an algebraic proof.

This relation of all odd numbers to each other has not (as far as I am aware) been remarked before; but it has occurred to me that possibly it may form part of the "mysteries of numbers" alluded to by FERMAT in that remarkable passage in which his theorem of the polygonal numbers was announced in a note to an edition of Diophantus, published after his death, p. 180*. The mysterious properties of numbers referred to by FERMAT must (of course) be connected with the theorem he was announcing; indeed he expressly refers to them as the source of his demonstration.

POSTSCRIPT.—May 20, 1858.

Since this paper was sent to the Society, some other theorems of a similar kind have occurred to me, in which the terms of a series (not arithmetical of the 1st order) have a similar relation with regard to the roots of the 4 squares into which they may be divided, that is, those which are equidistant from the middle (if the number of terms be even), or from the middle term (if the number of terms be odd), have the middle roots the same, and the exterior roots have an arithmetical relation to each other, varying with the distance from the centre, viz. the one being less and the other greater by the same quantity.

Thus, if any number of terms (exceeding 3) of either of the 2 series above-mentioned, viz. 1, 3, 9, 19, &c. $(2n^2+1)$, or 1, 5, 13, 25, &c. $(2n^2+2n+1)$, and, beginning with the 1st term, the successive differences of the terms be added "*inverso ordine*," a new series will be obtained possessing the property in question; thus the first seven terms of the 1st series are ${}^{0}1$, ${}^{2}3$, ${}^{4}9$, ${}^{6}19$, ${}^{8}33$, ${}^{10}51$, ${}^{12}73$; the differences are 2, 6, 10, 14, 18, 22; if the differences be added "*inverso ordine*," beginning with 22 instead of 2, the series becomes 1, 23, 41, 55, 65, 71, 73, each term of which may be divided into 4 squares, whose roots will be as follows:—

${}^{0}\mathbf{1}$	${}^{2}\mathbf{23}$	${}^{4}\mathbf{41}$	${}^{6}\mathbf{55}$	${}^{8}\mathbf{65}$	${}^{10}\mathbf{71}$	${}^{12}\mathbf{73}$
0, 0, 1, 0	+1, 2, 3, 3	0, 3, 4, 4	−3, 1, 6, 3	−2, 3, 4, 6	−3, 2, 3, 7	6, 0, 1, 6
		+2, 0, 1, 6	−1, 2, 5, 5	0, 0, 1, 8		
			+1, 1, 2, 7			

Here obviously the result is as stated above; 55 is the middle term; the terms equidistant from it have the same middle roots, and the difference between the other roots increases according to the distance from the middle term being 2, 4, 6. If 8 terms be so treated, there is no middle term; the result is similar, but the successive differences are 1, 3, 5, 7.

The other series, 1, 5, 13, 25, &c., gives a similar result.

The reason of these results is that the equidistant terms are always equal to the original corresponding term in the series, increased by the same number; thus,

41 and 65 = 9 + 32 and 33 + 32 respectively.

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* See LEGENDRE, Théorie des Nombres, 1st edit. p. 187.

There are certain numbers, which, added to the terms of the series in its ordinary state, convert it into a series whose differences have been added "*inverso ordine.*"

If the number of terms be 3, add 0 4 0
If 4, add 0 8 8 0
If 5, add 0 12 16 12 0
If 6, add 0 16 24 24 16 0
If 7, add 0 20 32 36 32 20 0
If 8, add 0 24 40 48 48 40 24 0
&c. &c.

The law of the formation of these numbers is obvious.

These numbers apply to both series, and to any consecutive terms in either; that is, [*e. g.*] 0, 12, 16, 12, 0, added to any 5 consecutive terms of either series, converts them into 5 terms whose differences have been added "*inverso ordine;*" and what is still more remarkable, the middle roots of the first eight (or indeed *n*) terms having their differences added "*inverso ordine,*" are the middle roots which answer for any eight [or *n*] consecutive terms, whose differences have been added "*inverso ordine*" through the unlimited extent of the whole series. Thus if the first 8 terms be formed with the differences added "*inverso ordine,*"

	1	**5**	**13**	**25**	**41**	**61**	**85**	**113**
add	0	24	40	48	48	40	24	0

^{1}**1**	^{3}**29**	^{5}**53**	^{7}**73**	^{9}**89**	^{11}**101**	^{13}**109**	^{15}**113**
0,0,0,1	0,2,4,3	−2,2,6,3	−1,0,6,6	−2,0,6,7	−5,2,6,6	−5,2,4,8	−7,0,0,8
	+2,0,0,5	−1,0,6,4	+1,2,2,8	0,2,2,9	−4,0,6,7	−3,0,0,10	
		+1,0,4,6			−2,0,4,9		
		+2,0,0,7			−1,0,0,10		

the indices and the roots of the squares will be as above.

Now take the 8 consecutive terms, beginning with 181:—

	181	**221**	**265**	**313**	**365**	**421**	**481**	**545**
add	0	24	40	48	48	40	24	0

^{19}**181**	^{21}**245**	^{23}**305**	^{25}**361**	^{27}**413**	^{29}**461**	^{31}**505**	^{33}**545**
−9,0,0,10	−9,2,4,12	−11,2,6,12	−10,0,6,15	−11,0,6,16	−14,2,6,15	−14,2,4,17	−16,0,0,1
	−7,0,0,14	−10,0,6,13	− 8,2,2,17	− 9,2,2,18	−13,0,6,16	−12,0,0,19	
		− 8,0,4,15			−11,0,4,18		
		− 7,0,0,16			−10,0,0,19		

the middle roots are the same for both.

These last matters add weight to the suggestion already made, that the properties of numbers referred to are connected with the "mysterious and abstruse" properties alluded to by FERMAT, as enabling him to prove the theorem he announced of the polygonal numbers.

IV. *A Sixth Memoir upon Quantics.* *By* ARTHUR CAYLEY, *Esq.*, *F.R.S.*

Received November 18, 1858,—Read January 6, 1859.

I PROPOSE in the present memoir to consider the geometrical theory: I have alluded to this part of the subject in the articles Nos. 3 and 4 of the Introductory Memoir. The present memoir relates to the geometry of one dimension and the geometry of two dimensions, corresponding respectively to the analytical theories of binary and ternary quantics. But the theory of binary quantics is considered for its own sake; the geometry of one dimension is so immediate an interpretation of the theory of binary quantics, that for its own sake there is no necessity to consider it at all; it is considered with a view to the geometry of two dimensions. A chief object of the present memoir is the establishment, upon purely descriptive principles, of the notion of distance. I had intended in this introductory paragraph to give an outline of the theory, but I find that in order to be intelligible it would be necessary for me to repeat a great part of the contents of the memoir in relation to this subject, and I therefore abstain from entering upon it. The paragraphs of the memoir are numbered consecutively with those of my former Memoirs on Quantics.

147. It will be seen that in the present memoir, the geometry of one dimension is treated of as a geometry of points in a line, and the geometry of two dimensions as a geometry of points and lines in a plane. It is, however, to be throughout borne in mind, that, in accordance with the remarks No. 4 of the Introductory Memoir, the terms employed are not (unless this is done expressly or by the context) restricted to their ordinary significations. In using the geometry of one dimension in reference to geometry of two dimensions considered as a geometry of points and lines in a plane, it is necessary to consider,—1°, that the word point may mean *point* and the word line mean *line*; 2°, that the word point may mean *line* and the word line mean *point*. It is, I say, necessary to do this, for in such geometry of two dimensions we have systems of points in a line and of lines through a point, and each of these systems is in fact a system belonging to, and which can by such extended signification of the terms be included in, the geometry of one dimension. And precisely because we can by such extension comprise the correlative theorems under a common enunciation, it is not in the geometry of one dimension necessary to enunciate them separately; it may be and very frequently is necessary and proper in the geometry of two dimensions, where we are concerned with systems of each kind, to enunciate such correlative theorems separately. It may, by way of further illustration, be remarked, that in using the geometry of one dimension in reference to geometry of three dimensions considered as a geometry of points, lines,

and planes in space, it would be necessary to consider,—1°, that the words point and line may mean respectively *point* and *line*; 2°, that the word line may mean *point in a plane**, and the word point mean *line*, viz. the expression points in a line mean *lines through a point and in a plane*; 3rd, that the word line may mean *line* and the word point mean *plane*, viz. the expression points in a line mean *planes through a line*. And so in using the geometry of two dimensions in reference to geometry of three dimensions considered as a geometry of points, lines, and planes in space, it would be necessary to consider,—1°, that the words point, line, and plane may mean respectively *point*, *line*, and *plane*; 2°, that the words point, line, and plane may mean respectively *plane*, *line*, and *point*. But I am not in the present memoir concerned with geometry of three dimensions. The thing to be attended to is, that in virtue of the extension of the signification of the terms, in treating the geometry of one dimension as a geometry of points in a line, and the geometry of two dimensions as a geometry of points and lines in a plane, we do in reality treat these geometries respectively in an absolutely general manner. In particular—and I notice the case because I shall have occasion again to refer to it—we do in the geometry of two dimensions include spherical geometry; the words plane, point, and line, meaning for this purpose, spherical surface, arc (of a great circle) and point (that is, pair of opposite points) of the spherical surface. And in like manner the geometry of one dimension includes the cases of points on an arc, and of arcs through a point.

148. I repeat also a remark which is in effect made in the same No. 4; the coordinates x, y of the geometry of one dimension, and the coordinates x, y, z and ξ, η, ζ of the geometry of two dimensions are only determinate to a common factor *près* (that is, it is the ratios only of the coordinates, and not their absolute magnitudes, which are determinate); hence in saying that the coordinates x, y are equal to a, b, or in writing $x, y=a, b$, we mean only that $x:y=a:b$, and we never as a result obtain $x, y=a, b$, but only $x:y=a:b$. And the like with respect to the coordinates x, y, z and ξ, η, ζ. (In the geometry of two dimensions, $x, y=a, b$, is for this reason considered and spoken of as a single equation.) But when this is once understood, there is no objection to treating the coordinates as if they were completely determinate.

On Geometry of One Dimension, Nos. 149 to 168.

149. In geometry of one dimension we have the line as a space or *locus in quo*, which is considered as made up of points. The several points of the line are determined by the coordinates (x, y), viz. attributing to these any specific values, or writing $x, y=a, b$, we have a particular point of the line. And we may say also that the line is the *locus in quo* of the coordinates (x, y).

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* It would be more accurate to say that the word line may mean *point-in-and-with-a plane*, viz. the *locus in quo* of lines through the point and in the plane. Added, June 16, 1859.—A. C.

150. A linear equation,

$$(*\between x, y)^1=0,$$

is obviously equivalent to an equation of the before-mentioned form $x, y=a, b$, and represents therefore a point. An equation such as

$$(*\between x, y)^m=0$$

breaks up into m linear equations, and represents therefore a system of m points, or point-system of the order m. The component points of the system, or the linear factors, or the values thereby given for the coordinates, are termed roots. When $m=1$ we have of course a single point, when $m=2$ we have a quadric or point-pair, when $m=3$ a cubic or point-triplet, and so on. The point-system is the only figure or locus occurring in the geometry of one dimension. The quantic $(*\between x, y)^m$, when it is convenient to do so, may be represented by a single letter U, and we then have $\mathrm{U}=0$ for the equation of the point-system.

151. The equation

$$(*\between x, y)^m=0$$

may have two or more of its roots equal to each other; or generally there may exist any systems of equalities between the roots of the equation, or what is the same thing, the system may comprise two or more coincident points, or any systems of coincident points. In particular, when the discriminant vanishes the equation will have a pair of equal roots, or the system will comprise a pair of coincident points; in the case of the quadric $(a, b, c\between x, y)^2=0$, the condition is $ac-b^2=0$, or as it may be written, $a, b=b, c$; in the case of the cubic

$$(a, b, c, d\between x, y^3)=0,$$

the condition is

$$a^2d^2-6abcd+4ac^3+4b^3d-3b^2c^2=0.$$

The preceding is the only special case for a quadric: for a cubic we have besides the special case where the three roots are equal, or the cubic reduces itself to three coincident points; the conditions for this are

$$ac-b^2=0, \quad ad-bc=0, \quad bd-c^2=0,$$

equivalent to the two conditions

$$a:b=b:c=c:d.$$

For equations of a higher order the analytical question is considered, and as regards the quartic and the quintic respectively completely solved, in my "Memoir on the Conditions for the Existence of given Systems of Equalities between the Roots of an Equation*."

152. Any covariant of the equation

$$(*\between x, y)^m=0,$$

equated to zero, gives rise to a point-system connected in a definite manner with the original point-system. And as regards the invariants, the evanescence of any invariant implies a certain relation between the points of the system; the identical evanescence of any covariant implies relations between the points of the system, such that the derived point-system obtained by equating the covariant to zero is absolutely indeterminate. The like remarks apply to the covariants or invariants of two or more equations, and the point-systems represented thereby.

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* Philosophical Transactions, vol. cxlvii. (1857), pp. 727–31.

153. In particular, for the two point-pairs represented by the quadric equations

$$(a\,,\,b\,,\,c\between x,\,y)^2=0,$$
$$(a',\,b',\,c'\between x,\,y)^2=0,$$

if the lineo-linear invariant vanishes, that is, if

$$ac'-2bb'+ca'=0,$$

we have the harmonic relation,—the two point-pairs are said to be harmonically related to each other, or the two points of the one pair are said to be harmonics with respect to the two points of the other pair. The analytical theory is fully developed in the "Fifth Memoir upon Quantics[*]." The chief results, stated under a geometrical form, are as follows:—

1°. If either of the pairs and one point of the other pair are given, the remaining point of such other pair can be found.

2°. A point-pair can be found harmonically related to any two given point-pairs.

154. The last of the two theorems gives rise to the theory of involution. The two given point-pairs, viewed in relation to the harmonic pair, are said to be an involution of four points; and the points of the harmonic pair are said to be the (double or) sibiconjugate points of the involution. A system of three or more pairs, such that the third and every subsequent pair are each of them harmonically related to the sibiconjugate points of the first and second pairs, is said to be a system in involution. In particular, for three pairs we have what is termed an involution of six points; and it is clear that when two pairs and a point of the third pair are given, the remaining point of the third pair can be determined. And in like manner for a greater number of pairs, when two pairs and a point of each of the other pairs are given, the remaining point of each of the other pairs can be determined. Two points of the same pair are said to be conjugate to each other; or if we consider two pairs as given, then the points of the third or any subsequent pair are said to be conjugate to each other in respect to the given pairs. This explains the expression sibiconjugate points; in fact, the two pairs being given, either sibiconjugate point is, as the name imports, conjugate to itself. In other words, any two pairs and one of the sibiconjugate points considered as a pair of coincident points, form a system in involution, or involution of five points.

155. The three point-pairs, $U=0$, $U'=0$, $U''=0$, will be in involution when the quadrics U, U', U'' are connected by the linear relation or syzygy $\lambda U+\lambda' U'+\lambda'' U''=0$. This property, or the relation

$$\begin{array}{ccc} a, & b & \\ a', & b', & c' \\ a'', & b'', & c'' \end{array} =0$$

Philosophical Transactions, vol. cxlviii. (1858), pp. 429–462.

to which it gives rise, might have been very properly adopted as the definition of the relation of involution, but I have on the whole preferred to deduce the theory of involution from the harmonic relation. The notion, however, of the linear relation or syzygy of three or more point-systems gives rise to a much more general theory of involution, but this is a subject that I do not now enter upon; it may, however, be noticed, that if $U=0$, $U'=0$ be any two point-systems of the same order, then we may find a point-system $U''=0$ of the same order, in involution with the given point-systems (that is, satisfying the condition $\lambda U+\lambda' U'+\lambda'' U''=0$), and such that the point-system $U''=0$ comprises a pair of coincident points; this is obviously an extension of the notion of the sibiconjugate points of an ordinary involution.

156. It was remarked in the Fifth Memoir, that the theories of the anharmonic ratio and of homography belong analytically to the subject of bipartite (lineo-linear) binary quantics; this may be further illustrated geometrically as follows: we may imagine two distinct spaces of one dimension or lines, one of them the *locus in quo* of the coordinates (x, y), and the other the *locus in quo* of the coordinates (x, y), which are absolutely independent of, and are not in anywise related to, the coordinates of the first-mentioned system. There is no difficulty in the conception of this; for we may in a plane or in space of three dimensions imagine any two lines, and study the relations of analogy between the points of the one line *inter se*, and the points of the other line *inter se*, without in anywise adverting to the space of two or three dimensions which happens to be the common *locus in quo* of the two lines. It is proper to remark, that in speaking of the spaces of one dimension, which are the *loci in quibus* of the coordinates (x, y) and (x, y) respectively, as being each of them a line, we imply a restriction which is altogether unnecessary; the words line and point may, in regard to the two figures respectively, be used in different significations; for instance, one of the spaces may be a *line* and the points in it *points*; while the other of the spaces may be a *point* and the points in it *lines*, or it may be a *line* and the points in it *planes*.

157. A lineo-linear equation,

$$(x-ay)(\mathrm{x}-\alpha\mathrm{y})=0,$$

denotes then the two points $(x, y=a, 1)$ and $(\mathrm{x}, \mathrm{y}=\alpha, 1)$ existing irrespectively of each other in distinct spaces, and only by the equation itself brought into an ideal connexion; and any invariantive relation between the coefficients of any such bipartite functions denotes geometrically a relation between a point-system in the space which is the *locus in quo* of the coordinates (x, y), and a point-system in the space which is the *locus in quo* of the coordinates (x, y); for instance, the equation

$$\begin{array}{llll} 1, & a, & \alpha, & a\alpha \\ 1, & b, & \beta, & b\beta \\ 1, & c, & \gamma, & c\gamma \\ 1, & d, & \delta, & d\delta \end{array} =0$$

is the relation of homography between the four points $(a, 1)$, $(b, 1)$, $(c, 1)$, $(d, 1)$ in the first line, and the four points $(\alpha, 1)$, $(\beta, 1)$, $(\gamma, 1)$, $(\delta, 1)$ in the second line. The analytical theory is discussed in the Fifth Memoir; and, in particular, it is there shown, that writing

$$\begin{aligned} \mathrm{A}&=(d-a)(b-c), & \mathfrak{A}&=(\delta-\alpha)(\beta-\gamma), \\ \mathrm{B}&=(d-b)(c-a), & \mathfrak{B}&=(\delta-\beta)(\gamma-\alpha), \\ \mathrm{C}&=(d-c)(a-b), & \mathfrak{C}&=(\delta-\gamma)(\alpha-\beta), \end{aligned}$$

then the condition may be expressed under any one of the forms

$$\mathrm{A} : \mathrm{B} : \mathrm{C} = \mathfrak{A} : \mathfrak{B} : \mathfrak{C},$$

equations which denote the equality of the anharmonic ratios of the two point-systems.

158. The number of points in each system may be four, or any greater number; the homographic relation is then conveniently expressed under the form

$$\left\| \begin{matrix} 1\,, & 1\,, & 1\,, & 1\,, & 1, & \ldots \\ a\,, & b\,, & c\,, & d\,, & e\,, & \\ \alpha\,, & \beta\,, & \gamma\,, & \delta\,, & \varepsilon\,, & \\ a\alpha, & b\beta, & c\gamma, & d\delta, & e\varepsilon, & \end{matrix} \right\| = 0.$$

The relation is such that given three points of the one system and the corresponding three points of the other system, then to any fourth point whatever of the first system there can be found a corresponding fourth point of the second system. It is to be observed, however, that two systems of four points homographically related to each other, always correspond together in four different ways, viz. the two systems being (a, b, c, d) and $(\alpha, \beta, \gamma, \delta)$; then if the four points of the first system are (a, b, c, d), the corresponding four points of the second system may be taken in the four several orders, $(\alpha, \beta, \gamma, \delta)$, $(\beta, \alpha, \delta, \gamma)$, $(\gamma, \delta, \alpha, \beta)$, $(\delta, \gamma, \beta, \alpha)$.

159. What precedes is not to be understood as precluding the existence of a relation between the spaces which are the *loci in quibus* of the coordinates (x, y) and (x, y) respectively: not only may these be spaces of the same kind, but they may be one and the same space or line; and the points of the two systems may then be points of the same kind; and further, the coordinates (x, y) and (x, y) may belong to the same system of coordinates, that is, the equations $(x, y=a, 1)$ and $(\mathrm{x}, \mathrm{y}=a, 1)$ may denote one and the same point.

160. If the two point-systems are systems of the same kind, and are in one and the same line, then there are in general two points of the first system which coincide each of them with the corresponding point of the second system; such two points may be said to be the sibiconjugate points of the homography. In particular, the two sibiconjugate points of the homography may coincide together.

161. A system in involution affords an example of two homographic systems in the same line; in fact, taking arbitrarily a point out of each pair, the points so obtained form a system which is homographic with the system formed with the other points of

the several pairs; and in this case the sibiconjugate points of the involution are also the sibiconjugate points of the homography. Thus if A and A′, B and B′, C and C′, D and D′ are pairs of the system in involution, then (A, B, C, D) and (A′, B′, C′, D′) will be homographic point-systems; and, as a particular case, (A, B, C, C′) and (A′, B′, C′, C) will be homographic point-systems. It is proper to notice that if F is a sibiconjugate point of the involution, then (A, B, F, F) and (A′, B′, F, F) are not (what at first sight they appear to be) homographic point-systems.

162. Imagine an involution of points; take on the line which is the *locus in quo* of the point-system a point O, and consider the point-system formed by the harmonics of O in respect to the several pairs of the involution; and in like manner take on the line any other point O′, and consider the point-system formed by the harmonics of O′ in respect to the several pairs of the involution; these two point-systems are homographically related to each other.—See Fifth Memoir, No. 111.

163. Two involutions may be homographically related to each other; in fact, take on the line which is the *locus in quo* of the first involution a point O, and consider the point-system formed by the harmonics of O in relation to the several pairs of the involution; take in like manner on the line which is the *locus in quo* of the second involution a point Q, and consider the point-system formed by the harmonics of Q with respect to the several pairs of the involution; then if the two point-systems are homographically related, the two involutions are said to be themselves homographically related: the last preceding article shows that the nature of the relation does not in anywise depend on the choice of the points O and Q. And it is not necessary that, as regards the two involutions respectively, the words line and point should have the same significations.—See Fifth Memoir, No. 111.

164. Four or more tetrads of points in a line may be homographically related to the same number of tetrads in another line. This is the case when the anharmonic ratios of the tetrads of the first system are homographically related to the anharmonic ratios of the tetrads of the second system. And it is not material which of the three anharmonic ratios of a tetrad of either system is selected, provided that the same selection is made for each of the other tetrads of the same system. The order of the points of a tetrad must be attended to, but there are in all four admissible permutations of the points of a tetrad, viz. if A, B, C, D are the points of a tetrad, then (A, B, C, D), (B, A, D, C), (C, D, A, B), (D, C, B, A) may be considered as one and the same tetrad. Any three tetrads whatever in the second system may correspond to any three tetrads of the first system; and then given a fourth tetrad of the first system, and three out of the four points of the corresponding tetrad of the second system, the remaining point of the tetrad may be determined. The words line and point need not, as regards the two systems of tetrads respectively, be understood in the same significations.—See Fifth Memoir, No. 112.

165. The foregoing theory of the harmonic relation shows that if we have a point-pair

$$(a, b, c \,)(\, x, y)^2=0,$$

the equation of any other point-pair whatever can be expressed, and that in two different ways, in the form

$$(a, b, c \between x, y)^2+(lx+my)^2=0\,;$$

the points $(lx+my=0)$ corresponding to the two admissible values of the linear function being in fact the harmonics of the point-pair in respect to the given point-pair $(a, b, c \between x, y)^2=0$, or what is the same thing, the sibiconjugate points of the involution formed by the two point-pairs (see Fifth Memoir, No. 105). The point-pair represented by the equation in question does not in itself stand in any peculiar relation to the given point pair $(a, b, c \between x, y)^2=0$; but when thus represented it is said to be inscribed in the given point-pair, and the point $lx+my=0$ is said to be the axis of inscription. And the harmonic of this point with respect to the given point-pair (that is, the other sibiconjugate point of the involution of the two point-pairs) is said to be the centre of inscription*.

166. We may, if we please, (x', y') and θ being constants, exhibit the equation of the inscribed point-pair in the form

$$(a, b, c \between x, y)^2(a, b, c \between x', y')^2 \sin^2\theta-(ac-b^2)(xy'-x'y)^2=0,$$

where we have for the axis of inscription and centre of inscription respectively, the equations

$$\begin{aligned} xy'-x'y \quad &=0,\\ (a, b, c \between x, y \between x', y')&=0. \end{aligned}$$

Or in the equivalent form,

$$(a, b, c \between x, y)^2(a, b, c \between x', y')^2 \cos^2\theta-\{(a, b, c \between x, y \between x', y')\}^2=0,$$

where we have for the axis of inscription and the centre of inscription respectively, the equations

$$\begin{aligned} (a, b, c \between x, y \between x', y')&=0,\\ xy'-x'y \quad &=0. \end{aligned}$$

167. The equivalence of the two forms depends on the identical equation

$$(a, b, c \between x, y)^2(a, b, c \between x', y')^2-\{(a, b, c \between x, y \between x', y')\}^2=(ac-b^2)(xy'-x'y)^2,$$

which is in fact the equation mentioned, Fifth Memoir, No. 95. If, for shortness, we write

$$\begin{aligned} (a, b, c \between x, y)^2&=00,\\ (a, b, c \between x, y \between x', y')&=01=10,\\ &\&c., \end{aligned}$$

then the equation may be represented in the form

$$\begin{matrix} 00, & 01 \\ 10, & 11 \end{matrix} =(ac-b^2) \begin{matrix} x, & y \\ x, & y' \end{matrix}$$

168. There is a like equation for the three sets (x, y), (x', y'), (x'', y''); the right-hand side here vanishes, for there are not columns enough to form therewith a determinant,

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[*] The words inscribed, inscription, are used not in opposition to, but as identical with, the words circumscribed, circumscription; and in like manner (*post*, Nos. 203 et seq.) as regards conics.

and the equation is

$$\begin{matrix} 00, & 01, & 02 \\ 10, & 11, & 12 \\ 20, & 21, & 22 \end{matrix} = 0,$$

an equation which may also be written in the form

$$\cos^{-1}\frac{01}{\sqrt{00}\sqrt{11}}+\cos^{-1}\frac{12}{\sqrt{11}\sqrt{22}}=\cos^{-1}\frac{02}{\sqrt{00}\sqrt{22}},$$

as it is easy to verify by reducing this equation to an algebraical form. The various formulæ have been given in relation to the establishment of the notion of distance in the geometry of one dimension, but it will be convenient to defer the consideration of this theory so as to discuss it in connexion with geometry of two dimensions.

On Geometry of Two Dimensions, Nos. 169 to 208.

169. In geometry of two dimensions we have the plane as a space or *locus in quo*, which is considered under two distinct aspects, viz. as made up of points, and as made up of lines. The several points of the plane are determined by means of the point-coordinates (x, y, z), viz. attributing to these any specific values, or writing $x, y, z=a, b, c$, we have a particular point of the plane; and in like manner the several lines of the plane are determined by the line-coordinates (ξ, η, ζ), viz. attributing to these any specific values, or writing $\xi, \eta, \zeta=\alpha, \beta, \gamma$, we have a particular line of the plane. And we may say that the plane is the *locus in quo* of the point-coordinates (x, y, z), and of the line-coordinates (ξ, η, ζ). It is not necessary to consider separately the analytical theories of point-coordinates and of line-coordinates; for the theory of the former in relation to points and lines respectively is identical with the theory of the latter in relation to lines and points respectively; but it is necessary to show how either system of coordinates, say the system of point-coordinates, is applicable to both points and lines, or in fact all loci whatever, and to explain the mutual relation of the two systems of coordinates.

170. Considering then point-coordinates, the equations

$$x, y, z=a, b, c,$$

determine, as already mentioned, a point.

A linear equation,

$$(*\between x, y, z)^1=0,$$

determines a line, viz. the line which is the locus of all the points, the coordinates of which satisfy this equation. And in like manner an equation

$$(*\between x, y, z)^m=0$$

determines a curve of the mth order, viz. the curve which is the locus of all the points, the coordinates of which satisfy this equation. In particular, an equation of the second degree,

$$(*\between x, y, z)^2=0,$$

determines a conic.

171. If the quantic breaks up into rational factors, then the equation of the curve is satisfied by equating to zero any one of these factors, or the curve breaks up into curves of a lower order, and the order of the entire curve is equal to the sum of the orders of the component curves. In particular, a curve of any order may break up into a system of lines, the number of lines being of course equal to the order of the curve, and any two or more of these lines may coincide with each other. A curve not thus breaking up into curves of a lower order is said to be a proper curve.

172. Returning to the linear equation and expressing the coefficients, the equation is

$$(\xi, \eta, \zeta \between x, y, z)=0,$$

or, what is the same thing,

$$\xi x+\eta y+\zeta z=0;$$

and we say as a definition, that the coordinates (line-coordinates) of this line are (ξ, η, ζ).

173. But the same equation, considering (x, y, z) as constant coefficients, and (ξ, η, ζ) as line-coordinates, is the equation of a point, viz. the point which is the locus (envelope) of all those points the coordinates of which satisfy the equation in question; and such point is precisely the point, the coordinates (point-coordinates) of which are (x, y, z). In fact, if (ξ, η, ζ) are considered as variable parameters connected by the equation $\xi x+\eta y+\zeta z=0$, then taking (X, Y, Z) as current point-coordinates, the equation $\xi X+\eta Y+\zeta Z=0$ is satisfied by writing (x, y, z) for (X, Y, Z); or the several lines the coordinates whereof are (ξ, η, ζ), all pass through the point (x, y, z).

174. Hence recapitulating, the equation

$$(\xi, \eta, \zeta \between x, y, z)=0,$$

or

$$\xi x+\eta y+\zeta z \quad =0,$$

considering (x, y, z) as current point-coordinates, and (ξ, η, ζ) as constant coefficients, is the equation of a line the coordinates (line-coordinates) of which are (ξ, η, ζ); and the same equation, considering (ξ, η, ζ) as current line-coordinates, and (x, y, z) as constant coefficients, is the equation of a point the coordinates (point-coordinates) of which are (x, y, z).

175. The expression, the point (a, b, c), means the point whose point-coordinates are (a, b, c); and in like manner the expression, the line (α, β, γ), means the line whose line-coordinates are (α, β, γ). The last-mentioned expression may, without any impropriety or risk of ambiguity, be employed when we are dealing with point-coordinates; but it is of course always allowable, and it is frequently better, to substitute for the definition the thing signified, and say the line having for its equation $\alpha x+\beta y+\gamma z=0$, or more briefly, the line $\alpha x+\beta y+\gamma z=0$. It will be proper to do this in the following articles, Nos. 176 to 184, which contain some formulæ in point-coordinates relating to the theory of the point and the line.

176. The condition that the point (a, b, c) may lie in the line

$$\alpha x+\beta y+\gamma z=0,$$

is of course

$$\alpha a+\beta b+\gamma c=0.$$

177. The equation of the line passing through the points (a, b, c), (a', b', c'), is

$$\left|\begin{array}{ccc} x, & y, & z \\ a, & b, & c \\ a', & b', & c' \end{array}\right|=0;$$

and if in this equation (a', b', c') are considered as indeterminate, we have the equation of a line subjected to the single condition of passing through the point (a, b, c). The equation contains apparently two arbitrary parameters, but these in fact reduce themselves to a single one.

178. The coordinates of the point of intersection of the lines

$$\alpha x+\beta y+\gamma z=0,$$
$$\alpha' x+\beta' y+\gamma' z=0,$$

are given by the equations

$$x, y, z=\beta\gamma'-\beta'\gamma,\quad \gamma\alpha'-\gamma'\alpha,\quad \alpha\beta'-\alpha'\beta;$$

and if in these equations we consider α', β', γ' as indeterminate, we have the coordinates of a point subjected to the single condition of lying in the line $\alpha x+\beta y+\gamma z=0$; the result, as in the last case, contains in appearance two arbitrary parameters, but these really reduce themselves to a single one.

179. The condition in order that the points (a, b, c), (a', b', c'), (a'', b'', c'') may lie in a line is

$$\left|\begin{array}{ccc} a, & b, & \\ a', & b', & c' \\ a'', & b'', & c'' \end{array}\right|=0,$$

which may also be expressed by the equations

$$a'', b'', c''=\lambda a+\mu a',\quad \lambda b+\mu b',\quad \lambda c+\mu c',$$

where λ, μ are arbitrary multipliers; these equations give therefore the coordinates of an indeterminate point in the line joining the points (a, b, c) and (a', b', c').

180. The condition that the lines

$$\alpha x+\beta y+\gamma z=0,$$
$$\alpha' x+\beta' y+\gamma' z=0,$$
$$\alpha'' x+\beta'' y+\gamma'' z=0$$

may meet in a point is

$$\left|\begin{array}{ccc} \alpha, & \beta, & \gamma \\ \alpha', & \beta', & \gamma' \\ \alpha'', & \beta'', & \gamma'' \end{array}\right|=0,$$

a relation which may also be expressed by the equations

$$\alpha'',\ \beta'',\ \gamma''=l\alpha+m\alpha',\quad l\beta+m\beta',\quad l\gamma+m\gamma',$$

where l, m are arbitrary multipliers; and substituting these values in the equation $\alpha''x+\beta''y+\gamma''z=0$, we have for the equation of a line subjected to the single condition of passing through the point of intersection of the lines $\alpha x+\beta y+\gamma z=0$, $\alpha'x+\beta'y+\gamma'z=0$, the equation

$$l(\alpha x+\beta y+\gamma z)+m(\alpha'x+\beta'y+\gamma'z)=0,$$

which is, in fact, at once obtained by the consideration that the values of (x, y, z) which satisfy simultaneously the equations $\alpha x+\beta y+\gamma z=0$ and $\alpha'x+\beta'y+\gamma'z=0$, satisfy also the equation in question.

181. The equation of the line passing through the point of intersection of the lines $\alpha x+\beta y+\gamma z=0$ and $\alpha'x+\beta'y+\gamma'z=0$, and also through the point (a, b, c), is obviously

$$\left.\begin{array}{ll} \alpha x+\beta y+\gamma z, & \alpha'x+\beta'y+\gamma'z \\ \alpha a+\beta b+\gamma c, & \alpha'a+\beta'b+\gamma'c \end{array}\right|=0,$$

which, or the equivalent form

$$\frac{\alpha x+\beta y+\gamma z}{\alpha a+\beta b+\gamma c}=\frac{\alpha'x+\beta'y+\gamma'z}{\alpha'a+\beta'b+\gamma'c},$$

is usually the most convenient one; but it is to be observed that the equation can also be written in the forms

$$\begin{array}{ccc} x\ , & y\ , & z \\ a\ , & b\ , & c \\ \beta\gamma'-\beta'\gamma, & \gamma\alpha'-\gamma'\alpha, & \alpha\beta'-\alpha'\beta \end{array}=0,$$

and

$$\begin{array}{ccc} bz-cy, & cx-az, & ay-bx \\ \alpha & \beta\ , & \gamma \\ \alpha' & \beta'\ , & \gamma' \end{array}=0,$$

or in the form

$$(\beta\gamma'-\gamma\beta')(bz-cy)+(\gamma\alpha'-\gamma'\alpha)(cx-az)+(\alpha\beta'-\beta\alpha')(ay-bx)=0,$$

which might also be represented by

$$\begin{array}{cccccc} \alpha, & \beta, & \gamma & x, & y, & z \\ \alpha', & \beta', & \gamma' & a, & b, & c \end{array}=0.$$

182. To find the coordinates of the point of intersection of the line joining the points (a, b, c), (a', b', c'), with the line $\alpha x+\beta y+\gamma z=0$, we have

$$x,\ y,\ z=\lambda a+\mu a',\quad \lambda b+\mu b',\quad \lambda c+\mu c',$$

where λ, μ are given by

$$\lambda(\alpha a+\beta b+\gamma c)+\mu(\alpha a'+\beta b'+\gamma c')=0.$$

The preceding are elementary formulæ of almost constant occurrence; it may be proper to add to them the formulæ which follow.

183. To find the equation of the line passing through the point of intersection of the lines

$$\alpha_1x+\beta_1y+\gamma_1z=0,\ \ \alpha_2x+\beta_2y+\gamma_2z=0,$$

and the point of intersection of the lines

$$\alpha_3x+\beta_3y+\gamma_3z=0,\ \ \alpha_4x+\beta_4y+\gamma_4z=0.$$

Write for shortness $u_1=\alpha_1x+\beta_1y+\gamma_1z$, &c.; then we have identically

$$\begin{matrix} u_1, & u_2, & u_3, & u_4 \\ \alpha_1, & \alpha_2, & \alpha_3, & \alpha_4 \\ \beta_1, & \beta_2, & \beta_3, & \beta_4 \\ \gamma_1, & \gamma_2, & \gamma_3, & \gamma_4 \end{matrix}\ =0,$$

and the two equations

$$\begin{matrix} u_1, & u_2, & & \\ \alpha_1, & \alpha_2, & \alpha_3, & \alpha_4 \\ \beta_1, & \beta_2, & \beta_3, & \beta_4 \\ \gamma_1, & \gamma_2, & \gamma_3, & \gamma_4 \end{matrix}\ =0, \qquad \begin{matrix} & & u_3, & u_4 \\ \alpha_1, & \alpha_2, & \alpha_3, & \alpha_4 \\ \beta_1, & \beta_2, & \beta_3, & \beta_4 \\ \gamma_1, & \gamma_2, & \gamma_3, & \gamma_4 \end{matrix}\ =0$$

are consequently equivalent to each other, and each of them represents the required line. It is easy to deduce the form

$$x\ \begin{matrix} \alpha_1, & \alpha_2 & & \\ & & \alpha_3, & \alpha_4 \\ \beta_1, & \beta_2, & \beta_3, & \beta_4 \\ \gamma_1, & \gamma_2, & \gamma_3, & \gamma_4 \end{matrix}\ +y\ \begin{matrix} \beta_1, & \beta_2, & . & \\ \alpha_1, & \alpha_2, & \alpha_3 & \\ ., & ., & \beta_3, & \beta_4 \\ \gamma_1, & \gamma_2, & \gamma_3, & \gamma_4 \end{matrix}\ +z\ \begin{matrix} \gamma_1, & \gamma_2, & & \\ \alpha_1, & \alpha_2, & \alpha_3 & \\ \beta_1, & \beta_2, & \beta_3, & \beta_4 \\ & & \gamma_3, & \gamma_4 \end{matrix}\ =0.$$

184. The condition in order that the points of intersection of the lines $u_1=0$, $u_2=0$, of the lines $u_3=0$, $u_4=0$, and of the lines $u_5=0$, $u_6=0$ (where, as before, u_1 denotes $\alpha_1x+\beta_1y+\gamma_1z$, &c.) may lie in the same line, is

$$\begin{matrix} \alpha_1, & \alpha_2, & \alpha_3, & \alpha_4, & ., & . \\ \beta_1, & \beta_2, & \beta_3, & \beta_4, & ., & . \\ \gamma_1, & \gamma_2, & \gamma_3, & \gamma_4, & ., & . \\ ., & ., & \alpha_3, & \alpha_4, & \alpha_5, & \alpha_6 \\ ., & ., & \beta_3, & \beta_4, & \beta_5, & \beta_6 \\ ., & ., & \gamma_3, & \gamma_4, & \gamma_5, & \gamma_6 \end{matrix}\ \Bigg|=0,$$

which is of course really symmetrical with respect to the six sets. The last formula was given by me, 'Cambridge Mathematical Journal,' t. iv. p. 18 (1849).

185. Instead of the term point of a curve, it will be convenient to use the term 'ineunt' of the curve.

The line through two consecutive ineunts of the curve is the tangent at the ineunt. The point of intersection of two consecutive tangents is the ineunt on the tangent.

The equation of a curve in point-coordinates, or as it may be termed the point-equation of the curve, is the relation which exists between the point-coordinates of any ineunt of the curve.

The equation of a curve in line-coordinates, or line-equation of the curve, is the relation which exists between the line-coordinates of any tangent of the curve.

186. It has been mentioned, that the order of a curve is the degree of its point-equation: in like manner the class of a curve is the degree of its line-equation; and in the same way that a curve, as represented by a point-equation, may break up into curves having the order of the entire curve for the sum of their orders, so a curve as represented by a line-equation may break up into curves having the class of the entire curve for the sum of their classes. And, in particular, a curve may break up into a system of points, the number of points being equal to the class of the curve, and two or more of these points may coincide together.

187. A line is a curve of the order one and class zero; a point is a curve of the order zero and class one. A proper conic is a curve of the order two and class two; but when the conic breaks up into a pair of lines, the class sinks to zero; and when the conic breaks up into a pair of points, the order sinks to zero. It is to be noticed that a point, or system of points, cannot be represented by an equation in point-coordinates, nor a line or system of lines by an equation in line-coordinates. We may say, in general, that a curve is both a point-curve and a line-curve, but a point or system of points is a line-curve only, and a line or system of lines is a point-curve only.

188. The points of intersection (common ineunts) of two curves are the points the coordinates of which satisfy simultaneously the point-equations of the two curves. Hence the number of common ineunts is equal to the product of the orders of the two curves; and, in particular if one of the curves be a line, the number of points of intersection (common ineunts) is equal to the order of the curve. In like manner the common tangents of the two curves are the lines the coordinates of which satisfy simultaneously the line-equations of the two curves. Hence the number of common tangents is equal to the product of the classes of the two curves; and, in particular, if one of the curves be a point, the number of common tangents (tangents to the curve through the point) is equal to the class of the curve. Since the tangent is the line through two consecutive ineunts, it besides meets the curve, assumed to be of the order m in $(m-2)$ points; and in like manner we may from any ineunt of a curve of the class n draw $(n-2)$ tangents to the curve.

189. The point-equation of a line passing through the points (x', y', z') and (x'', y'', z'') is, as already noticed,

$$\left.\begin{matrix} x, & y, & z \\ x', & y', & z' \\ x'', & y'', & z'' \end{matrix}\right| = 0.$$

Suppose that (x, y, z) are the coordinates of a point (ineunt) of the curve $U=0$, the coordinates of the consecutive ineunt will be $(x+dx, y+dy, z+dz)$, and the line joining these two points will be the tangent to the curve at the point (x, y, z). Take (X, Y, Z) as current point-coordinates, the equation of the tangent is

$$\left.\begin{array}{lll} X\;\;, & Y\;\;, & Z \\ x\;\;, & y\;\;, & z \\ x+dx, & y+dy, & z+dz \end{array}\right|=0,$$

or what is the same thing,

$$X(ydz-zdy)+Y(zdx-xdz)+Z(xdy-ydx)=0.$$

But since U is a homogeneous function of (x, y, z), we have

$$x\partial_x U+y\partial_y U+z\partial_z U=mU=0\,;$$

and since $(x+dx, y+dy, z+dz)$ is a point of the curve, we have

$$dx\partial_x U+dy\partial_y U+dz\partial_z U=0\,;$$

and from these two equations

$$ydz-zdy : zdx-xdz : xdy-ydx=\partial_x U : \partial_y U : \partial_z U,$$

and the equation of the tangent consequently is

$$X\partial_x U+Y\partial_y U+Z\partial_z U=0.$$

190. Take (ξ, η, ζ) as the line-coordinates of the tangent, then the equation of the tangent is

$$\xi X+\eta Y+\zeta Z=0\,;$$

and comparing the two forms, we have

$$\xi : \eta : \zeta=\partial_x U : \partial_y U : \partial_z U\,;$$

and if from these equations and the equation $U=0$ (the point-equation of the curve) we eliminate (x, y, z), we obtain an equation between (ξ, η, ζ), which is the line-equation of the curve. We may, if we please, present the system under the form

$$\begin{aligned} \partial_x U+\lambda\xi&=0, \\ \partial_y U+\lambda\eta&=0, \\ \partial_z U+\lambda\zeta&=0, \\ U=\;\;\;\;&\,0, \end{aligned}$$

or what is more simple, under the form

$$\begin{aligned} \partial_x U+\lambda\xi&=0, \\ \partial_y U+\lambda\eta&=0, \\ \partial_z U+\lambda\zeta&=0, \\ \xi x+\eta y+\zeta z&=0, \end{aligned}$$

and from either system eliminate x, y, z and λ.

191. If the point-equation of a conic be

$$(a, b, c, f, g, h \between x, y, z)^2 = 0,$$

then its line-equation is

$$\begin{array}{cccc} & & \eta, & \zeta \\ \xi & a, & h, & g \\ \eta & h, & b, & f \\ \zeta & g, & f, & c \end{array} = 0,$$

or writing

$$\begin{aligned} \mathfrak{A} &= bc - f^2, \\ \mathfrak{B} &= ca - g^2, \\ \mathfrak{C} &= ab - h^2, \\ \mathfrak{F} &= gh - af, \\ \mathfrak{G} &= hf - bg, \\ \mathfrak{H} &= fg - ch, \end{aligned}$$

and to complete the system,

$$\mathrm{K} = abc - af^2 - bg^2 - ch^2 + 2fgh,$$

then the line-equation of the conic is

$$(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H} \between \xi, \eta, \zeta)^2 = 0.$$

192. The quantities $\mathfrak{A}$, &c. satisfy the relations

$$\mathrm{K}^2 = \mathfrak{A}\mathfrak{B}\mathfrak{C} - \mathfrak{A}\mathfrak{F}^2 - \mathfrak{B}\mathfrak{G}^2 - \mathfrak{C}\mathfrak{H}^2 + 2\mathfrak{F}\mathfrak{G}\mathfrak{H},$$

$$\begin{array}{lll} \mathfrak{A}a + \mathfrak{H}h + \mathfrak{G}g = \mathrm{K}, & \mathfrak{H}a + \mathfrak{B}h + \mathfrak{F}g = 0, & \mathfrak{G}a + \mathfrak{F}h + \mathfrak{C}g = 0, \\ \mathfrak{A}h + \mathfrak{H}b + \mathfrak{G}f = 0, & \mathfrak{H}h + \mathfrak{B}b + \mathfrak{F}f = \mathrm{K}, & \mathfrak{G}h + \mathfrak{F}b + \mathfrak{C}f = 0, \\ \mathfrak{A}g + \mathfrak{H}f + \mathfrak{G}c = 0, & \mathfrak{H}g + \mathfrak{B}f + \mathfrak{F}c = 0, & \mathfrak{G}g + \mathfrak{F}f + \mathfrak{C}c = \mathrm{K}, \end{array}$$

and moreover

$$\begin{array}{ll} \mathrm{K}a = \mathfrak{B}\mathfrak{C} - \mathfrak{F}^2, & \mathrm{K}f = \mathfrak{G}\mathfrak{H} - \mathfrak{A}\mathfrak{F}, \\ \mathrm{K}b = \mathfrak{C}\mathfrak{A} - \mathfrak{G}^2, & \mathrm{K}g = \mathfrak{H}\mathfrak{F} - \mathfrak{B}\mathfrak{G}, \\ \mathrm{K}c = \mathfrak{A}\mathfrak{B} - \mathfrak{H}^2, & \mathrm{K}h = \mathfrak{F}\mathfrak{G} - \mathfrak{C}\mathfrak{H}. \end{array}$$

193. A system of points in a line is said to be a range, and a system of lines through a point is said to be a pencil. The theories of ranges and pencils, considered irrespectively of each other, are in fact a single theory, constituting the geometry of one dimension. It has been seen how in geometry of one dimension a range of points and a pencil of lines, although considered (as they must be considered) as existing in distinct spaces, may nevertheless stand in certain relations to each other. In geometry of two dimensions, the range and pencil may of course coexist in one and the same plane as their common *locus in quo*; and such coexistence occurs in fact very frequently: thus if we

have a line and a point, and if lines are drawn joining the point with the several points of the line, these lines constitute a pencil, and the points of the line constitute a range, and such pencil and range are homographically related.

194. The theory of homography in geometry of two dimensions may be made to depend upon the corresponding theory in geometry of one dimension, or what is the same thing, upon the homography of ranges or pencils. For consider two figures existing in distinct planes or spaces of two dimensions, any four points (not in a line) in the second figure may correspond to any four points (not in a line) in the first figure; and when this is so, we may, by the process about to be explained, given any other point of the first figure, construct the corresponding point of the second figure; and the two figures are then, by definition, homographically related. Suppose that the points A′, B′, C′, D′ of the second figure correspond respectively to the points A, B, C, D of the first figure, and let E be any other point of the first figure; suppose that E′ is the corresponding point of the second figure; the pencils AB, AC, AD, AE and A′B′, A′C′, A′D′, A′E′ should be homographic to each other, that is, E′ must lie on a given line through A′; and in like manner the pencils BA, BC, BD, BE and B′A′, B′C′, B′D′, B′E′ should be homographic to each other, that is, E′ must lie on a given line through B′. And then, as a theorem, CA, CB, CD, CE and C′A′, C′B′, C′D′, C′E′, or DA, DB, DC, DE and D′A′, D′B′, D′C′, D′E′ will be homographic pencils, that is, the construction will be a determinate one whichever two of the four points are selected for the points A and B. The foregoing construction leads to an analytical relation, which I think constitutes a better foundation of the theory. Consider the first plane as the *locus in quo* of the coordinates (x, y, z), and the second plane as the *locus in quo* of the coordinates (X, Y, Z), these two coordinate systems being absolutely independent of each other. Consider any point of the first plane and a corresponding point of the second plane such that its coordinates (X, Y, Z) are given linear functions of the coordinates (x, y, z) of the point in the first plane. Any figure whatever in the first plane gives rise to a corresponding figure in the second plane, and the two figures are said to be homographic to each other. To a point of the first figure there corresponds in the second figure a point, to a line a line, to a range of points or pencil of lines, a homographic range of points or pencil of lines; the line or point which is the *locus in quo* of the range or pencil in the one figure corresponding with the line or point which is the *locus in quo* of the range or pencil in the other figure. And generally, to any curve of any order and class in the first figure, and to its ineunts and tangents, there correspond in the second figure a curve of the same order and class, and the ineunts and tangents of such curve.

195. It is to be remarked, that it is not by any means necessary that the word or the words plane, point, and line, or consequently the words order and class, should have the same significations as regards the two figures respectively. The theory of homography, as above explained, in fact comprises what is commonly termed the theory of homography and also the theory of reciprocity.

196. Let the word plane have the ordinary signification as regards the two figures

respectively; and Suppose, first, that the words point and line, and therefore order and class, have also the ordinary significations as regards the two figures respectively: we have here the ordinary theory of homography, in which, to any range of points or pencil of lines in the first figure, there corresponds a homographic range of points or pencil of lines in the second figure, and to a curve of any order and class in the first figure there corresponds a curve of the same order and class in the second figure.

197. We may, as a specialization giving rise to further developments, assume that the two figures exist in one and the same plane. There is here in general a triangle, each of whose angles or sides, considered as a point or line in the first figure, corresponds to itself, considered as a point or line in the second figure: such triangle may be called the sibiconjugate triangle. Any one point of the plane, considered as belonging to the first figure, may correspond to any other point of the plane, considered as belonging to the second figure, and the second figure can be completely constructed by means of the sibiconjugate triangle and such pair of corresponding points. In certain special cases the sibiconjugate triangle becomes wholly or in part indeterminate; thus if the two figures are identical, each point of the plane, considered as belonging to the first figure, coincides with itself, considered as belonging to the second figure. But I reserve the further discussion of the theory of homography for another occasion.

198. Suppose, secondly, that in the foregoing general theory, as regards the first figure, the words point and line, and therefore order and class, signify *point* and *line*, *order* and *class*; while as regards the second figure, the words point and line signify *line* and *point* respectively, and therefore the words order and class, *class* and *order* respectively. We have in the present case the ordinary theory of reciprocity, viz. using all the words in the same significations as regards the two figures respectively; to a point in the first figure there corresponds in the second figure a line; to a line, a point; to a range of points or pencil of lines, a pencil of lines or range of points; to a curve of any order and class, and its ineunts and tangents, a curve of the same class and order, and the tangents and ineunts of such curve.

199. As a specialization giving rise to further developments, we may assume that the two figures exist in one and the same plane. In this case, the points which, considered indifferently as belonging to the first or the second figure, lie upon the corresponding lines in the second or first figure, generate a conic which may be termed the pole-conic; and the lines which, considered indifferently as belonging to the first or the second figure, pass through the corresponding points in the second or first figure, envelope a conic which may be termed the polar-conic, and these two conics have double contact with another. The further consideration of this subject is reserved for another occasion; but I remark that in the particular case where the two conics coincide, we have the ordinary theory of poles and polars in regard to a conic; a theory, which, in a different point of view, may be considered as arising out of the harmonic relation, and which must here be noticed.

200. Consider a conic and a point; any line through the point meets the conic in two

points (ineunts of the conic), and the harmonic in relation to these two points of the given point has for its locus a line which is the polar of the given point. The polar passes through the points of contact of the conic with the tangents through the given point.

In like manner considering the conic and a line; from any point of the line we may draw two tangents to the conic, and the harmonic of the given line with respect to the two tangents envelopes a point which is the pole of the given line. The pole is the point of intersection of the tangents of the conic at the points of intersection with the given line.

The polars of the several points of a line envelope a point which is the pole of the line; and the poles of the several lines through a point generate a line which is the polar of the point; and this proposition shows how the theory of poles and polars gives rise to a theory of reciprocity.

201. If the point-equation of a conic be

$$(a, b, c, f, g, h)(x, y, z)^2=0,$$

the point-equation of the polar with respect to this conic of the point (x', y', z') is

$$(a, b, c, f, g, h)(x, y, z)(x', y', z)=0.$$

But it has been seen that the line-equation of the same conic is

$$(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\xi, \eta, \zeta)^2=0,$$

and the line-equation of the pole with respect to this conic of the line (ξ', η', ζ') (that is, the line whose point-equation is $\xi'x+\eta'y+\zeta'z=0$) is

$$(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\xi, \eta, \zeta)(\xi', \eta', \zeta')=0,$$

in other words, the point-coordinates of the pole are

$$\mathfrak{A}\xi'+\mathfrak{H}\eta'+\mathfrak{G}\zeta', \quad \mathfrak{H}\xi'+\mathfrak{B}\eta'+\mathfrak{F}\zeta', \quad \mathfrak{G}\xi'+\mathfrak{F}\eta'+\mathfrak{C}\zeta'.$$

202. If $U=0$, $V=0$ be the point-equations of any two curves of the same order, then λ, μ being arbitrary coefficients,

$$\lambda U+\mu V=0$$

is the equation of a curve of the same order passing through the points of intersection (common ineunts) of the two curves; such curve is said to be in involution with the given curves. The discussion of the general theory of involution is reserved for another occasion.

203. In particular, if $U=0$ be the equation of a conic, and $P=0$, $Q=0$ the equations of two lines, then

$$U+\lambda PQ=0$$

is the equation of a conic passing through the points of intersection of the conic with the two lines; and if the two lines coincide, then

$$U+\lambda P^2=0$$

is the equation of a conic having double contact with the conic $U=0$ at its points of intersection with the line $P=0$. Such conic is said to be inscribed in the conic $U=0$;

the line $P=0$ is the axis of inscription; this line has the same pole with respect to each of the two conics, and the pole is termed the centre of inscription: the relation of the two conics is completely expressed by saying that the four common ineunts coincide in pairs upon the axis of inscription, and that the four common tangents coincide in pairs through the centre of inscription; it is consequently a similar relation in regard to ineunts and tangents respectively; and it is to be inferred *à priori*, that if $\Upsilon=0$ be the line-equation of the conic $U=0$, and $\Pi=0$ the line-equation of the centre of inscription, then the line-equation of the inscribed conic is $\Upsilon+\mu\Pi^2=0$.

204. To verify this, I remark that if the equation of the axis of inscription be

$$\xi' x+\eta' y+\zeta' z=0,$$

then (*ante*, No. 201) we have for the line-equation of the centre of inscription

$$\Pi=(\mathfrak{A},\ldots)(\xi,\eta,\zeta)(\xi',\eta',\zeta')=0.$$

The line-equation of the inscribed conic is in the first instance obtained in the form

$$(\mathfrak{A},\ldots)(\xi,\eta,\zeta)^2+\lambda(a,\ldots)(\eta\zeta'-\eta'\zeta,\ \zeta\xi'-\zeta'\xi,\ \xi\eta'-\xi'\eta)^2=0\,;$$

but we have identically,

$$(\mathfrak{A},\ldots)(\xi,\eta,\zeta)^2(\mathfrak{A},\ldots)(\xi',\eta',\zeta')^2-\{(\mathfrak{A},\ldots)(\xi,\eta,\zeta)(\xi',\eta',\zeta')\}^2=K(a,\ldots)(\eta\zeta'-\eta'\zeta,\zeta\xi'-\zeta'\xi,\xi\eta'-\xi'\eta)^2,$$

and the equation thus becomes

$$[K+\lambda(\mathfrak{A},\ldots)(\xi',\eta',\zeta)^2](\mathfrak{A},\ldots)(\xi,\eta,\zeta)^2-\lambda\{(\mathfrak{A},\ldots)(\xi',\eta',\zeta')(\xi,\eta,\zeta)\}^2=0,$$

which is of the form in question.

205. Take (x',y',z') as the point-coordinates of the centre of inscription, the equation of the axis of inscription is

$$(a,b,c,f,g,h)(x,y,z)(x',y',z')=0.$$

And we may, if we please, exhibit the equation of the inscribed conic in the form

$$(a,\ldots)(x,y,z)^2(a,\ldots)(x',y',z')^2\cos^2\theta-(\{a,\ldots)(x,y,z)(x',y',z')\}^2=0,$$

where θ is a constant. This equation may also be written

$$(a,\ldots)(x,y,z)^2(a,\ldots)(x',y',z')^2\sin^2\theta-(\mathfrak{A},\ldots)(yz'-y'z,\ zx'-z'x,\ xy'-x'y)^2=0,$$

the two forms being equivalent in virtue of the identity,

$$(a,\ldots)(x,y,z)^2(a,\ldots)(x',y',z')^2-\{(a,\ldots)(x',y',z')(x,y,z)\}^2=(\mathfrak{A},\ldots)(yz'-y'z,zx'-z'x,xy'-x'y)^2.$$

206. The line-coordinates (ξ',η',ζ') of the axis of inscription are

$$ax'+hy'+gz',\ hx'+by'+fz',\ gx'+fy'+cz',$$

and we thence deduce the relation

$$(\mathfrak{A},\ldots)(\xi',\eta',\zeta')^2=K(a,\ldots)(x',y',z')^2.$$

In order that the form

$$(a,\ldots)(x,y,z)^2(a,\ldots)(x',y',z')^2\cos^2\theta-\{(a,\ldots)(x',y',z')(x,y,z)\}^2=0$$

may agree with the originally assumed form

$$(a,\ldots)(x,y,z)^2+\lambda(\xi' x+\eta' y+\zeta' z)^2,$$

or what is the same thing,

$$(a,\ldots)(x,y,z)^2+\lambda\{(a,\ldots)(x,y,z)(x',y',z')\}^2=0,$$

we must have

$$\lambda=\frac{-1}{(a,\ldots)(x',y',z')^2\cos^2\theta},$$

which may also be written

$$\lambda=\frac{-\mathrm{K}}{(\mathfrak{A},\ldots)(\xi',\eta',\zeta')^2\cos^2\theta},$$

or what is the same thing,

$$\mathrm{K}+\lambda(\mathfrak{A},\ldots)(\xi',\eta',\zeta')^2-\lambda(\mathfrak{A},\ldots)(\xi',\eta',\zeta')^2\sin^2\theta=0\,;$$

and we thence, by a preceding formula, obtain the line-equation of the inscribed conic, viz.

207. The point-equation being

$$(a,\ldots)(x,y,z)^2(a,\ldots)(x',y',z')^2\cos^2\theta-\{(a,\ldots)(x,y,z)(x',y',z')\}^2=0,$$

or

$$(a,\ldots)(x,y,z)^2(a,\ldots)(x',y',z')^2\sin^2\theta-(\mathfrak{A},\ldots)(yz'-y'z,\ zx'-z'x,\ xy'-x'y)^2=0,$$

equivalent in virtue of

$$(a,\ldots)(x,y,z)^2(a,\ldots)(x',y',z')^2-\{(a,\ldots)(x,y,z)(x',y',z')\}^2=(\mathfrak{A},\ldots)(yz'-y'z,\ zx'-z'x,\ xy'-x'y)^2.$$

The corresponding forms of the line-equation are

$$(\mathfrak{A},\ldots)(\xi,\eta,\zeta)^2(\mathfrak{A},\ldots)(\xi',\eta',\zeta')^2\sin^2\theta-\{(\mathfrak{A},\ldots)(\xi,\eta,\zeta)(\xi',\eta',\zeta')\}^2=0,$$

and

$$(\mathfrak{A},\ldots)(\xi,\eta,\zeta)^2(\mathfrak{A},\ldots)(\xi',\eta',\zeta')^2\cos^2\theta-\mathrm{K}(a,\ldots)(\eta\zeta'-\eta'\zeta,\ \zeta\xi'-\zeta'\xi,\ \xi\eta'-\xi'\eta)^2=0,$$

equivalent to each other in virtue of the before mentioned identity,

$$(\mathfrak{A},\ldots)(\xi,\eta,\zeta)^2(\mathfrak{A},\ldots)(\xi',\eta',\zeta')^2-\{(\mathfrak{A},\ldots)(\xi,\eta,\zeta)(\xi',\eta',\zeta')\}^2=\mathrm{K}(a,\ldots)(\eta\zeta'-\eta'\zeta,\ \zeta\xi'-\zeta'\xi,\ \xi\eta'-\xi'\eta)^2.$$

208. Write for shortness

$$(a,\ldots)(x,y,z)^2=00,$$
$$(a,\ldots)(x,y,z)(x',y',z)=01=10,$$
&c.,

then we have identically,

$$\begin{vmatrix}00, & 01, & 02\\ 10, & 11, & 12\\ 20, & 21, & 22\end{vmatrix}=\mathrm{K}\begin{vmatrix}x, & y, & z\\ x', & y', & z'\\ x'', & y'', & z''\end{vmatrix}$$

and if the determinant on the right hand vanishes, that is if (x,y,z), (x',y',z'), (x'',y'',z'') are points in a line, then we have

$$\begin{vmatrix}00, & 01, & 02\\ 10, & 11, & 12\\ 20, & 21, & 22\end{vmatrix}=0,$$

an equation, which, as already remarked, is equivalent to

$$\cos^{-1}\frac{01}{\sqrt{00}\sqrt{11}}+\cos^{-1}\frac{12}{\sqrt{11}\sqrt{22}}=\cos^{-1}\frac{02}{\sqrt{00}\sqrt{22}}.$$

The foregoing investigations in relation to the inscribed conic are given for the sake of the application thereof to the theory of distance, and it has been necessary to make use of analytical formulæ of some complexity which are introduced out of their natural place.

On the Theory of Distance, Nos. 209 to 229.

209. I return to the geometry of one dimension. Imagine in the line or *locus in quo* of the range of points, a point-pair, which I term the Absolute. Any point-pair whatever may be considered as inscribed in the Absolute, the centre and axis of inscription being the sibiconjugate points of the involution formed by the points of the given point-pair and the points of the Absolute; the centre and axis of inscription *quà* sibiconjugate points are harmonics with respect to the Absolute. A point-pair considered as thus inscribed in the Absolute is said to be a *point-pair circle*, or simply a *circle*; the centre of inscription and the axis of inscription are termed the centre and the axis. Either of the two sibiconjugate points may be considered as the centre, but the selection when made must be adhered to. It is proper to notice that, given the centre and one point of the circle, the other point of the circle is determined in a unique manner. In fact the axis is the harmonic of the centre in respect to the Absolute, and then the other point is the harmonic of the given point in respect to the centre and axis.

210. As a definition, we say that the two points of a circle are equidistant from the centre. Now imagine two points P, P'; and take the point P'' such that P, P'' are a circle having P' for its centre; take in like manner the point P''' such that P', P''' are a circle having P'' for its centre; and so on: and again in the opposite direction, a point $P^{\backprime}$ such that $P', P^{\backprime}$ are a circle having P for its centre; a point $P^{\backprime\backprime}$ such that $P, P^{\backprime\backprime}$ are a circle having $P^{\backprime}$ for its centre, and so on. We have a series of points $\ldots P^{\backprime\backprime}, P^{\backprime}, P, P', P'', \ldots$ at equal intervals of distance: and if we take the points P, P' indefinitely near to each other, then the entire line will be divided into a series of equal infinitesimal elements; the number of these elements included between any two points measures the distance of the two points. It is clear that, according to the definition, if P, P', P'' be any three points taken in order, then

$$\text{Dist.}\,(P, P')+\text{Dist.}\,(P', P'')=\text{Dist.}\,(P, P''),$$

which agrees with the ordinary notion of distance.

211. To show how the foregoing definition leads to an analytical expression for the distance of two points in terms of their coordinates, take

$$(a, b, c\between x, y)^2=0$$

for the equation of the Absolute. The equation of a circle having the point (x', y') for its centre is

$$(a, b, c\between x, y)^2(a, b, c\between x', y')^2\cos^2\theta-\{(a, b, c\between x, y\between x', y')\}^2=0;$$

and consequently if (x, y), (x'', y'') are the two points of the circle, then

$$\frac{(a, b, c\between x, y\between x', y')}{\sqrt{(a, b, c\between x, y)^2}\sqrt{(a, b, c\between x', y')^2}}=\frac{(a, b, c\between x', y'\between x'', y'')}{\sqrt{(a, b, c\between x', y')^2}\sqrt{(a, b, c\between x'', y'')^2}},$$

an equation which expresses that the points (x'', y'') and (x, y) are equidistant from the point (x', y'). It is clear that the distance of the points (x, y) and (x', y') must be a function of

$$\frac{(a, b, c\between x, y\between x', y')}{\sqrt{(a, b, c\between x, y)^2}\sqrt{a, b, c\between x', y')^2}},$$

and the form of the function is determined from the before-mentioned property, viz. if P, P′, P″ be any three points taken in order, then

$$\text{Dist. }(\mathrm{P}, \mathrm{P}')+\text{Dist. }(\mathrm{P}', \mathrm{P}'')=\text{Dist. }(\mathrm{P}, \mathrm{P}'').$$

This leads to the conclusion that the distance of the points (x, y), (x', y') is equal to a multiple of the arc having for its cosine the last-mentioned expression (see *ante*, No. 168); and we may in general assume that the distance is equal to the arc in question, viz. that the distance is

$$\cos^{-1}\frac{(a, b, c\between x, y\between x', y')}{\sqrt{(a, b, c\between x, y)^2}\sqrt{(a, b, c\between x', y')^2}},$$

or what is the same thing,

$$\sin^{-1}\frac{(ac-b^2)(xy'-x'y)}{\sqrt{(a, b, c\between x, y)^2}\sqrt{(a, b, c\between x', y')^2}}.$$

It follows that the two forms

$$\begin{aligned}&(a, b, c\between x, y)^2(a, b, c\between x', y')^2\cos^2\theta-\{(a, b, c\between x, y\between x', y')\}^2=0,\\&(a, b, c\between x, y)^2(a, b, c\between x', y')^2\sin^2\theta-(ac-b^2)(xy'-x'y)\qquad=0,\end{aligned}$$

of the equation of a circle, each of them express that the distances of the two points from the centre are respectively equal to the arc θ; or, if we please, that θ is the radius of the circle.

212. When $\theta=0$, we have

$$xy'-x'y=0,$$

an equation which expresses that (x, y) and (x', y') are one and the same point. When $\theta=\frac{\pi}{2}$, we have

$$(a, b, c\between x, y\between x', y')=0,$$

an equation which expresses that the points (x, y) and (x', y') are harmonics with respect to the Absolute. The distance between any two points harmonics with respect to the Absolute is consequently a quadrant, and such points may be said to be quadrantal to each other. The quadrant is the unit of distance.

213. The foregoing is the general case, but it is necessary to consider the particular case where the Absolute is a pair of coincident points. The harmonic of any point whatever in respect to the Absolute is here a point coincident with the Absolute itself:

the definition of a circle is consequently simplified; viz. any point-pair whatever may be considered as a circle having for its centre the harmonic of the Absolute with respect to the point-pair; we may, as before, divide the line into a series of equal infinitesimal elements, and the number of elements included between any two points measures the distance between the two points. As regards the analytical expression, in the case in question $ac-b^2$ vanishes, or the distance is given as the arc to an evanescent sine. Reducing the arc to its sine and omitting the evanescent factor, we have a finite expression for the distance. Suppose that the equation of the Absolute is

$$(qx-py)^2=0,$$

or what is the same thing, let the Absolute (treated as a single point) be the point (p, q), then we find for the distance of the points (x, y) and (x', y') the expression

$$\frac{xy'-x'y}{(qx-py)(qx'-py')};$$

or introducing an arbitrary multiplier,

$$\frac{(q\alpha-p\beta)(xy'-x'y)}{(qx-py)(qx'-py')},$$

which is equal to

$$\frac{\beta x-\alpha y}{qx-py}-\frac{\beta x'-\alpha y'}{qx'-py'}.$$

It is hardly necessary to remark, that in the present case the notion of the quadrantal relation of two points has altogether disappeared, and that the unit of distance is arbitrary.

214. Passing now to geometry of two dimensions, we have here to consider a certain conic, which I call the Absolute. Any line whatever determines with the Absolute (cuts it in) two points which are the Absolute in regard to such line considered as a space of one dimension, or *locus in quo* of a range of points, and in like manner any point whatever determines with the Absolute (has for tangents of the Absolute through the point) two lines which are the Absolute in regard to such point considered as a space of one dimension, or *locus in quo* of a pencil of lines. The foregoing theory for geometry of one dimension establishes the notion of distance as regards each of these ranges and pencils considered apart by itself; in order to bring the different ranges and pencils in relation to each other, it is necessary to assume that the quadrant which is the unit of distance for these several systems respectively, is one and the same distance for each system (of course, when, as in the analytical theory, we actually represent the quadrant by the ordinary symbol $\frac{\pi}{2}$, the above assumption is tacitly made; but substituting the thing signified for the definition, and looking at the quadrant merely as the distance between two points, or as the case may be, lines, harmonically related to the point-pair, or as the case may be, line-pair, constituting the Absolute, the assumption is at once seen to be an assumption, and it needs to be made explicitly). But the assumption being made, the foregoing theory of distance in geometry of one dimension enables the comparison not only

of the distances of points upon different lines, or of lines through different points, but of the distances of points on a line and of lines through a point. The pole of any line in relation to the Absolute may be termed simply the pole, and in like manner the polar of any line in relation to the Absolute may be termed simply the polar, and we have the theorem that the distance of two points or lines is equal to the distance of their polars or poles, or what is the same thing, that the distance of two poles and the distance of the two corresponding polars are equal. And we may, as a definition, establish the notion of the distance of a point from a line, viz. it is the complement of the distance of the polar of the point from the line, or what is the same thing, the complement of the distance of the point from the pole of the line. The distance of a pole and polar is therefore the complement of zero, that is, it is the quadrant.

215. It has, by means of the preceding assumption as to the quadrant, been possible to establish the notion of distance, without the assistance of the circle, but this figure must now be considered. A conic inscribed in the Absolute is termed a circle; the centre of inscription (or point of intersection of the common tangents) and the axis of inscription (or line of junction of the common ineunts) are the centre and axis of the circle. All the points of a circle are equidistant from the centre; all the tangents are equidistant from the axis, and this distance is the complement of the former distance.

216. These properties of the circle lead immediately to the analytical expressions for the distances of points or lines in terms of the coordinates. In fact, take

$$(a, b, c, f, g, h \between x, y, z)^2=0$$

for the point-equation of the Absolute; its line-equation will be

$$(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H} \between \xi, \eta, \zeta)^2=0.$$

The point-equation of the circle having the point (x', y', z') for its centre, is

$$(a, \ldots \between x, y, z)^2 (a, \ldots \between x', y', z')^2 \cos^2\theta - \{(a, \ldots \between x, y, z \between x', y', z')\}^2=0,$$

or

$$(a, \ldots \between x, y, z)^2 (a, \ldots \between x', y', z')^2 \sin^2\theta - (\mathfrak{A}, \ldots \between yz'-y'z,\ zx'-z'x,\ xy'-x'y)^2=0,$$

from which (by the same reasoning as for the case of geometry of one dimension) it follows that the distance of the points (x, y, z), (x', y', z') is

$$\cos^{-1}\frac{(a, \ldots \between x, y, z \between x', y', z')}{\sqrt{(a, \ldots \between x, y, z)^2}\sqrt{(a, \ldots \between x', y', z')^2}},$$

or what is the same thing,

$$\sin^{-1}\frac{\sqrt{(\mathfrak{A}, \ldots \between yz'-y'z,\ zx'-z'x,\ xy'-x'y)^2}}{\sqrt{(a, \ldots \between x, y, z)^2}\sqrt{(a, \ldots \between x', y', z')^2}};$$

and it appears from the cosine formula (see *ante*, No. 208), that if P, P′, P″ be points on the same line, then we have, as we ought to have,

Dist. (P, P′) + Dist. (P′, P″) = Dist. (P, P″).

217. In like manner, the line-equation of the same circle, the line-coordinates of the

axis being (ξ', η', ζ'), is

$$(\mathfrak{A}, \ldots \between \xi, \eta, \zeta)^2(\mathfrak{A}, \ldots \between \xi', \eta', \zeta')^2 \sin^2\theta - \{(\mathfrak{A}, \ldots \between \xi, \eta, \zeta \between \xi', \eta', \zeta')\}^2 = 0,$$

or

$$(\mathfrak{A}, \ldots \between \xi, \eta, \zeta)^2(\mathfrak{A}, \ldots \between \xi', \eta', \zeta')^2 \cos^2\theta - \mathrm{K}(a, \ldots \between \eta\zeta' - \eta'\zeta, \zeta\xi' - \zeta'\xi, \xi\eta' - \xi'\eta)^2 = 0,$$

from which it follows that the distance of the lines (ξ, η, ζ) and (ξ', η', ζ') is

$$\cos^{-1}\frac{(\mathfrak{A}, \ldots \between \xi, \eta, \zeta \between \xi', \eta', \zeta')}{\sqrt{\mathfrak{A}, \ldots \between \xi, \eta, \zeta)^2}\sqrt{(\mathfrak{A}, \ldots \between \xi', \eta', \zeta')^2}},$$

or what is the same thing,

$$\sin^{-1}\frac{\sqrt{\mathrm{K}(a, \ldots \between \eta\zeta' - \eta'\zeta, \zeta\xi' - \zeta'\xi, \xi\eta' - \xi'\eta)^2}}{\sqrt{(\mathfrak{A}, \ldots \between \xi, \eta, \zeta)^2}\sqrt{(\mathfrak{A}, \ldots \between \xi', \eta', \zeta')^2}}.$$

218. And we may from the first formula of either set, deduce for the distance of the point (x, y, z) and the line (ξ', η', ζ'), the expression

$$\sin^{-1}\frac{\sqrt{\mathrm{K}}(\xi' x + \eta' y + \zeta' z)}{\sqrt{(a, \ldots \between x, y, z)^2}\sqrt{(\mathfrak{A}, \ldots \between \xi', \eta', \zeta')^2}},$$

as may be easily seen by writing $\mathfrak{A}\xi' + \mathfrak{H}\eta' + \mathfrak{G}\zeta', \ldots$ for x', y', z', or $ax + hy + gz, \ldots$ for ξ, η, ζ, and putting $\sin^{-1}$ for $\cos^{-1}$.

219. It may be noticed that there are certain lines, viz. the tangents of the Absolute, in regard to which, considered as a space of one dimension, the Absolute is a pair of coincident points; and in like manner certain points, viz. the ineunts of the Absolute, in regard to which, considered as a space of one dimension, the Absolute is a pair of coincident lines.

220. We may, in particular, suppose that the Absolute, instead of being a proper conic, is a pair of points. The line through the two points may be called the Absolute line; such line is to be considered as a pair of coincident lines. Any point whatever determines with the Absolute, two lines, viz. the lines joining the point with the two points of the Absolute; this line-pair is the Absolute for the point considered as a space of one dimension or *locus in quo* of a pencil of lines, and the theory of the distances of lines through a point is therefore precisely the same as in the general case. But any line whatever determines with the Absolute (meets the Absolute line in) a pair of coincident points, which pair of coincident points is the Absolute in regard to such line considered as a space of one dimension or *locus in quo* of a range of points, and the theory of the distance of points on a line is therefore the theory before explained for this special case. But we cannot, in the same way as before, compare the distances of points upon different lines, since we have not in the present case the quadrant as a unit of distance. The comparison must be made by means of the circle, viz. in the present case any conic passing through the two points of the Absolute is termed a circle, and the point of intersection of the tangents to the circle at the two points of the Absolute (or what is the same thing, the pole of the Absolute line in respect to the circle) is the centre of the circle. The Absolute line itself may, if it is necessary to do so, be considered as the axis of the circle. It

is assumed that the points of the circle are all of them equidistant from the centre, and by this assumption we are enabled to compare distances upon different lines. In fact we may, by a construction precisely similar to that of EUCLID, Book I. Prop. II., from a given point A draw a finite line equal to a given finite line BC, and thence also upon a given line through A, determine the finite line AD equal to the given finite line BC. Since the unit of distance for points on a line is arbitrary, we cannot of course compare the distances of points with the distances of lines. The distance of a point from a line does, however, admit of comparison with the distance of two points; we have only to assume as a definition that the distance of a point from a line is the distance of the point from the point of intersection of the line with the quadrantal line through the point.

221. As regards the analytical theory, suppose that the point-coordinates of the two points of the Absolute are (p, q, r), (p_0, q_0, r_0), then the line-equation of the Absolute is

$$2(p\xi+q\eta+r\zeta)(p_0\xi+q_0\eta+r_0\zeta)=0;$$

so that we have $\mathfrak{A}=2pp_0$, $\mathfrak{B}=2qq_0$, $\mathfrak{C}=2rr_0$, $\mathfrak{F}=qr_0+rq_0$, $\mathfrak{G}=rp_0+pr_0$, $\mathfrak{H}=pq_0+qp_0$, and thence $K=0$; but

$$K(a, b, c, f, g, h \between x, y, z)^2$$

$$\begin{vmatrix} x, & y, & z \\ p, & q, & r \\ p_0, & q_0, & r_0 \end{vmatrix}$$

where obviously

$$\begin{vmatrix} x, & y, & z \\ p, & q, & r \\ p_0, & q_0, & r_0 \end{vmatrix}=0$$

is the equation of the Absolute line.

222. The expression for the distance of the two points (x, y, z), (x', y', z') is given as the arc to an evanescent sine; but reducing the arc to its sine, and omitting the evanescent factor, the resulting expression is

$$\sqrt{2\begin{vmatrix} x, & y, & z \\ x', & y', & z' \\ p, & q, & r \end{vmatrix}\begin{vmatrix} x, & y, & z \\ x', & y', & z' \\ p_0, & q_0, & r_0 \end{vmatrix}} \div \begin{vmatrix} x, & y, & z \\ p, & q, & r \\ p_0, & q_0, & r_0 \end{vmatrix}\begin{vmatrix} x', & y', & z' \\ p, & q, & r \\ p_0, & q_0, & r_0 \end{vmatrix}$$

and the expression for the distance of the two lines (ξ, η, ζ), (ξ', η', ζ') is

$$\cos^{-1}\frac{(p\xi+q\eta+r\zeta)(p_0\xi'+q_0\eta'+r_0\zeta')+(p\xi'+q\eta'+r\zeta')(p_0\xi+q_0\eta+r_0\zeta)}{\sqrt{2(p\xi+q\eta+r\zeta)(p_0\xi+q_0\eta+r_0\zeta)}\sqrt{2(p\xi'+q\eta'+r\zeta')(p_0\xi'+q_0\eta'+r_0\zeta')}},$$

or what is the same thing,

$$\sin^{-1}\frac{(qr_0-rq_0)(\eta\zeta'-\eta'\zeta)+(rp_0-pr_0)(\zeta\eta'-\zeta'\eta)+(pq_0-qp_0)(\xi\eta'-\xi'\eta)}{\sqrt{2(p\xi+q\eta+r\zeta)(p_0\xi+q_0\eta+r_0\zeta)}\sqrt{2(p\xi'+q\eta'+r\zeta')(p_0\xi'+q_0\eta'+r_0\zeta')}};$$

and finally, the expression for the distance of the point x, y, z from the line (ξ', η', ζ'), reducing the arc to its sine and omitting the evanescent factor, is

$$(\xi' x+\eta' y+\zeta' z)\div\begin{vmatrix} x, & y, & z \\ p, & q, & r \\ p_0, & q_0, & r_0 \end{vmatrix}\sqrt{2(p\xi'+q\eta'+r\zeta')(p_0\xi'+q_0\eta'+r_0\zeta')}.$$

223. If in the above formula we put $(p, q, r)=(1, i, 0)$, $(p_0, q_0, r_0)=(1, -i, 0)$, where as usual $i=\sqrt{-1}$, then the line-equation of the Absolute is $\xi^2+\eta^2=0$, or what is the same thing, the Absolute consists of the two points in which the line $z=0$ intersects the line-pair $x^2+y^2=0$; the last-mentioned line-pair, as passing through the Absolute, is by definition a circle; it is in fact the circle radius zero, or an evanescent circle. If we put also the coordinate z equal to unity, then the preceding assumption as to the coordinates of the points of the Absolute must be understood to mean only $x:y:1=1:i:0$, or $1:-i:0$; that is, we must have x and y infinite, and, as before, $x^2+y^2=0$, or in other words, the Absolute will consist of the points of intersection of the line infinity by the evanescent circle $x^2+y^2=0$. With the values in question,

224. The expression for the distance of the points (x, y) and (x', y') is

$$\sqrt{(x-x')^2+(y-y')^2};$$

that for the distance of the lines (ξ, η, ζ) and (ξ', η', ζ') is

$$\cos^{-1}\frac{\xi\xi'+\eta\eta'}{\sqrt{\xi^2+\eta^2}\sqrt{\xi'^2+\eta'^2}}$$

$$=\sin^{-1}\frac{\xi\eta'-\xi'\eta}{\sqrt{\xi^2+\eta^2}\sqrt{\xi'^2+\eta'^2}},$$

which may also be written

$$=\tan^{-1}\frac{\xi}{\eta}-\tan^{-1}\frac{\xi'}{\eta'};$$

and the expression for the distance of the point (x, y) from the line (ξ', η', ζ') is

$$\frac{\xi' x+\eta' y+\zeta'}{\sqrt{\xi'^2+\eta'^2}},$$

which are obviously the formulæ of ordinary plane geometry, (x, y) being ordinary rectangular coordinates.

225. The general formulæ suffer no *essential* modification, but they are greatly simplified in form by taking for the point-equation of the Absolute,

$$x^2+y^2+z^2=0,$$

or what is the same, for the line-equation

$$\xi^2+\eta^2+\zeta^2=0.$$

In fact, we then have for the expression of the distance of the points (x, y, z), (x', y', z'),

$$\cos^{-1}\frac{xx'+yy'+zz'}{\sqrt{x^2+y^2+z^2}\sqrt{x'^2+y'^2+z'^2}};$$

for that of the lines (ξ, η, ζ), (ξ', η', ζ'),

$$\cos^{-1}\frac{\xi\xi'+\eta\eta'+\zeta\zeta'}{\sqrt{\xi^2+\eta^2+\zeta^2}\sqrt{\xi'^2+\eta'^2+\zeta'^2}};$$

and for that of the point (x, y, z) and the line (ξ', η', ζ'),

$$\sin^{-1}\frac{\xi'x+\eta'y+\zeta'z}{\sqrt{x^2+y^2+z^2}\sqrt{\xi'^2+\eta'^2+\zeta'^2}}.$$

226. Suppose (x, y, z) are ordinary rectangular coordinates in space satisfying the condition

$$x^2+y^2+z^2=1,$$

the point having (x, y, z) for its coordinates will be a point on the surface of the sphere, and (the last-mentioned equation always subsisting) the equation $\xi x+\eta y+\zeta z=0$ will be a great circle of the sphere; and since we are only concerned with the ratios of ξ, η, ζ, we may also assume $\xi^2+\eta^2+\zeta^2=1$. We may of course retain in the formulæ the expressions $x^2+y^2+z^2$ and $\xi^2+\eta^2+\zeta^2$, without substituting for these the values unity, and it is in fact convenient thus to preserve all the formulæ in their original forms. We have thus a system of spherical geometry; and it appears that the Absolute in such system is the (spherical) conic, which is the intersection of the sphere with the concentric cone or evanescent sphere $x^2+y^2+z^2=0$. The circumstance that the Absolute is a proper conic, and not a mere point-pair, is the real ground of the distinction between spherical geometry and ordinary plane geometry, and the cause of the complete duality of the theorems of spherical geometry.

227. I have, in all that has preceded, given the analytical theory of distance along with the geometrical theory, as well for the purpose of illustration, as because it is important to have the analytical expression of a distance in terms of the coordinates; but I consider the geometrical theory as perfectly complete in itself: the general result is as follows, viz. assuming in the plane (or space of geometry of two dimensions) a conic termed the Absolute, we may by means of this conic, by descriptive constructions, divide any line or range of points whatever, and any point or pencil of lines whatever, into an infinite series of infinitesimal elements, which are (as a definition of distance) assumed to be equal; the number of elements between two points of the range or two lines of the pencil, measures the distance between the two points or lines; and by means of the quadrant, as a distance which exists as well with respect to lines as points, we are enabled to compare the distance of two lines with that of two points; and the distance of a point and a line may be represented indifferently as the distance of two points, or as the distance of two lines.

228. In ordinary spherical geometry, the general theory undergoes no modification whatever; the Absolute is an actual conic, the intersection of the sphere with the concentric evanescent sphere.

229. In ordinary plane geometry, the Absolute degenerates into a pair of points, viz. the points of intersection of the line infinity with any evanescent circle, or what is the

same thing, the Absolute is the two circular points at infinity. The general theory is consequently modified, viz. there is not, as regards points, a distance such as the quadrant, and the distance of two lines cannot be in any way compared with the distance of two points; the distance of a point from a line can be only represented as a distance of two points.

230. I remark in conclusion, that, *in my own point of view*, the more systematic course in the present introductory memoir on the geometrical part of the subject of quantics, would have been to ignore altogether the notions of distance and metrical geometry; for the theory in effect is, that the metrical properties of a figure are not the properties of the figure considered *per se* apart from everything else, but its properties when considered in connexion with another figure, viz. the conic termed the Absolute. The original figure might comprise a conic; for instance, we might consider the properties of the figure formed by two or more conics, and we are then in the region of pure descriptive geometry: we pass out of it into metrical geometry by fixing upon a conic of the figure as a standard of reference and calling it the Absolute. Metrical geometry is thus a part of descriptive geometry, and descriptive geometry is *all* geometry and reciprocally; and if this be admitted, there is no ground for the consideration, in an introductory memoir, of the special subject of metrical geometry; but as the notions of distance and of metrical geometry could not, without explanation, be thus ignored, it was necessary to refer to them in order to show that they are thus included in descriptive geometry.

V. *On some Thermo-dynamic Properties of Solids. By* J. P. Joule, *LL.D., F.R.S., F.C.S., Hon. Mem. Phil. Soc. Cambridge, Vice-President of the Lit. and Phil. Soc. Manchester, Corresp. Mem. R.A. Sc. Turin, &c.*

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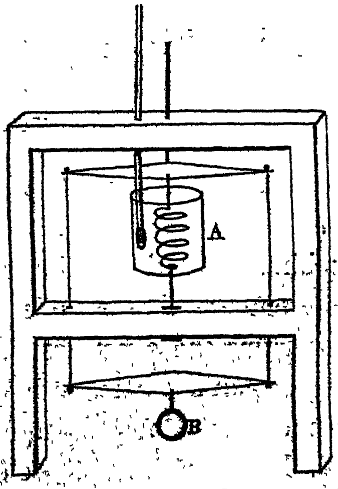
1. After finding the numerical relation between heat and work in 1843, it immediately occurred to me to investigate various phenomena in which heat is evolved by mechanical means, and of these one of the most interesting and important appeared to be the evolution of heat by the compression of elastic fluids. If the heat given out in this case proved to be the equivalent of the work spent, then the natural inference was that the elastic force of a gas and its temperature are owing to the motion of its constituent particles, both being proportional to the square of the velocity of the particles; and that the force of a falling body employed in compressing a gas is exhibited anew in the form of temperature. On the other hand it was possible, secondly, to conceive of an elastic fluid which would not give out any heat by compression. This would be the case if it was made up of mutually repelling particles, the temperature of which was that of the mass. The work required to compress the fluid would be the same on either hypothesis, but in one supposition the effect would be developed in actual energy, in the other in the potential form. Thirdly, we may suppose a fluid exhibiting as heat a portion of the force employed in its compression, and retaining the rest in the potential form. Or we may have a fluid giving out more heat than the equivalent of the work spent upon it when it is compressed and maintained at a constant temperature. Experiment proved that the heat actually evolved was very approximately that due to the compressing force. Nevertheless, it seemed desirable to demonstrate the possibility of a gas so constituted that heat shall not be evolved by its compression. It occurred to me that a bag full of elastic metallic springs would illustrate such a gas. If the springs were properly formed, the elasticity of the bag would follow the laws of gaseous pressure. To the question, Would such a bag evolve heat on compression? I could readily answer, no; for in the bending of a spring one part is extended while the other is compressed, and thus it might be expected that the thermal effect on the whole would be neutral. Still it seemed desirable to decide the point by experiment.

Fig. 1.



2. The apparatus I employed is represented by the adjoining sketch, where a spiral spring of tempered steel is seen immersed in the can A. By applying weights to a lever connected with the link B the spring could be compressed, and the heat evolved, if any, measured by the increase of temperature observed to take place in the water or mercury filling the can. The plan I pursued was to note the temperature successively; first, two minutes after the weight had been laid on; second, two minutes after the weight had been removed; and third, after two minutes more had elapsed. The mean of a number of these observations taken in succession, gave me,—first, the thermal effect resulting from the laying on of the weight and the atmospheric influence; second, that of the removal of the weight and the atmospheric influence; and third, the atmospheric influence alone.

3. The pressure applied was 318 lbs., which pushed down the spiral spring 1·136 inch. In the following summary of results, each number is the mean of eight or ten observations, given in terms of the graduation of a thermometer, of which each degree was equal to ·0558 of a degree Centigrade.

First Series. Spring immersed in 8 ozs. of water.

	Weight laid on.	Weight taken off.	Atmospheric influence.
First experiment	−·548	−·568	−·560
Second experiment	−·009	−·023	−·012
Third experiment	+·282	+·271	+·252
Mean	−·092	−·107	−·107

Second Series. Spring immersed in 7 lbs. of mercury.

	Weight laid on.	Weight taken off.	Atmospheric influence.
First experiment	+·180	+·175	+·169
Second experiment	+·066	+·092	+·059
Third experiment	+·041	+·039	+·020
Fourth experiment	−·060	−·059	−·053
Mean	+·057	+·062	+·049
Mean of both series	−·032	−·038	−·041

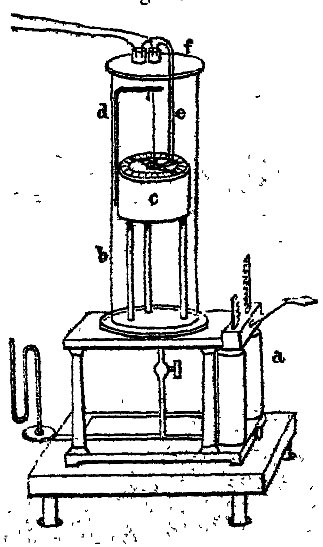
4. The capacity for heat of half a pound of water being about twice as great as that of 7 lbs. of mercury, I have divided the mean result of the second series by two, before combining it with the first series in a general mean. The mean of both series therefore represents the thermal effect in the capacity of half a pound of water. It indicates, on subtracting the effects of the atmosphere, a heating effect of ·009 after laying on the weight, and a heating effect of ·003 on taking off the weight. The highest of these numbers represents a temperature less than one thousandth of a degree Fahr.

5. Now the actual force expended in the compression of the spring was 14·268 foot-pounds, which is equivalent to 0°·037 Fahr. in half a pound of water, a thermal effect which would have been made manifest with the greatest facility. Hence it was

obvious that a gas might be conceived as so constituted that the heat evolved by its compression would be in no respect the equivalent of the mechanical force employed, and that therefore we had no right to assume such equivalency except as a hypothesis to be tested by experiment. Accordingly after this hypothesis had been proved by me approximately[*], Professor THOMSON devised the experiments[†] by which we have succeeded in defining the limits of its accuracy. The same philosopher has also applied his powerful analysis to the investigation of the thermo-elastic properties of matter[‡]. The results of the experiments I have just given an account of can only be considered as negative, but it has been decided by Professor THOMSON that the compression of a spring gives a certain, though excessively small thermal effect, owing to the almost exact counterpoise of heating and cooling effects on the compressed and extended sides. The method of obtaining appreciable results was obviously to examine these opposite effects separately. I have, therefore, on the suggestion of Professor THOMSON, undertaken some experiments with a view to ascertain the heat developed by longitudinal compression, and that absorbed on the application of tensile force.

6. At the outset it was obvious that a very delicate test of temperature would be required, and no means appeared to offer so many advantages as that of thermo-electricity. Professor J. D. FORBES had constructed a thermo-multiplier capable of detecting temperatures not exceeding one thousandth of a degree FAHRENHEIT. Adopting some of the refinements introduced by MELLONI and FORBES, I have simplified the instrument so as to render its construction and management very easy, and also increased its sensibility by immersing it into the vacuum of an air-pump. My thermo-multiplier is represented by the adjoining sketch (fig. 2), where *a* is the air-pump firmly clamped to a strong stool, the legs of which pass through holes in the laboratory floor and are driven into the ground beneath; *b* is a glass chimney receiver; *c* a block of wood supported on feet which rest on the pump plate; *d* a piece of glass rod fixed to the block, over which is thrown the filament which supports the astatic needles. Two thick copper wires (*e*) dip into mercury cups formed in the block, and, being carried out of the receiver through holes drilled in the ground glass plate *f*, are bent into two mercury cups placed on the top of the instrument. Pitch was employed to close all orifices air-tight.

Fig. 2.



7. The details of the astatic needles, which are poised according to the plan first suggested by Professor THOMSON, will be better understood by inspecting fig. 3. The needles

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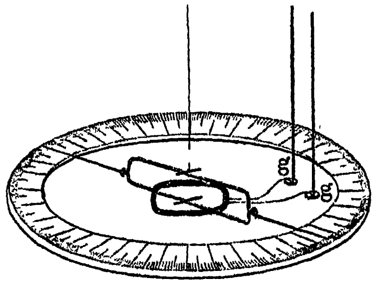
[*] Proceedings of the Royal Society, June 20, 1844; and Philosophical Magazine, May 1845.

[†] Philosophical Magazine, 1852, Supplement; Philosophical Transactions, 1853, Part III. p. 357, and 1854, Part II. p. 321.

[‡] Quarterly Mathematical Journal, April 1855.

are parts of one sewing-needle magnetized to saturation, one part being a little longer than the other, so as to exceed it in magnetic moment and give direction to the system. A piece of glass tube drawn very fine, and bent as represented in the sketch, is attached at right angles to the upper magnetic needle and serves as the pointer. The lower needle is hooked to the pointer by means of the fine glass tube to which it also is attached. The coil consists of twenty turns of silked copper wire, $\frac{1}{40}$th of an inch in diameter, the ends of which dip into the mercury cups *gg* formed in the block of wood.

Fig. 3.



8. In order still further to increase the sensibility of the instrument, a steel magnet one yard long, the permanency of which had been tested, was placed so as to counteract and almost entirely overcome the action of the earth's magnetism in the locality of the needle. A small telescope placed at the distance of a few yards, and looking obliquely downwards through the chimney glass at the graduated circle, completed the apparatus.

9. With air in the receiver at the atmospheric pressure, the mere standing at the distance of two yards on one side of the instrument, would in a short space of time cause the needle to travel through 10° in consequence of the currents of air produced by the unequal heating of the walls of the glass receiver. But when the air was reduced to a pressure of only half an inch in the mercury gauge, this did not take place, though still, when the hand was put in contact with the receiver, a very considerable deflection of the needle was speedily produced.

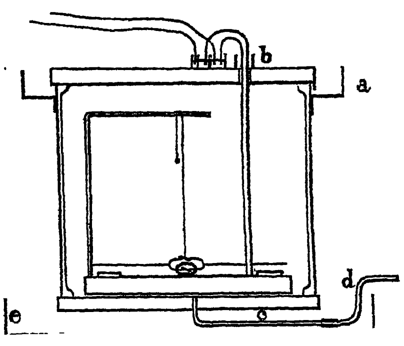
10. On working with my instrument, I was agreeably surprised to find that when the bar-magnet was placed so as to make the needle take up one minute in being deflected to a new position, no perceptible return swing of the needle took place, even when the rarefaction of the air was carried to half an inch pressure. If a small magnet was suddenly placed where it could deflect the needle 30°, the pointer would steadily travel towards that degree of deflection, and on arriving there would remain settled without any previous oscillation that could be discerned. When the time of a swing was reduced to 30″, a return swing was observed amounting to $\frac{1}{150}$, $\frac{1}{25}$, and $\frac{1}{18}$ of the first swing, according as the gauge was reduced to 1, $\frac{6}{10}$, and $\frac{1}{2}$ inch respectively.

11. As a test of the delicacy of the instrument arranged so as to give the swing in 45″, I may mention that, after increasing the resistance of the coil and its appendages a hundredfold by the addition of 100 yards of fine copper wire of $\frac{1}{40}$th of an inch diameter, a single degree Centigrade communicated to the junction bismuth and antimony produced a deflection of 2° 57′. Therefore, as it was quite possible to estimate a deflection amounting to 2′, it followed that a change of temperature in the junction bismuth and antimony directly connected with the multiplier would be estimable, if it were only $\frac{1}{8800}$ of a degree Centigrade.

12. An objection which might be raised on account of an air-pump being permanently

occupied, might be got rid of by the following arrangement:—A rim *a*, fig. 4, contains pitch or other cement to secure the joint between the chimney and top plate glass; a rim at *b* contains pitch to secure the orifices where the wires pass through the top plate. The bottom of the chimney rests upon a round metallic plate, into which a metal pipe *c* is screwed, which is attached by india-rubber tube to the glass tube *d*. This tube is hermetically sealed after the air of the receiver has been exhausted through it. All the lower joints are then rendered permanently tight, by putting the instrument in the shallow dish *e* filled with melted pitch.

Fig. 4.



Thermo-electricity of Iron in different states.

13. In my earliest experiments on the thermal effect of stretching a steel bar, I placed a copper wire in contact with the steel, and completed the circuit by an iron wire in contact with another part of the steel bar not under tension. I found anomalous results which I was ultimately able to refer to the strong thermo-electric relation between hard steel and iron. It was at once obvious that the existence of such marked differences between ferruginous metals in various states of aggregation or purity might render the thermo-multiplier a valuable test in the hands of practical men: I therefore allowed myself to be diverted awhile from the main object of inquiry in the endeavour to throw some light on so interesting a question.

14. Professor THOMSON has described* the changes in thermo-electric position which are produced by the various conditions under which metals are placed. He has shown that the process of hardening by plunging into cold water brings copper and most other metals nearer antimony in the thermo-electric series, but that iron is brought nearer bismuth: I find that in steel the change is in the same direction as in iron, but of enormously greater magnitude. In a softened state, the position of steel is about midway between copper and iron, but after hardening it by plunging it at a bright red heat into water, I have in some instances found it to be on the bismuth side of copper, the alteration of the thermo-electric position of the same specimen amounting to as much as $\frac{1}{15}$th of the entire range between bismuth and antimony. In all the specimens of wrought and cast iron I have examined, there is a notable effect in the same direction produced by plunging at bright red heat into water; but although a great change in the molecular state was thus occasioned, evinced by the iron bending with twice the difficulty it did when annealed, and the cast iron resisting the action of the file, either metal was only brought nearer bismuth by $\frac{1}{100}$ of the interval between bismuth and antimony. A fresh illustration of the extraordinary physical change produced in iron by its conversion into steel is thus afforded; and I believe that the

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* Philosophical Transactions, 1856, Part III. p. 722.

excellence of the latter metal might be tested by ascertaining the amount of change in thermo-electric condition which can be produced by the process of hardening.

15. The different varieties of cast iron I have tried present a surprising range of thermo-electric intensities, extending almost from that of wrought iron on the one hand, to that of German silver on the other. By the kindness of Professor F. C. CALVERT, I have been enabled to examine several interesting specimens, of which he has furnished me with the analysis. The general conclusion arrived at is, that the metal is brought nearer bismuth as the quantity of carbon in combination is increased, but much more so than would be the case if cast iron exhibited merely the combination of thermo-electric intensities which carbon and iron separately possess, at least if the intensity generally assigned to carbon is correct. The intensity of the junction wrought iron and highly carbonized cast iron is as much as one-fifth of the intensity of antimony and bismuth.

Thermo-electric Intensities of Metals, Alloys, &c.

16. In constructing the following Table, I employed the thermo-multiplier already described, furnished with a variable extra resistance. I examined first those metals whose thermo-electric qualities were most widely separated, and then those which lay intermediate and differed less from each other. Small arcs of deflection were observed, so that the deviation of the needle was a sufficiently correct measure of the intensity of the current, and constantly repeated comparisons were made with a standard thermo-electric junction of copper and iron included in the same circuit with the junctions experimented on.

17. *Scale of Thermo-Electric Intensities at* 12° *Centigrade.*

Antimony, specimen of commercial	100
Antimony, pure, prepared by Mr. CALVERT	98·06
Antimony, pure, prepared by Mr. CALVERT, not well annealed	95·95
Alloy consisting of 5 equivalents of antimony +1 equivalent of bismuth	83·75
Alloy consisting of 4 equivalents of antimony +1 equivalent of bismuth	80·50
Alloy consisting of 3 equivalents of antimony +1 equivalent of bismuth	78·94
Alloy consisting of 2 equivalents of antimony +1 equivalent of bismuth	68·79
Alloy consisting of 1 equivalent of antimony +1 equivalent of bismuth	45·51
Iron, thick wire, Professor CALVERT's	80·
Iron, thin wire	79·24
Iron, thick wire, very well annealed	78·24
Iron, thick wire, hardened by plunging at bright red into water	77·62
*Iron (Professor CALVERT's No. 1) drawn into wire	77·23
†Iron (Professor CALVERT's No. 1) after puddling, but previous to being drawn into wire	76·00
‡Iron (Professor CALVERT's No. 1), cast, previous to puddling	65·21

Iron (Professor Calvert's No. 1), cast, previous to puddling, hardened by plunging it at bright red into water	63·89
Iron (Professor Calvert's No. 1), cast, white fracture, very hard	70·92
Iron (Professor Calvert's No. 1), cast, black fracture	57·77
Steel, small file, annealed	73·15
Steel, small file, hardened by plunging at bright red into water	65·8
Steel, large file, annealed	70·28
Steel, large file, hardened by plunging at bright red into water	64·06
Iron, cast, annealed	63·25
Iron, cast, hardened by plunging at bright red into water	62·7
Iron, cast, hardened by plunging at bright red into water, another specimen	60·83
Iron and copper wires, faggot of, in proportion 1 iron to 5 copper	68·29
Iron and copper wires, faggot of, in proportion 2 iron to 5 copper	69·83
Iron and copper wires, faggot of, in proportion 4 iron to 5 copper	71·17
Iron and copper wires, faggot of, in proportion 8 iron to 5 copper	72·71
Iron and copper wires, faggot of, in proportion 16 iron to 5 copper	75·53
Zinc	67·94
Zinc, amalgamated	67·77
Copper deposited by electricity (brittle)	67·9
Copper, thin wire	67·14
Copper, another specimen	66·2
Copper, another specimen	65·92
Gold, pure	67·71
Silver deposited by electricity (brittle)	67·08
Silver, sheet	66·81
Tin, pure	65·46
Lead, commercial	65·29
Lead, pure	64·42
Platina, annealed	65·02
Platina, unannealed	64·87
Platina, fine wire	64·15
Aluminium	64·68
Mercury	61·74
German silver	51·88
German silver, hardened by hammering	49·51
Bismuth, pure, prepared by Professor Calvert	4·7
Bismuth, specimen of commercial	0

Professor Calvert's analysis of the specimens marked in the above Table *, †, and ‡, gave for the

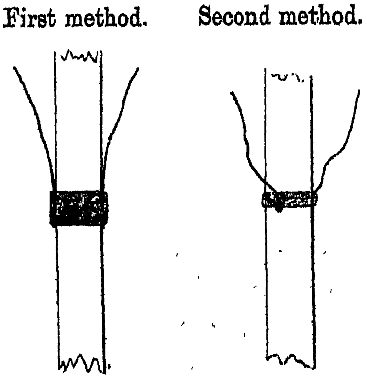
	Carbon.	Silicium.	Sulphur.	Phosphorus.	Iron.	Total.
Cast iron . . .	2·275	2·720	0·301	0·645	94·059	100
Puddled bar . .	0·296	0·120	0·134	0·139	99·311	100
Iron wire . . .	0·111	0·088	0·094	0·117	99·590	100

From the above digression I now return to the main subject of the present paper, by describing my

Experiments on the Thermal Effects of Tension on Solids.

18. All the metals employed, with the exception of lead, were in the form of cylindrical bars about a foot long and a quarter of an inch in diameter. The upper end of the bar was screwed into a piece of metal supported by a wooden framework, the lower end was attached to a lever, to the extremity of which weights could be hung without approaching the apparatus. In the first method the thermo-electric junction was made, as represented in the adjoining sketch, by binding to opposite sides of the bar copper and iron wires $\frac{1}{40}$th of an inch in diameter, hammered flat at the ends. In the second method, the junction of fine wires was inserted in a hole $\frac{1}{40}$th of an inch in diameter, bored through the centre of the bar, a small portion of wire being bound to the bar by means of cotton thread, but metallic contact prevented by an intervening slip of paper. The terminals of these wires were immersed in deep mercury cups formed in a solid block of wood, whence thick copper wires proceeded to the thermo-multiplier.

Fig. 5.



First method. Second method.

19. Immediately after each experiment on the effect of tension, the thermometric value of the deflections was ascertained by immersing the bar, to within one-third of an inch of the junction, in water of different temperatures. The deflections thus produced were about two-thirds of those occasioned by the same changes of temperature when the junction was completely immersed. The diminished effect in the former case is owing for the most part to the conduction of heat from the air by the thermo-electric wires. The experiments on tension were liable to be affected in the same way, but they were not subject to the loss arising from the conduction of heat from the surface of the bath to the junction. The error intervening from this latter circumstance could not be great, and was moreover in all probability almost exactly neutralized by a small error in the tension experiments, arising from the escape of $\frac{1}{14}$th of the thermal effect from the quarter-inch bars during the 40″ occupied by the swing of the needle.

20. *Iron.*—The weight of the bar per foot was ·1568 of a lb. The lever alone gave it a constant tension of 70 lbs. The additional tensions successively given were 194 lbs., 388 lbs., 583 lbs., and 1166 lbs., or as 1, 2, 3, and 6. The mean of five trials gave 20′·6 as the deflection indicating cold on applying the tension of 194 lbs., and 20′·2 as the

deflection in the opposite direction, indicating heat, when the weight was removed. With 388 lbs., I had 35′·8 and 42′·2: with 583 lbs., 56′·4 and 57′·8: and with 1166 lbs., 1° 55′·8 and 1° 58′·6. Hence it appeared that the quantity of cold produced by the application of tension was sensibly equal to the heat evolved by its removal; and further, that the thermal effects were proportional to the weights employed.

21. The above trials having been made without an accurate determination of the thermometric value of the deflections immediately afterwards (a practice I found it important to follow in order to obviate the effects of any alteration in the magnetic intensities of the earth, astatic needles, or controlling magnet, which might be taking place), I repeated the experiment, using the first method above described, and found a mean deflection of 1° 22′·8 with a tension of 775 lbs. The thermometric value of this deflection, determined in the manner already explained, was −0°·115 Centigrade.

22. Professor THOMSON, in his researches on the thermo-elastic properties of metals, has demonstrated the following formula as applicable to the phenomenon in question,—

$$H=\frac{t}{J}\times\frac{p}{1}\times\frac{e}{1}\times\frac{1}{s}\times\frac{1}{w},$$

where H is the thermal increase in degree Centigrade, t the temperature Centigrade from absolute zero, J the mechanical equivalent of the thermal unit, p the pressure applied in pounds, which in the case of tension is of course a negative quantity, e the longitudinal expansion per degree Centigrade, s the specific heat, and w the weight in pounds of a foot length of the bar*. Applied to the above experiment the formula gives

$$H=\frac{276{\cdot}7}{1390}\times\frac{-775}{1}\times\frac{1\dagger}{81200}\times\frac{1\ddagger}{{\cdot}11}\times\frac{1}{{\cdot}1568}=-0^\circ{\cdot}11017.$$

23. In another experiment by the first method, but with another thermo-electric junction, I obtained a thermal effect of 0°·124; in this case the formula gave

$$H=\frac{276}{1390}\times\frac{-775}{1}\times\frac{1}{81200}\times\frac{1}{{\cdot}11}\times\frac{1}{{\cdot}1568}=-0^\circ{\cdot}1099.$$

24. A third experiment, in which the second method was used, and in which I had the advantage of Professor THOMSON's assistance, gave a thermal effect of −0°·1007. In this case the formula gave

$$H=\frac{287}{1390}\times\frac{-725}{1}\times\frac{1}{81200}\times\frac{1}{{\cdot}11}\times\frac{1}{{\cdot}1568}=-0^\circ{\cdot}1069.$$

25. *Hard Steel.*—With stretching weights of 194, 388, and 775 lbs., I observed deflections of the needle amounting to 27′, 48′, and 1° 37′. Another experiment by the

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* The formula signifies in fact that the heat evolved by compressing a solid is equivalent to the work required to compress the volume of it due to temperature, just as in the case of a perfect elastic fluid, applied to which the expression becomes simplified to $\frac{p}{Jsw}$. M. CLAPEYRON has given a theoretical estimate of the heat disengaged by the cubical compression of iron (Scientific Memoirs, vol. iii. p. 373).

† LAVOISIER and LAPLACE.

‡ DULONG and PETIT.

first method to obtain the absolute thermal effect gave 53′, which was found to indicate a temperature of −0°·162. The theoretical result in this case was

$$H=\frac{275{\cdot}4}{1390}\times\frac{-775}{1}\times\frac{1^{*}}{81633}\times\frac{1^{\dagger}}{{\cdot}1024}\times\frac{1}{{\cdot}1499}=-0^{\circ}{\cdot}125.$$

26. *Cast Iron.*—The deflections produced by tensile forces of 194, 388, and 775 lbs. were 17′, 31′, and 59′·9 respectively. The thermal effect indicated by the last was −0°·1605. The formula gives

$$H=\frac{277{\cdot}4}{1390}\times\frac{-775}{1}\times\frac{1^{\ddagger}}{90139}\times\frac{1^{\S}}{{\cdot}1198}\times\frac{1}{{\cdot}1281}=-0^{\circ}{\cdot}112.$$

27. In another experiment I obtained a thermal effect of −·1481, theory giving

$$H=\frac{283}{1390}\times\frac{-784}{1}\times\frac{1}{90139}\times\frac{1}{{\cdot}1198}\times\frac{1}{{\cdot}1281}=-0^{\circ}{\cdot}115.$$

28. *Copper.*—With tensile forces of 191·6, 383·3, and 766·6 lbs. I obtained by the first method deflections of 21′, 41′, and 1° 12′·6 respectively, the absolute thermal effect indicated by the last being −0°·174. The formula gives

$$H=\frac{274{\cdot}9}{1390}\times\frac{-766{\cdot}6}{1}\times\frac{1^{\|}}{58200}\times\frac{1^{\P}}{{\cdot}095}\times\frac{1}{{\cdot}1781}=-0^{\circ}{\cdot}154.$$

29. With copper, and also with iron wires, I have observed the cooling effect of tension close up to the breaking-point. But whenever the pull occasioned permanent change of figure, the heat frictionally evolved overpowered the cooling effect.

30. *Lead.*—The bar employed was half an inch in diameter, and weighed 6308 grains to the foot. The stretching weights were, first, 193 lbs. with 70 lbs., the weight of the lever, constant; and second, 263 lbs., the weight of the lever itself being used along with the 193 lbs. laid on it. Using the first method, I obtained with the above weights deflections of 21′·5 and 30′·7, indicating thermal effects of −0°·0531 and −0°·0758. The formula gives in the two cases,

$$H=\frac{278{\cdot}5}{1390}\times\frac{-193}{1}\times\frac{1^{**}}{35100}\times\frac{1}{{\cdot}0303}\times\frac{1}{{\cdot}901}=-0^{\circ}{\cdot}0403,$$

and

$$H=\frac{278{\cdot}5}{1390}\times\frac{-263}{1}\times\frac{1}{35100}\times\frac{1}{{\cdot}0303}\times\frac{1}{{\cdot}901}=-0^{\circ}{\cdot}0550.$$

31. *Gutta Percha.*—The piece used was half an inch in diameter, and weighed 1246 grains per foot. The thermo-electric junction, formed of very thin copper and iron wires, was inserted in a slit made two-thirds through the substance of the gutta percha. The weights employed were, first, that of the lever alone amounting to 70 lbs.; and second, the lever with the further addition of 80 lbs., making in all 150 lbs. As with the metals, cold was produced when the gutta percha was stretched, and the heat restored when the stretching weight was removed. With the first-named tensile force a mean deflection of 16′·4 was observed, and with the second a mean deflection of 30′·2.

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[*] Smeaton. [†] Potter. [‡] Roy. [§] Myself.

[||] Lavoisier and Laplace. [¶] Dulong and Petit. [**] Lavoisier and Laplace.

In order to turn the above into thermal measure, the gutta percha was plunged into water of different temperatures, deep enough to immerse the part in which the thermo-electric junction was imbedded; the former plan of operation being inapplicable in this instance, on account of the low conducting power of the substance used. It was found in this way that the quantities of thermal effect due to the above deflections were $-0^{\circ}\cdot0284$ and $-0^{\circ}\cdot0524$, quantities, however, which are likely to err in defect by a very small quantity, owing to the nature of the method of finding the value of the deflections. The theoretical results, assuming as I did for the metals that the expansion of gutta percha by heat is the same whether suffering tension or not, are

$$H=\frac{276\cdot4}{1390}\times\frac{-70}{1}\times\frac{1}{6354}\times\frac{1}{\cdot402}\times\frac{1}{\cdot178}=-0^{\circ}\cdot0306,$$

and

$$H=\frac{276\cdot4}{1390}\times\frac{-150}{1}\times\frac{1}{6354}\times\frac{1}{\cdot402}\times\frac{1}{\cdot178}=-0^{\circ}\cdot0656.$$

32. The values just given for the expansion and specific heat of gutta percha are those which I arrived at by experiments on a part of the same sheet out of which the cylinder above used was made. The method pursued in obtaining the expansion was one I have frequently found very convenient. It consists in weighing the body in water at two temperatures, one as much below as the other is above the point of maximum density. In this way I found the linear expansion between $2^{\circ}\cdot4$ and $5^{\circ}\cdot8$ to be $\frac{1}{6354}$ per 1°, and the specific gravity to be 1·00462 at 4°. The specific heat at $4^{\circ}\cdot5$ was found by the method of mixtures to be ·402.

33. *India-rubber.*—The extraordinary physical properties of this substance appear to have been first remarked by Mr. GOUGH[*]. By placing a slip of it in slight contact with the edges of the lips, and then suddenly extending it, he experienced a sensation of warmth arising from an augmentation of the temperature of the rubber; and then by allowing the slip to contract again, he found that this increase of temperature could be destroyed in an instant. In his next experiment, he found that "if one end of a slip of caoutchouc be fastened to a rod of metal or wood, and a weight be fixed to the other extremity, in order to keep it in a vertical position, the thong will be found to become shorter with heat and longer with cold[†]." The third experiment described by Mr. GOUGH has indicated the means of using this substance in the production of textile fabrics; he says, "If a thong of caoutchouc be stretched, in water warmer than itself, it retains its elasticity unimpaired; on the contrary, if the experiment be made in water colder than itself, it loses part of its retractile power, being unable to recover its former figure; but let the thong be placed in hot water, while it remains extended for want of spring, and the heat will immediately make it contract briskly."

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[*] Memoirs of the Literary and Philosophical Society of Manchester, 2nd Series, vol. i. p. 288.

[†] Ibid. p. 292. From the context it appears that the weight was used to give tension, as well as to keep the slip vertical.

34. In addition to the above, my own experience has led me to the curious fact that a piece of india-rubber, softened by warmth, may be exposed to the zero of FAHRENHEIT for an hour or more without losing its pliability; but that a few days rest at a temperature considerably above the freezing-point will cause it to become rigid.

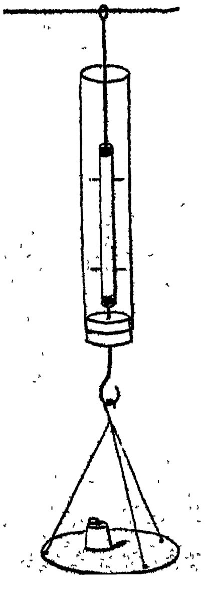
35. The ends of a piece of elastic india-rubber, about $\frac{4}{10}$ths of an inch square, were attached to an apparatus by means of which it could be stretched by various weights. A thermo-electric junction of thin copper and iron wires was placed in an orifice pierced through the centre of the slip; and at either side, at $\frac{6}{10}$ths of an inch distance from one another, pins were stuck which afforded the means of knowing the exact length to which the rubber was stretched. With small tensile forces no thermal effects were observed, but when the rubber was stretched by a weight of 6 lbs., a sensible deflection of the needle took place, indicating heat; the needle returning to zero, and thus indicating absorption of heat when the weight was removed. Experiments with different weights gave the following results:—

Stretching weight in pounds.	Length of rubber in inches		Deflection of thermo-multiplier.	Thermal effect indicated by deflection.
	When stretched.	After weight was removed.		
			° ′	°
0	0·59	0·59	0	0
2	0·67	Not observed.	0	0
4	0·77	Not observed.	− 1	−0·002
6	0·94	Not observed.	11	0·020
8	1·13	Not observed.	32	0·059
10	1·45	0·71	1 15	0·139
12	1·75	0·9	2 47	0·370
14	Not observed.	Not observed.	4 50	0·537
16	2·14	Not observed.	5 55	0·657
18	2·36	1·02	6 45	0·750

36. The low temperature (about 6° Centigrade) at which the above experiments were tried considerably interfered with the perfection of the elasticity of the india-rubber, which in consequence became gradually elongated on the removal of successively increasing weights. This circumstance no doubt interferes slightly with the absolute quantity of thermal effect recorded in the last column of the Table. Still the essential characteristics of the phenomena are plainly indicated, viz. that little heat, or even a reverse effect, is produced by moderate stretching weights, but that after a certain weight is reached, a rapid increase of thermal effect takes place.

37. The same piece of rubber was afterwards immersed in water in the manner represented by the adjoining sketch. A wire attached to the rubber at the upper extremity was made fast, and another wire affixed to the lower end passed through a cork in the glass tube, and being bent into a hook, supported a scale for weights. The temperature of the water in the tube was kept uniform by constant

Fig. 6.



additions and removals of water, which was kept well mixed by a current of bubbles of air blown from time to time through a narrow tube into the lower end. The positions of the pins, showing the length of the rubber, were ascertained by means of a graduated scale placed behind the tube, viewed through a telescope which was raised or depressed according to the level of the pins.

38. In my first experiment the stretching weight, inclusive of the weight of the tube, was 3 lbs., and the water in the tube being adjusted to 7° Cent., the length of the thong between the pins was 1·026 inch. The temperature of the water was then raised to 17°, and kept there for ten minutes, when the length was found to be reduced to 0·964 inch. A third observation at 28° gave the length 0·9. It was evident that this contraction was partly owing to the recovery by heat of the rubber from the constrained state in which it was left by the experiments of the preceding Table; for the lengths observed on lowering the temperature successively to 16°·6 and 7°·4, namely, 0·91 and 0·922, indicated a lengthening effect from the application of cold, only one quarter of the amount of the previous shortening by heat. I now used a stretching weight of 6 lbs., which immediately increased the length to 1·22, and in the course of thirty-six hours further increased it to 2·548 inches at the temperature 5°·4. Raising the temperature successively to 12° and 17°, the length was diminished to 2·527 and 2·459. Then on depressing it to 9°·5 and 4°·3, I observed 2·476 and 2·490. After leaving the same weight on for five hours longer, I observed lengths of 2·559, 2·514, and 2·47 with the temperatures 6°·2, 12°·4, and 20°·2, and 2·483 and 2·498 at temperatures successively depressed to 12° and 5°. After the lapse of twenty-two hours more, I found the lengths 2·624, 2·568, and 2·51 at the temperatures 6°·2, 13°·6, and 22°; and 2·523 and 2·548 on depressing the temperature to 13° and 6°·4. All these observations show the gradual elongation effected by tension, the tendency of heat to restore the original length, and the increase or decrease of elastic force with the elevation or depression of the temperature of the rubber.

39. I now reduced the stretching weight to 1 lb., being that of the tube and the water contained by it. The observed length was 1·63, and in forty-five minutes further decreased to 1·621. Afterwards it began to increase gradually; in five hours attaining to 1·635, in seventeen more to 1·724, and after a further lapse of forty hours to 1·751. These observations were at 7°·1. Then raising the temperature to 21°·6, the length was reduced to 1·191; and on depressing it again to 7°·4, the length was further reduced to 1·181. Keeping the rubber at the same temperature it still continued to contract, and in two hours its length was only 1·17. Then on raising the temperature to 23°, and afterwards depressing it to 7°·2, the length was successively reduced to 1·151 and 1·141. These and similar observations were continued for several days with like results, confirming my previous remarks, and also developing the curious fact, that when a piece of india-rubber, which has been previously gradually lengthened by stretching, is shortened by the temporary application of heat, the shortening effect continues to go on for some time, even if the rubber has been reduced to its previous temperature.

40. Although my experiments, already detailed, on the thermal effects of stretching india-rubber were not, on account of the imperfect elasticity of that substance at low temperatures, well calculated to give exact numerical results, the general conclusion derived from them is perfectly in accordance with Professor Thomson's formula, where, in this instance e, the expansion, being negative, the result indicated is a rise of temperature on the application of the tensile force.

41. When, by keeping india-rubber at rest at a low temperature for some time, it has become rigid, it ceases to be heated when stretched by a weight, and, on the contrary, a cooling effect takes place as in the metals and gutta percha. The experiment by which I ascertained this fact was made with a piece of india-rubber ¼ of an inch square, which had been exposed for several days to a temperature not lower than 5° Cent. In this state it was found that the application of a tensile force of 14 lbs. produced a deflection of 20′, indicating cold, and that a contrary deflection, indicating heat, was produced by removing the weight. After raising the temperature of the rubber to 15°, the elasticity of the rubber was restored, and the reverse phenomenon of heat on stretching produced, indicated by a deflection of 15′.

42. "*Vulcanized*" *India-rubber.*—I observed the principal physical characters of this substance before I was acquainted with Mr. Gough's discovery of the properties of simple india-rubber, above noticed. The superior permanency of the elasticity of rubber in combination with sulphur, and its unimpaired existence at low temperatures, rendered it better qualified for experiments in which accurate numerical results were desired.

43. I determined the *specific heat* of vulcanized india-rubber from portions of the specimens used in the experiments about to be detailed. The method of mixtures was employed, a quantity in small bits being raised to a given temperature in a bath of air, and then plunged into a thin copper can partly filled with cold water. The result, reliable to at least one-tenth of its amount, was 0·415.

44. The *expansion by heat* was found by weighing in water 2°·25 above and 2°·25 below the maximum density. The result showed a cubical dilatation of 0·000526 per degree Centigrade, an expansion greater than that of any other solid hitherto examined.

45. Mr. Gough has stated that the specific gravity of india-rubber is increased by stretching it; and to this he attributes the heat thereby evolved. The experiment, however, is a very delicate one, and it does not appear that this philosopher possessed the means of arriving at a reliable result. My experience with vulcanized india-rubber leads me to doubt the accuracy of Mr. Gough's conclusion. I weighed elastic bands alternately stretched and unstretched in water. The result of the first series of experiments was, that the bands unstretched had a specific gravity ·996057, but when pulled twice their natural length, ·994641. The second series gave me for the specific gravity of the unstretched bands ·990918, but when pulled to two and a half times the natural length, ·988483. The same vulcanized india-rubber bands were used in both series, and the diminution of the specific gravity in the second series is owing to the constant loss of sulphur taking place. On account, however, of the alternation of the experiments in

each series, this circumstance does not affect the conclusion that the action of tension on a band of vulcanized india-rubber sensibly diminishes its specific gravity.

46. In my experiments on the thermal effects of stretching vulcanized rubber, a piece about $\frac{3}{8}$ths of an inch square, and weighing 1452 grs. to the foot, was employed. The upper end was made fast, and to the lower, weights could be applied and removed at pleasure. A thermo-electric junction of thin copper and iron wires was inserted in a slit made in the direction of the length of the thong and near its middle. Also pins were stuck into the rubber at 4 inches distance from each other, in order to measure its length under various tensile forces. The following is a Table of results:—

Stretching weight in pounds.	Length of measured part, under tensile force, in inches.	Deflection of thermo-multiplier.	Thermal effect indicated by the deflection.
0	4	° ′ 0	° 0
2	4·06	−1·4	−0·003
4	4·12	−2	−0·004
7	4·3	−2·4	−0·004
14	4·8	−0·5	−0·001
21	5·21	7·9	0·014
28	5·87	29·1	0·053
35	6·6	51·9	0·095
42	7·25	1 15·7	0·137
49	7·75	1 39·3	0·180

47. The data for turning the deflections in the third column into the thermal effects in the fourth, were obtained by immersing the rubber, to a point above the place where the thermo-electric junction was inserted, in water of different temperatures.

48. These experiments developed the following facts:—1st. That the effect of laying on the weights was not sensibly different in quantity from the reverse effect of removing them. 2nd. That with light weights and a low temperature there was a slight cooling effect on the application of tensile force, which, first increasing with the weights laid on, ultimately changed into a heating effect, increasing much more rapidly than the stretching weight.

49. The temperature of the thong during the experiments was 7°·8. I found that at temperatures a few degrees higher, the reverse action with weak tensile forces did not take place, but that there was, on the contrary, a very slight heating effect.

50. Professor THOMSON suggesting to me that in those circumstances in which the rubber was heated by being stretched, it would contract its length under tension when its temperature was raised, I arranged the same piece with which the foregoing experiments were made in the manner already described in the case of common india-rubber. The method pursued was to observe the lengths at several temperatures successively raised, and then at similar temperatures descending. The necessity for this arose from the circumstance that a gradual elongation took place, particularly at the higher temperatures, and by taking the mean of the ascending and descending series, I hoped to eliminate the error thus arising. I give the following experiments in the order in which they were made:—

Tensile force in pounds.	Temperature successively raised.	Length in inches.	Temperature successively lowered.	Length.	Mean temperature of the two series.	Mean length.	Shortening per degree Cent. from 0°.
14	13°·4	4·899	9°·2	5·008	11°·3	4·953	$\frac{1}{2854}$
	22·5	4·894	21·5	4·979	22	4·936	
	32	4·888	30·8	4·952	31·4	4·920	
	48·6	4·888	……	……	48·6	4·888	
7	7·95	4·569	8	4·476	7·97	4·522	$\frac{1}{3536}$
	22	4·526	19·1	4·480	20·55	4·503	
	33·2	4·481	31·5	4·481	32·35	4·481	
	50·1	4·468	……	……	50·1	4·468	
21	2·8	5·497	6	5·661	4·4	5·579	$\frac{1}{2124}$
	18	5·468	17·4	5·612	17·7	5·540	
	33	5·458	32·4	5·540	32·7	5·499	
	50	5·459	……	……	50	5·459	
28	1·4	6·398	4·4	6·513	2·9	6·455	$\frac{1}{1001}$
	18	6·298	18	6·435	18	6·366	
	33·6	6·224	34	6·302	33·8	6·263	
	49·6	6·153	……	……	49·6	6·153	
35	1·8	7·467	3·4	7·589	2·6	7·528	$\frac{1}{887}$
	19	7·264	18	7·432	18·5	7·348	
	34	7·121	33	7·250	33·5	7·185	
	49·8	7·009	……	……	49·8	7·009	
42	−0·4	8·604	2·6	8·792	1·1	8·698	$\frac{1}{828}$
	17	8·415	16·6	8·575	16·8	8·497	
	34	8·195	32	8·327	33	8·261	
	50	8·020	……	……	50	8·020	

51. The above results completely justify the anticipation of Professor THOMSON, and, as we shall presently see, afford a remarkable confirmation of his theory. Before, however, instituting a comparison between theory and experiment, it will be proper to observe that the length of the rubber at the low temperatures beginning and ending each experiment show a gradual elongation. To this circumstance chiefly is it owing that so considerable a shortening effect took place with 7 lbs. tension; for a tensile force of 14 lbs. having been just previously tried, a certain amount of increased length of the nature of *set* was produced, which being taken out by heating the rubber under less tension, caused a greater apparent contraction by heat. On lowering the temperature again, a small contraction took place, which, being corrected for the gradual elongation of the rubber, gave me $\frac{1}{9009}$ as the expansion by heat under the tension of 7 lbs.

52. The following Table gives the theoretical estimate, compared with the experimental result, for each additional tensile force of 7 lbs. :—

Weight laid on in pounds.		$\frac{t}{J}$.	p.	e, or mean expansion by heat between permanent and temporary tension.	$\frac{1}{s}$.	$\frac{1}{w}$, or reciprocal of mean weight per foot.	Theoretical result, or product of the preceding five columns.	Experimental result taken from the foregoing Table.	Experimental result corrected for elongation of rubber by use.
Permanent.	Temporary.								
0	7	$\frac{281}{1390}$	-7	$+\frac{1}{9009}$	$\frac{1}{\cdot415}$	$\frac{1}{\cdot1917}$	−0°·002	−0°·004	−0°·004
7	14	$\frac{281}{1390}$	-7	$-\frac{1}{5003}$	$\frac{1}{\cdot415}$	$\frac{1}{\cdot1759}$	+0·004	+0·003	+0·003
14	21	$\frac{281}{1390}$	-7	$-\frac{1}{2435}$	$\frac{1}{\cdot415}$	$\frac{1}{\cdot1576}$	+0·009	+0·015	+0·016
21	28	$\frac{281}{1390}$	-7	$-\frac{1}{1360}$	$\frac{1}{\cdot415}$	$\frac{1}{\cdot1383}$	+0·018	+0·039	+0·043
28	35	$\frac{281}{1390}$	-7	$-\frac{1}{814}$	$\frac{1}{\cdot415}$	$\frac{1}{\cdot1194}$	+0·035	+0·042	+0·047
35	42	$\frac{281}{1390}$	-7	$-\frac{1}{656}$	$\frac{1}{\cdot415}$	$\frac{1}{\cdot1031}$	+0·050	+0·042	+0·050
0	42						+0·114	+0·137	+0·155

53. In the last two columns of the above Table are recorded the actual quantities of heat observed in the experiments given in (46), and the same corrected on account of the thong not having then been so much stretched by use. I am of opinion, however, that this circumstance would not cause any material difference in the thermal effect of a given strain, and that therefore the last column but one is the proper one to compare with theory, which it will be found to do quite as well as could have been expected.

54. After the experiment on the contraction by heat of the thong when it was stretched by 42 lbs., I removed all the weights except 2 lbs., and then a day or two afterwards observed as follows:—

Temperature successively raised.	Length.	Temperature successively lowered.	Length.	Contraction per degree of temperature raised.	Contraction per degree of temperature lowered.
−0°·4	4·445	3°·2	4·236	$\frac{1}{1212}$	$\frac{1}{8260}$
25	4·339	26	4·251		
50	4·26	50	4·26		

55. It will be observed that the effect of a rise of temperature in the above experiment was to take out the *set* which had been given to the thong by previous experiments at higher tension. Afterwards, the effect of cooling, as shown in column 4, is a contraction so great, as to prove that the *set* continued to be taken out after the heat which commenced its removal was withdrawn. That this was the case was proved by repeating the experiment several times, using the same weak tension of 2 lbs.; when the lengths at 8°·8 and 52°·8 were found to be 4·241 and 4·254 respectively, indicating an expansion of only $\frac{1}{14376}$ per degree.

56. On removing all tensile force, and applying a moderate warmth, the thong

returned to the same length it had at the commencement of my experiments with it, viz. exactly 4 inches.

57. In order to try the thermal effects of stretching vulcanized india-rubber for a greater range of tensions, I inserted a very fine thermo-electric junction into the breadth of an elastic ring 2 inches in diameter and weighing 30 grains. The ring, when pulled sufficiently to make the two sides parallel, became a double band 3·3 inches long, which elongated further to 6·8 inches with a tension of 2½ lbs. The results I arrived at are as follow:—

Permanent weight in pounds.	Length with permanent weight in inches.	Weight laid on in addition to the permanent weight.	Length with the additional weight.	Thermal effect in degree Centigrade.
0	3·3	2·5	6·8	0·110
2·5	6·8	2	10·9	0·242
4·5	11	2	14·6	0·330
6·5	14·8	2	17·2	0·132
8·5	17·8	2	18·5	0·088
10·5	18·9	2	19·4	0·068
12·5	19·7	2	20·1	0·004
14·5	20·5	4	20·9	0·001
18·5	21·1	2	21·3	0·009
20·5	21·7	4	22	0·044
24·5	22	4 band broke.		
			Sum ...	1·028

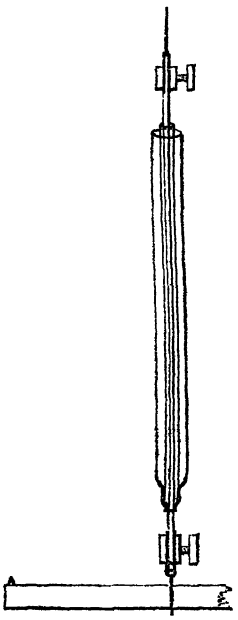
58. The Table shows that the rise of temperature occasioned by stretching vulcanized india-rubber bands up to their breaking-point is about 1° Centigrade, a quantity which I regard as erring somewhat in deficiency, owing to the attenuation of the band under high tension preventing the full communication of heat to the junction. The thermal effect is evidently greatest when the rubber receives the greatest elongation by an additional weight. When the band was stretched to six times its original length, little further elongation took place by increasing the tensile force, very little thermal effect being at the same time produced. I thought it possible that in this case a small cooling effect might exist, but after many trials did not succeed in finding it, owing probably to frictional generation of heat near the breaking-point.

59. *Wood.*—The physical properties of this substance have great interest, from its various applications to the wants and luxuries of mankind. On making experiments on the thermal effects of its extension and compression, I soon found anomalies which induced me to investigate at some length its expansion by heat, its elasticity, and the variations of these arising from such modifying influences as temperature, moisture, and tension.

60. The apparatus I employed will be understood from the following sketch, in which a wooden rod is represented as passing through a glass tube, which again passes through a wide tube of gutta percha. This latter fits tightly at its lower end on the glass tube, which again is made tight to the wood by means of a piece of india-

rubber tube. Mercury or water may be poured into the interstice between the glass tube and the wooden rod, and so serve as the means of convection of heat from the water with which the gutta-percha tube is filled. A stopcock enables one to change this water when desired. The ends of the rod are held by suitable clamps, which are attached, one to a firm upper support, the other to a lever of which one extremity has a knife-edge resting on a steel plate, and the other is furnished with a micrometer consisting of a glass plate, divided into $\frac{1}{200}$ths of an inch, examined by a microscope. The weight of the lever alone produced a tension of 35 lbs., which could be further increased by the addition of weights. When a very weak tension was desired, a lighter lever was employed.

Fig. 7.



61. Preliminary trials informed me of the time (never exceeding five minutes) required to bring the rod sensibly to the temperature of the water in the gutta-percha tube; I also found that the most ready means of obtaining uniform temperature in this water was to fill the tube twice, allowing a little intervening time. The observations of the micrometer were generally made at intervals of five minutes; the first observation being at a low temperature, the second at a high temperature, and the third at a low temperature again. The mean of the first and third observations gave me the length at the low temperature, the difference between which and the length at the high temperature, gave me the expansion of the rod. When a high tension was employed, gradual elongation frequently took place for so long a time, that I did not wait till it had entirely ceased. The system of observation employed was, however, such as prevented error arising from this circumstance; and since the gradual elongation was more rapid as the temperature was raised, the precaution was taken to expose the rod to the high temperature for the same space of time immediately before and after each micrometer observation at the high temperature.

62. *Bay Wood.*—The piece selected was well seasoned and very straight in the grain. Its length was 46·63 inches; the part passing through the glass tube and exposed to change of temperature measuring 33 inches. The diameter of the rod was $\frac{3}{8}$ths of an inch, and its weight, in the first instance, about 196 grains to the foot. Mercury poured into the inner glass tube formed the means of conducting heat from the surrounding water to the rod. The following Table contains the observed expansions after various intervals of time:—

Interval of time between the successive determinations.	Tension on the rod, in pounds.	Expansion per degree Centigrade.
	35	·000004612
A few hours	435	·000005665
Two or three days	35	·000004372
A few hours	435	·000005685
Two or three days	35	·000003952
A few hours	435	·000004699
Two or three days	35	·000004003
A few hours	435	·000004850
Mean for	35	·000004235
Mean for	435	·000005225

63. The above experiments manifest most strikingly the effect of tension in increasing the expansibility of wood, that with 435 lbs. being nearly one quarter greater than that with only 35 lbs. The gradual diminution of the coefficient perplexed me a good deal at first, particularly as the mercury in contact with the wood prevented the access of air to its sides, but I was ultimately able to refer it to the gradual absorption of moisture through the pores in the direction of the length of the rod.

64. The rod was now exposed to a tension of 435 lbs. for twenty-two days, at the end of which time its expansion by heat was found to be ·000004784. The weights were then removed so as to reduce the tension to 35 lbs. After remaining two days in this state, the first effect was found not to be expansion, but, on the contrary, a contraction amounting to ·000002408 per degree. Then the subsequent application of cold produced a contraction of ·000005705. On raising the temperature again after an interval of five minutes I found an expansion of ·000002170, and then on cooling, a contraction of ·000005017. After twenty minutes I found that the expansibility by heat, indicated by the mean effect of raising and depressing the temperature alternately, was ·000003575.

65. From the above it appears that wood has to a certain extent the property possessed by india-rubber, of returning from a state of strain on the application of heat. Mr. Hodgkinson has observed this property in iron; I have noticed it in whalebone, hair, leather, and to a slight extent in copper.

66. The increase of expansibility with tension suggested that the force of elasticity of wood decreases with a rise of temperature. To try this I affixed a graduated glass plate to the top of the glass tube (see fig. 7), while parallel and close to it another divided glass plate was affixed to the wooden rod. The extension of the rod under various tensile forces was thus read off by means of a microscope in $\frac{1}{200}$ths of an inch and fractions of the same appreciable to $\frac{1}{20}$th of the actual divisions. The mean of several series of trials gave me, at a low and high temperature, the following indications of the micrometer, with stretching weights increased by 100 lbs. at a time until 435 lbs., and then decreased by the successive removal of the weights again.

Tension.	Micrometer observations, the temperature of the wood being 15°·99.	Micrometer observations, the temperature of the wood being 41°·78.
35	0	0
135	4·199	4·381
235	8·539	8·914
335	12·890	13·415
435	17·135	17·743
335	12·830	13·392
235	8·579	8·968
135	4·373	4·602
35	0·231	0·268

67. $17·135-\frac{·231}{2}=17·02$, and $17·743-\frac{·268}{2}=17·609$, which are therefore the elastic strains in $\frac{1}{200}$ths of an inch, produced by a tension of 400 lbs. at the respective temperatures 15°·99 and 41°·78, the length of the rod being 33 inches. These results, reduced to unity of length and temperature, give the difference between ·000099953 and ·000103410, or ·000003459, as the quantity by which the expansion by heat of the wood ought to be increased by adding a tension of 400 lbs. Referring above, it will be seen that this increment is actually only about ·000001. I believe that two causes are concerned in producing this discrepancy. In the first place, I have reason to think that when tension is applied to wood and various other substances, a strain takes place of the nature of *set* besides the true elastic strain, which set is almost instantaneously taken out again on the removal of the tension; also that this temporary set increases with the temperature. Secondly, I think that when the expansion by heat of a body under tension is observed, the tendency of heat to take out *set* operating in a contrary direction to the expansive action, causes a slight apparent diminution of the latter. It will thus happen that the apparent decrease of elasticity by heat is greater than the real, while the apparent increase of expansion by heat, in consequence of the application of tension, is less than the real. These observations will be found to receive further confirmation from the phenomena of the elasticity of moistened wood, and also in the case of whalebone, a substance in which the characteristics of wood as regards *set* are very strongly developed.

68. Young's modulus of elasticity, deduced from the above data in terms of the length of a rod weighing one lb. to the foot, which would be extended by unity by the tensile force of one lb., is 5539710 and 5354349 for the temperatures 15°·99 and 41°·78 respectively.

69. The same rod of bay wood was now employed to determine the effects of hygrometric condition on its expansion and elasticity. Mercury or water was employed as the medium for conducting heat between the glass tube and the wood. When mercury was employed, some inconvenience resulted from its entry into the pores of the wood. Ultimately I found that the heat was conveyed with sufficient rapidity when air only occupied the narrow space intervening between the glass and the wood.

Remarks.	Weight of rod per foot in grains.	Medium of communication of heat between the glass tube and the rod.	Tension in pounds.	Limits of temperature between which the experiments were made.	Linear expansion or contraction per degree Centigrade.	YOUNG's modulus of elasticity or length of rod, 1 lb. per foot, which would be extended unity by one lb.
Moistened by immersion in water a short time. Experiment made half an hour afterwards	201	Mercury ...	35 435	42°·27 and 15°·2 42·3 and 16·01	·000002255 E. ·000004109 E.	
Moistened further by immersion under water forty hours. Experiment made shortly afterwards	227	Mercury ...	35 35 35 435	21·6 and 10·8 33 and 13·3 49·6 and 11·7 37·33 and 13·4	0 ·000000176 E. ·000000436 E. ·000001140 E.	 5721154 at 38°·2 6086956 at 12·4
Dried before a fire for twenty hours	196	Mercury ...	35 435	40·6 and 12·3 40·6 and 13·6	·000004555 E. ·000005481 E.	7472530 at 13
Immersed in alcohol for two days	215	Alcohol ...	35 435	31·5 and 12·8 37·05 and 14·72	·000002692 E. ·000003000 E.	
After immersion for one week in water, sp. gr. being then 0·82	291	Water	35 435	39·3 and 12·7 36·5 and 13·5	·000000388 C. ·000000327 E.	5948141 at 10·4 5242724 at 37
After it was kept warm before a fire for five days in frosty weather it became electrified on rubbing it with the hand, and retained its charge a long time	176	Mercury ...	35	39·3 and 11·7	·000004516 E.	7309582 at 13

70. *Deal.*—My next experiments were on a rod of St. John's Pine, perfectly free from knots and straight in the grain. Its diameter was ⅜ths of an inch, and the length subjected to thermal influence 33 inches, as before.

Remarks as to the condition of the wood.	Weight of rod per foot in grains.	Proximate specific gravity.	Medium of communication of heat between the glass tube and the rod.	Tension in pounds.	Limits of temperature between which the experiments were made.	Linear contraction or expansion per degree Centigrade.
Dried by remaining before a fire a few hours	132·4	......	Mercury ...	35 235	41°·7 and 13°·43 41·5 and 15·41	·000004277 E. ·000004379 E.
Moistened by immersion for a few hours in water.........................	172	·557	Water	35	48·7 and 15·36	·000000768 E.
Boiled in water, and then left immersed in cold water some hours ...	241·2	·761	Water	35 35	41 and 12·78 24·5 and 12·98	·000000377 C. ·000000360 C.
Left immersed in water three days longer	272·4	·854	Water	35	48·7 and 14·62	·000000636 C.
Left several days longer immersed in water	303·6	......	Water	35	12·7 and 2·7 36·4 and 7·5 40·8 and 28·8 63·6 and 38·6	·000000235 C. ·000000260 C. ·000000088 E. ·000000446 E.
Dried before a fire for several days, until it could be electrified by rubbing with the hand..........................	128·8	......	Air	35	37·16 and 11·57	·000004342 E.
Covered with pitch............................	141·8	......	Air	35	32·2 and 10·83	·000004030 E.

71. *Deal cut across the grain.*—The specimens were cut at right angles to the pores of the wood, but obliquely in respect to the concentric rings which indicate the growth of the tree. The length subjected to thermal influence was 12 inches.

Remarks on the condition of the wood.	Weight of rod per foot in grains.	Proximate specific gravity.	Medium of communication of heat between the glass tube and the rod.	Tension in pounds.	Limits of temperature between which the experiments were made.	Linear contraction or expansion per degree Centigrade.	YOUNG's modulus of elasticity or length of rod, 1 lb. per foot, which would be extended unity by 1 lb. tension.
				lbs. oz.	° °		
Dried before the fire for several days until it could be electrified by rubbing with the hand, and retain the charge a long time	204·2	·428	Mercury ...	4 5 16 5	36·9 and 7·35 38·5 and 8·9	·000041666 E. ·000045928 E.	203732
Boiled in water a few minutes, then left immersed in water for one day.........	593·2	1·122	Water	4 5 4 5	33·1 and 4·38 52 and 6·1	·000004497 C. ·000006455 C.	

72. The length of the wood saturated with water was 1·062 of its length in the dry state. In the saturated condition it broke after a tension of 17⅛ lbs. had been applied for half a minute. The specific gravity of the wood, considered apart from the air or water absorbed, was found to be 1·506.

73. The following Table contains the results of experiments with a rod of deal, similar to the last, also cut across the grain:—

Remarks on the condition of the wood.	Weight of rod per foot in grains.	Extreme length in inches.	Medium of communication of heat between the glass tube and the rod.	Tension in pounds.	Limits of temperature between which the experiments were made.	Linear contraction or expansion per degree Centigrade.	YOUNG's modulus of elasticity or length of rod, weighing 1 lb. per foot, which would be extended unity by 1 lb. tension.
Dried before a fire until it could be electrified by friction	193·5	15·625	Mercury	4·3	37°·5 and 10°·9	·000042470 E.	188120
On immersion in water, the wood absorbed in a few minutes sufficient moisture to make it weigh 243 grs. per foot. Its length was then 15·8; but after allowing it to rest two hours its length became 16·24, without receiving any additional water in the interim. Twelve hours afterwards its length was 16·3, and its weight per foot 228·2 grs.	228·2	16·3	Mercury	4·3	35·1 and 10·7	·000065153 E.	104600
Further immersion in water ...	270·5	16·594	Mercury	4·3	41·3 and 10·91	·000000380 E.	60033
Immersed several hours in water; specific gravity then was found to be ·891	446	16·625	Mercury	4·3	43·2 and 10·95	·000005404 C.	38257
Boiled in the exhausted receiver of an air-pump, and afterwards left immersed in water several hours; sp. gr. 1·143	573·4	16·7	Water ...	4·3 7·3	34·5 and 13·25 27·2 and 12·12	·000002201 C. ·000001105 C.	29618
Dried a little by placing it a short time before a fire ...	434·4 444·0	16·63 16·63	Air ...... Water ...	4·3 4·3	39·03 and 12·8 34·9 and 12·7	·000003159 E. ·000001561 C.	
Dried still further	358 370	16·62 16·62	Air ...... Water ...	4·3 4·3	37·38 and 13·3 43·8 and 15	·000010390 E. ·000001356 C.	

TABLE (continued).

Remarks on the condition of the wood.	Weight of rod per foot in grains.	Extreme length in inches.	Medium of communication of heat between the glass tube and the rod.	Tension in pounds.	Limits of temperature between which the experiments were made.	Linear contraction or expansion per degree Centigrade.	YOUNG's modulus of elasticity or length of rod, weighing 1 lb. per foot, which would be extended unity by 1 lb. tension.
Dried still further	231·5	16·48	Air	4·3	37°·7 and 10°·24	·000084212 E.	96170
Water poured between the glass tube and the rod. Then after a short time ...	......	......	Water ...	4·3	37·2 and 10·3	·000058836 E.	82430
After ten minutes more had elapsed	......	......	Water ...	4·3	40·9 and 12·15	·000019457 E.	
After another ten minutes	......	......	Water ...	4·3	37·6 and 12·3	·000004815 E.	
After another ten minutes	......	......	Water ...	4·3	42 and 12·3	·000001017 C.	
After another ten minutes; sp. gr. ·5646	272·3	16·656	Water ...	4·3	39·34 and 12·53	·000002951 C.	
			Air	4·3	37·5 and 9·7	·000000817 C.	
Dried some hours in a moderately heated oven	206	15·55	Air	4·3	41·5 and 9·77	·000038620 E.	
Immersed in oil under the exhausted receiver of an air-pump	329	15·54	Air	4·3	38·8 and 10·6	·000043670 E.	

74. The specific gravity of the wood, considered apart from the air or water in its pores, was in the above specimen found to be 1·595.

75. The following are deductions from the above experiments:—1st. That tension increases the expansibility of wood by heat. 2nd. The expansibility of dry wood cut across the grain is about ten times as great as its expansibility when cut in the direction of the grain. 3rd. The length of wood cut longitudinally is a little increased by the absorption of water; but when cut crossways, a very great augmentation, as is well known, takes place. 4th. When water is absorbed into the pores of wood cut longitudinally its expansibility decreases, until ultimately it passes into contraction by heat. 5th. When water is absorbed into the pores of wood cut across the grain, its expansibility appears to increase in the first instance, then decreases until it changes into contraction, which, after increasing with the absorption of water to a certain point, seems to diminish as the wood becomes completely saturated. 6th. Cut lengthways, the elasticity of wood appears to be lessened by the presence of water; when, however, it is cut across the grain, the effect of moisture is very greatly to impair the elastic force. 7th. A short period of immersion enables wood cut crossways to take up sufficient water to make it contract by heat; but, on the contrary, wet wood partially dried on the surface is expanded by heat. So that we may have contraction or expansion of wood charged to the same extent with water, according probably to peculiarities in the distribution of the water among the pores.

76. When wet wood, cut across the grain, is dried, it gets shorter very gradually, in consequence of a tendency to retain its former dimensions. The strain thus arising may be removed by raising the temperature, which speedily reduces the wood to the length due to its state of dryness. The same kind of effect takes place, but in the reverse

order, in wood which is taking up water. It does not immediately assume the length due to the state of hygrometry, but if heated the set is taken out, and the wood expands to its proper length.

77. Inasmuch as at a certain degree of humidity wood contracts in every direction with a rise of temperature, it might be inferred that on weighing it in water its specific gravity would be found to increase on the elevation of temperature. Such, however, is not the fact. A piece of saturated wood, being part of the specimen employed in the last series of experiments, was found to weigh 392·6 grs. in air, 42·26 grs. in distilled water at 0°, and 42·01 grs. in distilled water at 8°. Hence the cubical expansion was ·0000892 per degree, but calculated on the wood considered apart from the water in its pores, ·000396. Therefore on raising the temperature a decrease of specific gravity occurs simultaneously with a diminution in the external dimensions of the wood. This can only take place in consequence of a contraction of the surfaces of the walls of the cellular structure, while the actual bulk of the material of which they are composed is increased, a certain minute quantity of water exuding at the same time.

78. The phenomenon in question is therefore, I believe, owing to capillary attractions, which, diminishing with elevation of temperature, has the effect of removing a part of the swelled condition due to that action. Dr. YOUNG has given $\frac{1}{555}$ as the descent of water in a narrow tube due to a rise of temperature of 1° Cent. The fraction arrived at by M. C. WOLF is $\frac{1}{525 \cdot 67}$. To apply the latter estimate to the foregoing results, we find from the last Table that between extreme dryness and perfect humidity the wood increased in length from unity to 1·068. Hence, on the hypothesis that the contraction by heat is owing to diminution of capillary attraction, it ought to be equal to ·0001293 minus the expansion in the dry condition. An examination of the foregoing Tables will be found to afford ample support to this view.

79. *Ratan Cane.*—My observations of the expansion and elasticity of this substance are comprised in the following Table. The length exposed to thermal influence was 33 inches.

Remarks on the condition of the wood.	Weight of rod per foot in grains.	Medium of communication of heat between the glass tube and the rod.	Tension in pounds.	Limits of temperature between which the experiments were made.	Linear contraction or expansion per degree Centigrade.	YOUNG's modulus of elasticity, or length of rod, weighing 1 lb. per foot, which would be extended unity by 1 lb. tension.
Dried by placing it before a fire for a short time ...	150·2	Mercury ...	35 91	25°·3 and 11°·67 28·05 and 12·09	·000024750 E. ·000027000 E.	1094242 at 10°·6 918179 at 34 956720 at 29 1048546 at 12
Wetted by immersion under water for a few hours...	208	Water	35	26·6 and 11·04	·000001436 C.	
Dried by exposure before a fire for several hours....	145	Mercury ...	35	27·8 and 11·1	·000016740 E.	1214179 at 12

80. The following are the results of my experiments on rose, vine, and poplar. These were sprouts of recent growth, selected for their straightness. They were cut from

the tree in November, directly before the experiments. I have also inserted in the Table the expansion and elasticity of wheat-straw.

Kind of wood, &c.	Weight of rod per foot in grains.	Medium of communication of heat between the glass tube and the rod.	Tension in pounds.	Limits of temperature between which the experiments were made.	Linear expansion or contraction per degree Centigrade.	YOUNG's modulus of elasticity, or length of rod, weighing 1 lb. per foot, which would be extended unity by 1 lb. tension.
Rose wood in the green state, the bark not removed; sp. gr. ·638 ...	171·6	Water	35 91	46° and 14°·53 42 and 16	·000000240 E. ·000001101 E.	
Vine in the green state, bark not removed	116	Water	35	28·6 and 11·3	·000002121 E.	
Poplar in the green state, bark not removed	158·6	Water	35 40	23·2 and 10 39 and 9	·000000773 C. ·000000037 E.	
Wheat-straw dried before a fire	14·8	Air	5·2	33·4 and 11	·000004865 E.	3520625
Wheat-straw moistened by immersion in water ...	38·1	Air	5·2	16·8 and 12·13	·000000329 C.	982978

81. Cane, like wood cut in the direction of the grain, increases slightly in length in consequence of being moistened. The expansion by heat being diminished at the same time, it follows that the increment due to moisture is less when taken at a high than at a low temperature.

82. The following Table records my observations on the expansion and elasticity of paper, leather, and whalebone. The length exposed to thermal influence was 33 inches.

Remarks.	Weight per foot in grains.	Medium of communication of heat between the glass tube and the substance.	Tension in pounds.	Limits of temperature between which the experiments were made.	Linear expansion per degree Centigrade.	YOUNG's modulus of elasticity, or length, weighing 1 lb. per foot, which would be extended unity by 1 lb. tension.
A strip of cardboard one-third of an inch broad, dried before a fire	35	Air	5·2	39°·4 and 9°·4	·000015780	
A strip of cardboard one-third of an inch broad, moistened with water, which increased its length $\frac{1}{48}$th.	39·8	Air	5·2	40·3 and 11·5	·000026520	
A strip of cardboard one-third of an inch broad, moistened still further, which increased its length still further, $\frac{1}{120}$th.	47	Air	5·2	39·45 and 12·2	·000028950	
Cowskin leather dried before a fire. A narrow strip	17·55	Air	5·2	27·77 and 11·5	·000038720	227586
Cowskin leather moistened with water	19·95	Air	5·2	29 and 10·47	·000047116	167813
Cowskin leather moistened still further	25·8	Air	5·2	24·25 and 9·75	·000054401	118642
Whalebone, a strip one-third of an inch broad. Dry	60	Air	35 70	24·4 and 9·27 24·35 and 9·9	·000052200 ·000055020	1176554 at 10°
Whalebone, a strip one-third of an inch broad, moistened with water...	71·8	Air	5·2	23·7 and 11	·000024270	

83. It will be observed that an increase in the expansibility by heat of paper and leather was produced by moisture. It is probable, however, that had they been more thoroughly saturated, the expansion would again diminish, as in the case of wood cut across the grain, in which, although the expansibility in the first instance increased with the application of moisture, it ultimately changed into contraction when the wood became more completely saturated. I think that a cause tending to increase the expansibility in consequence of the application of moisture exists in every case; but that it is modified, and in wood sometimes completely overcome, by the effect of decreased capillary attraction with rise of temperature, to which I have already adverted.

84. Upon the facility with which hot moistened whalebone can be moulded into form, and the permanency with which it keeps the shape in which it is cooled, much of its use in the arts depends. When heat is applied to whalebone, thus cooled in a constrained state, it at once returns to its original shape. Even when cold moistened whalebone is strained, it returns almost immediately to its original dimensions on the application of heat. The slip of moistened whalebone employed in the experiments recorded in the last Table, 46 inches long and weighing 71·8 grains to the foot, became rapidly stretched 7 inches longer by a tension of 64 lbs. Left to itself after the tensile force was removed, it began to return gradually towards its original length, which however it at once assumed on immersion in hot water.

85. The apparent elasticity of whalebone is always somewhat less than the real, owing to the observed elongation, when tension is applied, consisting of a considerable amount of *set* as well as of elastic strain. Also when the tensile force is removed, the contraction which takes place consists of the return from *set* as well as of the elastic recoil. The interesting phenomena connected with imperfect elasticity might, I think, be very advantageously studied in this substance. The limits I had set myself did not, however, allow me to do more than obtain the following determinations of its elasticity in the dry state at two different temperatures, and with various intervals between the application or removal of tension and the corresponding observations.

Time allowed to elapse between laying on or taking off a tensile force of 35 lbs., and making the observation of length.	YOUNG's modulus of apparent elasticity, or length of rod, weighing 1 lb. per foot, which would be extended unity by 1 lb. tension.	
	At 10°·1.	At 30°·1.
$15''$	1176554	962450
$30''$	1147383	927100
$1'$	1119624	894545
$2'$	1080415	
$4'$	1043860	
$15'$	1028395	
1^h	922481	

86. *Thermal effect of tension on Wood.*—My first experiment was tried on a square rod of straight-grained pine, dried, and weighing 132·4 grains to the foot. A thermo-electric junction of thin copper and iron wires having been inserted into it, its upper extremity

was clamped to a firm support, and the lower attached to a lever giving a tension of 35 lbs. On applying an additional tension of 200 lbs., the motion of the needle of the thermo-multiplier indicated cold; on removing the tension, a reverse motion of equal extent indicated the evolution of heat. The mean of several trials gave me 9′·87, the thermal value of which, determined by immersing the junction in water of various temperatures, was ·01364. Owing to the large surface of the wood as compared with its capacity for heat, it was affected by the temperature of the surrounding atmosphere nearly three times as much as the quarter-inch metal bars. I found, in fact, that during the time occupied by a swing of the needle, viz. 45″, one-fifth of the thermal effect was lost. Adding, therefore, one quarter to ·01364, we find ·01705 for the thermal effect, which, it may be observed, errs a trifle in defect, owing to the conduction of a small portion of heat along the wires of the thermo-electric junction.

87. In order to arrive at the theoretical result, it was necessary to determine the specific heat of the wood in the condition it was in when used. This was done by raising the temperature of a faggot of it in a bath of mercury, and then plunging it in a thin copper can filled with mercury at the atmospheric temperature. The fall of temperature of the wood and the rise of temperature of the can and mercury, corrected for the atmospheric influence, indicated the specific heat; which, in the case of dried St. John's pine, was found to be ·3962, and in that of dried bay wood, ·3582.

88. The theoretical thermal effect of tension in the foregoing experiment, derived from the above data and the expansion by heat at the mean tension, is

$$H=\frac{279{\cdot}4}{1390}\times\frac{-200}{1}\times\frac{1}{231053}\times\frac{1}{{\cdot}3962}\times\frac{1}{{\cdot}0189}=-0^\circ{\cdot}02322.$$

89. *Bay Wood.*—The rod I employed was dried until it could be electrified by rubbing. A tensile force of 400 lbs. produced a deflection of 25′·8, which the test experiment on the junction proved to indicate 0°·0475. This, increased one quarter, gives ·0594 as the actual thermal effect at the moment of the application of the tension.

90. The theoretical result, using my own determination of the specific heat of the wood, and its expansion by heat at the mean tension 235 lbs., is

$$H=\frac{278}{1390}\times\frac{-400}{1}\times\frac{1}{196858}\times\frac{1}{{\cdot}3582}\times\frac{1}{{\cdot}01886}=-0^\circ{\cdot}06016.$$

91. *St. John's Pine, cut across the grain.*—This specimen was dried slowly until it became electrical by rubbing. A tensile force of 14 lbs. produced a deflection of 3′·4, indicating a thermal effect of ·00494. This, increased by one quarter, gives ·00617 as the cold produced by the tension. The theoretical result is

$$H=\frac{276}{1390}\times\frac{-14}{1}\times\frac{1}{23256}\times\frac{1}{{\cdot}3962}\times\frac{1}{{\cdot}03214}=-0^\circ{\cdot}0093.$$

92. *Bay Wood saturated with water.*—The specific gravity of this specimen was ·933; it weighed 333 grains per foot, 157 grains being imbibed water. The mean of twenty experiments gave me ·0073 of *heat* on applying a tension of 200 lbs., and ·0013,

likewise of heat, on removing the tensile force. There could be no doubt that in this instance the imperfect elasticity of the moist wood caused a considerable quantity of heat to be generated frictionally. It may therefore be safely concluded that the thermal effect, considered apart from the result of friction, was, as in the case of india-rubber, *one of heat on the application of tension, and cold on its removal.* Its actual value would be over-estimated by $\frac{\cdot 0073 - \cdot 0013}{2} = 0^{\circ}\cdot 003$, on account of the set communicated by tension being always greater than that taken out when the tension is removed. The approximate theoretical result is

$$H = \frac{277}{1390} \times \frac{-200}{1} \times -\frac{1}{2000000} \times \frac{1}{\cdot 68} \times \frac{1}{\cdot 0476} = 0^{\circ}\cdot 0006.$$

93. For the sake of ready comparison I have collected in the following Table the results of the foregoing experiments on the thermal effects of tension, placing by their side the results of Professor THOMSON's theory.

Material.	Experiment.	Theory.	Theoretical thermal effect of 1 lb. tension on a prism weighing 1 lb. to the foot, at the temperature 0° Cent.
Iron	—·115	—·110	—·0000220
Iron	—·124	—·110	
Iron	—·101	—·107	
Hard steel	—·162	—·125	—·0000235
Cast iron	—·160	—·112	—·0000168
Cast iron	—·148	—·115	
Copper	—·174	—·154	—·0000355
Lead	—·053	—·040	—·0001847
Lead	—·076	—·055	
Gutta percha	—·028	—·031	—·0000769
Gutta percha	—·052	—·066	
Vulcanized india-rubber	+·114	+·137	
Pine wood	—·017	—·023	—·0000021
Bay wood	—·059	—·060	—·0000028
Pine, cross-grained	—·006	—·009	—·0000213
Wet bay wood	+·003	+·001	+·00000015

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94. *Wrought Iron.*—A pillar 2 inches long and one-quarter of an inch in diameter had a fine hole bored through it, in which a thermo-electric junction of fine copper and iron wires was inserted, as described in § 18. It was placed under the lever which had served for the tension experiments, the weight of which alone gave a pressure of 50 lbs. When, in addition to this, a pressure of 1060 pounds was applied, a deflection of 21′·3, indicating heat, was produced; and a like deflection in the opposite direction, indicating cold, was produced by removing the pressing weight. The value of the deflection, ascertained by immersing the pillar, in water of various temperatures, to within one-eighth of an inch from the thermo-electric junction, was 0°·1517. The theoretical result is

$$H = \frac{287\cdot 8}{1390} \times \frac{1060}{1} \times \frac{1}{81200} \times \frac{1}{\cdot 11} \times \frac{1}{\cdot 1491} = 0^{\circ}\cdot 1645.$$

95. My next experiment was with a pillar of the same length, but half an inch in diameter, using a pressure of 1060 lbs. In pillars of this and greater diameters the experiment to test the value of the deflections was made by plunging the pillar in water, which rose a little above the orifice in which the thermo-electric junction was inserted. Thus in the present instance I found the deflection of 8′·94 to indicate a rise of temperature equal to 0°·0319. The theoretical result is

$$H=\frac{289}{1390}\times\frac{1060}{1}\times\frac{1}{81200}\times\frac{1}{\cdot 11}\times\frac{1}{\cdot 632}=0^{\circ}{\cdot}039.$$

96. Wishing to try higher pressures, I employed a very excellent hydraulic press, the pressure exerted by which I had ascertained by experiment as far as 2000 lbs. The friction amounted to only one-seventh of the exerted force. By supplying the correction thus indicated, I had a very convenient and tolerably accurate mode of applying pressure by simply placing weights on the handle of the pump. A pressure of 6458 lbs. applied in this way, gave a deflection of 42′ at the thermo-multiplier, placed at a distance of 40 yards, which deflection was found to indicate a thermal effect of 0°·2344. The theoretical result is

$$H=\frac{285{\cdot}5}{1390}\times\frac{6458}{1}\times\frac{1}{81200}\times\frac{1}{\cdot 11}\times\frac{1}{\cdot 632}=0^{\circ}{\cdot}235.$$

97. Next I tried a pillar 2 inches long and 1 inch in diameter. When a pressure of 1780 lbs. was applied to this by the lever, the needle was deflected 10′·6, indicating a rise of temperature of 0°·01803. The theoretical result is

$$H=\frac{290{\cdot}6}{1390}\times\frac{1780}{1}\times\frac{1}{81200}\times\frac{1}{\cdot 11}\times\frac{1}{2{\cdot}533}=0^{\circ}{\cdot}01642.$$

98. With the hydraulic press, pressures of 4154, 8762, and 20282 lbs., produced deflections of 9′, 21′·6, and 1° 15′·3 respectively, indicating temperatures of 0°·03157, 0°·07578, and 0°·2642. The theoretical results are—

$$H=\frac{289{\cdot}8}{1390}\times\frac{4154}{1}\times\frac{1}{81200}\times\frac{1}{\cdot 11}\times\frac{1}{2{\cdot}533}=0^{\circ}{\cdot}03822,$$

$$H=\frac{289{\cdot}8}{1390}\times\frac{8762}{1}\times\frac{1}{81200}\times\frac{1}{\cdot 11}\times\frac{1}{2{\cdot}533}=0^{\circ}{\cdot}08063,$$

and

$$H=\frac{289{\cdot}8}{1390}\times\frac{20282}{1}\times\frac{1}{81200}\times\frac{1}{\cdot 11}\times\frac{1}{2{\cdot}533}=0^{\circ}{\cdot}1866.$$

99. On applying a pressure of 47930 lbs., the pillar was squeezed to the length of 1·88 inch, giving out about 6°, or nearly the thermal equivalent of the work thus done. Then using the lighter pressure of 37744 pounds I obtained a deflection of 2° 6′, indicating a thermal effect of 0°·442. The theoretical result is

$$H=\frac{290}{1390}\times\frac{37744}{1}\times\frac{1}{81200}\times\frac{1}{\cdot 11}\times\frac{1}{2{\cdot}694}=0^{\circ}{\cdot}3266.$$

100. *Cast Iron.*—A pillar 2 inches long and one-quarter of an inch in diameter being

pressed by the lever to the extent of 730 lbs. produced a deflection of 41′, which was proved to indicate a rise of temperature equal to 0°·1123. The theoretical result is

$$H=\frac{287\cdot6}{1390}\times\frac{730}{1}\times\frac{1}{90139}\times\frac{1}{\cdot1198}\times\frac{1}{\cdot1281}=0^{\circ}\cdot1091.$$

Another experiment, with 1010 lbs. pressure on another bar, gave a thermal effect of 0°·1667, theory giving

$$H=\frac{288\cdot6}{1390}\times\frac{1010}{1}\times\frac{1}{90139}\times\frac{1}{\cdot1198}\times\frac{1}{\cdot1346}=0^{\circ}\cdot1443.$$

101. Pillar, 2 inches long and half inch in diameter. A pressure of 1060 lbs. applied by the lever, produced a deflection of 6′·5, indicating a thermal effect of 0°·0454. The theoretical result is

$$H=\frac{285\cdot3}{1390}\times\frac{1060}{1}\times\frac{1}{90139}\times\frac{1}{\cdot1198}\times\frac{1}{\cdot57}=0^{\circ}\cdot0353.$$

102. Using the hydraulic press, pressures of 1850, 4154, and 8762 lbs., produced thermal effects of 0°·0822, 0°·1883, and 0°·3423 respectively. The theoretical results are

$$H=\frac{285\cdot3}{1390}\times\frac{1850}{1}\times\frac{1}{90139}\times\frac{1}{\cdot1198}\times\frac{1}{\cdot57}=0^{\circ}\cdot0616,$$

$$H=\frac{285\cdot3}{1390}\times\frac{4154}{1}\times\frac{1}{90139}\times\frac{1}{\cdot1198}\times\frac{1}{\cdot57}=0^{\circ}\cdot1385,$$

and

$$H=\frac{285\cdot3}{1390}\times\frac{8762}{1}\times\frac{1}{90139}\times\frac{1}{\cdot1198}\times\frac{1}{\cdot57}=0^{\circ}\cdot2923.$$

Under a pressure of 20282 lbs. the pillar broke diagonally, the deflection of the needle indicating an evolution of 4°·16.

103. Pillar, 2 inches long, 1 inch in diameter. A pressure of 1781 lbs. applied by the lever, gave a thermal effect of 0°·01365. The theory gives

$$H=\frac{290\cdot8}{1390}\times\frac{1781}{1}\times\frac{1}{90139}\times\frac{1}{\cdot1198}\times\frac{1}{2\cdot352}=0^{\circ}\cdot01466.$$

Using the hydraulic press, pressures of 4154, 8762, and 20282 lbs. gave thermal effects of 0°·0364, 0°·0511, and 0°·1463 respectively, the theoretical results being

$$H=\frac{290}{1390}\times\frac{4154}{1}\times\frac{1}{90139}\times\frac{1}{\cdot1198}\times\frac{1}{2\cdot352}=0^{\circ}\cdot0341,$$

$$H=\frac{290}{1390}\times\frac{8762}{1}\times\frac{1}{20139}\times\frac{1}{\cdot1198}\times\frac{1}{2\cdot352}=0^{\circ}\cdot0719,$$

$$H=\frac{290}{1390}\times\frac{20282}{1}\times\frac{1}{90139}\times\frac{1}{\cdot1198}\times\frac{1}{2\cdot352}=0^{\circ}\cdot1666.$$

Under 47930 lbs. the pillar was squeezed to the length of 1·78 inch. Then with the same pressure I obtained a thermal effect of 0°·4708. The theoretical result is

$$H=\frac{290}{1390}\times\frac{47930}{1}\times\frac{1}{90139}\times\frac{1}{\cdot1198}\times\frac{1}{2\cdot643}=0^{\circ}\cdot3505.$$

104. *Copper.*—A pillar 2 inches long and one-quarter of an inch in diameter, being pressed by 717 lbs., gave a deflection of 10′·1, indicating a thermal effect of 0°·1359. The theoretical result is

$$H=\frac{292}{1390}\times\frac{717}{1}\times\frac{1}{58200}\times\frac{1}{\cdot 095}\times\frac{1}{\cdot 1744}=0^\circ\cdot 156.$$

105. Pillar, 2 inches long and half an inch in diameter. A pressure of 1325 lbs. applied by the lever, gave a deflection of 30′, indicating a rise in temperature of 0°·08316. The theoretical result is,

$$H=\frac{291\cdot 7}{1390}\times\frac{1325}{1}\times\frac{1}{58200}\times\frac{1}{\cdot 095}\times\frac{1}{\cdot 7207}=0^\circ\cdot 06972.$$

Pressures of 1850 and 4154 lbs., communicated by the hydraulic press, produced deflections of 24′·1 and 56′, indicating temperatures of 0°·1182 and 0°·2747 respectively. The theoretical results are

$$H=\frac{292}{1390}\times\frac{1850}{1}\times\frac{1}{58200}\times\frac{1}{\cdot 095}\times\frac{1}{\cdot 7207}=0^\circ\cdot 0974,$$

and

$$H=\frac{292}{1390}\times\frac{4154}{1}\times\frac{1}{58200}\times\frac{1}{\cdot 095}\times\frac{1}{\cdot 7207}=0^\circ\cdot 2188.$$

106. Pillar, 2 inches long and 1 inch in diameter. A pressure of 1792 lbs. communicated by the lever, gave a deflection of 9′·7, indicating a rise equal to 0°·0278. The theoretical result is

$$H=\frac{291\cdot 2}{1390}\times\frac{1792}{1}\times\frac{1}{58200}\times\frac{1}{\cdot 095}\times\frac{1}{2\cdot 953}=0^\circ\cdot 023.$$

Pressures of 4154, 8762, and 20282 lbs., communicated by the hydraulic press, produced deflections of 10′·6, 33′, and 1° 9′, which were found to indicate elevations of temperature of 0°·0494, 0°·1538, and 0°·3216 respectively. The theory gives

$$H=\frac{291}{1390}\times\frac{4154}{1}\times\frac{1}{58200}\times\frac{1}{\cdot 095}\times\frac{1}{2\cdot 953}=0^\circ\cdot 05322,$$

$$H=\frac{291}{1390}\times\frac{8762}{1}\times\frac{1}{58200}\times\frac{1}{\cdot 095}\times\frac{1}{2\cdot 953}=0^\circ\cdot 1122,$$

and

$$H=\frac{291}{1390}\times\frac{20282}{1}\times\frac{1}{58200}\times\frac{1}{\cdot 095}\times\frac{1}{2\cdot 953}=0^\circ\cdot 26.$$

A pressure of 57146 lbs. squeezed the pillar to the length of 1·32 inch. Then, when the full amount of set had taken place, the application of the same weight gave 2° 18′ of deflection, indicating a rise of temperature equal to 0°·643. The theoretical result is

$$H=\frac{291}{1390}\times\frac{57146}{1}\times\frac{1}{58200}\times\frac{1}{\cdot 095}\times\frac{1}{4\cdot 474}=0^\circ\cdot 4832.$$

107. *Lead.*—A pillar, 2 inches long and half an inch in diameter, being pressed by the

lever with a force of 350 pounds, produced a deflection of 12′·4, indicating a rise of temperature of 0°·0805. The theory gives

$$H=\frac{292}{1390}\times\frac{350}{1}\times\frac{1}{35100}\times\frac{1}{\cdot0303}\times\frac{1}{\cdot8796}=0^{\circ}\cdot0786.$$

Pillar, 2 inches long and 1 inch in diameter. A weight of 1500 lbs. applied by the lever, produced a deflection of 33′, indicating a thermometric rise of 0°·0944. Theory gives in this case,

$$H=\frac{290}{1390}\times\frac{1500}{1}\times\frac{1}{35100}\times\frac{1}{\cdot0303}\times\frac{1}{3\cdot717}=0^{\circ}\cdot0792.$$

4154 lbs. laid on by the hydraulic press, squeezed the pillar to the length of 1·8 inch. Then, when this set was fully established, the same pressure gave a deflection of 41′·6, indicating a temperature equal to 0°·1835. The theoretical result is

$$H=\frac{288\cdot6}{1390}\times\frac{4154}{1}\times\frac{1}{35100}\times\frac{1}{\cdot0303}\times\frac{1}{4\cdot13}=0^{\circ}\cdot1964.$$

108. *Glass.*—A cylindrical pillar of flint-glass, 10 inches long and $\frac{9}{10}$ths of an inch in diameter, had a thermo-electric junction of thin wires tightly bound to its side by cotton thread. A pressure of 900 lbs. applied by the lever, gave me a deflection of 10′·6, the thermometric value of which, estimated by immersing the pillar above the junction in water of various temperatures, proved to be 0°·01684. The theory gives

$$H=\frac{290\cdot8}{1390}\times\frac{900}{1}\times\frac{1}{124800}\times\frac{1}{\cdot19}\times\frac{1}{\cdot9576}=0^{\circ}\cdot0083.$$

In another experiment I obtained a deflection of 11′·3, indicating a rise of 0°·01774 when a weight of 1622 lbs. was laid on by means of the lever. In this case the theoretical result is

$$H=\frac{292}{1390}\times\frac{1622}{1}\times\frac{1}{124800}\times\frac{1}{\cdot19}\times\frac{1}{\cdot9576}=0^{\circ}\cdot015.$$

109. *Wood.*—A pillar of seasoned pine, 13 inches long and 1·4 inch in diameter, had a junction of fine copper and iron wires inserted into its centre. When 869 lbs. were laid on this pillar, a deflection of 6′·05 occurred, which was found to indicate a rise of temperature equal to 0°·0068. The theoretical result, taking my own results for expansion and specific heat, is

$$H=\frac{290\cdot4}{1390}\times\frac{869}{1}\times\frac{1}{238000}\times\frac{1}{\cdot4}\times\frac{1}{\cdot311}=0^{\circ}\cdot0061.$$

110. My next experiments were with a 3-inch cube of pine, which, being furnished with a junction of fine wires in its centre, had pressures of 4154 and 8762 lbs. applied by the hydraulic apparatus. The deflections obtained were 3′·8 and 5′·9, indicating thermal effects of 0°·0093 and 0°·0145. Theory gives in these cases,

$$H=\frac{290}{1390}\times\frac{4154}{1}\times\frac{1}{238000}\times\frac{1}{\cdot4}\times\frac{1}{1\cdot766}=0^{\circ}\cdot00516,$$

and

$$H=\frac{290}{1390}\times\frac{8762}{1}\times\frac{1}{238000}\times\frac{1}{\cdot4}\times\frac{1}{1\cdot766}=0^{\circ}\cdot01088.$$

111. When the same block was pressed, in a direction perpendicular to the grain, with a weight of 1792 lbs. communicated by the lever, a deflection of 37′·3 was produced; but when the pressure was removed, the deflection in the reverse direction amounted only to 31′·2. I believe that the excess in the result of pressure is owing to frictional evolution of heat through the imperfect elasticity of wood cut crossways to the grain. The mean, 34′·25, represents a thermal effect of 0°·0461. Theory gives

$$H=\frac{288\cdot6}{1390}\times\frac{1792}{1}\times\frac{1}{20160}\times\frac{1}{\cdot4}\times\frac{1}{1\cdot766}=0^\circ\cdot0261.$$

112. A second experiment gave a thermal effect of 0°·016, the theoretical result being 0°·0264. In this latter experiment the position of the block was reversed. I attribute the discordance between the two results to the difficulty I experienced in obtaining with my lever apparatus a perfectly even distribution of pressure over so large a surface as 3 inches square.

113. On exposing the block, still crossways to the grain, to a greater pressure by means of the hydraulic press, I obtained a deflection of 40′·5, indicating a rise of 0°·1 whenever a weight of 4154 lbs. was laid on, but observed no perceptible effect whatever when the pressure was removed. In this case the bulging of the wood under pressure extended to about half an inch. It was evident that frictional heat increased to a great extent the thermal effect of pressure, and that a similar though smaller frictional effect diminished the cooling effect on removal of pressure. The theoretical result, if bulging had not taken place, and supposing perfect elasticity, is

$$H=\frac{290}{1390}\times\frac{4154}{1}\times\frac{1}{20160}\times\frac{1}{\cdot4}\times\frac{1}{1\cdot766}=0^\circ\cdot0609.$$

114. *Vulcanized India-rubber.*—A pillar, 1·92 inch long, 1·22 inch in diameter, and weighing 692 grains, had a thermo-electric junction of thin copper and iron wires inserted into its centre. On applying pressure, the multiplier showed a rise of temperature, and when the pressure was removed, a depression took place, sensibly equal to the previous rise. The following is a Table of the results:—

Weight laid on the pillar, in pounds.	Deflection.	Heat by laying on the weight, or cold by removing it.
	° ′	°
28	3	0·0058
33	3·7	0·0072
47	6·7	0·0131
62	16	0·0312
93	32	0·0625
124	1 4·3	0·1254

With the pressure of 124 lbs. the pillar was compressed to the length 1·05 inch, and returned to 1·78 when the weight was removed.

115. The expansion of the pillar by heat, in the direction in which the pressure was applied, was found to be about $\frac{1}{1680}$ per degree Cent. Hence the theoretical result for a pressure of 124 lbs. would appear to be

$$H=\frac{287}{1390}\times\frac{124}{1}\times\frac{1}{1660}\times\frac{1}{\cdot 415}\times\frac{1}{\cdot 77}=0^{\circ}\cdot 05.$$

The excess of the actual result may be attributed to the central part, in which the junction was placed, bearing an undue share of pressure in consequence of the bulging of the waist of the pillar.

116. On examining the above Table, it will be remarked that the thermal effect increases more rapidly than the pressure. This is owing partly to the influence of a rise of temperature in increasing the elasticity of rubber under strain, to which we have already alluded, and partly to the unequal distribution of pressure to which I have just adverted. The longitudinal expansion of a pillar of vulcanized india rubber, 2·21 inches long and 1·44 inch in diameter, under various pressures, was found to be

Pressure in pounds.		Expansion per degree Cent. on the length under pressure.
0	. . . .	$\frac{1}{5700}$
55	. . . .	$\frac{1}{3730}$
135	. . . .	$\frac{1}{2228}$

117. Being desirous of ascertaining the thermal effects of higher pressures, I made the experiments tabulated below, on a pillar 1·7 inch long and 2·5 inches in diameter.

Pressure applied or removed, in pounds.	Heat on applying pressure, or cold on removing it, in degrees Centigrade.	Length of the pillar when under pressure.	Length of the pillar after the pressure was removed.
70	0°·011	......	1·7
113	0·015		
134	0·029		
162	0·036		
242	0·052		
275	0·078		
386	0·114		
547	0·130	1·43	
1426	0·384	1·00	
2742	0·750	·72	
4058	0·885	·64	
6692	1·192	·51	1·66
11958	1·463		
22400	1·426	·36	1·68

118. After a few experiments with the last pressure the rubber burst, being ruptured in a singularly symmetrical manner at the four quadrants of its circumference. Placed in hot water, it almost immediately regained its original shape.

119. I have collected in the following Table the results of the foregoing experiments on the thermal effects of pressing pillars. In reckoning the mean, the last result in each of the series for wrought iron, cast iron, copper, and lead is rejected on account of having been obtained with a pressure nearly up to the limit of strength, and after the form and structure of the pillar had been changed by the force to which it had been subjected.

Material	Diameter of pillar.	Experimental thermal effect	Theoretical thermal effect
	inch.	°	
Wrought iron	1/4	·152	·164
	1/2	·032	·039
	1/2	·234	·235
	1	·018	·016
	1	·032	·038
	1	·076	·081
	1	·264	·187
	1	·442	·327
	Mean	·115	·108
Cast iron	1/4	·112	·109
	1/4	·167	·144
	1/2	·045	·035
	1/2	·082	·062
	1/2	·188	·139
	1/2	·342	·292
	1	·014	·015
	1	·036	·034
	1	·051	·072
	1	·146	·167
	1	·471	·350
	Mean	·118	·107
Copper	1/2	·136	·156
	1/2	·083	·070
	1/2	·118	·097
	1/2	·275	·219
	1	·028	·023
	1	·049	·053
	1	·154	·112
	1	·322	·260
	1	·643	·483
	Mean	·146	·124
Lead	1/2	·080	·079
	1	·094	·079
	1	·183	·196
	Mean	·087	·079
Glass	2/10	·017	·008
	2/10	·018	·015
	Mean	·017	·011
Wood	1·4	·007	·006
	3	·009	·005
	3	·014	·011
	Mean	·010	·007
Wood cut across the grain	3	·046	·026
	3	·016	·026
	3	·050	·061
	Mean	·037	·038
Vulcanized india-rubber	1·22	·125	·050

120. The above results, as in the case of the tension experiments, indicate a slight excess of experiment over theory. I at first thought that this might be owing to the diminution of elastic force by heat in metals, of which I had not taken account as in wood, for in applying or removing tension the thermal effect would be increased in consequence of the increased expansion by heat in such a case. This cause would, however, diminish the thermal effect of the application or removal of pressure. The discrepancy must therefore be referred to experimental error, or to the incorrectness of the various coefficients which make up the theoretical results. Having, however, been led to believe that with a rise of temperature a certain change of elasticity takes place in metals, although too minute to be appreciated in the foregoing results, I made some experiments in which spirals weighted at one end were measured when exposed to cold air, and then again after they had been heated in the atmosphere of an oven. In the latter case there was considerable elongation, indicating, in the case of steel, a diminution of elasticity amounting to ·00041 per degree Centigrade, and in the case of copper, to ·00047. After I had made these experiments, I became acquainted with M. Kupffer's valuable researches on this subject, by which, using the method of vibrations, he finds the decrement of elasticity per degree Centigrade in steel and copper to be ·00047 and ·00048 respectively.

121. Another source of error exists, which, although not of sufficient amount to make a sensible alteration in the thermal effects of tension and pressure on metals, yet ought not to be neglected in a complete view of the subject. Mr. Hodgkinson has long ago shown that any force, however small, is able to produce a certain permanent deflection in a bar, and that this deflection increases rapidly with the force which has produced it. Professor Thomson has added the observation, that even after a metal has been exposed to great tensile force, its elasticity is not thereby rendered perfect for smaller degrees of stress. Thus he finds that when weights are successively hung to a wire so as gradually to increase its tension, and then successively removed, the wire never assumes immediately its just length, but is always shorter during the putting on of the weights than during their removal. Hence work is done on the wire which must necessarily evolve a certain quantity of heat; and if, as is probable, a greater quantity of work is thus done whilst the tensile force is being removed than whilst it is being applied, the result will be that the cold of tension will not be diminished to the same extent as the heat, in consequence of the removal of tension, will be increased, and so the mean thermal effect will be increased. On the other hand, it is probable that in the act of compression less work is done on the wire than during the removal of the compressing force, the result being that the mean thermal effect of applying and removing the pressure is lessened. The foregoing experiments do not afford sufficiently delicate tests to detect the excessively minute quantities of heat developed frictionally in the above manner.

122. Professor Thomson has pointed out that the dynamical theory of heat, with the modification of Carnot's principle introduced by himself and Clausius, show that "if

a spring be such that a slight elevation of temperature weakens it, and the full strength is recovered again with the primitive temperature, work done against that spring by bending or working in whatever way must cause a cooling effect." The quantity of cold expected was excessively small, yet I hoped to measure it by taking the mean of a large number of observations with the thermo-multiplier. I took a spiral of tempered steel wire $\frac{1}{9}$th of an inch thick, of which each convolution was $1\frac{1}{2}$ inch in diameter and one quarter of an inch distant from its neighbour. A thermo-electric junction was attached to one of the convolutions, and means were provided to compress or extend the spiral at pleasure without approaching it. In the case of a spiral stretched by a weight hung to it, the application of heat causes, as we have seen, a considerable elongation, in consequence of diminution of elasticity; but in the case of a spiral compressed by a weight laid on the top of it, the effect of the same cause is to diminish its length. Hence either the pulling out or the compression of the spiral must cause the absorption of heat, and the return of the spiral to its normal state must be accompanied by the evolution of heat.

123. The above thermal effects of bending a spring are evidently proportional to the square of the pressing or tensile force; for if these be increased, the elastic spring and the alteration of length by rise of temperature will be also proportionally increased.

124. Having arranged the thermo-multiplier so as to give one swing in 30″, I pursued the experiments as follows:—A weight of 7 lbs. was laid on the top of the spiral to compress it, and after 30″ had elapsed, the change in the position of the needle was noted. Then the weight was removed, and the needle observed after 30″ as before. One hundred such experiments were tried alternately. Afterwards I made another series of one hundred experiments, on the effect of stretching and removing the stretching force. The results are placed in the following Table, in which the signs + and − distinguish deflections indicating evolution and absorption of heat.

Deflection in minutes by compressing with 7 lbs.	Deflection on removal of compressing force.	Deflection in minutes by stretching with a force of 7 lbs.	Deflection on removing the tensile force.
−6	−2	0	−4
+1	+2	−4	−2
−1	−2	−8	−1
−1	0	−2	+1
0	+1	−3	+1
−4	+3	−3	−4
−6	−3	−4	0
+6	0	−2	+2
+2	+1	−6	−2
−1	0	+1	+2
0	+2	+1	−2
−3	−1	0	−3
−1	0	+1	−4
−4	0	0	−6
0	+6	−5	−1
−3	−1	−1	+4
0	0	−2	−1
−6	0	0	+3
−1	0	0	+5
−3	−1	+4	+3
−3	+5	−10	−1
−4	−2	−4	+7
+3	+5	−5	−3
+3	+3	−5	+4
−2	0	+4	+1
−1	+4	−5	−1
0	+5	+1	−1
0	+8	−6	+3
+4	0	−2	+1
−2	−1	0	+4
+2	+4	−3	+1
−1	+4	−3	+3
+2	+3	−2	+1
−1	+6	+4	+4
+2	+6	+2	+4
−2	+1	+6	+6
−5	−2	+3	+7
−2	+1	−6	−1
−1	+1	+2	−1
−3	−2	+2	+4
−4	−3	0	+1
−4	+2	−2	+1
−4	0	+2	+2
+1	+4	−1	0
0	+2	−2	+2
0	+6	−2	−2
−4	0	−1	0
0	+4	+1	+3
−7	−1	−1	+2
−6	+1	−1	+2
Mean...... −1·4	+1·38	−1·34	+0·88

125. The thermal value of 11° 6′ deflection being found to be 1°·63 Cent., the above deflections will indicate respectively ·00343 cold, ·00338 heat, ·00328 cold, and ·00215 heat, the average showing a quantitative thermal effect of 0°·00306. Using my own

coefficient for the diminution of the elastic force of steel by rise of temperature, I find for the theoretical result, in the case of compression,

$$H=\frac{283}{1390}\times\frac{7}{1}\times\frac{-1}{2379}\times\frac{1}{\cdot 11}\times\frac{1}{1\cdot 35}=-0^\circ\cdot 00403,$$

and in the case of extension,

$$H=\frac{283}{1390}\times\frac{-7}{1}\times\frac{1}{7500}\times\frac{1}{\cdot 11}\times\frac{1}{\cdot 428}=-0^\circ\cdot 00403,$$

the results being necessarily the same in both cases. The deficiency of the actual result is not great, and is on the side of the probable error, in consequence of the unavoidable loss of a portion of the thermal effect by conduction from the junction.

126. Thus even in the above delicate case is the formula of Professor THOMSON completely verified. The mathematical investigation of the thermo-elastic qualities of metals has enabled my illustrious friend to predict with certainty a whole class of highly interesting phenomena. To him especially do we owe the important advance which has been recently made to a new era in the history of science, when the famous philosophical system of BACON will be to a great extent superseded, and when, instead of arriving at discovery by induction from experiment, we shall obtain our largest accessions of new facts by reasoning deductively from fundamental principles.

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VI. *On the Thermal Effects of Compressing Fluids. By* J. P. Joule, *LL.D., F.R.S., F.C.S., Hon. Mem. Phil. Soc. Cambridge, Vice-President of the Lit. and Phil. Soc. Manchester, Corresp. Mem. R.A. Sc. Turin, &c.*

Received October 9,—Read November 25, 1858.

Professor William Thomson has published* a theoretical investigation of the subject of the present paper, in which he arrives at the formula $\theta=\frac{\mathrm{T}ep}{\mathrm{JK}}$, where θ is the increase of temperature, T the temperature from absolute zero, e the expansibility by heat, p the pressure in pounds on the square foot, J the mechanical equivalent of the thermal unit in foot-pounds, and K the capacity for heat in pounds of water, of a cubic foot of the fluid employed. He has also given a Table of theoretical results for the compression of water and mercury. The investigation being established on the basis of well-ascertained principles and facts, the correctness of the Table could not be reasonably doubted. Nevertheless, believing that an experimental inquiry would be interesting if not important, I have ventured to offer the following to the notice of the Royal Society.

The only previous experiments on the subject of which I am aware are those of M. Regnault. To his memoir on the Compressibility of Liquids, he appends a note† on the heat disengaged by the compression of water. The method employed by this celebrated physicist, though less delicate, is similar to that which I have adopted. One set of the junctions of a thermo-electric pile was placed in a copper vessel filled with water, to which a pressure of ten atmospheres could be instantaneously communicated by means of a reservoir of compressed air. The $\frac{1}{64}$th of a degree Centigrade could be detected by his thermo-multiplier. Nevertheless the conclusion arrived at was the negative one, that "the heat disengaged by a sudden pressure of ten atmospheres on water is unable to raise its temperature $\frac{1}{50}$th of a degree Centigrade."

In the absence of any statement to the contrary, we may consider that the temperature of the water compressed by Regnault was not above an ordinary one of the atmosphere, say 18° Centigrade. Thomson's formula gives a thermal effect of 0°·013 for a pressure of ten atmospheres at this temperature; and therefore the conclusion of Regnault above cited is strictly correct: indeed it is so as regards any temperature below 30°. It is to be regretted that he did not pursue his experiments a little further, for had he done so he would without fail have solved this, as he has done so many other more difficult problems, by showing the minute but nevertheless appreciable thermal effect which actually takes place at all temperatures but that of maximum or minimum density.

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* Proceedings of the Royal Society, June 18, 1857, vol. viii. No. 27, p. 566.

† Mémoires de l'Académie Royale des Sciences, xxi. p. 462.

The apparatus I employed consisted of a strong vessel of copper, 12 inches long and 4 inches in diameter. This vessel was connected at the upper part with a cylinder of $1\frac{5}{8}$ inch internal diameter, furnished with a piston, by means of which the requisite pressure could be laid on or taken off at pleasure. A thermo-electric junction of iron and copper wires was placed in the centre of the copper vessel, the orifice through which it was passed being made tight by means of a plug of gutta percha. The outer junction was immersed in a bath of water, and the induced currents were measured by the thermo-multiplier *in vacuo*, described in my former paper.

The method employed was alternately to lay on and remove weights from the piston and to examine the consequent deflection of the needle of the multiplier, from which, by means of data derived from experiments made from time to time to determine the amount of deflection arising from a change of temperature of the outer junction, the thermal effect sought could be readily deduced.

It was found that the needle took rather more than half a minute to assume a new deflection. I therefore fixed upon 40″ as the time allowed to elapse between the application or removal of pressure and the thermo-electric observation. It was at first suspected that the small cooling effect in consequence of the dilatation of the copper vessel by internal pressure might interfere with the effect sought for; but it was found on trial that a sudden application of heat to the outside of the copper vessel did not sensibly affect the temperature of the central part of the liquid in which the junction was plunged, until an interval of time had elapsed equal to twice that occupied by a swing of the needle. This source of error was therefore disregarded.

Another possible source of error occurred to me. Was the thermo-electric relation of the metals employed to form the junction sensibly altered by the influence of pressure? THOMSON has shown that such an alteration in the metals copper and iron accompanies the temporary strain produced by longitudinal tension*. This effect of temporary strain is however very minute, and, in the case of pressure uniformly applied in every direction, which we are now considering, is probably far too small to be appreciated by the most delicate tests. However, I made an attempt to ascertain whether it existed, by applying pressure when the temperature of the outer junction was widely different from that of the inner, the needle of the multiplier being kept in range by means of a controlling magnet. It was then found that the effect of pressure remained the same as before, and therefore the conclusion was necessarily arrived at, that the effect of pressure, uniformly applied, in altering the thermo-electric relation of the metals, was, if it existed at all, too small to produce any sensible error in the present experiments.

In the case of oil, it occurred to me to inquire whether, in consequence of the imperfect fluidity of that liquid, the full thermal effect was communicated to the junction (composed of wires $\frac{1}{20}$th of an inch in diameter) in the 40″ allowed for the swing of the needle. To settle this point a long series of experiments was made, in which the deflections after 40″ were alternately observed with deflections after three minutes. It was found that in the latter case the amount of deflection was one-tenth more than in the former; and therefore, in my experiments with this fluid, the deflections, observed at intervals of 40″, were increased one-tenth, as will be observed in the sequel.

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* Philosophical Transactions, 1856, p. 711.

As it was not convenient to apply a manometer to the apparatus during the experiments, I afterwards ascertained the pressure I had employed, by means of the indications of a carefully graduated air-gauge, when the piston was pressed down with the same weights I had employed in the experiments. The pressures thus obtained were nearly the same as those estimated from the weights laid on the piston, a small allowance being made for friction. The pressure given in the Table is that of the fluid after the weight had been laid on half a minute, minus the residuary pressure arising from friction of the piston after the weight was removed.

TABLE I.—Experiments on the Heat evolved by the Compression of Water.

Temperature in degree Centigrade. t.	Expansibility by heat per degree Centigrade. e.	Pressure in pounds on the square foot. p.	Capacity for heat of a cubic foot of liquid. K.	Deflections of the needle, each the mean of ten observations.	Value of deflections.	Experimental thermal effect.	Theoretical effect, or $\frac{(273+t)ep}{JK}$.
1°·2	−·000042	53634	62·43	− 5′·2 − 4·5 } Mean − 4′·85	596′=1°·03	−0°·0083	−0°·0071
5	·000016	53634	62·45	2·6	596=1·03	0·0044	0·0027
11·69	·000112	53634	62·45	12·6 15·8 } Mean 14·2	596=1·03	0·0244	0·0197
11·69	·000112	53634	62·45	9·5 9·8 } Mean 9·65	596=1·03	0·0166	0·0197
18	·000185	53634	62·41	17·2 20·9 } Mean 19·05	890=1·46	0·0312	0·0333
18·76	·000193	53634	62·41	14·3 17·1 20·6 19·0 } Mean 17·75	1454=2·58	0·0315	0·0347
30	·000300	53634	62·29	3·6 5·7 3·2 } Mean 4·17	204=2·66	0·0544	0·0563
31·37	·000303	33117	62·29	20·8 14·4 } Mean 17·6	554=1·24	0·0394	0·0353
40·4	·000396	33117	62·14	22·0 22·9 23·8 } Mean 22·90	694=1·36	0·0450	0·0476

TABLE II.—Experiments on the Heat evolved by the Compression of Sperm Oil.

Temperature in degree Centigrade. t.	Expansibility by heat per degree Centigrade. e.	Pressure in pounds on the square foot. p.	Capacity for heat of a cubic foot of the oil, in pounds of water. K.	Deflections of the needle, each the mean of ten observations.	Value of the deflections.	Experimental thermal effect increased by one-tenth.	Theoretical effect, or $\frac{(273+t)ep}{JK}$.
16°	·0007582	16777	29·83	32′·3 32·5 Mean 32′·4	950′=2°·11	0°·0792	0°·0886
17·29	·0007582	33117	29·83	64·3 66·0 Mean 65·15	867=2·04	0·1686	0·1758
16·27	·0007582	53634	29·83	108·7 109·3 Mean 109	950=2·11	0·2663	0·2837

The specific heat of the oil was found by the method of mixtures to be 0·5223 at 16°·5. Its expansion, determined by the weight of a volumenometer filled with it at various temperatures, proved to be ·0007582 at 21°·3. Its specific gravity at 0° was 0·915. It was important to ascertain its specific heat and expansion at a temperature near that which it had in the experiments, because, though quite transparent, it became considerably more fluid when the temperature was much raised. Such a gradual change of state in any viscous substance is accompanied by the absorption of "latent heat," and an increase in the rate of expansion, which are greatest at the temperature at which the change of state is most rapid.

VII. *On the Stratifications in Electrical Discharges, as observed in Torricellian and other Vacua.—Second Communication.* *By* John P. Gassiot, *V.P.R.S.*

Received December 9, 1858,—Read January 13, 1859.

60. In my former communication I stated that I had obtained several vacua tubes from M. Geissler of Bonn; shortly afterwards I had the pleasure of a visit from M. Plücker, the distinguished Professor of Physics in that University, when I ascertained that the experiments to which I referred as having been made in Germany, had been made by that gentleman in vacua-tubes constructed by M. Geissler.

During Professor Plücker's visit to this country, I witnessed, in the laboratory of the Royal Institution, his experiments on the action of a powerful electro-magnet on the negative luminous discharges from an inductive coil, which discharges *in vacuo*, under the influence of the magnet, "coincide in their direction with magnetic curves *;" this phenomenon is very clearly shown in Geissler's tubes, in which the wires are hermetically sealed at the extreme ends, projecting in a straight direction along the tube. Through the kindness of Mr. J. J. Griffin, I had the opportunity of experimenting with upwards of sixty of Geissler's vacua-tubes, in which many beautiful and novel results are produced; in some, for several seconds after the discharges had ceased, the tubes remained throughout their entire length highly phosphorescent. I have not been able to ascertain with accuracy what is the gas, which, however attenuated, must remain in each tube; but from most of them being constructed of a varied form, the discharge presents in the several portions of the same tube an entirely different appearance, not only in colour, but also in the form of the stratifications.

61. As I was desirous, during the progress of the experimental research I had entered on, to know the exact conditions under which each vacuum was obtained, and found, by comparison, that there was some uncertainty in the description of those I had received from M. Geissler, I reluctantly laid them aside, and for all the experiments I have to describe in this communication, each tube was charged and exhausted by myself or in my presence. The tubes were constructed by M. Casella, and marked with consecutive numbers, a note being taken of each as it was finally sealed; considerably above 100 were prepared, the major part similar in their form to those described in my former paper†; some of these, during experiment, were broken or otherwise destroyed, but

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* A translation of Professor Plücker's paper "On the Action of the Magnet upon the Electrical Discharge in Rarefied Gases," has since been published in the Philosophical Magazine for August 1858.

† Philosophical Transactions, 1858, Pl. I. fig. 3.

those that remain are as they were originally hermetically sealed. To describe all would in several instances be nearly a repetition, although I use some of the original numbers in this communication for the sake of future reference.

62. The terms Air, Hydrogen, Oxygen or Nitrogen Mercurial Vacua, denote, unless otherwise described, that previous to the insertion of the mercury each tube was filled either with air, hydrogen, oxygen, or nitrogen. The tubes were then exhausted by a good air-pump; another supply of air or of gas was admitted, and the tube again exhausted; this was generally repeated two or three times before the introduction of the mercury; any residue remaining after the tubes had been finally exhausted as a Torricellian vacuum would necessarily be mercurial vapour plus the remains of air or of the gas with which the tube had been originally filled.

The greater intensity of the discharge from the outer terminal of an induction coil, compared with that from the inner, whether the discharge is positive or negative, renders it advisable if not indispensable in many experiments that this point should be noted: I had intended to apply the terms inductric and inducteous (first used by FARADAY in 1838), the former to the outer and the latter to the inner; but finding this might lead to some misconception, in the present communication I use the terms *outer or inner positive*, *outer or inner negative*, to denote the condition of the discharge.

Continuous and reciprocating discharges.—These terms denote the peculiar conditions of the discharge from an induction coil, when taken in vacua-tubes. In a continuous discharge, the needle of a galvanometer placed in the circuit will be deflected; and the stratifications in the vacuum-tube will be also deflected by, and have a tendency to rotate as a whole round, the pole of a magnet.

In a reciprocating discharge, the needle of the galvanometer placed in the circuit is not deflected; such discharge in the vacuum-tube, whether stratified or otherwise, will be divided by a magnet, similar to the discharge when taken from a single terminal (54), which is always reciprocating, as is also the discharge under certain conditions to be described, where both wires of a vacuum-tube are attached to the terminals of the induction apparatus.

In the following experiments, unless when otherwise expressed, one wire in the vacuum-tube was connected with the outer and the other with the inner terminal of the coil.

Character of the Stratified Discharge under different conditions of rarefied Media.

63. If the tube is filled with dry hydrogen, and the discharge from the induction coil be made in the usual manner, no trace of the discharge (the wires being separated in the tube beyond the striking distance in air) will be observed until the gas is rarefied by the air-pump. The discharge first appears as a wavy line of bluish-grey colour; on continuing the exhaustion by the air-pump, assisting the rarefaction by gently heating,

the tube becomes filled with a luminous discharge to within about one inch of the negative wire; the stratifications appear gradually increasing in width, as the vacuum becomes more perfect; and if care is taken to continue the pumping, so as to prevent air being introduced, the tube can be sealed without the stratifications showing the slightest appearance of any redness. Dr. FRANKLAND subsequently suggested to me a mode of exhausting these tubes, which effectually prevents a possibility of the introduction of any air.

64. In a vacuum-tube exhausted by the air-pump alone I have not been able to observe any approach to the cloud-like stratifications which are always to be obtained in good Torricellian vacua. That they evidently do not depend either on the mercury or the tube being free from dirt, but are dependent on the perfection of the vacuum and the total absence of all trace of moisture, is evident; as in one instance the cloud-like stratifications were very distinct at the first discharge, although in the tube the mercury was very dirty, attaching itself in black patches on the sides; the dark band was nearly 10 inches in length while the mercury covered the negative wire. In another instance, during the process of the withdrawal of the mercury from a long tube, the discharges from the induction coil were continued, the wire in the upper portion being positive and the mercury negative; as the mercury descended the stratifications assumed the cloud-like appearance, each nearly one inch in length; on examining the tube traces of moisture were observable adhering to the sides near the negative wire; as soon as the mercury descended sufficiently down the tube, so as to expose the moisture to the vacuum, the cloud-like stratifications instantly changed to narrow bands. This tube was 38 inches in length, and the wires 32 inches apart, the internal diameter being about one inch; the proportion of moisture to the total capacity of the tube must consequently have been very minute.

65. Whenever a perceptible although minute residue of the gas or air remains in a Torricellian vacuum-tube, the stratifications are narrow and close; and if further portions of air or of gas are subsequently introduced, the stratifications gradually close until the discharge assumes the usual appearance of the aurora, as when taken in an air-pump vacuum from the electric machine. In the best Torricellian vacua which I have been enabled to obtain, the stratifications invariably assume the character of a distinct and clearly defined cloud-like discharge; these stratifications are generally more clearly defined towards the negative terminal, close to the dark band, than nearer to the positive.

66. If care be taken in the preparation of Torricellian vacuum-tubes by Mr. WELSH'S process*, it is immaterial whether the original air-pump vacuum be from air or from either of the gases I have enumerated (62); the last trace of air or gas being by this process sealed off in the bulb, the discharge in the mercurial vacua will invariably present the cloud-like stratifications. If any appreciable trace of air or gas was permitted to remain, I always found, if air, the stratifications had a red tinge, which by degrees

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* Philosophical Transactions, 1856, p. 507.

gave a bluish grey. In some instances oxygen gave the reddish tinge, in others grey, similar to hydrogen. This apparent anomaly I subsequently ascertained arose from the impurity of the oxygen.

67. If the extremity of a vacuum-tube is presented to the prime conductor of an electric machine, or to one of the terminals of an induction coil, a spark can be taken and the glass will be perforated. The perforation is extremely minute, but sufficient under the pressure on the vacuum to admit air or gas; but so slowly does the gas or air enter, that the experimentalist is enabled to note the gradual change which takes place during the progress of the discharges of the induction coil; the change (depending on the size of the perforation) sometimes occupied two or three hours. The extremity of a vacuum-tube after perforation was connected by means of tight-fitting gutta percha tubing to a glass cylinder containing fused chloride of calcium, through which air, hydrogen, oxygen, or nitrogen was permitted to pass into the vacuum; the result of many repeated experiments showed that with hydrogen or oxygen no change in the *colour* takes place; with air or nitrogen the colour of the stratifications changes from bluish grey to a fawn, and ultimately to a deep red tinge; and during this addition of gas or air, the cloud-like stratifications gradually close, becoming narrower and narrower until they are entirely destroyed, passing to a mere luminosity filling the entire tube, and finally into the wave discharge.

Condition of Discharge at Negative Terminal.

68. In some instances several seconds elapsed, during which the induction coil was in action before any luminous effect could be produced in the tube, the first luminous discharge filling the entire tube without any stratifications, the negative wire being surrounded with a brilliant *blue* phosphorescence; if the discharges are now continued for a few seconds, the cloud-like stratifications appear gradually becoming clear, and distinctly separated, leaving the dark band near the negative terminal 3 to 6 inches in length. At this time, and before any black deposit from the negative platinum wire (which now becomes intensely red) is deposited on the glass, a brilliant white luminous discharge (which for the sake of reference I name the White-tongue Discharge) is visible close to the negative wire; this tongue has a tendency to rotate round the pole of a magnet, and consequently assumes the appearance of being repelled or attracted as each pole is alternately presented; while a long brilliant phosphorescent discharge, proceeding from the upper part of the negative terminal towards the positive, is also similarly attracted or repelled by the magnet. This phosphorescent discharge can by manipulation be contracted or expanded by a magnet in a remarkable degree (for distinction I name this the Blue-tongue Discharge).

If the north pole of a magnet is placed close to the negative wire, and over the blue tongue discharge, the discharge will be elongated and repelled, the end thereof being forced on the opposite side of the tube, illuminating the portion on which it impinges with a brilliant phosphorescent blue light.

If the south pole be presented the discharge will be contracted, impinging on and brilliantly illuminating the portion of the tube round the negative wire.

69. In the apparatus* (Plate IX. fig. 7 of my last paper), when one wire is positive and the mercury negative, if the vibrating hammer of the induction coil be pressed by the finger so as to ensure a sudden disruption, the redness on the surface of the mercury instantly disappears, an intense white glow being substituted, while the dark band is nearly filled with stratifications. On carefully examining the discharge when in this state, it will be observed that on pressing the contact-breaker so as to ensure a more sudden disruption, the stratifications which now fill the space previously occupied by the dark band appear also to emanate from the surface of the mercury. If a portion of air $\frac{1}{2000}$ of the capacity of the tube be introduced, the stratifications assume a reddish tinge, and this redness increases as air is added, the redness on the surface of the mercury disappearing; if the discharges are taken from wire to wire on the introduction of a minute quantity of air, the negative wire loses its red glow and becomes intensely blue.

70. I have alluded to a remarkable phosphorescent *blue* tongue discharge from the negative wire (68), and stated that this discharge is strongly affected by the magnet.

While experimenting with many of the vacuum-tubes, I observed that this peculiar form of the discharge was always more distinct when the stratifications assumed the cloud-like appearance already described. When the stratifications are narrow and close this blue discharge is scarcely visible, although traces of it can sometimes be brought out by a magnet. In vacua which show narrow stratifications, fluorescence pervades the tube equally in all parts; but when the stratifications assume the cloud in a distinct form, *the fluorescence is almost entirely confined to the portion of the tube near the negative wire.* In my first experiments it appeared doubtful whether this blue tongue discharge might not be due to the reflexion on the sides of the glass, from the white-tongue discharge already described; but the result of many experiments made under a variety of conditions, lead me to the conclusion that it is a *distinct phenomenon*, having the appearance of being due to a force or action emanating from the negative terminal.

71. In one tube (Hyd. Merc. Vacuum, No. 47) the white tongue was peculiarly distinct, and the blue very brilliant and clear: in this tube the stratifications were much separated, exhibiting the large and long clouds from the positive terminal, to within four inches of the negative. The phosphorescence in the glass did not appear in any part except close to the negative wire, where it was very brilliant, illuminating that end of the tube with a deep blue colour. If during the discharges a horseshoe magnet is placed *under* the negative wire, the stratifications are brought down towards the magnet, and the white tongue is strongly deflected; on moving the magnet across the tube, the tongue is attracted or repelled according to the pole which is presented; while the blue discharge around the negative, although attracted and repelled, is so, comparatively with the white, in a far less degree.

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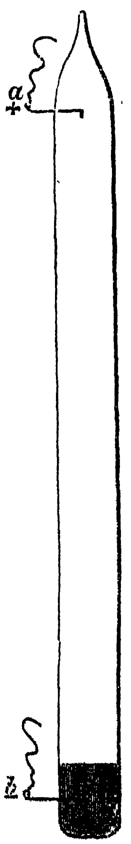
* Philosophical Transactions, 1858, Part I. Pl. I. fig. 7.

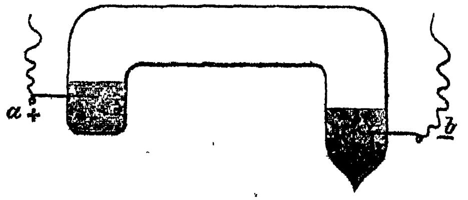
72. Fig. 1 represents a tube 22 inches long, 1$\frac{3}{4}$ inch internal diameter: the wires *a*, *b* are 19 inches apart; sufficient mercury remains in the tube to cover the lower wire, and in this manner one terminal is a wire of platinum and the other a surface of mercury. The tube when used is suspended by a string. In this apparatus, with a single cell of the nitric acid battery, particularly clear and distinct large cloud discharges are obtained, leaving a dark band of 14 to 15 inches in length. When a magnet is presented a little above the surface of the mercury, which in this apparatus I always made the negative terminal, the blue-tongue discharge is produced in a most brilliant and beautiful manner, while at the same time the cloud-like stratifications from the positive wire are brought down the tube. By manipulating with a magnet, the blue tongue can be made to extend up the tube, illuminating the side of the glass wherever it impinges with the phosphorescent blue rays. In this tube the vapour of mercury rises very rapidly; condensing on the sides of the glass, the condensation can be easily removed by the liquid mercury. In this tube, moreover, the redness does not appear on the surface of the negative mercury, which is covered with a bright lambent white glow.

Fig. 1.



73. Fig. 2 is another form of a vacuum-tube, 10 inches long, 1 inch internal diameter, in which each of the platinum wires *a* and *b* can be covered with mercury, so as to permit the surface of the metal to be used for both terminals at the same time. With one terminal of mercury and the other of wire, the white-tongue discharge is very distinct, and either with this arrangement or with both terminals of mercury, the blue phosphorescent discharge from the negative is shown in a very marked manner. When a horseshoe magnet is presented towards the cloud-like stratifications in this tube, the stratifications are attracted towards the magnet when the latter is approached in one direction, and repelled when presented in another. I shall, in the course of this communication, have occasion to refer to the form which this peculiar discharge assumes when the shape of the negative terminal is altered.

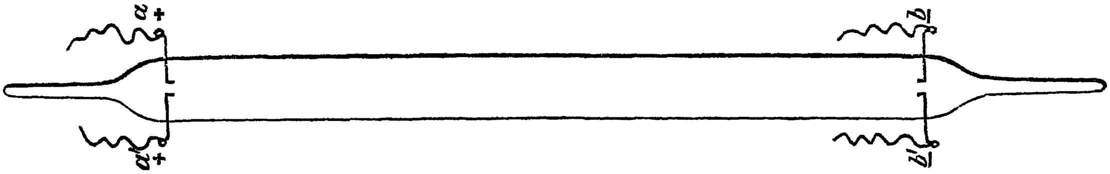
Fig. 2.



74. I have already stated (68) that in some instances several seconds elapsed during which discharges were made without producing any luminous effect, the stratifications ultimately appearing when the discharges had been further continued. The following experiment shows, that with the wire terminals the stratifications are much modified by the particular state of the negative terminal, while that of the positive does not influence them. Fig. 3. In this tube four wires, *a*, *a'*, *b*, *b'*, are hermetically sealed; *a* and *a'* were alternately attached to the positive terminal of the coil, and *b*, *b'* to the negative.

The discharges, whether from *a* or *a'* to *b* or *b'*, were at first luminous, filling the whole tube *without* stratifications. On the discharges being continued from *a* to *b* for a few seconds, the stratifications *appeared* of the cloud-like form, increasing in brilliancy, and leaving a dark negative band 3½ inches in length. The positive terminal of the coil was then attached to *a'*, and the discharge taken from *a'* to *b*; there was no alteration in the character of the stratifications; they remained as in the last experiment, clear and distinct. The negative terminal wire of the coil was then removed from *b* to *b'*, the discharge (luminosity without stratifications filling the entire tube) returned; the positive terminal wire of the coil was then removed from *a* to *a'*, with the same result.

Fig. 3.



The apparatus was allowed to remain in this state for one hour, when the experiment was repeated with the same result. On continuing the discharge from *a'* to *b'*, the stratifications appeared, and the tube now remains in this state. If *b* and *b'* are connected together by a wire so as to form one negative terminal, the stratifications are clearer and better defined than when the discharge is made to either separately, but there is not any increase or alteration if the *a a'* are connected. From these experiments it would appear that with platinum terminals the character of the stratifications is much modified by the altered condition of the negative wire, which it undergoes shortly after the discharges take place.

Stratified Discharge from both Terminals.

75. On one occasion I obtained a result, which, although as yet I have not been able to repeat with other tubes, yet with our present imperfect knowledge of the causes of the stratifications, may be worth recording. The following is an extract of a note taken at the time:—

In a short tube, 10 inches long, No. 13, the discharge presented the cloud-like stratifications. From the long time this tube had been used, and the direction of the discharges having been often altered, much metallic deposit had taken place at both terminals. In this state one end of the tube was perforated by an electrical spark while connected with the coil; as the discharges were continued, the stratifications gradually assumed greater distinctness, at first with a light fawn, and then of a slightly red colour, but they did not alter in their shape: after one hour the discharges were discontinued, and again renewed in about thirty minutes, when I observed that the red colour had increased, but the size of the stratifications remained unaltered. The discharges were further continued for about thirty minutes: during the whole of this experiment only one cell of the battery had been used. On trying the effect of ten cells, I observed two distinct series of stratifications, one from each terminal, separated in the centre of the tube by a dark band of about 1½ inch. In this state the tube was sealed, and remained for several days, during which time I repeatedly obtained the same double set of stratifications. Unfortunately, on one occasion I continued the discharges too long: one of the wires became heated and the tube fractured.

Influence of Temperature.

76. If the heat from the flame of a spirit-lamp is applied along *one side* of a Torricellian vacuum-tube in which the stratifications are narrow and close, that side of the tube aids the transmission of the discharge which now leaves the cooler for the heated side, the stratifications becoming clearer and wider apart. If a *section* of the tube is equally heated, that section will exhibit the stratifications much wider apart than the portion which has not been heated, where they appear to collapse; as the tube returns to its normal state of temperature, the stratifications throughout resume their original figure. If heat is applied to a section of a tube which exhibits the cloud-like stratifications, they lose their clear distinctness, becoming confused and intermingled; in this state the black from the negative wire appears to be more freely deposited; and when the flame of a spirit-lamp surrounds the negative wire, distinct discharges can be heard inside the tube near the negative, but none from the positive.

77. Through the kindness of Dr. Faraday, to whom I am much indebted for suggestions, advice, and personal assistance in the progress of these researches, I had the opportunity of making the following experiments in the laboratory of the Royal Institution:—

Fig. 4.

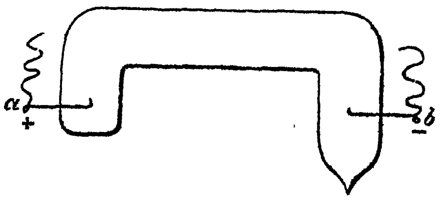
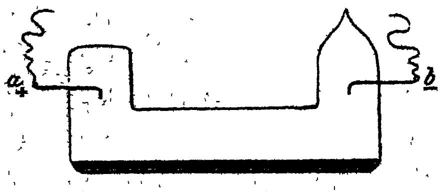


Fig. 4 represents an air Torricellian vacuum, the tube being bent in the form of the figure, 10 inches in length, and about 1 inch internal diameter. This tube was placed in a mixture of ice and salt, without altering the form of the stratifications, which were of the cloud-like character. It was then placed in a bath of ether, which, by solid carbonic acid, was gradually cooled to about −100°; all trace of the stratifications disappeared, the tube being filled with a luminous discharge, the negative wire no longer exhibiting the red glow.

Fig. 5.

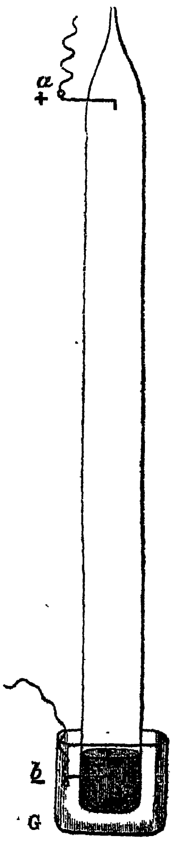


78. Fig. 5 represents a tube of the same dimensions, with sufficient mercury to cover both wires. In this tube the stratifications are narrow, but very clear and distinct; the centre of the tube was placed in the ether bath, leaving the ends and both wires exposed, the mercury remaining in the bend of the tube. The mercury soon froze, and the stratifications underwent considerable alterations, particularly near the positive terminal, where they were much wider apart; over the surface

of the mercury they appeared very brilliant from reflexion. The mercury was then allowed slowly to liquefy, and the position of the tube reversed, so as to permit the mercury to cover both wires, the larger end of the tube was heated by the spirit-lamp until the mercury boiled, passing over to and condensing at the other end. The stratifications entirely disappeared, the discharge passing in a direct line along the tube, and illuminating in its passage the lower side.

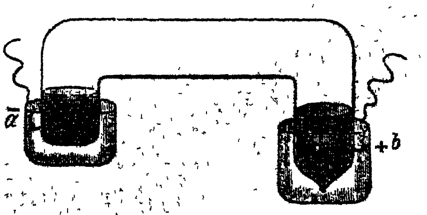
79. In the case of the tube represented in fig. 1, I have stated that with one cell of the nitric acid battery the discharges show two or three broad and distinct clouds (72); and if a magnet is brought near the negative mercury, eight or ten additional cloud-like stratifications are brought out equally clear and defined. In experimenting with this tube, I suspended it, as before, with a strong loop; in this manner it hangs freely in the air, the platinum wires being connected to the induction coil in the usual manner by helices of thin copper wire. The lower end of this tube was inserted in a glass vessel (fig. 6) containing ether to which a quantity of solid carbonic acid was from time to time added; owing to the large mass of mercury, some time elapsed before the metal froze; when the temperature of the ether was reduced to −85° Fahr., discharges from the induction coil being made, the stratifications were no longer visible, but a small luminous spot remained at the end of the positive terminal. On presenting the north pole of a magnet on one side of this luminous spot, or the south pole on the other, another luminous spot was visible, and on drawing the magnet down the tube the stratifications came out, but not so clearly separated as before. The blue negative discharge was perceptible, but not so defined as when the tube is in its normal state of temperature. The interior of the tube was progressively luminous from the negative mercurial terminal, the luminosity decreasing in intensity towards the positive wire. On removing the ether-bath, the mercury gradually liquefied; the temperature rose, and at +20° Fahr. the stratifications reappeared.

Fig. 6.



80. Fig. 7 represents a tube similar in form to fig. 4, but having, as in fig. 2, sufficient mercury to cover both wires. In this tube the cloud-like stratifications are very clearly defined. The white tongue, when the negative wire is exposed, is also very distinct, as is also the blue from the negative wire. Two glass vessels, containing ether, were placed so as to receive the two ends of the tube, the wires being covered with the mercury; solid carbonic acid was applied as before, the mercury in both ends quickly froze; on trying the temperature, one end indicated −98° and the other −102° Fahr.; the surfaces of the mercury, in the act of cooling, assumed a concave form, that at *a* being more depressed than that

Fig. 7.



at *b*; the negative terminal *a*, from its form concentrating the blue-tongue discharge on the upper surface of the tube, where it impinged, forming an oval-shaped spot remarkably rich in the fluorescent blue rays. This illuminated spot could be repelled or attracted, concentrated or diffused, according to the direction in which the magnet was presented. On removing the ether from *a*, and allowing the mercury to liquefy, a tremulous motion was purposely imparted to the mercury, while at *b* it remained rigid when kept frozen. By these means a vibrating or oscillating motion was imparted to the blue-tongue discharge, as it impinged on the glass; this discharge appearing as a focus of light, its form being regulated by that which the negative terminal assumed.

81. From these experiments in temperatures through a range of upwards of 700° Fahr., I obtained the following results:—

In a Torricellian vacuum which gave good cloud-like stratifications, no change could be observed when the temperature was lowered to +32°; but at a temperature of −102° all traces of stratifications were destroyed, and in this state the red or heated appearance of the negative wire disappeared, the discharge filling the entire vacuum with a white luminous glow; on the temperature being raised by the application of heat to the mercury, the stratifications reappeared. When the mercury was boiled, indicating heat of upwards +600°, the stratifications were all destroyed, but in this case the discharge passed along the mercury as it condensed in the cooler portion of the tube. When the mercury was frozen the stratifications disappeared, and the discharge did not illuminate the entire length of the tube, but merely the terminals; in this state, when a horseshoe magnet was brought near the tube, the cloud-like stratifications immediately appeared from the positive wire very distinct and large, but not so clearly separated as when the tube was at its normal temperature.

On Vacua free from Vapour of Mercury.

82. I was desirous to obtain a vacuum, if possible, more perfect, and particularly one free from any trace of mercury. In those with which I had to this time experimented, I believe the only medium remaining in the tube was mercurial vapour, for I had ascertained that the slightest trace of air altered the colour of the stratifications, and I found by using the process suggested by Mr. Welsh, I had obtained precisely the same result, whether the tubes were previously filled with air, hydrogen, or oxygen. In the experiments hitherto described in this, as well as in my former communication, I had obtained the best results by filling carefully prepared glass tubes with pure mercury, and thus forming good Torricellian vacua. Such vacua necessarily contain mercury in a state of vapour, and I assumed that the stratifications observable when electrical discharges are taken in such vacua, were "probably due to the pulsations or impulses of a force acting on this highly attenuated but resisting medium[*]." In a perfect vacuum all the effect due to such assumed resistance ought to be removed, and I anticipated that the discharge would either not pass from wire to wire, or it might pass in some other form than that

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[*] Philosophical Transactions, Part I. p. 14, 1858.

which has been hitherto observed. To test this opinion, I endeavoured to obtain a vacuum with fusible metal instead of mercury, but I could not succeed in getting it free from the presence of air, which was denoted by the colour of the discharge (66). I also prepared apparatus for a tin vacuum; but as similar difficulty would have occurred in this as with fusible metal, I laid the apparatus aside, and endeavoured to obtain a vacuum by inserting sodium in a closed glass vessel, previously charging it with pure oxygen; the stratified discharge was very clear, but the sodium as it melted adhering to the sides of the glass in irregular masses, conducted the discharge from the different surfaces, and the form of the stratifications was rendered so irregular that the results in this respect were unsatisfactory; I however succeeded in obtaining sufficient evidence to convince me that with better arrangements, sodium, and probably potassium with oxygen, will produce vacua which cannot fail to exhibit the stratified discharge with great brilliancy, and possibly by this process even a still more perfect vacuum may be obtained.

*Carbonic-acid Vacua**.

83. The experiments I now proceed to describe were made in glass tubes, many of which were of similar form and dimensions to those I had used with mercury, but the vacua were obtained by an entirely different process. In the preparation of these I am indebted to Dr. FRANKLAND, who not only suggested and explained to me the process to be adopted, but kindly devoted much of his valuable time to the superintending many of the experiments, which were made in his laboratory at St. Bartholomew's Hospital. Fig. 8, Plate IX. represents a tube in which the platinum wires *a* and *b* are hermetically sealed; in the narrow portion A, one or two sticks of caustic potash are inserted, one end of the tube being connected by means of gutta-percha tubing to a vessel in which pure carbonic acid gas is generated; the other end passes through a tight-fitting collar into a vessel containing a little mercury under a receiver attached to the plate of a good air-pump. A portion of the air is first expelled by the rush of the carbonic acid; after this the passage of the gas is stopped at B, and the tube is exhausted by means of the air-pump. The mercury (M) acting as a valve, prevents any risk of air entering into the tube; the process is repeated two or three times, when, if the discharge from the induction apparatus is found to be entirely free from any trace of redness, it may be assumed that all the air has been expelled, and the tube can be then prepared for sealing. When it is full of the gas, the glass is sealed at B. The tube is again exhausted to the utmost limit of the capability of the pump, and finally sealed at C. The apparatus is now ready for experiment.

84. In carbonic-acid vacua obtained by this process, the discharge from an induction coil is at first in a wave line, strongly affected by the magnet or by the hand when placed on the tube; in this state the discharge does not generally present the stratified appearance, or if present the stratifications are only near the positive terminal; sometimes in the course of a few minutes, but often not until several days have elapsed, the

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* Dr. ANDREWS of Belfast was, I believe, the first to describe a method of obtaining carbonic acid vacua in the receiver of an air-pump (Philosophical Magazine, February 1852, p. 104).

stratified discharge becomes visible, and as the carbonic acid is absorbed by the potash, the stratifications gradually appear more clearly defined. I have obtained clearly defined stratifications when a little moisture was perceptible in the tube, this moisture probably assisting the absorption of the carbonic acid by the caustic potassa; as the stratifications gradually become more distinct and divided, they assume a conical form, and lastly the cloud-like appearance (64) of the best mercurial vacua; after this, under some conditions, the stratified appearance of the discharge will entirely cease, the whole length of the tube being filled with faint luminosity; when in this state, if the outside of the tube is touched by the finger, pungent electrical discharges arise, and sparks one-eighth of an inch in length can be elicited. On testing this discharge by a magnet, it will divide in a similar manner to the discharge when taken from a single wire; on continuing the discharges, the peculiar blue phosphorescence previously noticed at the negative (68) is often perceivable at *both* terminals (87); and when in this state, if a galvanometer is introduced in the circuit, the needle is not deflected, proving that the discharge is not continuous but reciprocating; but if during the discharges the potash in the tube is heated, the stratifications become perceptible, and the needle of the galvanometer will be deflected, showing that a continuous discharge is now present: this experiment is fully described in another part of this paper (88).

85. I have often found it difficult, and at times impracticable, to obtain precisely the same results in carbonic-acid vacua; they are probably modified, not only by the relative purity of the gas and of the potash, but also by the quantity of moisture. When the gas was passed through chloride of calcium or through strong sulphuric acid, the stratifications appeared more quickly. When moisture had been purposely added, as by placing a piece of asbestos moistened with boiled distilled water within the tube, the cloud-like stratifications were not obtained, although the stratifications of the conical form were remarkably clear: in a vacuum-tube thus prepared, No. 61, the caustic potash remains in the centre, the asbestos being placed in the narrow end; the potash has absorbed the moisture, leaving the asbestos to all appearance perfectly dry. The potash in this tube has been repeatedly heated; when heat is applied the stratifications at first become clearer; they gradually lose the conical form and become narrow. As the heat continues, all trace of stratifications or discharges over the potash is destroyed, that portion nearest the negative is blackened (possibly by carbon being evolved). If the tube is permitted to cool, the stratifications gradually reappear, until they resume their normal state; on carefully examining the potash, a minute hole is observable in the end of it nearest the positive wire, apparently showing a direction of force in the discharge.

On Non-conducting Vacua.

86. I have stated that under certain conditions the stratifications entirely disappeared, and as this result is, I believe, new, and has been confirmed by three separate experiments, I will briefly describe two as they are entered in my note-book. On the 12th of June, two tubes were prepared in the laboratory of St. Bartholomew's Hospital; in each, exclusive of the caustic potash, some fused chloride of calcium was introduced; in addi-

tion to these, in one tube, No. 66, small crystals of Sulphur were placed, and in the other, No. 67, pieces of Selenium. The tubes were then charged with carbonic acid, and exhausted as before. No. 66, on the 21st of June, showed clear narrow stratifications, the potash was then heated; on the 26th the stratifications had increased, assuming the conical form, clear and defined; some of the sulphur was melted, condensing at each end of the tube; as it cooled the stratifications which had been destroyed reassumed their conical form. The stratifications improved very slowly, and on the 14th of July they assumed the large cloud-like appearance, similar to those of the mercury vacuum (63). The caustic potash was then again heated, and the stratifications became very narrow; from this time they gradually improved, and on the 28th, on trying the discharge, I found that it no longer passed. The potash was then heated and the discharges continued, the tube shortly became illuminated; this was followed by large cloud-like stratifications, then by the conical and narrow; on allowing the potash to cool, the tube resumed its insulating state.

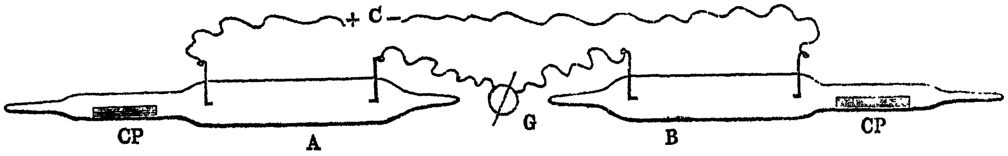
87. Selenium, No. 67.—From the 12th to the 19th of June the discharge was a wave line; on the 20th the potash was heated; on the 22nd stratifications appeared of a greenish tinge, inclined to the conical form; from the 23rd to the 28th they assumed the cloud-like character; on the 28th the selenium was melted; the stratifications were then of a reddish tinge, but they remained nearly of the same form, the vapour of the selenium condensing very quickly in the cooler portions of the tube. On the 6th of July the end of the tube was placed in a freezing mixture, and the tube was heated in order to free the sides thereof from a quantity of selenium which had deposited and thus obscured the view of the discharge. On the following day (7th July) the discharge illuminated the tube *without* stratification, and on grasping the tube with the hand both wires were surrounded with the intense blue discharge (84), which, under ordinary conditions, only appears at the negative. On the 11th of July, on heating the caustic potash, the blue phosphorescence at the positive wire was destroyed, and the cloud-like stratifications reappeared. This tube was subsequently placed aside, after having been heated two or three times, for a month, when, on trying the discharge on the 4th of October, I ascertained that the discharge no longer passed, the vapour which had evolved by heating the caustic potash having been again absorbed.

88. When a vacuum-tube is in a state to show the stratified discharge, it is so good a conductor that sparks from the outer terminal of the induction coil will pass to one of its wires (the other being attached to the inner terminal) one inch in length through air; but when no signs of a luminous discharge can be observed through the tube, or merely faint luminosity without stratifications, the spark is scarcely visible, although it will pass from the outer terminal of the coil through air to a wire attached to a glass rod, or to one insulated by shell-lac, showing that a vacuum-tube in this state is a more perfect insulator of the electrical discharge than air. The following experiment will perhaps be considered as conclusive as to vacua under this condition insulating the discharge.

Fig. 9 represents two tubes and a galvanometer attached to the induction coil C.

A, carbonic acid vacuum-tube, similar to the sulphur or selenium one (86, 87); B, vacuum-tube (which shows stratification when discharges are passed); G, a delicate

Fig. 9.



galvanometer placed at a sufficient distance to preclude any action on the needle from the iron core in the induction coil. If discharges from the induction coil C are now made, there will not be a trace of any luminous discharge in A; in B, the stratified discharges are reciprocating, and, when tested by a magnet, will be found to divide; they are merely discharges from the terminal of the coil to which one of the wires is attached, the needle of the galvanometer remains stationary.

If in this state heat is applied to (CP) the caustic potash in A, in a few seconds the stratified discharges will be visible; the needle of the galvanometer will be deflected, and the stratifications in B will at the same time assume the usual appearance of the conductive discharge.

Vacua-tubes prepared with Arsenious Acid, Bromine, Iodine, Pentachloride of Antimony, Bichloride and Bisulphide of Carbon.

89. ARSENIOUS ACID, No. 68.—A few crystals being first introduced with some caustic potash and chloride of calcium, the arsenious acid being kept separate, the tube was charged with carbonic acid gas and exhausted as before (83); the discharge at first appeared in the form of a wave line of a greenish tinge; the potash being heated, in a few days the discharge assumed a tendency to the cloud-like stratifications; the arsenious acid was then heated, the stratifications immediately narrowed, a brilliant luminous wave discharge passing through them from wire to wire; for about three months a gradual change took place, until the discharge again assumed the cloud-like, stratified appearance; on heating the potash, the stratifications suddenly became clear and defined with a greenish tinge, showing remarkably brilliant bands.

90. BROMINE, No. 69.—The bromine was inserted in a small glass bulb, and placed within the vacuum-tube previous to its being charged with carbonic acid and exhausted. In about ten days the cloud-like stratifications were visible, when the glass bulb was broken; the tube instantly filled with bromine vapour so dense that the discharge would not pass. On the following day the potash had absorbed a portion of the vapour, and the discharge was visible with very narrow stratifications; these gradually improved, and in twenty-seven days assumed the cloud-like appearance. On the twenty-ninth day, finding the stratifications becoming indistinct, while the blue tongue at the negative remained peculiarly fine and extremely sensitive to the magnet, I sealed off that portion of the glass tube in which the caustic potash had been placed; the discharge up to the

present time retains the same appearance, showing the blue tongue expanded or contracted by the magnet in a very beautiful and clear manner.

91. Iodine, No. 70.—Crystals of iodine were placed in a sealed glass bulb within the tube with caustic potassa; it was then charged with carbonic acid gas and exhausted as before; the cloud-like stratifications appeared in about eleven days; the bulb was broken and the heat applied to a few of the crystals of iodine; at first a wave discharge was visible, passing through the vapour without stratifications; in a few days stratifications were visible, and the tube has remained ever since in the same state. If the contact breaker is pressed so as to cause a more sudden disruption in the discharge, a wave line brilliantly illuminated can be obtained, passing conjointly with and through the narrow stratifications, as in the arsenious-acid tube (89). When the discharge is taken from one terminal in vapour of iodine, the difference of the action between the outer and inner terminal is very remarkable; when the former is positive, if the tube is grasped by the hand, the discharge shows stratifications between the hand to the discharging wire; from the inner terminal no stratifications can be seen, but only a faint luminosity divisible by a magnet.

92. Pentachloride of Antimony, No. 71.—The small glass bulb containing the pentachloride having been inserted in the tube as in the previous experiment, was broken after the cloud-like stratifications had appeared, when all trace of stratification ceased, the discharge passing in the form of an intensely brilliant wave line; the gradual progress first to the narrow and ultimately to the cloud-like stratifications was subsequently very slow, for it was upwards of two months before the latter appeared.

93. Bichloride and Bisulphide of Carbon, Nos. 73 & 74.—These were introduced into the separate tubes in glass bulbs, as in the previous experiments; the bulbs were broken as before after the discharges had assumed the cloud-like appearance; but although every means was adopted, first by melting the potash, and then applying freezing mixtures to condense the vapour; it remained under every condition so dense, that, after the glass bulbs were broken, the discharge could not pass even in the form of the wave line.

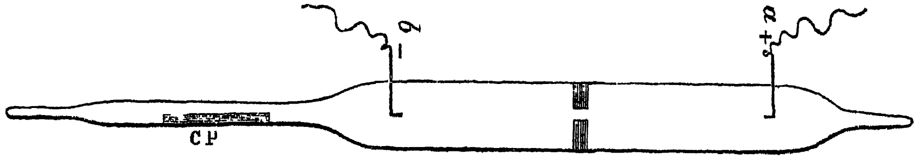
All the experiments hitherto described were made in glass tubes of about 1 inch internal diameter, similar to those of my previous experiments with Torricellian vacuums, in which I had been necessarily limited as to the size of the tubes by the difficulty of manipulating with large masses of mercury; with carbonic-acid vacua this difficulty no longer arose, and I was induced to try the effect that could be obtained in larger vessels.

94. Fig. 10, Plate IX. represents a large glass vessel, egg-shaped, the globular portion being 18 inches in length and 7 inches in diameter; the wires are 22 inches apart, the caustic potash being placed in the narrow end; total length 25 inches. It was charged with carbonic acid and exhausted in the usual manner; a portion of the potash being heated by a spirit-lamp, in about two months the discharge assumed in a very marked manner the character of large and distinct clouds, most clearly and separately defined;

these are strongly affected by induction as the hand approaches the globe, presenting a very striking appearance; there is a slight tinge of red, showing that a very minute quantity of air remains: the cloud-like stratifications *extend to the entire diameter of the vessel.*

95. Fig. 11 represents another form of apparatus. This glass tube, No. 81, is 12 inches long, and about 1 inch internal diameter; it has a glass division in the centre, perforated with a hole about $\frac{1}{8}$th of an inch in diameter, my object being to ascertain in what manner the stratified discharge would be affected during its passage through the aperture; the wires are 10 inches apart. I had already ascertained that if an electrical discharge passes through a vacuum-tube a portion of which is of very small dimensions, the glass is very much heated, while that portion of the glass which is wider remains cool. In the apparatus fig. 11, if *a* is positive and *b* negative when the discharge shows narrow striæ, those near the diaphragm in the *a* positive side are indistinct, while on the *b* negative side the striæ are very clearly defined, although the luminous portion is much contracted; when the discharge has assumed the cloud-like appearance, the aperture only affects them by contracting the cloud, which passes immediately through it. In this tube I had at one time succeeded in entirely insulating the discharge, the same as in both those in which sulphur and selenium were placed (86), (87), and I subsequently repeated this experiment with the same tube in the presence of Dr. FRANKLAND and his assistants, in his laboratory; but a few days afterwards, on heating the potash, the discharge returned to, and has ever since retained the cloud-like appearance,

Fig. 11.



96. I had always found there was apparently much less deposit on the sides of the glass from brass than from platinum wires; this was so decided, as to make me doubtful whether this deposit was not in all cases due to the platinum to which the brass wire is attached. In a tube, No. 82, with brass terminal wires, on the interior of which, from long continuance of the discharge, I observed much deposit had taken place, on examination Dr. FRANKLAND found this deposit was entirely brass in a minute state of division, the metal being deposited precisely the same as the platinum (37), translucent when examined by direct light, but presenting by reflected light a dark metallic lustre.

Condition of Discharge at Negative Terminal (continued).

97. That the stratified discharge is considerably influenced and modified by the particular condition of the negative terminal in a vacuum-tube, is evident by the experiment, where it did not appear when new platinum wires were used, until such wires were corroded by continuous discharges (74). I have also invariably found that if brass wire

terminals were used, the stratifications were visible at the first discharge; the only explanation I can offer is, that the stratifications may be influenced by the presence of the very minute points in the brass wire used in my tubes, while the new smooth platinum did not produce them until corroded by the discharges.

98. When the negative terminal is of liquid mercury, whatever be the length to which the mercury is elongated, its entire surface is covered with a luminous white glow; if it is made positive, it is only the extreme point from which the illumination proceeds (58). If a tube with mercury is placed in a perpendicular position, the blue tongue is very large, and the surface of the mercury (which in this experiment must be made negative) is covered with a lambent white glow (72). This phenomenon appearing to me to be intimately connected with the blue-tongue discharge (68), which is always present under certain conditions of the stratified discharge, I determined on examining a phenomenon that presented the appearance of a distinct form of discharge from that at the positive terminal; for this object I had several vacua-tubes constructed, in which the platinum, as well as brass wires hermetically sealed, were more or less protected by being covered with glass tubing.

99. After many trials, I ascertained that if the negative wire is protected by glass tubing open at the end about one-eighth of an inch beyond the point of the wire in the tube, no stratifications can be observed in the discharge, which in such cases merely exhibits a luminous glow; for this experiment it is necessary that the orifice in the tube should be contracted in order to prevent the wire being exposed, and the discharge from the coil should also be of moderate intensity; as if the intensity is much reduced, either by the battery or by manipulation with the contact breaker, faint stratifications in the discharges may at times be observed. In a tube thus constructed, No. 90, a discharge appears to emanate from the negative wire, issuing with great intensity through the orifice; and if the wire and the tubing are a little inclined, the discharge will impinge against the side of the vacuum-tube (fig. 12), brilliantly illuminating the spot on which it impinges, in a similar manner to the blue-tongue discharge (68, 70); if this discharge is continued for a few seconds, that portion of the tube on which the discharge impinges will be sensibly heated; and if a magnet is presented, the spot will be contracted with one pole, and with the other the discharge will be bent in a manner so that its extreme portion will itself impinge on the other side of the tube, illuminating and heating it as above. If the experiment is made by reducing the intensity of the discharge, so that stratifications from the positive terminal are observable, these stratifications vanish, as the discharge, which apparently proceeds from the negative terminal, is forced by the magnet along the tube (fig. 13). In this experiment there is the appearance of a direction of a force emanating from the negative terminal as well as one from the positive, while from the latter we have, under certain conditions, as I have already described, a direction of force from the positive to the negative, centring to the axis of stratification* (57).

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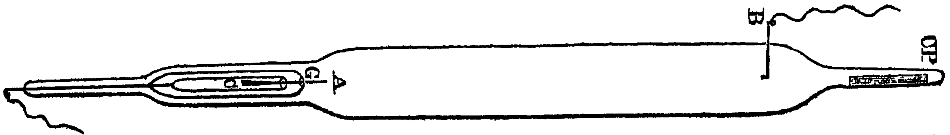
* In a recent communication from Professor W. Thomson to Mr. Joule, read before the Literary and

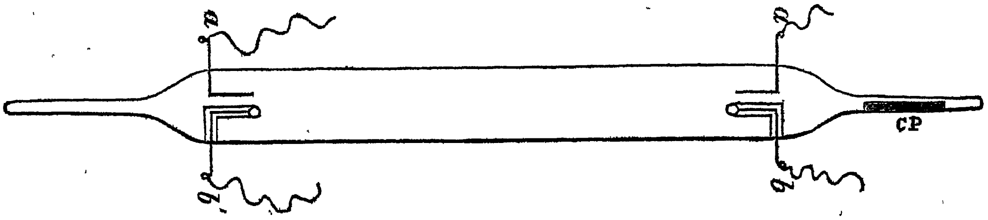
100. I have already alluded to the difficulty of obtaining separate carbonic acid vacua in which the stratified discharge presents in all respects precisely the same appearance; and in order that I might examine the discharge with protected as well as non-protected negative terminals in vacua made under precisely the same conditions, I had two glass tubes constructed. No. 101, fig. 14, represents one, a carbonic acid vacuum-tube, with caustic potash inserted in the narrow portion CP; the wire A can, by inclining the tube, be dropped into the inner case of brass tubing C, which is itself protected by a glass tubing G; the brass tubing is soldered to a platinum wire P, and thus hermetically sealed; B, the other wire, which is fixed, is also of brass. When both wires are exposed, the stratified discharges show the clear cloud-like appearance so often described, whether A is positive or negative; but if A is made negative and the tube is inclined so as to let the wire drop into the brass tubing, almost all trace of stratifications immediately disappear, that which remains being evidently due to the portion of wire and the brass tube which is exposed through the orifice; in this state each discharge is accompanied by a ringing noise from within the brass tube: reversing the direction of the discharge, A becomes positive and B negative; the stratifications are immediately perceivable, and the ringing noise in the brass tube ceases.

Fig. 14.



101. In the other tube, No. 104 (fig. 15), four wires are hermetically sealed; *a*, *a'* are exposed; *b*, *b'* are protected by being placed in glass tubing, extending about one-eighth of an inch beyond the points of the metal. When discharges are taken from *a* to *a'*, or when *b* or *b'* is positive and *a* or *a'* negative, the stratifications are clear and distinct; and by heating the potassa they can be altered from the cloud to the conical shape; but under whatever condition the tubes may be, the stratifications entirely cease if *b* or *b'* is made negative; in this condition no trace of stratifications can be observed, *the negative discharge issuing through the orifice as from a jet.*

Fig. 15.



102. In vacuum-tubes, particularly in some obtained by the carbonic acid process, I

Philosophical Society of Manchester, the former gentleman alluded to the remarkable difference in the effects of positive and negative electrifications.

have often observed beyond the clear cloud-like stratifications towards the negative, several faint but clearly-defined striæ or bands; when repeating Mr. GROVE's experiment, of allowing the discharge to be intercepted between two metallic points attached to the coil*, I observed that however short might be the interval by which the discharge was thus separated, these faintly illuminated striæ or bands always vanished, leaving the clear clouds without change of form; for this experiment I use my micrometer-electrometer, described by me in the Philosophical Transactions, Part I., 1840, and in which, by means of the micrometer screw, I can obtain a separation of $\frac{1}{5000}$th of an inch.

103. Most striking and beautiful effects of the stratified discharge were obtained in a large glass cylinder, of about 4½ inches internal diameter, in which brass wires were hermetically sealed about 20 inches apart. The caustic potash being inserted in the narrow portion, the cylinder was charged with carbonic acid gas, and exhausted in the usual manner. With one wire attached to the outer positive terminal of the coil, the stratifications (large cloud-like form) are very clearly defined and most remarkably sensitive to inductive action, by the approach of the hand; on the four fingers being successively placed on the stratifications, the stratifications disappear in succession; but while in this state, if a powerful electro-magnet is put in action close to the negative, the stratifications reappear and pass by the fingers of the hand, which during the experiment rest on the glass; the power of the magnet overcoming the inductive resistance. The stratifications in this apparatus can be separated for a considerable space by placing both hands on two separate portions of the cylinder; and if one hand is placed within about 2 inches of the positive wire, where two or three clouds are then visible, and the other hand is placed on the cylinder about 4 to 6 inches nearer the negative, a large single cloud will be brought out in a very striking manner, showing that even where stratifications are no longer visible, they can be elicited by the mere effect of induction. If the wire previously attached to the outer is connected with the inner terminal of the coil, the effects of induction, by the approach or even on touching the cylinder, cannot be obtained, but the power of the magnet to draw out a further number of the stratifications is equally strong as before.

General Conclusions.

104. From the absorption of carbonic acid by caustic potash, we not only obtain a far more perfect vacuum than the Torricellian, but the process of absorption is so gradual, that we are enabled to examine with much accuracy the different phases which the electrical discharge assumes. With the air-pump vacuum, a discharge that will not pass one inch in air will, in the form of a wave-discharge, pass from wire to wire through 30 or 40 inches, or even more; and as the vacuum improves by the absorption of the carbonic acid, the discharge gradually fills the tube with a luminous glow, showing narrow stratifications commencing at the positive wire, the negative wire being surrounded with a blue glow; as the vacuum continues to improve, the narrow stratifications extend throughout the discharge to about one inch of the negative wire, where

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* See note at end of this paper.

they terminate abruptly in a dark space, the negative wire becoming intensely red; in this state, if the contact-breaker is pressed so as to make the discharge more sudden, stratifications extend to the negative wire, which loses its red and assumes a luminous white appearance. As the absorption of the carbonic acid continues, the stratifications become more distinct and separated, and the dark space often extends, in a tube of 20 inches in length, to 6 or 8 inches; they then assume a conical form, showing, when the contact-breaker is pressed, an apparent direction of force from the positive wire, centring on the axis of the stratified discharge *; the stratifications are then of the cloud-like appearance, precisely the same form as in the best Torricellian vacua. Under some conditions the stratifications gradually disappear, the glass tube is filled with faint luminosity, which, when tested by placing a galvanometer in the circuit, is found to be no longer a conductive but a reciprocating discharge. When the process was carried still further, vacua-tubes were obtained, in which even this luminosity could not be perceived except with a stronger charge, as by attaching the condenser. In addition to a galvanometer, another vacuum-tube (which under the ordinary action of the coil showed stratifications) was introduced into the circuit and no light perceived in it; thus *the non-transferring condition of a vacuum has been experimentally ascertained* †.

105. Whatever may be the length of the stratified discharge, it will always be found to be in the same electrical condition; from its great length the positive would appear to be the dominant, as it is equally long whether taken from the outer or inner terminal. In my previous paper I stated that I was inclined to the opinion "that the stratifications arose from pulsations or impulses of a force acting on highly attenuated matter;" and I am the more confirmed in this opinion, as, *if the pulsations or vibrations are greatest in bright bands and least in the obscure, this system of interference or of pulsations would also account for the entire absence of stratifications when the air or gas is not sufficiently rarefied, as well as when the vacuum becomes nearly perfect*, while the graduations of narrow to the broad cloud-like stratifications can be thus satisfactorily explained.

106. The absence of a dark band and of stratifications when the sides of the negative wire are completely protected by glass tubing, is strong evidence that both must in some way be governed by the portion of the negative metallic surface which is exposed; by carefully protecting the negative wire, we not only destroy all trace of stratifications, but *we obtain evidences of apparently a direct force from the negative terminal.* If the negative wire is uncovered and placed in a direct line with the positive, the negative discharge is entirely in a lateral direction, as is denoted by the metallic deposit on the

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* This direction of force in the positive discharge is also noted by the perforation in the stick of potassa (85).

† While preparing this communication for the Royal Society, I observed that Professor PLÜCKER states that, after passing the discharge in a tube containing oxygen for about 1¼ hour, "the discharge through the tube began to be discontinuous, till at last only single separate discharges occurred; finally, the current entirely ceased to traverse the tube." I have since repeated the experiment in tubes carefully prepared by Dr. FRANKLAND, continuing the discharges for several hours, but without obtaining this result.

sides of the vacuum-tubes; and PLÜCKER has shown that it arranges itself between the poles of a powerful electro-magnet in the line of magnetic force. In this communication I have shown not only the appearance of the direction of a force as evinced when passing through a narrow orifice, but that wherever this discharge impinges heat is evolved. At present I do not alter the opinion that I ventured to offer, that the dark band may be due to interference; but the new facts which have been elicited in relation to the negative discharge require a further, and probably an extended experimental investigation, and I am at present engaged in endeavouring to test the accuracy of the opinion I have offered, by means of apparatus now in the progress of construction.

107. I am aware that in this paper I have entered somewhat more fully into detail, when describing my experiments, than is usual in communications to the Royal Society, but I have been induced to be thus minute, as the subject is novel; and by explaining some of my experiments in detail, it may afford to others the means of examining directly, and possibly more correctly, into the nature of that mysterious force which we denominate Electricity, and which is in many respects so nearly related to heat, magnetism, and other forces, each being, as Mr. GROVE has long since shown, convertible into each other; while the fact that a vacuum so perfect can be obtained in a closed vessel containing such a substance as hydrate of potassa, would excite a hope that the limit to vaporation, the existence of which FARADAY* and others have, if not proved, at least rendered so probable, may be determined, and even its consequences exhibited by direct experiment.

JOHN P. GASSIOT.

Clapham Common,
December 8, 1858.

Note.—Received January 11, 1859.

Since I sent my communication to the Royal Society, I have again obtained a non-conducting vacuum without the introduction of sulphur or selenium (95). In the extremity of a tube, No. 140, 40 inches long, 1⅞ inch internal diameter, four pieces of caustic potash were inserted; the tube was then charged with carbonic acid, exhausted and hermetically sealed, as already described; in this state the potash was carefully heated by a spirit-lamp until it melted; the tube being slowly turned, the potash on gradually cooling covered the surface of that portion of the tube in which it had been placed; in this manner it presented a large surface; in a short time the remainder of the carbonic acid was absorbed, and the vacuum thus formed no longer conducted the discharge.

If two vacuum-tubes which conduct are attached, one to the inner and the other to the outer terminal of an induction coil, without being otherwise connected with each other, a reciprocating discharge will take place in each, that from the outer being far more vivid; if the circuit is now completed, either by a wire or by a tube which conducts, the stratifications in each tube become clear and distinct, the dark discharges becoming as visible as if the circuit with the coil had been completed by a single tube.

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* Philosophical Transactions, 1826, Part III. p. 484.

If the tube which conducts is replaced by one such as I have described in this Note, it will be found that the discharges in the two tubes attached to the coil are reciprocating, while in that with which they are connected no luminous discharge will be perceptible, until heat is applied to the caustic potassa, when in a short time flashes of light will be observed, and then the cloud-like stratifications, the discharge in the two tubes at the same time being no longer *reciprocating*, but *continuous.*

If a vacuum-tube is connected by flexible wires to the terminals of an induction coil, the tube can, when held by the hand, be moved to and fro with a rapid motion; if while in this state the discharges are made they will appear separated, forming in darkness a very brilliant fan-like figure. In my former communication* I have shown that, whatever might be the length of a luminous stratified discharge in a vacuum-tube, "the full intensity of the discharge is visible at a single contact, exhibiting 80 to 100 stratifications (36)." When a single disruption of the primary circuit is made in the manner I have described (30), while the vacuum tube is moved to and fro with rapidity, or if by an apparatus which I have had constructed, the tube is rotated in a plane, only a single discharge showing stratifications will be visible; the discharge takes place in that part of the circle in which the tube happens to be situated, at the instant of the disruption of the primary circuit; but when the vibrating contact breaker of the coil is substituted for the single disruption (30), the motion of the tube gives it the appearance of illuminated spokes in a rotating wheel, the discharges being separated according to the velocity of the rotation.

I venture to offer this experiment as an evidence that the stratified discharge *in vacuo* is entirely due to a single disruption of the primary circuit.

Jan. 10, 1859.

Note.—Received February 17, *read March* 3, 1859.

Since my communication to the Royal Society (read January 13) I have repeated the experiment of Mr. GROVE, wherein by an interruption of the secondary circuit through air the stratifications of the electrical discharge in a vacuum-tube are destroyed; the results I have obtained, and which I now proceed to describe, tend to confirm me in the opinion I originally ventured to offer, that the stratifications arise from the effect due to pulsation, or impulses of a force acting on highly attenuated matter, and are not due to a conflict of two currents *in the manner* explained by Mr. GROVE, which his experiment of simply making an interruption of the secondary circuit† has led him to suggest; these results also show that the varied form observable in the stratifications *mainly* depend on the greater or less density of the matter remaining in the tube.

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* Philosophical Transactions, 1858, Part I.

† Philosophical Magazine, 1858, p. 18.

I have already stated that the striæ can be obtained by the electrical machine[*] if a Leyden jar is discharged through a vacuum-tube; they may at times be observed, but the discharge generally passes in the form of a wave of light of dazzling brightness; this has also been observed by Messrs. Quet and Seguin[†]; but if the intensity of the discharge is reduced by means of a wet string, the striæ can be developed in a vacuum-tube as clearly and as distinctly as from the induction coil; this led me to suppose that when the circuit is interrupted, as in the experiment of Mr. Grove, the absence of striæ is due to the heightened intensity of the discharge.

Having at this time a considerable number of vacuum-tubes at my command, I have been enabled to repeat Mr. Grove's experiment, and to test the above supposition in a variety of ways, and I invariably found that with the best vacuum-tubes which show large cloud-like stratifications, I could obtain the result described by Mr. Grove with greater certainty; for instance, in my large cylinder (103) which I exhibited at the meeting of the Royal Society on January 13, and with which Mr. Grove subsequently repeated his experiment at the Royal Institution on January 28, the stratifications are destroyed by an interruption of about one-eighth of an inch, although with other tubes the distance must not only be considerably increased, but the stratified and non-stratified discharges, "notwithstanding all the care used, pass in irregular succession." Now if the disappearance of the stratifications be due, as suggested by Mr. Grove, to the cutting off of the feeble current by the interruption introduced into the circuit, it remains to be explained why a space of one-eighth of an inch abolishes the stratifications in one tube, while three times that space will fail to do so in another.

With a vacuum-tube in which the stratifications were entirely destroyed when the secondary circuit was interrupted, I found they were always restored when the space thus interrupted was completed by *means of a wet string*; but as this experiment is not strictly analogous to the reduced discharge of a Leyden jar, it was varied by making a second interruption in the circuit and closing this by a wet string; in this manner the discharge passed through the space in air, but being retarded by the string the stratifications were again visible.

This experiment appears to be fatal to the theory of Mr. Grove; for the introduction of a wet string *in a distant portion of the circuit* cannot possibly help a feebler current to cross the interval opposed to it; it is quite evident that in this case, as well as in that of the Leyden jar, the appearance of the striæ depends not upon the conflict of "secondary and tertiary" currents, but simply as to the manner in which the discharge passes.

I also ascertained that when, by means of an interrupted discharge, the stratifications are destroyed, they are reproduced in a carbonic acid vacuum-tube, when heat is applied to the caustic potash; in this experiment the increased resistance arises from the greater density of the matter formed in the tube; if this be correct, the discharge, when passed simultaneously through two tubes, in each of which the attenuated matter possesses different densities, would produce different effects; if one tube exhibits with the ordinary discharge the cloud-like character, and the other narrow striæ, in the former the current, when interrupted through space in air, would pass without stratification, and in the latter they would be retained.

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[*] Philosophical Transactions, 1858, Part I. p. 10.

[†] Comptes Rendus, December 13, 1858.

Two tubes were so arranged, one showing the cloud discharge, in the other (filled with the vapour of iodine) in which the stratifications are clearly defined but very narrow; by means of my micrometer-electrometer the secondary circuit could be interrupted or completed with great precision. When the secondary circuit is interrupted by separation with the electrometer to about one-eighth of an inch, the large cloud stratifications in the tube disappeared, the discharges passing "with uniform luminous flow," as described by Mr. GROVE; but no alteration is perceptible in the iodine tube, in which the stratifications continue as before, even when the separation extends to half an inch; it is manifest that this result is also irreconcileable with the explanation offered by Mr. GROVE, as, if the alleged feeble current were cut off in one tube, it would be cut off in both.

J. P. G.

Clapham Common,
February 14, 1859.

VIII. *On the Vertebral Characters of the Order* Pterosauria, *as exemplified in the Genera* Pterodactylus (Cuvier) *and* Dimorphodon (Owen). *By Professor* Owen, *F.R.S., Superintendent of the Natural History Departments in the British Museum.*

Received February 23,—Read March 24, 1859.

Although the skeletons of the extinct Flying Saurians of the mezozoic strata have been discovered in a more complete condition than those of any other contemporary Reptiles, they have not, hitherto, owing to the delicate texture and commonly crushed state of the bones, afforded satisfactory observations of the structure of the vertebræ. Yet the vertebral characters of a saurian skeleton are of peculiar interest to the Palæontologist and Comparative Anatomist, on account of the many and strongly-marked differences which they present in the extinct members of the Reptilian class.

In the existing species, the articular terminal surfaces of the centrum, with the exception of those of the Geckos, Rhynchocephalus*, and of some single vertebræ in the column of other Reptiles†, are concave in front and convex behind. But in extinct Reptiles some genera (*Streptospondylus*) show reverse positions of the cup and ball; others (*Ichthyosaurus*) show both surfaces concave; others show both surfaces slightly concave (*Teleosaurus*); others show both surfaces flat (some *Plesiosauri*); others are subconcave behind and flat before (*Cetiosaurus*); with many minor modifications. It is only on arriving at the uppermost of the mezozoic series of rocks in an ascending survey, that we find Lacertian genera (*Mosasaurus* and *Leiodon*‡ of the Chalk) and crocodilians (*Crocodilus basifissus*, from the Greensand of New Jersey§) presenting the procœlian‖ type of vertebra which prevails in tertiary and modern reptiles.

No fossil vertebra from secondary rocks has come under my observation or knowledge with characters of the articular ends of the centrum which distinguish the vertebræ of birds, viz. a concavity in one direction and a convexity in the other, the directions being reversed at the two ends of the centrum.

All the foregoing considerations have tended to invest the question of the vertebral characters of the Pterodactyles with peculiar interest; and, especially, seeing the adaptation of their reptilian type of structure for flight, in reference to carrying out the comparison of their skeleton with that of birds. I have therefore thought the summary of observations might not be unworthy the attention of the Royal Society, which I have made, as opportunities presented themselves, for some years past, on the vertebral characters of different species of the order *Pterosauria.*

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* Catalogue of the Osteological Series, Museum of the Royal College of Surgeons, 4to. 1853, p. 142.

† Biconcave fifth cervical vertebra in *Chelone Mydas*; biconvex first caudal vertebra, Crocodile.—Catalogue of the Physiological Series, Museum of the Royal College of Surgeons, 4to. vol. i. 1832, pp. 52, 53.

‡ History of British Fossil Reptiles, 4to. p. 188.

§ "Notes on Remains of Fossil Reptiles discovered in Greensand Formations of New Jersey," Quarterly Journal of the Geological Society, vol. v. 1849, p. 380.

‖ Report on British Fossil Reptiles, 8vo. 1841, p. 65.

Before entering upon such summary, I may briefly allude to the recorded observations on the subject. Baron Cuvier, in his celebrated memoir establishing the reptilian character of the *Ornithocephalus* of Soemmering, which he, thereupon, proposed to call 'Pterodactyle,' almost restricts himself to a comparison of the number of the cervical and dorsal vertebræ in the Pterodactyle and Bird[*]. Five of the cervical vertebræ of *Pterodactylus longirostris* are stated to be "large and prismatic like those of long-necked birds"[†]. With regard to the dorsal vertebræ, "the spinous processes of the anterior ones are a little longer, those of the posterior ones are short and cut square[‡]."

The able editors of the posthumous edition of the 'Ossemens Fossiles' found nothing to add, in 1836, to the above-cited brief notices by the Baron on the vertebræ of the Pterodactyle, nor has anything material been with certainty determined and stated since that time.

The *Pterodactylus suevicus* described and figured by Professor Quenstedt in 1855, was perhaps in the best condition to have thrown some additional light upon the subject; but the most important remark relates to one of the dorsal vertebræ, in which "the articular surfaces of the body are convex at the back end, as in the Crocodile, concave at the fore part. At least so it seems;" writes the author[§].

In regard to the cervical vertebræ, the author says, "they seem not to have the trochlear joints as in Birds, yet a small cavity may be observed on the articular surfaces[‖]."

Accordingly, Professor Pictet, in the able summary of the characters of extinct reptiles in the last edition of his 'Paléontologie' (8vo, 1857), limits his notice of the vertebral characters of the Pterodactyles to their number in the different regions of the spine.

From observations made on species of *Pterosauria,* extending from the period of the Lias, as exemplified by the *Dimorphodon macronyx,* to the Upper Greensand, as exemplified by the *Pterodactylus Sedgwickii* and *Pter. Fittoni,* I am now able to state that, both with respect to the cervical and dorso-lumbar vertebræ, the terminal articular surfaces of the vertebral bodies are simply concave anteriorly (Plate X. fig. 1), convex posteriorly (*ib.* fig. 2)*; and that they consequently manifest the earliest known instance of the 'procœlian' type which now prevails in the reptilian class. But in no other reptile are those articular surfaces so narrow vertically in proportion to their breadth as they are in the cervical vertebræ of the *Pterosauria* (*ib.* figs. 1, 2, 12–14, *b*, *c*): in the dorsal series the cup and ball present more ordinary saurian proportions.

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[*] Ossemens Fossiles, ed. 8vo. tom. x. (1836), p. 215, 222.

[†] "On y voit cinq vertèbres, grandes et prismatiques comme celles des oiseaux à long cou," tom. cit. p. 232.

[‡] "Les apophyses épineuses antérieures sont un peu plus longues; les postérieures sont courtes et coupées carrément," *ib.* p. 233. The figures show that the author is not speaking of two kinds of spinous processes on the same vertebra.

[§] "Die Gelenkfläche der Wirbelkörper war auf der Hinterseite convex, vie beim Crokodil, vorn dagegen concav. So scheint es wenigstens."—Quenstedt, Ueber *Pterodactylus suevicus* im Lithographischen Schiefer Würtembergs, 4to. 1855, p. 45.

[‖] "Sie scheinen nicht das Nussgelenk wie bei Vögeln zu haben, obwohl man eine kleine Grube auf der Gelenkfläche wahrnimmt," *ib.* p. 40.

This fact, being established, will give new and increased interest to the working out and examination of any detached vertebræ from secondary rocks which resemble in texture and size those of Birds and Pterodactyles. It has been alleged, for example, on microscopic characters of the osseous tissue, that certain pneumatic wing-bones found fossil in Stonesfield slate, are those of birds. But every such vertebra from that lower oolitic bed which I have yet seen, has the simple terminal articular concavity or convexity characteristic of the Pterodactyle. A single specimen showing the interlocking joint, *i. e.* a transverse concavity and vertical convexity, or the reverse, would establish the fact of the existence of birds during the period of deposition of the stratum containing such fossil vertebra. There is but one exception, so far as I know, in the whole class of birds, to the foregoing type of vertebral structure; this occurs in a bird (*Aptenodytes*) which does not fly, but has the wing-bones dense, solid, and modified in form to serve the office of a fin, and is exemplified in but a small part of the vertebral column. In the third to the eighth dorsal vertebræ inclusive, the fore-part of the centrum is simply convex (*ib.* fig. 22, *c*), the hind part concave, and this part is concave in the second dorsal; but the fore-part of this dorsal vertebra is concavo-convex, as in the cervical vertebræ of the Penguin, and in the cervico-dorsal vertebræ of all other birds. But, besides the reverse positions of the cup and ball in the above dorsal vertebræ of the Penguin, as compared with the dorsal vertebræ of the Pterodactyle, the latter might be distinguished by the absence of the large bifurcate hypapophysis (*ib.* fig. 22, *hy*) which projects from the dorsal vertebræ of the Penguin; the inferior surface of the dorsal vertebræ in the Pterodactyle being smooth and simply convex transversely, and slightly concave longitudinally. The anterior cup becomes more shallow in the last lumbar and first sacral vertebræ of the Pterodactyle; the confluent articular surfaces of the sacral vertebræ are flattened. In the caudal vertebræ of the Pterodactyle the anterior transversely-elliptical cup and posterior ball are resumed at the articular end of the centrum; at least in most of the anterior caudal vertebræ of the great *Pterodactylus Sedgwickii* (*ib.* figs. 36, 37).

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* VON MEYER was led to believe from the crushed specimen of *Pterodactylus Gemmingi*, described and figured in his 'Palæontographica,' Erster Bd. 4to. 1851, p. 10, that both articular surfaces of the bodies of the cervical vertebræ were concave; and that the hinder surface of a dorsal vertebra was not convex. But this error was due to the state of the specimen. See also his 'Reptilien aus dem Lithographischen Schiefer,' fol. 1859, p. 68.

The cervical vertebræ average seven in number, counting the coalesced atlas and axis as one. They are characterized by their superior size, and especially length, as compared with the other vertebræ in the same skeleton; but differ in different species in this respect, being longest in those Pterodactyles with long, light and slender jaws, shortest in those with shorter, stronger and thicker jaws. The third to the sixth inclusive preserve the same, or nearly the same, length; the seventh becomes shorter, and exhibits other modifications transitional to the dorsal vertebræ.

The ordinary cervical vertebra in *Pterodactylus Sedgwickii* (figs. 1–10) has a long depressed centrum (*b–c*) coalesced with the neural arch (*n*), slightly concave lengthwise at the sides, slightly convex lengthwise below, at least in the fourth or fifth cervical (fig. 5). It developes a pair of short obtuse processes (p'), which form the posterior inferior angles, and between these the under surface is concave transversely; the articular ball (*b*) seems as if tilted up above the level of these processes. There is a low obtuse hypapophysis (*h*) at the fore-part, the base of which is in some cervicals prolonged backward a short way as a rudimental ridge. A large 'foramen pneumaticum' (figs. 5, 7, 8, *o*) perforates the middle of the side of the vertebra at the line of junction of the centrum and neural arch; it conducts chiefly to the large cancelli in the centrum.

The neural arch is low; broader, but shorter, than the centrum. Viewed from above (fig. 9), it presents a subquadrate form, deeply emarginate before and behind, less concave at the sides, and with the four angles produced to form the zygapophyses. The articular surfaces of the anterior pair (*z*) look upward, inward and forward; those of the posterior pair (z') downward, outward and backward: both are flat. The anterior zygapophyses (fig. 10, *z*) project on a level with the sides of the anterior articular cup, some way in advance of it, the base of the process being divided by an oblique notch from the cavity. The posterior zygapophyses (fig. 7, z') do not extend quite to the hinder ball; they project above its level; there is a tuberosity (*m*) above the articular surface in many vertebræ. The neural canal (fig. 6, *n*) is slightly expanded at its outlets, subcylindrical in transverse section, about half the length of the centrum; consequently leaving a large proportion of the spinal cord unprotected by bone at the intervals of the successive neural arches. The neural spine (fig. 8, *ns*) is moderately produced, compressed and truncate; its base is coextensive with the summit of the arch, and it contracts as it rises, with a sharp anterior border. Not any of the cervical vertebræ possess the pleural elements which give the great transverse anterior breadth to the cervicals in the bird, and complete there the large vertebrarterial foramen on each side: nor is the hypapophysis double, forming a transverse pair, as in most of the neck-vertebræ of birds. The processes answering to p' in the Pterodactyle are rudimental, when present, in the bird.

The chief differences in seemingly answerable cervical vertebræ, in different individuals, perhaps species, of large Pterodactyles from the Upper Greensand, are seen in the degree of depth or depression of the centrum, with corresponding differences in the form of the foramen pneumaticum, as shown in figs. 5, 7 and 8; with some slight differences in the hypapophysis affecting the lower contour of the centrum, and in the proportions of the articular surfaces of the zygapophyses.

In the sixth cervical vertebra the centrum becomes flatter below, and an anterior parapophysis (*p*) sometimes extends to the under part of the prezygapophysis (*z*), circumscribing a small arterial foramen, as in fig. 11.

In the last cervical (fig. 12) the centrum becomes much shorter: a diapophysis (*d*) is more distinctly developed behind the prezygapophysis (*z*), and these, with the parapophysis now completing a larger arterial canal, form a protuberance resembling that in the bird, on each side the fore-part of the vertebra.

In the dorsal vertebræ (figs. 13–21) the centrums grow deeper as they shorten, with a proportionate expansion of the articular cup, as in fig. 17, and ball, fig. 20. The posterior zygapophyses (fig. 14, *z'*) are more approximated, project from a higher level, are shorter, with the articular surfaces rather convex and looking more outward, and more distinct from the tubercle above (fig. 15, *z'*, *m'*). The neural spine (*ns*) is thicker. There is a small pneumatic foramen behind the base of each diapophysis. The walls of the neural arch increase in height, expanding as they rise, to send out the diapophyses (*d*) to which the free ribs articulate. These ribs are slender and hollow, as in Serpents (fig. 23, *a*).

In what appears to be a lumbar vertebra (fig. 24) the neural arch again sinks, and the strong diapophysis (fig. 24, *d*) projects far, without supporting a rib. In the first sacral vertebra (fig. 25) the fore-part of the neural arch (*d*) forms a broad transversely extended vertical wall above the small neural canal (*n*), and the strong transverse process (*d.*.*p*) sinks or extends its base vertically down to the lower border of the inferiorly flattened centrum (fig. 25, *c* and fig. 26, *c*). Fig. 27 shows parts of the bodies of three anchylosed sacral vertebræ of a neocomian Pterodactyle, the first having an anterior concave articulation (*ib. a*), as in the larger specimen, fig. 25; the groove for the passage of the nerve notches the back part of the parapophysis close to the line of suture with the second sacral vertebra. In this vertebra the corresponding nerve-notch is more advanced, leaving a short sutural surface behind, indicative of a position of the neural arch crossing for a short extent the line of junction of the second with the third sacral centrum. The parapophyses (*p*) of the second vertebra are sent off almost on a level with the lower surface of the centrum, which is almost flat.

Atlas and axis vertebræ of the Pterodactyle (figs. 28–34).

These vertebræ early coalesce with each other, but I have been able to separate them in a young specimen.

The atlas consists of a centrum (figs. 28, 29, 30, *c*) and of two styliform neurapophyses (ib. *na*); in one specimen there seemed also to be a very small flattened neural spine. The centrum is so short as to be discoid: it is flat where it joins and becomes anchylosed to the axis, and is concave for the occipital tubercle: this cup is circular (fig. 28, *c*).

The neurapophyses, resting on each side of the upper half of the centrum of the atlas, converge and articulate above with two small tubercles (figs. 31 and 32, *z*) on the fore-part of the neural arch of the axis, almost meeting but not uniting above the neural canal (fig. 28, *n*).

The body of the axis (figs. 29, *cx*) is eight times larger than that of the atlas: it expands posteriorly, and terminates by a transversely elliptical ball (*b*) at the upper part of that end, and in a pair of thick short obtuse diverging apophyses (p') at the lower part. There is a rudimental hypapophysial ridge (fig. 33, *hy*) from the middle and toward the fore-part of the under surface of the centrum. There is no trace of a hypapophysis, free or anchylosed, below the body of the atlas.

The centrum of the axis-vertebra is confluent with the neural arch: at the middle of the side, apparently crossing the line of junction, is a large subcircular aperture (fig. 29, *o*), which leads directly into the widely cancellous structure of the bone below the neural canal. This vacuity, as in the other cervicals, answers to the 'foramen pneumaticum' in the vertebræ of birds, and doubtless admitted a production from the air-cells extending along the neck of the Pterodactyle into the cancelli of the osseous tissue. The neural arch (*nx*) rests upon the three anterior fourths of the centrum; it expands as it passes backward; and there, also, as it rises, until it sends off from each posterior angle the zygapophysis (z'), which has a tubercle above, and a flat articular surface below, looking downward and a little outward and backward. The small tubercles at the fore-part of the neural arch (fig. 31, *z*), to which the neurapophyses of the atlas are ligamentously connected, may be the stunted homologues of anterior zygapophyses. The neural spine begins by a low ridge between those tubercles, increasing rapidly in thickness behind; but it has not been preserved in its full height in any specimen.

In the small atlas and axis (figs. 32—34) the line of suture between the bodies of these two vertebræ is distinct. In a somewhat larger specimen, the centrum of the atlas was separable by a smart blow, and showed the true anterior surface of that of the axis (fig. 31); it is very slightly concave with a submedian prominence. A vertical section of the anchylosed atlas and axis is shown in fig. 30: the neural canal (*n*) expands at its posterior outlet: the large cancelli of the centrum are filled by the matrix.

On comparing the atlas and axis of the Pterodactyle with that of the Bird, the Ostrich for example, the atlas in the bird is represented by the neurapophyses which have coalesced below with a hypapophysis, forming an irregular ring of bone. The centrum has coalesced with that of the axis forming a small prominence convex anteriorly, and filling up the vacuity at the upper part of the cup excavated in the fore-part of the hypapophysis: the neurapophyses are broad in the bird, and overlap the anterior zygapophyses of the axis: they meet above the neural canal, but retain the separating fissure there in the Ostrich. The centrum of the axis is broader before than behind. A short process, like a connate pleurapophysis from the fore-part of the centrum, unites with a diapophysis from the neural arch to form an arterial canal. The pneumatic foramen is behind the diapophysis, and conducts to the cancellous tissue of the neural arch. The centrum is produced into a strong hypapophysis below the posterior articular surface, but not expanded laterally into transverse processes, answering to those marked p' in the Pterodactyle. The hinder articular surface of the centrum of the axis of the bird is convex transversely, but concave vertically, not simply convex, as in the Pterodactyle; thus a portion of the vertebra of that reptile, notwithstanding its pneumatic structure, might be distinguished from the vertebra of a bird.

The caudal region of the spine is that which, in the *Pterosauria*, as in the Bats and most other natural groups, is subject to most variety. In *Pterodactylus* (*Rhamphorhynchus*) *Gemmingi*, v. MEYER, from Solenhofen slate, the tail is very long, and most of the long and slender vertebræ, between thirty and forty in number, seem to have coalesced into a stiff style for supporting an expanded interfemoral membrane. In *Pterodactylus longirostris*, CUV. the tail is short, and the small and short vertebræ are free. In the large Pterodactyles from the Cambridge Greensand, it would seem that the tail was of moderate length and the vertebræ are free. The collections examined by me included fifteen caudal vertebræ.

The largest of these (figs. 35–37) measures 1½ inch in length; it is slightly contracted in the middle; the fore-part of the under surface is a little produced; the back part almost flat between the rudimental processes (p'): the shallow anterior concavity (fig. 36, *c*) has resumed its transversely elliptical shape, and the hinder convexity is defined below by a shallow groove connecting the processes (p'). There is no pneumatic foramen, unless a small hole on each side the hinder outlet of the neural canal have served as such. The neural arch is long and low, of one piece with the centrum, which extends beyond it posteriorly; it sends off short obtuse zygapophyses before and behind: those in front extend beyond the cup of the centrum; the surfaces on those behind look downward and backward. The base of the spine is coextensive with the summit of the arch, but is narrow. The neural canal is much contracted. There is no indication of a hæmal arch, either by articular or fractured anchylosed surfaces. The diameter of the middle of this vertebra is 6 lines.

The next caudal vertebra in size measures 1 inch 5 lines. The base of the neural spine begins two lines behind the fore-part of the arch, but terminates nearer the hind part. The nerve-grooves notch the hinder zygapophyses.

Three more slender caudal vertebræ present each a length of 1 inch 3 lines: the diameter at the middle is 5 lines in one, 4 in a second, and 3½ in the third vertebra, showing that they become more slender without losing length. A caudal vertebra, 3 lines across the middle, appears to have been nearly an inch in length; but both extremities are injured.

The most perfect vertebræ of other species of Pterodactyles from older formations which I have as yet had opportunities of examining, are those of a rather large species from the oolitic slate at Stonesfield. Dr. BUCKLAND kindly transmitted to me, some years ago, two drawings of one of these vertebræ (figs. 38 and 39). They are more slender in-proportion to their length than in the larger species from the Greensand. The articular cup and ball of the centrum have the same relative position and transverse breadth. The hypapophysis (*hy*, fig. 39) is further from the anterior border. The neural arch has the same superior breadth, compared with the centrum. There seems to have been rather more development of the process uniting the base of the anterior zygapophysis to the centrum.

The cervical vertebræ of the *Pterodactylus* (*Dimorphodon*) *macronyx*, from the lower lias of Dorsetshire, present the same type of structure.

EXPLANATION OF PLATE.

PLATE X.

All the figures are of the natural size, and with the exception of fig. 22 (*Aptenodytes patachonica*), belong to the Pterodactyle; and of these, all, save figs. 23, 38, and 39, are from the Upper Greensand, near Cambridge. The details are explained in the text.

Fig. 1. Front view
Fig. 2. Back view
Fig. 3. Under view } of a middle cervical vertebra, *Pterodactylus Sedgwickii.*
Fig. 4. Upper view
Fig. 5. Side view
Fig. 6. Vertical longitudinal section of a similar but more mutilated vertebra.
Fig. 7. Side view of a middle cervical vertebra, *Pterodactylus Fittoni.*
Fig. 8. Side view of probably fifth cervical vertebra, *Pterodactylus Fittoni.*
Fig. 9. Upper view } of a middle cervical vertebra, *Pterodactylus Fittoni.*
Fig. 10. Under view
Fig. 11. Front view of sixth cervical vertebra, *Pterodactylus Fittoni.*
Fig. 12. Front view of seventh cervical vertebra, *Pterodactylus Fittoni.*
Fig. 13. Front view } of last cervical or first dorsal vertebra, *Pterodactylus Sedgwickii.*
Fig. 14. Back view
Fig. 15. Side view } of a dorsal vertebra of a smaller Pterodactyle, with the transverse
Fig. 16. Front view } processes broken off.
Fig. 17. Side view of a more posterior dorsal vertebra.
Fig. 18. Under surface } of centrum of large posterior dorsal vertebra.
Fig. 19. Hinder surface
Fig. 20. Back view of a middle or posterior dorsal vertebra.
Fig. 21. Under surface of the dorsal vertebra, fig. 16.
Fig. 22. Front view of fourth dorsal vertebra, *Aptenodytes patachonica.*

Fig. 23. A rib of *Pterodactylus Bucklandi*, from Stonesfield Oölite.
Fig. 24. Front view of a lumbar vertebra, *Pterodactylus Sedgwickii.*
Fig. 25. Front view of first sacral vertebra, *Pterodactylus Sedgwickii.*
Fig. 26. Under view of first and second sacral vertebra, *Pterodactylus Sedgwickii.*
Fig. 27. Front and under view of centrums of first three sacral vertebræ of a smaller, probably younger, Pterodactyle.
Fig. 28. Front view } of atlas and axis vertebræ, *Pterodactylus Sedgwickii.*
Fig. 29. Side view }
Fig. 30. Vertical longitudinal section of atlas and axis, *Pterodactylus Sedgwickii.*
Fig. 31. Front view of axis vertebra, *Pterodactylus Sedgwickii.*
Fig. 32. Side view }
Fig. 33. Back view } of atlas and axis vertebræ, *Pterodactylus Fittoni.*
Fig. 34. Under view }
Fig. 35. Under view } of an anterior caudal vertebra, *Pterodactylus Sedgwickii.*
Fig. 36. Front view }
Fig. 37. Upper view } of a cervical vertebra, *Pterodactylus Bucklandi*, from Stonesfield
Fig. 38. Under view } Oolite.

IX. *On the Comparison of Hyperbolic Arcs. By* CHARLES W. MERRIFIELD. *Communicated by the Rev. Dr.* BOOTH, *F.R.S.*

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AN application of JACOBI's second theorem—the imaginary transformation—has led me to a formula which reduces the comparison of the arcs of hyperbolas to the same facility as that of elliptic arcs. The transformation is so easy and obvious, that I have had some hesitation in publishing it; but I observe that my result was not noticed by LEGENDRE, or by Professor MOSELEY, or in any more recent work which I have seen. Some of its applications, too, are worthy of remark.

I shall use the ordinary notations:—

$$\Delta(\theta,\varphi)=(1-\sin^2\theta\sin^2\varphi)^{\frac{1}{2}},\quad \mathrm{F}(\theta,\varphi)=\int\frac{d\varphi}{\Delta(\theta,\varphi)},\quad \mathrm{E}(\theta,\varphi)=\int\Delta(\theta,\varphi)d\varphi.$$

The functional equation

$$\mathrm{F}\varphi_1+\mathrm{F}\varphi_2-\mathrm{F}\varphi_3=0\quad\ldots\ldots\ldots\ldots\quad(\text{A.})$$

is satisfied, as is well known, by either of the three trigonometrical equations—

$$\cos\varphi_3=\cos\varphi_1\cos\varphi_2-\sin\varphi_1\sin\varphi_2\sqrt{(1-\sin^2\theta\,.\sin^2\varphi_3)}\quad\ldots\ldots\quad(1.)$$

$$\cos\varphi_2=\cos\varphi_1\cos\varphi_3+\sin\varphi_1\sin\varphi_3\sqrt{(1-\sin^2\theta\,.\sin^2\varphi_2)}\quad\ldots\ldots\quad(2.)$$

$$\cos\varphi_1=\cos\varphi_2\cos\varphi_3+\sin\varphi_2\sin\varphi_3\sqrt{(1-\sin^2\theta\,.\sin^2\varphi_1)}.\quad\ldots\ldots\quad(3.)$$

Dividing each of these by $\cos\varphi_1\,.\cos\varphi_2\,.\cos\varphi_3$, and transposing, they become

$$\sec\varphi_3=\sec\varphi_1\sec\varphi_2+\tan\varphi_1\tan\varphi_2\sqrt{(1+\cos^2\theta\,.\tan^2\varphi_3)}\quad\ldots\ldots\quad(4.)$$

$$\sec\varphi_2=\sec\varphi_1\sec\varphi_3-\tan\varphi_1\tan\varphi_3\sqrt{(1+\cos^2\theta\,.\tan^2\varphi_2)}\quad\ldots\ldots\quad(5.)$$

$$\sec\varphi_1=\sec\varphi_2\sec\varphi_3-\tan\varphi_2\tan\varphi_3\sqrt{(1+\cos^2\theta\,.\tan^2\varphi_1)}.\quad\ldots\ldots\quad(6.)$$

It will be noticed, that we might pass from one set to the other, directly, by substituting $\sec\varphi$ for $\cos\varphi$, $\sqrt{-}\tan\varphi$ for $\sin\varphi$, and $\cos\theta$ for $\sin\theta$. These substitutions constitute JACOBI's second theorem. They convert

$$\frac{d\varphi}{(1-\sin^2\theta\,.\sin^2\varphi)^{\frac{1}{2}}}\text{ into }\frac{\sqrt{-1}\,.\,d\varphi}{(1-\sin^2\theta\,.\sin^2\varphi)^{\frac{1}{2}}},$$

and

$$(1-\sin^2\theta\,.\sin^2\varphi)^{\frac{1}{2}}d\varphi\text{ into }\frac{\sqrt{-1}\,.\,d\varphi}{\cos^2\varphi}(1-\sin^2\theta\,.\sin^2\varphi)^{\frac{1}{2}}.$$

Now, calling $\int(1-\sin^2\theta\,.\sin^2\varphi)^{\frac{1}{2}}d\varphi$, $\mathrm{E}\varphi$, we know that

$$\mathrm{E}\varphi_1+\mathrm{E}\varphi_2-\mathrm{E}\varphi_3=\sin^2\theta\sin\varphi_1\,.\sin\varphi_2\,.\sin\varphi_3.\quad\ldots\ldots\ldots\quad(7.)$$

If, therefore, we make the above substitutions in this equation, and divide by $\sqrt{-1}$, we have, making $H\phi=\int\frac{d\phi}{\cos^2\phi}(1-\sin^2\theta.\sin^2\phi)^{\frac{1}{2}}$,

$$H\phi_1+H\phi_2-H\phi_3=-\cos^2\theta.\tan\phi_1.\tan\phi_2.\tan\phi_3. \quad . \quad . \quad . \quad . \quad . \quad (8.)$$

Moreover, since we also have

$$F\phi_1+F\phi_2-F\phi_3=0,$$

it is evident that these equations remain true, if we put for

$$E\phi,\ E\phi+k.F\phi,$$

or for

$$H\phi,\ H\phi+k.F\phi,$$

k being any constant whatever.

If we make $k=-\sin^2\theta$, $U\phi=H\phi-\sin^2\theta.F\phi$ represents the arc of a hyperbola.

In fact, if $\frac{x^2}{\sin^2\theta}-\frac{y^2}{\cos^2\theta}=1$ be the equation to a hyperbola, and we make the ordinate $y=\cos^2\theta.\tan\phi$, we have the abscissa $x=\frac{\sin\theta}{\cos\phi}\sqrt{(1-\sin^2\theta\sin^2\phi)}$. From these we may obtain by differentiation,

$$\int\sqrt{(dx^2+dy^2)}=\int\frac{\cos^2\theta}{\cos^2\phi}\frac{d\phi}{\sqrt{(1-\sin^2\theta.\sin^2\phi)}}$$

$$=\int\frac{1-\sin^2\theta\sin^2\phi}{\cos^2\phi\sqrt{(1-\sin^2\theta.\sin^2\phi)}}.d\phi-\int\frac{\sin^2\theta.d\phi}{\sqrt{(1-\sin^2\theta.\sin^2\phi)}},$$

or $U\phi=H\phi-\sin^2\theta.F\phi$.

If we make $\sin\tau=\sin\theta.\sin\phi$, τ is the angle which the normal of the hyperbola makes with the axis of x. If we change the variable from ϕ to τ, we have

$$U=\sin^2\theta.\cos^2\theta.\int\frac{d\tau}{(\sin^2\theta-\sin^2\tau)^{\frac{3}{2}}},$$

an equation which bears a remarkable analogy to the arc of the ellipse referred to its tangent,

$$E_1-E=\cos^2\theta.\int\frac{d\tau}{(1-\sin^2\theta.\sin^2\tau)^{\frac{3}{2}}}.$$

It may be worth while to remark, that θ, the angle of the modulus, represents, in the ellipse, the eccentricity, while in the hyperbola it represents the angle between the asymptote and the ordinate.

For the comparison of hyperbolic arcs, therefore, we have the equation

$$U\phi_1+U\phi_2-U\phi_3=-\cos^2\theta.\tan\phi_1.\tan\phi_2.\tan\phi_3, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.)$$

answering to the equation for elliptic arcs,

$$E\phi_1+E\phi_2-E\phi_3=\sin^2\theta.\sin\phi_1.\sin\phi_2.\sin\phi_3. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.)$$

Formula (8.) may be derived from the equations (4.), (5.), (6.) in exactly the same way that formula (7.) is derived from the equations (1.), (2.), (3.)*.

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* For the details, see LEGENDRE, 'Fonctions Elliptiques,' vol. i. p. 43, or MOSELEY "On Definite Integrals," Encyclopædia Metropolitana, 'Pure Mathematics,' vol. ii. p. 497.

In particular, if $\varphi_3=\frac{1}{2}\pi$, we have, for the complementary functions, $U\varphi_1+U\varphi_2=\infty$, as it ought to be, since the whole length of the curve is infinite.

For duplication, making $\varphi_1=\varphi_2=\omega$, we have

$$U\varphi_3-2U\omega=(\cos\theta.\tan\omega)^2\tan\varphi_3.$$

The same formula serves for bisection, if we obtain ω from φ_3 by the help of the elliptic equations.

Equation (8.) is easily verified at the extremes, making $\theta=0$,

$$H\varphi=\int\frac{d\varphi}{\cos^2\varphi}=\tan\varphi,$$

and we have the known theorem

$$\tan\alpha+\tan\beta-\tan(\alpha+\beta)=-\tan\alpha.\tan\beta.\tan(\alpha+\beta).$$

If we make

$$\theta=\frac{\pi}{2},\ \Delta\varphi=\cos\varphi,\ \text{and}\ H\varphi=\int\frac{d\varphi}{\cos\varphi},$$

whence

$$\int\frac{d\varphi_1}{\cos\varphi_1}+\int\frac{d\varphi_2}{\cos\varphi_2}-\int\frac{d\varphi_3}{\cos\varphi_3}=0,\ \ldots\ldots\ldots\ldots \quad (10.)$$

which is also a particular case of $F\varphi_1+F\varphi_2-F\varphi_3=0$, depending on the particular equation

$$\sec\varphi_3=\sec\varphi_1\sec\varphi_2+\tan\varphi_1\tan\varphi_2,\ \ldots\ldots\ldots \quad (11.)$$

which I should call the MERIDIONAL EQUATION, from its connexion with the common formula for meridional parts, and with certain curves on MERCATOR'S Chart, which I have discussed elsewhere.

I have taken the trouble of deducing (8.) from (4.), (5.), (6.) directly, but the process is so exactly parallel to Mr. MOSELEY'S work, at vol. ii. p. 497 of the work above cited, that it would be unnecessary to insert it here.

A simpler verification may be found as follows: differentiating with regard to φ the expression $\tan\varphi.\Delta\varphi$, we have

$$\frac{d}{d\varphi}(\tan\varphi.\Delta\varphi)=\frac{\Delta\varphi}{\cos^2\varphi}-\frac{\sin^2\theta\sin^2\varphi}{\Delta\varphi}=\frac{\Delta\varphi}{\cos^2\varphi}+\Delta\varphi-\frac{1}{\Delta\varphi},$$

whence, by integration (no constant needed, since each term vanishes with φ),

$$\tan\varphi.\Delta\varphi=H\varphi+E\varphi-F\varphi.\ \ldots\ldots\ldots\ldots \quad (12.)$$

If we now add the equations (7.) and (8.) and subtract the equation

$$F\varphi_1+F\varphi_2-F\varphi_3=0,$$

we should have, substituting (12.),

$$\tan\varphi_1.\Delta\varphi_1+\tan\varphi_2.\Delta\varphi_2-\tan\varphi_3.\Delta\varphi_3=\sin^2\theta.\sin\varphi_1.\sin\varphi_2.\sin\varphi_3-\cos^2\theta.\tan\varphi_1.\tan\varphi_2.\tan\varphi_3. \quad (13.)$$

This equation may be easily verified by using the values of $\Delta\varphi_1$, $\Delta\varphi_2$, and $\Delta\varphi_3$ obtained directly from the equations (1.), (2.), (3.), and clearing by means of the quadratic to which they all lead,

$$1+2\cos\varphi_1.\cos\varphi_2.\cos\varphi_3=\cos^2\varphi_1+\cos^2\varphi_2+\cos^2\varphi_3+\sin^2\theta.\sin^2\varphi_1.\sin^2\varphi_2.\sin^2\varphi_3\ \ .\ \ . \quad (14.)$$

Equation (8.) leads to a formula for the direct reduction of the logarithmic integral of the third kind, whose parameter is negative and greater than unity. It is the exact analogue of LEGENDRE's formula for the reduction of the same integral where the parameter is negative and less than unity, pp. 153, 154 of his third volume on Elliptic Functions. The reduction is of some importance, because on it depends the possibility of tabulating those functions, which would otherwise require a table of *treble* entry, too cumbrous to attempt.

Let ω_1 and ω_2 be two amplitudes, such that, for the common modulus θ, we have

$$\left.\begin{aligned} F\omega_1 &= F\phi + F\alpha \quad . \quad . \quad . \quad . \\ F\omega_2 &= F\phi + F\alpha \quad . \quad . \quad . \quad . \end{aligned}\right\} (a).$$

We must have simultaneously

$$\left.\begin{aligned} H\omega_1 + H\alpha - H\phi &= -\cos^2\theta \tan\alpha \tan\phi \tan\omega_1 \quad . \quad . \quad . \quad . \\ H\phi + H\alpha - H\omega_2 &= -\cos^2\theta \tan\alpha \tan\phi \tan\omega_2 \quad . \quad . \quad . \quad . \end{aligned}\right\} (b),$$

and also, putting for shortness $\delta\phi$ for $\surd(1+\cos^2\theta\tan^2\phi)$,

$$\left.\begin{aligned} \tan\omega_1 &= \frac{\tan\phi\cos\alpha\delta\alpha - \tan\alpha\cos\phi\delta\alpha}{1-\cos^2\theta\tan^2\alpha\tan^2\phi} \quad . \quad . \quad . \\ \tan\omega_2 &= \frac{\tan\phi\cos\alpha\delta\alpha + \tan\alpha\cos\phi\delta\alpha}{1+\cos^2\theta\tan^2\alpha\tan^2\phi} \quad . \quad . \quad . \end{aligned}\right\} (c).$$

Let us next consider the function

$$\Omega = V\omega_2 - V\omega_1 = \int\frac{d\omega_2}{\Delta\omega_2} H\omega_2 - \int\frac{d\omega_1}{\Delta\omega_1} H\omega_1.$$

If we regard α as constant, we obtain from equations (a.),

$$\frac{d\omega_2}{\Delta\omega_2} = \frac{d\phi}{\Delta\phi} = \frac{d\omega_1}{\Delta\omega_1},$$

whence

$$\Omega = \int\frac{d\phi}{\Delta\phi}(H\omega_2 - H\omega_1).$$

Now formulæ (b.) and (c.) give

$$H\omega_2 - H\omega_1 = 2H\alpha + \cos^2\theta\tan\alpha\tan\phi(\tan\omega_2 + \tan\omega_1),$$

and

$$\tan\omega_2 + \tan\omega_1 = \frac{2\tan\phi\cos\alpha\Delta\alpha}{1-\cos^2\theta \,.\, \tan^2\alpha \,.\, \tan^2\phi};$$

whence

$$\tfrac{1}{2}\Omega = \int\frac{d\phi}{\Delta\phi}\left\{H\alpha + \frac{\cos^2\theta\sin\alpha\tan^2\phi\delta\alpha}{1-\cos^2\theta\tan^2\alpha\tan^2\phi}\right\},$$

and, after a few reductions, we find

$$\tfrac{1}{2}\Omega = \left(H\alpha - \frac{\cos^2\theta\tan\alpha}{\Delta\alpha}\right)F\phi + \frac{\cos^2\theta\tan\alpha}{\Delta\alpha}\int\frac{1}{1-(1+\cos^2\theta\tan^2\alpha)\sin^2\phi}\cdot\frac{d\phi}{\Delta\phi};$$

or, transposing,

$$\int\frac{1}{1-(1+\cos^2\theta\tan^2\alpha)\sin^2\phi}\cdot\frac{d\phi}{\Delta\phi} - F\phi = \frac{\Delta\alpha}{\cos^2\theta\tan\alpha}\{\tfrac{1}{2}(V\omega_2 - V\omega_1) - H\alpha . F\phi\}.$$

No constant is needed, since $V\phi$ is an even function of ϕ.

One lesson we may learn from this process is, that the proper expression for the negative parameter greater than unity is $-(1+\cos^2\theta\tan^2\alpha)$. In geometrical researches this remark will probably lead to simplicity. LEGENDRE has deliberately avoided the discussion of this form of the parameter*. His reason was, that the complete integral presents itself in the form of $\infty-\infty$.

The tabulation of the function $V\omega$ would only require a table of double entry.

It may be as well to notice that the equations (*a*.), (*b*.), (*c*.) are solved by auxiliary arcs as follows:

Assume

$$\tan\eta_2=\tan\phi\Delta\alpha,\quad \tan\eta_1=\tan\alpha\Delta\phi,$$

then

$$\omega_1=\eta_2-\eta_1\quad,\quad \omega_2=\eta_2+\eta_1.$$

It is needless to remark that JACOBI'S transformation does not enable us to reduce the integral of the circular form. The difficulty which we here encounter, is exactly analogous to that which presents itself in the reduction of the cubic equation of ordinary algebra. In fact, if we were to apply JACOBI'S transformation to *one only* of α or ϕ, the auxiliary arcs just mentioned would give values of ω of the form $\eta\pm\eta'\sqrt{-1}$, and the difficulty would depend upon the interpretation of $F(\eta\pm\eta'\sqrt{-1})$.

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* See Fonctions Elliptiques, vol. i. p. 71. sect. 53.

X. *On the Thermo-dynamic Theory of Steam-engines with dry saturated Steam, and its application to practice. By* WILLIAM JOHN MACQUORN RANKINE, *C.E., LL.D., F.R.SS.L. & E., Pres. Inst. Eng. Scot., Regius Professor of Civil Engineering and Mechanics in the University and College of Glasgow.*

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Introduction.

IT was demonstrated independently from the laws of Thermo-dynamics, by Professor CLAUSIUS and the author of this paper, in 1849[*], that when steam or other saturated vapour in expanding performs work by driving a piston, and receives no heat from without during that expansion, a portion of it must be liquefied.

That theoretical conclusion has since been amply confirmed by experience in actual steam-engines; for it has been ascertained that the greater part of the liquid water which collects in unjacketed cylinders, and which was once supposed to be wholly carried over in the liquid state from the boiler (a phenomenon called "priming"), is produced by liquefaction of part of the steam during its expansion; and also that the principal effect of the "*jacket*," or annular casing enveloping the cylinder, filled with hot steam from the boiler, which was one of the inventions of WATT, is to prevent that liquefaction of the steam in the cylinder.

That liquefaction does not, when it first takes place, directly constitute a waste of heat or of energy; for it is accompanied by a corresponding performance of work. It does, however, afterwards by an indirect process, diminish the efficiency of the engine; for the water which becomes liquid in the cylinder, probably in the form of mist and spray, acts as a distributer of heat and equalizer of temperature, abstracting heat from the hot and dense steam during its admission into the cylinder, communicating that heat to the cool and rarefied steam which is on the point of being discharged, and thus lowering the initial pressure and increasing the final pressure of the steam; but lowering the initial pressure much more than the final pressure is increased. Accordingly, in all cases in which steam is expanded from a high down to a low pressure, it has in practice been found necessary to envelope the cylinder in a steam-jacket[†]. The liquefaction which would otherwise have taken place in the cylinder, takes place in the jacket instead, where the presence of the liquid water produces no bad effects, and that water is returned to the boiler.

In double-cylinder engines, where the expansion of the steam begins in a smaller cylinder and finishes in a larger, the usual practice is to have steam-jackets round both cylinders; but in a few examples in which the smaller cylinder alone is jacketed, the liquefaction is found to be almost wholly prevented, showing that the steam during its passage from the smaller to the larger cylinder receives sufficient heat, either directly from the small cylinder or indirectly by conduction from the smaller to the larger cylinder (which is in close contact with the small cylinder), to prevent any appreciable portion of it from condensing.

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[*] POGGENDORFF's 'Annalen,' 1850; Edinburgh Transactions, vol. xx.

[†] Unless the steam is superheated (Sept. 1859).

The only *exact* formulæ hitherto published for the work performed by the steam on the piston, viz. those contained in a paper on thermo-dynamics by the author of this paper, which was received by the Royal Society in 1853 and published in the Philosophical Transactions for 1854, and those contained in a paper by Professor CLAUSIUS, published in POGGENDORFF's 'Annalen' for 1856 (the same results having been independently arrived at in both papers), are adapted to cylinders *without steam-jackets.*

It is obviously desirable that exact formulæ should be deduced from the principles of thermo-dynamics, for the action of steam in jacketed cylinders also; and the present paper is intended to supply that want. In the first place, those fundamental equations of thermo-dynamics, to which it is afterwards necessary to refer, are briefly recapitulated: then the exact formulæ for the action of steam, and the corresponding expenditure of heat, in jacketed cylinders are deduced from them, and exemplified by numerical results:—then is explained a convenient approximation to the exact formulæ, founded on the facts, that within the limits usual in practice, the pressure of saturated steam varies nearly as the seventeenth power of the sixteenth root of its density; and that the expenditure of heat in a jacketed cylinder is nearly equal to fifteen and a half times the product of the initial pressure and volume of the steam:—and, lastly, are given examples of the application of the formulæ to the engines of three steam-vessels recently experimented on by the author, and a comparison of the results of the formulæ with those of experiment.

Summary of previously-known Principles and Formulæ.

The following is the GENERAL EQUATION OF THERMO-DYNAMICS which has been known since 1849–50[*]:—

$$H=\int t d\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

H denotes the *quantity of heat* which must be communicated to a mass of matter in order to make it undergo a given series of changes of volume and elastic pressure, expressed in *foot-pounds of energy*, of which 772 are equivalent to one degree of FAHRENHEIT in one pound of water, as proved by Mr. JOULE[†].

t is *absolute temperature*, reckoned from the absolute zero which corresponds to total privation of heat. It has from the first been conjectured that the scale of absolute temperature coincides with that of a perfect-gas thermometer, the absolute zero being the temperature corresponding to total absence of elastic pressure. On that supposition the specific heat of air under constant pressure was predicted in 1850 as being probably 0·24 of that of water, or thereabouts*; and M. REGNAULT, in 1853, ascertained it by experiment to be 0·238. The long series of experiments by Messrs. JOULE and THOMSON on the thermic effects of currents of elastic fluids†, have proved still more conclusively that if the scale of absolute temperature and that of the perfect-gas thermometer differ, it must be by quantities so small that they have not yet been measured.

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[*] Edinburgh Transactions, vol. xx.; and POGGENDORFF's 'Annalen,' 1850.

[†] Philosophical Transactions, 1850.

The expansion of a perfect gas from 32° FAHR. to 212° FAHR. being in the ratio 1:1·365, the absolute zero is

$$\frac{180^\circ}{0{\cdot}365}=493^\circ{\cdot}2 \text{ Fahr.}$$

below the temperature of melting ice; or

$$t \text{ in degrees of Fahr.}=461^\circ{\cdot}2+\mathrm{T},$$

T being the temperature on the ordinary FAHRENHEIT'S scale.

φ is a function which remains constant when the mass under consideration either performs work by expansion, or undergoes compression, without receiving or emitting heat. Its value is

$$k.\text{hyp. log}\, t+\int\frac{dp}{dt}\,dv; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

where k is the real specific heat of the substance, expressed in foot-pounds of energy per degree of temperature; p is the elastic pressure of the mass per unit of area when it occupies the volume v; the differentiation $\frac{dp}{dt}$ is performed on the supposition of v being constant, and the integration on the supposition of t being constant.

Another form of the function φ, which is convenient in certain calculations, is as follows:—

$$\varphi=\left(k+\frac{p_0v_0}{t_0}\right)\text{hyp. log}\, t-\int_0^p\frac{dv}{dt}\,dp, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

where $\frac{p_0v_0}{t_0}$ is the constant value of the ratio $\frac{pv}{t}$ for the substance under consideration, in the perfectly gaseous state‡.

The function φ is sometimes called the *Thermo-dynamic function*.

One of its properties is as follows:—that when the series of changes of pressure and volume to which the integration of equation 1 is applied constitute a *cycle*, so that the mass returns in the end to its primitive volume and pressure, then for a complete cycle

$$\int(t_1-t_2)d\varphi=\int(\varphi_1-\varphi_2)dt=\int(p_1-p_2)dv=\int(v_1-v_2)dp, \quad . \quad . \quad . \quad . \quad (4.)$$

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* Edinburgh Transactions, vol. xx.

† Philosophical Transactions, *passim*.

‡ $k+\frac{p_0v_0}{t_0}$ is the specific heat of the substance *under constant pressure*, in the perfectly gaseous state, expressed in units of energy.

in which t_1 and t_2 are the two values of t corresponding to one given value of ϕ, and the other similar symbols have analogous meanings.

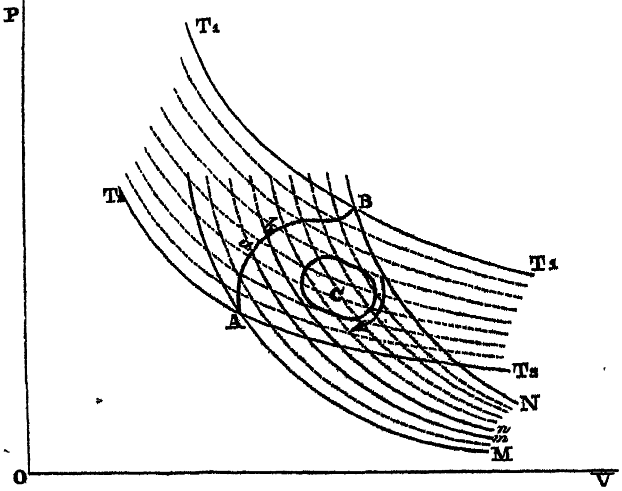
Equation 4, expressed in words, means—*The heat which disappears during a cycle of operations is equal to the work performed.* Another mode of expressing the principle of equation 4 is as follows:—

$$dt.d\phi=dp.dv. \qquad \text{(4 A.)}$$

The following is a summary of that graphic representation of the General Equation of Thermo-dynamics which was first demonstrated in the Philosophical Transactions for 1854.

In fig. 1, let OP, OV be rectangular axes of coordinates; and let ordinates measured parallel to OP and OV respectively represent pressures and volumes, so that areas represent quantities of energy. Let the coordinates of a line of any figure, such as AB, represent a series of changes of pressure and volume undergone by an elastic body. Let AM, BN be curves traversing A and B, of the class called *adiabatic* curves, or curves of no transmission; that is, curves whose coordinates show the law of variation of the pressure and volume of the body when it neither receives nor emits heat. Those curves are indefinitely extended both ways, and OV is their asymptote.

Fig. 1.



Then *the heat which the body receives during the change from* A *to* B, *is represented by the area contained between the line* AB *and the curves* AM *and* BN, *indefinitely extended in the direction* OV.

To show the connexion between this and the algebraical expression of the same law in equation 1, let ϕ be a function which is constant for a given adiabatic curve, and of such a nature that $dt.d\phi=dp.dv$. Divide the area MABN into an indefinite number of indefinitely-narrow bands by a series of adiabatic curves. Let *mabn* be one of these bands. Let the difference of the values of the function ϕ for the curves *am*, *bn*, be $d\phi$; let t be the absolute temperature corresponding to the element *ab* of the line AB; then—

$$\text{area } mabn=td\phi; \quad \text{area MABN}=\int td\phi.$$

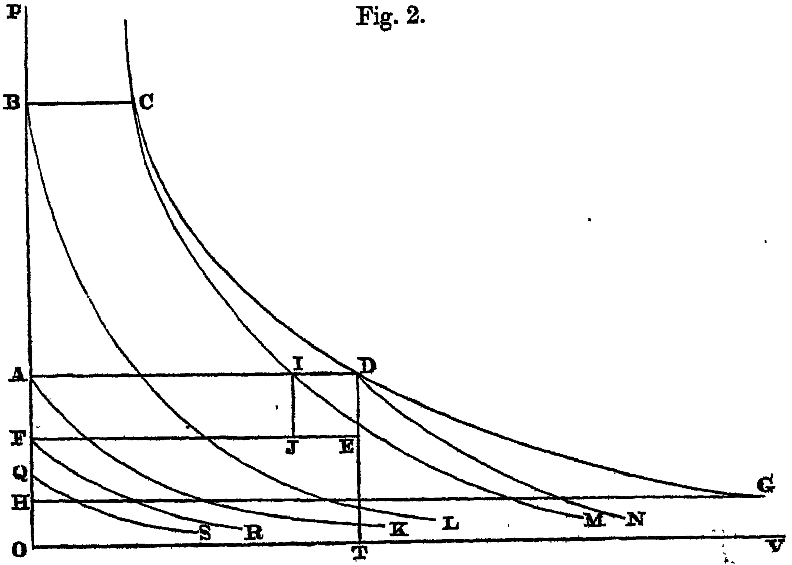
Let $T_1 T_1$, $T_2 T_2$, and the dotted lines between them, be *isothermal lines*, each of which by its coordinates represents the relation between the pressure and volume of the body for a particular uniform temperature. Then the scale of Absolute Temperatures is such, that a series of isothermal lines, corresponding to a series of equal divisions upon that scale, divides the band between any pair of adiabatic curves into equal areas.

A *cycle of changes* is represented by a closed figure, such as C; and the area of that figure represents the heat transformed into mechanical energy, or the mechanical energy transformed into heat, according as the cycle of changes takes place in the direction represented by the arrow, or in the contrary direction. That area is the quantity expressed by equation 4.

The following are the results of applying the general equation of thermo-dynamics to fluids which are in the act of changing from the liquid to the gaseous state, or nascent vapours.

In fig. 2, let the line $\overline{BC}$ parallel to OV represent the increase of volume which a

Fig. 2.



given fluid mass undergoes in changing from the liquid to the gaseous state under a pressure represented by the ordinate $\overline{OB}=p$. Through B and C draw a pair of indefinitely extended adiabatic curves, BL, CIM; then the area LBCM represents the *latent heat of evaporation* of the fluid mass under the pressure p.

To find the algebraical expression for that latent heat, it is to be considered, that in applying to this case the formula $H=td\phi$, t is the absolute temperature of the *boiling-point* corresponding to the pressure p, and is *constant*; so that

$$H=t(\phi_c-\phi_b);$$

ϕ_c and ϕ_b being the values of the thermo-dynamic functions for the curves CM and BL respectively. It is next to be considered, that because t is constant, the term of equation 2 which depends on t alone, is the same in ϕ_c and ϕ_b, so that it disappears from their difference, which consequently becomes simply

$$\varphi_c-\varphi_b=\int\frac{dp}{dt}\cdot dv;$$

and that because $\frac{dp}{dt}$ is constant during the evaporation, the value of the above integral is $v\cdot\frac{dp}{dt}$, where v denotes the *excess of the volume of the vapour above that of the liquid*, or *the increase of volume in the act of evaporation*; so that, finally, the latent heat of evaporation under the pressure p, in units of mechanical energy, is

$$H=vt\cdot\frac{dp}{dt}. \qquad (5.)$$

Now the relation between the pressure p and boiling-point t for water and various other fluids has been ascertained by the experiments of M. REGNAULT, and expressed in tables and formulæ; and for water, the relation between the boiling-point t and latent heat of evaporation H, has also been ascertained by M. REGNAULT'S experiments in ordinary units of heat, which by the aid of "JOULE'S equivalent" can be reduced to units of energy; consequently the INCREASE OF VOLUME OF ONE POUND OF WATER IN EVAPORATING is given by the equation

$$v=\frac{H}{t.\frac{dp}{dt}}. \qquad (6.)$$

The volume of a given weight of liquid water at all ordinary pressures is so small, as compared with that of the same weight of steam, that for practical purposes v may without material error be considered as representing the *volume of one pound of steam.*

The latent heat of evaporation of one pound of water at the boiling-point t in foot-pounds of energy, is given with sufficient accuracy by the formulæ

$$H=a-bt, \qquad (7.)$$

where $a=1109550$ foot-pounds,
and $b=540\cdot4$ foot-pounds per degree of FAHRENHEIT.

The pressure of saturated steam is given with great exactness, at all boiling-points at which it has yet been measured, by the formula

$$\text{com.}\log p=A-\frac{B}{t}-\frac{C}{t^2} \qquad (8.)$$

(first published in the Edinburgh Philosophical Journal, 1849). The values of the constants A, B, C, given as revised, in the Philosophical Magazine for December 1854, are as follows:—

For pounds on the square foot, $A=8\cdot2591$.
For pounds on the square inch, $A=6\cdot1007$.
For absolute temperatures in degrees of FAHRENHEIT,

$$\log B=3\cdot43642; \quad \log C=5\cdot59873.$$

In using equation 6, the unit of volume and the unit of pressure to be employed depend on each other. The following are examples:—

Unit of Pressure.	*Unit of Volume.*
Pound on the square foot.	Cubic foot.
Pound on the square inch.	Prism one foot long by one inch square, or $\frac{1}{144}$ cubic foot.

In either of the above cases, quantities of energy and of heat are expressed in foot-pounds.

The latent heat of so much steam as occupies a unit of volume more in the gaseous state than it did in the liquid state is obviously

$$t\frac{dp}{dt}=p\left\{\frac{B}{t}+\frac{2C}{t^2}\right\}\text{ hyp. log } 10 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.)$$

(hyp. log 10=2·3026 nearly).

In every case of the working of steam which occurs in practice, the volume of the liquid water is so small a fraction of the volume of the steam, that it may be neglected without sensible error. When this is done, the indicator-diagram of a steam-engine working perfectly, and without transmission of heat to or from the steam in the cylinder, may be represented in the following manner.

In fig. 2, let $\overline{OB}=p_1$ represent the "pressure of admission" at which the steam is admitted into the cylinder; t_1 the corresponding boiling-point:—

$\overline{BC}=v_1$ the volume of one pound of steam when admitted:—

$\overline{OA}=p_2$ the final pressure of the steam in the cylinder at the end of the expansion; t_2 the corresponding boiling-point:—

$\overline{OF}=p_3$ the "pressure of exhaustion" at which the steam is expelled from the cylinder; t_3 the corresponding boiling-point.

Let AID and FJE be parallel to OV and BC. Draw the adiabatic curves CIM, BL, FR. Then CI will be the *curve of expansion* of the steam, and $\overline{AI}=s_2$ will represent the volume occupied by one pound at the end of the expansion. The work of one pound of steam on the piston will be represented by the area FABCIJF, consisting of the parts

$$\text{ABCIA}=\int_{p_2}^{p_1} s\,dp\text{; and AIJF}=s_2(p_2-p_3). \quad . \quad . \quad . \quad . \quad . \quad . \quad (10.)$$

The symbol s is used to denote the volume occupied by one pound of the mixture of steam and liquid water which the cylinder contains at any given time during the expansion, s_2 being the final value of that volume.

Let CDG be a curve whose ordinates parallel to OV represent the volumes of one pound of dry saturated steam at the pressures represented by its ordinates parallel to OP. Then $\text{AD}=v_2$ is the volume which one pound of steam would occupy at the end of the expansion, but for the partial liquefaction; and the proportion of the steam liquefied is

$$\frac{\overline{\mathrm{ID}}}{\overline{\mathrm{AD}}}=\frac{v_2-s_2}{v_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11.)$$

The expenditure of heat per pound of steam (supposing the feed-water to be supplied to the boiler at the temperature of exhaustion, t_3) is represented by the indefinitely-prolonged area RFBCM, and consists of two parts,—

The sensible heat RFBL$=\mathrm{J}(t_1-t_3)$, where J is the specific heat of liquid-water, 772 lb. per degree of FAHRENHEIT, and the latent heat LBCM$=$H (see eq. 7.) (12.)

If the feed-water is supplied to the boiler, not at the temperature of exhaustion, t_3, but at some lower temperature, t_4, the latter temperature must be substituted for the former in the formulæ.

The application of these principles to steam-engines without jackets has been fully explained and exemplified in the paper already referred to in the Philosophical Transactions for 1854.

Calculations respecting such engines in ordinary cases are facilitated by approximate formulæ, founded on the fact that within the usual limits of pressure, viz. with $p_1=$ from one to twelve atmospheres, the coordinates of the curve CM are related by the equation

$$p \propto s^{-\frac{10}{9}} \text{ nearly*}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.)$$

The expenditure of heat can be roughly computed to within about $\frac{1}{30}$ of the truth, by the formula

$$v_1(13\tfrac{1}{3}p_1+4000)\text{*}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14.)$$

v_1 being in cubic feet and p_1 in pounds on the square foot*.

Theory of the Work, Heat, and Efficiency of Dry Saturated Steam.

In the following investigation it is assumed that the steam while expanding receives just enough of heat to prevent any part of it from condensing, without super-heating it. This assumption is founded on the fact, that dry steam is a bad conductor of heat as compared with liquid water, or with cloudy steam, and that after cloudy steam has received enough of heat to make it dry or nearly dry, it will probably receive little more.

The assumption is justified by the fact, that its results are confirmed by experiment.

The symbol v is used to denote the volume of one pound of steam in *cubic feet*, and the symbol p to denote pressure in *pounds on the square foot*; so that pressure in pounds on the square inch is denoted by $\frac{p}{144}$.

In fig. 2, as before, let CDG be the curve whose coordinates represent the volumes and pressures of dry saturated steam.

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* Manual of Applied Mechanics, Art. 656.

Let $\overline{\mathrm{OB}}=p_1$, and $\overline{\mathrm{BC}}=v_1$, represent the pressure and volume of admission, and t_1 the corresponding absolute temperature:—

Let $\overline{\mathrm{OA}}=p_2$, and $\overline{\mathrm{AD}}=v_2$, represent the pressure and volume at the end of the expansion, and t_2 the corresponding absolute temperature; then

$$\frac{v_2}{v_1}=r \text{ is the \textit{ratio of expansion}, and}$$

$$\frac{v_1}{v_2}=\frac{1}{r} \text{ the \textit{effective cut-off}.}$$

Let $\overline{\mathrm{AF}}=p_3$ be the pressure of exhaustion, and t_3 the corresponding absolute temperature;

Let t_4 be the absolute temperature of the feed-water; and

Let $\mathrm{OQ}=p_4$ be the corresponding pressure.

The work of one pound of steam is represented by the area of the diagram ABCDEFA, consisting of

$$\left.\begin{array}{l} \text{the area ABCDA}=\int_{p_2}^{p_1} v\,dp, \text{ and} \\ \text{the area ADEF } =v_2(p_2-p_3); \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15.)$$

while the expenditure of heat per pound of steam is represented by the area contained between the line QFABCD, and the two indefinitely extended adiabatic curves, QS, DN, and may be distinguished into the following parts:—

$$\left.\begin{array}{ll} \text{the sensible heat SQBL} & =\mathrm{J}(t_1-t_4) \\ \text{the latent heat of evaporation, LBCM}=\mathrm{H}_1; & \\ \text{the latent heat of expansion} & =\mathrm{MCDN}. \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (16.)$$

Thus it appears that the work of one pound of dry saturated steam exceeds that of one pound of steam which expands from the same initial pressure to the same final pressure without receiving heat, by an amount represented by the area JICDEJ, while the expenditure of heat is greater by the quantity represented by the area MCDN.

To find the area ABCDA, which represents part of the work, the value of v corresponding to any value of p is to be taken from equation 6, and that of H, the corresponding latent heat of evaporation, from equation 7, giving

$$v=\frac{a-bt}{t\frac{dp}{dt}},$$

which being multiplied by $\frac{dp}{dt}\,dt$, and integrated between t_1 and t_2, the initial and final temperatures of the expanding steam, we obtain for the area ABCDA,

$$\int_{p_2}^{p_1} v\,dp=\int_{t_2}^{t_1}\left(\frac{a}{t}-b\right)dt=a\,\text{hyp.}\log\frac{t_1}{t_2}-b(t_1-t_2); \quad . \quad . \quad . \quad . \quad . \quad . \quad (17.)$$

to which adding the rectangle ADEFA, the WORK OF ONE POUND OF STEAM is found to be

$$W=\int_{p_2}^{p_1} v\,dp+v_2(p_2-p_3)=a\,\text{hyp.}\log\frac{t_1}{t_2}-b(t_1-t_2)+v_2(p_2-p_3); \quad . \quad (18.)$$

in which

$a=1109550$ foot-pounds; $b=540\cdot4$ foot-pounds per degree of FAHRENHEIT.

The MEAN EFFECTIVE PRESSURE, or work per unit of volume traversed by the piston, is

$$w=W\div v_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18\text{A}.)$$

The heat expended per pound of steam, by a different mode of division from that given in the formulæ 16, is computed as follows:

Part of the sensible heat, SQAK$=J(t_2-t_4)$;

Latent heat of evaporation at the temperature t_2, KADN$=H_2=a-bt_2$;

Work performed between the temperatures t_1 and t_2, ABCDA$=\int_{p_2}^{p_1} v\,dp$ as in equation 17.

The addition of those quantities gives for the whole EXPENDITURE OF HEAT PER POUND OF STEAM in foot-pounds of energy,

$$\mathfrak{H}=J(t_2-t_4)+a-bt_2+\int_{p_2}^{p_1} v\,dp=J(t_2-t_4)+a\left(1+\text{hyp.}\log\frac{t_1}{t_2}\right)-bt_1 \quad . \quad . \quad . \quad (19.)$$

($J=772$ foot-pounds per degree of FAHRENHEIT).

The heat expended per unit of space traversed by the piston is equivalent to a pressure whose intensity is

$$h=\mathfrak{H}\div v_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19\text{A}.)$$

The EFFICIENCY of the steam is the ratio

$$E=W\div\mathfrak{H}=\frac{w}{h}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20.)$$

of the work performed by the steam on the piston to the heat expended on the steam; and that ratio having been determined, the available heat of a pound of fuel may be computed from the indicated work per pound of fuel, or *vice versâ*, by means of the equation,

$$\frac{\text{available heat}}{\text{indicated work}}=\frac{1}{E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21.)$$

In the practical use of equations 18, 18A, 19, 19A, 20, and 21, the usual data are,—the initial pressure p_1, the ratio of expansion r, the pressure of exhaustion p_3, and the temperature of the feed-water t_4. From p_1, by the aid of equations 6, 7, 8, 9, or of tables, are to be found t_1 and v_1. Then

$$rv_1=v_2;$$

and from v_2, by the aid of the same equations, or of tables, are to be found t_2 and p_2, and thus are completed the data for the use of equations 18 and 19.

Let $\overline{OH}=p_0$ represent the pressure, and $\overline{HG}=v_0$ the volume, of a pound of steam at some standard temperature, such as that of melting ice ($t_0 = 32° + 461°{\cdot}2 = 493°{\cdot}2$ FAHR.); and let

$$U = \int_{p_0}^{p} v\,dp = a \text{ hyp. log } \frac{t}{t_0} - b(t - t_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22.)$$

be the area contained between HG and another parallel ordinate of the curve CDG, corresponding to the absolute temperature t.

Then by the aid of tables of the function U, the equations 18 and 19 can be put into the following form:—

$$\left.\begin{aligned} W &= U_1 - U_2 + v_2(p_2 - p_3); \\ \mathfrak{H} &= U_1 - U_2 + J(t_2 - t_4) + a - bt_2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23.)$$

Tables of the values of p, v, and U, for every ninth degree of FAHRENHEIT's scale from 32° to 428° above the ordinary zero, have been calculated, and are now being printed*. As an example of the results contained in them, the following extract is given for every thirty-sixth degree from 104° to 392° FAHRENHEIT.

Extract from Table.

T.	t.	p.	v.	U.
104°	565°·2	152·6	312·8	112290
140	601·2	414·3	122·0	161340
176	637·2	987·6	53·92	206410
212	673·2	2116·4	26·36	247950
248	709·2	4152	14·00	286290
284	745·2	7563	7·973	321780
320	781·2	12940	4·816	354670
356	817·2	20990	3·057	385200
392	853·2	32520	2·025	413580

For the purpose of interpolating intermediate numbers in such tables, the *logarithms* of p and v are more convenient than those numbers themselves, as their successive differences are more nearly uniform.

Approximate Formulæ.

As the formulæ of the preceding section require in their use a considerable amount of calculation, and in some cases the solution of transcendental equations by trial and error (unless special tables are at hand), it is desirable to have, for the purpose of solving ordinary practical problems, approximate formulæ of a more simple kind. Those which will now be explained were arrived at by a process of trial, based upon a table of the results of the exact formulæ; and their agreement with the exact formulæ, and with experiment, has been tested for initial pressures ranging from 30 to 120 lbs. on the square inch, and for ratios of expansion ranging from *four* to *sixteen*.

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* In a work "On the Steam-Engine and other Prime Movers."

They may therefore be applied with confidence to engines working within those limits, and probably somewhat above them; but for pressures much exceeding 120 lbs. on the square inch, and ratios of expansion exceeding 16, it is advisable for the present to use the exact formulæ.

The foundation of the approximate formulæ for work, and for mean effective pressure, is the fact, that for pressures not exceeding 120 lbs. on the square inch, or 17280 lbs. on the square foot, the equation of the curve CDG is very nearly

$$p \propto v^{-\frac{17}{16}}. \qquad \text{(24.)}$$

This equation is very convenient in calculation, because the sixteenth root can be extracted, with great rapidity, to a degree of accuracy sufficient for practical purposes, by the aid of a table of squares alone; and by a little additional labour, without any tables whatsoever.

Let r, as before, be the ratio of expansion; then evidently,

Final pressure $p_2 = p_1 . r^{-\frac{17}{16}}$; . . . . (25.)

Gross work per lb. steam = area OBCDTO $\left.\begin{aligned} &= \mathrm{W} + p_3 v_2 = \int_{p_2}^{p_1} v\,dp + p_2 v_2, \\ &= p_1 v_1 (17 - 16 r^{-\frac{1}{16}}) = p_1 v_2 (17 r^{-1} - 16 r^{-\frac{17}{16}}) \end{aligned}\right\}$; . . . (26.)

Effective work per lb. steam = area FBCDEF $\left.\begin{aligned} &= \mathrm{W} = \int_{p_2}^{p_1} v\,dp + (p_2 - p_3) v_2 \\ &= v_2 \{ p_1 (17 r^{-1} - 16 r^{-\frac{17}{16}}) - p_3 \} \end{aligned}\right\}$; . . . (27.)

Mean gross pressure $= \frac{\mathrm{W}}{v_2} + p_3 = p_1 (17 r^{-1} - 16 r^{-\frac{17}{16}})$; . . . . (28.)

Mean effective pressure, or work per cubic foot, $w = \frac{\mathrm{W}}{v_2} = p_1 (17 r^{-1} - 16 r^{-\frac{17}{16}}) - p_3$. . (29.)

It is evident, that if the pressure of exhaustion p_3 be given, and any two out of the following three quantities,—the initial pressure p_1, the mean effective pressure w, the ratio of expansion r,—the fourth quantity can be calculated directly, if it is one or other of the pressures, p_1, w, or by approximation, if it is the ratio of expansion r.

The approximate formula for the expenditure of heat, in foot-pounds per pound of steam, which has been found by trial to agree very closely with the exact formula within the limits already specified, and when the feed-water is supplied at a temperature of from 100° to 120° FAHR., is as follows:

$$\mathfrak{H} = 15\tfrac{1}{2} . p_1 v_1 = \frac{15\frac{1}{2} p_1 v_2}{r}; \qquad \text{(30.)}$$

so that the *heat expended per cubic foot of space traversed by the piston*, or, the pressure per square foot of piston to which the expenditure of heat is equivalent, is

$$h = \frac{\mathfrak{H}}{v_2} = \frac{15\frac{1}{2} . p_1}{r}. \qquad \text{(31.)}$$

This gives for the *efficiency*,

$$\mathrm{E} = \frac{\mathrm{W}}{\mathfrak{H}} = \frac{w}{h} = \frac{17 - 16 r^{-\frac{1}{16}}}{15\frac{1}{2}} - \frac{r p_3}{15\frac{1}{2} p_1}; \qquad \text{(32.)}$$

by means of which, when the work of one pound of coal is known, its available heat can be computed, and *vice versâ*, as with the exact formula.

Tables of the ratios given by the equations from 25 to 29 for various ratios of expansion have been computed, and are in course of being printed. The following are examples of the results contained in them:—

Expansion r.	Effective cut-off $1 \div r$.	Mean gross pressure ÷ initial pressure, $(w+p_3) \div p_1$.
20	0·05	0·186
10	0·10	0·314
$6\frac{2}{3}$	0·15	0·417
5	0·20	0·505
4	0·25	0·582
$3\frac{1}{3}$	0·30	0·648
$2\frac{6}{7}$	0·35	0·707
$2\frac{1}{2}$	0·40	0·756
$2\frac{2}{9}$	0·45	0·800
2	0·50	0·840
$1\frac{9}{11}$	0·55	0·874
$1\frac{2}{3}$	0·60	0·900
$1\frac{7}{13}$	0·65	0·929
$1\frac{3}{7}$	0·70	0·945
$1\frac{1}{3}$	0·75	0·960
$1\frac{1}{4}$	0·80	0·976

Comparison of Theory with Experiment.

In comparing the results of formulæ for the expansive working of steam with those of the indicator-diagrams of engines, it is not to be expected that the indicated pressures corresponding to particular volumes during, or at the end of, the expansion, will closely agree with those given by calculation; because considerable deviations of the line marked on the diagram, alternately upwards and downwards, arise from the friction of the indicator, from elastic vibration of the indicator-spring, and from oscillations of the steam itself. In the course of a complete stroke, however, those deviations neutralize each other, so that the indicated *mean effective pressure*, if the theory is sound, ought to agree with that given by theory within the limit of errors of observation.

About half a pound on the square inch, or 72 lbs. on the square foot, may be considered as an ordinary limit of error in indicator-diagrams.

Two examples of the application of the exact formulæ, and four of the application of the approximate formulæ, to actual engines, are annexed; and the results of the formulæ are compared with those of experiment.

EXAMPLE I.—Paddle-steamer of 820 tons displacement, with a pair of double-cylinder engines of 744 indicated horse-power.

DATA:—

	Bottom of cylinders.	Top of cylinders.
	lb. per square inch.	
$p_1\div144$	33·7	34·3
$p_3\div144$	4·0	4·0
r	$4\frac{1}{6}$	$6\frac{1}{4}$

$T_4=t_4-461{\cdot}2$=about 104° FAHR.

RESULTS by exact formula:—

	Bottom of cylinders.	Top of cylinders.
$v_2=rv_1$	50·375	74·4
$p_2'\div144$	7·367	4·867
W	109552	117338
$\frac{W}{144v_2}=w\div144$	15·1	10·95

Mean . . . . 13·03

OBSERVED mean effective pressure, lb. on the inch . . . 13·10

Difference . . . $-$ 0·07

$\mathfrak{H}$	906989	925678
$\frac{\mathfrak{H}}{144v_2}=h\div144$	125	86·4

Mean . . . . 105·7

$E=\frac{w}{h}$	0·121	0·127

$$\text{Mean efficiency}=\frac{\text{mean } w}{\text{mean } h}=0{\cdot}123.$$

EXAMPLE I., calculated by approximate formula.

DATA:—

	lb. per inch.
Mean . . . $p_1\div144$	34
Mean . . . $p_3\div144$	4
Mean . . . $\frac{1}{r}$	0·2

RESULTS:—

	lb. per inch.
$w\div144$, Calculated	13·17
Observed	13·10
Difference	+ 0·07
$h\div144$	105·4
E	0·125

Example II.—Screw-steamer of about 700 tons displacement (?), with engine of 226 indicated horse-power.

Data:—

	Bottom of cylinders.	Top of cylinders.
	lb. on the square inch.	
$p_1 \div 144$	108½	104
$p_3 \div 144$	3·3	4·0
r	16	14

$T_4 = t_4 - 461{\cdot}2 = 122°$ Fahr. nearly.

Results by exact formula:—

	Bottom of cylinders.	Top of cylinders.
$v_2 = rv_1$	64·27	58·52
$p_2 \div 144$	5·6	6·3
W	191437	182108
$\dfrac{W}{144 v_2} = \dfrac{w}{144}$	20·7	21·6
Mean	21·15	

Observed mean effective pressure, lb. on the inch . . 21·0

Difference . . . + 0·15

$\mathfrak{H}$	975301	966524
$\dfrac{\mathfrak{H}}{144 v_2} = h \div 144$	105	115
Mean	110	
$E = w \div h$	0·196	0·188

Mean efficiency $= \dfrac{\text{mean } w}{\text{mean } h}$ . . . . . 0·192

Example II., calculated by approximate formula.

Data:—

	lb. per inch.
Mean . . . $p_1 \div 144$	106¼
Mean . . . $p_3 \div 144$	3·65
Mean . . . $\dfrac{1}{r}$	0·067

Results:—

	lb. per inch.
$w \div 144$, Calculated	21·05
Observed	21·00
Difference	+ 0·05
$h \div 144$	110
E	0·192

EXAMPLE III.—Paddle-steamer of 1100 tons displacement, with a pair of engines of 1176 indicated horse-power.

In this and the following example, some of the data were not obtained with sufficient precision to make it worth while to use the exact formulæ; and therefore the approximate formulæ alone are employed.

DATA:—

		lb. per inch.
Mean . . .	$p_1 \div 144$	38
Mean . . .	$p_3 \div 144$	$3\frac{1}{2}$
Mean . . .	$\frac{1}{r}$	0·15

RESULTS:—

		lb. per inch.
$w \div 144$,	Calculated	12·35
	Observed	12·2
	Difference	+ 0·15
$h \div 144$		88·3
E		0·14

EXAMPLE IV.—The same steamer as in Example III., with only one of her two boilers at work; indicated horse-power, 568.

DATA:—

		lb. per inch.
Mean . . .	$p_1 \div 144$	32
Mean . . .	$p_3 \div 144$	3
Mean . . .	$\frac{1}{r}$	0·1

RESULTS:—

		lb. per inch.
$w \div 144$,	Calculated	7·05
	Observed	7·0
	Difference	+ 0·05
$h \div 144$		49·6
E		0·142

XI. *On the Double Tangents of a Plane Curve.* *By* A. CAYLEY, *Esq.*, *F.R.S.*

Received March 17,—Read April 14, 1859.

IT was first shown by PLÜCKER on geometrical principles, that the number of the double tangents of a plane curve of the order m was $\tfrac{1}{2}m(m-2)(m^2-9)$: see the note, "Solution d'une question fondamentale concernant la théorie générale des Courbes," *Crelle*, t. xii. pp. 105–108 (1834), and the "Théorie der algebraischen Curven" (1839). The memoir by HESSE, "Ueber die Wendepuncte der Curven dritter Ordnung," *Crelle*, t. xxviii. pp. 97–107 (1844), contains the analytical solution of the allied easier problem of the determination of the points of inflexion of a plane curve. In the memoir, "Recherches sur l'élimination et sur la théorie des Courbes," *Crelle*, t. xxxiv. pp. 30–45 (1847), I showed how the problem of double tangents admitted of an analytical solution, viz. if $U=0$ is the equation of the curve, L, M, N the first derived functions of U, and

$$D=\alpha(M\partial_z-N\partial_y)+\beta(N\partial_x-L\partial_z)+\gamma(L\partial_y-M\partial_x)$$

(where α, β, γ are arbitrary), then the points of contact of the double tangents are given as the intersections of the curve $U=0$, with a curve the equation whereof is in the first instance obtained under the form $[Y]=0$; $[Y]$ being a given function of D^2U, D^3U, ..D^mU of the degree m^2-m-6 in respect of (α, β, γ), the degree

$$m^3-2m^2-10m+12$$

in respect of (x, y, z), and the degree m^2+m-12 in respect of the coefficients of U. It was necessary, in order that the points of intersection should be independent of the arbitrary quantities (α, β, γ) that we should have identically

$$[Y]=A.U+N.\Pi U,$$

N being of the degree m^2-m-6 in (α, β, γ), and consequently ΠU a function of (x, y, z) without (α, β, γ). Guided by HESSE's investigation for the points of inflexion, I asserted that it was probable that N was of the form $(\alpha x+\beta y+\gamma z)^{m^2-m-6}$; which being so, ΠU would be of the degree $(m-2)(m^2-9)$ in respect of (x, y, z), and the degree m^2+m-12 in respect of the coefficients, and I was thus led to the theorem, "On trouve les points de contact des tangentes doubles en combinant avec l'équation de la courbe une équation $\Pi U=0$, de l'ordre $(m-2)(m^2-9)$ par rapport aux variables et de l'ordre m^2+m-12 par rapport aux coefficients—c'est à dire, puisqu'il corresponde deux points de contact à une tangente double, le nombre de ces tangentes est égal à $\tfrac{1}{2}m(m-2)(m^2-9)$: théorème démontré indirectement par M. PLÜCKER."

HESSE, in the memoir "Ueber Curven dritter Ordnung, &c.," *Crelle*, t. xxxvi. pp. 143–176 (1848), showed how the components D^2U, D^3U, .. D^mU of $[Y]$ could each of them be expressed in a simplified form, and he thus effected the actual reduction of $[\Upsilon]$ to the form $\Lambda . U+(\alpha x+\beta y+\gamma z)^{4(m-3)}R$, where R still contained the arbitrary quantities (α, β, γ) in the degree $(m-2)(m-3)$. In particular for a quartic curve, the equation $R=0$ was shown to be

$$3Q_2Q_4-Q_3^2=0,$$

where the left-hand side is of the degree 2 in (α, β, γ) and the degree 16 in (x, y, z); and which should therefore by means of the equation $U=0$ be reducible so as to contain the factor $(\alpha x+\beta y+\gamma z)^2$.

JACOBI's paper, "Beweiss des Satzes, dass eine Curve n-ten Grades im allgemeinen $\frac{1}{2}n(n-2)(n^2-9)$ Doppeltangenten hat," *Crelle*, t. xl. pp. 237–260 (1850), did not, I think, materially advance the solution of the question. In a letter to JACOBI, dated the 30th December, 1849, published at the conclusion of the last-mentioned paper, HESSE gave the equation of the curve of the 14th order for the points of contact of the double tangents of a quartic, viz. in my notation,

$$(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\partial_x H, \partial_y H, \partial_z H)^2-H(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\partial_x, \partial_y, \partial_z)^2 H=0,$$

and the demonstration is given in HESSE's paper, "Ueber die ganzen homogenen Functionen von der dritten und vierten Ordnung zwischen drei Variabeln," *Crelle*, t. xli. pp. 285–292 (1851), and is reproduced in Mr. SALMON's Treatise on the Higher Plane Curves (1852). Two very interesting memoirs by HESSE and STEINER, *Crelle*, t. xlix. (1855), relate to the *geometrical* theory of the double tangents of a quartic, and it is not necessary to refer to them more particularly. It is to be observed that the curve which determines the points of contact of the double tangents is not absolutely determinate; for we may, it is clear, in the place of $\Pi U=0$, write $\Pi U+M . U=0$, where M is an arbitrary function of the proper degree: a very elegant transformation in the case of the quartic is given in HESSE's paper, "Transformation der Gleichung der Curven 14ten Grades, welche eine gegebene Curve 4ten Grades in den Berührungspuncten ihrer Doppeltangenten schneiden," *Crelle*, t. lii. pp. 97–103 (1856).

Mr. SALMON's work above referred to, contains the fundamental theorem of the *tangential* of a cubic, viz. a tangent to a cubic meets the cubic in a third point which lies on the second or line polar of the point of contact with respect to the Hessian. In my "Memoir on Curves of the Third Order*," I gave an identical equation relating to the tangential of a cubic, but which is not there exhibited in its proper form; this was afterwards effected by Mr. SALMON, in the paper "On Curves of the Third Order†." The equation, as given by Mr. SALMON, is in the notation of the present memoir,

$$-\mathfrak{H} . U+\tfrac{1}{3}\mathbb{D}\mathfrak{H} . DU-\tfrac{1}{3}DH . \mathbb{D}\Upsilon+H . \Upsilon=0,$$

an equation which in fact puts in evidence the last-mentioned theorem for the tangential of a cubic.

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* Philosophical Transactions, vol. cxlvii. (1857), pp. 415–446, art. No. 37.

† Ibid. vol. cxlviii. (1858), pp. 535–541.

The idea occurred to me of considering, in the case of the higher plane curves, the *tangentials* of a given point of the curve, viz. the points in which the tangent again meets the curve; for by expressing that two of these tangentials were coincident, we should have the condition that the given point is the point of contact of a double tangent. But I was not able to complete the solution.

Finally, Mr. SALMON discovered the equation of a curve of the order $m-2$, which by its intersections with the tangent at the given point determines the tangentials, and by expressing that the curve in question is touched by the tangent, he was led to a complete solution of the Double-tangent problem. Mr. SALMON'S result is given in the note, "On the Double Tangents to Plane Curves," in the Philosophical Magazine for October 1858. The discovery just referred to led me to the investigations of the present memoir, in which it will be seen that I obtain, for a curve of any order whatever, the identical equation corresponding to the before-mentioned equation obtained by Mr. SALMON in the case of a cubic; which identical equation puts in evidence the theorem as to the tangentials of the curve, and may thus be considered as containing in itself the solution of the Double-tangent problem: the identical equation is besides interesting for its own sake, as a part of the theory of ternary quantics.

1. Mr. SALMON'S solution of the problem of double tangents is based upon the following analytical determination of the tangentials of any point of the curve.

Let

$$\Upsilon=(*\between X, Y, Z)^n=0$$

be the equation of the given curve, (X, Y, Z) being current coordinates; and let (x, y, z) be the coordinates of a point on the curve, so that we have

$$U=(*\between x, y, z)^n=0,$$

a condition satisfied by the coordinates of the point in question.

Then the tangent

$$V=(X\partial_x+Y\partial_y+Z\partial_z)U=0$$

at the point (x, y, z), meets the curve besides in $(n-2)$ points, which are the tangentials of the given point (x, y, z), and which are determined as the intersections of the tangent $V=0$ with a certain curve,

$$\Omega=(\dagger\between x, y, z)^{n-2}=0.$$

2. To express the equation of this curve, let U_1, U_2, .. be the successive emanants of U, taken with the facients of emanation $(x_{\prime}, y_{\prime}, z_{\prime})$, viz.

$$U_1=\frac{1}{n}(x_{\prime}\partial_x+y_{\prime}\partial_y+z_{\prime}\partial_z)U,$$

$$U_2=\frac{1}{n(n-1)}(x_{\prime}\partial_x+y_{\prime}\partial_y+z_{\prime}\partial_z)^2U,$$

where it should be noticed that the numerical determination is such, that putting (x, y, z) for $(x_{\prime}, y_{\prime}, z_{\prime})$, then U_1, U_2, .. become respectively equal to U. Suppose also that $H, H_1, H_2, \ldots$ are the Hessians of $U, U_1, U_2, \ldots$, viz. H is the determinant formed with the second derived functions of U with respect to (x, y, z), H_1 the like determinant with the second derived functions of U_1 with respect to the same quantities (x, y, z); and so on. Moreover let $D^{n-2}H, =(X\partial_x+Y\partial_y+Z\partial_z)^{n-2}H$, denote the $(n-2)$thic emanant of H with respect to the current coordinates (X, Y, Z) as facients of emanation; and similarly let $D^{n-2}H_1$, $D^{n-2}H_2, \ldots$ denote the $(n-2)$thic emanants of $H_1, H_2, \ldots$ in respect to the same facients of emanation—it being understood that in all these functions, $(x_{,}, y_{,}, z_{,})$ are after the differentiations to be replaced by (x, y, z). It is to be observed that U_r is of the degree $(n-r)$ in (x, y, z), and consequently H_r of the degree $3(n-2-r)$; hence $D^{n-2}H_r$ is of the degree $3(n-2-r)-(n-2), =2(n-2)-3r$, which implies that $r \ngtr \frac{2}{3}(n-2)$, for otherwise $D^{n-2}H_r$ would be identically equal to zero. Upon replacing $(x_{,}, y_{,}, z_{,})$ by (x, y, z), $D^{n-2}H_r$ (r satisfying the above condition) becomes of the degree $2(n-2)$ in (x, y, z), and it is obviously of the degree 3 in the coefficients of U, and of the degree $(n-2)$ in the current coordinates (X, Y, Z).

3. This being premised, we have

$$\Omega=(\dagger)(X, Y, Z)^{n-2}$$

$$=D^{n-2}H-\frac{n-1}{1}D^{n-2}H_1+\&c.=0$$

for the equation of the curve of the order $(n-2)$, which by its intersection with the tangent gives the tangentials of the given point; the numerical coefficients are the binomial coefficients of the order $(n-1)$ taken with the signs $+$ and $-$ alternately, and the series is continued as long as the terms do not vanish, that is, if as before r denote the suffix of H, for so long as $r \ngtr \frac{2}{3}(n-2)$; but of course the value will not be altered by continuing the series to $r=n-1$. In particular, for the quartic we have

$$\Omega=D^2H-3D^2H_1,$$

for the quintic

$$\Omega=D^3H-4D^3H_1+6D^3H_2,$$

and so on. The function Ω, like the several component terms, is of course of the degree 3 in the coefficients of U, and of the degree $2(n-2)$ in (x, y, z).

4. It is to be remarked that the formula applies to a cubic; we have here simply $\Omega=DH$, which agrees with a result already mentioned. It may be noticed also that in the general case the formula gives at once the condition for the points of inflexion; in fact, if the point (x, y, z) be a point of inflexion, then one of the tangentials must coincide with this point, or the equation $\Omega=0$ will be satisfied by writing therein (x, y, z) for (X, Y, Z); but when this is done $D^{n-2}H$, $D^{n-2}H_1$ &c. reduce themselves (to numerical factors *près*) to H, and the equation becomes simply $H=0$, which is the well-known condition for the points of inflexion.

5. If two of the tangentials coincide, or what is the same thing, if the tangent $V=0$ touches the curve $\Omega=0$, then the point (x, y, z) will be the point of contact of a double tangent. The equation which expresses the condition in question, treating therein

(x, y, z) as current coordinates, is consequently that of a curve, intersecting the given curve (now represented by $\mathrm{U}=0$) in the points of contact of the double tangents. The process leads to a determinate form $\Pi\mathrm{U}=0$, of the curve in question, but of course any curve whatever, $\Pi\mathrm{U}+\mathrm{M}.\mathrm{U}=0$, will intersect the curve $\mathrm{U}=0$ in the points of contact of the double tangents.

6. I write for the moment

$$\begin{aligned}\Omega&=(\mathrm{A},\ldots\between \mathrm{X}, \mathrm{Y}, \mathrm{Z})^{n-2}=0,\\ \mathrm{V}&=\xi\mathrm{X}+\eta\mathrm{Y}+\zeta\mathrm{Z} \qquad =0,\end{aligned}$$

for the two equations; the coefficients $(\mathrm{A},\ldots)$, as already mentioned, are of the degree $2(n-2)$ in (x, y, z) and of the degree 3 in the coefficients of U; or as we may express it,

$$\mathrm{A},\ldots=(a,\ldots)^{1}(x, y, z)^{2(n-2)}.$$

In like manner ξ, η, ζ are of the degree $(n-1)$ in (x, y, z), and the degree 1 in the coefficients of U, or we may write

$$\xi, \eta, \zeta=(a,\ldots)^{1}(x, y, z)^{n-1}.$$

7. The equation which expresses that the line $\mathrm{V}=0$ touches the curve $\Omega=0$, is $F\Omega=0$, where the facients of the Reciprocant $F\Omega$ are the coefficients (ξ, η, ζ) of the linear function. This equation is of the form

$$(\mathrm{A},\ldots)^{2(n-3)}(\xi, \eta, \zeta)^{(n-2)(n-3)}=0\ ;$$

or attending to the forms of $(\mathrm{A},\ldots)$ and (ξ, η, ζ), it is of the form

$$(a,\ldots)^{6(n-3)+(n-2)(n-3)}(x, y, z)^{4(n-2)(n-3)+(n-1)(n-2)(n-3)}=0,$$

or what is the same thing, the form

$$(a,\ldots)^{(n+4)(n-3)}(x, y, z)^{(n-2)(n^2-9)}=0,$$

viz. the curve through the points of contact of the double tangents is a curve of the order $(n-2)(n^2-9)$, and its equation contains the coefficients of the equation $\mathrm{U}=0$ of the given curve in the degree $(n+4)(n-3)$. And since each double tangent corresponds to two points of contact, the number of double tangents is $\tfrac{1}{2}n(n-2)(n^2-9)$. This agrees with the before-mentioned results.

8. The whole problem is thus reduced to the demonstration of Mr. SALMON's expression for the curve $\Omega=0$. To fix the ideas, consider the case of a quartic curve $\Upsilon=(*\between \mathrm{X}, \mathrm{Y}, \mathrm{Z})^4=0$, and let the function $\mathrm{U}=(*\between x, y, z)^4$ (or as for shortness we may write it, $\mathrm{U}=(x, y, z)^4$) and certain of its emanants be represented as follows, viz.—

$$\begin{aligned}a&=(x, y, z)^4 &,\\ b&=(x, y, z)^3(\mathrm{X}, \mathrm{Y}, \mathrm{Z})&,\\ c&=(x, y, z)^2(\mathrm{X}, \mathrm{Y}, \mathrm{Z})^2&,\\ d&=(x, y, z)\,(\mathrm{X}, \mathrm{Y}, \mathrm{Z})^3&,\\ e&=\qquad\quad (\mathrm{X}, \mathrm{Y}, \mathrm{Z})^4&;\end{aligned}$$

$$\begin{aligned}
a' &= (x,y,z)^3 \quad . \quad . \quad (X',Y',Z'),\\
b' &= (x,y,z)^2(X,Y,Z)(X',Y',Z'),\\
c' &= (x,y,z)(X,Y,Z)^2(X',Y',Z'),\\
d' &= \quad . \quad . \quad (X,Y,Z)^3(X',Y',Z'),\\[1ex]
a'' &= (x,y,z)^2 \quad . \quad . \quad (X',Y',Z')^2,\\
b'' &= (x,y,z)(X,Y,Z)(X',Y',Z')^2,\\
c'' &= \quad . \quad . \quad (X,Y,Z)^2(X',Y',Z')^2,
\end{aligned}$$

where (X', Y', Z') are new arbitrary facients; but, as before, (X, Y, Z) are taken to be current coordinates, and (x, y, z) the coordinates of the given point on the curve:

$e=0$ is the equation of the curve;

$d=0$, the equation of the first or cubic polar of the point (x, y, z);

$b=0$, the equation of the last or line polar of the point (x, y, z), or what is the same thing (the point being on the curve), the tangent of the curve at this point;

$a=0$, the condition which expresses that the point is on the curve.

9. Imagine now an identical equation,

$$\text{I}a+\text{II}b+\text{III}d+\text{IV}e=0;$$

then, since $a=0$, we have

$$\text{II}b+\text{III}d+\text{IV}e=0;$$

and if in this equation we write $b=0$, $e=0$, it becomes $\text{III}d=0$, that is, the points of intersection of the curve $e=0$ and the tangent $b=0$ lie on one or other of the curves $d=0$, $\text{III}=0$. But the points in question do not lie on the curve $d=0$, consequently they lie on the curve $\text{III}=0$.

10. To explain the law of formation of the multipliers I, II, III, IV, I form the matrix

$$\begin{pmatrix} a, & b, & c, & d; & a', & b', & c' \\ b, & c, & d, & e; & b', & c', & d' \\ a', & b', & c', & d'; & a'', & b'', & c'' \end{pmatrix},$$

and then we have

$$\text{I}= \begin{vmatrix} d, & c, & b' \\ e, & d, & c' \\ d', & c', & b'' \end{vmatrix} + \begin{vmatrix} d, & b, & c' \\ e, & c, & d' \\ d', & b', & d'' \end{vmatrix},$$

$$\text{II}= \begin{vmatrix} d, & c, & a' \\ e, & d, & b' \\ d', & c', & a'' \end{vmatrix} - \begin{vmatrix} d, & b, & b' \\ e, & c, & c' \\ d', & b', & b'' \end{vmatrix} - \begin{vmatrix} d, & a, & c' \\ e, & b, & d' \\ d', & a', & c'' \end{vmatrix}$$

$$\text{III}= \begin{vmatrix} a, & b, & c' \\ b, & c, & d' \\ a', & b', & c'' \end{vmatrix} - \begin{vmatrix} a, & c, & b' \\ b, & d, & c' \\ a', & c', & b'' \end{vmatrix} - \begin{vmatrix} a, & d, & a' \\ b, & e, & b' \\ a', & d', & a'' \end{vmatrix}$$

$$\mathrm{IV}=\begin{vmatrix} a, & b, & b' \\ b, & c, & c' \\ a', & b', & b'' \end{vmatrix}+\begin{vmatrix} a, & c, & a' \\ b, & d, & b' \\ a', & c', & a'' \end{vmatrix}$$

values which, as I proceed to show, satisfy the identical equation

$$\mathrm{I}a+\mathrm{II}b+\mathrm{III}d+\mathrm{IV}e=0.$$

11. We have in fact

$$\begin{aligned}\mathrm{I}= {} & d\,(db''-c'^2+cc''-b'd') \\ & +e\,(b'c'-b''c+b'c'-bc'') \\ & +d'(cc'-db'+bd'-cc'),\end{aligned}$$

where the last line is

$$bd'^2-db'd';$$

$$\begin{aligned}\mathrm{II}= {} & d(b'c'-a''d+b'c'-b''c+a'd'-bc'') \\ & +e\,(ca''-a'c'+bb''-b'^2+ac''-a'c') \\ & +d'(a'd-b'c+b'c-bc'+bc'-ad'),\end{aligned}$$

where the last line is

$$da'd'-ad'^2;$$

$$\begin{aligned}\mathrm{III}= {} & a\,(-b'd'+cc''+c'^2-b''d+b'd'-ea'') \\ & +b\,(bc''-b'c'+b''c-b'c'+a''d-a'd') \\ & +a'(cc'-bd'+b'd-cc'+a'e-b'd),\end{aligned}$$

where the last line is

$$-ba'd'+ea'^2;$$

and

$$\begin{aligned}\mathrm{IV}= {} & a(b''c-b'c'+da''-b'c') \\ & +b\,(b'^2-bb''+a'c'-a''c) \\ & +a'(bc'-b'c+cb'-a'd),\end{aligned}$$

where the last line is

$$ba'c'-d'^2.$$

And these values may be expressed as follows:—

$$\begin{array}{lll}\mathrm{I}= & a(\;0 &) \\ & +b(\;d'^2 &) \qquad (12) \\ & +d(\;b''d+c''c-b'd'-c'c'-d'b') & \qquad (13) \\ & +e(-b''c-c''b+b'c'+c'b' &), \qquad (14) \\ \mathrm{II}= & a(-d'^2 &) \qquad (21) \\ & +b(0 &) \\ & +d(-a''d-b''c-c''b+a'd'+b'c'+c'b'+d'a') & \qquad (23) \\ & +e(\;a''c+b''b+c''a-a'c'-b'b'-c'a' &), \qquad (24)\end{array}$$

$$
\begin{array}{rll}
\mathrm{III}= & a(-b''d-c''d+b'd'+c'c'+d'b' \qquad\qquad) & (31) \\
 & +b(\ \ a''d+b''c+c''b-a'd'-b'c'-c'b'-d'a') & (32) \\
 & +d(\ \ 0 \qquad\qquad\qquad\qquad\qquad\qquad) & . \\
 & +e(-a''a+a'a' \qquad\qquad\qquad\qquad), & (34) \\
\mathrm{IV}= & a(\ \ b''c+c''b-b'c'-c'b' \qquad\qquad) & (41) \\
 & +b(-a''c-b''b-c''a+a'c'+b'b'+c'a' \quad) & (42) \\
 & +d(\ \ a''a-a'a' \qquad\qquad\qquad\qquad) & (43) \\
 & +e(\ \ 0 \qquad\qquad\qquad\qquad\qquad\qquad), & .
\end{array}
$$

which are of the form

$$
\begin{aligned}
\mathrm{I}&=a\ 0\ +b(12)+d(13)+e(14),\\
\mathrm{II}&=a(21)+b\ 0\ +d(23)+e(24),\\
\mathrm{III}&=a(31)+b(32)+d\ 0\ +e(34),\\
\mathrm{IV}&=a(41)+b(42)+d(43)+e\ 0\ ,
\end{aligned}
$$

where (12)=−(21) &c., and which therefore satisfy the equation

$$\mathrm{I}a+\mathrm{II}b+\mathrm{III}d+\mathrm{IV}e=0.$$

12. The equation of the curve which by its intersection with the tangent gives the tangentials, is

$$
\mathrm{III}=-\begin{vmatrix} a, & b, & c' \\ b, & c, & d' \\ a', & b', & c'' \end{vmatrix}-\begin{vmatrix} a, & c, & b' \\ b, & d, & c' \\ a', & c', & b'' \end{vmatrix}-\begin{vmatrix} a, & d, & a' \\ b, & e, & b' \\ a', & d', & a'' \end{vmatrix}=0,
$$

the degrees of which are

in the coefficients of U, 3,
in $(x\ , y\ , z\ )$ . . . 6,
in (X, Y, Z) . . . 4,
in (X′, Y′, Z′) . . . 2;

and it only remains to divest this equation of a factor which it contains,

$$
\begin{vmatrix} x\ , & y\ , & z \\ \mathrm{X}, & \mathrm{Y}, & \mathrm{Z} \\ \mathrm{X}', & \mathrm{Y}', & \mathrm{Z}' \end{vmatrix}^2,
$$

which being thrown out, the equation will be independent of (X′, Y′, Z′) and of the degrees

in the coefficients of U, 3,
in $(x\ , y\ , z\ )$ 4,
in (X, Y, Z) 2,

and will in fact be the before-mentioned equation $\Omega=\mathrm{D}^2\mathrm{H}-3\mathrm{D}^2\mathrm{H}_1=0$.

13. Write for shortness,

$$\begin{vmatrix} x\,, & y\,, & z \\ \mathrm{X}, & \mathrm{Y}, & \mathrm{Z} \\ \mathrm{X}', & \mathrm{Y}', & \mathrm{Z}' \end{vmatrix} = \Lambda,$$

it is to be shown that

$$\mathrm{III} = -\tfrac{1}{2}\Lambda^2(\mathrm{D}^2\mathrm{H} - 3\mathrm{D}^2\mathrm{H}_1).$$

14. To effect this I remark that we have identically

$$\begin{vmatrix} a, & b, & a' \\ b\,, & c\,, & b' \\ a', & b', & a'' \end{vmatrix} = \Lambda^2\mathrm{H};$$

and I proceed to operate upon this equation with $\mathrm{D} = \mathrm{X}\partial_x + \mathrm{Y}\partial_y + \mathrm{Z}\partial_z$.

I notice that

$$a, b, c, d, e; \quad a', b', c', d'; \quad a'', b'', c''$$

are in regard to (x, y, z) of the degrees

$$4, \quad 3, 2, 1, 0; \qquad 3, \quad 2, 1, 0; \qquad 2, \quad 1, 0;$$

or what is the same thing, since for the case in hand $n=4$, of the degrees

$$n, n-1, \ldots \;\; ; \;\; n-1, n-2, \ldots; \;\; n-2, n-3, .$$

and we have

$$\mathrm{D}a = nb, \; \mathrm{D}b = (n-1)c, \ldots \quad \mathrm{D}a' = (n-1)b', \; \mathrm{D}b' = (n-2)c', \ldots \quad \mathrm{D}a'' = (n-2)b'', \ldots$$

15. In the determinant

$$\begin{vmatrix} a, & b, & a' \\ b\,, & c\,, & b' \\ a', & b', & a'' \end{vmatrix}, = \Lambda^2\mathrm{H},$$

the degrees of the terms (other than each top term, the degree of which is higher by unity) in the several columns are $n-1$, $n-2$, $n-2$; if then we operate on the determinant with D, and as regards the top terms we write

$$\begin{aligned} \mathrm{D}a &= b + (n-1)b, \\ \mathrm{D}b &= c + (n-2)c, \\ \mathrm{D}a' &= b' + (n-2)b', \end{aligned}$$

we have in the first place a term

$$\begin{vmatrix} b\,, & c\,, & b' \\ b\,, & c\,, & b' \\ a', & b', & a'' \end{vmatrix}$$

which vanishes, and next the terms

$$(n-1)\begin{vmatrix} b\,, & b\,, & a' \\ c\,, & c\,, & b' \\ b', & b', & a'' \end{vmatrix} + (n-2)\begin{vmatrix} a, & c\,, & a' \\ b\,, & d, & b' \\ a', & c', & a'' \end{vmatrix} + (n-2)\begin{vmatrix} a, & b\,, & b' \\ b\,, & c\,, & c' \\ a', & b', & b'' \end{vmatrix}$$

the first of which vanishes. On the right-hand side $D\Lambda=0$ identically, and therefore $D.\Lambda^2H=\Lambda^2DH$, or we have

$$(n-2)\begin{vmatrix} a, & c, & a' \\ b, & d, & b' \\ a', & c', & a'' \end{vmatrix} +(n-2)\begin{vmatrix} a, & b, & b' \\ b, & c, & c' \\ a', & b', & b'' \end{vmatrix} =\Lambda^2DH.$$

16. I repeat the operation D: we have

$$\left.\begin{array}{l} (n-2)(n-1)\begin{vmatrix} b, & c, & a' \\ c, & d, & b' \\ b', & c', & a'' \end{vmatrix} +(n-2)(n-1)\begin{vmatrix} b, & b, & b' \\ c, & c, & c' \\ b', & b', & b'' \end{vmatrix} \\ +(n-2)(n-3)\begin{vmatrix} a, & d, & a \\ b, & c, & b \\ a', & d', & a'' \end{vmatrix} +(n-2)(n-2)\begin{vmatrix} a, & c, & b' \\ b, & d, & c' \\ a', & c', & b'' \end{vmatrix} \\ +(n-2)(n-2)\begin{vmatrix} a, & c, & b' \\ b, & d, & c' \\ a', & c', & b'' \end{vmatrix} +(n-2)(n-3)\begin{vmatrix} a, & b, & c' \\ b, & c, & d' \\ a', & b', & c'' \end{vmatrix} \end{array}\right\}=\Lambda^2D^2H\,;$$

or collecting the different terms,

$$(n-2)(n-3)\begin{vmatrix} a, & b, & c' \\ b, & c, & d' \\ a', & b', & c'' \end{vmatrix} +2(n-2)^2\begin{vmatrix} a, & c, & b' \\ b, & d, & c' \\ a', & c', & b'' \end{vmatrix} +(n-2)(n-3)\begin{vmatrix} a, & d, & a' \\ b, & e, & b' \\ a', & d', & a'' \end{vmatrix} +(n-2)(n-1)\begin{vmatrix} b, & c, & a' \\ c, & d, & b' \\ b', & c', & a'' \end{vmatrix} =\Lambda^2D^2H.$$

17. A little consideration will show that in this equation we may write $n-1$ for n, and H_1 for H. In fact, putting for a moment $\delta=x_{\prime}\partial_x+y_{\prime}\partial_y+z_{\prime}\partial_z$, we have corresponding to the equation

$$\begin{vmatrix} a, & b, & a' \\ b, & c, & b' \\ a', & b', & a'' \end{vmatrix}=\Lambda^2H,$$

this other equation,

$$\begin{vmatrix} \delta a, & \delta b, & \delta a' \\ \delta b, & \delta c, & \delta b' \\ \delta a', & \delta b', & \delta a'' \end{vmatrix}=\Lambda^2H_1,$$

where ultimately $(x_{\prime}, y_{\prime}, z_{\prime})$ are to be replaced by (x, y, z). We may operate upon this equation with D, D^2, ... as before, the only difference being that in the first instance δa, δb, &c. are as regards (x, y, z) of degrees lower by unity than a, b, &c., that is $n-1$ must be substituted throughout in the place of n; and when at the end of the process $(x_{\prime}, y_{\prime}, z_{\prime})$ are replaced by (x, y, z), then δa, δb, &c. become equal to a, b, &c., from which the truth of the asserted proposition is manifest.

18. Hence writing $n=4$, we have

$$2\begin{vmatrix} a, & b, & c' \\ b, & c, & d' \\ a', & b', & c'' \end{vmatrix} +8\begin{vmatrix} a, & d, & a' \\ b, & e, & b' \\ a', & d', & a'' \end{vmatrix} +2\begin{vmatrix} a, & d, & a' \\ b, & e, & b' \\ a', & d', & a'' \end{vmatrix} +6\begin{vmatrix} b, & c, & a' \\ c, & d, & b' \\ b', & c', & a'' \end{vmatrix} = \Lambda^2 D^2 H,$$

$$2\begin{vmatrix} a, & d, & a' \\ b, & e, & b' \\ a, & d', & a'' \end{vmatrix} +2\begin{vmatrix} b, & c, & a' \\ c, & d, & b' \\ b', & c', & a'' \end{vmatrix} = \Lambda^2 D^2 H_1;$$

and hence

$$2\left\{\begin{vmatrix} a, & b, & c' \\ b, & c, & d' \\ a', & b', & c'' \end{vmatrix} + \begin{vmatrix} a, & c, & b' \\ b, & d, & c' \\ a', & c', & b'' \end{vmatrix} + \begin{vmatrix} a, & d, & a' \\ b, & e, & b' \\ a', & d', & a'' \end{vmatrix}\right\} = \Lambda^2(D^2H-3D^2H_1),$$

which is the required equation,

$$\text{III}=-\tfrac{1}{2}\Lambda^2(D^2H-3D^2H_1).$$

19. It is to be added, that the equation for $\Lambda^2 DH$ gives $\text{IV}=\frac{1}{2}\Lambda^2 DH$; the values of II and I are at once obtained from those of III and IV by interchanging (x, y, z) and (X, Y, Z). Hence if we represent by $\mathfrak{H}$, $\mathfrak{D}$, &c. the values which H, D, &c. assume by this interchange, we may write

$$\begin{aligned} \text{I} &= -\tfrac{1}{2}\Lambda^2\mathfrak{D}\mathfrak{H}, \\ \text{II} &= +\tfrac{1}{2}\Lambda^2(\mathfrak{D}^2\mathfrak{H}-3\mathfrak{D}^2\mathfrak{H}_1), \\ \text{III} &= -\tfrac{1}{2}\Lambda^2(D^2H-3D^2H_1), \\ \text{IV} &= +\tfrac{1}{2}\Lambda^2 DH; \end{aligned}$$

and the identical equation,

$$\text{I}a+\text{II}b+\text{III}d+\text{IV}e=0,$$

gives therefore

$$-\mathfrak{D}\mathfrak{H}\,.\,U+\tfrac{1}{4}(\mathfrak{D}^2\mathfrak{H}-3\mathfrak{D}^2\mathfrak{H}_1)DU-\tfrac{1}{4}(D^2H-3D^2H_1)\mathfrak{D}\Upsilon+DH\,.\,\Upsilon=0,$$

which is of itself sufficient to put in evidence the property that the curve $D^2H-3D^2H_1=0$ gives by its intersections with the tangent $DU=0$, the tangentials of the point (x, y, z). The last-mentioned equation is the equation for a quartic corresponding to Mr. SALMON'S equation

$$-\mathfrak{H}\,.\,U+\tfrac{1}{3}\mathfrak{D}\mathfrak{H}\,.\,DU-\tfrac{1}{3}DH\,.\,\mathfrak{D}\Upsilon+H\,.\,\Upsilon=0$$

for the cubic $U=0$.

20. It is worth while to give the investigation of the equation for the cubic; the matrix is

$$\begin{pmatrix} a, & b, & c; & a', & b' \\ b, & c, & d; & b', & c' \\ a', & b', & c'; & a'', & b'' \end{pmatrix}$$

and the identical equation is

$$\mathrm{I}a+\mathrm{II}b+\mathrm{III}c+\mathrm{IV}d=0,$$

where

$$\mathrm{I}=\begin{vmatrix} c, & b, & b' \\ d, & c, & c' \\ c', & b', & b'' \end{vmatrix},$$

$$\mathrm{II}=-\begin{vmatrix} c, & b, & a' \\ d, & c, & b' \\ c', & b', & a'' \end{vmatrix}-\begin{vmatrix} c, & a, & b' \\ d, & b, & c' \\ c', & a', & b'' \end{vmatrix},$$

$$\mathrm{III}=-\begin{vmatrix} a, & b, & b' \\ b, & c, & c' \\ a', & b', & b'' \end{vmatrix}-\begin{vmatrix} a, & c, & a' \\ b, & d, & b' \\ a', & c', & a'' \end{vmatrix},$$

$$\mathrm{IV}=\begin{vmatrix} a, & b, & a' \\ b, & c, & b' \\ a', & b', & a'' \end{vmatrix}.$$

or, as we may express them,

$$\begin{array}{rll}
\mathrm{I}= & a(\ 0\ ) & . \\
& +b(\ c'^2\ ) & (12) \\
& +c(b''c-b'c'-c'b') & (13) \\
& +d(-b''b+b'b'\ ) & (14) \\
\mathrm{II}= & a(\ -c'^2\ ) & (21) \\
& +b(\ 0\ ) & . \\
& +c(-a''c-b''b+a'c'+b'b'+c'a') & (23) \\
& +d(\ a''b+b''a-a'b'-b'a'\ ) & (24) \\
\mathrm{III}= & a(-b''c+b'c'+c'b'\ ) & (31) \\
& +b(\ a''c+b''b-a'c'-b'b'-c'a') & (32) \\
& +c(\ 0\ ) & . \\
& +d(-aa''\ \ +a'^2\ ) & (34) \\
\mathrm{IV}= & a(b''b\ \ -b'b'\ ) & (41) \\
& +b(-a''b-b''a+a'b'+b'a'\ ) & (42) \\
& +c(\ a''a\ \ -a'a'\ ) & (43) \\
& +d(\ 0\ ) & .
\end{array}$$

which verify the identical equation. We have $\mathrm{III}=-\Lambda^2\mathrm{DH}$, $\mathrm{IV}=\Lambda^2\mathrm{H}$, and thence $\mathrm{II}=+\Lambda^2\mathfrak{D}\mathfrak{H}$, $\mathrm{I}=-\Lambda^2\mathfrak{H}$, and thence the equation in question,

$$-\mathfrak{H}.\mathrm{U}+\tfrac{1}{3}\mathfrak{D}\mathfrak{H}.\mathrm{DU}-\tfrac{1}{3}\mathrm{DH}.\mathfrak{D}\Upsilon+\mathrm{H}.\Upsilon=0.$$

21. One other example will be sufficient to render manifest the law of the formation of the multipliers I, II, III, IV.

In the case of a sextic curve we have the matrix

$$\left(\begin{array}{cccccc|ccccc} a, & b, & c, & d, & e, & f; & a', & b', & c', & d', & e' \\ b, & c, & d, & e, & f, & g; & b', & c', & d', & e', & f' \\ a', & b', & c', & d', & e', & f'; & a'', & b'', & c'', & d'', & e'' \end{array}\right),$$

the identical equation is

$$\mathrm{I}a+\mathrm{II}b+\mathrm{III}f+\mathrm{IV}g=0;$$

and the expressions for the multipliers I, II, III, IV are—

$$\mathrm{I}=\begin{vmatrix} f, & e, & b' \\ g, & f, & c' \\ f', & e', & b'' \end{vmatrix}+\begin{vmatrix} f, & d, & c' \\ g, & e, & d' \\ f', & d', & c'' \end{vmatrix}+\begin{vmatrix} f, & c, & d \\ g, & d, & e \\ f', & c', & d'' \end{vmatrix}+\begin{vmatrix} f, & b, & e' \\ g, & c, & f' \\ f', & b', & e'' \end{vmatrix},$$

$$\mathrm{II}=-\begin{vmatrix} f, & e, & a' \\ g, & f, & b' \\ f', & e', & a'' \end{vmatrix}-\begin{vmatrix} f, & d, & b' \\ g, & e, & c' \\ f', & d', & b'' \end{vmatrix}-\begin{vmatrix} f, & c, & c' \\ g, & d, & d' \\ f', & c', & d'' \end{vmatrix}-\begin{vmatrix} f, & b, & d' \\ g, & c, & e' \\ f', & b', & d'' \end{vmatrix}-\begin{vmatrix} f, & a, & e' \\ g, & b, & f' \\ f', & a', & e'' \end{vmatrix},$$

$$\mathrm{III}=-\begin{vmatrix} a, & b, & e' \\ b, & c, & f' \\ a', & b', & e'' \end{vmatrix}-\begin{vmatrix} a, & c, & d' \\ b, & d, & e' \\ a', & c', & e'' \end{vmatrix}-\begin{vmatrix} a, & d, & c' \\ b, & e, & d' \\ a', & d', & e'' \end{vmatrix}-\begin{vmatrix} a, & e', & b' \\ b, & f', & c' \\ a', & e'', & b'' \end{vmatrix}-\begin{vmatrix} a, & f, & a' \\ b, & g, & b' \\ a', & f', & a'' \end{vmatrix},$$

$$\mathrm{IV}=\begin{vmatrix} a, & b, & d' \\ b, & c, & e' \\ a', & b', & d'' \end{vmatrix}+\begin{vmatrix} a, & c, & c' \\ b, & d, & d' \\ a', & d', & c'' \end{vmatrix}-\begin{vmatrix} a, & d, & b' \\ b, & e, & c' \\ a', & d', & b'' \end{vmatrix}-\begin{vmatrix} a, & e, & a' \\ b, & f, & b' \\ a', & e', & a'' \end{vmatrix}.$$

22. We have in fact

$$\begin{array}{rlll}
\mathrm{I}= & a\ 0 & & . \\
 & +b(-ge'' & +f'f' \quad) & (12) \\
 & +f(\ b''f+c''e+d''d+e''c & -b'f'-c'e'-d'd'-e'c'-f'b' \quad) & (13) \\
 & +g(-b''e-c''d-d''c & +b'e'+c'd'+d'c'+e'b' \quad) & (14) \\
\mathrm{II}= & a(\ e''g & -f'f' \quad), & (21) \\
 & +b\ 0 & & . \\
 & +f(-a''f-b''e-c''d-d''e-e''b+a'f'+b'e'+c'd'+d'c'+e'b'+f'a') & & (23) \\
 & +g(\ a''e+b''d+c''c+d''b & -a'e'-b'd'-c'c'-d'b'-e'a' \quad) & (24) \\
\mathrm{III}= & a(-b''f-c''e-d''d-e''c & +b'f'+c'e'+d'd'+e'c'+f'b' \quad) & (31) \\
 & +b(\ a''f+b''e+c''d+d''c+e''b-a'f'-b'e'-c'd'-d'c'-e'b'-f'a') & & (32) \\
 & +f\ 0 & & . \\
 & +g(-a''a & +a'^2 \quad), & (34)
\end{array}$$

$$\begin{array}{llllr}
\mathrm{IV}=a(& b''e+c''d+d''c & -b'e'-c'd'-d'c'-e'b' &) & (41)\\
\phantom{\mathrm{IV}=}+b(& -a''e-b''d-c''c-d''b & +a'e'+b'd'+c'c'+d'b'+e'a' &) & (42)\\
\phantom{\mathrm{IV}=}+f(& a''a & -a'^2 &) & (43)\\
\phantom{\mathrm{IV}=}+g & 0 & & , & .
\end{array}$$

which are of the form

$$\begin{aligned}
\mathrm{I}&=a\ 0\ +b(12)+f(13)+g(14),\\
\mathrm{II}&=a(21)+b\ 0\ +f(23)+g(24),\\
\mathrm{III}&=a(31)+b(32)+f\ 0\ +g(34),\\
\mathrm{IV}&=a(41)+b(42)+f(43)+g\ 0,
\end{aligned}$$

where $(12)=-(21)$ &c., and the equation

$$\mathrm{I}a+\mathrm{II}b+\mathrm{III}f+\mathrm{IV}g=0$$

is consequently satisfied.

23. The expression

$$\mathrm{III}=-\left|\begin{array}{ccc} a, & b, & e' \\ b, & c, & f' \\ a', & b', & e'' \end{array}\right|-\left|\begin{array}{ccc} a, & c, & d' \\ b, & d, & e' \\ a', & c', & d'' \end{array}\right|-\left|\begin{array}{ccc} a, & d, & c' \\ b, & e, & d' \\ a', & d', & c'' \end{array}\right|-\left|\begin{array}{ccc} a, & f, & a' \\ b, & g, & b' \\ a', & f', & a'' \end{array}\right|$$

leads to

$$\mathrm{III}=-\Lambda^2(\mathrm{D}^4\mathrm{H}-5\mathrm{D}^4\mathrm{H}_1+10\mathrm{D}^4\mathrm{H}_2),$$

and consequently the equation of the curve which by its intersections with the tangent determines the tangentials of a point of a sextic, is

$$\mathrm{D}^4\mathrm{H}-5\mathrm{D}^4\mathrm{H}_1+10\mathrm{D}^4\mathrm{H}_2=0.$$

24. In the general case of a curve of the order n the matrix is

$$\begin{array}{c}
\left(\begin{array}{ccccc} a_0, & a_1, & a_2 & \ldots & a_{n-1}; \qquad a'_0, \ a'_1 \ \ldots \ a'_{n-2} \end{array}\right)\\
\left|\begin{array}{ccccc} a_1, & a_2, & a_3 & \ldots & a_n \ ; \qquad a'_1, \ a'_2 \ \ldots \ a'_{n-1} \\ a'_0, & a'_1, & a'_2 & \ldots & a'_{n-1}; \qquad a''_0, \ a''_1 \ \ldots \ a''_{n-2} \end{array}\right|
\end{array}$$

where, in analogy with what precedes,

$$\begin{aligned}
a_0\ &=(\mathrm{X},\ \mathrm{Y},\ \mathrm{Z})^n\qquad\qquad ,\\
a_1\ &=(\mathrm{X},\ \mathrm{Y},\ \mathrm{Z})^{n-1}(x,\ y,\ z),\\
&\ \ \vdots\\
a_{n-1}&=(\mathrm{X},\ \mathrm{Y},\ \mathrm{Z})\ \ (x,\ y,\ z)^{n-1},\\
a_n\ &=\qquad\qquad\quad (x,\ y,\ z)^n\ ,
\end{aligned}$$

and similarly for the accented letters, so that

$a_0\ =0$ is the equation of the curve;

$a_1\ =0$ is the equation of the first or $(n-1)$thic polar;

$a_{n-1}=0$ is the equation of the last or line-polar, or what is the same thing, since (x, y, z) is a point on the curve, the tangent at this point;

$a_n\ =0$, the condition which expresses that (x, y, z) is a point of the curve;

and we have to form the identical equation

$$\mathrm{I}a_0+\mathrm{II}a_1+\mathrm{III}a_{n-1}+\mathrm{IV}a_n=0.$$

25. If, for shortness, the columns of the last-mentioned matrix are represented by

$$1,\ 2,\ 3\ \ldots\ n,\ (1),\ (2)\,..\,(n-1),$$

and the determinants formed with these columns respectively by a corresponding notation $\{1, 2, (1)\}$, $\{1, 2, (2)\}$, &c., then the expressions for the multipliers I, II, III, IV are as follows, viz.

$$\begin{aligned}
\mathrm{I}&=\ \ \{n, n-1, (2)\}+\{n, n-2, (3)\}\ldots+\{n, 2, (n-1)\}\\
\mathrm{II}&=-\{n, n-1, (1)\}-\{n, n-2, (2)\}\ldots-\{n, 2, (n-2)\}-\{n, 1, (n-1)\},\\
\mathrm{III}&=-\{1, 2, (n-1)\}-\{1, 3, (n-2)\}\ldots-\{1, n-1, (2)\}-\{1, n,\ \ (1)\ \},\\
\mathrm{IV}&=\ \ \{1, 2, (n-2)\}+\{1, 3, (n-1)\}\ldots+\{1, n-1, (1)\};
\end{aligned}$$

the truth of the identical equation being shown, as in the foregoing special cases, by the transformation of the multipliers into the form

$$\begin{aligned}
\mathrm{I}&=a_0\ \ 0\ +a_1(12)+a_{n-1}(13)+a_n(14),\\
\mathrm{II}&=a_0(21)+a_1\ \ 0\ +a_{n-1}(23)+a_n(24),\\
\mathrm{III}&=a_0(31)+a_1(32)+a_{n-1}\ \ 0\ +a_n(34),\\
\mathrm{IV}&=a_0(41)+a_1(42)+a_{n-1}(43)+a_n\ \ 0\ ,
\end{aligned}$$

where $(12)=-(21)$ &c.: the required expressions may be written down without difficulty.

26. Proceeding then to reduce the equation

$$\mathrm{III}=-\{1,2,(n-1)\}-\{1,3,(n-2)\}\ldots-\{1,n-1,(2)\}-\{1,n,(1)\},$$

we have the equation

$$\{1,2,(1)\}=\Lambda^2\mathrm{H},$$

which is to be successively operated on with D. The degrees (less unity) of the columns

$$1,\quad 2,\,..\ n-1, n,\ \ (1),\quad (2),\,..\,(n-1),$$

are

$$n-1, n-2,\,..\,1,\quad 0, n-2,\ n-3,\,..\ \ 0;$$

and the rule is to operate on each column of the determinant, multiplying by the degree less unity, and increasing the symbolical number by unity. Thus

$$\begin{aligned}
\mathrm{D}\{1,2,(1)\}&=(n-1)\{2,2,(1)\}+(n-2)\{1,3,(1)\}+(n-2)\{1,2,(2)\}\\
&=\qquad\qquad\qquad\qquad\quad (n-2)\{1,3,(1)\}+(n-2)\{1,2,(2)\},
\end{aligned}$$

since $\{2, 2, (1)\}$ vanishes identically. The following Table shows the mode of effecting the operations:—

H	1			= 1	$[n-2]^0 \quad [n-2]^0$	12(1)
DH	1			= 1	$[n-2]^0 \quad [n-2]^1$	12(2)
		1		1	$[n-2]^1 \quad [n-2]^0$	13(1)
D²H	1			= 1	$[n-2]^0 \quad [n-2]^2$	12(3)
	1	1		2	$[n-2]^1 \quad [n-2]^1$	13(2)
		1		1	$[n-2]^2 \quad [n-2]^0$	14(1)
			1	1	$[n-2]^1 \quad [n-2]^0 \quad [n-1]^1$	23(1)
D³H	1			1	$[n-2]^0 \quad [n-2]^3$	12(4)
	2	1		3	$[n-2]^1 \quad [n-2]^2$	13(3)
	1	2		3	$[n-2]^2 \quad [n-2]^1$	14(2)
		1		1	$[n-2]^3 \quad [n-2]^0$	15(1)
	1		2	3	$[n-2]^1 \quad [n-2]^1 \Big\} [n-1]^1$	23(2)
		1	1	2	$[n-2]^2 \quad [n-2]^0$	24(1)
D⁴H	1			= 1	$[n-2]^0 \quad [n-2]^4$	12(5)
	3	1		4	$[n-2]^1 \quad [n-2]^3$	13(4)
	3	3		6	$[n-2]^2 \quad [n-2]^2$	14(3)
	1	3		4	$[n-2]^3 \quad [n-2]^1$	15(2)
		1		1	$[n-2]^4 \quad [n-2]^0$	16(1)
	3		3	6	$[n-2]^1 \quad [n-2]^2$	23(3)
	2	3	3	8	$[n-2]^2 \quad [n-2]^1 \Big\} [n-1]^4$	24(2)
		2	1	3	$[n-2]^3 \quad [n-2]^0$	25(1)
			2	2	$[n-2]^2 \quad [n-2]^0 \quad [n-1]^2$	34(1)
D⁵H	1			= 1	$[n-2]^0 \quad [n-2]^5$	12(6)
	4	1		5	$[n-2]^1 \quad [n-2]^4$	13(5)
	6	4		10	$[n-2]^2 \quad [n-2]^3$	14(4)
	4	6		10	$[n-2]^3 \quad [n-2]^2$	15(3)
	1	4		5	$[n-2]^4 \quad [n-2]^1$	16(2)
		1		1	$[n-2]^5 \quad [n-2]^0$	17(1)
	6		4	10	$[n-2]^1 \quad [n-2]^4$	23(4)
	8	6	6	20	$[n-2]^2 \quad [n-2]^3$	24(3)
	3	8	4	15	$[n-2]^3 \quad [n-2]^2 \Big\} [n-1]^1$	25(2)
		3	1	4	$[n-2]^4 \quad [n-2]^1$	26(1)
	2		8	10	$[n-2]^2 \quad [n-2]^1 \Big\} [n-1]^2$	34(2)
		2	3	5	$[n-2]^3 \quad [n-2]^0$	35(1)
D⁶H	1			= 1	$[n-2]^0 \quad [n-2]^6$	12(7)
	5	1		6	$[n-2]^1 \quad [n-2]^5$	13(6)
	10	5		15	$[n-2]^2 \quad [n-2]^4$	14(5)
	10	10		20	$[n-2]^3 \quad [n-2]^3$	15(4)
	5	10		15	$[n-2]^4 \quad [n-2]^2$	16(3)
	1	5		6	$[n-2]^5 \quad [n-2]^1$	17(2)
		1		1	$[n-2]^6 \quad [n-2]^0$	18(1)
	10		5	15	$[n-2]^2 \quad [n-2]^4$	23(5)
	20	10	10	40	$[n-2]^2 \quad [n-2]^3$	24(4)
	15	20	10	45	$[n-2]^3 \quad [n-2]^2 \Big\} [n-1]^1$	25(3)
	4	15	5	24	$[n-2]^4 \quad [n-2]^1$	26(2)
		4	1	5	$[n-2]^5 \quad [n-2]^0$	27(1)
	10		20	30	$[n-2]^2 \quad [n-2]^2$	34(3)
	5	10	15	30	$[n-2]^3 \quad [n-2]^1 \Big\} [n-1]^2$	35(2)
		5	4	9	$[n-2]^4 \quad [n-2]^0$	36(1)
			5	5	$[n-2]^3 \quad [n-2]^0 \quad [n-1]^3$	45(1)
D⁷H	&c.					

where the first three columns show the numbers which give, by the addition of the numbers in the same horizontal line, the numerical coefficients of the factorials which multiply the different terms of H, DH, &c., and where in the last column 12(1), &c. are written for shortness in the place of $\{1, 2, (1)\}$, &c.

27. It is clear that we have in general

$$
\begin{array}{lllll}
\Lambda^2 D^r H = & & 1[n-2]^0[n-2]^r & \{1, 2, (r+1)\} \\
& + & \frac{r}{1}\ [n-2]^1[n-2]^{r-1} & \{1, 3, \ (r)\ \} \\
& + & \frac{r.r-1}{1.2}\ [n-2]^2[n-2]^{r-2} & \{1, 4, (r-1)\} \\
& & \vdots & \\
& + & 1\ [n-2]^r[n-2]^0 & \{1, r+2, (1)\} \\
\hline
+[n-1]^1 & \left\{\begin{array}{l} +R'_0 \\ +R'_1 \\ \vdots \\ +R'_{r-2} \end{array}\right. & \begin{array}{l} [n-2]^1\ [n-2]^{r-2} \\ [n-2]^2\ [n-2]^{r-3} \\ \\ [n-2]^{r-1}[n-2]^0 \end{array} & \begin{array}{l} \{2, 3, (r-1)\} \\ \{2, 4, (r-2)\} \\ \\ \{2, r+1, (1)\} \end{array} \\
\hline
+[n-1]^2 & \left\{\begin{array}{l} +R''_0 \\ \vdots \\ +R''_{r-4} \end{array}\right. & \begin{array}{l} [n-2]^2\ [n-2]^{r-4} \\ \\ [n-2]^{r-2}[n-2]^0 \end{array} & \begin{array}{l} \{3, 4, (r-3)\} \\ \\ \{3, r, \ (1)\ \} \end{array} \\
\hline
& & \vdots & \\
+[n-1]^{\frac{1}{2}r} & \left\{\begin{array}{l} +R_0^{\frac{1}{2}r} \\ r \text{ even; or } r \text{ odd} \end{array}\right. & [n-2]^{\frac{1}{2}r}\ [n-2]^0 & \{\tfrac{1}{2}r+1\ , \tfrac{1}{2}r+2\ , (1)\} \\
+[n-1]^{\frac{1}{2}(r-1)} & \left\{\begin{array}{l} +R_0^{\frac{1}{2}(r-1)} \\ +R_0^{\frac{1}{2}(r+1)} \end{array}\right. & \begin{array}{l} [n-2]^{\frac{1}{2}(r-1)}[n-2]^1 \\ [n-2]^{\frac{1}{2}(r+1)}[n-2]^0 \end{array} & \begin{array}{l} \{\tfrac{1}{2}(r+1), \tfrac{1}{2}(r+3), (2)\} \\ \{\tfrac{1}{2}(r+1), \tfrac{1}{2}(r+5), (1)\}; \end{array}
\end{array}
$$

and the general term is

$$[n-1]^\delta\ R_s^\delta[n-2]^{\delta+s}[n-2]^{r-2\delta-s}\{\delta+1, \delta+2+s, (r-2\delta+1-s)\},$$

where s extends from $s=0$ to $s=r-2\delta$, and δ from $\delta=0$ to $\delta=\frac{1}{2}r$ or $\frac{1}{2}(r-1)$, according as r is even or odd. The expression for the coefficients R_s^0 is

$$R_s^0=\frac{[r]^s}{[s]^s},$$

and that of the other coefficients R_s^δ ($\delta=$ or <1) is not required for the present purpose.

28. According to a remark already made, the expressions for D^rH_1, D^rH_2 &c. are at once obtained from that for D^rH by merely writing $n-1$, $n-2$, &c. in the place of n: it is however to be noticed, that the quantity within the [ ] must not be negative, and that on its becoming so, the factorial is to be omitted.

29. I write now

$$+s=\alpha,\quad r-2\delta-s=\beta,$$

and I consider the expression

$$\mathrm{D}^r\mathrm{H}-\frac{n-1}{1}\mathrm{D}^r\mathrm{H}_1+\&\mathrm{c}.;$$

the general term of which is

$$\mathrm{R}^\delta_s\{\delta+1,\ \delta+2+s,\ (r-2\delta+1-s)\}\times\left\{\begin{array}{l}\qquad [n-1]^\delta[n-2]^\alpha[n-2]^\beta\\ -\frac{n-1}{1}[n-2]^\delta[n-3]^\alpha[n-3]^\beta\\ +\qquad \&\mathrm{c}.\end{array}\right\},$$

or as this may be written, putting $q=n-\delta-1$,

$$\mathrm{R}^\delta_s\{\delta+1,\ \delta+2+s,\ (r-2\delta+1-s)\}\times[n-1]^\delta\left\{\begin{array}{l}\quad [n-2]^\alpha[n-2]^\beta\\ -\frac{q}{1}[n-3]^\alpha[n-3]^\beta\\ \quad +\ \&\mathrm{c}.\end{array}\right\}.$$

30. I assume $r\not>n-2$, we have then $\alpha+\beta=r-\delta\not>n-\delta-2$, and therefore $\alpha+\beta<q$. The general term of the series in {} is

$$(-)^\vartheta\frac{[q]^\vartheta}{[\vartheta]^\vartheta}[n-2-\vartheta]^\alpha[n-2-\vartheta]^\beta,$$

where the terms for which $n-2-\vartheta$ is negative are to be excluded, or what is the same thing, the series is not to be continued beyond $\vartheta=n-2$. But observing that $[q]^\vartheta$ vanishes for $\vartheta>q$, that is, $\vartheta>n-\delta-1$, it is in fact the same thing whether the series is continued indefinitely or only to the term for which $\vartheta=n-\delta-1$, and we may consistently with the condition $\vartheta\not>n-2$, continue the series as far as $\vartheta=n-\delta-1$, except in the case $\delta=0$, when by doing so we include the term corresponding to $\vartheta=n-1$, which in virtue of the condition ought to be excluded. The expression for the term in question is $(-)^{n-1}[-1]^\alpha[-1]^\beta$; hence if the sum of the series continued to the proper point is S, the sum continued indefinitely (in the particular case $\delta=0$) is $\mathrm{S}+(-)^{n-1}[-1]^\alpha[-1]^\beta$, but in every other case the sum continued indefinitely is simply S. And by a well-known theorem in finite differences, the sum continued indefinitely is in fact zero. That is, except in the case $\delta=0$, we have

$$\mathrm{S}=0,$$

but in the excepted case

$$\mathrm{S}+(-)^{n-1}[-1]^\alpha[-1]^\beta=0;$$

or observing that $\alpha+\beta(=r-\delta)$ is in this case $=r$, and transforming the factorials, we have

$$\mathrm{S}=(-)^{n-r}[\alpha]^\alpha[\beta]^\beta,$$

or substituting for α and β their values,

$$\mathrm{S}=(-)^{n-r}[s]^s[r-s]^{r-s}.$$

31. Hence the general term of

$$D^rH-\frac{n-1}{1}D^rH_1+\&c.$$

vanishes except for $\delta=0$, but when $\delta=0$, its value is

$$R^0_s\{1,\,2+s,\,(r+1-s)\}\times(-)^{n-r}[s]^s[r-s]^{r-s};$$

or observing that R^0_s is equal to $[r]^r\div[s]^s[r-s]^{r-s}$, the value is simply

$$(-)^{n-r}[r]^r\{1,\,2+s,\,(r+1-s)\},$$

that is, we have

$$\begin{aligned}&D^rH-\frac{n-1}{1}D^rH_1+\&c.\\&\quad=(-)^{n-r}[r]^r\,S_s\{1,\,2+s,\,(r+1-s)\},\end{aligned}$$

the summation in respect to s extending from $s=0$ to $s=r$. In particular, giving to r the values $n-2$ and $n-1$, and attending to the expressions for III and IV, we find

$$\Lambda^2\left(D^{n-2}H-\frac{n-1}{1}D^{n-2}H_1+\&c.\,.\,.\right)=-[n-2]^{n-2}\,\mathrm{III},$$

$$\Lambda^2\left(D^{n-3}H-\frac{n-1}{1}D^{n-3}H_1+\&c.\,.\,.\right)=-[n-3]^{n-3}\,\mathrm{IV}.$$

32. The equation $\mathrm{III}=0$ belongs to the curve which by its intersections with the tangent, gives the tangentials of a point of the curve $U=0$. Hence the equation of the curve in question is

$$D^{n-2}H-\frac{n-1}{1}D^{n-2}H_1+\&c.=0,$$

which is Mr. SALMON's theorem, leading to the solution of the problem of double tangents.

33. The expressions for I and II are obtained from those of IV and III by interchanging (X, Y, Z) and (x, y, z), and reversing the sign. Hence if, as before, $\mathfrak{H}$, $\mathfrak{D}$, &c. denote the values which H, D, &c. assume by this interchange, we have

$$\Lambda^2\left(\mathfrak{D}^{n-2}\mathfrak{H}-\frac{n-1}{1}\mathfrak{D}^{n-2}H_1+\&c.\,.\,.\right)=[n-2]^{n-2}\,\mathrm{II},$$

$$\Lambda^2\left(\mathfrak{D}^{n-3}\mathfrak{H}-\frac{n-1}{1}\mathfrak{D}^{n-3}H_1+\&c.\,.\,.\right)=[n-3]^{n-3}\,\mathrm{I},$$

and the identical equation

$$\mathrm{I}a_0+\mathrm{II}a_1+\mathrm{III}a_{n-1}+\mathrm{IV}a_n=0$$

becomes therefore

$$\left.\begin{aligned}&\left(\mathfrak{D}^{n-3}\mathfrak{H}-\frac{n-1}{1}\mathfrak{D}^{n-3}\mathfrak{H}_1+\&c.\right)U\\&+\frac{1}{n(n-2)}\left(\mathfrak{D}^{n-2}\mathfrak{H}-\frac{n-1}{1}\mathfrak{D}^{n-2}\mathfrak{H}_1+\&c.\right)DU\\&-\frac{1}{n(n-2)}\left(D^{n-2}H-\frac{n-1}{1}D^{n-2}H_1+\&c.\right)\mathfrak{D}\Upsilon\\&\left(D^{n-3}H-\frac{n-1}{1}D^{n-3}H_1+\&c.\right)\Upsilon\end{aligned}\right\}=0,$$

which is the general identical equation referred to in the introduction to the present memoir.

34. It is to be noticed that for $n=3$, the equation is

$$(\mathfrak{H}-2\mathfrak{H}_1)\mathrm{U}+\tfrac{1}{3}\mathfrak{D}\mathfrak{H}.\mathrm{DU}-\tfrac{1}{3}\mathrm{DH}.\mathfrak{D}\Upsilon-(\mathrm{H}-2\mathrm{H}_1)\Upsilon=0.$$

But we have $\mathrm{H}_1=\mathrm{H}$, and in like manner $\mathfrak{H}_1=\mathfrak{H}$, and the equation thus becomes

$$-\mathfrak{H}\mathrm{U}+\tfrac{1}{3}\mathfrak{D}\mathfrak{H}.\mathrm{D}\Upsilon-\tfrac{1}{3}\mathrm{DH}.\mathfrak{D}\Upsilon+\mathrm{H}\Upsilon=0.$$

And so also for $n=4$, the equation is

$$(\mathfrak{D}\mathfrak{H}-3\mathfrak{D}\mathfrak{H}_1)\mathrm{U}+\tfrac{1}{8}(\mathfrak{D}^2\mathfrak{H}-3\mathfrak{D}^2\mathfrak{H}_1)\mathrm{DU}-\tfrac{1}{8}(\mathrm{D}^2\mathrm{H}-3\mathrm{D}^2\mathrm{H}_1)\mathfrak{D}\Upsilon-(\mathrm{DH}-3\mathrm{DH}_1)\Upsilon=0.$$

But we have in general $\mathrm{DH}_1=\frac{n-3}{n-2}\mathrm{DH}$, and therefore in the present case $\mathrm{DH}_1=\tfrac{1}{2}\mathrm{DH}$, and consequently $\mathfrak{D}\mathfrak{H}_1=\tfrac{1}{2}\mathfrak{D}\mathfrak{H}$, and the equation thus becomes

$$-\mathfrak{D}\mathfrak{H}.\mathrm{U}+\tfrac{1}{4}(\mathfrak{D}^2\mathfrak{H}-3\mathfrak{D}^2\mathfrak{H}')\mathrm{DU}-\tfrac{1}{4}(\mathrm{D}^2\mathrm{H}-3\mathrm{D}^2\mathrm{H}_1)\mathfrak{D}\Upsilon+\mathrm{DH}.\Upsilon=0,$$

which agree with the results previously obtained for the two particular cases.

XII. *On the Resistance of Glass Globes and Cylinders to Collapse from external pressure; and on the Tensile and Compressive Strength of various kinds of Glass.*

By W. Fairbairn, *Esq., C.E., F.R.S., and* Thomas Tate, *Esq.*

Received May 3,—Read May 12, 1859.

The recently published experiments upon the collapse of tubes of wrought iron[*], led to results so novel and so much at variance with the ordinary rules of practice, as to exemplify anew the caution and diligence which are requisite in investigating the physical laws of nature, in order to arrive at just conclusions in regard to the properties of materials and their most effective distribution for the purposes of construction.

In the experiments alluded to, it was clearly shown that the prevailing ideas of the strength of vessels subjected to a uniform external force were erroneous and at variance with the laws of resistance to collapse under such circumstances; whilst in practice the prevalence of error in this matter had led to serious and sometimes fatal accidents, arising out of the construction of vessels of inadequate strength to sustain the pressures placed upon them. These errors, it is hoped, need no longer be perpetuated: the experiments on wrought iron indicated a means of increasing the strength of boiler flues and other vessels of that material, subjected to a collapsing force, to any required amount; and this was the immediate practical application of the general law then discovered, that the strength of cylindrical vessels, exposed to a uniform external force, varied inversely as the length between the rigid ends.

The results deduced from the experiments on tubes composed of riveted plates were so important as to suggest further inquiry, under the same conditions of rupture, but with other materials, differing in their physical properties from wrought iron. The joints in the tubes employed in those experiments were defects the influence of which might be determined by experiments upon homogeneous vessels. The ductile yielding character of wrought iron suggested the extension of the experiments to hard, rigid materials, more capable of retaining their form under pressure.

To fulfil the conditions thus sought for, *glass* was selected for experiment, as a material differing totally in character from wrought iron, and on that account well fitted to supply data for extending our knowledge of the laws of collapse. Of vitreous structure, rigid, elastic, and brittle, and of low tenacity, it possessed the further advantage of being easily obtained and blown into homogeneous vessels of the required forms. But there were other reasons which had weight in making this selection. Our acquaintance with the strength of glass, in the various forms in which it is employed in the arts and in scientific research, is very limited; and often as it is employed in circumstances in which it is exposed to pressure, few attempts have been made to register observations of its strength. Some researches on the density of steam at high pressures, now in progress, led to an examination of the subject, and added, to other reasons for testing its powers of resistance, the immediate necessity of knowing more of its properties before it could be trusted in those experimental inquiries.

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[*] Philosophical Transactions, 1858, pp. 389—413. Erratum in that paper: at page 408, in the place of "and by an obvious transformation," *read* "and when $k=\cdot043$, by an obvious transformation".

Our knowledge of the cohesive properties of glass is so defective, that, to arrive at satisfactory and complete results, it was deemed advisable to ascertain by direct experiment its tenacity, or resistance to a tensile strain, its resistance to a crushing force, and, in the form of globes and cylinders, to determine its resistance to an internal bursting force and to an external pressure tending to produce rupture by collapse. The results of the experiments upon glass globes and cylinders will, it is believed, form decided contributions to our present knowledge relative to the laws which determine the strength of materials. One remarkable result is that the law expressing the resistance of glass cylinders to compression is precisely similar to that which has been established for sheet-iron cylinders.

The glass experimented upon was of three kinds, known commercially as

Best Flint-glass,
Common Green Glass, and
Extra White Crown-glass.

The flint-glass obtained from Messrs. MOLINEAUX, WEBB, and Co., Manchester, was made of sand, oxide of lead, and carbonate of potash, in the following proportions:—

Sand	54 per cent.
Red oxide of lead	22 per cent.
Carbonate of potash . . .	24 per cent.

This glass is of a fine transparent character, fusible, and of a high specific gravity, due to the large per-centage of lead in its composition.

The green and crown-glass were obtained from Messrs. CHANCE Brothers of Birmingham, and were made of sand, soda, and lime in the following proportions:—

Common Green Glass.

Sand	100 parts
Sulphate of soda	42 parts
Carbonate of lime	45 parts

This is a hard infusible glass of a green colour, transparent, but of a less density than the flint-glass.

White Crown-glass.

Sand	100 parts.
Carbonate of soda	38 parts.
Lime	11 parts.

A clear transparent glass, hard under the action of the grindstone, and highly infusible.

The specific gravity of these different kinds of glass varies greatly; the following Table gives the result of several determinations:—

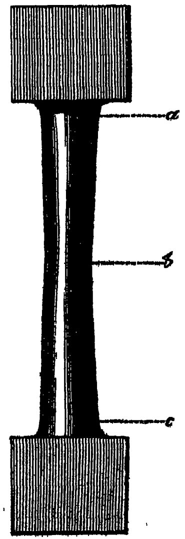
Specific Gravity of Glass.

		Mean.
Best flint-glass	3·0787	3·0782
Best flint-glass	3·0777	
Common green glass	2·5279	2·5284
Common green glass	2·5289	
White crown-glass	2·4498	2·4504
White crown-glass	2·4510	

SECTION I.—TENACITY OF GLASS.

A few experiments were made upon the tenacity of glass, by tearing specimens asunder by a direct tensile strain. These results, however, owing to the following circumstances, are not so satisfactory as could be wished. In breaking glass by the method adopted for other materials, namely, by suspending weights to it, there is danger that its great rigidity and brittleness may occasion its fracture before the entire cohesive force has been balanced by the strain applied, from the vibration of laying on the weights. In the experiments upon globes, however, in which a uniform water pressure was employed, the tenacity of the glass was ascertained with more accuracy, and any failure in the present experiments is therefore the less to be regretted. The glass to be broken by a tensile strain was obtained of the form shown in fig. 1, drawn smaller at the middle to secure fracture at that part. These specimens were fixed in a pair of wrought-iron shackles, and rested by their shoulders upon a thick india-rubber washer placed on the turned faces of the shackles. In this state they were suspended to a firm support, and a scale-pan attached to the lower shackle. Weights were then added, with the greatest care, till the specimen fractured. In this manner the following results were obtained:—

Fig. 1.



Experiment 1.

Annealed Flint-glass.

Least diameter	0·57 in.
Least area	0·255 sq. in.

Breaking weight, 583 lbs.=2286 lbs. per square inch.

The fracture took place at *a*, fig. 1, and presented a regular smooth convex surface. No notch had been cut in this specimen.

Experiment 2.

Annealed Flint-glass.

Least diameter . . . . 0·50 in.
Least area . . . . . 0·196 sq. in.
Breaking weight, 499 lbs.=2540 lbs. per square inch.

Broke in the notch at *b*, fig. 1, which in this case was cut by the grindstone. It is possible that the exterior coat of glass may be stronger than its core, in which case the above specimen was weakened. In the next experiments the specimens were drawn thinner by heat.

Experiment 3.

Common Green Glass.

Least diameter . . . . 0·53 in.
Least area . . . . . 0·220 sq. in.
Breaking weight, 639 lbs.=2896 lbs. per square inch.

Experiment 4.

White Crown-glass.

Least diameter . . . . 0·54 in.
Least area . . . . . 0·229 sq. in.
Breaking weight, 583 lbs.=2545 lbs. per square inch.

Broke at shoulder *a*, fig. 1.

The following Table exhibits at one view the results of these experiments, which, notwithstanding the objections to the method by which they were obtained, appear to be consistent with each other.

TABLE I.—Tensile Strength of Glass Bars.

Description of glass.	Area of specimen, in inches.	Breaking weight, in lbs.	Tenacity per square inch, in lbs.	Tenacity per square inch, in tons.
Flint-glass	0·255	583	2286	1·02
	0·196	499	2540	1·13
Green glass	0·220	639	2896	1·29
Crown-glass	0·229	583	2546	1·14

It may be observed here, in anticipation, that the tensile strength is much smaller in the case of glass fractured by a direct strain in the form of bars, than when burst by internal pressure in the form of thin globes. This difference is no doubt mainly due to the fact that thin plates of this material generally possess a higher tenacity than stout bars, which, under the most favourable circumstances, may be but imperfectly annealed. There is also a considerable discrepancy between the strength of green and crown-glass when in the form of bars and when in the form of globes. In the case of the bars, the results are as 1·0 to 1·13 in favour of green glass, whilst in the case of the globes, the results are as 1·0 to 1·2 in favour of the crown-glass. These discrepancies may, however, be accounted for from the different condition of the material in relation to annealing in the two cases, or from an imperfect bedding of the specimen, causing a distortion of the strain out of the direction of the axis of the specimen, or from accidental vibration in laying on the weights.

Section II.—RESISTANCE OF GLASS TO CRUSHING.

Fig. 3.

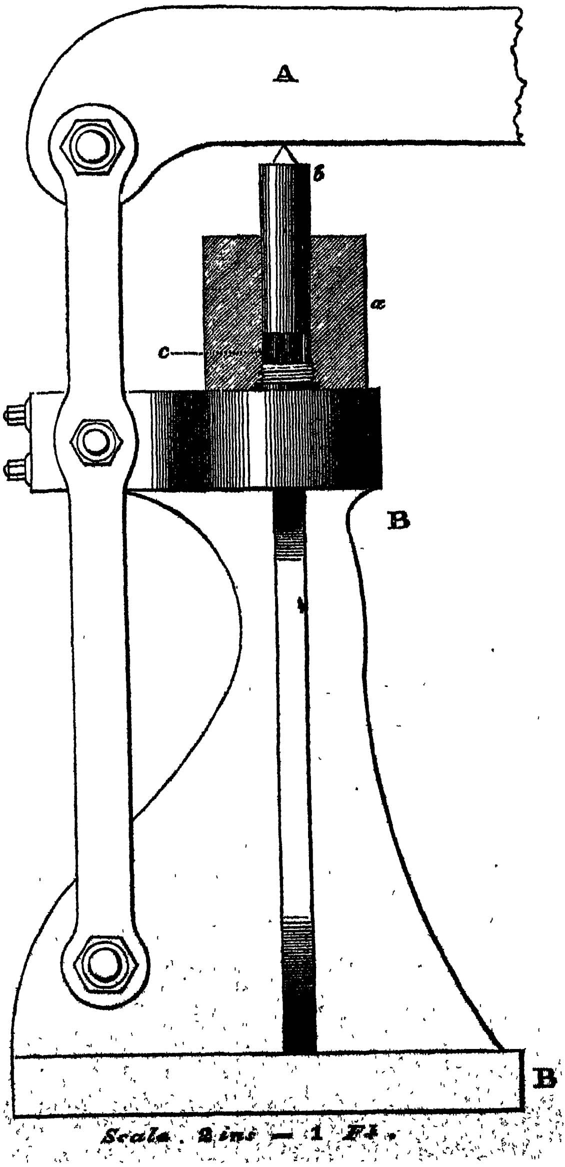
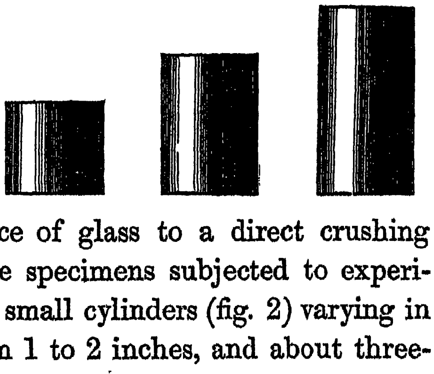


Fig. 2.



The next series of experiments was instituted with a view of determining the powers of resistance of glass to a direct crushing force. The specimens subjected to experiment were small cylinders (fig. 2) varying in length from 1 to 2 inches, and about three-quarters of an inch in diameter. They were placed for the purpose of crushing within the box *a* (fig. 3), thin packings of soft lead being interposed between the glass and the parallel crushing surfaces of the box and its solid steel piston *b*; in this way a firm and uniform bearing surface was secured, and the crushing force was applied perpendicularly in the direction of the axis of the specimen. Fig. 3 exhibits the general arrangement of the crushing apparatus, consisting of a lever A, 8 feet long, supported on a strong cast-iron base B, B. The crushing force obtained by placing weights in the scale-pan hung at the extremity of the lever is transmitted through the piston *b*, to the specimen to be crushed, *c*.

Flint-glass Cylinders.

Experiment 1.

Diameter	0·85 inch.
Area	0·5674 square inch
Height	1·0 inch.

Placed between india-rubber packings.

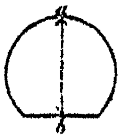
This specimen having been slightly fractured at an early stage in the experiment, it was taken out and the fractured side ground flat preparatory to another trial.

Experiment 2.

The same specimen as in Experiment 1.

Diameter	0·85 inch.
Height of segment (*ab*)	0·73 inch.
Area	0·555 inch.
Height of specimen	1·0 inch.

Fig. 4.



Weights added, in lbs.	Total weight, in lbs.	
1321	1321	
2080	3401	
896	4297	
896	5193	
896	6089	
896	6985	
896	7881	
448	8329	
448	8777	
448	9225	Fractured
224	9449	
224	9673	
224	9897	
224	10121	
224	10345	
224	10569	
224	10793	
224	11017	
224	11241	
224	11465	
224	11689	
224	11913	
224	12137	
224	12361	
224	12585	
112	12697	
112	12809	
112	12921	
112	13033	Crushed

Fractured with 9,225 lbs.=16,621 lbs. per square inch.
Crushed with 13,033 lbs.=23,483 lbs. per square inch.

It will not be necessary again to repeat in detail the steps by which the weights were augmented, as these were similar in every case. In the succeeding experiments the weights at which the specimens fractured and crushed are alone given.

Experiment 3.

Diameter	0·69 inch.
Area	0·3739 square inch.
Height	1·0 inch.

Fractured with 11,465 lbs.=30,661 lbs. per square inch.

Crushed with 13,033 lbs. =34,854 lbs. per square inch.

Experiment 4*.

Diameter	0·70 inch.
Area	0·3848 square inch.
Height	1·55 inch.

Crushed suddenly with 5193 lbs.=13,494 lbs. per square inch.

Experiment 5.

Diameter	0·83 inch.
Area	0·541 square inch.
Height	1·6 inch.

Crushed suddenly with 11,241 lbs.=20,775 lbs. per square inch.

Experiment 6.

Diameter	0·68 inch.
Area	0·3631 square inch.
Height	2·05 inches.

Crushed with 11,913 lbs.=32,803 lbs. per square inch.

Green Glass Cylinders.

Experiment 7.

Diameter	0·77 inch.
Area	0·466 square inch.
Height	1·0 inch.

Fractured with 6933 lbs.=14,888 lbs. per square inch.

Crushed with 10,516 lbs.=22,583 lbs. per square inch.

Experiment 8.

Diameter	0·76 inch.
Area	0·454 square inch.
Height	1·5 inch.

Fractured with 8126 lbs.=17,883 lbs. per square inch.

Crushed with 15,891 lbs.=35,029 lbs. per square inch.

---

* This experiment is so evidently anomalous, that there can be little doubt that the bedding surfaces were not parallel; hence this result is omitted in the following averages.

Experiment 9.

Diameter . . . . 0·79 inch.
Area . . . . 0·4901 square inch.
Height . . . . 2·0 inches.

Sustained a weight of 18,634 lbs.=38,015 lbs. per square inch, without being crushed. At this point the deflection of the lever was so great, that it was considered dangerous to proceed. On removing the specimen to a heavier lever, it crushed with a force of 12,000 lbs. The larger weight, however, had been fairly supported.

Crown-glass Cylinders.

The two cylinders of crown-glass were slightly rounded towards the edge of the bearing surfaces, which reduced the area directly subjected to the crushing force. It is therefore probably most accurate to take the less area in reducing the results.

Experiment 10.

Diameter . . . . {0·72 inch at middle.
{0·68 inch at ends.
Area . . . . 0·363 square inch.
Height . . . . 1·5 inch.

Crushed suddenly with 14,100 lbs.=38,825 lbs. per square inch.

Experiment 11.

Diameter . . . . {0·80 inch at middle.
{0·76 inch at ends.
Area . . . . 0·454 square inch.
Height . . . . 1·0 inch.

Crushed suddenly with 10,516 lbs.=23,181 lbs. per square inch.

Arranging the above results together, we obtain the following general Table of the results of the experiments:—

Table II.—Summary of Results of Experiments on the Resistance of Annealed Glass Cylinders to Crushing.

Description of glass.	Height of cylinder, in inches.	Area of cylinder, in inches.	Weight causing fracture, in lbs.	Crushing weight, in lbs.	Weight per sq. in. to cause fracture. lbs.	Weight per sq. in. to crush. lbs.
Flint-glass	1·00	0·555	9,225	13,033	16,621	23,483
	1·00	0·374	11,465	13,033	30,661	34,854
	1·60	0·541	......	11,241	......	20,775
	2·05	0·363	......	11,913	......	32,803
Green glass	1·00	0·466	6,933	10,516	14,888	22,583
	1·50	0·454	8,126	15,891	17,888	35,029
	2·00	0·490	......	18,634	......	38,015
Crown-glass......	1·0	0·454	......	10,516	......	23,181
	1·5	0·363	......	14,100	......	38,825

Taking the means of the above values, and reducing the weights to tons, we have—

Table III.—Mean Compressive Resistance of Glass.

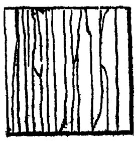
Description of glass.	Height of cylinder, in inches.	Crushing weight per square inch,		Mean crushing weight per square inch,	
		in lbs.	in tons.	in lbs.	in tons.
Flint-glass	1·0	29,168	13·021	27,582	12·313
	1·6	20,775	9·274		
	2·0	32,803	14·644		
Green glass............	1·0	22,583	10·081	31,876	14·227
	1·5	35,029	15·628		
	2·0	38,015	16·971		
Crown-glass	1·0	23,181	10·348	31,003	13·840
	1·5	38,825	17·332		

The mean resistance of glass to a crushing force is therefore, from the above experiments, equivalent to 13·460 tons per square inch. Assuming the above numbers to represent the comparative values of each kind of glass, and taking flint-glass as the standard, we have their respective strengths as follow:—

Green glass . . .	1152
Crown-glass . . .	1124
Flint-glass	1000

The specimens were crushed almost to powder from the violence of the concussion, when they gave way; it however appeared that the fractures occurred in vertical planes, splitting up the specimen in all directions. This characteristic mode of disintegration has been noticed before, especially with vitrified brick and indurated limestone. The experiments following on cubes of glass which were exposed to view during the crushing process, illustrated this subject further: cracks were noticed to form some time before the specimen finally gave way; then these rapidly increased in number, splitting the glass into innumerable irregular prisms of the same height as the cube; finally these bent or broke, and the pressure, no longer bedded on a firm surface, destroyed the specimen. The annexed ideal sketch (fig. 5) may give some notion of the fractures of a cube, supposing all the particles were restored to their position after crushing.

Fig. 5.



The specimens employed in the following experiments were cut from the square heads of the pieces of glass employed in the experiments on tensile strain (fig. 1). These pieces were approximately cubical; and their size prevented their insertion in the box *a* (fig. 3); they were therefore crushed between parallel steel discs, exposed to view. The crushing was more gradual, and was not effected so completely in these experiments as in those on small cylinders, the fragments being in every case larger after the conclusion of the experiment: it must further be recollected, in comparing these with the preceding experiments, that the cylinders were cut off, of the required length, from rods of glass drawn out when molten to the diameter desired, so as to retain the first-cooled exterior skin of glass, which is probably of greater tenacity than the interior; on the other hand, the cubes were cut from the centre of larger lumps of glass, and were possibly in a state of imperfect annealing.

Flint-glass Cubes.

Experiment 12.

Area =0·96×0·97 inch =0·9312 square inch.
Height =1·15 inch.
Crushed suddenly with 13,257 lbs.=14,235 lbs. per square inch

Experiment 13.

Area =0·99×0·98 inch=0·9702 square inch.
Height =1·16 inch.
Crushed with 12,809 lbs.=13,202 lbs. per square inch.

Experiment 14.

Area =0·98×1·02 inch=0·9996 square inch.
Height =1·10 inch.
Crushed with 13,257 lbs.=13,262 lbs. per square inch.

Experiment 15.

Area =0·98 inch×0·98 in.=0·9604 square inch.
Height =1·10 inch.
Fractured with 6537 lbs.=6806 lbs. per square inch.
Crushed with 11,353 lbs.=11,820 lbs. per square inch.

Green Glass Cubes.

Experiment 16.

Area =1·0×0·98 inch=0·98 square inch.
Height =1 0 inch.
Crushed with 20,059 lbs.=20,468 lbs. per square inch.

Experiment 17.

Area =0·99×1·2 inch=1·188 square inch.
Height =1·0 inch.
Crushed with 23,535 lbs. =19,945 lbs. per square inch.

Crown-glass Cube.

Experiment 18.

Area $=0·82\times0·92$ inch$=0·7534$ square inch.
Height $=0·9$ inch.
Crushed with 16,475 lbs.=21,867 lbs. per square inch.

This crushed suddenly after bearing the weight some time, and was reduced almost to powder.

Table IV.—Summary of the Results of Experiments on the Resistance of Cut Glass Cubes to Compression.

Description of glass.	Area of specimen, in square inch.	Crushing weight, in lbs.	Resistance to crushing per square inch,	
			in lbs.	in tons.
Flint-glass	0·9312	13,257	14,235	6·355
	0·9702	12,809	13,202	5·894
	0·9996	13,257	13,262	5·921
	0·9604	11,353	11,820	5·276
Green glass......	0·9800	20,059	20,468	9·116
	1·1880	23,535	19,945	8·904
Crown-glass	0·7534	16,475	21,867	9·762

Hence the mean resistance to crushing of cubes of glass is equivalent to a weight of—

	lbs.
For flint-glass	13,130
For green glass	20,206
For crown-glass	21,867
Mean	18,401

Comparing these with the preceding results on glass cylinders, we have the mean resistance of the former experiments to the mean resistance of the above as 30,153 : 18,401, or as 1·6 : 1.

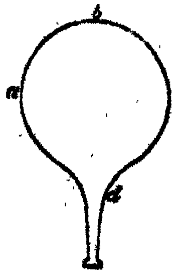
General Observations relative to the Results of the Experiments on the Resistance of Glass Cylinders and Cubes to Crushing.

With iron and some other materials, when a short column undergoes a pressure in the direction of its length, rupture takes place in a plane having a determinate angle to the axis of the column, this plane being the section of least resistance. Neglecting the friction of the surfaces, Coulomb found this angle to be 45°; and allowing on an average 10° for the limiting angle of friction, the angle of the plane of rupture may be taken at 55°. To fulfil this condition, the length of the column to be crushed should be at least three times its radius: when the length greatly exceeds this limit, the rupture will be effected by the tendency of the column to bend; and when the length is within this limit, the force requisite to produce rupture will be increased in consequence of the irregular form of the line of fracture. These theoretical deductions have been confirmed by experiments made upon columns of iron, wood, bone, stone, and other materials. The results of the experiments here recorded, however, show that when the length of the cylinder does not greatly differ from three times its radius, the resistance to a crushing force is pretty nearly a constant, viz. on an average 12·313 tons per square inch in the case of flint-glass, 14·227 tons in the case of green glass, and 13·84 tons in the case of crown-glass. But, according to COULOMB's law, the cubes of flint-glass (their lengths being considerably less than three times their semi-diameters) should have presented higher powers of resistance than the cylinders; this discrepancy is probably owing to the injury which the glass had sustained in the process of cutting, and to the imperfect annealing of glass when cast in the form of cubes and cylinders.

SECTION III.—RESISTANCE OF GLASS GLOBES AND CYLINDERS TO INTERNAL PRESSURE.

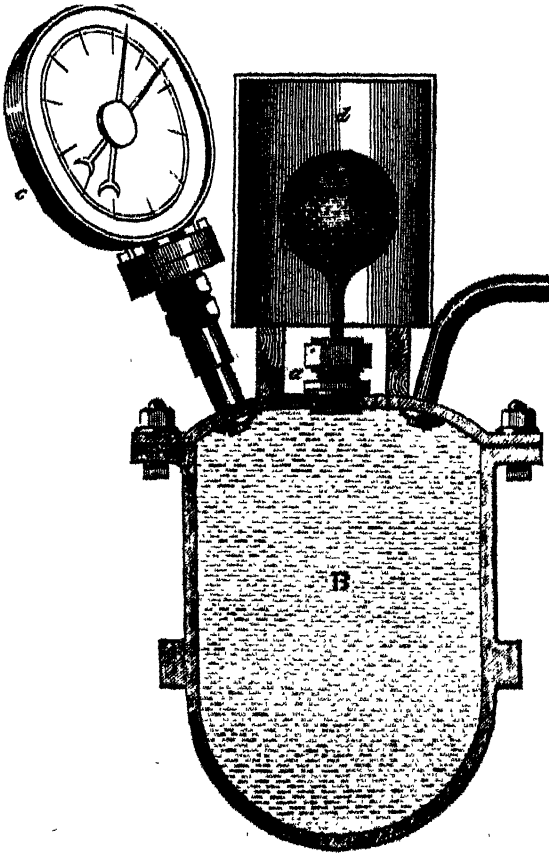
In the following experiments it has been sought not only to determine the law of resistance to internal pressure, which is already well known from theoretical considerations, but to ascertain the direct tensile strength of the glass (of which the bursting pressure is a function) by a method free from many of the objections to that described in Section I. The bursting pressure of cylindrical and spherical vessels is well known to be in the ratio of the tenacity of the material, other things being the same, and the determination of the tensile strength upon this principle, presents in the case of glass peculiar advantages. As glass can be obtained in tolerably perfect spheres, and as the fracture of these may be effected by a uniform water pressure, increasing slowly and regularly without vibration, there is a better chance of ascertaining the ultimate resistance of the material, from the absence of those shocks and irregularities which are inseparable from any process depending upon the piling up of weights, however carefully conducted.

Fig. 6.



In making these experiments, a number of glass globes were procured of varying size and thickness. The stems were then flanched out by the blowpipe (fig. 6), and the diameter having been carefully measured, they were ready for experiment. To effect their rupture, each globe, *k* (fig. 7), was attached by means of a stuffing-box (*a*) to the cover of a strong wrought-iron boiler H, and was enclosed by the iron cylindrical vessel *d*, to prevent the dispersion of the fragments when rupture took place. In the stuffing-box the flanch of the stem of the globe was bedded upon vulcanized india-rubber, in such a manner as to secure a watertight attachment without impeding the access of the water to the interior of the globe. [illegible] was connected with a hydraulic pump by means of the pipe *b*, and an accurate [illegible] Schaeffer construction was fixed to the boiler to register the pressure. [illegible] it will be seen that as the pumping was continued the water would rise in the globe, compressing the air in its interior, progressively, up to the point at which the resistance of the glass was overcome by the expansive force of the fluid; at that point explosion would take place, the pressure in pounds per square inch being noted both by the eye of the observer and by the maximum finger of the gauge.

Fig. 7.



In glass globes generally, the upper half of the sphere *a*, *b*, *c* (fig. 6) is the most spherical, and is approximately uniform in thickness, being however thinnest at *b*, and thickening gradually downwards towards the stem, the lower half (*a*, *d*, *c*) being considerably the strongest. Hence it happened, in several cases (in fact in every case in which the point could be determined with certainty from the condition of the fragments), that the globes ruptured first at *b*, the lines of fracture radiating in every direction, passing round the globe as meridians of longitude, and splitting it up into thin bands, varying from $\frac{1}{20}$th to $\frac{1}{8}$th inch in width. In the case of some elongated ellipsoids, it appeared that the fractures occurred horizontally, or perhaps obliquely, from the condition of the fragments attached to the stem. In most cases, however, it was not clear from the fragments which had been the direction of the fracture, although the mode of rupture was the same in every case.

To ascertain the thickness, several specimens were selected from the thinnest fragments, and each being measured separately by a micrometer screw of fifty threads to the inch and reading on a graduated head to $\frac{1}{1000}$th of an inch, the minimum thickness was assumed as that of the part which ruptured, and has been employed in reducing the results.

It must also be observed that the globes were usually slightly elliptical, in some cases seriously so; the vertical diameter, *b d* (using the same form of expression as before), being generally less than the horizontal, *a c*. In the following Tables the two diameters are given in each case.

Flint-glass.

Experiment 1.

Globe a. Diameters 4·0 and 3·98 inches.

$$\text{Thicknesses measured}\left\{\begin{array}{c}\text{In parts of an inch.}\\ 0{\cdot}0230\\ 0{\cdot}0256\\ 0{\cdot}0284\\ 0{\cdot}0244\\ 0{\cdot}0302\\ 0{\cdot}0250\\ 0{\cdot}0240\end{array}\right\}\text{Minimum} = 0{\cdot}024\text{ inch.}$$

Bursting pressure =84 lbs. per square inch.

Experiment 2.

Globe b. Diameters 4·0 and 3·98 inches.

$$\text{Thicknesses measured}\left\{\begin{array}{c}\text{In parts of an inch.}\\ 0{\cdot}034\\ 0{\cdot}032\\ 0{\cdot}031\\ 0{\cdot}0254\\ 0{\cdot}0256\\ 0{\cdot}031\\ 0{\cdot}028\end{array}\right\}\text{Minimum} = 0{\cdot}025\text{ inch.}$$

Bursting pressure =93 lbs. per square inch.

Experiment 3.

Globe c. Diameter 4·0 inches.

$$\text{Thicknesses measured}\left\{\begin{array}{c}\text{In parts of an inch.}\\ 0{\cdot}040\\ 0{\cdot}0406\\ 0{\cdot}039\\ 0{\cdot}039\\ 0{\cdot}038\end{array}\right\}\text{Minimum} = 0{\cdot}038\text{ inch.}$$

Bursting pressure =150 lbs. per square inch.

Experiment 4.

Globe d. Diameters 4·5 and 4·55 inches.

$$\text{Thicknesses measured}\left\{\begin{array}{c}\text{In parts of an inch.}\\ 0{\cdot}0620\\ 0{\cdot}0694\\ 0{\cdot}0581\\ 0{\cdot}0626\\ 0{\cdot}0564\\ 0{\cdot}0614\\ 0{\cdot}0604\\ 0{\cdot}0580\end{array}\right\}\text{Minimum} = 0{\cdot}056 \text{ inch.}$$

Burst with 280 lbs. per square inch.

Experiment 5.

Globe e. Diameters 5·1 and 5·12 inches.

$$\text{Thicknesses measured}\left\{\begin{array}{c}\text{In parts of an inch.}\\ 0{\cdot}0580\\ 0{\cdot}059\\ 0{\cdot}0586\\ 0{\cdot}0634\\ 0{\cdot}0620\\ 0{\cdot}059\end{array}\right\}\text{Minimum} = 0{\cdot}058 \text{ inch.}$$

Burst with 184 lbs. per square inch.

Experiment 6.

Globe f. Diameter 6·0 inches.

$$\text{Thicknesses measured}\left\{\begin{array}{c}\text{In parts of an inch.}\\ 0{\cdot}060\\ 0{\cdot}066\\ 0{\cdot}060\\ 0{\cdot}059\\ 0{\cdot}0592\\ 0{\cdot}0592\end{array}\right\}\text{Minimum} = 0{\cdot}059 \text{ inch.}$$

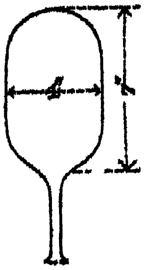
Burst with 152 lbs. per square inch.

Experiment 7.

Cylinder g. Diameter 4·05 inches. Length 7·0 inches.

$$\text{Thicknesses measured}\left\{\begin{array}{c}\text{In parts of an inch.}\\ 0{\cdot}079\\ 0{\cdot}081\\ 0{\cdot}090\\ 0{\cdot}086\\ 0{\cdot}086\\ 0{\cdot}079\\ 0{\cdot}086\end{array}\right\}\text{Minimum} = 0{\cdot}079 \text{ inch.}$$

Fig. 8.



Burst with 282 lbs. per square inch.

Green Glass.

Experiment 8.

Globe k. Diameters 4·95 and 5·0 inches.

In parts of an inch.

$$\text{Thicknesses measured}\left\{\begin{array}{l}0{\cdot}029\\0{\cdot}024\\0{\cdot}026\\0{\cdot}022\\0{\cdot}025\\0{\cdot}023\\0{\cdot}024\\0{\cdot}0225\end{array}\right\}\text{Minimum} = 0{\cdot}022\text{ inch.}$$

Bursting pressure = 90 lbs. per square inch.

Experiment 9.

Globe l. Diameters 4·95 and 5·0 inches.

In parts of an inch.

$$\text{Thicknesses measured}\left\{\begin{array}{l}0{\cdot}024\\0{\cdot}023\\0{\cdot}022\\0{\cdot}0196\\0{\cdot}023\\0{\cdot}022\\0{\cdot}020\\0{\cdot}020\end{array}\right\}\text{Minimum} = 0{\cdot}020\text{ inch.}$$

Burst with 85 lbs. per square inch.

Experiment 10.

Globe m. Diameters 4·0 and 4·05 inches.

In parts of an inch.

$$\text{Thicknesses measured}\left\{\begin{array}{l}0{\cdot}020\\0{\cdot}0205\\0{\cdot}0202\\0{\cdot}021\\0{\cdot}018\\0{\cdot}020\\0{\cdot}023\\0{\cdot}0205\\0{\cdot}0215\\0{\cdot}020\end{array}\right\}\text{Minimum} = 0{\cdot}018\text{ inch.}$$

Bursting pressure = 84 lbs. per square inch.

Experiment 11.

Globe n. Diameters 4·0 and 4·03 inches.

In parts of an inch.

$$\text{Thicknesses measured}\left\{\begin{array}{l}0{\cdot}016\\0{\cdot}019\\0{\cdot}018\\0{\cdot}017\\0{\cdot}019\\0{\cdot}016\\0{\cdot}016\end{array}\right\}\text{Minimum}=0{\cdot}016\text{ inch.}$$

Bursting pressure =82 lbs. per square inch.

Crown-glass.

Experiment 12.

Globe p. Diameters 4·2 and 4·35 inches.

In parts of an inch.

$$\text{Thicknesses measured}\left\{\begin{array}{l}0{\cdot}0252\\0{\cdot}0270\\0{\cdot}0272\\0{\cdot}030\\0{\cdot}0252\\0{\cdot}0256\end{array}\right\}\text{Minimum}=0{\cdot}025\text{ inch.}$$

Burst with 120 lbs. per square inch.

Experiment 13.

Globe q. Diameters 4·05 and 4·20 inches.

In parts of an inch.

$$\text{Thicknesses measured}\left\{\begin{array}{l}0{\cdot}028\\0{\cdot}0236\\0{\cdot}0256\\0{\cdot}0236\\0{\cdot}0212\\0{\cdot}0210\end{array}\right\}\text{Minimum}=0{\cdot}021\text{ inch.}$$

Bursting pressure =126 lbs. per square inch.

Experiment 14.

Globe r. Diameters 5·9 and 5·8 inches.

In parts of an inch.

$$\text{Thicknesses measured}\left\{\begin{array}{l}0{\cdot}0334\\0{\cdot}020\\0{\cdot}0244\\0{\cdot}017\\0{\cdot}0172\\0{\cdot}016\end{array}\right\}\text{Minimum}=0{\cdot}016\text{ inch.}$$

Burst with 69 lbs. per square inch.

Experiment 15.

Globe s. Diameters $\begin{cases}\text{horizontal} = 6{\cdot}0 \text{ inches.}\\ \text{vertical} = 6{\cdot}3 \text{ inches.}\end{cases}$

In parts of an inch.

Thicknesses measured $\left\{\begin{array}{l}0{\cdot}024\\ 0{\cdot}020\\ 0{\cdot}0228\\ 0{\cdot}0204\\ 0{\cdot}0270\\ 0{\cdot}0262\end{array}\right\}$ Minimum = 0·020 inch.

Bursting pressure = 86 lbs. per square inch.

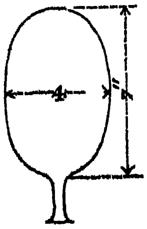
Experiment 16.

Ellipsoid t. Diameters 4·1 and 7·0 inches.

In parts of an inch.

Thicknesses measured $\left\{\begin{array}{l}0{\cdot}0180\\ 0{\cdot}0208\\ 0{\cdot}0220\\ 0{\cdot}0160\\ 0{\cdot}0184\\ 0{\cdot}0170\\ 0{\cdot}0220\end{array}\right\}$ Minimum 0·016 inch.

Fig. 9.



Bursting pressure = 80 lbs. per square inch.

Experiment 17.

Ellipsoid v. Diameters 4·0 and 7·0 inches.

In parts of an inch.

Thicknesses measured $\left\{\begin{array}{l}0{\cdot}0206\\ 0{\cdot}0208\\ 0{\cdot}0224\\ 0{\cdot}0190\\ 0{\cdot}0206\\ 0{\cdot}022\\ 0{\cdot}021\\ 0{\cdot}0254\end{array}\right\}$ Minimum = 0·019 inch.

Bursting pressure = 109 lbs. per square inch.

Summing up the preceding results, they are arranged in the following Table:—

Summary of Results.

TABLE V.—Resistance of Glass Globes to internal pressure.

Number of experiment.	Description of glass.	Diameter, in inches.	Thickness, in parts of an inch.	Bursting pressure, in pounds per square inch.
I.	Flint-glass ...	4·0 × 3·98	0·024	84
II.		4·0 × 3·98	0·025	93
III.		4	0·038	150
IV.		4·5 × 4·55	0·056	280
V.		5·1 × 5·12	0·058	184
VI.		6	0·059	152
VIII.	Green glass ...	4·95 × 5·0	0·022	90
IX.		4·95 × 5·0	0·020	85
X.		4·0 × 4·05	0·018	84
XI.		4·0 × 4·03	0·016	82
XII.	Crown-glass...	4·2 × 4·35	0·025	120
XIII.		4·05 × 4·2	0·021	126
XIV.		5·9 × 5·8	0·016	69
XV.		6·0 × 6·3	0·020	86

TABLE VI.—Resistance of Glass Cylinders and Ellipsoids to internal pressure.

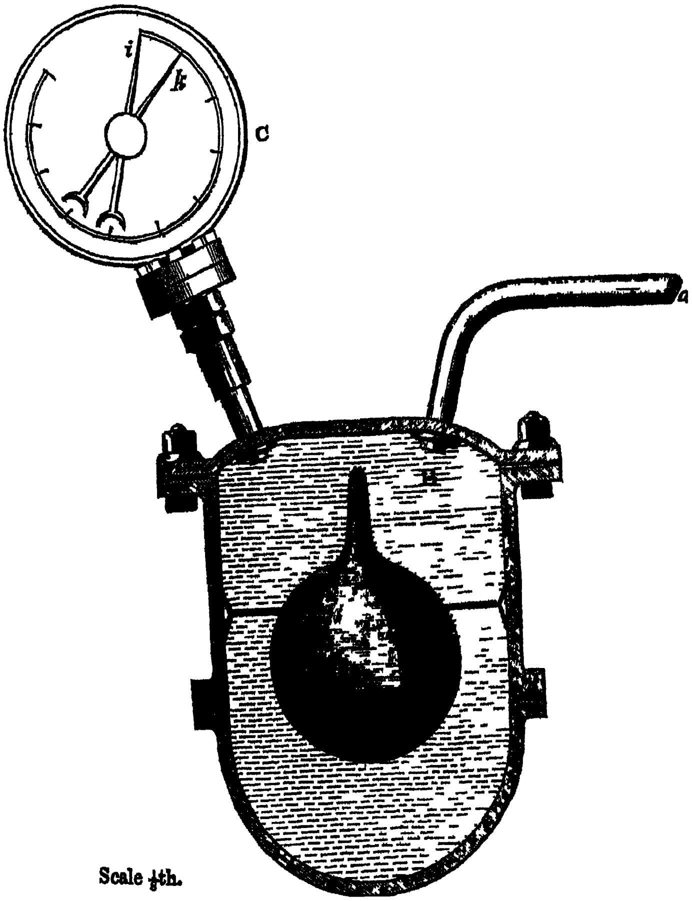
Number of experiment.	Description of glass.	Form of vessel.	Diameter, in inches.	Thickness, in parts of an inch.	Bursting pressure, in pounds per square inch.
VII.	Flint-glass	Cylinder	4·05 × 7·0	0·079	282
XVI.	Crown-glass.........	Ellipsoid	4·1 × 7·0	0·016	80
XVII.	Crown-glass.........	Ellipsoid	4·1 × 7·0	0·019	109

SECTION IV.—ON THE RESISTANCE OF GLASS GLOBES AND CYLINDERS TO AN EXTERNAL PRESSURE.

The following experiments are in continuation of, and supplementary to the researches on the collapse of wrought-iron vessels already alluded to. In this aspect they are the most important in their bearings and the most novel of any in the present memoir.

The method of conducting them did not differ in any essential detail from that pursued in the researches upon wrought-iron tubes, described in a former paper. A number of globes of varying dimensions were procured, and hermetically sealed by means of the blowpipe. In this state they were fixed in the interior of the strong wrought-iron boiler B (fig. 10) (capable of sustaining a pressure of about 2500 lbs. per square inch), in the position shown at A. The boiler or vessel B communicated by means of the pipe *a*, with a hydraulic force-pump having a plunger of three-quarters of an inch diameter, so that a uniform pressure of about 1000 lbs. per square inch could easily be obtained. In order to register the pressure, gauges of the Schaeffer construction (C) were employed, as before, affording, within small limits of error, certain and accurate indications of the increase of pressure obtained by the pump. The collapse of the glass vessel was made known by a loud report, and by the instant recession of the moveable finger of the gauge *i*; the maximum pressure obtained was marked by a second finger *k*, and also, to prevent error from any accidental cause, by the eye of the observer*.

Fig 10.



During the collapse the globes were reduced to the smallest fragments; in some cases a great part almost to powder, by the violence of the concussion. Hence in these experiments no indication could be found of the mode in which the globes had given way, nor of the direction of the primary lines of fracture.

After the globe had been ruptured, the fragments were carefully collected, and a selection having been made of the thinnest, they were measured, as before, by means of a micrometer-screw. The minimum thickness thus determined has been assumed for the thickness of the point of rupture in the calculations.

---

* In pressure-gauges of the Schaeffer construction a corrugated steel plate measures the force, by expanding under pressure. The indications are communicated by a rack and pinion to the hand of the gauge which moves over a face plate graduated by trial. In principle this gauge does not materially differ from the aneroid barometer.

Flint-glass.

Experiment 1.

Globe A. Diameters 5·05 and 4·76 inches.

$$\text{Thicknesses measured}\left\{\begin{array}{c}\text{In parts of an inch.}\\0{\cdot}0170\\0{\cdot}0192\\0{\cdot}0190\\0{\cdot}0218\\0{\cdot}0220\\0{\cdot}0146\\0{\cdot}0178\\0{\cdot}0154\end{array}\right\}\text{Minimum }=0{\cdot}014\text{ inch.}$$

Collapsing pressure=292 lbs. per square inch.

Experiment 2.

Globe B. Diameters 5·08 and 4·7 inches.

$$\text{Thicknesses measured}\left\{\begin{array}{c}\text{In parts of an inch.}\\0{\cdot}0210\\0{\cdot}0200\\0{\cdot}0180\\0{\cdot}0200\\0{\cdot}0194\\0{\cdot}0188\\0{\cdot}0192\\0{\cdot}0196\end{array}\right\}\text{Minimum }=0{\cdot}018\text{ inch.}$$

Collapsing pressure=410 lbs. per square inch.

Experiment 3.

Globe C. Diameters 4·95 and 4·72 inches.

$$\text{Thicknesses measured}\left\{\begin{array}{c}\text{In parts of an inch.}\\0{\cdot}0214\\0{\cdot}0246\\0{\cdot}0208\\0{\cdot}0220\\0{\cdot}0266\\0{\cdot}0222\\0{\cdot}0226\end{array}\right\}\text{Minimum }=0{\cdot}022\text{ inch.}$$

Collapsing pressure =470 lbs. per square inch.

Experiment 4.

Globe D. Diameter 5·6 inches.
Minimum thickness =0·020 inch.
Collapsing pressure =475 lbs. per square inch.

Experiment 5.

Globe E. Diameters 8·22 and 7·45 inches.

$$\text{Thicknesses measured}\left\{\begin{array}{c}\text{In parts of an inch}\\ 0{\cdot}0152\\ 0{\cdot}0118\\ 0{\cdot}0122\\ 0{\cdot}0100\\ 0{\cdot}0106\\ 0{\cdot}0128\\ 0{\cdot}0108\\ 0{\cdot}0108\\ 0{\cdot}0110\\ 0{\cdot}0102\end{array}\right\}\text{Minimum }=0{\cdot}010\text{ inch.}$$

Collapsing pressure =35 lbs. per square inch.

Experiment 6.

Globe F. Diameters 8·2 and 7·2 inches.

$$\text{Thicknesses measured}\left\{\begin{array}{c}\text{In parts of an inch.}\\ 0{\cdot}0124\\ 0{\cdot}0138\\ 0{\cdot}0126\\ 0{\cdot}0116\\ 0{\cdot}0120\\ 0{\cdot}0120\end{array}\right\}\text{Minimum }=0{\cdot}012\text{ inch.}$$

Collapsing pressure =42 lbs. per square inch.

Experiment 7.

Globe G. Diameters 8·2 and 7·4 inches.

$$\text{Thicknesses measured}\left\{\begin{array}{c}\text{In parts of an inch.}\\ 0{\cdot}0160\\ 0{\cdot}0144\\ 0{\cdot}0166\\ 0{\cdot}0148\\ 0{\cdot}0144\\ 0{\cdot}0158\\ 0{\cdot}0164\\ 0{\cdot}0150\end{array}\right\}\text{Minimum }=0{\cdot}015\text{ inch.}$$

Collapsing pressure =60 lbs. per square inch.

Experiment 8.

Globe H. Diameters 4·0 and 3·98 inches.
Minimum thickness = 0·024 inch.

This globe sustained unbroken a pressure of 900 lbs. per square inch.

Experiment 9.

Globe I. Diameter 4·0 inches.
Minimum thickness = 0·025 inch.

This globe sustained unbroken a pressure of 900 lbs. per square inch.

Experiment 10.

Globe K. Diameter 6·0 inches.
Minimum thickness = 0·059 inch.

This globe remained unbroken with a pressure of 1000 lbs. per square inch.

Experiment 11.

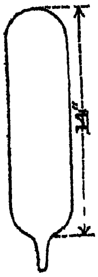
Cylinder L. Diameter 3·09 inches.
Length 14 inches.

In parts of an inch.

$$\text{Thicknesses measured}\left\{\begin{array}{l}0{\cdot}0243\\0{\cdot}0241\\0{\cdot}0235\\0{\cdot}0238\\0{\cdot}0241\\0{\cdot}0352\end{array}\right\}\text{Minimum} = 0{\cdot}024 \text{ inch.}$$

Collapsing pressure = 85 lbs. per square inch.

Fig. 11.



Experiment 12.

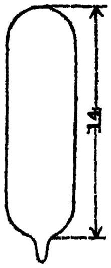
Cylinder M. Diameter 3·08 inches.
Length 14 inches.

In parts of an inch.

$$\text{Thicknesses measured}\left\{\begin{array}{l}0{\cdot}0320\\0{\cdot}0324\\0{\cdot}052\\0{\cdot}0316\\0{\cdot}0326\\0{\cdot}0322\end{array}\right\}\text{Minimum} = 0{\cdot}032 \text{ inch.}$$

Collapsing pressure = 103 lbs. per square inch.

Fig. 12.



Experiment 13.

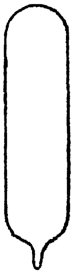
Cylinder N. Diameter 3·25 inches.
Length 14 inches.

In parts of an inch.

Thicknesses measured
0·0452
0·0436
0·0472
0·0422
0·0426
0·0436
0·0452
Minimum = 0·042 inch.

Collapsing pressure = 175 lbs. per square inch.

Fig. 13.



Experiment 14.

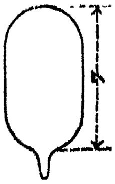
Cylinder O. Diameter 4·05 inches.
Length 7·0 inches.

In parts of an inch.

Thicknesses measured
0·0454
0·0384
0·0368
0·0344
0·0348
0·0392
0·0348
Minimum = 0·034 inch.

Collapsing pressure = 202 lbs. per square inch.

Fig. 14.



Experiment 15.

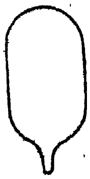
Cylinder P. Diameter 4·05 inches.
Length 7 inches.

In parts of an inch.

Thicknesses measured
0·0502
0·0464
0·0460
0·0464
0·0510
0·0498
0·0558
Minimum = 0·046 inch.

Collapsing pressure = 380 lbs. per square inch.

Fig. 15.



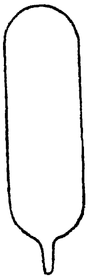
Experiment 16.

Cylinder Q. Diameter 4·06 inches.
Length 13·8 inches.

In parts of an inch.

$$\text{Thicknesses measured}\left\{\begin{array}{l}0{\cdot}0448\\0{\cdot}050\\0{\cdot}0474\\0{\cdot}0490\\0{\cdot}0476\\0{\cdot}0470\\0{\cdot}053\\0{\cdot}0434\end{array}\right\}\text{Minimum}=0{\cdot}043\text{ inch.}$$

Fig. 16.



Collapsing pressure =180 lbs. per square inch.

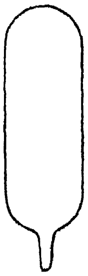
Experiment 17.

Cylinder R. Diameter 4·02 inches.
Length 13·8 inches.

In parts of an inch.

$$\text{Thicknesses measured}\left\{\begin{array}{l}0{\cdot}0678\\0{\cdot}0664\\0{\cdot}0728\\0{\cdot}0718\\0{\cdot}0660\\0{\cdot}0644\\0{\cdot}0706\\0{\cdot}0682\end{array}\right\}\text{Minimum}=0{\cdot}064\text{ inch.}$$

Fig. 17.



Collapsing pressure =297 lbs. per square inch.

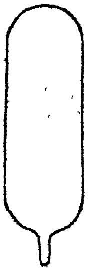
Experiment 18.

Cylinder S. Diameter 3·98 inches.
Length 14·0 inches.

In parts of an inch.

$$\text{Thicknesses measured}\left\{\begin{array}{l}0{\cdot}0792\\0{\cdot}0774\\0{\cdot}0762\\0{\cdot}0812\\0{\cdot}0828\\0{\cdot}0848\\0{\cdot}0836\\0{\cdot}0778\\0{\cdot}0766\end{array}\right\}\text{Minimum}=0{\cdot}076\text{ inch.}$$

Fig. 18.

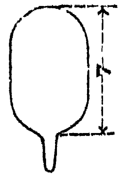


Collapsing pressure =382 lbs. per square inch.

Experiment 19.

Cylinder T. Diameter 4·05 inches.
Length 7·0 inches.

Fig. 19.



	In parts of an inch.	
	0·079	
	0·081	
	0·090	
Thicknesses measured	0·086	Minimum = 0·079 inch.
	0·086	
	0·079	
	0·086	

This cylinder remained unbroken after sustaining a pressure of 500 lbs. per square inch.

Experiment 20*.

Cylinder V. Diameter 4·2 inches.
Length 22 inches.

	In parts of an inch.	
	0·063	
	0·056	
	0·057	
Thicknesses measured	0·055	Minimum = 0·055 inch.
	0·057	
	0·055	
	0·056	

Collapsed with 120 lbs. per square inch.

Experiment 21*.

Cylinder W. Diameter 4·1 inches.
Length 21·5 inches.

	In parts of an inch.	
	0·0535	
	0·051	
	0·055	
	0·052	
Thicknesses measured	0·053	Minimum = 0·051 inch.
	0·060	
	0·057	
	0·0525	
	0·055	

Collapsed with 129 lbs. pressure per square inch.

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* The experiments marked with an asterisk were not originally included in the calculations; but the results are strictly in conformity with those previously reduced.

Experiment 22*.

Cylinder X. Diameter 4·2 and 4·1 inches.
Length 22 inches.

$$\text{Thicknesses measured}\left\{\begin{array}{l}\text{In parts of an inch.}\\ 0{\cdot}0455\\ 0{\cdot}047\\ 0{\cdot}047\\ 0{\cdot}049\\ 0{\cdot}047\\ 0{\cdot}047\\ 0{\cdot}052\\ 0{\cdot}047\\ 0{\cdot}049\\ 0{\cdot}047\\ 0{\cdot}054\\ 0{\cdot}046\end{array}\right\}\text{Minimum} = 0{\cdot}0455 \text{ inch.}$$

Collapsing pressure $=125$ lbs. per square inch.

Green Glass.

Experiment 23.

Globe Z. Diameters 5·0 and 5·02 inches.

$$\text{Thicknesses measured}\left\{\begin{array}{l}\text{In parts of an inch.}\\ 0{\cdot}015\\ 0{\cdot}013\\ 0{\cdot}0125\\ 0{\cdot}019\\ 0{\cdot}016\\ 0{\cdot}018\\ 0{\cdot}021\\ 0{\cdot}0126\\ 0{\cdot}013\end{array}\right\}\text{Minimum} = 0{\cdot}0125 \text{ inch.}$$

Collapsed with 212 lbs. per square inch.

TABLE VI.—Summary of the Results of Experiments upon the Resistance of Glass Globes to an External Pressure.

Number of experiments.	Description of glass.	Diameters, in inches.	Minimum thickness, in inches.	Collapsing pressure per square inch, in lbs.
I.	Flint-glass ...	5·05 and 4·76	0·014	292
II.		5·08 and 4·7	0·018	410
III.		4·95 and 4·72	0·022	470
IV.		5·6	0·020	475
V.		8·22 and 7·45	0·010	35
VI.		8·2 and 7·2	0·012	42
VII.		8·2 and 7·4	0·015	60
VIII.		4·0 and 3·98	0·024	(900)*
IX.		4·0	0·025	(900)*
X.		6·0	0·059	(1000)*
XXIII.	Green glass	5·0 and 5·02	0·0125	212

TABLE VII.—Summary of Results of Experiments on the Resistance of Glass Cylinders to an External Force.

Number of experiment.	Description of glass.	Diameters, in inches.	Length, in inches.	Minimum thickness, in inches.	Collapsing pressure per square inch, in lbs.
XI.	Flint-glass ...	3·09	14·0	0·024	85
XII.		3·08	14·0	0·032	103
XIII.		3·25	14·0	0·042	175
XIV.		4·05	7·0	0·034	202
XV.		4·05	7·0	0·046	380
XVI.		4·06	13·8	0·043	180
XVII.		4·02	13·8	0·064	297
XVIII.		3·98	14·0	0·076	382
XIX.		4·05	7·0	0·079	(500)†
XX.		4·20	22·0	0·055	120
XXI.		4·10	21·5	0·051	129
XXII.		4·15	22·0	0·046	125

* These globes remained unbroken.

† Remained unbroken.

SECTION V.—REDUCTION OF THE PRECEDING RESULTS.

I. *Generalization of the Results of Experiments on the Resistance of Glass Globes and Cylinders to External Pressure.*

Let us assume—

$P=$ the external pressure in pounds per square inch to produce rupture.

$D=$ the diameter of the globe or tube, as the case may be, in inches.

$k=$ the thickness of the glass in inches.

$p=$ the pressure P reduced to unity of thickness, viz. $k=\cdot01$ inch.

C, α, β, constants to be determined from the data supplied by the experiments.

Then for the globes we assume

$$P=\frac{Ck^{\alpha}}{D^{\beta}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

and for the cylinders

$$P=\frac{C'k^{\alpha}}{D^{\beta}L^{\alpha'}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

where the exponent of the thickness is the same in both formulæ.

Hence we find for globes of the same diameter and also for cylinders of the same length and diameter,—

$$\alpha=\frac{\log P_1-\log P_2}{\log k_1-\log k_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

Taking the results of Experiments I. and II., we find $\alpha=1\cdot35$; from V. and VII. we find $\alpha=1\cdot33$; from XI. and XII. we find $\alpha=1\cdot28$; from XVI. and XVII. we find $\alpha=1\cdot26$; and from XV. and XIV. we find $\alpha=2$. Hence we get for the mean value of α,

$$\alpha=\tfrac{1}{5}\{1\cdot35+1\cdot33+1\cdot28+1\cdot26+2\}=1\cdot4.$$

Again, the following formula enables us to reduce the pressure P, of the cylinders as well as of the globes, to unity of thickness,

$$\log p=\log P-\alpha\log(100\,k) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

Making these calculations, we obtain the following Tables of results:—

TABLE VIII.—Reduction of the Results of Experiments on the Resistance of Glass Globes to unity of thickness.

Number of experiment.	D. Diameter, in inches.	*k*. Thickness, in inches.	P. Collapsing pressure, in lbs. per square inch.	*p*. P reduced to unity of thickness.
I.	5·05	·014	292	178
II.	5·08	·018	410	168
III.	4·95	·022	470	156
IV.	5·60	·020	475	180
V.	8·22	·010	35	35
VI.	8·20	·012	42	32·54
VII.	8·20	·015	60	34·01

TABLE IX.—Reduction of the Results of Experiments on the Resistance of Glass Cylinders to unity of thickness.

Number of experiment.	D. Diameter, in inches.	L. Length, in inches.	k. Thickness, in inches.	P. Collapsing pressure, in lbs. per square inch.	p. P reduced to unity of thickness.
XI.	3·09	14	0·024	85	27·36
XII.	3·08	14	0·032	103	20·23
XIII.	3·25	14	0·042	175	23·47
XIV.	4·05	7	0·034	202	36·41
XV.	4·05	7	0·046	380	44·86
XVI.	4·06	13·8	0·043	180	23·36
XVII.	4·02	13·8	0·064	297	22·10
XVIII.	3·98	14	0·076	382	22·33

Let D_1, p_1 be put for the data derived from Experiment 1; D_2, p_2 for the data derived from Experiment 2, and so on; then we get from equation (1.),

$$\beta=\frac{\log p_1+\log p_2+\log p_3-(\log p_5+\log p_6+\log p_7)}{\log D_5+\log D_6+\log D_7-(\log D_1+\log D_2+\log D_3)} \quad . \quad . \quad . \quad . \quad . \qquad (5.)$$

$$\beta=\frac{\log p_1-\log p_7}{\log D_7-\log D_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6.)$$

$$\beta=\frac{\log p_2+\log p_3+\log p_4-(\log p_5+\log p_6+\log p_7)}{\log D_5+\log D_6+\log D_7-(\log D_2+\log D_3+\log D_4)} \quad . \quad . \quad . \quad . \quad . \qquad (7.)$$

From equation (5.) we find $\beta=3·43$; from equation (6.) we find $\beta=3·25$; and from equation (7.) we find $\beta=3·56$; and the mean of these values gives

$$\beta=\tfrac{1}{3}(3·43+3·25+3·56)=3·4.$$

For the value of the constant C, we find

$$\log C=\tfrac{1}{7}(\log p_1+\ldots\ldots+\log p_7)+\frac{\beta}{7}(\log D_1+\ldots\ldots+\log D_7)+2\alpha, \quad . \quad . \quad . \qquad (8.)$$

whence we find C=28,300,000.

Substituting the values of α, β, C thus obtained in the general formula (1.), we get

$$P=28{,}300{,}000\times\frac{k^{1·4}}{D^{3·4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9.)$$

which is the general formula for calculating the strength of flint-glass globes subjected to external pressure. In order to facilitate calculation, this formula may be written

$$\log P=4·6518+1·4\log(100\,k)-3·4\log D. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10.)$$

Calculating the value of P by this formula from the data of Experiment XXIII., viz. D=5 and k=·0125, we find

$$\log P=4·6518+1·4\log 1·25-3·4\log 5=\log 258,$$

that is P=258 lbs. Now this would be the crushing pressure supposing the globe to be flint-glass; but the crushing pressure given by the experiment is 212 lbs.; hence it appears that the resistance of green glass to external pressure differs very little from that of flint-glass. The following Table will show how nearly formula (9.) represents the results of the experiments on glass globes.

TABLE X.—Results of Experiments on the Resistance of Glass Globes to External Pressure.

Number of experiment.	D.	*k*.	P by experiment.	P by formula (9).	Proportional error by formula.
I.	5·05	·014	292	292	0
II.	5·08	·018	410	408	$-\frac{1}{100}$
III.	4·95	·022	470	580	$+\frac{1}{4}$
IV.	5·6	·020	475	340	$-\frac{1}{4}$
V.	8·22	·010	35	35	0
VI.	8·2	·012	42	44·8	$+\frac{1}{20}$
VII.	8·2	·015	60	61·3	$+\frac{1}{50}$
VIII.	4·0	·024	900*	1370	...
IX.	4·0	·025	900*	1450	...
X.	6·0	·059	1000*	1218	...
XXIII.	5·02	·0125	212†	258	$+\frac{1}{5}$

The lengths of the cylinders of Experiments XI., XII., and XIII. are the same, and their diameters are nearly equal to one another. The same observation applies to the cylinders of Experiments XIV. and XV., and also to Experiments XVI. and XVII. In order, therefore, to reduce the pressures p to uniformity of diameter, we may assume, for such small differences, that D varies as $\frac{1}{\mathrm{P}}$. These reductions being made, we may obtain the following Table:—

TABLE XI.—Reduction of the Results of Experiments on Glass Cylinders to uniformity of Diameter, &c.

Number of experiment.	D. Diameter, in inches.	L. Length, in inches.	p. P reduced to unity of thickness.	p D L.
11, 12, 13.	3	14	24·81	1042
14, 15.	4·05	7	40·63	1151
16, 17.	4·05	13·8	22·67	1260
18.	3·98	14	22·33	1240

$$\text{Mean value of } p\ \mathrm{L\ D} = \frac{4)4693}{1173}$$

Here the continued product of the pressure, diameter, and length is shown to be very nearly a constant quantity, the thickness of the glass being the same, that is for $k=\cdot01$. Hence we have

$$\mathrm{C}=\frac{p\mathrm{DL}}{\cdot01^{\alpha}}. \qquad (11.)$$

Now the mean value of p D L is 1173, as shown in Table XI., and $\alpha=1\cdot4$, as determined by equation (3.); hence we find

---

* Remained unbroken.

† Green glass.

$$C=\frac{1173}{\cdot01^{1\cdot4}}=740{,}000.$$

Substituting these values of the constants in equation (2.), we get

$$P=740{,}000\,\frac{k^{1\cdot4}}{DL}, \qquad \text{(12.)}$$

which is the general formula for calculating the strength of glass cylinders subjected to external pressure, within the limits indicated by the experiments, that is, provided their length is not less than twice their diameter, and not greater probably than six times their diameter. This law of strength is precisely similar to that found for sheet-iron tubes.

For convenience of calculation, this formula may be written

$$\log P=3\cdot06923+1\cdot4\log(100\,k)-\log(DL) \qquad \text{(13.)}$$

The following Table will show how nearly formula (12.) represents the results of the experiments on glass cylinders:—

Table XII.—Results of Experiments on the Resistance of Glass Cylinders to External Pressure.

Number of experiment.	D.	L.	k.	P by experiment.	P by formula.	Proportional error by formula.
XI.	3·09	14	·024	85	86	$+\frac{1}{80}$
XII.	3·08	14	·032	103	138	$+\frac{1}{3}$
XIII.	3·25	14	·042	175	192	$+\frac{1}{9}$
XIV.	4·05	7	·034	202	227	$+\frac{1}{10}$
XV.	4·05	7	·046	380	351	$-\frac{1}{13}$
XVI.	4·06	13·8	·043	180	161	$-\frac{1}{9}$
XVII.	4·02	13·8	·064	297	284	$-\frac{1}{20}$
XVIII.	3·98	14	·076	382	361	$-\frac{1}{18}$
XIX.	4·05	7	·079	500	747	Unbroken.
XX.	4·2	22	·055	120	138	$+\frac{1}{7}$
XXI.	4·1	21·5	·051	129	130	$+\frac{1}{129}$
XXII.	4·2	22	·0455	125	107	$-\frac{1}{7}$

Comparative Strength of Glass and Sheet-iron Cylinders subjected to an External Pressure tending to produce Collapse.

The formula of strength for sheet-iron cylinders, after reducing L to inches, is

$$P'=806{,}300\times12\times\frac{k^{2\cdot19}}{LD}.$$

Now for cylinders of the same diameter, length, and thickness, we find, by dividing equation (12.) by the above,

$$\frac{P}{P'}=\frac{\cdot0764}{k^{\cdot79}}. \qquad \text{(14.)}$$

When $k = \cdot 043$, as in most of the experiments on iron, then $\frac{\mathrm{P}}{\mathrm{P}'} = \frac{11}{12}$; that is, in this case, the strengths of the two cylinders will be nearly equal to one another.

II. *Generalization of the Results of the Experiments on the Resistance of Glass Globes, Cylinders, and Ellipsoids to Internal Pressure.*

Let D = the diameter of the globe or cylinder, as the case may be.
k = the thickness of the material in inches.
a = the longitudinal sectional area of the material in square inches; that is, in the direction of the line of rupture, or line of minimum strength.
A = the longitudinal section in square inches.
P = the bursting pressure in lbs. per square inch.
T = the tenacity of the material in lbs. per square inch.

Then we find

$$\mathrm{P} = \frac{a\mathrm{T}}{\mathrm{A}}; \quad \ldots \ldots \ldots \ldots \quad (15.)$$

$$\therefore \quad \mathrm{T} = \frac{\mathrm{PA}}{a}; \quad \ldots \ldots \ldots \ldots \quad (16.)$$

that is, $\frac{\mathrm{PA}}{a}$ is a constant for vessels of the same material. This theoretical deduction is fully confirmed by the results of these experiments, as arranged in the following Table:—

Table XIII.—Resistance of Glass Globes, Cylinders, and Ellipsoids to an Internal Pressure.

No. of exper.	Description of glass.	D.	k.	P.	Value of $\mathrm{P}\frac{\mathrm{A}}{a}$.	Mean value of $\mathrm{P}\frac{\mathrm{A}}{a}$.
1.	Flint-glass ……	4 and 3·98	·024	84	3500	4200
2.		4 and 3·98	·025	93	3710	
3.		4	·038	150	3950	
4.		4·5 and 4·55	·056	280	5650	
5.		5·1 and 5·12	·058	184	4050	
6.		6	·059	152	3870	
7.		4·05 and 7	·079	282	4660	
8.	Green glass ……	4·95 and 5	·022	90	4040	4800
9.		4·95 and 5	·020	85	5280	
10.		4 and 4·05	·018	84	4690	
11.		4 and 4·03	·016	82	5150	
12.	Crown-glass ……	4·2 and 4·35	·025	120	5120	6000
13.		4·2 and 4·05	·021	126	6190	
14.		5·9 and 5·8	·016	69	6300	
15.		6 and 6·3	·020	86	5290	
16.		4·1 and 7	·016	80	6360	
17.		4 and 7	·019	109	6900	

Hence we have the tenacity of glass,—

lbs. per sq. in.

$T=4200$, for flint-glass,

$T=4800$, for green glass,

and $T=6000$, for crown-glass.

The general equation (15.) then becomes—

lbs. per sq. in.

$$P=4200\times\frac{a}{A}, \text{ for flint-glass,}$$

$$P=4800\times\frac{a}{A}, \text{ for green glass,}$$

$$P=6000\times\frac{a}{A}, \text{ for crown-glass.}$$

For globes of uniform diameter and thickness these formulæ become—

lbs. per sq. in.

$$P=16{,}800\times\frac{k}{D}, \text{ for flint-glass,}$$

$$P=19{,}200\times\frac{k}{D}, \text{ for green glass,}$$

$$P=24{,}000\times\frac{k}{D}, \text{ for crown-glass.}$$

III. *Generalization of the Results of Experiments on the Tensile and Compressive Resistances of Glass.*

Mean tenacity (T_1) of glass in the form of bars

$$=\tfrac{1}{4}(2286+2540+2890+2540)=2560 \text{ lbs. per square inch;}$$

Mean tenacity (T'_1) of glass in the form of thin plates

$$=\tfrac{1}{3}(4200+4800+6000)=5000 \text{ lbs. per square inch;}$$

$$\therefore \quad \frac{T'_1}{T_1}=\frac{5000}{2560}=2 \text{ nearly,}$$

that is, the tenacity of glass in the form of thin plates is about twice that of glass in the form of bars.

Mean resistance (T_2) of glass to compression

$$=\tfrac{1}{3}(27582+31876+31003)=30{,}150 \text{ lbs. per square inch;}$$

$$\therefore \quad \frac{T_2}{T_1}=\frac{30150}{2560}=11{\cdot}8 \text{ nearly,}$$

that is, the ultimate resistance of glass to a crushing force is about twelve times its resistance to extension.

IV. *Resistance of Rectangular Glass Bars to a Transverse Strain.*

Let $l =$ the length of the bar supported at the ends and loaded in the middle.
$\mathrm{W} =$ breaking weight in lbs.
$\mathrm{K} =$ area of the whole transverse section.
$\mathrm{D} =$ the whole depth of the section.
$d, d_1 =$ the respective distances of the top and bottom edges from the neutral axis.
$\mathrm{T}_1 =$ the tensile resistance of the material in lbs. per square inch.
$\mathrm{T}_2 =$ the compressive resistance of the material in lbs. per square inch.

Then we have "TATE's strength of material" equations (27.) and (6.)—

$$\mathrm{W}=\tfrac{4}{3}\cdot\frac{\mathrm{T}_1 d_1 \mathrm{K}}{l}, \text{ and } \frac{\mathrm{T}_1}{\mathrm{T}_2}=\frac{d}{d_1};$$

hence we get

$$\mathrm{W}=\tfrac{4}{3}\cdot\frac{\mathrm{T}_1.\mathrm{T}_2}{\mathrm{T}_1+\mathrm{T}_2}\times\frac{\mathrm{K}.\mathrm{D}}{l}=\mathrm{C}\frac{\mathrm{K}.\mathrm{D}}{l}, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (17.)$$

where the constant

$$\mathrm{C}=\tfrac{4}{3}\cdot\frac{\mathrm{T}_1.\mathrm{T}_2}{\mathrm{T}_1+\mathrm{T}_2}=\tfrac{4}{3}\times\frac{2560\times 30150}{32710}=3140 \text{ nearly.}$$

Substituting this value of the constant, equation (16.) becomes

$$\mathrm{W}=3140\cdot\frac{\mathrm{K}.\mathrm{D}}{l}, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (18.)$$

which expresses the transverse strength of a rectangular bar of glass supported at the ends and loaded in the middle.

XIII. *On the Atomic Weight of Graphite. By* B. C. Brodie, *F.R.S., Professor of Chemistry in the University of Oxford, and President of the Chemical Society.*

Received May 12,—Read May 12, 1859.

The term Graphite has been indiscriminately applied to many varieties of native carbon of very different properties. The graphite of New Brunswick differs but little in appearance from anthracite coal. The graphite of Greenland is not very dissimilar, but possesses rather more metallic lustre. However, among these varieties of carbon, two may be especially distinguished,—by a superior degree of metallic lustre, by their structure, and other well-defined properties. In the following paper, the term Graphite is limited to these two varieties, which may be further distinguished as "lamellar" and "amorphous."

The lamellar graphite is found in great abundance in Ceylon, whence large quantities are annually imported into this country. It appears in commerce in masses, sometimes of the weight of many pounds, of a brilliant metallic lustre, and possessing a distinct fibrous structure. It is very difficult by mechanical processes to bring this graphite to a fine state of division; however, by a prolonged grinding in water, it may be reduced to minute flat plates. This graphite is associated with quartz. A deposit of the same variety of graphite has recently been discovered at Travancore. The graphite from Travancore has no fibrous structure, but is in the form of slightly coherent, minute plates. Specimens of a similar graphite have also been given to me, from Moreton Bay in Australia, in a matrix of quartz, and from Ticonderoga, in the State of New York, associated with olivine and sphene. When cast iron is dissolved in acid, a residue is left of about 4 per cent. of carbon in the form of graphite. This graphite also consists of minute brilliant plates, and is perfectly similar in its appearance and properties to the lamellar variety of native carbon. Amorphous graphite is found in Borrowdale in Cumberland, and is also largely imported into this country from Germany, probably from Griesbach near Passau, but I am unable to speak with certainty as to the locality whence it comes. It appears as a powder of a silvery grey colour, soft to the touch, and which rubbed on paper gives a brilliant metallic streak. This graphite is much softer than the other variety, and therefore better adapted for the manufacture of pencils.

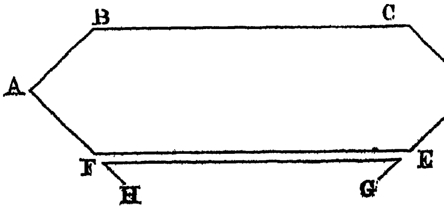
The graphite used in my experiments was in all cases carefully purified by boiling with acids, and by fusion with hydrate of potash in a silver crucible; unless otherwise mentioned, it is to be understood to be the variety from Ceylon. The graphite thus prepared leaves an almost inappreciable residue, and gave to analysis 99·96 of carbon. Its specific gravity was ascertained in two determinations as 2·25 and 2·26.

The following investigation was undertaken with the view of ascertaining the weight with which carbon in the form of graphite enters into combination. With this end I examined its reactions.

The first experiment in which I succeeded in eliciting a difference in the chemical reactions of the different forms of carbon, was, in the case of the action upon them, of a mixture of concentrated nitric and sulphuric acids. When finely-divided carbon, in the form of lampblack, or charcoal from the decomposition of sugar, is heated with a mixture of 1 of nitric and 4 of sulphuric acids, the carbon is rapidly oxidized, and a black substance is formed, soluble in the concentrated acid, but precipitated on the addition of water. This substance is insoluble in acids and saline solutions, but is soluble in pure water and in alkalies. It is accompanied with other products which render its purification difficult. When the graphite of Ceylon is treated in a similar manner, the result is very different: the graphite becomes of a beautiful purple colour, and falls to pieces in the fluid. The substance, after the acid has been washed from it by water, has much the appearance of the graphite itself, but is darker in colour. It was found on analysis to contain the elements of sulphuric acid combined with oxygen, with hydrogen, and with a large amount of carbon. My efforts to procure this substance of a constant composition have been unavailing; it is insoluble in all reagents; it may be boiled with a strong solution of potash, without separation of sulphuric acid, and with slight, if any, alteration of weight; when heated it undergoes a remarkable change; gases are given off in the interior of the substance, which swells up in a most singular manner and is reduced to the minutest state of division. The residue consists of carbon, which has the appearance and the structure of the lamellar graphite. A similar oxidation takes place when the nitric acid in the above experiment is replaced by other oxidizing agents, such as bichromate of potash and chlorate of potash. These experiments established one point of importance, the existence of a peculiar compound of carbon in the form of graphite. The discovery of this substance led me to turn my attention to the oxidation of graphite. I found that graphite, when heated with a mixture of nitric acid and chlorate of potash, increased in weight, and that the substance formed was, on the application of heat, disintegrated with evolution of gas. The disintegrated substance differed but little in appearance from the original graphite. The analysis of the body resulting from different preparations, and with a difference in the time of oxidation, gave no constant result. These difficulties led me to further experiments. I found that when the substance formed by the treatment of the graphite with the oxidizing mixture was washed free from the salts produced in the reaction, and dried at 100° and again oxidized, it gradually underwent a change in appearance, until, after the fourth or fifth repetition of the process, the whole of the graphite was converted into a substance of a light yellow colour, consisting of minute transparent and brilliant plates. Analysis showed that this change was attended with a gradual alteration of the constitution of the substance, but that, finally, a time arrived when treatments with the oxidizing mixture produced no further change. It is remarkable that this result cannot be produced by one prolonged treatment; before the oxidation can proceed, the original conditions must be restored.

The details of this process are as follows:—A portion of graphite is intimately mixed with three times its weight of chlorate of potash, and the mixture placed in a retort. A sufficient quantity of the strongest fuming nitric acid is added, to render the whole fluid. The retort is placed in a water-bath, and kept for three or four days at a temperature of 60° C. until yellow vapours cease to be evolved. The substance is then thrown into a large quantity of water, and washed by decantation nearly free from acid and salts. It is dried in a water-bath, and the oxidizing operation repeated with the same proportion of nitric acid and of chlorate of potash until no further change is observed: this is usually after the fourth time of oxidation. The substance is ultimately dried, first *in vacuo*, and then at 100° C. A modification of the process which may be advantageously adopted, consists in placing the substance with the oxidizing mixture in flasks exposed to sunlight. Under these circumstances the change takes place more rapidly, and without the application of heat.

These crystals, when examined with the microscope, are perfectly transparent, and exhibit beautiful colours by the agency of polarized light. Professor MILLER of Cambridge, who was good enough to examine them, has communicated to me the following observations:—"The crystals, though not absolutely too small to be measured, are too thin and too imperfect to admit of measurement with the reflective goniometer. I have examined them under a microscope, for the purpose, if possible, of making out at least the system of crystallization to which they belong. Their system appears to be either the prismatic or the oblique,—most likely the former*. Sometimes the crystals appear broken, as in the annexed figure. The straightness of the fractured edges (in the direction EF) evidently indicated the existence of a cleavage in that direction. The crystals are so extremely thin in a direction perpendicular to the paper on which the above figure is traced, that it is impossible to obtain any reflexion, except from the faces parallel to the plane of the paper."



The crystals, on the application of heat, are decomposed with ignition. Gases are evolved, and a black residue is left of a substance resembling in appearance finely divided carbon.

I have not discovered any reagent by which these crystals may be dissolved, and they admit of no process of purification. The explosion which the crystals undergo on the application of heat render special methods of analysis necessary, and I have taken, with regard to the analysis, every precaution which suggested itself. Neither chlorine nor nitrogen could be detected.

The substance was placed in a platinum boat in a porcelain tube and burnt in a

[*] Measurements of the crystals, by means of the microscopic goniometer, which have subsequently been made by Mr. A. H. CHURCH, of Lincoln College, have determined with certainty that the crystals belong to the one or other of these systems.

current of oxygen, the gases evolved being passed over heated oxide of copper. At the extremity of the tube, where the oxygen entered, was placed a stopper of asbestos, and a second stopper between the substance and the oxide of copper. The ordinary potash-bulbs were replaced by two tubes filled with pumice moistened with concentrated solution of potash.

In order to ascertain in the most rigorous manner the point at which the constitution of the compound became fixed, it was analysed after each successive oxidation. It will be seen that the analyses became constant upon the fourth treatment; after which, the most prolonged action of the oxidizing mixture produced no further change.

The following are the results per cent. of the analyses of the same preparation of graphite, treated in the first instance once, in the second, twice, with the oxidizing mixture:—

	I.	II.
Carbon . . .	69·67	67·79
Hydrogen . . .	1·48	1·84
Oxygen . . .	28·85	30·37
	100·00	100·00

The following are the results of analysis after the constitution of the body may be regarded as fixed. Analyses 1 and 2 are of a substance prepared by four treatments; analyses 3 and 4, of a substance prepared by five treatments; analysis 5, of a substance prepared by six treatments; analyses 6 and 7, of a substance prepared by seven treatments; analysis 8 is of a substance dried *in vacuo*, the previous preparations having been dried at 100°; analysis 9 is a preparation from the amorphous graphite of Cumberland, the previous preparations having been from the graphite of Ceylon. I have also prepared the same compound from the graphite of cast iron.

	Substance taken.	Carbonic acid formed.	Water formed.
1.	0·23375	0·52025	0·02925
2.	0·233	0·51875	0·03775
3.	0·24475	0·545	0·04175
4.	0·27275	0·609	0·0465
5.	0·209	0·46	0·036
6.	0·25275	0·5635	0·042
7.	0·23825	0·5325	0·04025
8.	0·2555	0·56875	0·0435
9.	0·216	0·48	0·0355

These analyses correspond to the following per-centage composition:—

	1.	2.	3.	4.	5.	6.	7.	8.	9.
Carbon	60·70	60·74	60·73	60·88	60·47	60·80	60·94	60·71	60·69
Hydrogen	1·37	1·80	1·89	1·88	1·91	1·80	1·87	1·89	1·75
Oxygen	37·93	37·46	37·38	37·24	37·62	37·40	37·19	37·40	37·56
	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

The mean of these analyses (omitting the hydrogen of the first analysis, which is evidently faulty) is—

Carbon . . .	60·74
Hydrogen . .	1·85
Oxygen . . .	37·41
	100·00

A correction has yet to be made in these numbers. The graphite from which the substance is prepared leaves on combustion a slight ash, and during the prolonged treatment of the substance a further portion of incombustible matter is introduced, probably from the glass of the vessels in which the operations were conducted, which, as I have ascertained, may be estimated at 0·5 per cent. When this correction is made, we have for the per-centage constitution of the substance—

Carbon . . .	61·04
Hydrogen . .	1·85
Oxygen . . .	37·11
	100·00

The formula $C_{11}H_4O_5$ corresponds to this result. It requires—

C_{11} . . .	132	61·11
H_4 . . .	4	1·85
O_5 . . .	80	37·04
	216	100·00

This substance has the following properties. It is insoluble in water containing acids or salts, but is very slightly soluble in pure water. The crystals, placed upon litmus paper, have a feeble acid reaction. It combines with alkalies. When agitated with dilute ammonia, it is converted into a transparent jelly, but is not dissolved. On the addition of acids it is separated unaltered from this combination in the form of a gelatinous mass resembling silica, which, dried under the air-pump, appears as a slightly yellow and spongy body of the same weight as the substance originally treated with ammonia. The crystals treated with deoxidizing agents are readily decomposed. When a solution of sulphide of ammonium or of potassium is poured upon the dry substance, a crackling sound is heard, and a body is ultimately formed possessing the metallic lustre and general appearance of graphite itself. Changes similar in appearance take place on boiling the substance with an acid solution of protochloride of copper and of protochloride of tin. The substances formed in these processes admit of no process of purification, and I have not been able to procure them in a state of purity.

This body belongs very distinctly to the class of acids, but from the insolubility of its salts and the facility with which they are decomposed, I have not been able to procure them pure. The following determinations, however, indicate it to be bibasic. A portion of the moist substance shaken up with baryta-water, washed out, and dried at 100°, gave a compound containing 21·19 per cent. of barium. The same substance suspended in water, and decomposed by a current of carbonic acid, gave a substance which, dried at 100°, contained 13·30 per cent. of barium. The formula $C_{11}H_3BaO_5$ requires 24·13 per cent. of barium. The formula $C_{22}H_7BaO_{10}$ requires 13·73 per cent. of barium. These substances are excessively hygroscopic, and when heated, explode with yet greater violence than the original body. The compound itself may be termed graphic acid.

The residue which is left upon the decomposition by heat of graphic acid, although resembling carbon in appearance, is not that element, but contains a considerable amount of oxygen. I made many experiments with a view of procuring this substance in a pure state,—both by decomposing the compound, alone, in an atmosphere of nitrogen, and by decomposing it mixed with a large quantity of chloride of sodium, with the view of moderating the violence of the decomposition, and causing it to take place at the lowest possible temperature. My endeavours were unsuccessful, until it occurred to me to effect the decomposition in a fluid medium, by which the particles of the substance would be separated from one another and elevation of temperature precluded. The fluid which I selected for the experiment was the mixture of hydrocarbons of high boiling-points from the Rangoon naphtha. It was purified by rectification over sodium, and boiled at about 270° C. When graphic acid is heated in this fluid, a considerable quantity of water is formed, which distils over between 100° and 200°, and which is accompanied at first by a slight, and ultimately by a brisk evolution of gas. This gas was in one experiment collected and examined: it consisted solely of carbonic acid. But water and carbonic acid are not the exclusive products of this reaction. The hydrocarbon becomes of a deep red colour, and when filtered and distilled, leaves a black carbonaceous residue. I have found no means of separating the body formed from the naphtha in which it is dissolved.

In the following experiments the substance was heated in the naphtha for three or four hours, at about 250° C., until all perceptible evolution of gas had ceased. The graphic acid employed was weighed, and also the residue of its decomposition. The residue was collected on a weighed filter, washed out with ether and with alcohol, and dried at 100°.

Expt. I. 1·024 grm. of graphic acid gave 0·684 grm. of residue.
Expt. II. 1·0854 grm. of graphic acid gave 0·7248 grm. of residue.
Expt. III. 2·1805 grms. of graphic acid gave 1·4607 grm. of residue.

This corresponds to a loss on 100 parts of the substance taken—in Experiment I. of 66·78, in Experiment II. of 66·77, in Experiment III. of 66·98 parts.

These substances, burned in a current of oxygen, gave the following results. In these and all following analyses the residual ash was weighed, and its amount deducted from the substance taken.

	Substance taken.	Carbonic acid formed.	Water formed.
1. Expt. I.	0·311	0·9147	0·0148
2. Expt. II.	0·4237	1·2372	0·022
3. Expt. III.	0·3573	1·0404	0·0178
4. The same . .	0·3391	0·9857	0·0185

These results correspond to the following per-centage composition:—

	1.	2.	3.	4.
Carbon	80·19	79·64	79·40	79·27
Hydrogen	0·52	0·57	0·55	0·60
Oxygen	19·29	19·79	20·05	20·13
	100·00	100·00	100·00	100·00

When this substance was carefully examined under the microscope, traces of transparent crystals could still be perceived, similar in appearance to the original substance. To preclude the possibility of an error from the imperfect decomposition of the substance, the following experiments were made, in which the substance was submitted in the naphtha to a prolonged action of heat, for fourteen hours.

Expt. IV. 3·9185 grms. of substance gave 2·577 grms. of residue.
Expt. V. 17·175 grms. of substance gave 11·2965 grms. of residue.

In Experiment IV. the temperature was raised to 240° C., in Experiment V. to 220° C. These results correspond to a loss on 100 parts of the substance—in Experiment IV. of 65·76, in Experiment V. of 65·77 parts.

The analyses of these substances gave the following result:—

	Substance taken.	Carbonic acid formed.	Water formed.
1. Expt. IV. . . .	0·377	1·1126	0·0210
2. The same . . .	0·4766	1·4068	0·0255
3. Expt. V. . . .	0·3835	1·1327	0·022

These analyses give the following per-centage composition of the substance:—

	1.	2.	3.
Carbon	80·48	80·50	80·56
Hydrogen	0·62	0·59	0·63
Oxygen	18·90	18·91	18·81
	100·00	100·00	100·00

In this substance, placed under the microscope, no traces of the transparent plates of the original body could any longer be perceived. The result, therefore, of heating the substance for the ten additional hours had been to increase the per-centage of carbon in the residue from about 0·5 to 1 per cent. It is evident that the decomposition is arrested at about this point; and if we take the mean, as may reasonably be done, of three analyses of the first and the three of the second series of experiments, as representing the constitution of the body, we have the following as its per-centage composition:—

Carbon	80·13
Hydrogen	0·58
Oxygen	19·29
	100·00

This corresponds to the formula $C_{22}H_2O_4$, which has the following composition:—

C_{22} . .	264	80·00
H_2 . .	2	0·60
O_4 . .	64	19·40
	330	100·00

There is every reason to believe that, although the decomposition is difficult to regulate, this same body is formed by the simple application of heat to the substance without the intervention of the naphtha. The result of one experiment, in which the graphic acid had been carefully heated in an air-bath to 280° until it ceased to lose weight, gave to analysis—carbon 80·36, hydrogen 0·71. In other experiments numbers approximating to these were obtained.

We have not the data to determine with certainty the precise mode of the formation of this body. The simplest hypothesis which coincides with the ascertained facts, is that 7 equivalents of the graphic acid give by their decomposition 6 of the new body. We have $7(C_{11}H_4O_5)=1512$ and $3(C_{22}H_2O_4)=990$ and $1512:990::100:65\cdot48$, 65·48 being the amount of residue found.

On this view it would be possible to form the new substance by the elimination of water, carbonic acid, and carbonic oxide from the original body, according to the equation

$$7(C_{11}H_4O_5)=3(C_{22}H_2O_4)+6H_2O+6CO_2+5CO.$$

The slight excess of carbon in the last series of analyses indicates the presence of some compound containing a higher per-centage of carbon. It will be rendered evident, by the following experiments, that we have only to heat this substance under slightly different circumstances to determine a further decomposition. Portions of the substances last analysed were placed in a platinum boat in a glass tube, through which was passed a current of nitrogen, and which was heated in an air-bath to a temperature of 250°. The substance was weighed before and after the experiment.

Expt. 1. ·9087 grm. of the substance resulting from Expt. IV. gave a residue of 0·883 grm.

Expt. 2. 0·9209 grm. of the substance resulting from Expt. V. gave a residue of 0·8968 grm.

This corresponds to a residue on 100 parts of the substance taken—in Expt. 1 of 97·16, in Expt. 2. of 97·38 parts.

Water is given off from the substance during the process, which was collected in a sulphuric acid tube and estimated. It corresponded, in the two experiments, to 2·26 and 2·30 per cent. The quantity formed was, however, too small for correct estimation, and these numbers are undoubtedly too high. Traces of probably carbonic oxide, amounting to something under 1 per cent., are also given off. The action is perfectly definite. The experiment was continued until the substance ceased to lose weight; in the last two hours the loss in weight was not above 1 milligramme.

The following are the results of the analysis of the bodies:—

	Substance taken.	Carbonic acid formed.	Water formed.
1. Expt. I.	0·367	1·1051	0·0163
2. The same . . .	0·3783	1·1366	0·0139
3. Expt. II. . . .	0·3979	1·1938	0·0158
4. The same . . .	0·3927	1·1724	0·0155

These numbers give the following per-centage composition:—

	1.	2.	3.	4.
Carbon . . .	82·12	81·95	81·80	81·41
Hydrogen . .	0·49	0·41	0·44	0·43
Oxygen . . .	17·39	17·64	17·76	18·16
	100·00	100·00	100·00	100·00

The substance derived from the preceding body, by the elimination of 1 atom of water from 3 atoms of the substance, would have the formula $C_{66}H_4O_{11}$. $3(C_{22}H_2O_4) = C_{66}H_4O_4 + H_2O$.

The calculated composition of this body is as follows:—

C_{66} . .	792	81·48
H_4 . .	4	0·41
O_{11} . .	176	18·11
	972	100·00

Every 100 parts of the substance taken would leave a residue of 98·18 parts, and the water formed would amount to 1·80 part. The amount of carbonic oxide formed in the preceding experiments is so small, that it can hardly be regarded as an integral part of the decomposition; and if this amount be added to the residue, we arrive at precisely the theoretical numbers.

By a greater elevation of temperature the substance undergoes further change, with the loss of carbonic acid and carbonic oxide. But it may be exposed to a red heat for several hours in a current of nitrogen and only undergo a very partial decomposition, the residual substance containing a considerable portion both of hydrogen and oxygen.

Buff and **Wöhler**, in their researches "On the Graphitoidal Form of Silicon*," discovered a remarkable series of compounds derived from it. When hydrochloric acid gas is led over this form of silicon at a low red heat, a volatile liquid is formed containing silicon, chlorine, and hydrogen, to which they assign the formula $Si_2H_2Cl_3$. With hydriodic acid gas a similar reaction takes place, with the formation of the corresponding compound $Si_2H_2I_3$. In water, these substances decompose with the production of a compound of silicon, hydrogen, and oxygen, of the formula $Si_4H_4O_5$. The analyses are unfortunately neither so concordant nor so exact as might be desired; but the formulæ are derived from the consideration of a system of decompositions, which hardly leaves room for any other hypothesis. The general properties of the substance $Si_4 H_4 O_5$, correspond very closely with those of the graphite compound as separated from its combinations by an acid. It is described as a white and voluminous body, which floats upon water, in which it is very slightly soluble. Heated to a temperature of above 300°, it is decomposed with ignition and evolution of hydrogen. The ratio of the number of equivalents of hydrogen to those of oxygen is as 4 to 5. It is formed exclusively from graphitoidal silicon, as the graphite compound from graphitoidal carbon. From these considerations we may infer that the graphite compound is the same term in the system of carbon as the silicon compound in the system of silicon. When we proceed to state this analogy in the formula of the substance, we are led to very remarkable conclusions.

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* Liebig's 'Annalen,' vol. civ. p. 94.

The total weight of graphite, which in this compound is combined with the 4 atoms of hydrogen and the 5 of oxygen, is 132. If we assume that this weight is, like the corresponding weight, 84 of silicon, to be divided into 4 parts, we arrive at the number 33 as the atomic weight of graphite. Representing this weight by the letters Gr, the formulæ of the substances $C_{11} H_4 O_5$ and $C_{22} H_2 O_4$, and $C_{66} H_4 O_{11}$, become

$$Gr_4 \; H_4 O_5$$
$$Gr_8 \; H_2 O_4$$
$$Gr_{24} H_4 O_{11}.$$

We are acquainted with a property of graphite by which this theory may be tested.

According to the law of DULONG and PETIT, which has been confirmed by the exact researches of REGNAULT, the specific heats of the elemental bodies vary inversely with their atomic weights. With the atomic weights of the elemental bodies as generally received, the elements are divided into two classes, the one in which the product of the specific heat into the atomic weight is approximately 3·3, the other in which this product is approximately 6·6. This law expresses the only common physical property by which these weights are characterized. To this law, however, there is one remarkable exception, namely, the specific heat of carbon in its various forms. The following are the specific heats of carbon as determined by REGNAULT:—

	Specific heats.
Diamond	0·14687
Native graphite	0·20187
Graphite from the blast furnace	0·19702
Carbon from gas retorts	0·2036
Carbon from anthracite	0·201
Wood charcoal	0·2415
Animal charcoal	0·26085

In no single case, whether the atomic weight of carbon be assumed as 6 or as 12, does carbon conform to the law. The product of the specific heat of graphite into the atomic weight is with the weight 6, 1·2, with the weight 12, 2·4. If, however, we assume the atomic weight of graphite as 33, we have for the product of the specific heat into the atomic weight the number 6·6, which is according to the law; this product being the same as the product of the specific heats into the atomic weights of the elements phosphorus, antimony, arsenic, bismuth, and iodine.

This form of carbon should be characterized by a name marking it as a distinct element. I propose to term it Graphon.

In the formulæ assigned by BUFF and WÖHLER to the compounds of silicon, the atomic weight has been assumed as 21. If the atomic weight of silicon be assumed as 28, these relations no longer appear; the atomic weight of graphite must be assumed as 44, and the product of the specific heat into the atomic weight would be 8·8. We are thus enabled to decide upon the atomic weight of the graphitoidal silicon. It is greatly to be regretted that those chemists who have had a sufficient quantity of this substance in their possession have omitted to determine its specific heat. There is much reason to believe that in its other forms silicon, like carbon, has a distinct atomic weight. The vapour-density of the chloride of silicon leads us to the weight 28; and it is worthy of remark, that the atomic weight 44, which in the system of carbon corresponds to the weight 28 of silicon, would, assumed as the atomic weight of diamond, render its specific heat conformable to the law, since $44 \times \cdot 147 = 6\cdot 46$. The relation is also singular which exists between the atomic weight of graphon and the atomic weights of the elements boron, silicon, and zircon, it being precisely the kind of numerical relation which exists between the weights of analogous elements.

Boron	11
Silicon	21
Graphon	33
Zircon	66

The weight 21 of silicon must be considered as still open to correction. The results arrived at by DEVILLE*, from the oxidation of graphitoidal silicon, indicate a higher number.

Finally, these considerations lead us to the remarkable inference that carbon in the form of graphite functions as a distinct element; that it forms a distinct system of combinations, into which it enters with a distinct atomic weight, the weight 33. Analogy would lead us to a similar conclusion with regard to the elements boron and silicon. How far this inference is to be extended to the allotropic forms of other elements experiment alone can decide.

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* DEVILLE obtained from 100 parts, 205·3 of silica. Calculated from these numbers, the atomic weight of silicon would be 22·7, approximating to the former atomic weight of BERZELIUS, 22·5. The subject requires further investigation.

XIV. *On the Physical Phenomena of Glaciers.*—Part I. *Observations on the Mer de Glace.*

By John Tyndall, *F.R.S., Membre de la Société Hollandaise des Sciences; la Société de Physique et d'Histoire Naturelle de Genève; la Société Philomathique de Paris; Mitglied der Naturforschenden Gesellschaft, Zürich, der Gesellschaft zur Beförderung der Gesammt-Naturwissenschaften, Marburg, der K. Leop. Akad. der Naturforscher, Breslau, and Professor of Natural Philosophy at the Royal Institution.*

Received May 20,—Read May 20, 1858.

§ 1.

The Philosophical Transactions for 1857 contain a paper by Mr. Huxley and myself upon the Structure and Motion of Glaciers. The observations on which that paper was founded extended over a very brief period, and hence arose the desire, on my part, to make a second expedition to the Alps, in which I regret to say my friend was unable fully to join. The phenomena of the Mer de Glace being those on which the most important theoretic views of the constitution and motion of glaciers are based, I wished especially to make myself acquainted by personal observation with these phenomena. Six weeks of the summer of 1857 were accordingly devoted to the examination of this glacier. For the purpose of observing its motion, bearings and inclinations, and also of determining its width at various points, I took with me an excellent 5-inch theodolite, and a surveyor's chain; for both of which I am indebted to the kindness of the Director-General of the Geological Survey, and to Professor Ramsay. I propose to divide the investigation into two parts, the first of which forms the subject of the following paper, while the second will be the subject of a future communication. It gives me great pleasure here to record my grateful sense of the able and unremitting assistance rendered me throughout the entire period of the observations, by my friend Mr. T. A. Hirst, whose name indeed, had he permitted it, I should gladly have seen associated with my own at the head of this paper.

§ 2. *On the Motion of the Mer de Glace.*

Our first observation of the motion of the Mer de Glace was made on the 14th of July. On the steep terminal incline of the Glacier de Bois we singled out a tall pinnacle of ice, the front edge of which was perfectly vertical. In coincidence with this edge I fixed the vertical wire of our theodolite, and after three hours found that the ice cliff had moved downwards, the cross hairs being now projected against the face of the cliff several inches above its edge.

Our first line across the glacier was set out upon the 17th of July. The mode of proceeding in all such cases was this:—the theodolite was placed beside the glacier, quite

Second Line (BB′).—Mean Daily Motion.

No. of stake.	Motion in inches.	No. of stake.	Motion in inches
West 1	7½	17	22½
2	10¾	18	21
3	12¼	19	22½
4	14½	20	20½
5	14½	21	×
6	16	22	×
7	16¾	23	24½
8	17½	24	×
9	19	25	21¾
10	19½	26	×
11	19½	27	×
12	21	28	22¼
13	21	29	22¾
14	21	30	25¼
15	22½	East 31	25¾
16	22½		

The stakes marked thus × were fixed by the eye, their positions being such that they could not be seen by the theodolite. Some of them were placed in deep glacial hollows, where, without an instrument, it was difficult to keep them in the same vertical plane. The slight uncertainty thus arising induced me finally to reject them. The gradual augmentation of velocity from the side towards the centre is very manifest; but it will be observed that stake 31, which stood upon the Talèfre side of the glacier, moved quickest of all. The difference in favour of the latter side is, however, much less than it was lower down.

The reason why in the two cases just considered the terminal stake towards the eastern side of the glacier shows no retardation, is, that the state of the ice, and the position of the theodolite, were not such as to enable us to continue the line of stakes completely across the glacier to the eastern side, and hence the observations could not show the retarding influence of that side. In setting out the third line CC′, therefore, Mr. Hirst took up a position on the Chapeau side of the valley, from which the vision across the glacier was quite uninterrupted by ridges or other obstacles, while the crevasses were not impracticable. One of the fixed termini of this line was the corner of a window of the Montanvert Hotel. There were twelve stakes planted along the line, and the motion of these during twenty-four hours, from the 20th to the 21st of July, was as follows:—

Third Line (CC′).—Mean Daily Motion.

	East.											West.
No. of stakes.	1	2	3	4	5	6	7	8	9	10	11	12
Motion....	19½	22¾	28¾	30¼	33¾	28¼	24½	25	25	18	×	8½

Stake No. 1 was fixed in the ice, close to the eastern side of the glacier, and the retarding influence of this side is quite manifest from the measurements. A glance, however, reveals a fact confirmative of the former measurements; the daily motion of the extreme eastern stake is 14½ inches behind the maximum, while the motion of the extreme western stake is 25½ inches behind it. The stake No. 5, which moved at the maximum rate, was also much nearer to the eastern than to the western side of the ice-stream; the observation therefore corroborates those already made as regards the position of the point of maximum motion.

How then is the fact to be accounted for, that the point of maximum motion of the Mer de Glace is thus thrown towards its eastern boundary? Reflection suggested to me that the effect might be due to the curvature of the valley through which the Mer de Glace moves. At the place where the foregoing observations were made the glacier bends, turning its concave side to the Montanvert, and its convexity towards the Chapeau. M. RENDU insists on the complete analogy of the phenomena of a river and those of a glacier; and the idea has been to a great extent corroborated by the measurements of Professor FORBES and M. AGASSIZ; but let us make a bolder application of the analogy than any of them contemplated, confining our view to the influence of curvature merely. The point of maximum motion of a river moving through a channel similar to that occupied by the Mer de Glace, would lie on that side of the centre of the channel towards which the river turns its convex curvature. Can this be the case with the ice? If so, the place of maximum motion ought to be different where the glacier bends in the opposite direction. Fortunately the Mer de Glace itself enables us to bring this idea to a test.

Higher up the valley, and opposite to the passages called "Les Ponts," such a band occurs. Here the convexity is turned towards the Montanvert or western side of the valley. A line was set out across this portion of the glacier on the 25th of July, and its measurement upon the 26th gave the following results:—

Fourth Line (DD′).—Mean Daily Motion.

	East.															West.
No. of Stakes.	1	2	3	4	5	6	9	10	11	12	13	14	15	16	17
Motion...	6½	8	12½	15¼	15½	18¾	19½	21	20½	23¼	23¼	21	22¼	17¼	15

After the setting out of this line, its length was measured by Mr. HIRST; and found to be 39 chains 25 links, which, as each chain is equal to 22 yards, gives 863 yards as the width of the Mer de Glace opposite the first "*Pont.*" A mark on the rock crossed by this *pont* constituted indeed one of the fixed termini of the line.

For the sake of stricter discussion, a copy of the notes of this measurement faces the next page.

The stakes along the line are marked thus, ⊙. The fixing of them commenced at the Echellets or eastern side of the valley, and they were numbered *from* this side: the measurement, on the contrary, commenced at the "Pont." Hence it is that the 17th stake was the first encountered in the measurement. This stake stood at a distance of 326 links, or nearly 72 yards from the edge of the glacier. Stake No. 1 at the other end of the line stood close beside the lateral moraine at the eastern side of the glacier.

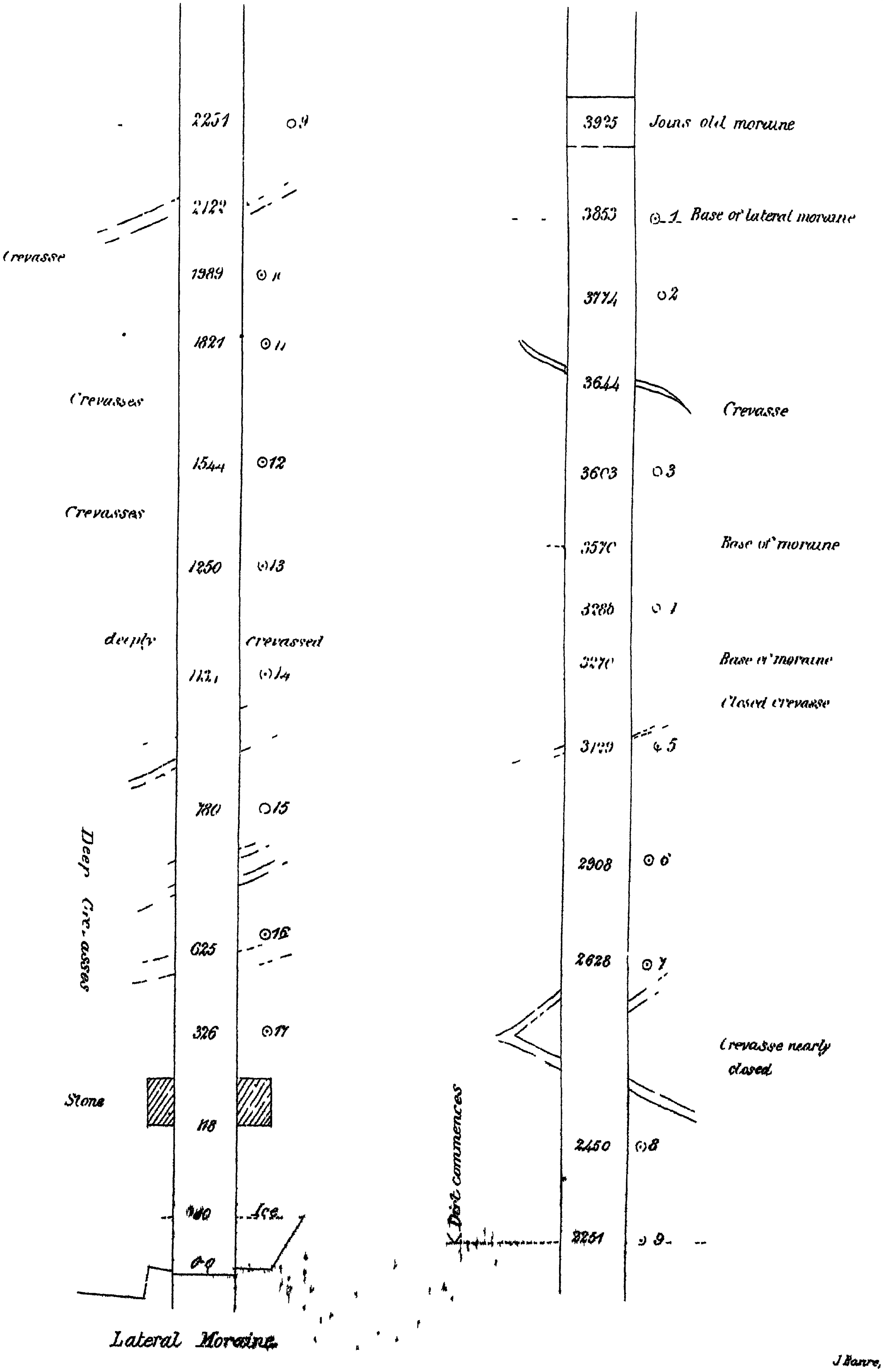
Referring to the notes, it will be seen that the place of maximum movement occurs between the stakes 12 and 13, the former at a distance of 1544 links, and the latter at a distance of 1250 links from the western side of the glacier. The mean of these is 1397 links; consequently, as the entire width is 3925 links, the point of maximum motion is here 1131 links nearer to the western than to the eastern side of the Mer de Glace. The dirt also which marks the junction of the portion of the ice derived from the Col du Géant, with that derived from the other tributaries, is crossed at the distance 2251; hence the place of maximum motion occurs at a point 854 links *west of the dirt*, while on the lines set out lower down the point of maximum motion was far in upon the dirt, eastward from the junction. The position of the point of maximum motion changes, therefore, in exact accordance with the explanation given above.

But the question is capable of still closer examination. The notes enable us to compare a number of points at the eastern side of the glacier with others, situated at the same respective distances from the western side. Let us call every pair of points, one of which is situated as far from the eastern boundary as the other is from the western, *corresponding points*. The corresponding points along our fourth line may then be ranged as follows:—

	S.	V.	S.	V.	S.	V.	S.	V.	S.	V.	
West....	17—	15 ;	16—	17¼;	15—	22¼;	13—	23¾;	12—	23¼	...(A)
East	3—	12½;	4—	15¼;	5—	15½;	7—	18¼;	9—	19½	

The numbers under the letter S are those of the stakes, those under V are the corresponding velocities. It will be seen that in each case the point on the western portion of the glacier moves quicker than the corresponding point on the eastern side. As a whole, therefore, the western side moves more speedily than the eastern, which is the reverse of what was observed lower down, but quite demonstrative of the explanation which refers the effect to the curvature of the valley.

An inspection of the notes also shows, that at the place where the fourth line crossed the glacier, the crevasses are found chiefly upon the portion derived from the Glacier du Géant. The dirt which announces the position of the other tributaries of the Mer de Glace is crossed at the distance 2251; and after this distance we find the remark "crevasse nearly closed," "closed crevasse;" so that not only is the eastern side of the glacier here less crevassed than the western, but crevasses previously formed are partially, or wholly closed up. The shifting of the place of strain consequent on the change of curvature, carried naturally along with it the shifting of the crevasses. It may be inferred from the notes that the measurement of such a line is not without its difficulties.



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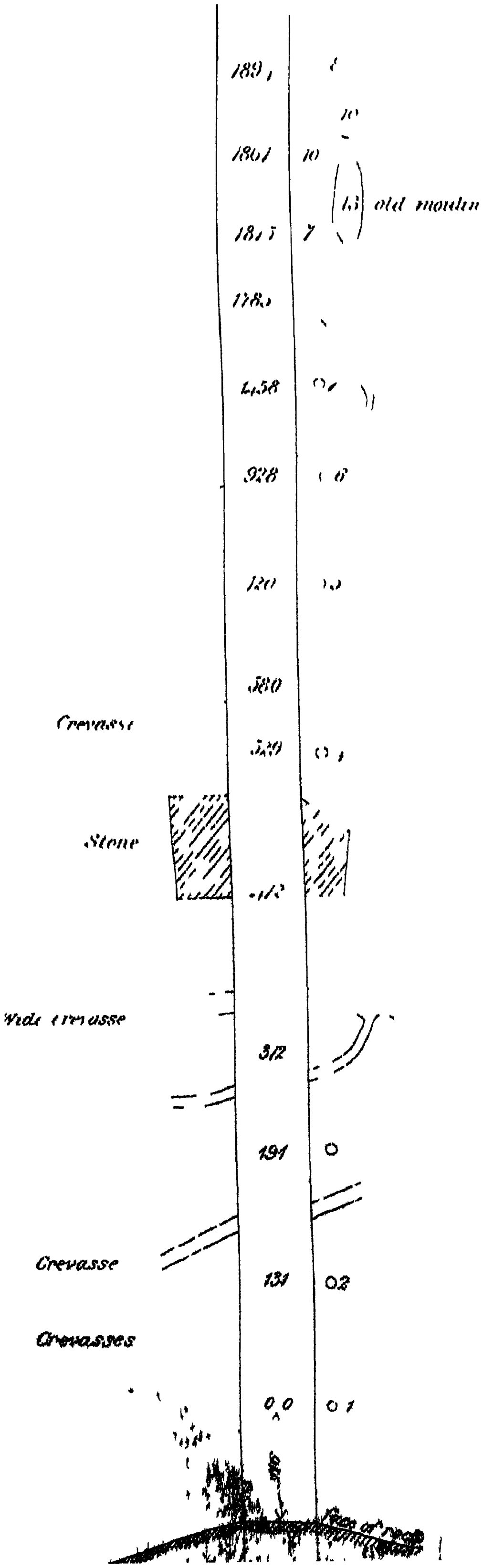
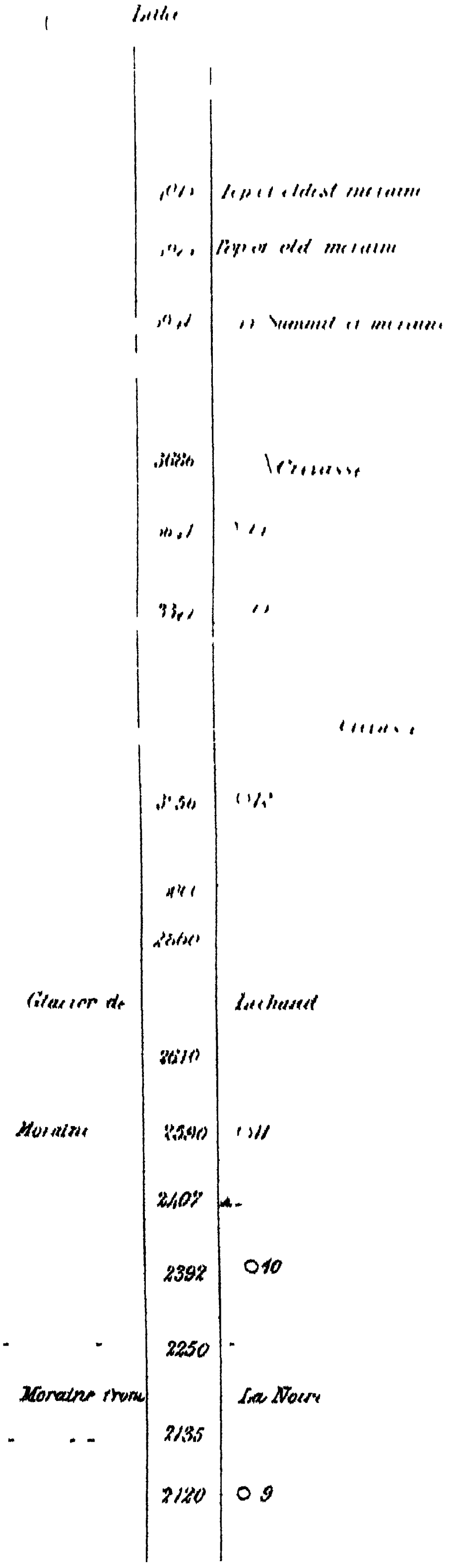


189
8
10
1861 10
13 old moulin
1815 7
1785
1458 07
928 06
120
580
Crevasse
520 0
Stone
412
Wide crevasse
312
191 0
Crevasse
131 02
Crevasses
00 01
Face of rock



Lake
Top of oldest moraine
Top of old moraine
Summit of moraine
3086 Crevasse
Crevasse
Glacier de Lachaud
2860
2610
Moraine 2590 011
2407
2392 010
2250
Moraine from La Noire
2135
2120 09

Our next line (EE′) stretched across the glacier from the promontory of Trelaporte to the base of the Aiguille du Moine. The instrument being placed upon a grassy slope above the promontory, the line was set out on the 28th of July. The Trelaporte end of this line was immediately under the station marked G* on the Map of Professor FORBES; the displacements of the stakes were measured on the 31st of July, and were found to be as follows:—

Fifth Line (EE′).—Mean Daily Motion.

	West.														East.
No. of Stakes.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Motion......	11¼	13½	12¾	15	15¼	16	17¼	19¼	19¾	19	19½	17½	16	14¾	10

The first of these stakes was about 80 feet distant from the face of the rock at Trelaporte; the 15th was on the lateral moraine, which moved along with the ice at the opposite side of the valley. The retarding influence of both sides is very clearly shown, the motion of the central stakes being nearly twice that of the extreme ones. As a whole, the rate of motion is slower here than at the "Ponts" or at the Montanvert.

This line was also chained by Mr. HIRST; a copy of his notes, showing the distances along the line at which the stakes were set, faces this page.

The chaining commenced at a point 116 links distant from the face of the rock at Trelaporte. Adding these 116 to the distance 3941, we have 4057 links, or 893 yards for the width of this portion of the Mer de Glace. The point of maximum motion occurs at stake No. 9, which is 2236 links distant from the rock at Trelaporte, or more than one-half the distance across; that is to say, the point of maximum motion is here nearer to the Talèfre side than to the Géant side of the glacier. Here, again, we have a result different from that obtained with our fourth line; and if we look to the sketch-map we shall see the reason. Between the fourth and fifth lines the Mer de Glace has passed a point of contrary flexure; and here at Trelaporte the convex side of the glacier is turned towards the base of the Aiguille du Moine.

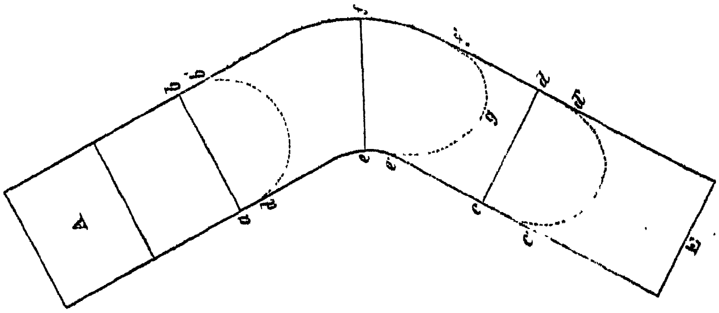
Taking the 116 links at the commencement into account, the following pairs of stakes may be regarded as corresponding points:—3 and 14; 4 and 12; 7 and 10; the small numbers referring to stakes at the western, and the large numbers to stakes at the eastern side of the glacier. The relative motions of these points are as follows:—

West....... 3—12¾; 4—15; 7—17¼.
East ...... 14—14¾; 12—17½; 10—19.

Comparing this Table with Table A, we observe a reverse result; in the latter case the western stakes moved most swiftly; here the eastern ones do so; the deportment of the ice is the same as at the places intersected by our three first lines, and the curvature of the valley is also similar.

From the foregoing observations the following law of glacier motion is derived:—*When a glacier moves through a sinuous valley, the locus of the point of maximum motion does not coincide with a line drawn along the centre of the glacier, but always lies on the* *convex side of the central line. It is therefore a curve more deeply sinuous than the valley itself, and crosses the axis of the glacier at each point of contrary flexure*.*

Fig. 2.



The law may be illustrated by the following experiment:—A, fig. 2, is a box filled with fine mud, which by raising a sluice in front flowed into the curved trough AE. A line *ab* was drawn upon the mud above the bend, a second line, *cd*, below the bend, and a third, *ef*, at the bend. The distortions of these three lines by the motion of the mud downward will reveal the position of the point of maximum motion at the particular places where they are drawn. The line *ab* was distorted to *a′b′*, the summit of the curve being exactly in the centre of the trough, thus proving that the centre was the place of maximum motion. The same was true of the line *cd*, which was distorted to *c′d′*. The line *ef* was distorted to *e′f′*, the summit *g* of the curve being nearer to the side *bfd* of the trough, this proving the point of maximum motion to lie towards that side.

I scarcely know a case more calculated to impress the mind both with the yielding power of ice to pressure, and the magnitude of the forces brought into play in the motion of glaciers, than the crushing of the three tributaries of the Mer de Glace through the throat of the valley between Trelaporte and the base of the Aiguille du Moine. Not wishing to trust the eyes in the estimation of distances here, each of the three confluent branches was measured. The width of the Glacier du Géant, a short distance above the Tacul, was found to be 5155 links, or 1134 yards. The width of the Glacier de Léchaud, just before its junction with the Talèfre, was found to be 3725 links, or 825 yards. That of the Talèfre, before it is influenced by the pressure of the Léchaud, that is, across the ice-cascade, was found, approximately, to be 2900 links, or 638 yards. Adding all together, we find the sum of the widths of the three branch glaciers to be 2597 yards. At Trelaporte these three branches *are forced through a gorge* 893 *yards wide*; and our measurements show that it passes through with a velocity of nearly 20 inches a day!

Limiting our view to one of the glaciers thus compressed, the facts appear still more astonishing. Previous to its junction with the Talèfre, the Glacier de Léchaud has a width of thirty-seven chains and a half. In passing through the jaws of the granite vice at Trelaporte, this broad ice river is squeezed to a driblet *less than four chains in width*! This fact illustrates the relation of the size and power of a glacier to the quantity of snow drainage which supplies it. The Talèfre has its basin, and the Géant has its vast plateau, from which the respective glaciers derive nutrition; but the Léchaud is fed by two or three couloirs merely, which descend principally from the Mont Mallet and Les Jorasses. The Géant, in the struggle for place at Trelaporte, takes up more than half the valley, and the others come in the order of the drainage which supplies them.

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* If the [illegible] line between X and Y on the sketch map represents a line drawn along the centre of the glacier, the [illegible] will represent the locus of the point of maximum motion.

The velocity of the Mer de Glace at Trelaporte being about 20 inches, it seemed probable that the velocity of the Glacier du Géant above the Tacul, and also of the Léchaud above its junction with the Talèfre, would be considerably less, in consequence of the greater width at these places. This proved to be the case. On the 29th of July a line was set out across the Glacier du Géant, a little above the Tacul. There were ten stakes in this line, and their motions reduced to twenty-four hours were as follows:—

Sixth Line (HH').—Mean Daily Motion.

No. of Stakes.	1	2	3	4	5	6	7	8	9	10
Motion......	11	10	12	13	12	12¾	10½	10	9	5

The velocity here is considerably under that of the Mer de Glace at Trelaporte.

On the 1st of August we set out a line across the Glacier de Léchaud immediately above where it is joined by the Talèfre. The line commenced at the side of the glacier beneath the block of stone called the Pierre de Béranger, and ran perpendicular to the axis of the glacier to the other side. The displacements were measured on the 3rd of August: reduced to twenty-four hours, they are as follows:—

Seventh Line (KK').—Mean Daily Motion.

No. of Stakes.	1	2	3	4	5	6	7	8	9	10
Motion......	4½	8¼	9½	9	8½	7½	6¼	8½	7	5½

The stakes 8 and 9 were at opposite sides of a "moulin," which was found to share the general motion of the glacier. A new crevasse crossed our line above 8 and below 9, and the greater advance of stake No. 8 was probably owing to the yielding which this crevasse permitted. The rates of motion, it will be observed, are still less than those upon the Glacier du Géant.

Were the Glacier de Léchaud subjected to no waste during its descent, and did no accumulation take place at any point, equal quantities of ice would pass through all its cross sections in the same time. The compression which takes place at Trelaporte is not a change of *volume* but of *form*. The mass is squeezed laterally, and no doubt expands vertically. Comparing the velocities and widths at Trelaporte and opposite the Pierre de Béranger, we should be led to the result that the depth of the Glacier de Léchaud at the former place would, if no waste had taken place, be at least four and a half times its depth at the latter. The loss of ice by superficial and subglacial melting must materially modify this result; but some interesting observations might be made in connexion with the point, and I think one result of such observations would be the establishment of the comparative shallowness of the Glacier de Léchaud.

There is another characteristic of glacier motion which was predicted by Professor Forbes, before any observations had been made upon the point, and afterwards confirmed both by his own measurements and those of M. Martins,—I allude to the fact that the glacier is not only retarded by its sides, but by its bottom, the superficial ice thus moving more quickly than that in contact with the bed of the glacier.

Objections have been made to both the measurements alluded to, and I was therefore desirous to submit the question to a new test. The experiments which I have to record were made upon the face of an ice precipice, which offered a rare opportunity for an observation of the kind. The face formed the eastern boundary of the Glacier du Géant near the Tacul, was about 140 feet in height, and nearly vertical. I requested Mr. Hirst to place two stakes, one at the top and the other at the bottom of this precipice. This was done on the 3rd of August; and on the 5th it was found that the stake at the top had moved through 12½ inches, while that at the bottom showed an advance of 6 inches only. There was some uncertainty regarding this latter result, on account of the danger incurred by the assistant, from the stones which fell incessantly from the top of the precipice, and which compelled him to retreat several times before the measurement could be effected.

I was reluctant, however, to leave an observation of the kind with a shade of uncertainty attached to it. On the 11th of August, therefore, I fixed myself two stakes, one at the top and the other at the bottom of the precipice, and feeling strongly impressed with the importance of ascertaining the motion of a point midway between top and bottom, I cut steps in the ice, climbed the face of the precipice, pierced the ice with an auger, and drove a stake firmly into it. Until Monday the 17th of August I was unable to reach the place again. On this day I penetrated through dense fog and snow to the Tacul, and found the highest of the three stakes standing, but the two lower ones were buried in a heap of snow which lay at the base of the precipice. On the following day the perilous process above described had to be repeated; and on Tuesday the 20th of August the displacements were measured. Reduced to twenty-four hours, the motion of the three stakes was found to be as follows:—

	inches.
Top stake	6·00
Middle stake	4·59
Bottom stake	2·56

The distance from the top of the ice-wall to its base was found, by measurement with a rope, to be 140·58 feet, but it was not quite perpendicular at its upper portion; the height of the middle stake from the ground was 35 feet, and of the bottom one 4 feet. It is therefore proved by these measurements that the bottom of the ice-wall at the Tacul moves with less than half the velocity of the summit; while the deportment of the intermediate stake shows how the velocity increases from the bottom upwards.

§ 3. *On the Cause of Glacier Motion.*

The various theories which have been advanced to account for the progression of glaciers are too well known to need detailed discussion here. SAUSSURE, and some before him, thought that the glacier slid along its bed[*]. CHARPENTIER thought that the motion was due to the freezing of water in capillary fissures, and the consequent swelling of the contents of these fissures. Other hypotheses have been advanced without producing any deep impression. It has been objected to SAUSSURE's theory, that were it true, glaciers must slide down with an accelerated motion; but reflection alone would deprive this objection of weight, and an experiment of Mr. HOPKINS completely refutes it[†]. When incessantly checked by the surface over which they slide, even avalanches may, and do, sometimes descend with a uniform motion. The motion of a man in walking down stairs is on the whole uniform, but it is actually made up of an aggregate of small motions, each of which is accelerated. It is easy to conceive that ice moving over an uneven bed, will, when it is released from one opposing obstacle, be checked by another, and its motion thus be rendered sensibly uniform. So many obstacles exist along the bed of a glacier, that sudden slipping forwards of the mass through any considerable distance is not to be expected. But the real weak point of SAUSSURE's theory, though partly true, is its inability to account for many facts observed since his time. The theory of CHARPENTIER, though not always fairly represented, has been shown to be untenable.

The facts submitted to our consideration are briefly as follows:—We see the glacier winding through a valley, squeezing itself through a gorge, and widening where it has room. We see that the centre moves more quickly than the sides, and the top more quickly than the bottom; and the next demand of the mind is for a general principle which shall unite these facts, and from which they shall follow as physical corollaries. Professor FORBES seeks this principle in the *viscosity* of the ice. Ice, according to him, is a substance resembling treacle, honey, or tar, and the observed phenomena are a consequence of this property. In this assumption consists what is called the *viscous theory*[‡].

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[*] I hardly think, however, that SAUSSURE would have subscribed to some of the interpretations of his theory now extant.

[†] See HOPKINS in Philosophical Magazine, vol. xxvi. p. 4. Were it not that this objection is thoughtlessly repeated in every work upon glaciers, I would not dwell upon it here. The objection drawn from the deportment of secondary glaciers lying on steep slopes is also very commonly dwelt upon, but it is equally without weight; and applies with at least as much force to the viscous theory as to the theory of SAUSSURE.

[‡] The name of M. RENDU will always be honourably associated with the theory of glacier motion. He first drew attention to the power of the glacier to move through a sinuous valley, to narrow and widen and behave like lava or like "a soft paste." He conjectured also that the centre would move more quickly than the sides. In fact he appears to have had a correct conception of almost all that the subsequent observations of Professor FORBES established. I regret to say that I have not been able to obtain M. RENDU's original memoir.

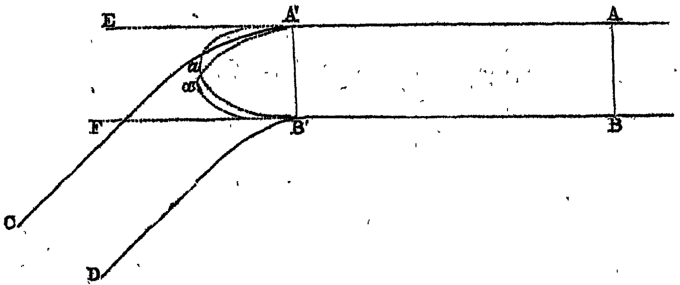
August 1859.—Thanks to my Zürich friends, I have recently had the pleasure of reading M. RENDU's paper, the perusal of which has confirmed my estimate of his sagacity. Had this gentleman been a philosopher instead of an ecclesiastic, we should doubtless have heard more about his claims than we have hitherto done.

Before entering upon the examination of this theory, I would ask permission to make the following remarks:—I am aware that the paper published by Mr. HUXLEY and myself has produced considerable diversity of opinion among scientific men. Some, whose opinions are entitled to every respect, regard the views there advocated, and the experiments there described, as consistent with and explanatory of the viscous theory; while others, of equal eminence, believe that if the views referred to be sound, the viscous theory can no longer be maintained. Under these circumstances it behoves me to state distinctly the point of view from which I intend to examine the theory, submitting myself completely to the public sense as to whether this point of view be the correct one or not. Both the terms and the illustrations made use of by Professor FORBES have diffused ideas regarding the physical qualities of ice which render a strict examination of the subject essential. Let me here briefly state what I understand by viscosity, and what I, and other more competent persons, at one time believed to be a demonstrated property of ice*.

By viscosity, I understand that property of a semifluid body which permits of its being drawn out when subjected to a force of *tension*, the particles of the substance taking up new positions of equilibrium, so that when relieved from the strain the substance has no distortion to recover from. A capacity to change the form under crushing *pressure* is not, I think, a test of viscosity; for this power is possessed by substances, to which we should never think of applying the term viscous.

In examining whether glaciers possess the power of yielding to tension like viscous bodies, I would refer:—1. To the shifting of the place of strain by the curvature of the valley, to which I have already referred. Let ABCD, fig. 3, embrace a curved portion of a glacial valley, and let AB be a linear element of the glacier transverse to its axis.

Fig. 3.



As the ice descends AB becomes curved in consequence of the quicker movement of its centre. Did the valley continue straight in the direction of E and F, the point of maximum velocity would, after a certain time, be found at *a*, midway between the lines AE, BF; but the curving of the valley throws the point *a* to *a'*, and thus increases the strain upon the branch *a'*A' of the curve, while it diminishes the strain upon *a'*B'. The consequence of this difference of action upon the two branches, is that the side of the glacier which is subjected to the augmented tension does not yield to the strain as a viscous body would do, but *breaks*. In the words of Professor FORBES, the glacier at this place becomes "*excessively crevassed.*" This fact, therefore, as far as it goes, is opposed to the idea of viscosity as above defined.

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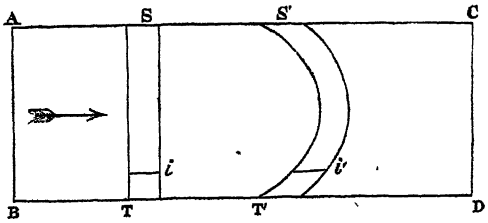
* "*Gluey tenacity*" is the quality which I have heard ascribed to ice by intelligent and cultivated persons.

2. The fact that the centre of a glacier moves more quickly than the sides, is that on which the viscous theory is chiefly based: let us examine the circumstances connected with this motion, availing ourselves while doing so both of the figure and the reasoning of Mr. HOPKINS. Let ABCD, fig. 4, be a sloping canal, into which is poured a quantity of treacle, honey, tar, or melted caoutchouc, all of which have been referred to as illustrative of the character of ice; and let the mass move down the slope in the direction of the arrow. Let ST be a narrow segment of the viscous substance; this segment, as it moves downwards, will take the form S'T'. Supposing T*i* to be a square element of the mass, it will be distorted lower down into the lozenge T'*i*', and the line T*i* will become T'*i*'. Now the analogy between such a substance and ice fails in this respect; in the viscous mass the short diagonal of the square *stretches* to the long one of the lozenge, but, in the glacier, the ice breaks at right angles to the tension, and *marginal crevasses* are formed. It was by means of the simple diagram here sketched that Mr. HOPKINS showed why the marginal crevasses of a glacier are inclined towards its source[*]. This fact, therefore, so far as it goes, is also opposed to the idea of viscosity.

Fig. 4.



But it is known that in the case of a substance confessedly viscous, a sudden shock or strain may produce fracture. Professor FORBES justly urges, "that sealing wax at moderate atmospheric temperatures, will mould itself (with time) to the most delicate inequalities of the surface on which it rests . . . . . but may, at the same time, be shivered to atoms by a blow with a hammer[†]." Hence, in order to estimate the weight of the objection, that the glacier breaks when subjected to strain, we must know the conditions under which the force is applied.

The fifteenth station on the line (EE') at Trelaporte stands on the lateral moraine of the glacier; between it and the fourteenth, a distance of 300 links, or 190 feet, intervenes, and within this distance the glacier suffers its maximum strain. Let AB (fig. 5) be the side of the glacier, and let the direction of motion be that indicated by the arrow. Let *abcd* be a square element of the glacier with a side of 190 feet. The whole square moves downwards with the glacier, but the side *bd* moves quickest. The point *a* moves 10 inches, the point *b*, 14·75 inches in twenty-four hours, the differential motion thus amounting to an inch in five hours. Let *ab'cd'* be the shape of the figure after five hours' motion, the distance *bb'*=*dd'* being 1 inch; then the line *ab* would be extended to *ab'*, and the line *cd* to *cd'*.

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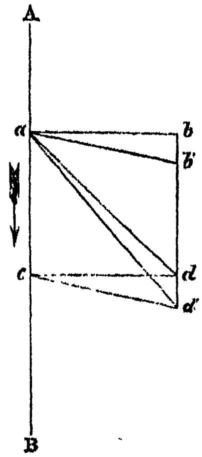
[*] Philosophical Magazine, vol. xxvi. page 160.

[†] Philosophical Magazine, Fourth Series, vol. x. p. 201. Proceedings of the Royal Society, June 14, 1855.

Fig. 5.



But the extension of *these* lines does not mark the *maximum strain* to which the ice is subjected. Mr. Hopkins has shown this strain to take place along the line *ad*, which encloses an angle of 45° with the side of the glacier. In five hours, then, this line, if capable of yielding, would be stretched to *ad'*.

In the right-angled triangle *abd'* we have *ab*=2280 inches, *bd'*=2281, and hence we find *ad'* to be 3225·1 inches; the diagonal *ad* is 3224·4 inches; and the amount of yielding required from the ice is that the latter line shall be extended by five hours' gradual strain to the length of the former.

This is the utmost demand made upon the presumed viscosity of the ice, but the substance is unable to respond to it: instead of stretching, it breaks, and copious fissures are the consequence. It must not be forgotten that the evidence here adduced merely proves what ice *cannot* do; what it *can* do in the way of viscous yielding we do not know. There is no experiment on record, with masses great or small, to show that the substance possesses, in any measurable degree, that power of being drawn out which is the very essence of viscosity.

Further, the case here referred to is not solitary, but typical. I dare say every single glacier of the first order would furnish proofs of the absence of viscosity equally cogent with that here brought forward. The marginal crevasses of glaciers usually result from an incapacity on the part of the ice to respond to a demand upon its viscosity, not greater than that just cited *.

When a person unaccustomed to glacier life observes, from a safe distance, the profound fissures by which the ice is intersected, the question sometimes arises, "what if one of these chasms should suddenly open beneath the traveller's feet?" There is, however, no fear of this. The crevasses, when first formed, are exceedingly narrow, and they open with extreme slowness. While standing one evening, in company with Mr. Hirst, on the Glacier du Géant, both of us were startled by a sound like a heavy explosion in the body of the glacier, underneath the place where we stood. This was instantly followed by a succession of loud cracks, accompanied by a low singing noise. The ice continued cracking for an hour; but notwithstanding the manifest breaking of the glacier, which was to some extent awe-inspiring, we could not, for a long time, detect any trace of rupture. The escape of air-bubbles from the surface first informed us of the position and direction of the incipient crevasse, for such it was. It was so narrow that the thinnest blade of my penknife would not enter it.

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* It may, however, be urged that I do not know how much the ice observed in the locality referred to had been stretched before it arrived there. Extend an elastic string to the point of breaking, and a small additional force would break it; but this latter small extension would be no measure of the extensibility of the string. To this I reply, that it is the very essence of a viscous mass to accommodate itself to the forces which act upon it, so that in each new position the texture of the substance shall be in a state of equilibrium. If such a mass be broken it will have no distortion to recover from. The idea that a glacier is typified by such a string as that referred to, has been expressly rejected by the ablest advocates of the viscous theory; in proof of which I would refer to the lucid note of Dr. Whewell, in the 26th volume of the Philosophical Magazine, page 172. Cases may occur where the lateral yielding produced by the *pressure* along *bc*, fig. 5, may satisfy the *strain* along *ad*; in such a case no marginal crevasses would be formed.

On another occasion, our guide, while engaged in setting out one of our lines, observed the ice to break beneath his feet, and a rent to propagate itself suddenly, with loud cracking, to a distance of 50 or 60 yards across the glacier. These fissures are produced by tension, and the velocity with which they widen is a measure of the amount of relief demanded by the glacier. The crevasse last alluded to required several days to attain a width of 3 inches, and the opening of the one on the Glacier du Géant was far slower than this. This is their general character. They form *suddenly* and open *slowly*, and both facts are demonstrative of the non-viscosity of the ice. *For were the substance capable of stretching, even at the small rate at which they widen, there would be no necessity for their formation*[*].

There is another point of view from which the question of viscosity may be examined; but as the observations which bear upon it possess a general value, I will devote a special section to them; choosing afterwards those which more particularly apply to the case now under consideration.

§ 4. *On the Inclinations of the Mer de Glace.*

By calculation from heights and distances, Professor Forbes obtained approximately the inclinations of some portions of the Mer de Glace[†], but no direct observations on the subject have been hitherto made. On the 4th of August we transported our theodolite to the Jardin, for the purpose of ascertaining its inclination, and that of the Glacier du Talèfre. From the green space on which visitors to the place usually repose, the angle of elevation to the top of the Jardin is 24° 7′, and from the same place downwards to the bottom of the Jardin the inclination is 30°. From the bottom of the Jardin, for some distance along its medial moraine, the ice is nearly level, its inclination being only 21′. A succession of slopes then follows, enclosing with the horizon the following angles of depression:—3° 5′; 4° 25′; 6° 50′; 8° 5′ and 9° 40′, which last brings us to the brow of the ice cascade. The inclination of the fall is 25°,—producing a line drawn along the centre of the cascade until it cuts the moraine between the Talèfre and Léchaud: the inclination along this line, from the base of the cascade downwards, is 7° 30′.

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[*] For an interesting account of the formation of a number of new crevasses, see Agassiz, 'Système Glaciaire,' p. 310.

[†] Travels, p. 117.

The descent of the ice through this gorge from the basin of the Talèfre, is adduced by Professor FORBES as an illustration "which will appear to the impartial reader almost a demonstration" of the principle of viscosity. "The ice is compact," he urges, "and almost without fissures. . . . . The open crevasses which commence a little above AB are turned towards the basin*." The line AB here referred to is actually in the jaws of the gorge, and apparently at a considerable distance below where the ice enters it. The description certainly would not apply to the ice of the year 1857. Long before reaching the summit of the fall the most skilful iceman would find himself in difficulties. We proceeded as far as we dared amid the pits and chasms into which the glacier is torn, and which followed each other so speedily, that the ridges between the fissures were often reduced to mere plates and wedges, which were in many instances bent and broken by the lateral pressure. At some places vertical forces seemed to have acted upon the mass, and turned huge pyramids so far round as to place the structural veins at right angles to their normal position. Looking downwards towards the summit of the cascade, the ice was frightfully riven. The glacier descends the cascade itself in wedges, pyramids, and columns, which latter often fall with a sound like thunder, and crush to pieces the ice crags below them. After this description I do not think that the case is likely to be accepted as a demonstration of the viscosity of ice.

I now pass on to the inclinations of the Glacier du Géant. For some distance below the base of the so-called *Seracs* the irregularities of the glacier render an estimate of its general inclination somewhat difficult, but I should judge it to be about 13°. From the end of this steeper portion, two slopes, one of 4° 37′, and the other of 3°, bring us to the Tacul, and from this point to the bottom of the ice valley at Trelaporte we have the following series of inclinations:—2° 15′; 3° 15′; 5° and 9°; thence to the Grand Moulin the slope is 3° 30′, and afterwards, down the glacier to a point nearly opposite to the Grande Cheminée below l'Angle, the inclinations are 3° 10′; 5°; 6° 25′, and 4°. The glacier then descends a slope of 9°, and afterwards passes the Montanvert at an inclination of 4° 45′. Below the Montanvert it falls steeply for some distance, the inclination being 16°. Between the base of this slope and the brow which marks the termination of the Mer de Glace and the commencement of the Glacier des Bois, the slope is 5° 10′. The ice afterwards descends an incline of 22° 20′ in a state of great dislocation. From the base of this incline the general inclination of the lower portion of the glacier is 10°.

A brief reference to the Glacier de Léchaud will complete this portion of our subject. The upper portion of the glacier, to the base of the steep snow slopes which rear themselves against the Grande Jorasse, has an inclination of 4° 29′. Opposite to the icefall of the Talèfre, the inclination, for a short distance, is 3° 17′, and afterwards down to the Tacul, where the Léchaud and Géant join, the slope is 5° 22′.

I will now endeavour to show the theoretic significance of the observations above recorded, referring in the first place to the great terminal slope of the Glacier des Bois, down which the ice is shot in crags, pinnacles, wedges, and castellated masses, all tossed together in the utmost confusion. Regarding this portion of the glacier, Professor FORBES writes as follows:—"Escaping from the rocky defile between the promontory of the Montanvert and the base of the Aiguille de Dru, it pours in a cascade of icy fragments, assuming the most fantastic forms, into the valley beneath." Above the fall the ice is compact: Professor FORBES compares it to the dark unruffled swell of swift water rushing to precipitate itself in a mass of foam over a precipice.

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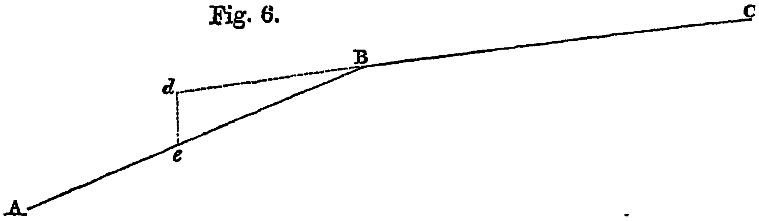
* "Reply to HOPKINS," Philosophical Magazine, 1845, vol. xxvi. p. 415.

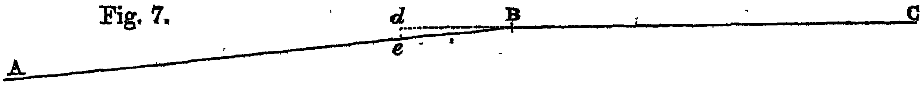
In fig. 6 I have protracted the inclination of the fall and of the glacier above it, one of them, BC, making an angle of 5° 10′, and the other, BA, an angle of 22° 20′ with the horizon. Supposing the ice to pursue the direction which it had previous to reaching the fall, it would, at the end of a certain time, reach the point *d* *; but the ice is not rigid enough to do this, and the mass descends to *e*. Now if it be the viscosity of the substance which has carried it in a certain time from B to *d*, that same property ought, one would think, to enable it to drop down the vertical *de* without breaking. But so far from its being able to do this, the glacier descends the slope BA as "a cascade of icy fragments." The fact, therefore, adds its evidence to that already adduced against the viscosity of the substance.

Fig. 6.



But the case will appear much stronger when we revert to other slopes upon the Mer de Glace. For example; the inclination of the glacier above l'Angle is 4°: it subsequently descends a slope of 9° 25′, and in doing so is so much fissured as to be absolutely impassable. The chasms cut the glacier from side to side, and present clear vertical faces of great depth †. Subtracting the smaller of the above angles from the larger, the difference, 5° 25′, gives the *change* of slope which produces the chasms. In fig. 7 the two adjacent slopes are protracted to a proper scale. Now the velocity of the glacier here, in the direction of its length, is to the vertical velocity with which it would have to sink to reach its bed, as B*d*: *de*, or as the cosine of 5° 25′ is to its sine, or as 996 : 94, or, in round numbers, as 10 : 1. Hence if it be viscosity which enables the mass to move from B to *d* in a certain time, the same property ought, one would think, to permit it to *sink* through the space *de*, which is only one-tenth of B*d*, in the same

Fig. 7.



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* I here assume that the general inclination of the surface of the glacier changes in accordance with that of its bed, which will hardly be questioned.

† I once found myself alone upon this portion of the glacier towards the close of a day's work, and experienced great difficulty in escaping from the entanglement of chasms in which I had involved myself.

time. But this is not the case. In accommodating itself to the change of inclination, the glacier breaks and is fissured in the manner described.

The change of inclination last mentioned, so far from marking the limit at which transverse crevasses begin to be formed, is sufficient to produce chasms of great magnitude, and in most inconvenient numbers. Higher up the glacier, transverse crevasses are produced by a change of inclination from 3° 10′ to 5°. If this change be accurately protracted, the mere inspection of it will illustrate more forcibly than words can do the absence of the power of viscous yielding on the part of the ice.

Looked at broadly, then, two classes of facts address themselves to the attention of the glacier investigator; one entirely in accordance with the idea of viscosity, and the other as entirely opposed to it. The affirmers and deniers of the viscous theory have perhaps been influenced too exclusively by one or the other of these classes of phenomena. The analysis of the facts gives the result, that where *pressure* comes into play we have the evidences of apparent viscosity*, but where *tension* is active we have evidences of an opposite kind. One of these classes of effects is as undeniable as the other, and hence the true theory of glaciers must render an account of both.

When the mountain snow is first moistened, it becomes more coarsely granular; these granules abut against each other, and hold air and water in their interstices. But as successive layers press upon the mass, the granules are squeezed more closely together; rupture and liquefaction, succeeded by regelation, take place at the points of abutment; water and air are expressed by the process, and the mass becomes more and more consolidated. But although powerfully squeezed, each portion of the deeper ice is surrounded on all sides by a resistent mass; it is thus compelled to yield very gradually to the pressure and moves slowly through into the valley of *écoulement*. As far as external appearances go, there is, of course, almost a perfect similarity between such an action and one due to viscosity.

But when a force of tension is applied, the case is wholly different. That intestine mobility which characterizes a truly viscous body, and enables one molecule to move round another while clinging to it, or one particle to advance while another slides in laterally to supply its place, being absent, the only way in which such a body can meet the requirements of a strain is by breaking, the fissures widening as the strain continues.

Thus, I think, we take account of all the facts adduced in proof of viscosity, and also furnish a satisfactory explanation of the other set of facts on which the opponents of the viscous theory have hitherto based their arguments.

Royal Institution, May 1858.

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* The ingenious experiment of Mr. CHRISTIE with a bomb-shell filled with water and submitted to a freezing temperature, belongs, of course, to this class of effects.

XV. *On the Veined Structure of Glaciers; with observations upon White Ice-seams, Air-bubbles and Dirt-bands, and remarks upon Glacier Theories.* By JOHN TYNDALL, *F.R.S., Professor of Natural Philosophy, Royal Institution.*

Received February 24,—Read February 24, 1859.

§ 1. *Introduction.*

On the 20th of May, 1858, I communicated a paper to the Royal Society, containing an account of observations made upon the Mer de Glace of Chamouni. In addition to the questions there discussed, another of great importance occupied my attention during my sojourn at the Montanvert, and that was the *veined structure* of the ice. To obtain information on this head, I visited almost every portion of the Mer de Glace and its tributaries; I examined the Talèfre and Léchaud glaciers, and spent several days amid the *seracs* of the Glacier du Géant. To investigate the connexion, if any, between the structure of the glacier and the stratification of its névé I ascended the Col du Géant, and afterwards inspected the magnificent ice-sections exhibited in the dislocations of the Grand Plateau and other portions of Mont Blanc.

During this investigation my convictions were by no means fixed; cases strongly suggestive of the influence of pressure, in producing the structure, came before me, and again other cases appeared which suggested, with almost equal force, the influence of stratification. The result, however, of the observations on the Mer de Glace was a strong opinion that *pressure* was the true cause of the phenomenon.

But I could not help feeling that the facts and arguments which I was in a position to bring forward would still leave the question an open one. They might influence the opinions of others, as they had influenced mine; but I had nothing to advance on which the mind could rest with perfect certainty. In short, neither the Mer de Glace nor its tributaries furnished facts capable of completely deciding the question. The subject being one on which a great deal had been written and retracted, I was unwilling to swell the bulk of the literature connected with it, while a possibility remained that what I had to say upon the subject might also require withdrawal. I therefore thought it better to wait another year; to extend the range of my observations, to visit glaciers in which the mechanical conditions of strain and pressure were different from those of the Mer de Glace. Thus by varying the circumstances, and observing Nature at work under different conditions, I hoped to confer upon the investigation the character and precision of an *experimental* inquiry.

The course of the inquiry in 1858 was as follows:—I first examined the glaciers of Grindelwald; crossing the Strahleck, I ascended the lower glacier of the Aar to the

Grimsel, thence to the glacier of the Rhone, thence to the great Aletsch glacier, in the neighbourhood of which I remained eight days. I afterwards spent eleven days at the Riffelberg, and explored the entire system of glaciers between the Monte Rosa and Mont Cervin. I thence proceeded to the Matmark Alp, and remained for five days in the vicinity of the Allalein glacier; I afterwards visited the Fée glacier, and completed the expedition by a visit to the Mer de Glace and its tributaries, and a second ascent to the summit of Mont Blanc.

The present paper contains the evidence derived from the sources thus opened to me; and I shall take these sources in the order in which they come before me; the evidence is therefore necessarily of a varied character, and it will I think be found conclusive. Besides those sections which are immediately devoted to the subject of structure, the paper contains others on the cause of the *flattening* of the air-bubbles in glacier ice, on the problem of glacier motion, and on the origin and cause of the Dirt-bands of the Mer de Glace.

§ 2. *General Aspect of the Veined Structure.*

The general appearance of the veined structure is well known. The ice of glaciers, especially midway between their mountain sources and their extremities, is of a whitish hue, owing to the number of small air-bubbles enclosed in the mass—the residue, doubtless, of that air which was originally entangled in the snow of which the ice is composed. Through the general whitish mass, however, at some places, blue veins are drawn, so numerous indeed, in some cases, as to cause the blue ice to predominate over the white. A laminated appearance is thus conferred upon the ice, the cause of the blueness being, that for some reason or other, the bubbles distributed throughout the general mass do not exist in the veins, or exist there in much smaller numbers.

In different glaciers, and in different portions of the same glacier, these veins exist in different stages of perfection. On the clean walls of some crevasses, and in the channels worked in the ice by glacial streams, they present a most beautiful appearance. They are not to be regarded as a partial phenomenon, or as affecting the constitution of glaciers to a small extent only. Vast masses of some glaciers are thus affected: by far the greater part of the Mer de Glace, and its tributaries, is composed of this laminated ice. The lower portion of the glacier of the Rhone, from the base of the ice cascade downwards, is entirely composed of it, and numerous similar cases might be cited.

To observe the structure of a glacier it is not even necessary to see the blue veins. Those who have ascended Snowdon, or wandered among the hills of Cumberland, or even walked in the environs of Leeds or other towns in Yorkshire and Lancashire, where the stratified sandstone of the district is used for architectural purposes, will have observed the exposed edges of the slate rocks, and of the stratified sandstone, to be grooved and furrowed by the action of the weather. In fact some portions of such rocks withstand the action of the atmosphere better than others, and these more resisting portions stand out in ridges while the softer portions between them are worn away. An effect exactly similar is observed upon the surface of the glacier. The laminated ice, exposed to the sun, and to wasting atmospheric influences, melts in a manner similar to the wasting of the rocks; little grooves and little ridges are formed upon the surface of the glacier, the latter being due to the more resisting ice, while the grooves are produced by the melting of the less resisting mass between them.

The consequence of this is, that the light dirt scattered by winds and avalanches over the surface of the glacier is gradually washed into the little grooves, thus forming fine lines, which to the practised eye are an infallible indication of the structure of the ice underneath. Visitors to the Jardin have ample occasion to observe these striæ, for they are finely shown upon the surface of the Mer de Glace between the Augle and Trelaporte. When they are followed until they are intersected by a fissure or a stream, it is seen that the superficial groovings always mark the direction of the veined structure within the glacier.

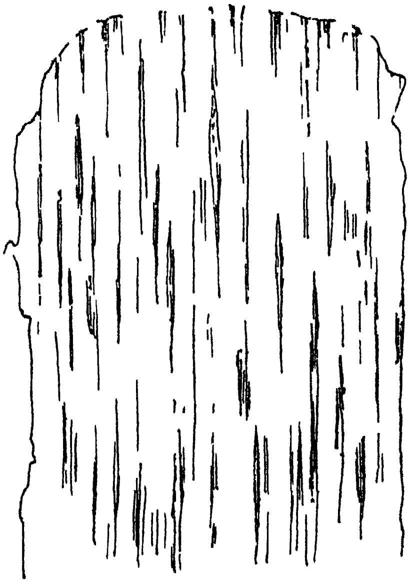
§ 3. *Structure and Stratification:—Marginal Structure.*

Opinions at present are very diverse as to the origin of these veins. Professor FORBES first regarded them as being caused by the freezing of water which filled fissures in the ice, but he now discards the notion of freezing, and supposes the "incipient fissures" to be closed by "time and cohesion." M. AGASSIZ despairs of rendering an account of them, but calls them "bands of infiltration." The Messrs. SCHLAGINTWEIT have also treated the question, but with no greater success. In the paper published by Mr. HUXLEY and myself, pressure is referred to as the probable cause of the phenomenon, but we were unable at the time to furnish proofs of this. Apart from those who give public expression to their views upon the subject, I know that there are many who reject the pressure theory, and adopt instead of it the explanation that the blue veins of the glacier are merely the continuation of the strata of the névé; a view which has recently been upheld by Mr. JOHN BALL in the Philosophical Magazine. The matter indeed so stands, that in a recent *résumé* of glacier investigations, Professor MOUSSON of Zürich omitted the subject of structure altogether, for the express reason that the question is still in complete obscurity.

I will not take up the time of the Society in discussing the vague, involved, and often absurd explanations which have been given of the blue veins of glaciers, but state broadly that the question now rests between the pressure theory, and that of stratification. Taking the parallel geological phenomena, the question then is, Does the veined structure of glaciers correspond to the *stratification* or the *cleavage* of rocks?

In reply to this question, I will remark, in the first place, that the veins are not always, nor even generally, such as we should expect from stratification. The latter ought to furnish us with distinct planes extending parallel to each other for great distances through the glacier; this is by no means the general character of the veins. We observe blue streaks, some a few inches, some a foot, and some several feet in length upon the walls of the same crevasse, and varying from a fraction of an inch to several inches in thickness. In many cases the blue spaces are definitely bounded, giving rise to the lenticular structure described by Mr. Huxley and myself, but more usually they lose themselves as pale washy streaks in the general mass of the white ice. In fig. 1, I have endeavoured to give an idea of a very common aspect of the veined structure. Such a structure is not that which we should expect from bedding.

Fig 1.



Again, taking the Glacier du Géant as a representative case, we have first of all the slopes of the Col du Géant, the collectors of the snow by which the glacier is formed. The fissures on these slopes exhibit beautifully, to a certain depth, the horizontal stratification. The lines of bedding may be seen as far down as the summit of the great ice-fall between the Rognon and the Aiguille Noire; and on the castellated masses at the summit of this fall, to which the name *seracs* has been applied, the lines of stratification may be distinctly seen. Escaped from the confusion of the fall, the glacier flows gently through a long valley towards its junction with the Léchaud and Talèfre at the Tacul. Throughout the entire length of this glacier the planes of the structure are *vertical* or nearly so; sometimes they dip a little forward, but at other places they dip an equal quantity *backward.* Now let the mind figure, if it can, an agency which, as the mass descends the fall, shall turn up the horizontal strata of the Col du Géant and set them vertical, without a single break, throughout *the entire length* of the Glacier du Géant, and I imagine the effort to conceive of such an agency will be followed by the conviction that the change indicated is inconceivable.

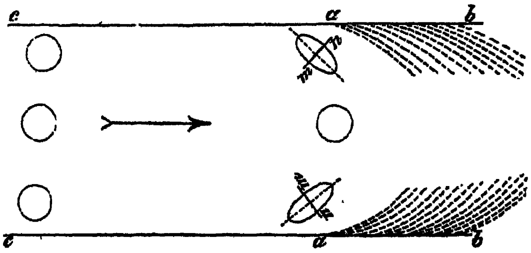
Further, we often find, in the central portions of a glacier, the structure feeble, or scarcely developed at all, while at the sides it is well developed. This is often the case where the glacier moves through a valley of tolerably uniform inclination, and where no medial moraines occur to complicate the phenomenon. But if the veins mark the bedding, there seems to be no reason why we should not find them as clearly defined at the centre as at the sides; the fact, however, certainly is that we do *not* so find them.

Let me here show the true significance of this fact. If a plastic substance, such as mud, flow down a sloping canal, the central portions will flow more quickly than the lateral ones which are held back by friction. Now the flow may be so regulated that a circle stamped upon the central portion of such a mud-stream shall move downwards *without sensible distortion*, thus proving that the central mud is neither compressed nor stretched longitudinally; for if the former, the circle would be *squeezed* to an ellipse with its major axis transverse to the axis of the stream; and if the latter, it would be *drawn out* to an ellipse with its major axis parallel to the line of flow. A similar absence of longitudinal compression exists in many glaciers, and *in such ice-streams there is no transverse central structure developed.*

But let a circle be stamped upon the mud-stream near to its side; owing to the speedier flow of the centre, this circle must be distorted to an ellipse, because the part of the circle furthest from the side moves more quickly than the part nearest the side. Hence we shall have an ellipse formed with its major axis inclined downwards, indicating that the mud is compressed in one direction and expanded in another. An exactly similar state of things occurs in many glaciers; the ice near the sides is subjected to a pressure and tension like that here indicated, and we have marginal crevasses as the result of the tension, while the veined structure is, at all events, found associated with the pressure.

Fig. 2 will perhaps render my meaning more intelligible, in which *cb*, *cb* represents the sides of a glacier moving in the direction of the arrow. Here, while the central circle retains its shape, the side ones are squeezed and drawn out to ellipses. Marginal crevasses occur parallel to the lines *m n*, or perpendicular to the tension, while the dotted lines mark the direction of the blue veins which are at right angles, or nearly so, to the crevasses. I have dotted the line marking the direction of the structure along the margins *ab*, *ab*. In connexion with this point, I would refer to the instructive papers of Mr. HOPKINS*, who has shown that in glaciers which move through valleys of uniform width, the directions of maximum pressure and tension are at right angles to each other, each of them enclosing an angle of 45 degrees with the side of the glacier.

Fig. 2.



I have simply said that the structure in the case described is "associated with the pressure;" thus confining myself within the strict limits of the facts. But what has been said shows that the pressure theory affords, at all events, a possible solution of a difficulty, which, without violence to fact, is inexplicable upon the hypothesis of stratification; the difficulty, namely, that a finely developed structure often exists along the margin of a glacier, while it is excessively feeble, or entirely absent, in the central portions.

§ 4. *Transverse Structure.—Glaciers of Grindelwald, the Rhone, &c.*

In many cases, however, the structure is not thus limited to the margins, but sweeps across the glacier from side to side, without interruption, being as well developed at the centre as at the margins. The stratification theory is wholly incompetent to account for this; the pressure theory requires that to produce this transverse structure the glacier must, at some portion of its route, have been forcibly compressed longitudinally. It was not till after my return from the Mer de Glace in 1857, that the full mechanical significance of a *change of inclination* in the glacier occurred to me.

Bend a prism of glass, we have compression on one side and extension on the other, with a neutral axis between, the mechanical conditions of the mass being shown by its action on polarized light. The same is true of any other substance,—the concave surface of the bent prism is compressed. Now at the bases of steep glacier slopes, where the inclination suddenly changes, we have a case of this bending, and along with it a thrust of the mass behind. The concave surface is turned towards us, and that surface is thrown into a state of compression corresponding to the thrust, and to the change of inclination. Hence it occurred to me, that the bases of the ice-falls, where the requisite change of inclination occurs, were likely to be the manufactories of the transverse structure. The experience of 1858 completely verified this idea.

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* Philosophical Magazine, 1845, xxvi. p. 148. See also Proceedings of the Royal Institution, vol. ii. p. 324.

In illustration of my position I will take a representative case; and to render my observations capable of being easily checked, I will choose one of the most accessible glaciers in the Alps,—the lower glacier of Grindelwald. One portion of this glacier descends from the Viescherhörner; but there is another branch which descends from the Schreckhorn, Finsteraarhorn and Strahleck, and it is to this latter branch that I now wish to direct attention.

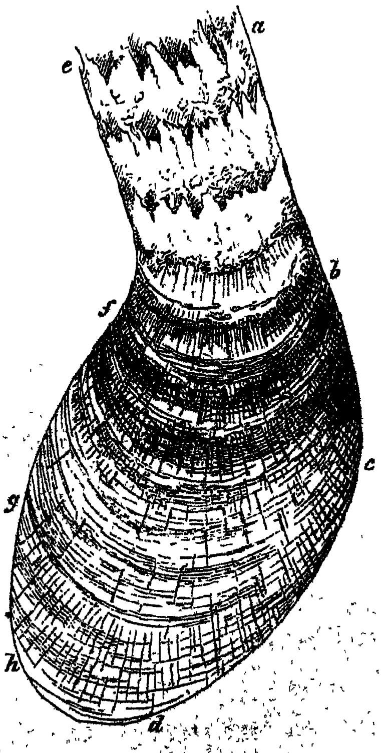
Walking up this glacier from its place of junction with the tributary from the Viescherhörner, we come at length to the base of an ice-fall which forbids further advance upon the ice. Let the glacier be here forsaken, and let the flanking mountain side, either right or left, be ascended, until a position is attained which affords a complete view of the fall and of the glacier stretching downwards from the base of the fall. The view from such a position will furnish a key to the development of the transverse structure.

It is, in point of fact, a grand *experiment* which Nature here submits to our inspection. The glacier, descending from its névé, reaches the summit of the fall and is broken transversely as it crosses the brow. It descends the fall as a succession of broken cliffy ridges, with transverse hollows between them. In these latter the ice débris and the dirt collect, partially choking up the fissures formed in the first instance. Carrying the eye downwards along the fall, we see, as we approach the base, these sharp ridges toned down, and a little below the base they dwindle into rounded protuberances which sweep, in curves, across the glacier. At the centre of the fall there is not a trace of the true structure to be observed. At the base of the fall it *begins* to appear,—at first feebly, but soon becomes more pronounced; until finally, at a short distance below the fall, the eye can follow the structural groovings right across the surface of the glacier, while the mass underneath has become correspondingly laminated in the most beautiful manner.

It is difficult to convey, by writing, the force of the evidence which the actual observation of this great experiment places before the mind. The ice at the base of the fall has to bear the powerful thrust of the descending mass; but more than this, the sudden change of inclination which it suffers throws its upper portion into a state of violent longitudinal compression. The protuberances are squeezed more closely together, the hollows between them wrinkle up in submission to the pressure; the whole aspect of the glacier here gives evidence of the powerful exertion of the latter force; and exactly at the place where *it* is exerted, the structure makes its appearance, and being once manufactured, is sent onwards, giving a character to other portions of the ice-stream which have no share in its production.

An illustration, perhaps equally good and equally accessible, is furnished by the glacier of the Rhone. Above the great icefall which the traveller descending from the Furca has to his right, the horizontal bedding is exhibited in a more or less perfect manner, to a certain depth, upon the walls of the huge and numerous crevasses here existing. I have also examined this fall from both sides, and an ordinary mountaineer will find no difficulty in reaching a spot nearly opposite the centre of the fall, from which both the fall itself and the glacier below it are distinctly visible. Here a similar state of things to that already presented to his view reveals itself. The fall is *structureless*; the cliffy ridges are separated from each other by transverse hollows, following each other in succession down the slope; those ridges are toned down to protuberances at the base of the fall, becoming more and more subdued, until low down the glacier the transverse swellings disappear. As in the case of the Grindelwald glacier, the squeezing of the protuberances and of the spaces between them is visibly manifested. *Where this squeezing commences the transverse structure also commences*, and in a very short distance reaches perfection. All the ice that forms the lower portion of the glacier has to pass through this *structure mill* at the base of the fall, and the consequence is that *it is all laminated.*

Fig. 3.



The case will be better appreciated by reference to figs. 3 and 4, the former being a sketch, in plan, and the latter a sketch in section of a part of the ice-fall and of the lower portion of the glacier of the Rhone. *a e b f* is the gorge of the fall, and *f b* its base. The transverse ridges are shown crossing the fall, being subdued at the base to protuberances, which gradually disappear further down the glacier. The "structure" sweeps across the glacier in the direction of the fine curved lines. On the plan I have also endeavoured to show the radial crevasses of the glacier; they are at right angles, or nearly so, to the structure. As would be inferred by those acquainted with what I have already written upon the influence of curvature, the side *b c d* of the glacier is much more violently crevassed than the side *f g h*.

Fig. 4 shows the cliffy ridges of the fall, and of the rounded protuberances below it, in section. The shading lines below denote the structure. The protuberances are so powerfully squeezed in some cases that they scale off at their surfaces. Fig. 5 is a representation of such scaling off which I have observed at the bases of several cascades, including those of Grindelwald, the Rhone, the Rognon, and the Talèfre, each of which has also its "structure mill" at the base of its cascade. Fig. 5*a* is an example of scaling off which I have produced by artificial pressure.

Fig. 4

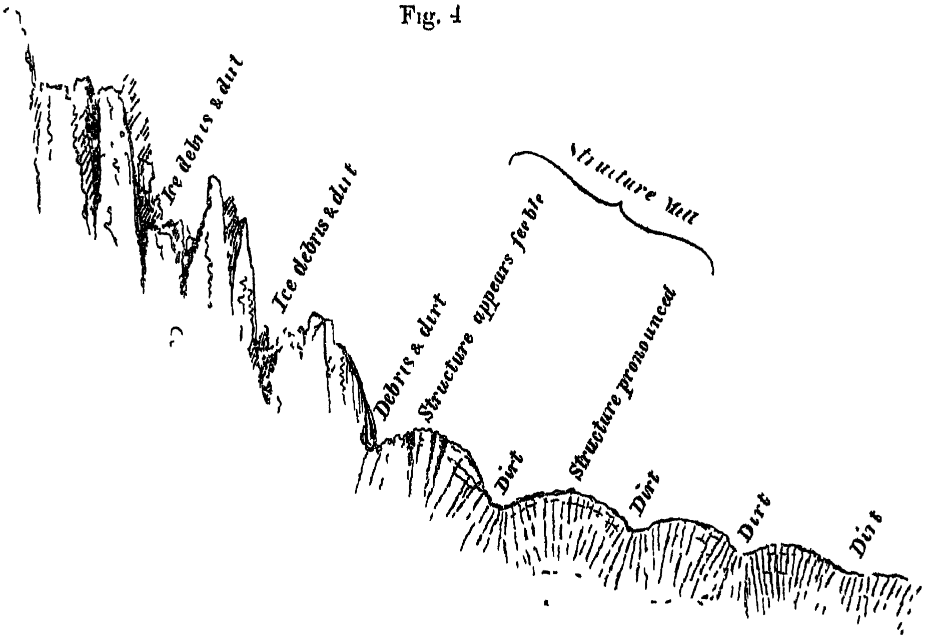


Fig. 5.

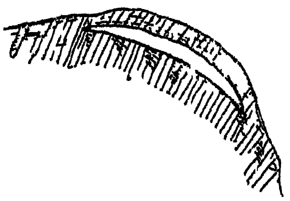
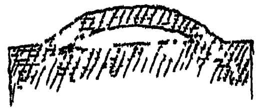


Fig 5 *a*.



It is to be borne in mind that the structure once formed prolongs itself into places which have no part in its formation; it would therefore be hasty to infer the relationship of structure and pressure from an observation of them at a particular portion of the glacier. I have sometimes seen the veined structure parallel to the crevasses for a short distance: there are some transverse crevasses on the Glacier du Géant a little above Trelaporte which illustrate this; but it would be altogether erroneous to infer from this that the law which makes the structure perpendicular to the pressure, and hence, as a general rule, transverse to the crevasses, finds an exception here. It is perfectly manifest that the structure which is brought into this unusual relationship to the crevasses has been developed far higher up; that the change of conditions from longitudinal pressure to longitudinal strain is too weak and transitory to obliterate it. To effect obliteration, a force commensurate with that which produced the structure must be brought into play, and at the place now referred to no such force exists.

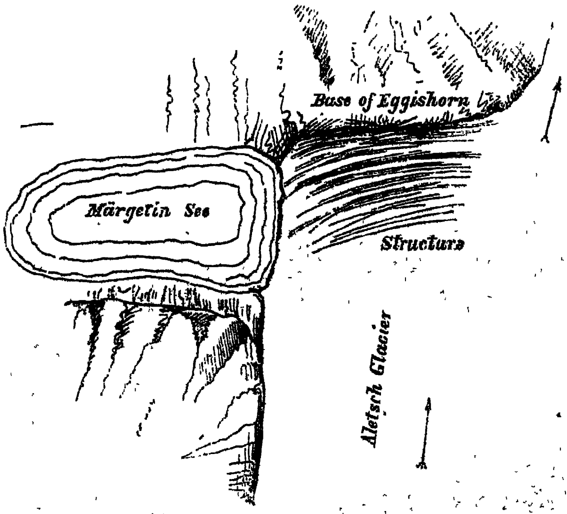
§ 5. *The Aletsch Glacier.*

Having made the foregoing observations upon the glacier of the Rhone, I proceeded to the Aletsch glacier, and during a residence of eight days at the hotel upon the slope of the Æggischhorn, made frequent excursions upon the ice. I had never previously seen this grand ice-stream, and my interest in it, at the time of my visit, was greatly augmented by the arguments which Mr. JOHN BALL had founded upon its deportment against the pressure theory of the veined structure. I shall here limit myself to a few brief remarks upon this subject.

I have already stated, and this must be particularly remembered, that the veined structure often appears in places which have no share in its production. The longitudinal structure in the centre of the stream of the Aletsch, for four miles above the base of the Æggischhorn, is not due to the lateral pressure endured by the glacier during these four miles. It is due, as Mr. BALL himself suggests, to the mutual thrust of the branch glaciers, which unite to form the trunk stream; and, once formed by this thrust, it perpetuates itself throughout a great portion of the trunk stream.

But it is urged against this view, that pressure exerted in new directions—the longitudinal pressure, for example, endured by the stream in its descent, and acting through long periods, ought,—if pressure has the power ascribed to it, to obliterate the first structure. Now here, again, it must be remembered that it is the portions of the ice near the bed of the glacier that yield, and that the upper portions of the ice, in many cases, are simply *floated* upon the moving under portions. Were the uniform "long reach" referred to by Mr. BALL strictly examined, it would, in all probability, be found that the ice near the surface is no more compressed than a log of timber would be if placed upon the glacier, and permitted to share its motion downwards.

Fig. 6.



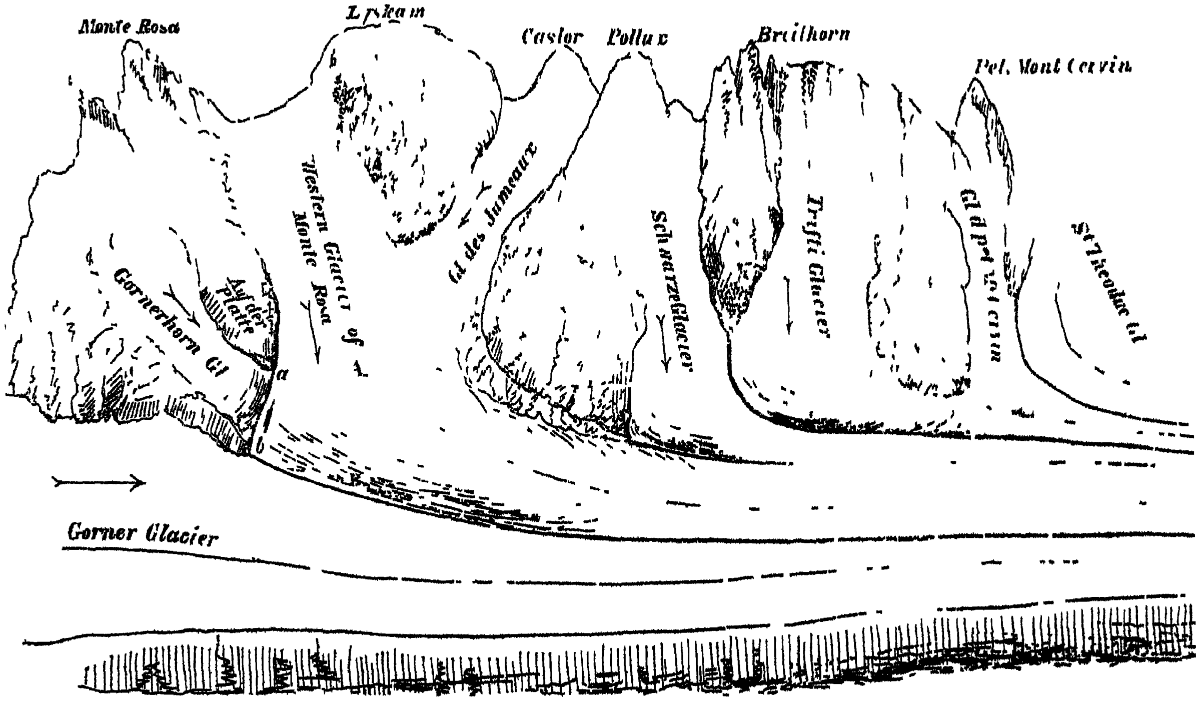
I may sum up by saying that a close examination of the glacier satisfied me, not only that it presented no phenomena which were at variance with the pressure theory, but also exhibited some which, as far as I could see, were perfectly fatal to the theory of stratification. The state of the ice at the base of the Æggischhorn, as shown in fig. 6, is certainly quite in harmony with the pressure theory; another fact observed upon the glacier shall be referred to at a future page.

§ 6. *Glaciers of Monte Rosa.*

I will next endeavour to describe the phenomena of structure exhibited in the system of glaciers in the neighbourhood of Monte Rosa. The general mechanical conditions of these glaciers will be evident to an observer stationed upon the Görner Grat, a point of view well known to travellers, and famous for the magnificence of the panorama which it commands.

As the observer stands here, facing Monte Rosa, the great Görner glacier, coming down from the heights of the old Weissthor at his left, flows beneath him. It is joined, in its course, by a series of glaciers from the sides of the opposite range of mountains. First of all comes the western glacier of Monte Rosa, which really ought to give its name to the trunk stream, as it is the most considerable of its tributaries. Into the glacier of Monte Rosa, and before the latter reaches the trunk valley, a glacier from the Twins, Castor and Pollux, pours its contents. Afterwards we have the Schwarze glacier, which lies between the Twins and the Breithorn; then the Trifti glacier, which lies upon the flank of the Breithorn, and afterwards the glaciers of the little Mont Cervin and of St. Theodule. The accompanying sketch (fig. 7) will render intelligible what I have to say regarding these glaciers.

Fig. 7.



The small Görnerhorn glacier, which comes down the sides of Monte Rosa, is a very singular one. In comparison with the western glacier of Monte Rosa its mass is insignificant, and it is abruptly cut off by the latter along the line *a b*, a moraine occurring here, which may be regarded as forming, at once, the *terminal* moraine of the one glacier and the *lateral* moraine of the other. Thus the smaller glacier coming down the mountain side abuts against its more powerful neighbour, and we should infer from the inspection of the glacier that its terminus is subjected to great pressure.

Let the observer now suppose himself transported to the Görnerhorn glacier, at some distance above the terminal moraine *a b*; he will find there the transverse structure, if at all developed, excessively feeble and defective; let him now walk downwards towards the moraine *a b*: every step he takes brings him to a place where the ice is subjected to a greater pressure, and every step also brings him to a better structure: both phenomena go hand in hand. At the end of the glacier, alongside the terminal moraine and under it, the structure is finely developed. If the observer now cross the glacier and ascend the rocks called *Auf der Platte*, from which he can command a near view of the Görnerhorn glacier, and embrace a large portion of it, he will be able to observe the gradual perfecting of the structure as the region of pressure is approached. Towards the extremity of the glacier the surface becomes wrinkled, the groovings denoting the structure become more and more pronounced, the dirt striæ being more closely squeezed together; and from these external aspects he may infer, with certainty, the gradual perfecting of structure within the glacier.

The western glacier of Monte Rosa next commands our attention. This great stream occupies the valley between Monte Rosa and the Lyskamm, receiving the snows of the opposite sides of both. The branch of the Görner glacier coming down from the Weissthor throws itself across the flow of its powerful neighbour, and deflects the latter, both of them afterwards moving together down the trunk valley, with a moraine, as usual, between them.

Before quitting the "Platte," we will suppose that the observer has endeavoured to form some idea of the mechanical conditions of the Monte Rosa glacier. He would see the mass arrested in its descent by the Görner glacier, and compelled to accompany the latter. A certain component of the weight of the glacier is borne by the ice where it comes into contact with the Görner glacier. The observer would infer, from mere inspection, that if the structure be due to pressure, it ought to be most fully developed near the moraine which separates the Monte Rosa from the Görner glacier.

If he now pass from the "Platte" to the ice, and cross to the centre of the Monte Rosa glacier to A, he will find the structure there excessively feeble, if at all developed. Let him now walk straight down the glacier towards B, where the pressure is most intense. Every step he takes downwards brings him to more perfectly veined ice; and I am not acquainted with a more splendid example of laminated structure than that exhibited by this glacier along the moraine, and for some distance from it, at its southern side.

The system of glaciers which next come under review are exceedingly instructive. In no place in the whole range of the Alps are the effects of pressure and the phenomena of structure more strikingly exhibited. I have endeavoured, in the sketch, to render the aspect of these glaciers intelligible. The Schwarze glacier moves down a steep mountain slope, and welds itself to the Monte Rosa glacier at the bottom. But the great mass of this latter enables it to pursue its way without being compelled to swerve sensibly by its feebler neighbour. The latter is forced to bend abruptly, and from a wide irregularly-shaped field of névé, it is squeezed between the Trifti and the Monte Rosa glaciers to the narrow band represented in the figure, and moves thus downwards.

The Trifti glacier itself is perhaps a still more striking illustration of the power of ice to yield when subjected to pressure for a long period. The aspect of the real glacier is much the same as that shown upon the sketch. *It* also is compelled to change its direction, and to flow as a narrow stripe along the trunk valley, being hemmed in between the strip of the Schwarze glacier and that of the glacier of the little Mont Cervin. A beautiful system of bands is to be seen at the lower portion of this glacier.

The inspection of the sketch will show better than words the modifications of shape which the lower portion of the glaciers undergo by the pressure of their higher portions, and the resistance of the trunk stream. They are turned aside, firmly welded together, and form a series of parallel narrow bands, separated from each other by moraines. They are all well seen either from the Görnergrat or the summit of the Riffelhorn.

I have examined each of these glaciers, and find the same to be true of all of them. High up the structure is feeble; as we descend it becomes more pronounced, and at the places where the tributaries join the trunk, and the ice has to bear the full thrust of the mass behind it, we have a finely developed structure.

§ 7. *Coexistence of Structure and Stratification.—The Furgge Glacier.*

The evidence of the association of pressure and glacier lamination which I have thus far laid before the Society, will, I think, be admitted to be very strong. I have no hesitation in saying that the stratification theory has nothing to urge at all to be compared with it in point of cogency. Still I cannot help feeling how a critical and well-informed mind might weaken the force of what I have adduced. Difficult as the conception is, it might be urged that the structure, so fully developed near the margins of glaciers, may be due to a turning up of the strata edgeways, in consequence of a wide névé being squeezed into a narrow channel,—just as a sheet of paper, if forced through a groove less than itself in width, would turn up at its edges. It might also be urged that the structure developed alongside and under the medial moraines, is due to the placing side by side of these folded-up strata; the perfect welding of both and the clearer development of the structure being conceded as possible consequences of the mutual pressure. This indeed is Mr. Ball's view of the subject; and M. Agassiz assumes such a folding-up of the strata of the Unteraar glacier. With regard to the transverse structure also, it might be said that we do not know how the interior of the mass is affected in descending the ice-falls. The mind, it is true, finds great difficulty in conceiving of any agency which could set the strata which were horizontal above the fall, vertical below it; still this difficulty may be due to our ignorance of the mechanical conditions of the mass during its descent. In this way it would be quite possible to fritter away conviction to a mere opinion, and hence arose a strong desire on my part, either to confirm these surmises, or to place the pressure theory, once for all, beyond the power of such attacks.

One conclusive observation is still wanting to establish the analogy between glacier lamination and the cleavage of slate rocks. In the latter case the arrangement of the strata has been traced by their organic remains; and, indeed, stratification has often been visibly exhibited coexistent with cleavage, both crossing each other at a high angle. If a similar state of things could be detected upon a glacier, it would at once lay the axe to the root of all the scruples above referred to, and place the pressure theory upon an unassailable basis. The consciousness of this was sufficient to stimulate me in the search of such evidence.

I had visited all the glaciers hitherto mentioned, and others not mentioned, without obtaining more than one clear case of the kind: this case I observed upon the Aletsch glacier on the 6th of August. Not far from the junction of the Middle Aletsch glacier with the trunk stream, a crevasse exposed a wall of ice 50 or 60 feet in height, upon which the stratification was exposed, and cutting the stratification at a high angle were the groovings which marked the true veined structure. The association was distinct; my friend Professor Ramsay was with me at the time; I drew his attention to the fact, and to him the case was perfectly conclusive. Thus the Aletsch glacier, which had been referred to by Mr. Ball as furnishing evidence against the pressure theory, gave us a fact, which, as far as I could see, was perfectly fatal to the theory of stratification.

But the case was solitary, and although inspiriting at the moment, its effect upon the mind became feeble as time passed, and no repetition of the observation occurred. I had remained at the Riffel from the 9th to the 18th of August, exploring all the adjacent glaciers, and adding each day to my stock of knowledge; but I met no case in which the structure and the bedding were so clearly and independently exhibited, as to leave an adherent of the stratification theory no room for doubt. Wednesday the 18th of August was to be my last day at the Riffel, and it was devoted to the examination of the Furgge glacier, which occupies the space between the pass of St. Theodule and the Matterhorn.

Crossing the valley of the Görner glacier, I climbed the opposite mountain slope, and passing the Schwarze See, soon came upon the glacier referred to. I walked up it until I found myself in a kind of *cul de sac*, flanked by precipitous ice-slopes, and opposed in front by a cascade composed of four high terraces of ice. The highest terrace was composed principally of broken cliffs and peaks of ice, and it had let some of its frozen boulders fall upon the platform of the second terrace, where they stood like rocking-stones on the point of falling. The whole space at the foot of the fall was covered with quantities of crushed ice, while some coherent masses, upwards of 200 cubic feet in volume, were cast to a considerable distance down the glacier.

Upon the face of the terraces the stratification of the névé was beautifully shown. Above the fall the névé extends as a frozen plain, quite undisturbed, so that the bedding took place with great regularity; and being broken through for the first time at the summit of the fall, the lines of stratification were peculiarly well defined and beautiful.

Towards the right of the fall, looking upwards, this was particularly the case; for here no pressure had been exerted upon the beds sufficient to contort them or to rupture their continuity.

The figure of a vast lake pouring its waters over a rocky barrier, which curves convexly upwards, thus causing the water to rush down it, not only longitudinally over the vertex of the curve, but also laterally over its two arms, will convey to the mind a tolerably correct conception of the appearance of the fall. Towards the centre the ice was powerfully squeezed; the beds were *bent*, and their continuity often ruptured, so as to exhibit faults; but they were as plain, and as easily traced, as in any other portion of the fall. I thought I saw structural groovings running at a high angle to the stratification. Had the question been an undisputed one I should have felt *sure* of this, for the groovings were such as always mark the structure. The place being dangerous, I first observed it from a little distance through my opera-glass; but at length, resigning the instrument to my guide, and leaving him to watch the tottering blocks overhead, and to give me warning in case of their giving way, I went forward to the base of the fall, peeled the grooved surface away with my axe, and *found the true veined structure underneath*, running, in this case, nearly at right angles to the stratification.

The superficial groovings were not uniformly distributed over the whole face of the terrace, but occurred here and there where the ice had yielded most to the pressure. I examined several of these places, and in each instance found the superficial grooving to be the exponent of the true veined structure underneath, the structure being in general nearly *vertical*, while the lines of bedding were *horizontal*. The coarse bands which marked the division of the beds were also seen underneath, when the surface of the ice was removed. Having perfectly, and with deliberation, satisfied myself of these facts, I made a speedy retreat; for the ice blocks were most threatening, and the time of day that at which they fall most frequently.

We now resolved to try the ascent of the glacier to the right; it was much riven, but perfectly practicable to a good iceman. To me it was also perfectly delightful; in fact, as regards the relationship of structure and stratification, this glacier taught me more than all the others I had visited taken together. Our way lay through fissures which exposed magnificent sections, and every step forward added further demonstration to what I had already observed at the base of the fall. The bedding was perfectly distinct, and the structure equally so, the one being at a high angle—sometimes at a right angle—to the other. Among those crevasses the pressure was in some cases greater than on the fall, and the structure proportionally more pronounced. The crumpling of the beds demonstrated the exercise of the pressure, and the structure went straight through such crumplings, thus furnishing me with numerous parallels to the case observed by Professor Sedgwick, Mr. Sorby, and others, of the passage of slaty cleavage through contorted beds. Indeed I question whether the phenomena of cleavage and bedding, in the case of slate rocks, were ever exhibited, side by side, with a distinctness equal to that of the stratification and "structure" of ice in the present instance.

Fig. 9 represents a crumpled portion of the ice, with the lines of lamination passing through those of bedding at a high angle. Fig. 10 represents a case where a fault occurred, the veins at both sides of the line of dislocation *ab* being inclined towards each other. The lines *mn*, *mn* represent of course the lines of bedding, and the lines crossing them the structure. These observations are conclusive as regard the claims of the rival theories of structure and stratification*.

Fig. 9.

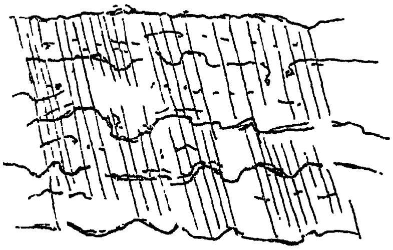
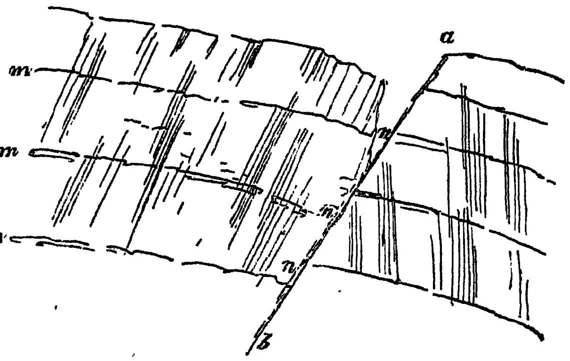


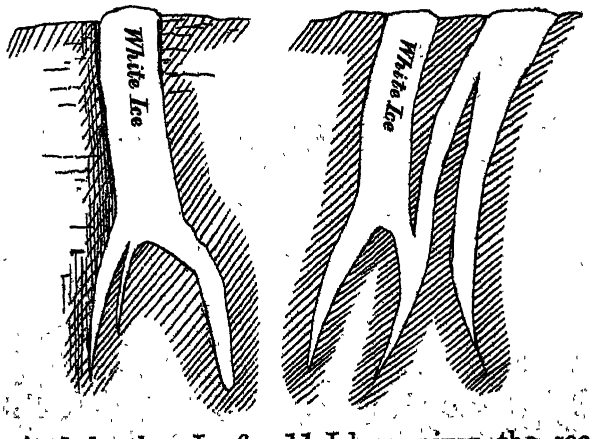
Fig. 10.



§ 8. *On the White Ice-seams of the Glacier du Géant, and their relation to the Veined Structure.*

From an elevated point at Trelaporte I observed a remarkable system of white bands sweeping across the Glacier du Géant in the direction of the structure. From one of the moraines near the junction of the three tributary glaciers, the same system of bands present a very striking appearance. They consist of a hard white ice, more resistant than the general mass of the glacier, and in some cases rising to a height of three or four feet above the surface. On close examination I found that they penetrated the glacier only to a limited depth. In fig. 11 I have given the sections of two of these

Fig. 11.



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* While correcting this proof, I find a case of this kind figured by M. Agassiz in the atlas to his 'Système Glaciaire,' pl. 8. fig. 3.

veins, about 15 feet deep, which were exposed on the walls of a crevasse high up the glacier. They constituted a kind of *inverted glacier trap*, and I was led to a knowledge of their origin in the following way.

In one of my earliest visits to the base of the ice-fall of the Talèfre, I observed a curious disposition of the veined structure on the walls of some of the crevasses: fig. 12 represents one case of the kind, and fig. 13 another, and numerous similar ones find a place in my note-book. In the former case the veins fell *backward* as well as forward, being vertical through the central portion of the curve. In fig. 13 the position of the veins varies in a very short distance from the vertical to the horizontal.

Fig. 12.

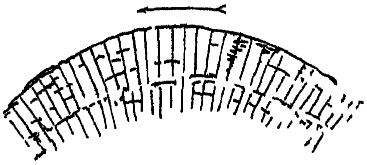
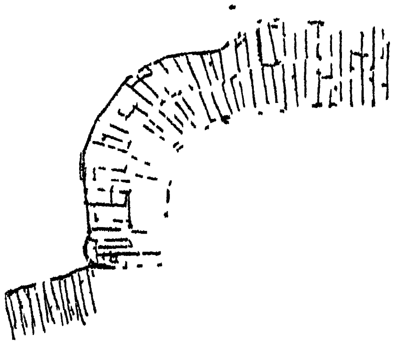


Fig. 13



I found that the portions of ice which showed the phenomena, formed, when seen from a point of view sufficiently commanding, a part of a system of crumples or protuberances which swept round the base of the fall, between the moraine which descends along it from the Jardin, and its highest lateral moraine. I have already referred to the protuberances which sweep across the Strahleck branch of the Lower Grindelwald glacier, and of those of the glacier of the Rhone: those to which I now refer were of the same character.

Right and left from the position where the crumples were most pronounced they gradually became subdued, shading off to a mere undulating surface; the sides of a crevasse intersecting this surface longitudinally presented the structural arrangement shown in fig. 14. It will be observed that the directions of the veins change in accordance with the undulations of the surface.

Supposing the squeezing of the mass to become so violent that the gentle undulations shall become steep crumples, the deviation of the structure from parallelism with itself would, of course, be augmented. This prepares us to understand the exact phenomena observed at the base of the Talèfre cascade. Fig. 15 represents a series of crumples following each other in succession at the place referred to; at the base of each I found a vein of *white* ice, *a*, *a*, wedged into the mass. This interrupted the continuity of the structure; the abrupt change in its direction at opposite sides of the white band being, as shown in the figure, in every case observed.

I found that the width of the seams was exceedingly irregular, varying, at different portions of the same seam, between 6 inches and 3 or 4 feet. I also found that a seam sometimes became forked so as to form two branches, which thinned gradually off until

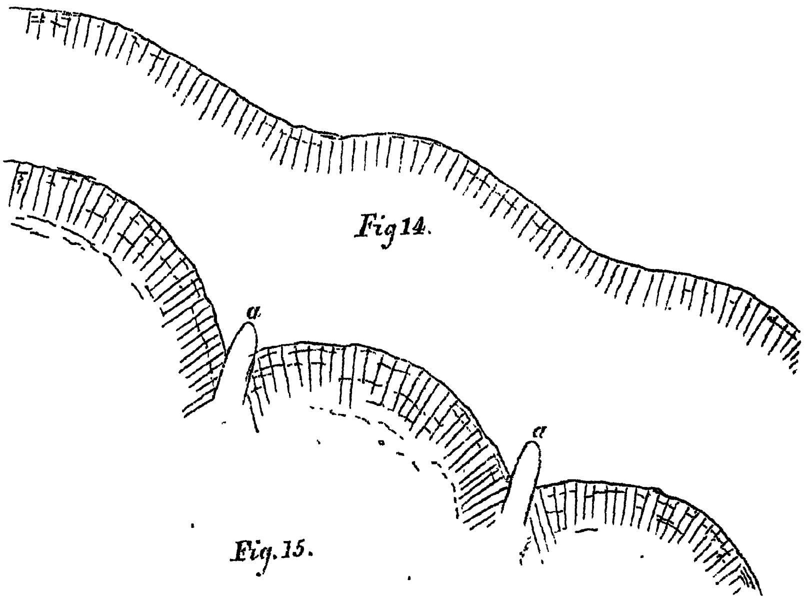
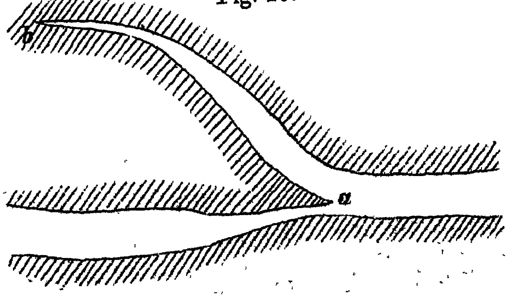


Fig 14.

Fig. 15.

they finally vanished. Fig. 16 is an example of this kind: the seam was divided at the point *a*; one of its branches *ran up the face of the crumple*, thinned off and disappeared at *b*; the other widened considerably, but finally thinned off and also vanished.

Fig. 16.



Along the bases of the crumples the fillets of water which poured down their faces were collected and flowed. The streams thus formed ran in many cases alongside the existing veins of white ice, and had worn for themselves deep channels in the glacier. The thought soon suggested itself, that the seams themselves were formed by the gorging up of those channels by snow in winter, and the subsequent consolidation of this snow during the descent of the glacier. Indeed the channels of the streams seemed the exact matrices of the seams of white ice*.

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* The fact of one branch of a vein running up the face of a crumple, seems to prove that the ice, which at one time constitutes the *base* of a crumple, does not always remain so; the bases of the crumples are sometimes *lifted up* by the squeezing. The *horizontal* structure at the fronts of many of the crumples seems due to a local forcing forward of one protuberance over that next below it. Were the matter tested by strict measurement, I think it would be found that different portions of the crumples move downwards with different velocities. According to this view, upon the general motion of the glacier there are *local* motions superposed.

I afterwards traced the seams of white ice of the Glacier du Géant to their origin amid the ridges and hollows at the base of the great ice-fall of Le Rognon. In some cases the seams opened out into two branches, which, after remaining for some distance separate, would unite again so as to enclose a little glacial island; at other places lateral branches were thrown off from the principal seam, presenting the form of a glacier stream which had been fed by tributary branches. Fig. 17 is the plan of an actual stream observed at the base of the ice-fall; fig. 18 is the plan of a seam of white ice observed the same day lower down the glacier; their relationship is evident. I may remark that I have observed other seams produced by the gorging of *crevasses* with snow, and the subsequent closure of the fissures.

Fig. 17.

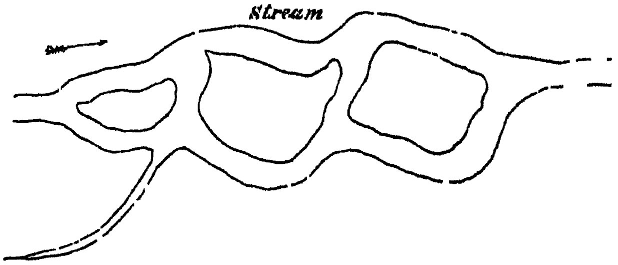
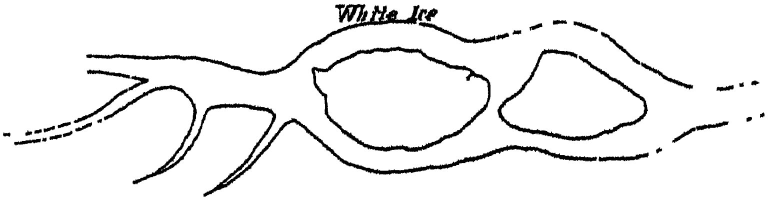


Fig. 18.



Considering the place where they are formed, these channels cannot escape compression; but let me remove all uncertainty on this point, by proving that not only at the base of the seracs, but throughout almost its entire length, the Glacier du Géant is in a state of longitudinal compression.

The first proof I have to offer is that the transverse undulations of the glacier, to which reference has been so often made, become gradually *shorter* as they descend. A series of three of them, measured along the axis of the glacier on the 6th of August 1857, gave the following respective lengths—955, 855, and 770 links, the shortest undulation being the furthest down the glacier. Now these undulations, as I shall subsequently show, are due to a regularly recurrent action, and are doubtless originally of the same length; that the lower ones are shorter than the higher must therefore be due to compression.

The following observation is, however, more conclusive. About three-quarters of a mile above the Tacul, and to the left as we ascend, there is a green patch upon the craggy mountain side. From this spot, as a station, I set out with a theodolite a line (No. 1) transverse to the axis of the glacier.

From a station lower down, chosen in a couloir along which the stones are discharged from the end of a secondary glacier which hangs upon the slope of Mont Tacul, I set out a second line (No. 2) transverse to the axis of the glacier.

A third line (No. 3) was set out across the glacier about a quarter of a mile still lower*.

The mean daily motion of the centres of these three lines is given in the annexed Table, and also their distances apart.

Mean daily motion of three points upon the axis of the Glacier du Géant.

	inches.	Distances apart.
No. 1	20·55	2477 links.
No. 2	15·43	2215 links.
No. 3	12·75	

The advance of the hinder lines upon these in front is most strikingly shown by these measurements; and the proof that the Glacier du Géant is in a state of longitudinal compression is thus complete.

Here then we have a vast ice press, and here we have the pure snow filling the transverse channels of the streams. We are thus furnished with an experimental test on a grand scale of the pressure theory of the veined structure. In 1857 I examined a great number of these seams of white ice, and found in many of them *a finely developed lenticular structure.* In 1858 I also examined the seams, and found some of them "ribboned" in the most exquisite manner by the blue veins; indeed I had never seen the veins more sharply and beautifully developed.

This structure was observed in portions of the seams at and near the centre of the glacier, where the differential motion observed at the sides does not exist. This fact, I think, throws grave difficulties in the way of any theory which makes the veined structure *dependent on differential motion*, and more especially a theory which requires "*a very considerable amount of this differential motion* to produce any sensible degree of stratification in the vesicles."

§ 9. *On the flattening of Air-bubbles in Glacier Ice, and its relation to the Veined Structure.*

Those who have given their attention to the subject, know that the bubbles contained in glacier ice are, in general, not spherical, but *flattened*; and that from their shape conclusions of the greatest import have been drawn regarding the internal pressures of glaciers.

M. Agassiz draws attention to this subject in the following words:—"The air-bubbles undergo no less curious modifications; in the neighbourhood of the névé, where they are most numerous, those which one sees on the surface are all spherical or ovoid, but by degrees they begin to be flattened, and near the end of the glacier there are some that are so flat *that they might be taken for fissures when seen in profile.* The drawing, fig. 10, represents a piece of ice detached from the gallery of infiltration; all the bubbles are greatly flattened. But what is most extraordinary is, that far from being uniform, *the flattening is different in each fragment*; so that the bubbles, according to the face which they offer, appear either very broad or very thin. I know of no more significant fact than this, *since it demonstrates that each fragment of ice is capable of undergoing in the interior of the glacier a proper displacement independently of the movement of the whole.*

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* These three lines are drawn upon the sketch map at page 268 of Part I. of these researches (FF′, GG′, HH′).

"The same flattening of the bubbles," continues M. Agassiz, "is found at a greater depth. While engaged in my boring experiments, I observed attentively the fragments of ice brought up by the borer. I found in them almost flat bubbles, perfectly similar to those of the fragment figured above, at all depths from 10 to 65 metres. It follows *hence* that *a strong pressure is exercised on the interior of the glacier.*"

The description of the "flattening" here given is correct: all observers agree in corroborating it, and every observer with whom I am acquainted draws substantially the same conclusion from the phenomenon that M. Agassiz does. Professor Thomson's speculation upon the subject is particularly refined and ingenious.

Mr. John Ball converts the flattening of the bubbles into evidence against the pressure theory of the structure in the following way:—"As Agassiz has pointed out," writes Mr. Ball, "and I have frequently verified his observations upon this point, though the air-cavities show traces of compression reducing them to the form of flattened lenses, the directions in which they are flattened are most various, *and show no constant relationship to the planes of the veined structure.* Here then we have direct evidence that separate portions of the ice have been acted upon by pressure sufficient in amount to modify their internal arrangement, but that these pressures have not acted in the same, or nearly the same direction."

Granting the inference that the observed flattening "furnishes direct evidence" of pressure, the foregoing argument would, I confess, be a very formidable one. If the bubbles are thus flattened by pressure, and if the veined structure, as I contend, be the result of pressure, and approximately at right angles to the direction of the force, we ought to have the bubbles squeezed out in planes parallel to the structure. The fact that the bubbles are not so squeezed out, would then afford a strong presumption that the structure is not produced by pressure. I expect, however, to be able to prove that the shape of the bubbles is *not* a "direct evidence" of pressure, as hitherto assumed; and I think, as I do so, it will be seen how necessary it is to associate experiment with an inquiry of this kind, if we would read aright our observations.

In a paper in the Philosophical Transactions on the Physical Properties of Ice, I have shown that when a sunbeam traverses a mass of ice, the latter melts at innumerable points in the track of the beam, and that each portion melted assumes the form, not of a globule, but of a flower of six petals. The planes in which these flowers are formed are independent of the shape of the mass and of the direction of the beam through it; they are always formed *parallel to the surface of freezing.*

This is a natural consequence of the manner in which the particles of ice are set together by the crystallizing force. By the slow abstraction of heat from water its particles build themselves into these little stars, and by the introduction of heat into a mass so built the architecture is taken down in a reverse order. In watching the formation of artificial ice, by the machine of Mr. HARRISON referred to in my paper, I have seen little solid stars formed, by freezing, which were the exact counterparts of the little liquid stars formed by melting. So far as I can see, the complementary character of the phenomena is perfectly natural, and presents no difficulty to the mind in conceiving of it.

When the beam is intense, and its action continued for some time, the flowers expand, so as to form liquid plates within the mass. Looked at edgeways, these liquid spaces appear like fine lines; which proves that the melting is not symmetrical laterally and vertically, but that the ice melts in the planes of freezing much more readily than at right angles to these planes.

If an air-bubble exists within ice, and if the ice melts at the concave surface of this bubble, as might be expected from the foregoing facts, the ice will so yield that the composite cell of air and water will not be spherical, even though the bubble of air may originally have been so. In the planes of freezing the mass yields most readily, and the cavity containing the air and water will appear *as if flattened by a force acting perpendicular to these planes.* This is not a deduction merely, but an observation which I have made in a hundred different cases.

What I have here said applies to ordinary lake ice; but glacier ice has no definite "planes of freezing." The substance is first snow, which sometimes, it is true, falls regularly in six-rayed crystals, as observed by myself on the summit of Monte Rosa; but it is usually disturbed by winds, while falling, and whirled and tossed by the same agency after it has fallen; the mountain snow is often melted, mixed with water and refrozen. Even after it has become consolidated it is often shattered in descending precipitous slopes. In such ice definite planes of crystallization are, of course, not to be expected.

If we suppose a mass of lake ice to be broken up into fragments, and these fragments thrown together confusedly and regelated in their new positions to a continuous mass, we have an exact image of the character of the glacier ice in which this flattening of the bubbles in different directions has been observed.

In the paper already referred to, I have given a sketch of a piece of ice composed of such segments, and have described the effects obtained with it. That ice was sold to me as Norway lake ice. I am not aware whether glacier ice is ever imported into this country from Norway; but if it be, the piece in question must, I think, have belonged to it. It is so like all the glacier ice that I have examined since that time, and so unlike all the lake ice, that I feel little hesitation in saying that it belonged to the former*. No matter how coherent and optically continuous a mass of ice may be, a condensed sunbeam would at once tell us whether it belonged to a lake or to a glacier.

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* Perhaps formed from the connecting together of confused fragments.

I have given in fig. 19 a sketch of a piece of ice taken from the end of the great Allalein glacier, on the Swiss side of the Monte Moro. On reference to M. AGASSIZ's figure, it will be quite manifest that we are both dealing with the same phenomenon; we have the division of the ice into "angular fragments," the flattening of the "bubbles," and the non-parallelism of their directions in the different fragments.

Fig. 19

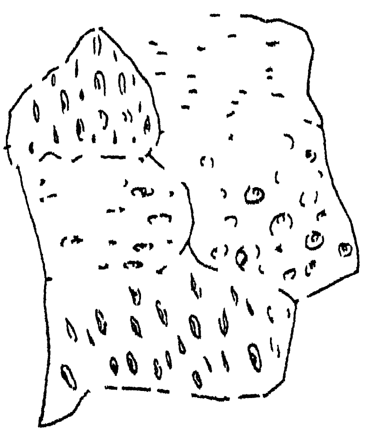


Fig. 20 is a sketch of a piece of ice which showed the veined structure. The line AB was parallel to the veins, and it will be seen that the "bubbles" are inclined to this line at different angles, and in different azimuths. The circles indicate, of course, that the "bubbles" were there parallel to the horizontal face of the slab, while the *lines* indicate that they were perpendicular. In one case the bubbles are seen in plan, in the other case in section. The ellipses show the bubbles foreshortened where their planes are oblique to the surface of the slab.

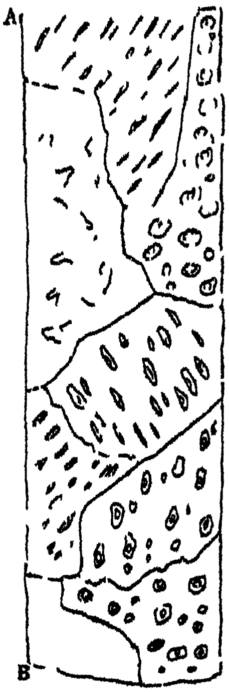
Associated with the air-bubbles, and usually beyond comparison more numerous in ice taken from the "ends" of glaciers, were the round liquid disks which I have described in my paper on the Physical Properties of Ice. Associated with each liquid disk was a *vacuous spot*, which shone with exceeding lustre when the sunbeams fell upon it. That the spots were vacuous, and not bubbles of air, I proved by permitting them to collapse under warm water; the collapse was complete, and no trace of air arose from them.

These, I doubt not, are the "bubbles" observed by M. AGASSIZ "near the end of the glacier," and which were "so flat that they might be taken for fissures when seen in profile."

These "vacuum disks," as I have usually called them, were invaluable as indicators of the planes of crystallization. When a condensed sunbeam was sent through the mass, the six-petalled flowers, which always indicate the planes referred to, started into existence parallel to the disks. Consequently, as the beam passed through different fragments, flowers were formed, in different planes, along the track of the beam.

True air-bubbles, associated with water, also occurred in these masses of ice, and such composite cells were always flattened out in the planes of the vacuum disks.

Fig. 20



The fact then is that many of the so-called air-bubbles are not air-bubbles at all, and that the so-called "*flattening*" is in reality no flattening at all; and that pressure, in the sense hitherto conceived, has had nothing whatever to do with the shape of these bubbles. In glacier ice, as in lake ice, their shape is determined by the crystalline architecture. The conclusion that they were *squeezed* flat seems to have been drawn by M. AGASSIZ, and reproduced by subsequent writers, without due regard to the difficulties associated with it. That the pressures of a glacier are so parcelled out as to squeeze *contiguous fragments* of ice, not exceeding a cubic inch in size, in all possible directions, is so improbable, that reflection alone must throw great difficulties in the way of its acceptance.

It is with some diffidence that I here venture to express an opinion upon a question that I have not specially examined; but it appears to me probable that the decomposition of glacier ice into large granules, regarding which so much has been written, may be connected with the foregoing facts. The ice of glaciers is sometimes disintegrated to a great depth; causing it to resemble an aggregate of jointed polyhedra more than a coherent solid. I was very near losing my life in 1857 on the Col du Géant by trusting to such ice; and last summer I found vast masses of it at the end of the Allalein glacier. Blocks a cubic yard and upwards in volume, fell to pieces to their very centres on being overturned; they were an aggregate of granules, whose average volume scarcely exceeded a cubic inch. From the constitution which the foregoing observations assign to glacier ice, this disintegration seems natural. The substance is composed of fragments which are virtually crystallized in different planes; and it is not to be expected that the union along the surfaces, though they may be *invisible* when the ice is sound, is as intimate as that among the different parts of a mass homogeneously crystallized. Besides, ice no doubt, and all uniaxal crystals, expands by an augmentation of temperature, differently in different directions, and hence a differential motion of the particles on both sides of one of the above surfaces when the volume of the substance is changed by heat or cold is unavoidable. Such surfaces then would become surfaces of discontinuity, and perhaps produce that granular condition which has occupied so much of the attention of observers.

§ 10. *Physical Analysis of the Veined Structure.*

The relation of pressure and structure has been shown in the foregoing pages, but the mode in which the pressure acts remains yet to be considered. As regards their causes, slaty cleavage and slaty structure have been reduced to one and the same; but as regards the operation of that cause, no two things can, I imagine, in some respects at least, be more different.

In a note at page 336 of the 'Proceedings of the Royal Society' for January 1857, I refer to an experiment in which a clear mass of ice was caused by pressure to resemble a piece of fissured gypsum, and I there promised the full details of the experiment in due time. In my paper on the Physical Properties of Ice this promise is fulfilled; I have shown how a mass of compact ice may be liquefied by pressure, in parallel planes perpendicular to the direction of the force, and explained the effect by reference to the ingenious deductions of Mr. JAMES THOMSON from CARNOT'S maxim.

Let the attention now be fixed on the state of a glacier at the base of one of the ice-falls where it is bent so as to throw its surface into a state of longitudinal compression. According to the above experiments, the glacier whose temperature is 32° FAHR. must here be liquefied *in flats*, perpendicular to its axis. A liquid connexion is thus established between all the air-bubbles which are intersected by any one flat, and a means for the escape of this air from the glacier is thus furnished. The water produced is also partially expressed, partially absorbed by capillary attraction of the adjacent bubble ice, and partially refrozen when the pressure is relaxed. It is, I think, perfectly manifest that such a process, each step of which may be illustrated by experiment, must result in the formation of the blue veins*.

All the experiments and observations recorded in the paper on the Physical Properties of Ice were made with reference to the glaciers, and one experiment there recorded illustrates the present point more forcibly than any words could do,—it is that in which I have thrown a prism of ice into a state of compression, which brings one side of it into the exact condition of the glacier at the base of an ice-fall. Fig. 21 is precisely the same as that given at page 226 of my paper, the prism being placed horizontal instead of vertical, so as to show its bearing upon the present point more distinctly. The original shape of the piece of ice is given in fig. 22, which by compression between the surfaces AB and CD is reduced to the shape and condition of fig. 21. The vertical lines represent the planes of liquefaction, and they correspond exactly to the planes of the blue veins in the glacier. The application of the same principles to all cases where pressure comes into play is sufficiently obvious.

Fig. 21.

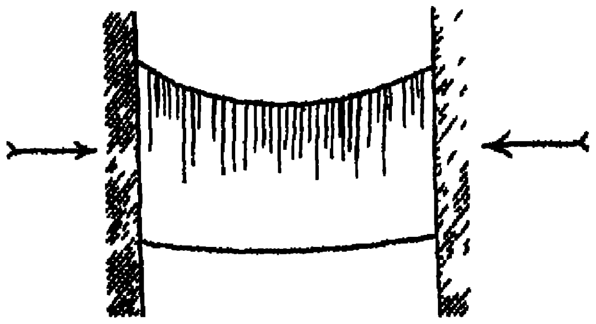
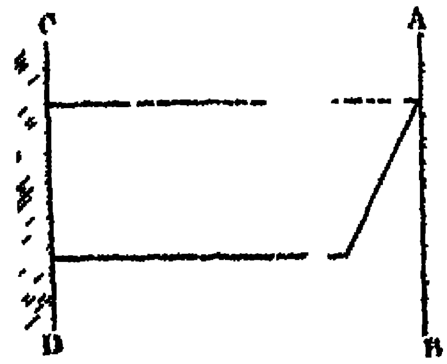


Fig. 22.



In the experiments with the hydraulic press, the portion of ice between each two liquefied flats transmits the pressure without sensible yielding. We have no difficulty in conceiving that the same holds true of glacier ice, and that pressure may be transmitted through one portion of a bubbled mass of ice and produce the liquefaction of another portion, without sensible distortion of the bubbles contained in the former. One of the objections which have been urged against the pressure theory is thus, I conceive, completely answered,—the objection, namely, that the white ice which transmits the pressure ought to have its bubbles flattened. Indeed this objection continued to be of weight only so long as it was imagined that the observed flat bubbles had been squeezed to this shape; a notion, which I think will no longer be entertained.

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* The mechanical actions which accompany the development of ordinary slaty cleavage, must, I think, also manifest themselves to some extent in the glacier.

§ 11. *Remarks on Glacier Motion.*

It is only by slow degrees that we master from actual observation, a problem so large as that presented by the glaciers; the muscular labour alone being such as to render the expenditure of a considerable amount of time unavoidable. The examination of the various questions connected with glaciers, has been therefore, in my case, distributed over some years, and not until last summer was I able to devote the requisite attention to the subject of the present section, which, however, is essential to a right comprehension of the physics of glacier motion.

It would be a problem eminently worthy of any geologist, to lay down upon a trustworthy map of Switzerland the directions of the striæ on the rocks over which ancient glaciers have moved; and to one who sees its importance and desires exact information upon this subject, it must be a matter of surprise that nothing of the kind, in a systematic way, has yet been attempted. A suitable map furnished with such lines of direction, carefully and conscientiously drawn, would impart more satisfactory information than all the volumes that ever have been, or ever will be written upon the subject. Here is a piece of work loudly calling for accomplishment, and one on which any young geologist may base an honourable reputation.

Mr. Hopkins, I believe, was the first to urge the existence of *roches polies* at the ends of existing glaciers and along the continuations of existing glacier valleys as an evidence in support of the sliding theory. That such facts exist is known to every body, and that the rocks are thus polished and rounded by the glaciers sliding over them is incontrovertible. Let a traveller, if he wish to obtain a wealth of information upon this subject, transport himself to the terminus of the Unteraar glacier, and walk thence down the valley through which the river Aar now flows. On all sides he will obtain the most striking evidence that the base of the valley was once the bed of the glacier. The rocks are polished and striated, and present at some places the appearance of huge rounded mounds, which, at first sight, would appear to offer an insuperable barrier to the motion of the glacier, but which show by their aspect that the ice actually moved over them, grinding off their angles and furrowing their summits and sides. All along the valley towards Meyringen, similar evidences exist. In fact, the phenomenon is very common, and admitted on all hands.

The conclusion which Mr. Hopkins has drawn from these facts is unavoidable; the glaciers must have *slidden* over the rocks on which such traces are left. To an eye a little practised in those matters, the precise limits reached by the ancient glaciers are perfectly visible. The junction of the rounded and abraded portions of the mountains, with those portions which in ancient times rose over the then existing ice, is perfectly distinct; and I should say in the valley of the Aar reaches to a height of more than a thousand feet above the present bed of the river. The valley of Saas, in the Canton de Valais, furnishes magnificent examples of the same kind.

At all places, from the base of the ancient glacier to its surface, sliding must have occurred; the evidence of it is perfectly irresistible. The summit of the Grimsel pass constituted the bed of an ancient névé; and the groovings and polishings, at the very summit of the pass, show that the ancient névés, as well as the ancient glaciers, slid upon their beds. In company with my friend Professor Ramsay, and assisted by his great experience, I visited the sites of other ancient névés, and found the same true of all of them; they all slid more or less over their beds.

No investigator of glacier motion can shut his eyes to those facts, nor refuse to give them their proper weight. *The sliding theory is beyond doubt to some extent true*; and many of the objections raised against it, and still repeated in works intended to instruct the public, are altogether futile.

Here, as in other cases, we find that the extreme facts have been dwelt upon principally by rival theorists, and coexistent truths have, by partial treatment, been rendered apparently hostile to each other. It is perfectly certain that a glacier *changes its form* by pressure like a plastic mass, but it is equally true that it *slides over its bed*.

§ 12. *On the Dirt-bands of the Mer de Glace.*

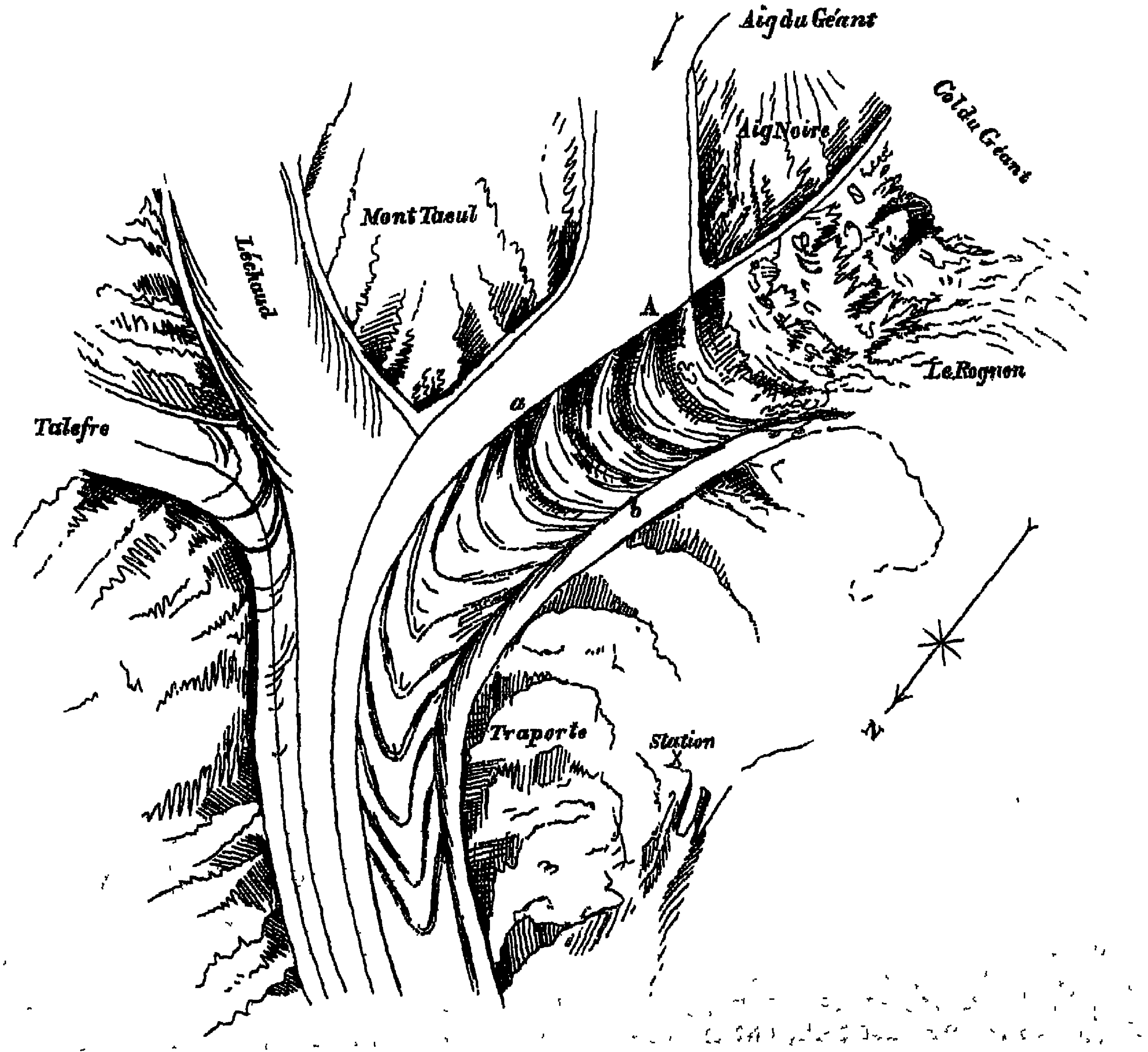
In walking over the Mer de Glace, we soon observe differences in the distribution of the dirt upon its surface; but while standing on the glacier itself, no orderly arrangement of the dirty and clean spaces is observed. From a point, however, which commands a view of a large portion of the glacier, it is seen that the dirty spaces are arranged so as to form a series of broad brown curves, which follow each other in succession down the glacier. They were first observed by Professor Forbes from the heights of Charmoz, on the 24th of July, 1842, and from the same place, on the 16th of July, 1857, I observed them. Last summer I counted eighteen of them from the same position, and this agrees with the number observed by Professor Forbes. This agreement, after an interval of sixteen years, proves the regularity of their occurrence.

These bands were different from anything of the kind I had previously seen, and I felt that the explanation given of the "dirt-bands" observed by Mr. Huxley and myself would not completely account for those now before me. They were perfectly detached from each other, and resembled sharp hyperbolas with their vertices pointing downwards.

From Charmoz, however, I could only see the existence of the bands, but could not see their origin. To observe this, I climbed a summit to the left of a remarkable cleft in the mountain range between the Aiguille du Charmoz and Trelaporte, which strikes most visitors to the Montanvert. The place is that referred to by Professor Forbes at page 84 of the 'Travels,' and which he was prevented from reaching by the fire of stones which a secondary glacier sent down upon him. From the pile of stones on the summit I, however, infer that he or some of his guides must have reached the place. Fig. 23 represents what I observed from this excellent station.

From the base of the Aiguille du Géant and the Periades, a glacier descends which is separated by the Aiguille and promontory of La Noire from the great glacier which descends from the Col du Géant. A small moraine is formed between both, beside which the letter *a* stands in the diagram. The glacier descending from the Col is bounded on the west by the small moraine *b*, and between *b* and the side of the valley is another little glacier derived from one of the lateral tributaries.

Fig. 28.



With regard to the "dirt-bands," the following significant fact at once revealed itself. *The dirt-bands extended over that portion of the Glacier du Géant only which lay between the moraines a* and *b*, or, in other words, were confined to the ice which had descended the great cascade between Le Rognon and La Noire. It was perfectly evident that the cascade was in some way the cause of the bands.

The description which I have already given of the ice-fall of the Rhone and of the Strahleck arm of the Lower Grindelwald glacier, applies generally to the fall of the Glacier du Géant. The terraces, however, are here larger, and the protuberances at the base of the fall of grander proportions. These latter are best seen from a point near A upon the Glacier du Géant; they are steepest on that side, in consequence of the oblique thrust of the western tributaries of the glacier. All that I have said regarding the toning down of the ridges to rounded undulations which sweep in curves across the glacier, applies here also. Referring to the section of the glacier of the Rhone in fig. 4, it will be seen that the word "dirt" is written opposite to each hollow. In fact the depressions between the protuberances are, to some extent, the collectors of the fine superficial dirt. This is also the case upon the Glacier du Géant; but here I noticed that the *frontal slopes* of the protuberances were also covered with a fine brown mud. Lower down the glacier the swellings disappear, but the dirt retains its position upon the ice, and afterwards constitutes the *dirt bands* of the Mer de Glace.

A remarkable change in the form of the bands occurs where the glacier is forced through the neck of the valley at Trelaporte. They sweep across the Glacier du Géant in gentle curves with their convexity downwards; but in passing Trelaporte the arms of the curves are squeezed more closely together, the vertices are pushed sharply forwards, so that on the whole the bands resemble a series of hyperbolas which tend to coincide with their asymptotes.

Looking down from the Convercle upon the Glacier du Talèfre, a series of swellings like those upon the Glacier du Géant are observed. Along the intervening hollows streams run, and sand and dirt are collected, forming the rudiments, so to speak, of a series of dirt-bands; but these latter never attain anything like the precision of those upon the Mer de Glace. I saw no such bands upon the Léchaud, for here the necessary ice-fall is absent: if bands at all exist on this glacier, they must, I imagine, be of a very rudimentary and defective character.

I will not occupy the time of the Society in describing my various expeditions up the Glacier du Géant in connexion with those bands; but one circumstance, to which the definite printing of the bands is mainly due, must be mentioned. The Glacier du Géant lies nearly north and south, being only 14 degrees east of the true north. Standing with his back to the Col du Géant, an observer looks northward, and consequently the frontal slopes of the protuberances to which I have referred have a *northern aspect*. They therefore retain the snow upon them long after it has been melted from the general surface of the glacier. The summer of 1857 was unusually warm in the Alps, but its great heat was not sufficient entirely to remove the snow. No doubt, in colder summers, the snow is retained upon the slopes all the year round. Now this snow becomes the collector of a fine brown mud, which is scattered over the surface of the glacier. It catches the substance transported by the little rills and retains it. The edges of the snow still remaining, when I was on the glacier, were exceedingly black and dirty; and in many cases the entire surface of the snow appeared as if fine peat mould had been strewn over it. Lower down the glacier this snow melts, but it leaves its sediment behind it, and to this sediment the distinctness of the dirt-bands of the Mer de Glace is mainly due.

The regularity of the bands depends on the regularity with which the glacier is broken, and the ridges or terraces formed as it passes over the brow of the fall. It is the toning down of these ridges which produces the undulations, which are to some extent modified by the squeezing at the base of the fall; and it is the undulations which produce the bands. Thus the latter connect themselves with the transverse fracture of the glacier as it crosses the brow of the fall.

In the figure I have given the general aspect of the bands, but not their number. Thirteen of them exist on the Glacier du Géant. I may add that the bearing I have assigned to this glacier differs from that assigned to it on the map which accompanies Professor FORBES's 'Travels on the Alps,' and which I had with me at the Montanvert. The reason is, that on the map the true north is drawn on the wrong side of the magnetic north, thus making the "Declination" easterly instead of westerly. I have since learned that this error is corrected in the smaller work of Professor FORBES.

It has been affirmed that the dirt-bands cross some of the medial moraines of the Mer de Glace, and they are thus drawn upon the map of Professor FORBES. Were this correct, my explanation would be untenable; but the fact is, that the bands are confined to the Glacier du Géant from beginning to end.

Royal Institution, February 1859.

XVI. *On the Fossil Mammals of Australia.*—Part I. *Description of a mutilated Skull of a large Marsupial Carnivore* (Thylacoleo carnifex, Owen), *from a calcareous conglomerate stratum, eighty miles S.W. of Melbourne, Victoria. By Professor* Owen, *V.P.R.S. &c., Superintendent of the Natural History Departments in the British Museum, and Fullerian Professor of Physiology in the Royal Institution of Great Britain.*

Received September 18,—Read December 16, 1858.

In a Report, No. X., on the Geology of the Basin of the Condamine River, by the Rev. W. B. Clarke, to the Honourable the Colonial Secretary of Australia, dated 14th October, 1853, is the following passage:—"It is probable that Mr. Stutchbury, whose studies in palæontology fit him for the search, will be so fortunate as to find the remains of an animal indicated by Professor Owen*, in the year 1842, of a carnivorous kind, for, as he says, 'some destructive species of this kind must have coexisted, of larger dimensions than the extinct *Dasyurus laniarius*, the ancient destroyer of the now equally extinct Kangaroo, *Macropus Titan*, &c., whose remains were discovered in the bone-caves of Wellington Valley.' There were some fragments in the immense heap of osseous matter accumulated by Mr. Turner, which appeared likely to belong to such a carnivorous giant, but they were too small and imperfect to deserve conjectural description. The discovery of what *must have existed* cannot be altogether incapable of demonstration, and, therefore, such a verification of Professor Owen's anticipation is to be hoped for on many grounds."—p. 6.

Now, although such verification has come to hand, I admit that the absolute terms in which the anticipation was expressed merit the mild rebuke implied by the italics in which those terms are emphasized in the quotation from the 'Report' by the accomplished geologist of Australia. Eighteen years of scientific experience have engendered a more cautious tone in referring to inductive probabilities.

The evidence of a large carnivorous marsupial, from pliocene formations in Australia, reached me not many years after my determination of the still larger herbivorous marsupial, *Diprotodon australis*†, which first suggested the idea of the coexistence. That evidence was received in the year 1846 with the accompanying letter from my esteemed friend and correspondent Dr. Hobson, of Melbourne:—

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* Letter to Editor of 'Annals of Natural History,' November 1st, 1842.

† Zoological Appendix to 'Mitchell's Three Expeditions into the Interior of Australia,' 8vo, 1838, vol. ii. p. 362.

"Bona Vista, New Melbourne, 25 January 1846.

"MY DEAR SIR,—I send you, by Captain BURRELL of the 'Achilles,' a box which contains some interesting fossil bones, from a lake eighty miles south-west of Melbourne. They were discovered and kindly forwarded to me by Mr. W. ADENEY, who has a sheep-station on the banks of the lake. I have since visited the lake, which is called by the aborigines 'Colungoolac.' It is very shallow, indeed almost dry in autumn, its muddy bottom being covered with a pretty thick deposit of common salt of excellent quality. This is the case in most of those in this part of Australia. The whole of this part of the country is volcanic, and probably these salt lakes are the deeper parts of the ancient sea. There is one, however, called 'Parrumbat,' which appears to be the crater of an extinct volcano. Its waters are from eighteen to twenty fathoms deep, with abrupt and almost perpendicular escarpments, except at two points, which appear to have been the outlets to streams of lava. The sides are regularly stratified, and consist apparently of condensed scoriæ. The strata are singularly undisturbed and perfectly parallel, except in those places where large globular pieces of compact lava have fallen, and here their direction has been altered, as indicated in this rough diagram. As these are some of the features of the country in which these bones are found, I think, perhaps, it may not be uninteresting to mention them. The fragment of skull and incisor I hope may be new to you.

"I sent you about a year ago a box of the Mount Macedon fossils, by Captain FORDYCE of the brig 'Athens.'

(Signed) "HENRY HOBSON."

The 'skull' consisted of the cranial part (Plates XI. XIII. and XV. fig. 1), similar in size and in the development of the temporal ridges and fossæ to that of a Lion. The 'incisor' was a large tooth with a trenchant or incisive crown, implanted, with a small tubercular tooth, in a portion of the right superior maxillary bone, including part of the orbit and lacrymal bone (Plate XI. fig. 1, p_4, and fig. 2).

The latter specimen gave decisive confirmation of the carnivorous character of the fossil, the 'incisor' tooth (p_4) answering in shape and function to the great sectorial or 'carnassial' (Plate XV. fig. 4, p_4), and the tubercular tooth (fig. 1, m_1) to the small tubercular molar (fig. 4, m_1) of the Lion*; being situated, as in that animal, on the inner side of the back part of the sectorial tooth. Fortunately the nasal process of the maxillary in the detached facial portion of the skull of the *Thylacoleo* fitted a surface at the fore-part of the cranium in such a way as to demonstrate that it formed part of the same skull, completing the lower half of the orbit (Plate XI. fig. 1, o'), of which the upper half (o) remains in the cranial portion of the skull.

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* The real homologies of these teeth can only be determined by specimens of young *Thylacoleo*, showing [illegible] of development and change of the dentition; the symbols here only indicate the conformity of [illegible] function to p 4 and m 1 in *Felis*.

The upper sectorial tooth of the fossil (Plate XI. figs. 1 and 2, p_4) is larger than that of the largest Lion or Tiger which I have seen, and than that of the great extinct Lion (*Felis spelæa*, Plate XIV. fig. 4, p_4). Its antero-posterior extent is 2 inches 3 lines, that in *Felis spelæa* being 1 inch 7 lines. The greatest diameter of the upper tubercular tooth (Plate XI. fig. 2, *b*), which is at right angles to that of the sectorial one, is 7½ lines, that of the Lion (Plate XIV. fig. 4, m_1) averaging 6 lines.

The upper sectorial tooth of the Felines is divided into a 'blade' and 'tubercle,' the latter being developed from the inner side of the base of the fore-part of the crown, and being supported by a fang which makes an extension of the socket inwards at right angles to the rest of the socket. A portion of the fossil tooth has been broken away at this part (Plate XI. fig. 2, p_4), but apparently little more than the enamel; and the socket certainly shows no inward extension indicative of a 'tubercle' so large and distinct as in the Felines (Plate XIV. fig. 4, p_4): the crown of the sectorial in *Thylacoleo* is thicker here than in the rest of its extent, and has been slightly convex on the inside as on the outside of this part of the tooth; but there appears to have been no distinct lobe or tubercle, and I conclude that the crown of the upper great sectorial in the *Thylacoleo* consists exclusively of the 'blade.' The trenchant edge of this is not notched as in the Felines where it is trilobate (Plate XII. fig. 1, p_4), but is even and uniform, describing a very feeble concavity lengthwise (Plate XI. fig. 1, p_4). In the specimen it has been worn to a sharp edge by the play of the blade of a similar sectorial obliquely upon its inner side. The outer side of the crown is convex vertically, wavy lengthwise, being in this direction gently concave at the mid-part, convex at each end, with minor undulations of the surface near the base. The inner side of the crown is gently concave vertically at its mid-part, slightly undulated, but mainly convex lengthwise. The anterior border of the crown is formed by a subdentate ridge, sloping with a slight convexity downward and backward, in vertical extent 1 inch: the crown gradually decreases in this diameter to its back part, which ends in the form of a low protuberance. The tooth is strongly implanted by, apparently, an undivided base coextensive with the crown. I have not thought fit to mutilate the unique fossil to determine the depth and precise character of this implantation. The thickest part of the tooth is 8 lines.

The tooth which most nearly corresponds with the sectorial of *Thylacoleo* is the penultimate upper molar of *Sarcophilus* (*Dasyurus*) *ursinus* (Plate XIV. fig. 2). In this tooth the 'blade' forms the chief part of the crown; it is concave externally, convex internally lengthwise; its edge is entire, slightly concave; but it is associated with an anterior lobe and antero-internal tubercle, wanting in the fossil.

The tubercular tooth (*ib.* fig. 1, *m*, Plate XI. fig. 2, *b*) in *Thylacoleo* is on the inner side of, and at right angles with the sectorial tooth, but is almost half an inch in advance of the hind end of that tooth: in *Felis* (Plate XIV. fig. 4, *m*) it is close to that end. In *Thylacoleo* this tooth consists of a principal portion next the sectorial, and a small lobe (Plate XI. fig. 2, *b*) forming the inner or 'mesial' end of the crown; the principal part rises to a low obtuse point, with a middle longitudinal depression between two convexities, on the outside: the inside slopes forward gradually to the base so as to represent, and act as, a crushing surface. Besides being relatively larger, this 'tubercular' tooth is more deeply and firmly implanted than in the Lion, whence is due its preservation in the present fossil, a circumstance which is very rare in *Felis spelæa*. In no Feline does this tooth present the accessory lobe, as in *Thylacoleo*. In the presence of this lobe at the inner end of the crown, the last small molar in *Sarcophilus* presents a closer resemblance to the same tooth in *Thylacoleo*; but the principal lobe is more pointed and trenchant in the small existing marsupial carnivore, and the whole tooth is so situated that its outer end is visible in a side view. The firm implantation of the last small molar, and its shape, are significant of the affinity of *Thylacoleo* to *Sarcophilus*.

In the Felines the outer wall of the maxillary above the socket of the sectorial tooth is perforated by the large antorbital foramen: it is not so perforated in *Thylacoleo*. The canal for the suborbital nerve and vessels is relatively smaller in *Thylacoleo* (Plate XI. fig. 2, *c*), and must open some way in advance of the socket of the penultimate tooth, as it does in *Sarcophilus*. From that socket to the orbit the outer surface of the maxillary is smooth and even, first gently concave, then as gently convex: it does not show the zygomatic protuberance which intervenes in *Sarcophilus*. The vertical extent of this part of the maxillary is 2 inches, being nearly the same as in the Lion. The border of the orbit is sharper and more produced, especially at the lower and fore part (*ib.* fig. 1, *o'*), than in the Lion.

Sufficient of the palatal part of the maxillary is preserved in this fragment to afford a very significant character of the nature and affinities of the *Thylacoleo*. In most *Marsupialia*, and in all the carnivorous species, the bony palate is interrupted by large vacuities opposite the antepenultimate and penultimate molars. In all placental *Carnivora* the bony palate is here entire; it shows, at least, only a small oblique nervo-vascular foramen at the suture between the palatine and maxillary; and the roof of the mouth is extended by bone some way behind the last molar tooth. In the present specimen of the *Thylacoleo* is preserved the smooth rounded outer border (Plate XI. fig. 2, and Plate XIV. fig. 1, *d*) of a large palatal vacuity opposite the hinder half of the penultimate tooth, and at a distance of 14 lines from it transversely. Such a vacuity extends opposite the penultimate and antepenultimate molar in *Sarcophilus* (Plate XIV. fig. 2).

I now proceed with the description of the larger, cranial, portion of the present fossil, before returning to another character in the smaller portion which I regard as decisive of its marsupial affinities. The cranial part of the skull shows a broad and low occipital surface (Plate XV. fig. 1); the sides (Plate XI. fig. 1) excavated by large temporal fossæ (27'), with their ridges meeting at a low and short parietal crest (Plate XVI. *t'*); the upper surface expanding, in front of this, to a very broad, almost flat interorbital region, 11. The post-orbital processes, 12, with the zygomatic arches, 27, and part of the basis cranii, are broken away. The extreme length of this portion of skull is 8 inches; the least breadth of the cranium, at the temporal fossæ, is 2 inches 2 lines.

The upper border of the occipital foramen (Plate XV. fig. 1) is as broad as in the *Felis spelæa*, and broader than in most of the existing species of Lion or Tiger; it does not present the pair of processes that characterize it in those large placental *Carnivora*. As the occiput rises from this border it slopes forward with a slight concave curve to the ridge, $_{3}$, $_{3}$ (Plate XIII. figs. 1, 3, 8), dividing the occipital from the upper plane of the skull: transversely the occiput is concave in the middle and slightly convex on each side, with a surface marked by musculo-tendinous insertions; the median depression is partly bisected by a vertical ridge (Plate XV. fig. 1, *f*), on each side of which there is a venous foramen. The breadth of the occiput on the level of the upper border of the foramen magnum is 5 inches; its height from the same border 2 inches 2 lines. In *Felis spelæa* the breadth of this part is 3 inches 4 lines; its height being 2 inches 8 lines. The *Sarcophilus* (Plate XV. fig. 2) much more nearly resembles the *Thylacoleo* in its low and broad occiput.

The major part of the basioccipital is broken away (Plate XIV. fig. 1); the anterior portion, which has coalesced with the basisphenoid (*ib.* $_{5}$), forms with it, not a platform extending horizontally forward, as in placental *Carnivora*, but a bent surface forming a curve convex downward as it extends forward; this character is seen in the *Dasyurus macrurus* and in many Kangaroos; but the convexity at the junction of the basioccipital and basisphenoid, $_{5}$, appears to have been greater in the *Thylacoleo*. The base of the left occipital condyle, $_{2}$, remains; and in the fossa anterior to it, are the orifices of three precondyloid foramina (*g*), as in the majority of *Marsupialia*, including the *Dasyuri*; they unite to form a single hole internally in the *Thylacoleo*. In the placental *Carnivora* the precondyloid canal is single at both ends, and commonly opens externally into the jugular foramen (*Viverridæ*, *Hyæna*, *Felis*), or close to it, as in the Dog.

The jugular foramen (*i*) is bounded behind by a notch in the exoccipital, forming the margin turned towards the tympanic, $_{28}$, and which margin is extended further in advance of the precondyloid foramina than in the Dog or any placental Carnivore in which those foramina do not communicate with the jugulars; in this respect the *Dasyuri* and many other marsupials resemble the *Thylacoleo*.

The bones composing the complex framework of the organ of hearing are strikingly different in the placental and marsupial *Carnivora*. In the Cat, Dog, Hyæna, Civet, Otter, Bear, the tympanic bulla is formed by the inflated petrosal with which the true tympanic bone has coalesced; in the marsupials the petrosal remains comparatively small, and is confined chiefly, if not wholly, to the function of a capsule of the internal organ of hearing; the tympanic bulla is excavated in the inflated base of the alisphenoid; and the tympanic bone itself continues a free and distinct ossicle, which, in the Dasyures and Thylacine, is a small thick semicylindrical canal with smooth obtuse margins, and its concavity looking backward and upward.

On the right side of the fractured base of the fossil skull in question, the small compact petrosal (Plate XIV. fig. 1, $_{16}$) is exposed; it is similar to that in the *Dasyurus*, being grooved longitudinally at its inner and under side, the lower border of the groove forming a sharp edge, above which, on the inner side of the petrosal, the foramina auditoria interna pierce the bone. On both sides the tympanic sinuses in the alisphenoid, 6, are exposed; and their concordance with those in the *Dasyuri* is very clearly exemplified on the left side, in which the tympanic bone, 28, is preserved, showing its characteristic shape and relative position behind and external to the alisphenoid bulla, 6.

The canal of the meatus (*k*) external to the tympanic, is excavated in the outwardly produced base of the zygoma, behind the postglenoid process (*l*), for an extent resembling that in the *Dasyuri*, but much greater than in the Dog or other placental *Carnivora*.

Another character distinctive of the marsupial order is the position of the entocarotid canal (*m*), which perforates the outer and back part of the basisphenoid, 5*: this orifice is lodged in a fossa between the basioccipito-sphenoid and the bulla auditoria in *Thylacinus* and *Dasyurus* (*ib.* fig. 2, *m*), and it presents exactly the same position, and perforates the same part of the basisphenoid, in *Thylacoleo*.

In the genus *Felis* the entocarotid enters the base of the skull at the fore-part of the *foramen jugulare*, notching the part of the petro-tympanic bulla at the fore-part of that foramen. In the Hyæna, as in the Viverrines, the entocarotid notches or perforates the tympanic bulla in advance of the jugular foramen close to the side of the basioccipital: it perforates the same part of the tympanic bulla in the Otter and other Mustelines.

The foramen ovale pierces the base of the alisphenoid immediately anterior to the bulla in the marsupial *Carnivora*, and is divided by a ridge from the carotid canal in the *Dasyuri*; it presents the same relations in *Thylacoleo* (Plate XIV. fig. 1, *n*), and the base of the ridge (ib. *s*) also remains to show the existence of that character.

The interval between the foramen ovale (Plate XI. fig. 1, *n*) and foramen rotundum (ib. *p*) is relatively much greater in the marsupial than in the placental *Carnivora*. In the genus *Felis*, they are separated from each other only by the base of the ridge or rising of bone extending from the ectopterygoid towards the glenoid cavity, and the foramen is on the same transverse line with the anterior boundary of that articulation; in the Hyæna, Viverrines, and Dog, it is a little in advance of the same boundary; in the Otter it opens externally into a fossa common to it with the foramen lacerum anterius (or for. ophthalmicum). In the Thylacine and Dasyures the foramen rotundum is distinct both within and without the cranial cavity from the foramen lacerum anterius, and is far in advance of the glenoid cavity. It presents the same relative position in the *Thylacoleo* (Plate XI. fig. 1, *p*).

In *Felis* the foramen rotundum is larger than the foramen opticum; in *Dasyurus* it is much smaller; and this is the case also with the *Thylacoleo* (Plate XI. fig. 1), although the foramen opticum (*q*) is relatively smaller than in the *Dasyurus ursinus*.

[illegible] carotid canals pierce the body of the sphenoid, as in Birds, and terminate in the skull very close [illegible] the *sella turcica*."—Zoological Transactions, vol. ii. (October, 1838), p. 390. See also Mr. [illegible] and minute account of the "Foramina at the Base of the Skull" in Zoological Pro- [illegible]

In the marsupial *Carnivora* the basisphenoid is relatively longer than in the placental *Carnivora*, and, at its posterior part, it sends a ridge downwards from that part of each lateral margin which is not underlapped or covered by the base of the alisphenoid, the suture of which long continues distinct. These ridges, with the alisphenoid, render the whole under surface of the basisphenoid canaliculate, or concave transversely: the basisphenoid is flat beneath in the placental *Carnivora*, and that part of the base of the skull is made canaliculate by the development of the ectopterygoid plate from the alisphenoid: these plates exist likewise in the marsupials, but, as they extend backwards to join the alisphenoidal bullæ, they diverge from the basisphenoid ridges and are external to them.

Sufficient of the base of our fossil skull remains to demonstrate this characteristic marsupial structure: the basisphenoid, though convex lengthwise beneath, is concave transversely by the production from the lateral margins of its hinder part of the same ridges (*r*, *r*) as those of the Thylacine and Dasyures, and in the degree of concavity more resembles the latter: the commencement of the outer ectopterygoid ridge (*s*) of the alisphenoid is preserved, diverging as it extends backwards from its anterior junction with the basisphenoid ridge.

The sutures between the alisphenoids and basisphenoid still remain, indicating the great antero-posterior extent of the former, and the degree to which they underlap the basisphenoid, leaving only a strip 2½ lines broad exposed at its junction with the presphenoid, 9; and gradually diverging as they extend backward, the basisphenoid, 5, being one inch and a half in breadth at their hinder borders.

The characters of the base of the cranium here displayed by the *Thylacoleo*, and the greater retention of the typical elementary construction of the skull, would be sought for in vain in any mammalian *Carnivora*, save those of the marsupial order.

In the placental *Carnivora*, the superoccipital region, defined below by a boundary line drawn across the upper ends of the condyles, is almost as high as it is broad, and in rising from the foramen magnum it curves slightly backward. In the marsupial *Carnivora* the same region so defined is much broader than it is high, especially in the Dasyures (Plate XV. fig. 2); in these the occiput is vertical; it inclines a little forward from the foramen magnum in the Thylacine. All these characters are repeated in the *Thylacoleo*; the occiput being relatively as broad as in *Sarcophilus*, and the superoccipital sloping more forwards than in the Thylacine before it rises vertically to the occipital crest; thus departing in a greater degree from the placental type, and manifesting, as might be expected from the superior general size of the skull, in a more marked manner, the inferiority of development of the brain. In every natural group or family of the warm-blooded Vertebrata the brain is proportionally less as the animal is larger, and its osseous case makes a smaller part of the entire skull.

In the marsupial *Carnivora* the brain is relatively much smaller than in the placental *Carnivora*, and the lateral walls of the cranial cavity make a smaller protuberance or convexity at the temporal fossæ. In the remarkable skull under comparison, the sides of the cranial cavity make no protuberance whatever into the temporal fossæ; they have been moulded solely in obedience to the pressure of the enormous temporal muscles, and present a uniform concavity towards the temporal fossæ. The cranial walls here (Plate XIII. fig. 1, *27*) show as little indication of the brain within as in a cold-blooded reptile: amongst the mammalian *Carnivora* the *Thylacoleo* is unique in this respect; and in the diminutive relative size of its cerebral organ, it is approached only by the Thylacine and the largest existing species of Dasyure.

In the *Das. ursinus* the apparent breadth of the cranial chamber is here greater than it actually is, by reason of the swelling out of the squamosal above the root of the zygoma through the extension therein of tympanic air-cells; and similar air-cells are exposed on the right side of the fossil *Thylacoleo* (Plate XI. fig. 1, *c*); but I know of no species of placental Carnivore in which the squamosal is so modified.

Another equally instructive marsupial character is exhibited by the bony outlet of a vein (ib. *s*), which conducts part of the blood from the lateral sinus to the outer and back part of the cranium: this venous foramen is situated behind the root of the zygoma and above the meatus auditorius in the Thylacine (Plate XII. fig. 2, *s*) and Dasyures. A similar diverticular vein is present in certain placental *Carnivora*, and has its external outlet behind the glenoid cavity and in front of the meatus auditorius, as *e. g.* in the Dog and Otter; there is a small venous outlet on the outside of the tympanic bulla in the Cat and Hyæna; but in no placental Carnivore is such a venous foramen present behind, or piercing the ridge continued backward from the root of, the zygoma.

In the *Thylacoleo* this venous foramen (*s*) is present in nearly the same relative position as in the marsupial *Carnivora*, posterior, viz., to the commencement of the ridge or hind root of the zygoma; in the Dasyure it is below the upper margin of the ridge; in the Thylacine it is posterior to the beginning of the ridge; in the *Thylacoleo* it is posterior and superior to the beginning of the ridge. Thus in the same degree in which the *Thylacoleo* departs in this particular from the largest existing marsupial *Carnivora*, it differs from the placental *Carnivora*, in all of which the foramen, besides its other differences of position, is quite below the zygomatic ridge in question.

The interorbital part of the upper surface of the cranium (Plate XIII. fig. 1, *11*) is remarkable in the marsupial *Carnivora* for its great breadth, especially as compared with that of the cerebral portion of the cranium; the transverse diameter of this part at the middle and highest part of the upper border of the squamosals is, in the *Dasyurus ursinus*, less than half the same diameter of the narrowest part of the interorbital portion of the cranium. In the Tiger, Lion, and *Felis spelæa* the diameter of the interorbital space is one-seventh less than that of the cranium of the *Thylacoleo*, taken across the same part as in the *Dasyurus*. In the *Thylacoleo* the least diameter of the interorbital surface is 2 inches 10 lines; the diameter of the cranium opposite the middle of the upper border of the squamosals, *27*, is 1 inch 3 lines.

The broad interorbital platform of the *Thylacoleo*, with a broad and shallow depression, and [illegible] slight lateral convexities at its anterior half, passing posteriorly into an almost flattened surface, decreasing to the point where the temporal ridges (*t'*) meet above the parietal, forms, in contrast with the contracted cerebral part of the cranium, a conspicuous marsupial character of the skull.

In the *Thylacoleo* the squamosal (Plate XI. fig. 1, *27*) extends forward in the temporal fossa nearly half-way between the root of the zygoma and the postorbital process, and two-thirds of the way upward, *27'*, between the root of the zygoma and the parietal ridge; its contour is almost semicircular. In the Felines the squamosal extends a very little way, if at all, in advance of the base of the zygoma, and does not ascend half-way from that part to the parietal ridge (Plate XII. fig. 1, *27*). It is only in the marsupial *Carnivora* that we find those proportions of the squamosal which characterize the *Thylacoleo*.

On the inner wall of the right orbit the fronto-lacrymal suture shows that the lacrymal bone, *73*, was of large size, that it formed the anterior half of that wall, and extended upon the upper part of the skull, forming apparently the anterior superorbital protuberance, besides extending forward upon the facial part of the skull, as far as that part anterior to the orbit has been preserved in the fossil. The lacrymal bone presents the same relative dimensions and extent in the largest existing *Dasyurus* (*D. ursinus*), in which the lacrymal duct pierces, not the orbital, but the facial, plate of the lacrymal bone, and is consequently outside the orbit.

In the Lion, the *Felis spelæa* (Plate XII. fig. 1), and other placental *Carnivora* in which the lacrymal bone is best developed, it is almost confined to the orbit, its most forward portion forming about the middle third of the anterior margin of the orbit, where it developes a slight protuberance; its orbital plate, moreover, does not attain that part of the inner wall of the cavity where it is so conspicuous in the *Thylacoleo* and *Dasyurus*, but extends backward along the lower part of the inner wall to join the orbitosphenoid. The lacrymal foramen, *73'*, is within the orbit.

The *Thylacinus* (Plate XII. fig. 2), which retains the marsupial proportions of the lacrymal bone, has an intraorbital perforation, besides two antorbital ones, *73'*: in most marsupials there are only the two antorbital lacrymal holes, and in the *Dasyuri* there is only one lacrymal foramen, which is outside and in front of the orbit; the *Thylacoleo* (Plate XI. fig. 1, *73'*) resembles the *Dasyuri* in its single antorbital perforation of the lacrymal bone, and this is one of the decisive marks of its marsupiality.

The postorbital process has been fractured on both sides; but on the left sufficient is preserved to show that the hind and front sides meet at a right angle, and form a ridge at its under part.

Sufficient of the articular surface (Plate XIV. fig. 1, *8*) for the lower jaw is preserved on the left side of the skull of the *Thylacoleo*, to show that it had a greater antero-posterior extent than in *Felis spelæa*, and was flatter at its fore-part, the margin there not being so produced: the same character is shown in *Dasyurus ursinus*. The post-glenoid process is fractured.

I am indebted to Mr. Samuel Stutchbury, F.L.S., for a cast of a portion of a right ramus of a lower jaw of a large Carnivore, a fossil which he obtained at Hodgson's Creek, Darling Downs, during his geological survey of that district of Australia in the year 1853.

This portion (Plate XI. fig. 3; Plate XIII. figs. 4 and 5) fortunately includes the carnassial and tubercular teeth, and by the correspondence of these in shape and size with the answerable teeth (p_4, m_1) in the upper jaw of *Thylacoleo* (Plates XI. and XIV. fig. 1), I believe it to belong to the same species.

The lower carnassial tooth consists only of the 'blade,' which is thickest anteriorly, with an even trenchant edge, describing a slight concavity lengthwise, and obliquely abraded by the play of the upper tooth upon the outer side of the edge. The outside of the tooth (Plate XI. fig. 3, p_4) is convex lengthwise, and also vertically at the fore-part: the inside (Plate XIII. fig. 4, p_4) is concave lengthwise, except near the fore-part; vertically it is convex at the base and concave above, the base being slightly grooved vertically. The anterior margin of the tooth, which is the highest, is bounded by a vertical ridge. The length of the crown is 1 inch 8 lines; the height at its fore-part is 9 lines; the thickness at the same part is 6 lines: the height and thickness of the crown diminish toward the back part. The small tubercular tooth (Plate XI. fig. 3; Plate XIII. figs. 4 and 5, m_1) is immediately behind the preceding: it consists, more distinctly than the one above, of an anterior principal lobe and a small posterior one; the anterior lobe is convex on both sides, subconical, with a worn obtuse summit. The antero-posterior extent of the crown is 7 lines; the same extent of both the teeth just equals that of the upper sectorial, and the lower tubercular is so situated as to play, in lateral movements of the jaw, upon the upper tubercular. Behind the tubercular in place there is the socket of a rudimental one, which cannot have exceeded 3 lines in its longest diameter (ib. m_2). From this socket the coronoid process begins to rise, sloping upward and backward. It is broken off within half an inch of its origin. The fore-part of the fossa for the implantation of the temporal muscle is preserved, showing by its depth the strength of that muscle; the anterior boundary of the fossa is vertical and convex forwards. The ramus of the jaw preserves the same depth from the last socket to the fore-part of that of the sectorial tooth. The symphysis (Plate XIII. fig. 4, *s*) begins behind, at a vertical line dropped from a little in advance of the middle of the sectorial, p_4; it is of a wide oval form. To judge from the cast, but little of the jaw appears to have been broken away from the fore-part of the symphysis. The upper and fore-part shows the alveolus and base of a tooth (Plate XI. fig. 3, *c*) which has projected obliquely upward and forward. It is separated by an interspace of 3 lines from the sectorial, and would seem to be the sole tooth in advance of it. If the ramus be really produced at the upper part of the symphysis further than is indicated by the present cast, it may have contained one or more incisors, and the broken tooth in question may be the lower canine. If, however, this be really the foremost tooth of the jaw, it would appear to be one of a pair of large incisors, according to the marsupial type exhibited by the *Macropodidæ* and *Phalangistidæ*. However this may ultimately prove to be, the molar series in each ramus of the lower jaw is reduced to the enormous sectorial and two small tuberculars; and it would seem, therefore, in the upper jaw, to be reduced to the single sectorial and single tubercular on each side. It is possible that a canine may have existed in the upper jaw as much longer and larger than that below, as the upper canine is in the extinct *Machairodus*. But sufficient is demonstrated in the above-described fossils to make known the most anomalous dental system in carnivorous Mammalia, whether placental or marsupial.

There is an interesting resemblance between the known dentition of the lower jaw of *Thylacoleo* and that of the small extinct mammal from the Purbeck strata called *Plagiaulax* by Dr. Falconer*; and the resemblance would be closer should the broken tooth in the lower jaw of *Thylacoleo* here described prove to be the foremost one. Certainly no other known mammal shows two posterior tubercular teeth so similar to those in *Plagiaulax minor*, in their relative size to each other, to the trenchant tooth in front and to the ramus of the jaw, as does the *Thylacoleo*.

The anterior orifice of the dentary canal (Plate XI. fig. 3, *o*) appears, in the cast, to have been in the fossa, on the outside of the jaw, between the socket of the sectorial tooth and the one anterior to it. As much of the lower border of the jaw as is preserved is straight. Not enough of the back part of the jaw remains to indicate the form or direction of the angle. But the lower jaw of *Thylacoleo* must have been singularly short in proportion to its depth and breadth, and a like extreme shortness of the muzzle or facial part of the skull may be inferred.

With the above-described portions of the cranium of the *Thylacoleo*, I received from my friend Dr. Hobson a portion of a sectorial tooth with one of the fangs. It was so similar in the character of the crown to the great sectorial in place, that I had no doubt about the genus to which it belonged, but only as to whether it was a smaller anterior sectorial of the upper jaw, or the sectorial of the lower jaw. Mr. Stutchbury's specimen has settled that doubt. The tooth (Plate XI. figs. 4, 5 and 6) is the hinder half, with the hinder fang of the left sectorial of the lower jaw. The characteristic markings and undulations or grooves of the enamel, and the thickness of this substance where it is exposed by the abrasion of the trenchant edge, are carefully shown in the figures.

In existing carnivorous mammals the ferocity of the species is in the ratio of the 'carnassiality' of the sectorial molar, *i. e.* of the predominance of the 'blade' over the 'tubercle;' and this ratio is shown more particularly in the upper sectorial, in which, as the tubercular part enlarges, the species becomes more of a mixed feeder, and is less devoted to the destruction of living prey. From the size and form of the carnassials of *Thylacoleo*, especially of the upper one, we may infer that it was one of the fellest and most destructive of predatory beasts.

The metacarpal bone (Plate XIII. figs. 6, 7 and 8) is here figured, as it resembles in its shape that of a large carnivorous animal, and may possibly belong to the *Thylacoleo*. The figures preclude the necessity of verbal description. It is from a freshwater deposit in Darling Downs, Australia.

On the occasion of a visit to London, in 1848, by the able comparative anatomist and palæontologist M. Paul Gervais, at the period when the supposed marsupial character of the *Pterodon* or *Hyænodon* of the Miocene deposits of Auvergne, Gard, and Vaucluse was under discussion, I took the opportunity to point out to M. Gervais certain characters deducible from the 'foramen caroticum' and 'foramen lacrymale' bearing on this question, and illustrated my conclusions by reference to the then unique carnivorous fossil which I had a short time before received from Australia.

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* Proceedings of the Geological Society, March, 1857.

The estimable author of the 'Zoologie et Paléontologie Françaises,' 4to, 1848–52, enters the genus *Thylacoleo* in the Table of Fossil Mammalia according to their geographical arrangement[*]; and in his remarks on those of Australia (Nouvelle-Hollande), he writes, "Ses dépôts pliocènes ou pleistocènes ont fourni des Grands Kangaroos, un grand Wombat[†], diverses autres espèces congénères de celles d'à présent, les genres de *Diprotodon* et *Nototherium* qui étaient aussi des Marsupiaux, mais dont les allures et la taille approchaient de celles de nos grands pachydermes diluviens, et le Dasyurien, plus grand que le Lion, que M. Owen nomme *Thylacoleo*[‡]."

I cite this passage in testimony of the date of my determination of the marsupial nature of the great carnivorous Australian fossil, and of the imposition of its generic name; because the portion of the lower jaw with the carnassial and tubercular teeth of the same extinct species, which was obtained by my friend Mr. Stutchbury during the period in which he was fulfilling his valuable duties as "Geological Surveyor" of the colony of Australia, is alluded to under the name *Schizodon* in a Report to the Colonial Secretary, dated "Darling Downs, 1st October, 1853."

If this generic name had had priority of the one given by me to the same extinct genus, it must have been suppressed, since *Schizodon* had been previously applied in 1829 to a genus of fishes, which still retains it, by Agassiz[§]; to a genus of mammals by Mr. Waterhouse, in 1842; and, slightly modified as *Schizodus*, to a genus of mollusks by Mr. King. Of course the two latter applications, like that by Mr. Stutchbury, must fall into the subordinate rank of synonyms.

The additional fossil of the *Thylacoleo* discovered by Mr. Stutchbury is a very welcome one. It was not, indeed, sufficient to guide the Colonial geologist to an idea of the order of *Mammalia* to which it belonged; and Mr. Stutchbury concludes his brief notice of the fossil by the remark, "Its affinities had better be left for future discussion, as it is probable that further search may bring to light more remains illustrative of this very singular animal[||]."

Such remains had, however, been obtained by Mr. Adeney, and had been transmitted to me eight years previously; and the chief conclusion as to the affinities of the animal to which they belonged, had been indicated by the term *Thylacoleo*, i. e. Marsupial or

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[*] *Op. cit.* vol. i. p. 190.

[†] That, viz., which is alluded to as being "at least four times as large as either of the known existing species," in my Memoir on the existing Species of *Phascolomys*, of July 1845, Trans. Zool. Soc. vol. iii. p. 306.

[‡] *Op. cit.* vol. i. p. 192.

[§] Selecta Genera et Species Piscium Brasiliensium, 4to, 1829.

[||] Papers relative to Geological and Mineralogical Surveys, 1853, p. 10.

Pouched Lion[*], which conclusion was based on the characters and comparisons of those fossil remains detailed in the foregoing pages.

A desire to exhaust every needful and available subject of comparison has occasioned the long delay in communicating descriptions of the present selection of fossil remains of Australian Mammals.

The concurrence in them of so many cranial characters found only in the *Marsupialia*, will be deemed, I apprehend, demonstrative of the marsupial nature of the *Thylacoleo*; and, amongst existing *Marsupialia*, the *Sarcophilus* or *Dasyurus ursinus*—at present the largest existing species of its genus—seems to me to have the nearest affinities to the *Thylacoleo*, although the interval be still very great between them.

Description of the Plates.

PLATE XI.

Fig. 1. Side view of the cranium and part of the upper jaw of the *Thylacoleo carnifex*:—nat. size.

Fig. 2. Inside view of part of the upper jaw, showing both the sectorial and tubercular molars of ditto.

Fig. 3. Outside view of part of the lower jaw of the *Thylacoleo carnifex*.

Fig. 4. Inside view of part of the left lower carnassial tooth of the *Thylacoleo carnifex*.

Fig. 5. Outside view of the same specimen.

Fig. 6. Upper view of the same specimen.

PLATE XII.

Fig. 1. Side view of the skull of the *Felis spelæa* (from European Bone-cave):—half nat. size.

Fig. 1 *a*. Outline of the sutures between the nasals, 15, and frontal, 11, and between the superior maxillary, 21, and the frontal, 11, showing the backward extension of the maxillaries, which distinguishes the Lion from the Tiger:—nat. size.

Fig. 2. Outline of the skull of the *Thylacinus Harrisii*:—nat. size.

PLATE XIII.

Fig. 1. Upper view of the cranium of the *Thylacoleo carnifex*:—two-thirds nat. size.

Fig. 2. Upper view of the cranium of the *Dasyurus* (*Sarcophilus*) *ursinus*:—nat. size.

Fig. 3. Upper view of the cranium of the *Thylacinus Harrisii*:—nat. size.

Fig. 4. Inside view of part of the lower jaw of the *Thylacoleo carnifex*:—nat. size.

Fig. 5. Upper view of the same fossil.

Fig. 6. Side view of a metacarpal of a carnivorous quadruped; from Australian pleistocene.

Fig. 7. Proximal end of the same.

Fig. 8. Distal end of the same.

[*] From *θύλακος, marsupium*; *λέων, leo*.

PLATE XIV.

Fig. 1. Base view of the mutilated fossil cranium of the *Thylacoleo carnifex*:—nat. size.
Fig. 2. Base view of the cranium of the *Dasyurus* (*Sarcophilus*) *ursinus*:—nat. size.
Fig. 3. Base view of the cranium of the *Thylacinus Harrisii*:—nat. size.
Fig. 4. Part of the palate, with the sectorial (*p* 4) and tubercular (*m* 1) molars of the Lion (*Felis Leo*):—nat. size.

PLATE XV.

Fig. 1. Occiput of the *Thylacoleo carnifex*:—nat. size.
Fig. 2. Occiput of *Dasyurus* (*Sarcophilus*) *ursinus*:—nat. size.
Fig. 3. Occiput of *Thylacinus Harrisii*:—nat. size.
Fig. 4. Occiput of *Felis spelæa*:—nat. size.

The letters and figures are explained in the text.

XVII. *On Colour-Blindness.* *By* William Pole, *F.R.A.S.*, *F.G.S.*, *Mem. Inst. C.E.*, *Professor of Civil Engineering, University College, London. Communicated by* Charles Manby, *F.R.S.*

Received March 24,—Read April 7, 1859*.

1. My object in this paper is to state my own case of Colour-Blindness, which I believe to be one of the most decided on record; and to show that the general phenomena attending this defect of vision are more simple, uniform, and consistent than is generally supposed.

2. Some apology is perhaps necessary for a colour-blind person undertaking to treat, even indirectly, of the subject of colours; on which the views formed from his own experience must necessarily be very limited. It is not, however, my intention to trespass upon the province of the normal-eyed. All I wish to do is to give a statement, as explicit and accurate as I can, of the facts connected with the sensations of myself and others similarly situated, leaving these facts to be discussed by others more competent to deal with them.

And as far as the correct determination of these facts is concerned, it seems to me that we, the colour-blind, are really in a better position for investigating their nature than the normal-eyed.

Such an investigation must be based entirely on a comparison of the impressions experienced respectively by two distinct classes of individuals. The normal-eyed person experiences one set of sensations, the colour-blind person a set entirely different; neither can see what the other sees; and each must therefore draw his ideas of the other's impressions solely from the description communicated to him.

Now in such case, supposing it is necessary for one of the parties so far to understand both sets of phenomena as to describe faithfully the difference between them, it is evident that he will be most favourably situated who can procure the best testimony from the other side; and this is certainly the position of the colour-blind. The greatest difficulty in dealing with the subject has been hitherto, and still is, the limited and imperfect nature of the evidence obtainable from those labouring under the defect; and this is natural for several reasons. In the first place the number of persons afflicted is very small, only (as far as can at present be computed) about two or three in every hundred. Of these again but a small proportion know of their defect; still fewer have cared to examine themselves upon it; and there are only a much smaller minority still, who, being accessible, have qualified themselves to explain their views in such a manner as to represent them faithfully. For it is important to consider that it is by no means an easy thing for a colour-blind person to explain his impressions to a normal-eyed one in language free from misapprehension; and I shall hereafter have occasion to show that for this reason erroneous inferences may often be drawn from perfectly honest testimony.

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* This paper is in the main identical with a former paper by the author, received June 7 and read June 19, 1856 (see 'Proceedings,' vol. viii. p. 172), which is here presented in an abridged and modified form, with the addition of the results of some experiments with Professor Maxwell's colour-top, made by the author in the interval.

But let him be the inquirer instead of the witness, and the case is wholly reversed; he may collect evidence about colours in every possible shape, from the scientific investigations of the philosopher to the artless remarks of the child; he need not go far for an answer to any question he may choose to put, however abstruse; and as to common facts, almost every one he meets is a competent witness from whom he may gather them.

I conceive, therefore, that the careful and patient investigation by any colour-blind person of his own case, supposing him to embrace the opportunities open to him for ascertaining clearly and fully the ordinary facts and impressions regarding colour in general, is likely to lead to a more faithful representation of the phenomena, than when, as is usually the case, the comparison is undertaken by the normal-eyed.

3. It is not necessary for me to enter into the optical theory of colours generally. I only require the means of illustrating, in the simplest manner possible, the facts I have to bring forward; and this, I conceive, may be sufficiently done by reference to well-known colours in ordinary use. And as it is convenient to adhere to some definite system of colour-nomenclature, I propose to adopt, for the purposes of the paper, the popular hypothesis that there are three primary colours, red, blue, and yellow, by combinations of which all hues in nature may be supposed to be formed.

4. The existence of a defect in the eye, which gives rise to an abnormal vision of colours, is now well established. Its nature has been discussed, and descriptions of cases have been published, within the last few years, by various writers; but it will be sufficient here to refer to the latest and most complete work on the subject, namely, that by Professor Wilson of Edinburgh*, who has made this defect his particular study.

5. Dr. Wilson considers colour-blindness, existing in eyes otherwise normal, as of three kinds:—

A. Inability to discern any colour, properly so called; so that black and white, *i. e.* light and shade, are the only variations of tint perceived. Cases of this total blindness to colour are very rare.

B. Inability to discriminate the nicer shades (hues) of the more composite colours. This is so frequent as to be apparently rather the rule than the exception; and more than one of our most eminent artists have expressed to me their conviction that there are very few people who have a perfect appreciation of the nicer distinctions of colour. It is, however, probable that this often arises rather from want of education of the eye, or want of ability to describe the impressions perceived, or both combined, than from any physical defect of the visual organs. My own eyesight is very sensitive to minute differences of tone or hue[*], in regard to those colours which I can see.

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* Researches on Colour-Blindness. By George Wilson, M.D., F.R.S.E. &c. Edinburgh, 1855.

C. The third variety is the only one with which we have now to do. It is a real physical defect in vision, of a milder character than the first kind, but much more common; and its outward manifestation is a natural inability to distinguish between many of the colours most marked and distinct to normal eyes. It is yet an open question what degrees of severity may exist in this defect; but its most complete form is that called *dichromic* vision, of which my own case is a very perfect example.

6. I was born in 1814. My eyesight, in every other respect but that of colour, is remarkably good. I am very fond of bright colours, and of scenes and pictures that contain them.

I believe I was about eight or ten years old when the mistaking of a piece of red cloth for a green leaf betrayed the existence of some peculiarity in my ideas of colour; and as I grew older, continued errors of a similar nature led my friends to suspect that my eyesight was defective; but I myself could not comprehend this, insisting that I saw colours clearly enough, and only mistook their names.

I was articled to a civil engineer, and had to go through many years' practice in making drawings of the kind connected with this profession. These are frequently coloured, and I recollect often being obliged to ask, in copying a drawing, what colours I ought to use; but these difficulties left no permanent impression, and up to a mature age I had no suspicion that my vision was different from that of other people. I frequently made mistakes, and noticed many circumstances in regard to colours which temporarily perplexed me[†]; but I still adhered to my first opinion, that I was only at fault in regard to the names of colours, and not as to the ideas of them; and this opinion was strengthened by observing that the persons who were attempting to point out my mistakes, often disputed among themselves as to what certain hues of colour ought to be called.

I was nearly thirty years of age, when a glaring blunder, persisted in by me in opposition to the positive evidence of others, led me seriously to suspect that my vision of colours must be defective; and this suspicion once admitted, it was soon confirmed and strengthened by further observation. I cannot recollect what process of investigation I followed, but I succeeded in determining, from my own case, the general facts now known to characterize the defect, and made out its principal feature to be an insensibility to one of the primary colours. I subsequently became acquainted with the accounts of similar cases, and have now the advantage of a large fund of information collected on the subject, which enables me to analyse and express my sensations in a much more complete way than I could have done a few years ago.

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[*] I use the word *hue* always to denote a distinct variety of colour. The variations in illumination or shade which may be given to any hue by its mixture with white or black, I call *tones*. The light tones produced by diluting it with white are technically called "tints;" the dark ones formed by darkening it with black are called "shades." Thus the addition, to any pigment, of white or black will only alter its *tone*; the addition of any other colour will alter its *hue*.

[†] I recollect in particular, having wondered why the beautiful rose light of sunset on the Alps, which threw my friends into raptures, seemed all a delusion to me.

It will be convenient first to describe what I may call the *symptoms* of the malady, or the effects it produces on my judgment of colours; and then to endeavour to trace more accurately how these effects arise.

7. The symptoms of my colour-blindness, as manifested by the errors I commit in judging of colours, are so varied and heterogeneous in character, that it would be an endless task to follow the usual practice of particularizing the mistakes to which they have led me. I therefore prefer stating them in a generalized list, such as it might be supposed a careful normal-eyed person would prepare, who had been examining me fully, and had reduced the result of his observations into the most condensed form.

Symptoms of Colour-Blindness.

A. Blue and yellow are always perfectly distinguished, even in their lightest or darkest tones, and are never confounded with each other.

B. Only these two colours are seen in the spectrum, the blue corresponding to the more, and the yellow to the less refrangible rays. The red space is seen as yellow.

C. Red is often confounded with black, or more properly grey.

D. It is also confounded with orange, and (E.) with yellow.

F. Also with green. This is the most common symptom of all.

G. Also with brown, (H.) with blue, and (I.) with violet.

K. Crimson and pink appear to have no relation to the idea of red derived from vermilion or a soldier's coat.

L. Red is frequently identified, when of a full tone, and when the hue is scarlet, or some other tending towards orange.

M. Green is a colour most perplexing to the patient, who cannot be said generally to manifest any definite sensation about it at all.

N. It is not only confounded with red, but also with black, white, or grey.

O. Also with orange; (P.) with yellow; (Q.) with blue; (R.) with violet; and (S.) with brown.

T. Orange is confounded with yellow.

V. Violet is confounded with black or grey, and (W.) with blue.

X. Light or dark tones of certain colours are more liable to mistake than full tones.

8. Symptoms such as these, in their apparently hopeless complexity, furnish all the information usually obtained from a colour-blind witness under his examination by a normal-eyed investigator. I propose, however, now to go further, and to state, in a more definite and logical manner, what my sensations of colour really are, and how they may be made to explain and reconcile the anomalous effects above alluded to.

In doing this it is necessary to have some standard specimens to refer to, and the most convenient for the purpose I have yet met with are the "Cercles Chromatiques" and "Gammes Chromatiques" lately brought out by M. CHEVREUL*.

The former consist of ten graduated colour-circles, one representing the full colours, the rest the same colours in different darkened shades. We require only the first circle, which is divided into seventy-two radial sectors, each coloured with a different graduated hue. Three of these sectors, at 120° apart, contain the full primary colours, yellow, blue, and red, and half-way between them are the three secondary compounds, green, violet, and orange. The intermediate spaces are filled with different gradations, and the diagram, therefore, presents a complete re-entrant circle of hues, of great variety, and capable of identification with great exactitude.

For the representation of toned colours, M. CHEVREUL has prepared a series of what he calls "Gammes Chromatiques," each consisting of twenty-two oblong divisions ranged one over the other. One division, about the middle of the row, contains a certain full colour, which is toned upwards in various shades to black, and downwards in various tints to white. The Gamme thus represents twenty different *tones* of the given colour, from the lightest tint to the darkest shade.

I shall also have occasion to refer to Mr. MAXWELL's method of representing colour-sensations in a mathematical form, by the aid of his ingenious "Colour-Top," described in the Transactions of the Royal Society of Edinburgh, vol. xxi. art. xviii.

9. I have already stated that my vision is perfectly *dichromic*; *i. e.* while that of normal-eyed persons is compounded of *three* primary colour-sensations, mine is formed of only *two*.

This is the usual theoretical manner of defining the defect; but in a practical point of view it may be desirable somewhat to modify the definition. The more important colour-compounds may be practically considered as giving distinct impressions, and on this principle we may say, that while normal-eyed persons have *seven* colour-ideas (red, blue, yellow, green, orange, violet, and white), the dichromically colour-blind have only *three*, namely, two primary colours, and the secondary one produced by their combination.

The first mention of the dichromic explanation of colour-blindness is in a letter from Sir JOHN HERSCHEL to Dr. DALTON, which, though dated 2nd of May, 1833, was only published in HENRY's 'Life of DALTON' in 1854. I have little more to do than to show the application, to my own case, of the theory there so simply and admirably laid down, and which I believe has never yet been followed out so completely as is necessary to explain the varying phenomena observed.

10. The first and most positive fact I am cognizant of is, that there are two distinct colours which I really do see, and which I can at once identify with those called, by the normal-eyed, *blue* and *yellow*. The pigments ultramarine and chrome-yellow, or the parts of the solar spectrum near the lines D and F of FRAUNHOFER, excite the colour-

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* Paris, chez Digeon, Rue Galande, No. 65, 1855. M. CHEVREUL guarantees the perfectly similar reproduction of the colours in all copies.

sensations I am capable of, most fully and completely, and form with each other the strongest contrast it is possible for me to conceive.

I can, of course, never know whether my abstract sensations of these colours are the same as those of the normal-eyed; but this is of no moment, for we can only judge of our ideas by their manifestation; and if I find that I can understand and agree in all that is said of blue and yellow by the world in general, and that they can also understand and agree in all that my impressions lead *me* to say about blue and yellow, I have a sufficient proof that I see those colours as others do. This experience is general among the colour-blind, who one and all identify blue and yellow as the colours about which they are most certain, and which they never run the slightest risk of confounding with each other.

11. The fact next in importance is, that my ideas of the third primary colour, *red*, do not agree with those of the normal-eyed.

The term red is so indefinite, in ordinary language, that I have had much trouble to find out by an example, what is usually considered the pure colour, free from yellow on the one hand, or blue on the other. Many well-educated persons differ much on this point, and I have myself often detected considerable variation between examples of red which normal-eyed people have declared to correspond. I believe, however, that the pigment *carmine* is usually understood to be the best artificial representation that can be had of the true unmixed colour.

Now this red is by no means invisible to me; it conveys a very marked sensation of colour, by which I am perfectly able to identify, in a great number of instances, bodies of this hue. If therefore my testimony ended here, there would be no reason for inferring that I am blind to red, or have much defect in my vision regarding it. But when I examine more closely what I do really see in the cases referred to, I am obliged to come to the conclusion that the sensation I perceive is not one that I can identify separately, but is simply a modification of one of my other sensations, namely *yellow*. It is in fact yellow shaded with black or grey—a dark yellow.

This I can prove to myself in several ways. First, I find that among the various tones in the gamme of yellow, there is one, namely that numbered 16, which perfectly matches, to my eye, the "rouge" in the circle. Secondly, I can make very nearly this appearance of red to my eye, by mixing 1 part by measure of lampblack with 5 or 6[*] of chrome-yellow powder, so as to darken it to the required tone. And thirdly, I can produce, on Mr. MAXWELL's colour-top, a tolerable match to the carmine card, by a combination of about 90 parts black with 10 parts chrome.

12. It may be safely stated, therefore, that what is ordinarily called red, is distinctly visible to my colour-blind eye, not as an individual colour, but as a shade of yellow.

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[*] These quantities are only given approximately to illustrate the fact; they are very difficult to get accurately. The proportions by mixing powders are very different to those required by rotation to produce the same hue; and even when this has been tolerably arrived at, the quality of the pigments has much influence on the appearance of the match.

Now if I follow the Chevreul circle, starting from red, and going round, in the direction of a watch-hand, towards blue, in every division which I pass, the sensation of yellow becomes fainter and fainter, and at the same time the darkening shade becomes more and more powerful, until very soon the yellow disappears altogether, and nothing but a dark grey or perfectly colourless hue remains. This point is still within the limit of what may be called *red*; and thus I arrive at the fact that there *is* a hue of red which, to me, is, as a colour, absolutely invisible. It is a well-defined crimson, not much differing from crimson-lake; and its place on the circle is very near the division called "red-violet," probably the one marked 5 immediately beyond it; and it is perfectly matched to my eye by the division 17 on the gamme of neutral grey. I can also imitate it by mixing about 3½ or 4 parts of carmine powder with 1 of ultramarine; or on the colour-top, 86 parts of carmine combined with 14 of ultramarine will about match a grey compound of 7 white and 93 black.

This, to me, neutral hue of red is not contained in the spectrum, but lies among the hues which are wanting to connect the red and violet ends together, and make the image re-entrant. The red end conveys to me a decided sensation of yellow, and the violet end a decided sensation of blue.

13. I can now easily fill up my description of that portion of the circular diagram lying between red-violet and yellow. The appearance to my eye of the whole of this, simply corresponds with that of the upper part of the yellow gamme, *i. e.* it presents a series of successive *shades* of yellow, commencing with the full colour, and gradually darkening down, by the abstraction of yellow and the addition of black, till it arrives at the colourless limit before described. Thus, "yellow-orange" appears yellow slightly darkened; "red-orange" or scarlet, still more; carmine-red, again more; and red-violet or crimson is all darkness with no yellow at all. The gradations of *hue* visible to the normal eye, are to my vision represented by gradations of *shade* only, the element of red acting, not as a chromatic agent, but simply as a darkening power*.

14. The appearance to me of the *violet* division of the diagram, or the part lying between blue and red-violet, is perfectly analogous to that of the part just described. The blue I see perfectly, but the various tints of violet are to me only darkened blue; for example, the "violet" division of the Chevreul circle perfectly corresponds with No. 18 of the blue gamme, which is simply blue+black; and I can make by mixtures of powders, or by rotation of the coloured disks, combinations of blue and black which shall exactly match others of blue and red. And this of course may be expected to be so; for since I have described the effect of pigmental red as only that of a darkening

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* This expression must of course be only understood as regards the effect obtained in the pigments; what the intensity of direct light, of my red neutral hue, would be, I cannot say. It is probable that in regard to the degree of illumination generally, my sensations are normal. To an ordinary observer a crimson pigment conveys much less light than a yellow one; I appreciate the loss of light, but fail to see the accompanying effect of colour; hence what may be to him redness and darkness combined, is to me darkness only.

agent, its action, when added to blue, will not be to alter the hue, but only the shade. Thus, in the diagram, beginning from the vivid blue and going towards the red, I see the blue gradually diminish, and the shade gradually increase, till it meets the red-violet, as before, at which point all colour is gone.

15. I have now gone over two-thirds of the colour-circle, namely, from yellow round through red to blue; the remaining third comprising the hues which to the normal-eyed are known as *green*. The appearances this division presents to the colour-blind eye are very simple.

Starting from the vivid blue, and passing round in the direction of a watch-hand, I find the sensation of colour gradually diminish, and an impression of darkness gradually increase, until, at about the second or third division beyond "bleu vert," the blue has entirely disappeared, and nothing is left but a neutral grey, which matches, to me, No. 18 on the grey gamme. Beyond this the illumination begins to increase again, and at the same time a sensation of *yellow* begins to enter; the light and the colour both gradually heightening as I advance, until at the division "jaune" the darkening influence has entirely disappeared, and the full normal yellow hue is obtained.

Thus the appearance, to me, of the green division of the diagram corresponds exactly with that of the red or opposite one. First, I find one particular hue (full green to the normal-eyed), which to me is entirely neutral and colourless, and visible only as a dark grey; secondly, all the greens on the blue side of this neutral are represented to me by shades of blue, and can be matched to my eye by blue *plus* black; and thirdly, all the greens on the yellow side are simply shades of yellow, and can be perfectly imitated by darkening this colour. I am, in fact, as totally blind to green as to red; an element of the malady which, I think, has not hitherto received the attention it deserves.

16. But here a curious question arises, whether these two kinds of blindness have any connexion with each other? We have seen that insensibility to red induces also a want of perception of its compounds, orange and violet; but can this in any way affect the vision of green, a colour into which red can scarcely be supposed to enter?

It is difficult to answer the question without assuming something in regard to the nature of the colour green. If, according to the popular theory, it be supposed to be a combination of blue and yellow, our white, which must, as Sir JOHN HERSCHEL has observed in his letter to DALTON, be produced by the equilibrium of the two colours we see, is not in reality white at all, but *green*! Now green is only a *colour* to the normal-eyed, inasmuch as it is contrasted with their white light; but since we know no such contrast, we have no perception of green as a distinct colour, and our green, white, and grey become synonymous terms.

It is certainly rather a startling thought that, if this explanation be true, a portion of mankind exist all their lives in an atmosphere of green without knowing it; that their sunlight is green, their snow green, their grey green; everything in short about them green which is not blue or yellow; and the invisibility of white is no doubt a harder lesson to be learned than that of red. But this is no argument against the fact; we are in no more anomalous position than any man would be if shut up from his birth in a room with green windows; he could form of himself no idea that the light outside his prison was of a different hue from that within; he would call it white if he heard others do so; and if it were as practicable to open our eyes as it would be his to the true aspect of nature, no doubt we should be quite as much astonished as he could be, at what had remained so long unseen.

17. If, however, we ought to consider, according to a doctrine now strongly supported, that blue and yellow combined properly make white, and that green is an independent colour, we can only accept the fact as it stands, that the colour-blind must be insensible to *two* primary or independent colour-sensations, red and green, instead of to one only, as has been generally supposed.

But since it is very difficult for me to conceive that my two colours, blue and yellow, are not also independent sensations, the idea would seem to suggest itself of a *tetra*-chromatism in the formation of the normal colour-scale; or at least that *four* standard colours must be selected in order to enable us to describe all other colours as mixtures of them. This question, however, it is out of my province to discuss further.

18. To recapitulate, in a brief form, the appearance to my eyes of CHEVREUL's colour-circle, I may describe it as having one point full yellow, another full blue, two points colourless grey, and all the rest simply different tones of blue or yellow. The following diagrams will explain this even to those who have not the colour-circle itself to refer to. (See next page.)

And thus I come back to the statement, that I have only three sensations of colour, namely, yellow, blue, and that produced by their combination.

19. It may be serviceable to those who may wish to compare my case with others, to state still more definitely my perceptions of colour according to the mathematical system of Mr. MAXWELL, before alluded to.

Proceeding on the principle that all my colour-sensations are compounded of two elements, blue and yellow, I take cards coloured with the most perfect examples of these I can find, namely, ultramarine and "pale or lemon chrome*," and form with them one of the circles of the colour-top, while in the other circle I place the colour to be matched. I find in this way I can match every colour on the set of cards, by simply varying the proportions of blue and yellow; only, since the *intensity* of the blue and yellow combination is generally different from that of the matched colour, white or black must be used in addition, to make the resemblance perfect. The colour-circle I suppose to be divided into 100 parts.

The neutral combination of blue and yellow is, to my vision, as follows:—

$$\text{(I.)}\quad 38\tfrac{1}{2}\text{ Y.} + 61\tfrac{1}{2}\text{ Ult.} = 64\text{ Black} + 36\text{ White.}$$

The combinations for other colours are—

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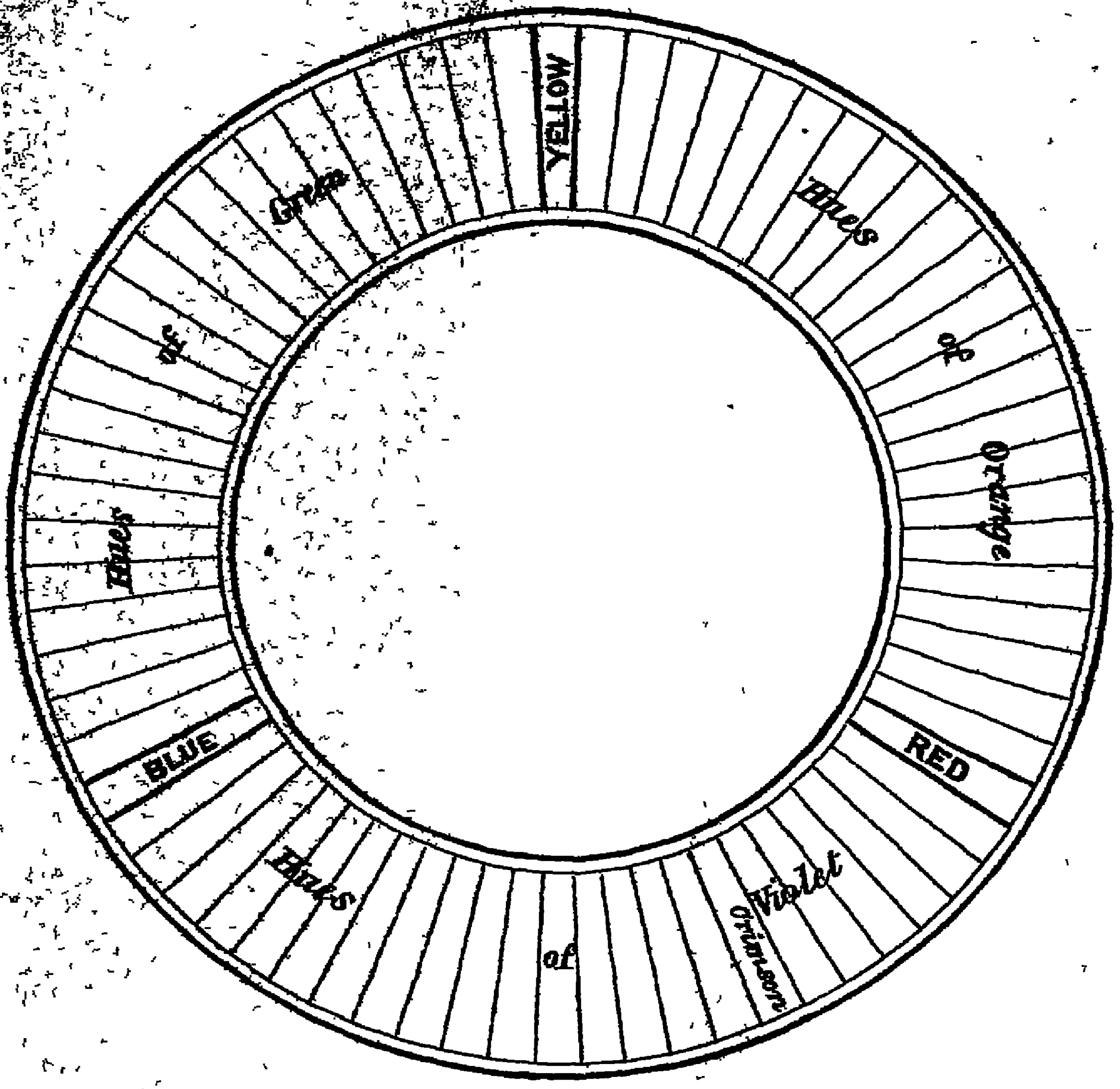
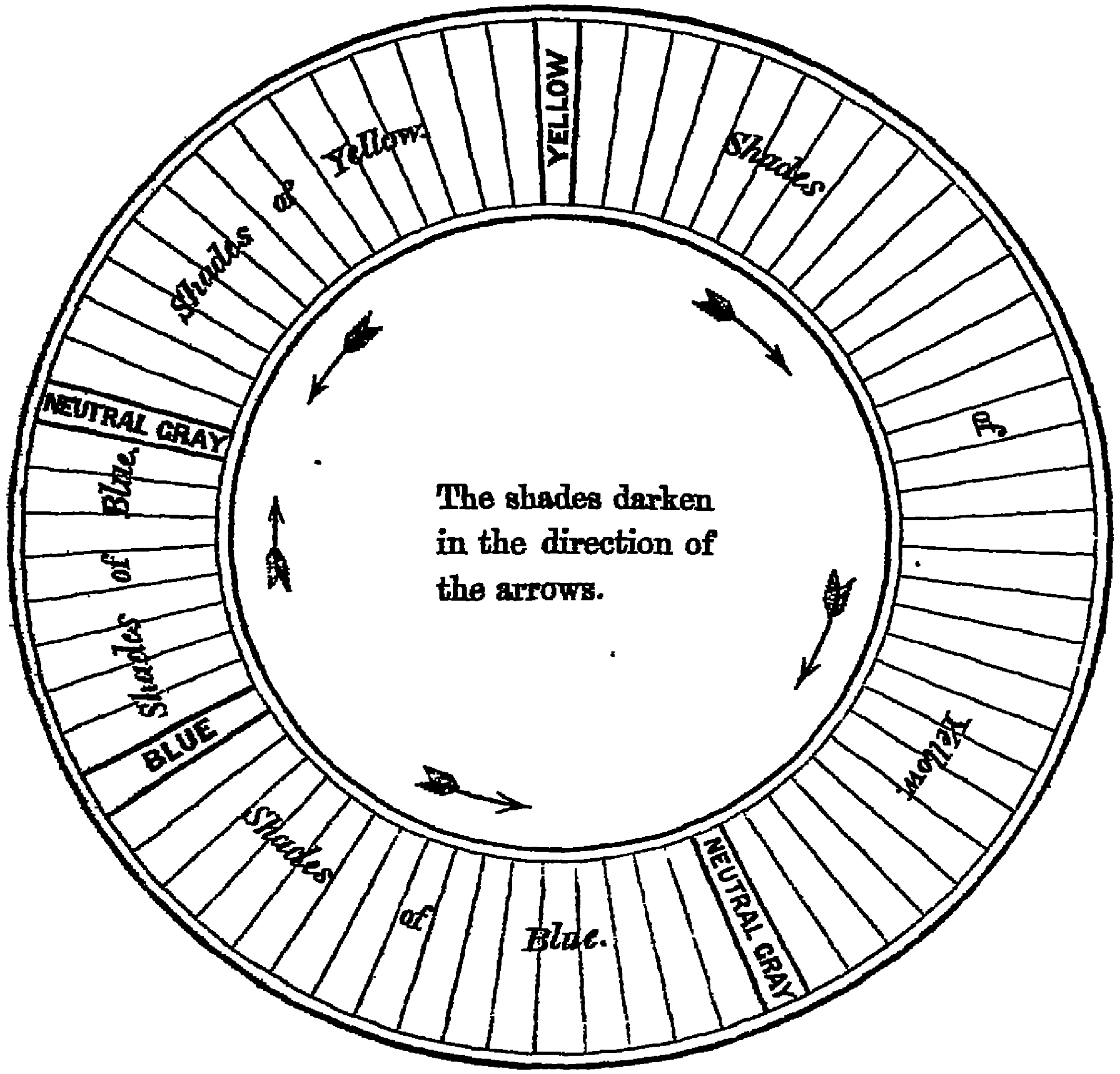
* These experiments were tried in December 1858, with a colour-top and cards prepared by Mr. BRYSON, Optician, Edinburgh.

DIAGRAMS OF CHEVREUL'S COLOUR-CIRCLE.

As it appears to the Normal-eyed.



As it appears to the Colour-blind.



Yellow side of the Neutral.

(II.) 62 Y. + 38 Ult. = 78 Orange chrome + 22 White.
(III.) 51 Y. + 49 Ult. = 88 Emerald green + 12 Wh.
(IV.) 48½ Y. + 51½ Ult. = 70 Vermilion + 30 Wh.
(V.) 42½ Y. + 57½ Ult. = 66 Carmine + 34 Wh.
(VI.) 42½ Y. + 57½ Ult. = 68½ Brunswick gr. + 31½ Wh.
(yellow shade)

Blue side of the Neutral.

(VII.) 20 Y. + 80 Ult. = 65 Blue verditer + 35 Black.

Now as all colours on the yellow side of the neutral are to me only shades of yellow, it follows that they may all be matched more directly by yellow only, darkened with black or grey as the match of intensity may require. The following are the equations thus produced:—

(VIII.) 50 Y. + 50 Blk. = 100 Orange chrome.
(IX.) 23 Y. + 58 Blk. + 19 Wh. = 100 Emerald green.
(X.) 23½ Y. + 76½ Blk. = 100 Vermilion.
(XI.) 10 Y. + 90 Blk. = 99 Carmine + 1 White.
(XII.) 9½ Y. + 87½ Blk. + 3 Wh. = 100 Brunswick green.

And similarly for the blue:—

(XIII.) 70 Ult. + 30 Wh. = 100 Blue verditer.

All the yellow colours may be neutralized by adding blue, and the blue ones by adding yellow. The following are the equations:—

(XIV.) 73 Emerald gr. + 27 Ult. = 70½ Black + 29½ White.
(XV.) 73 Vermilion + 27 Ult. = 83½ Blk. + 16½ Wh.
(XVI.) 86 Carmine + 14 Ult. = 93 Blk. + 7 Wh.
(XVII.) 87 Brunswick gr. + 13 Ult. = 90 Blk. + 10 Wh.
(XVIII.) 68 Blue verditer + 32 Yellow = 51 Blk. + 49 Wh.

The following are some equations of red and green matched with each other:—

(XIX.) 84 Vermilion + 16 White = 85 Emerald gr. + 15 Blk.
(XX.) 75 Carmine + 10 Y. + 15 Wh. = 76 Emerald gr. + 24 Blk.
(XXI.) 96 Carmine + 4 White = 100 Brunsw. gr. (nearly).

The different classes of equations above given may be compared theoretically with each other. An example will best show how this is done.

Take equation (III.) for emerald green, and add to both sides as much yellow as will neutralize the 49 parts of ultramarine, which, from equation (I.), is found = about 31 parts. Then

51 Y. + 49 Ult. + 31 Y. = 88 Em. gr. + 12 Wh. + 31 Y.

But by equation (I.),

$$49 \text{ Ult.} + 31 \text{ Y.} = 51 \text{ Blk.} + 29 \text{ Wh.};$$

$$\therefore \quad 20 \text{ Y.} + 51 \text{ Blk.} + 17 \text{ Wh.} = 88 \text{ Em. gr.};$$

or, which is the same thing,

$$23 \text{ Y.} + 58 \text{ Blk.} + 19 \text{ Wh.} = 100 \text{ Em. gr.},$$

which is equation (IX.).

I have in this way tested the whole of the equations both by theory and observation, and the correspondence of the two is very satisfactory.

20. I may now point out how easily and perfectly the simple fact of dichromic vision may be made to explain all the phenomena, or as I have called them symptoms, collected in art. 7, and which appear, when considered alone, so anomalous and contradictory.

A. See art. 10.

B. The explanation of the appearance of the spectrum is precisely analogous to that of the colour-circle. The division between the yellow and blue colours I believe to be about half-way between the lines *b* and F of FRAUNHOFER.

C, N. The neutral hues of red and green appear grey.

D, E, F, G, O, P, S, T. Since certain hues of both red and green, as well as all kinds of orange and brown, appear shades of yellow, these five colours are confounded with each other. Red and green are the two most common colours in nature, and as the hues of both most frequently met with, lie on the yellow side of the neutral, the mistaking of red and green is such a universal symptom of colour-blindness as to constitute the "shibboleth" of the disorder. Bright grass-green and scarlet may be taken as perhaps the most common representatives of the two colours, and these, to me at least, present almost the same identical shade of yellow.

H. The red which is confounded with blue is always crimson, or its light tint pink, never any of the scarlet or orange varieties; and it is obvious that any of the red hues on the blue side of the neutral will be seen by the colour-blind only as blue. With regard to pinks, it would appear that diluted or thinly applied crimson dyes give generally a bluer impression than the full colours. For example, taking the gamme of red-violet, and comparing it with that of grey, I find that No. 8 of the former will perfectly match No. 10 of the latter, *i. e.* No. 8 of the crimson scale is to me neutral and colourless. But all above 8, *i. e.* all the fuller tones, decidedly incline to yellow, and all below, *i. e.* all lighter tints, as decidedly to blue; so that, seeing Nos. 2 or 3 alone, I should have no hesitation in calling them blue, though of a very dull and imperfect kind. And it must be observed that the blue for which the colour-blind mistake crimson and pink, is always described by them as dirty and faded; never so decided as to stand comparison with the pure colour, even diluted. I am told that the circumstance of certain coloured substances appearing to change in *hue* when reduced in thickness or diluted, is one familiar to the normal-eyed.

J. Violet being to us only dark dull blue, its confusion with crimson, and that of its tint lilac with pink, come under the explanation above given.

K. The remark, first made by DALTON, that crimson and pink appear to have no relation to the idea of red derived from vermilion or a soldier's coat, must now be perfectly intelligible. Crimson appears dark grey, pink dirty blue, but scarlet is a very decided yellow, and these are therefore as unrelated to each other as the yellow to grey of normal eyes.

L. All scarlet hues of red are very distinctly visible as dark yellow.

M. The hopeless perplexity into which the colour-blind are thrown in regard to green may be easily inferred from art. 15. It is to them what white or grey is to the normal-eyed, *i. e.* it represents no colour at all. The only cases therefore where it becomes visible as a colour, are those where it has a preponderance of yellow or blue in its composition.

Q, R, W. All violets, and all greens on the blue side of the neutral, appear dark blue.

V. If the red element in violet is much stronger than the blue, the latter may be overlooked, and the hue will then appear colourless or grey.

X. This only applies to compound hues, in which the blue or yellow element (which alone gives the appearance of colour to our eyes) is often present in quantities so small as scarcely to bear toning without becoming imperceptible. With the simple colours blue and yellow, I believe our vision is quite as acute as that of the normal-eyed, even for the faintest or the darkest tones.

21. Having now described my own case, the question arises how far this conforms with other instances of colour-blindness, and to what extent it may be considered as a type of the defect in general.

It will be impossible for any one to examine the various cases on record without noticing many points of strong resemblance to the characteristics of my own vision, as enumerated in art. 7; but I am not aware that, till now, the whole of these varied and apparently incongruous symptoms have ever been represented as combining in one person. Each individual has contributed his portion to the list, by telling the story of his own experience; but I scarcely suppose it has been conceived possible that any case should exist which would comprehend them all together, or that phenomena so different could be brought under any simple law. Hence an opinion has prevailed that there are many varieties of colour-blindness, differing much in character as well as in severity, each being denoted by its own peculiar symptoms, and each therefore requiring special classification.

Professor ELIE WARTMANN of Lausanne, in a paper on Colour-Blindness, translated in TAYLOR'S Scientific Memoirs for 1846, after enumerating several different classifications of the disease given by different continental authorities, appears himself to be of the opinion that there are as many varieties of the defect as of individuals who are affected with it, so that, as he expresses it, no classification is possible.

Dr. WILSON does not go to this length, but he appears to consider his cases as varying much in severity, and inclines strongly to the opinion that the number of instances where the vision is perfectly dichromic, *i. e.* where the true sensation of red is altogether imperceptible, are very few.

It is with diffidence I would dissent from the opinion of such an authority; but as this is a point confessedly obscure, I do not hesitate to record my own impression that the cases of perfect dichromatism are much more common than Dr. WILSON supposes, and that, in fact, the majority of cases where well-defined symptoms occur, may be, and probably are, purely dichromic.

22. The opinion in favour of the varied character of the disorder is doubtless suggested by the nature of the testimony; for on examining it closely we discover differences in the descriptions which would seem to indicate considerable variety between the sensations of different parties. But granting, without hesitation, that each individual may have faithfully described what he thinks he sees, there are some considerations which must ever render it necessary to use great caution in interpreting the testimony of the colour-blind, and which, if not allowed for, may lead to the deduction of very erroneous inferences from what they say.

23. In the first place they must be very liable to associate, almost indissolubly, the true normal name of a colour with the sensation it conveys to their minds, *whatever that sensation may be*; and they may therefore easily be led to speak of that colour as if they saw it like other people, although the sensation they refer to may be really of quite a different nature to that which the name implies. A colour-blind person will be especially loth to believe that certain colours, which he hears about and talks about every hour of the day, can be invisible to him. Objects of these hues will probably present to his mind *some* ideas of colour (though not the true ones), and he may naturally imagine therefore that he does see them, and may give his description accordingly.

And this source of error is very much enhanced by the fact that it is not an easy matter always to refer different *tones* of any one colour to the same colour-sensation; so that a modification of tone, if considerable, may be easily supposed to be a different colour. The sensations excited by the higher, the middle, and the lower divisions of any one of the "gammes chromatiques" are so different from each other, that it requires no small degree of reflection and judgment to be able to identify the difference as due to black or white only, independent of any other colouring agent.

I believe this difficulty is also felt by the normal-eyed, and the popular nomenclature of colours furnishes illustrations of the fact, different tones of the same colour having often different names, and being treated as separate colours. Pink and crimson, lilac and violet, are well-acknowledged examples of this; and a dark shade of orange is called brown, which generally passes for a separate colour. Persons not well versed in the principles of colour, are often reluctant to admit that these variations of tone are really the same hue. Hence we may easily see what a great probability there is that the colour-blind may acquire the habit of attaching the names of different colours to what are in reality only varieties of the same sensation; and as this habit dates from their infancy, and is encouraged by their every-day communication with the world, it is much more difficult to get rid of than might be supposed. The sufferer may find himself continually blundering, but he must go through a very rigid self-examination before he can trace this to the fact that some of the principal ideas he has all his life held upon colour are mere delusions.

Taking red as an example; it is in the highest degree natural that persons who are continually seeing this colour under the appearance of dark yellow, should imagine that the latter sensation (which is certainly very distinct from that of full yellow) is what corresponds to the term red, and the notion that they cannot really see red at all is one they may have the greatest difficulty in comprehending. Hence the very general assertion by the colour-blind that they *do* see red, an assertion which I think has been far more readily accepted than it ought to be*.

My own experience is very decided on this point. It is only after long and careful investigation I have come to the conclusion that my sensations of colour are limited to blue and yellow. But before I found this out, that is, for nearly thirty years of my life, I firmly believed that what I now know to be only differences in tone of one or other of these, were different colours, and hence I was in the habit of talking of red, crimson, scarlet, green, brown, purple, pink, orange, &c., not of course with the confidence of the normal-eyed, but still with a full belief that I *saw* them. If therefore at that time any scientific man had examined me, I should have given him a description of my case which I now, after more careful study, know would have been entirely wrong. I should have told him, among other incorrect statements, that I saw red objects of a full tone, such as vermilion, soldiers' coats, &c., perfectly well; and I could, if necessary, have supported my assertion by naming correctly a great variety of bodies having this colour, which indeed I am in the habit of doing every day. It would have been inferred, with great appearance of truth, that I was really impressible with the red sensation; but I now see what an erroneous inference this would have been.

24. Another source of confusion in interpreting the descriptions of the colour-blind, is the want of due appreciation of the different sensations that may be produced on their minds by modified *hues* of the same general colour. The normal-eyed person considers green, for example, as always green, whether it be yellow-green, neutral green, or blue-green; whatever the particular "shade of green," as it is called, it still has in his eye the distinguishing character of *greenness*, which cannot be hidden or disguised by any predominance of blue or yellow it may contain. But with the colour-blind this identifying characteristic of greenness is wanting; and hence several patients, speaking of green, may, by each having reference in his own mind to some different hue of the colour, describe it in the most contradictory terms. One may say, with perfect sincerity, that green appears to him like red, another that it looks yellow, a third blue,

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* Red is a more common colour than dark yellow, and hence the preference, by the colour-blind, of the former name for the common sensation. A great variety of bodies are known to be red by habit and association, and are for this reason often named correctly.

a fourth black, a fifth orange, a sixth violet;—from which the normal-eyed examiner, impressed with the unity of greenness, may naturally infer that each person is suffering under a different species of the disorder; while by proper interpretation these anomalous descriptions would only convey the expression of one consistent truth, and one perfectly uniform defect of vision.

25. Guided by these considerations, as arising out of my own experience, I have carefully examined all the published cases accessible to me, and I generally find that I can understand and enter fully into the feelings of the witnesses; can put myself in their places, and see, as it were, with their eyes; and, recollecting what my ideas of my own sensations once were, I feel I can adopt by far the greater part of the testimony as my own. The explanation of my case comprehends and accounts for, with trifling exceptions, the whole varied range of symptoms described by others; and I think there are fair grounds for believing that the few discrepancies that exist are but apparent, and admit of being satisfactorily explained away, on the grounds above alluded to. It appears to me, therefore, a reasonable inference, that the great majority of cases of colour-blindness hitherto described, correspond in general nature with my own.

26. And I think there must be great difficulty in finding any hypothesis, except that of perfect dichromic vision, which will satisfactorily account for the facts observed. Take for instance the most common symptom, the confounding of red and green. If it be assumed that *either* of the colours presents its proper sensation to the colour-blind, it is difficult to avoid the inference that this colour ought always to be recognized; but this is not so, red and green being generally *both* mistaken.

It has been sometimes thought that red may be visible only in a partial degree; but I can scarcely conceive what a partial vision of red can mean, or what mistakes of colour it ought to give rise to; for it would be only a perversion of language to say that red is partially visible when it appears yellow, green, or grey. If red is really the most positive and individual of all colours, as it is usually described, one would think that *any* perception of its true sensation should lead to a much more perfect and consistent identification of it than is generally found in the colour-blind. I am not only sure of my own insensibility to red, but I cannot see, in the testimony of other colour-blind persons, any satisfactory evidence that the true sensation of the colour is really perceptible to them.

It has also been imagined that though *full* colours may be properly discriminated, they may lose their individuality in their light or dark tones; but I cannot see any proof of this supposition, or that, if admitted, it would afford any sufficient explanation of the phenomena.

It would be desirable to direct further inquiries to this part of the subject, in order to discover whether the apparent varieties in the cases are merely due to the nature of the testimony, or whether they really indicate true variations in the character of the vision. If the latter, it would surely not be difficult to reduce them to some kind of consistent classification, and to determine, with more precision than hitherto, what are the real chromatic perceptions causing the different classes of symptoms observed.

POSTSCRIPT.

Since the foregoing paper was written, I have had the opportunity of examining three colour-blind persons, and have found that the vision of each is perfectly dichromic, corresponding precisely in general character with my own. But the remarkable feature has presented itself, that the *coefficients* of the colour-top equations vary considerably in the different cases. Thus, for example, although my equation XV. will always hold in its *general* form,

$$m \text{ Vermilion} + n \text{ Ult.} = p \text{ Black} + q \text{ White},$$

yet the values of m, n, p, and q will vary for different individuals.

I have also found reason to suspect that the values of the coefficients may vary to some extent, even in the *same* individual, at different times.

I hope to investigate both these points more fully.

October 1859.

XVIII. *On the Ova and Pseudova of Insects.*

By JOHN LUBBOCK, *Esq.*, *F.R.S.*, *F.L.S.*, *F.G.S.*

Received November 10,—Read December 9, 1858.

IF, in his celebrated work on "Alternations of Generations," Professor STEENSTRUP has not succeeded in explaining the phenomena of asexual reproduction, he has at least the great merit of having brought together many interesting observations, the relations of which had remained unrecognized up to his time. The value of his suggestions is well shown by the number of memoirs which in the last few years have appeared on this subject, and by their having produced a discussion in which almost every naturalist has taken some part. It is, however, perhaps not going too far to say that as yet no satisfactory explanation of the phenomena has been suggested, and that we are now just as far from knowing as we were twenty years ago, what are the different conditions under which some eggs remain undeveloped unless they are brought under the influence of the spermatozoa, while others contain within themselves the power of producing young without the necessity of any external stimulus. Still, though we have been unable to obtain any insight into the philosophy of the subject, we have in this period collected together a great mass of facts which will perhaps ere long lead us to some satisfactory conclusion.

In a paper "On the Double Method of Reproduction in *Daphnia*[*]," I lately endeavoured to show that eggs and buds are in fact identical, that they are the two extremes of a long series, and that therefore every intermediate gradation between them will probably exist or has existed in nature. I however suggested that it would probably "be convenient to apply some distinguishing name to those eggs which do not require impregnation as a necessary antecedent to development," and Professor HUXLEY has since proposed to call them Pseudova[†].

This name seems to me very appropriate, and I intend therefore to adopt it, merely repeating that we cannot, in my opinion, draw any definite line between eggs on the one hand and pseudova on the other, and that in the Bee and many Lepidoptera the same body is capable of becoming either the one or the other.

In the above-mentioned memoir, I described at length the development of the agamic eggs, or pseudova, of *Daphnia*, in order to show that it is essentially the same as that of the ordinary eggs of a Crustacean. Professor HUXLEY, however, has since presented to the Linnean Society an excellent work on the ova and pseudova of *Aphis*, and shown that in this genus at least there are important differences between the ovarian development of the ova, and that of the so-called internal buds or pseudova; and he suggested to me that it would be desirable to examine the formation of the ova in *Coccus hesperidum*, an insect which, according to Dr. LEYDIG[*], offers in this respect several very remarkable peculiarities.

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[*] Philosophical Transactions, 1857.

[†] Linnean Transactions, vol. xxii. p. 111.

This investigation was completed, the drawings were all made, and the results written out almost exactly in their present form, by the beginning of June last, but I delayed the publication of them, in order to present at the same time some observations on the pseudova of *Cynips* and on the ovarian ova of the Insecta generally.

In the mean time Professor LEUCKART published a paper on the same subject in MOLESCHOTT'S 'Untersuchungen zur Naturlehre des Menschen und der Thiere' for 1858[†], in which he anticipated the greater part of what I had to say about *Coccus*. It was, however, very satisfactory to me to find my observations confirmed by so eminent a naturalist, and the more so as in a great many points the descriptions and conclusions of Dr. LEYDIG seemed to me to be inaccurate. Although, therefore, most of the following description of the development of the pseudova of *Coccus* agrees with what has been published by Professor LEUCKART, yet when two such eminent anatomists came to very different conclusions, it seemed to me that the results obtained by a third observer were worthy of publication.

It is well known that in *Aphis* the self-fertile individuals are viviparous, and the differences which have been pointed out by Professor HUXLEY, and more recently by Professor LEUCKART (*l. c.*), between the development of the ovarian product in the oviparous and viviparous forms, depend perhaps more on the different nature of the body that is to be produced than on the presence or absence of impregnation. According to Professors HUXLEY and LEUCKART, the vitelligenous cells are very distinct in the oviparous *Aphis*, while they are not developed or are inconspicuous in the viviparous form. The latter is, I believe, the true state of the case; it is admitted that there are in the upper chamber of the egg-tube in the viviparous form, certain round cells, originally identical with the one which has developed itself into the germinal vesicle, and I consider these to represent the vitelligenous cells.

In *Coccus* also we have one species, *C. persicæ*, which is truly oviparous, and in which the eggs when laid do not contain an embryo, while in *Lecanium hesperidum* the pseudova are rapidly developed in the ovary, and when laid contain a fully-formed larva, which emerges from the egg-shell in a few hours. This latter species is therefore very nearly viviparous. Now it is remarkable, that whereas in the oviparous *C. persicæ* the vitelligenous cells (Plate XVIII. figs. 11 & 12) are very conspicuous, in *C. hesperidum* they are often with difficulty perceptible. Acetic acid generally renders their walls visible; but if an ovary is examined in syrup, the majority of the egg-tubes show hardly a trace of the vitelligenous cells. In order, however, to compare the formation of the pseudova in *Coccus* and *Cynips* with that of the ova in insects generally, it is necessary to give an account of the latter, because up to the present time very little has been written on the subject, and that little is almost unknown in this country. Thanks, indeed, to the indefatigable labours of M. Léon Dufour and others, we are tolerably well acquainted with the external form of insect-ovaries, but very few writers have paid any attention to the histology and internal structure of these organs. In this country, indeed, Dr. Allen Thomson[*] is almost the only one who has written on this matter, and he was so limited in time and space, that he was unable to enter at length into this portion of his subject.

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[*] Archiv für Wissen. Zool. Bd. v. p. 1.

[†] Zur Kenntniss des Generationswechsels und der Parthenogenesis bei den Insekten.

I propose, therefore, first, to give an account of the process of egg-formation throughout the Insecta, and then to describe the pseudova of *Coccus*, *Cynips*, and *Solenobia*, in order to show that, as far as we are yet acquainted with these secrets of nature, the two processes are perfectly identical.

In all female insects there are two ovaries, each generally consisting of several egg-tubes opening into a common chamber, called the uterus. At its posterior end the uterus contracts into the oviduct. The two oviducts converge to the middle line of the body, where they unite to form the egg-canal, to which various appendages are usually attached. The upper or anterior end of each of the egg-tubes is kept in its place by a connecting filament, which is generally attached to the dorsal vessel, either separately or after having united with the other filaments belonging to the same ovary.

The egg originates and generally attains its full size in the egg-tubes, which differ very much in different groups of insects. In all the large orders, except perhaps the Lepidoptera and Heteroptera, we find some species in which they are very few, and others in which they are very numerous, as, for instance, among Coleoptera, in *Lytta vesicatoria*[†]; among Orthoptera, in *Acheta domestica*; among Neuroptera, in *Libellula*; among Diptera, in the *Tipulidæ* and *Culicidæ*; among Homoptera, in *Coccus*; and among Hymenoptera, in *Apis mellifica*, which has about 170[‡].

On the other hand there are, among the Coleoptera, in *Lixus*[§] and *Anthonomus*[§], only two, in *Hypophlœus*[§] and *Lathridius porcatus*[‖] only three; among Orthoptera, in which, on the whole, they are most numerous, a small species of *Locusta* had only six; among Neuroptera, *Psocus* has only five; among Diptera, *Melophagus* has only two[¶]; among Homoptera, *Schizoneura corni*[**] and *Aphis padi*[**] have respectively only two and three; and among Hymenoptera, in *Chelonus* only two; and even in *Apis mellifica* the workers have only from two to twelve, the general number being five or six[††].

A very remarkable instance of this difference occurs in the *Forficulidæ*, in which family *F. auricularia* possesses a large number of very short egg-tubes, whilst *Labidura gigantea* has only five, which however are much longer[‡‡].

According to Burmeister[§§], the ovaries in *Ephemera* and *Stratiomys* are simple bags in which the germs of the eggs are contained. This form of ovary, if it did really exist, would be a type entirely aberrant from that of the Insecta generally. BURMEISTER, however, adds, that "egg is linked to egg by an exceedingly fine filament;" and LÉON DUFOUR[*] gives a very different description of the ovary in *Ephemera*. Each ovary, he says, consists of a prodigious number of small egg-tubes inserted into the superior wall of a common calyx or uterus, as is the case in *Libellula*. I have little doubt that this account is correct, and that in *Stratiomys* also the egg-tubes are in reality present, but have been overlooked.

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[*] Todd's Encyclopædia, Article 'Ovum.'

[†] Lacordaire, Introduction to Entomology, vol. ii. p. 333.

[‡] Leuckart, *l. c.* p. 421.

[§] Frey and Leuckart, Lehrbuch zur Zootomie.

[‖] Stein, Der weiblichen Geschlectsorgane der Kafer, p. 27.

[¶] Leuckart, Zur Entwickelung der Pupiparen, p. 8.

[**] Leuckart, *l. c.* p. [illegible]

[††] Leuckart, Zur Kenntniss, p. 421.

[‡‡] See Léon Dufour, Ann. des Sc. Nat. 1828.

[§§] Manual of Entomology, translated by Shuckard, p. [illegible]

The number of egg-tubes is most constant in the Lepidoptera, almost all of which possess four[†]. *Pterophorus*, however, is said to have only three[‡].

The Heteroptera also vary but little in this respect. *Hydrometra*[†], *Aradus*[§], and *Gerris* have four; *Nepa* and *Ranatra*[§] five; *Notonecta*[†] six or seven; *Coreus*[§], *Pentatoma*[§], *Alydus*[§], *Scutellera*[§], *Pyrrhocoris*[§], *Lygæus*[§], *Cimex*[§], *Reduvius*[§], *Pelogonus*[§], *Corixa*[§], and *Naucoris cimicoides*[§] seven; while *Naucoris aptera* has only five[§].

In the Homoptera, on the contrary, the number varies greatly. *Schizoneura corni* has only two; *Aphis padi*[‖] three or four; *Aphrophora spumaria*[¶], eight; *A. salicina*[§], fifteen; *Ledra*[§] and *Dorthesia*[§], ten to twelve; *Psylla*[†] and *Cercopis*[§], about thirty; *Cicada*[†], from fifty to sixty; and *Coccus* an unlimited number.

The egg-tubes are numerous in *Locusta migratoria*[‡], *Phasma gigas*[‡], *Gryllotalpa vulgaris*[‡], *Acheta domestica*, the *Mantodea*[‡], and in most Orthoptera; but in a small species of *Locusta* I found only six, and the *Blattæ* have eight[**]. *Œdipoda*[**] *biguttula* has six or seven, and *Œ. cærulescens*[**] fourteen; *Tetrix*[**] *subulata* has twenty; *Œcanthus*[**], eight to ten; and *Ephippigera vespertina*[**] thirty to forty.

Among the Neuroptera the egg-tubes are extremely numerous in *Ephemera*[††], *Perla*[††], *Libellula striolata*, *Æschna cyanea*, and the *Libellulidæ* generally. There are also a good many in *Sialis*[‡‡], *Limnephilus* and *Phryganea*[‡‡]; and in one of the *Leptoceridæ* I found over forty. *Termes* has thirty[†]; in *Panorpa communis* I found ten, which is the number also in *Myrmeleo*[††] and *Hemerobius*[††]; and the minute genus *Psocus* has only five.

We are indebted to STEIN (*l. c.*) for an excellent work on the female generative organs of the Coleoptera. He found three egg-tubes only in one species, *Lathridius porcatus*; four is also an uncommon number, but exists in *Hister sinuatus*, *Platysoma frontale*, *Dromius truncatellus*, *Scydmænus tarsatus*, *Triphyllus bifasciatus*. In *Engis humeralis* and *Trox sabulosus* there are five; six in *Clivina arenaria*, *Notiophilus aquaticus*, *Noterus crassicornis*, in many Brachelytra, and in many Lamellicorns. Seven egg-tubes occur in

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[*] Recherches sur les Orthoptères, Hyménoptères, et Neuroptères. Mém. de l'Institut, 1841, p. 582.

[†] FREY and LEUCKART, *l. c.*

[‡] BURMEISTER, *l. c.* p. 186.

[§] LÉON DUFOUR, Sur les Hémiptères, Mém. d. Sav. Etran. 1833.

[‖] LEUCKART, Zur Kenntniss, &c. pp. 335, 342.

[¶] BURMEISTER, *l. c.*

[**] LÉON DUFOUR, Mém. d. Sav. Etrangers, 1841.

[††] LÉON DUFOUR, Mém. de l'Institut, 1841.

[‡‡] BURMEISTER, *l. c.* p. 185.

Pterostichus vernalis, *Aphodius fimetarius*, *Cassida equestris*, and *Coccidula pectoralis*; eight in *Bembidium femoratum* and *Læmophlœus monilis*; nine in *Nitidula obsoleta* and *Diaperis boleti*; ten to twelve in *Cicindelinæ*, *Carabus granulatus*, *hortensis* (and *violaceus*); twelve in *Clerus formicarius*; fourteen to sixteen in *Hyphydrus ovatus*, *Hydroporus palustris*, *Coccinella globosa* and *quinquepunctata*; about twenty in many *Chrysomelidæ*; twenty-five to thirty in *Elater aterrimus* and *Coccinella septempunctata*; thirty to forty in *Allecula morio*, *Lagria hirta*, *Colymbetes fuscus*, *Acilius sulcatus*, and most of the large species of *Dytiscus*. The largest number observed by Stein was fifty-two, in *Helops caraboides*.

According to Frey and Leuckart (*l. c*), *Lixus* and *Anthonomus* have only two egg-tubes, *Hypophlœus* only three, most Longicorns eight to ten, and most *Buprestidæ* twenty to thirty. Burmeister (*l. c.*) ascribes seven tubes to *Silpha atrata*, seven to ten to *Tenebrio*, *Leptura* and *Saperda*, from ten to fifteen to *Dytiscus*, *Staphylinus*, *Hydrophilus*, *Cerambyx*, and *Lamia tristis*, and thirty to *Blaps mortisaga*.

According to Burmeister, *Sarcophaga carnaria* possesses only one egg-tube, and *Melophagus ovinus*[*] has only two; but in the majority of the Diptera they are short and very numerous.

In the Hymenoptera *Chelonus* has only two, and *Odynerus* only three egg-tubes. According to Frey and Leuckart, *Anthidium*, *Crabro*, *Chrysis*, and *Xylocopa* have also three; but Léon Dufour attributes four to the two latter genera. *Anthophora*, *Crocisa*[†], *Melecta*[†], *Vipio*[†], and *Bombus*[†] have four: Frey and Leuckart indeed attribute eight to *Bombus*, and Burmeister[‡] from seven to ten; but I only found four in *B. terrestris* and *B. muscorum*.

Nomada[†] and *Sapyga*[†], and according to Frey and Leuckart, all the *Ichneumonidæ*, have five, but I find ten in *Ophion luteum* (Plate XVII. fig. 6). *Leucospis*[†], *Chalcis*[†], and *Eulophus*[†], five or six; *Formica rufa*[‖], 100 to 120; *F. nigra*, thirty to forty; *Vespa vulgaris*, seven; *Psithyrus*[†], eight; *Pimpla*[†], *Paniscus*[†] and *Diapria*[†], eight to ten; *Athalia*[§] has twelve, but most of the *Tenthredinidæ*[†] only ten; *Myrmica* and *Xiphydria*[†], twenty; *Banchus*[†], twenty-five; *Cynips*, a considerable number; and *Apis mellifica*[‖] as many as 170.

The number of egg-germs which are produced in each egg-tube offers also many variations, from *Coccus* on the one hand, in which there is only one, to *Pontia* on the other, in which there are, according to Herold[¶], more than a hundred.

The number is not, however, so easy to determine, as it might at first sight appear to be; for as the eggs are produced successively, there must in every case be a time when only one egg-germ is present, though this stage of development is generally past before the insect arrives at the perfect state. Moreover, before the last egg begins to grow the first one has generally already left the ovarian tube. These sources of error have already led to several misstatements in works on insect anatomy. There is, however, I believe, in every species, except perhaps when the egg-germs are very numerous, a certain normal number which are present in each egg-tube when the oldest, and therefore the posterior egg-germ has attained to maturity, and is ready to descend into the uterus. Still in most egg-tubes the egg-germs become so "small by degrees and gradually less," that it is difficult if not impossible to say exactly how many are present. Where, therefore, the exact number is not mentioned, it must not be inferred that the number varies, but only that it was not more accurately determined. Probably, however, when the number is considerable, it admits of certain variations, even in the same species.

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[*] Leuckart, *l. c.*

[†] Léon Dufour, Sur les Orthoptères, &c., Mém. de l'Institut, 1841.

[‡] Burmeister, *l. c.* p. 187.

[§] Frey and Leuckart, *l. c.*

[‖] Leuckart, *l. c.*

[¶] See Herold, Entwickelungsgeschichte Schmetterlinge, 1815, pl. 81.

In the Lepidoptera the egg-germs are very numerous, varying from twelve to more than a hundred. In the Orthoptera they are rather fewer. BURMEISTER*, indeed, ascribes only three to *Gryllus migratorius*, and six to *Blatta*; but in *B. orientalis* I found rather more than twenty. He does not mention to what species he alludes, but there is seldom much variation in this respect in species of the same genus, so that probably he examined a young specimen in which only a few egg-germs had as yet been produced.

In the Hymenoptera the eggs are rather less numerous in each egg-tube. *Chrysis* and *Xylocopa* are said to have only three; *Bombus terrestris*† only six (but this again I doubt, as in *B. muscorum* I have found in July from twelve to fifteen); in *Cynips lignicola* there are thirteen; *Apis mellifica*‡ has seventeen, and in some *Ichneumonidæ* they are still more numerous.

In *Pulex irritans* I found fourteen egg-germs occupying the whole width of the egg-tube, and a large number which had not yet arrived at that stage.

Among the Neuroptera the number of egg-germs in each egg-tube is nearly the same as in the Hymenoptera. In *Æschna* there are about fifteen; in *Libellula striolata* about the same number; in *Panorpa* about twelve; in *Chrysopa* about ten; in *Psocus* only five.

The egg-development of the Coleoptera has been carefully described by STEIN, from whose excellent work I have extracted many details concerning this order. He finds (*l. c.* p. 32) the greatest number of egg-chambers in the *Curculionidæ*, where frequently as many as twenty are present. Thus, *Brachyderes incanus* has twenty-one, *Hylobius abietis* and *Sitones lineatus* eighteen.

In the *Cyphonidæ* also the number is considerable; *Cyphon pubescens* has as many as fifteen. In *Telephorus dispar* he found seven or eight; in the *Cicindelidæ*, *Carabidæ*, and *Hydrocantharidæ* also, each tube has several egg-germs. In *Colymbetes fuscus* there are twelve; the number in *Notiophilus aquaticus*, *Carabus granulatus*, and *Cicindela campestris* is seven.

The remaining families of this great order have less than five egg-germs in each tube.

The egg-development in the Diptera has been little studied, and from the small size of the ovaries, and indeed of the generative organs generally, it presents great difficulties. Each egg-tube contains generally very few egg-germs, though in a small species belonging to the *Syrphidæ*, and I believe to the genus *Cheilosia*, there were as many as nine. In *Melophagus ovinus* LEUCKART has figured three, and I have found the same number in *Musca* and *Eristalis tenax*. In *Tipula* and *Culex* I found only one egg-germ besides the terminal chamber, which is present throughout this order as in the Coleoptera.

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* [illegible]

† Ibid.

‡ BURMEISTER, l. c.

In the Hemiptera the egg-germs are few in number, and each egg-tube has a large terminal chamber.

In the Homoptera but few species have been examined. They appear, however, to agree with the Hemiptera in having few egg-germs, and a terminal chamber containing vitelligenous cells. In *Coccus hesperidum* and *C. persicæ* I found only one egg-germ, in *Aphrophora spumaria* three; and LEYDIG and HUXLEY[*] have figured five in an *Aphis*[†].

Histology.

The egg-tubes consist of two membranes. I did not, indeed, succeed in seeing the outer muscular one clearly in every species I examined, and in some insects all the egg-tubes are enveloped by one common covering; but in most instances the outer membrane was distinctly visible between each two egg-chambers, where, unlike the inner membrane, it is not contracted, but retains nearly the same diameter, only tapering gradually from the most mature egg-germ up to the youngest. It is figured in Plate XVI. figs. 3 and 9, and Plate XVII. fig. 7.

On the inner side of the internal membrane lie a number of cells. These form in most parts a continuous layer; but in those insects which have between each of the egg-germs a small group of the remarkable vitelligenous cells, presently to be described, the epithelial cells at these parts of the egg-tube are scattered more sparingly, and are less distinctly visible. These epithelial cells take probably an active part in the secretion of the yelk, and subsequently in that of the chorion.

Between each of the egg-germs in Lepidoptera, Hymenoptera, Geodephaga, Hydradephaga, and Neuroptera (except the Libellulina), is situated a group of cells, which increase in size from the anterior to the posterior end of the egg-tube.

These cells, which, adopting the convenient name proposed by Professor HUXLEY, I propose to call the vitelligenous cells, were first noticed by HEROLD[‡], who however describes them as rings, and offers no opinion on their homologies or function. STEIN, in his excellent work to which I have already had occasion to allude, gives a faithful description of these cells, and expresses a decided opinion that they secrete yelk-matter. HERMANN MEYER[§] also has described these bodies in the egg-tube of *Saturnia Carpini*; but he considers them abortive ova; an opinion, which, although adopted by Dr. ALLEN THOMSON[‖], appears to me quite untenable.

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[*] HUXLEY, *l. c.* pl. 40. fig. 1.

[†] Zeitschrift für Wissenschaftliche Zoologie, 1849, pl. 5.

[‡] De Generatione Insectorum in Ovo.

[§] Zeitschrift für Wissenschaftliche Zoologie, 1849.

[‖] Article 'Ovum,' p. 113, Todd's Cyclopædia.

Dr. LEYDIG *, in his paper on the Agamic Reproduction of *Aphis*, and Professor HUXLEY (*l. c.* pl. 36) have figured in the terminal chamber certain round, nucleated cells, any one of which may develope itself into a germinal vesicle, and so pass down the tube and form around itself the future egg. These so-called cells are probably homologous with the nuclei of the vitelligenous cells.

Dr. LEYDIG also, in his paper on *Coccus hesperidum*, figures certain large nucleated cells in the upper chamber of the short egg-tubes, but having overlooked the true germinal vesicle, he seems to have (if I understand him right) considered these bodies as the equivalents of the germinal vesicle, whereas in fact they are true vitelligenous cells, differing neither in position nor appearance from the type present in the other Homoptera.

Professor LEUCKART also (*l. c.*) has described these cells in *Aphis*, *Coccus*, and *Solenobia lichenella*, and adopts in reference to them the same opinion as Professors STEIN and HUXLEY.

In adopting the term "vitelligenous" as applicable to these bodies, I have expressed my opinion of their function; an opinion, I may add, which was adopted independently by Professor STEIN, Professor HUXLEY, and myself. In proof of this view I would refer to the egg-tubes of any Lepidopterous or Hymenopterous insect, or to HEROLD's figs. 15, 16, 14, 13 and 12 (*l. c.* pl. 1). It will there be seen that although the vitelligenous cells increase *individually* in size, as does the yelk-mass, yet that the latter constantly grows at the expense of the former, which become fewer in number, and finally disappear altogether.

Professor STEIN has observed that in *Acilius sulcatus*, in which the yelk is brightly coloured, the vitelligenous cells are of the same hue; and in all insects the contents of these cells closely resemble the mass of yelk substance.

Professor HUXLEY has observed in *Aphis*, and I have noticed in certain Hemiptera (see Plate XVII. figs. 7 and 9), that a tube, or channel, leads down from the terminal chamber into the second and third egg-chambers, which seems evidently intended to convey the yelk-matter to the developing eggs.

Finally, if, as Professor STEIN also remarks, we press the vitelligenous cells out of one of the egg-chambers, we shall generally find some of them in which the cell-wall is almost entirely absorbed, so that on the application of but slight pressure the contents spread in all directions.

The two theories respecting the nature of these cells are not quite so opposite to one another as would at first sight appear to be the case. In their earliest stage the egg-cell and the vitelligenous cells cannot be distinguished from one another; and no one, I think, who has carefully examined the upper part of the egg-tube in any Hemipterous or Dipterous insect, can fail to be of the same opinion. The egg-tube contains indeed at this end cells which are neither vitelligenous nor egg-cells, but which are capable of becoming under certain circumstances either the one or the other.

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* Zeitschrift für Wissenschaftliche Zoologie, 1849, pl. 5.

Professor STEIN, indeed, appears to consider that the vitelligenous cell is homologo with the germinal vesicle, and that the yelk is deposited round it without being enclose at first in any distinct membrane.

In the Diptera (Plate XVII. fig. 8, and Plate XVI. fig. 9) and the Hemiptera (Pla XVII. fig. 7) it is, I think, evident that the germinal vesicle corresponds to the nucle of the vitelligenous cell, and that the yelk-mass is in the early stages of egg-formati enclosed in a membranous envelope or cell, homologous with the cell-wall of the vite ligenous cell, and like it destined soon to be absorbed, but which in the mean time I pr pose to call the egg-cell; and even in the Lepidoptera, Hymenoptera, Geodephaga, a Neuroptera, though the matter is not quite so clear, any one who examines careful the upper end of the egg-tubes will, I think, come to the same conclusion.

Our knowledge of the modes of egg-formation in the Insecta is perhaps hardly y sufficient to enable us to generalize with much confidence; the following Table exhibi however, the present state of our information on the subject; and though many of t aberrant forms, as for instance *Thrips* and the *Strepsiptera*, are not included, and it therefore very incomplete, yet it will probably be found correct as far as it goes. T only two cases in which any families differ greatly, as regards the vitelligenous cel from the remainder of the order to which they belong, are the Geodephaga a Hydradephaga from the remainder of the Coleoptera, and the Libellulina from t Neuroptera.

Egg-tubes	Several groups of vitelligenous cells, one group to each egg-chamber.			
Without	Vitelligenous cells.	Contained in a terminal chamber		Coleoptera (generally).
				Homoptera.
				Heteroptera.
		Absent.		Orthoptera.
				Pulex.
				Libellulina.
		One to each egg		Forficula.
With	Egg-chamber	Not constricted		Diptera.
		Constricted in the middle		Lepidoptera.
				Hymenoptera.
				Geodephaga and Hydradephaga.
				Neuroptera (except Libellulina).

In thus dividing the Insecta into two sections, according to whether or no each eg germ carries with it a group of vitelligenous cells, we get a classification which is from natural, inasmuch as it separates the Libellulina from the rest of the Neuropte *Pulex* from the Diptera, and the two first sections of the Coleoptera from the remainc of that order.

It is rather difficult to decide in which group the genus *Forficula* (Earwig) ought be placed, since each egg appears to consist of an egg-cell and one vitelligenous cell on

The second section, containing the insects which possess groups of vitelligenous c forming a part of each egg-germ, combines most of the insects in which the power flight is largely developed; but the Libellulina, which are pre-eminent in this respe belong to the other series.

The development of the prothorax appears to have some curious connexion with t presence of alternate groups of vitelligenous cells. LACORDAIRE* says, "Si le prothor

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* Introduction à l'Entomologie, p. 326.

a acquis un développement extraordinaire, et s'est en quelque sorte séparé du mésothorax et du métathorax, on aura le thorax d'un Coléoptère, d'un Dermaptère, d'un Orthoptère et d'un Hémiptère. Si au contraire le prothorax est réduit à des dimensions très-exiguës, et que le mésothorax intimement uni au métathorax ait pris un accroissement énorme, on aura celui d'un Hyménoptère, d'un Lépidoptère et d'un Diptère." And again, in page 332, after having described the prothorax of the Coleoptera, Dermaptera, Orthoptera and Hemiptera, he says, "Avant de considérer le prothorax des mêmes ordres sous un point de vue plus en rapport avec ses usages dans la nomenclature, il est essentiel de connaître les modifications qu'il subit chez les Hyménoptères, les Lépidoptères, les Diptères et quelques Neuroptères."

The Diptera, Hymenoptera, and Lepidoptera, thus nearly allied to one another by the structure of the thorax, also agree very nearly in the type of egg-development.

In the Libellulina also (Dragon-flies), the prothorax is rather large and distinct from the rest of the thorax.

The Geodephaga and Hydradephaga, however, are subversive of this curious coincidence, which would otherwise have been as complete as that of the coexistence of rumination and cloven feet; or the inward bending of the condyle of the lower jaw, with the other characters of the Marsupialia.

In the Orthoptera we find, on the whole, perhaps the simplest type of egg-formation which occurs in insects, the large vitelligenous cells being entirely absent, and their functions probably monopolized, instead of being only shared, by the small cells which line the membrane of the egg-tube.

The number of egg-chambers in each egg-tube is generally numerous; in *Blatta orientalis* (Plate XVI. fig. 1) amounting to as many as twenty-three. In each of the egg-chambers the germinal vesicle is easily visible, lying nearly in the centre, and possessing a distinct macula. The germinal vesicle in *Blatta orientalis* (Cockroach) is rounded in the upper chambers, but in the lower ones is somewhat elongated transversely. The macula germinativa is small, round and simple, as is the case in *Acheta*. In a species of *Locusta* (Grasshopper) I found the germinal vesicle in the lower chambers, lying near the posterior end of the egg; in other words, near the end which is turned towards the vulva. In *Acheta domestica* (Cricket), as in *Blatta*, the macula is a small round vesicle $\frac{1}{3000}$ in diameter in the lowest egg-chamber in which it is visible, and gradually diminishing to $\frac{1}{6000}$. In the lower chambers a minute nucleolus can be seen in it, which is about $\frac{1}{4000}$ in diameter. I never found in *Acheta* any other vesicular structures in the germinal vesicle, but there is in the anterior or smallest germinal vesicles, a conspicuous, dark, granular mass, which both WAGNER and STEIN* seem to have taken for the true macula. This dark granular mass is single in about the first four egg-chambers, and often conceals the true macula; it then gradually breaks up, and in the three or four most mature egg-germs is reduced to a cloudy mass of very fine granules.

[illegible] egg-formation which is represented in Plate XVI. fig. 1, is found in *Blatta*,

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* *Loc. cit.* p. 49.

Gryllus, *Locusta*, and *Acheta*, and probably prevails also through the *Phasmidæ** and *Mantidæ*, which two latter families, however, I have as yet had no opportunity of examining. It is, however, quite possible, if not probable, that we may find in them some approach to the peculiar type exhibited by the remarkable genus *Forficula* (Earwig).

In the Libellulina the egg development is remarkably similar to that of the Orthoptera, with which it agrees in the absence of the large vitelligenous cells, and of any terminal germ-chamber. The egg-tubes are excessively numerous, and I have generally found from twelve to twenty eggs in each.

The macula of *Æschna* (Dragon-fly) consisted apparently of several small vesicles, which gradually increased in size in unequal ratios, until at length one large one became pre-eminent. The end of each egg-tube terminates in a chord-like tube.

In *Pulex* (Flea) also the type appears to be the same. There is here the same absence of the vitelligenous cells and of the terminal germ-chamber. If, however, a young specimen be examined, fewer egg-chambers will be found; and at the end of the egg-tube there will be a false, or rather perhaps a temporary germ-chamber, destined eventually to be entirely occupied by egg-germs, and to lose gradually its special character. In the most advanced tube I found fourteen egg-germs occupying the whole width of the bore, besides a great many others in less forward stages.

The germinal vesicle is dark, large, circular, and very distinct. I could not see in it any macula; but in two or three cases, shortly before its disappearance, it looked as if it was breaking up into a number of small ovate bodies.

It is very remarkable, that in spite of the numerous affinities existing between *Pulex* and the *Diptera* (Gnats and Flies), the mode of egg-formation in these two groups is entirely different.

In the Hemiptera (Plate XVII. fig. 7), the egg-tube terminates in a large terminal chamber full of round cells, each of which can apparently become an egg-cell or a vitelligenous cell. In this type, therefore, the vitelligenous cells, instead of being divided into small groups, one at the posterior end of each of the egg-cells, are all collected into a common germ-chamber at the end of the egg-tube.

I was for some time in doubt whether the cells in this common germ-chamber did really contribute to the formation of the yelk. In *Aphis*, however, in which the same type of ovary exists, Professor HUXLEY† discovered, as before remarked, a duct or passage leading from the germ-chamber through the anterior, to the more developed egg-chambers. A similar structure exists in *Nepa* (Water Scorpion) (Plate XVII. fig. 7), and some other Hemiptera; and as there can, I think, be no doubt that it is really a duct through which the yelk-substance descends to the growing egg, it follows that some at least of the cells in the terminal germ-chamber do really fulfil the office of vitelligenous cells.

In the common *Nepa cinerea* I once found as many as five of these vitelliferous ducts (Plate XVI. fig. 10) in an egg-tube containing seven egg-germs, so that it seems probable that each egg-germ has its own peculiar vitelliferous duct, and that the duct is obliterated when the egg has received a sufficient amount of yelk.

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* MÜLLER, Nova Acta Nat. Cur. xii. 2.

† *Loc. cit.*

I could not see in the common germ-chamber more than one sort of cell, except that near the posterior end some few are evidently changing (Plate XVII. fig. 7) into egg-cells; and I conclude, therefore, that each of the cells in the germ-chamber is capable of becoming either vitelligenous or egg-cells. Those which develope themselves into egg-cells gradually swell, and their nucleus also increases in size and becomes the germinal vesicle; after awhile a small cell-like macula makes its appearance in this latter.

The common germ-chamber is at its posterior end somewhat contracted, and at this place one or two narrow egg-germs generally lie transversely across, occupying the whole width of the tube. The next egg-germ is somewhat square-shaped, as are the three following; the seventh is somewhat barrel-shaped; and the eighth shows on one side the projection which is so conspicuous in the mature egg, and on which the micropyle is situated. I found also at this period (end of September) some few egg-tubes in which the projection was also commencing in the penultimate chamber. In the four anterior egg-germs the germinal vesicle occupied a central position, but in the more mature egg-chambers it laid alternately, first on one side and then on the other, close to the layer of epithelial cells. It is worthy of remark, that in all my specimens the lateral projection was situated on the same side as the germinal vesicle.

In the oldest egg-germs the yelk contained numerous granules and small oil-globules. In each ovary there are five egg-tubes.

In the Coleoptera, excepting the Geodephaga and Hydradephaga, we find the same type of ovarian tube as in the Hemiptera, but the terminal germ-chamber is generally smaller in proportion. The cells contained in the germ-chamber are apparently of the same nature as in the Hemiptera, and probably therefore secrete part of the yelk-substance. I have not yet, however, met with the yelk-duct. Since, however, I became familiar with this structure in *Nepa*, &c., I have been able to examine but few Coleoptera.

In *Telephorus* (Plate XVI. fig. 4) the epithelial cells are small and indistinct, but in a *Galeruca* I found them very large.

I have examined only few species of Coleoptera, since STEIN has devoted so much attention to them, that, as far as the female generative organs are concerned, there is no order with which we are better acquainted.

In the Homoptera I have examined *Aphrophora spumaria*, *Coccus hesperidum*, and *C. persicæ*; Professor HUXLEY (*l. c.*) has described a species of *Aphis*, and Professor LEUCKART three or four allied forms. All these present the same type of ovary as the Hemiptera; but the terminal germ-chamber is generally smaller, and the cells contained in it much less numerous, being in fact, in *C. hesperidum*, only three in number. In[illegible] this genus, as there is only one egg in each tube, there is in fact a group of [illegible] to each egg-chamber; and this form of ovary forms an intermediate [illegible] found in the Hemiptera and the Hymenoptera.

The latter part of this paper will contain a full account of the formation of the egg in *C. hesperidum*, to which therefore I need not here allude in greater detail.

In *Forficula auricularia* the egg-tubes are short and very numerous; each contains a large egg more or less mature, and two or three small rudimentary ones. The large egg is pear-shaped, and the other two germs, together with the membrane containing them, may not unfitly be compared to the stalk.

The large egg-germs consist of two parts. The posterior part consists of the yelk, and encloses the germinal vesicle; the yelk-mass contains dark granules and small oil-globules. The anterior part is lighter, browner, more homogeneous, and appears to consist of a single large vitelligenous cell. In this anterior portion several vermiform masses may be perceived, but I am unwilling at present to offer any suggestion as to their origin and functions. At first the anterior portion occupies the larger portion of the egg-chamber, but the posterior portion grows both absolutely and relatively larger, until at length it fills almost, or altogether, the whole of the egg-chamber. I believe that I am correct in considering the anterior portion as a yelk-cell, but I was never able to see the nucleus.

The terminal, smaller part of the egg-tube is not represented in WIEDEMANN's 'Archiv für Zoologie et Zootomie,' Bd. xi. tab. 111; it has, however, been described by STEIN*. It consists of two or three egg-germs, each situated in a chamber which is separated from the next by a rather slight constriction, and each consisting of two parts, the posterior or egg-cell, and the anterior or vitelligenous cell. Sometimes, however, there were two vitelligenous cells to each egg-cell. In some of the more advanced of these egg-germs an indistinctly bordered granular nucleus was visible. This small terminal portion of the egg-tube is bent down on to the large egg, so that it might easily be overlooked. At an earlier period in the summer, as for instance in July, the difference of size between the last and penultimate egg-germs is not nearly so decided as it becomes towards the end of August.

M. LÉON DUFOUR also, in his paper on the *Forficulidæ*†, has overlooked the small terminal chambers of the egg-tube, which consequently he considers as consisting of only one egg-chamber. In the same paper he describes and figures the ovary of *Labidura gigantea*, which differs in the most remarkable manner from that of *Forficula auricularia*, and consists of five long egg-tubes enclosing twenty, or a lesser number of egg-germs. It is impossible to determine, either from the description or the figure, upon what type the egg-germ is formed, but there is no reason to suppose that the process is different from that which occurs in the small species.

It is very interesting to meet with such an important difference in the ovaries of two species which are in other respects so nearly allied, and to find thus in *Forficula* a type of ovary entirely different either from the Coleoptera on the one hand, or the Orthoptera on the other. If the terminal chamber is really absent, *Labidura gigantea* resembles the latter order rather than the former.

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* *Loc. cit.* p. 89.

† Annales des Sciences Naturelles, 1828, p. 358.

The Neuroptera (with the exception of the Dragon-flies) offer the next step towards the type which prevails in the Lepidoptera, &c. Each egg-chamber is generally divided by a constriction, the posterior portion containing the yelk, germinal vesicle, and germinal spot; and the anterior part enclosing in *Panorpa communis* generally two vitelligenous cells, in *Psocus* three, in one of the *Leptoceridæ* four, and in *Chrysopa perla* five or six. Besides being few in number, the vitelligenous cells in the Neuroptera are somewhat indistinct, especially in *Panorpa*, and do not present a well-defined nucleus.

In *Panorpa* also, and in one of the *Leptoceridæ*, it seemed to me evident that the whole egg-cell, and not only the germinal vesicle, was equivalent to the vitelligenous cell.

In the Diptera* I have only examined *Cheilosia*, *Eristalis*, *Musca*, and the *Tipulidæ* and *Culicidæ*; nor am I aware of any other observations on the ovary of this order except those of Loew, which I have not been able to obtain, of Leuckart on *Melophagus*, and Swammerdam on *Stratiomys*, which latter refer rather to the external form than to the histological structure. Probably, however, we shall find in the remainder of the order, as in the forms already examined, that the ovarian tubes are very short and extremely numerous. In a small species of *Cheilosia*, indeed (Plate XVI. fig. 9), there were as many as eight egg-chambers, but in *Musca* and *Eristalis* there are only two or three, and in *Culex* and *Tipula* only one or two.

In all cases, besides the egg-chambers, there is a terminal germ-chamber. The layer of epithelial cells is early and strongly developed, and the constriction between each two egg-chambers is very deep. In many cases, indeed, there is a considerable interval between each two egg-chambers, which gives to the egg-tubes a remarkable and peculiar appearance. There is no constriction between the germ- and yelk-portions of the egg-chamber, but the two together form a round mass, which gradually becomes oval, and finally more or less elongated, according to the shape of the mature egg. The vitelligenous cells are large, polygonal from mutual pressure, and in some cases provided with a distinct nucleus.

It is worthy of observation, that this type of egg-formation is very different from that of *Pulex*, or indeed of any other insects. The external membrane of the egg-tube may generally be seen forming a series of bridges, projecting from one egg-chamber to another. The vitelligenous cell which lies at the posterior end of the egg does not at first differ from the others. Gradually, however, it grows darker, and its nucleus becomes the germinal vesicle. It is, however, only in the two lowest egg-chambers that this, which evidently corresponds to what I have called the egg-cell, becomes much larger than the true vitelligenous cells. At this stage it appears to lose its true cell-wall and to become a mere yelk-mass, which, as the vitelligenous cells gradually disappear, absorbs the yelk-matter supplied by them, and finally occupies the whole egg-chamber.

In the early part of September the ovary of *Eristalis tenax* is a white body, shaped like a cucumber. It consists of a large number of egg-tubes, each formed of two chambers, one more or less spherical, and the other a miniature representation of the ovary itself, as far as shape is concerned.

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* [illegible] and Wellich have described a partial segmentation as occurring in the eggs of Diptera; [illegible] mistake, into which they may have been led by the vitelligenous cells.

The external membrane is in this species very distinct, and the internal structure of the egg-tube cannot distinctly be seen until it has been removed. It is, however, easily torn off, and the egg-tube will then be generally found attached by the upper part, and with the end next to the spherical chamber floating loose. The part of the egg-tube also below the spherical chamber is very thin, and tapers rapidly to a very delicate filament, while the upper part retains to the end a much more considerable diameter. These facts puzzled me a little at first, and made me doubt for awhile which was the ovarian end of the egg-duct. I soon found, however, that the germinal vesicle was always at the side of the egg-germ which was turned away from the cylindrical chamber; and a specimen occurred in which the lower part of the cylindrical chamber was beginning to separate itself from the remainder, and to form a new egg-germ (Plate XVII. fig. 8). Towards the middle of October, all, or nearly all the egg-tubes were in this stage. The vitelligenous cells contained no distinct nucleus, and varied much in size, some being larger, and others smaller than the egg-cell.

In the Geodephaga and Hydradephaga, STEIN has already observed that we find a type of egg-formation differing entirely from that of the other Coleoptera, and closely resembling that of the Hymenoptera, from which indeed it only differs (so far as our knowledge at present extends) in possessing a terminal germ-chamber.

This terminal germ-chamber is, however, less largely developed in the present group than in most other Coleoptera, and in old specimens, indeed, is very much reduced in size.

Plate XVII. fig. 1 represents one of the egg-tubes of *Carabus violaceus*, magnified thirty times. At the lower or posterior end was a glandular part containing a yellow mass, which STEIN, apparently with justice, considers as analogous with the "corpus luteum." The lowest egg-chamber was very opake, so that no structure could be perceived in it. In the second or penultimate egg-chamber, and the two following, the constriction separating the germ-chamber from the yelk-chamber is well marked; the three following have a constriction separating them from one another, but the chambers themselves have an entire margin; and still nearer to the end are several egg-germs, which are not sufficiently large to occupy the whole diameter of the egg-tube.

Plate XVIII. fig. 18^3 represents a germinal vesicle of *Pterostichus melanarius*. In this specimen the macula germinativa contains five large equal-sized vesicles. I have found other specimens presenting very nearly the same appearance, but it is by no means usual, and was not present in two other germinal vesicles belonging to the same egg-tube. In the smallest of these (Plate XVIII. fig. 18^1), the germinal vesicle contained two small equal-sized vesicles. In the second (Plate XVIII. fig. 18^2), the germinal vesicle contained three vesicles, one rather large and the other two smaller.

The vitelligenous cells present nearly the same appearance as in the egg-tube of a

Hymenopterous insect. At first small and round, they become larger in each successive egg-chamber, and are gradually forced to take a polygonal form, by the pressure which they mutually exert upon one another. The nucleus continues round, and is somewhat darker than the surrounding cell. The vitelligenous cells in each chamber are not, however, all of the same size; on the contrary, they vary a good deal in this respect, and those which lie nearest to the egg-germ are larger than those at the anterior extremity.

The yelk itself is clear, transparent, and contained, in the lower egg-germs of the specimen figured, a cloud of reddish-brown granules just behind the germinal vesicle. In most specimens, however, I found these granules collected all round the germinal vesicle, so as to obscure it very much. The epithelial cells are not much developed in *Carabus*, perhaps owing to their function being in part fulfilled by the vitelligenous cells.

The macula germinativa, as is I believe the case in all insects, has the form of a round vesicle. In the second chamber of the figure the germinal vesicle was about $\frac{8}{800}$ in diameter, and the macula about $\frac{1}{800}$. In the other egg-germs they became gradually smaller. The macula germinativa seemed to be composed of a number of small oval masses, enclosed apparently by a very delicate membrane. In one case it appeared to me as if the macula was broken up, and these little bodies were floating about loose in the germinal vesicle. Besides the true macula, each germinal vesicle contained a number of smaller vesicles, which seemed to become more numerous in the larger egg-germs. They were none of them, however, nearly so large as the true macula.

In *Steropus madidus*, also, the germinal vesicle contains some of these small vesicles, which however are not so numerous as in *Carabus violaceus*. In young egg-germs of *Steropus*, two or three of the vesicles are at first sometimes of equal, or very nearly equal size. Gradually, however, one of them surpasses the rest, and becomes the macula germinativa. Probably, therefore, these small vesicles are of no great functional importance.

The development of the eggs in the Lepidoptera has attracted the attention of naturalists more than has been the case in any other order. Heroldt, Meyer, Stein, and Leuckart have all given more or less accurate figures and descriptions of these organs. The type is essentially the same as in the Geodephagous beetles, and differs merely in the smaller number of egg-tubes, with a greater number of egg-chambers in each, and in the absence of any terminal chamber.

As in the Geodephaga, the vitelligenous cells are large and polygonal, and each presents a distinct nucleus. The constrictions are deep, but the egg-chambers follow one another immediately, without ever presenting any long narrow portion, as in some Diptera; and the yelk-cell grows much more rapidly than in that order.

In the pseudovaria of *Solenobia lichenella* the egg-tubes are, according to Leuckart, very short, but otherwise the type of formation is precisely the same as in any of the more normal orders of the group. The mode of egg-formation exemplified in Plate XVIII. fig. 17, except as far as regards the shortness of the tube, is precisely the same as that of any other Lepidopterous insect.

[illegible] many instances have been established of unfecundated eggs proving to be fertile; and as no doubt can exist that these eggs also were capable of fertilization, it seems to follow that they must have been produced in the ordinary manner.

An opinion is prevalent among entomologists in this country, that caterpillars hatched from such eggs as these are weak, and can seldom be reared,—as if the vital force was indeed sufficient to carry the egg up to the point of hatching, but was by that exertion almost exhausted*.

In the Hymenoptera the process of egg-development very much resembles that of the Lepidoptera, but the ovaries differ generally in possessing more numerous egg-tubes, each with a fewer number of egg-germs. They differ from the Diptera, and agree with the Lepidoptera, Geodephaga, Hydradephaga, and most of the Neuroptera, in having each egg-chamber divided transversely into two parts. The vitelligenous cells are generally round, as in most of the Neuroptera; but sometimes they are polygonal, as in the Diptera and other orders possessing alternate groups of these cells. The nucleus of the vitelligenous cells is very indistinct, and can seldom be perceived plainly. In each chamber the vitelligenous cells differ considerably in size, being larger and darker towards the lowest end. The yelk is generally opake, and not nearly so pale and transparent as in some other groups. These circumstances are not favourable to observations on the germinal vesicle, which, however, appears to be in most instances (Plate XVII. figs. 2, 4, 5) a round vesicle, of rather small size in proportion to the size of the egg.

Plate XVII. figs. 2 and 3, represents two egg-tubes of *Cynips lignicola*, which I shall describe more fully; and I therefore content myself with mentioning here, that the process in this group offers no essential difference, and that there are no peculiarities in the formation of these pseudova, except those which arise from their very peculiar form. It is, however, of course possible, that when we shall have obtained a deeper insight into the mysterious processes of embryonic development, some differences will be detected between these pseudova and ordinary eggs.

The egg-formation in *Chelonus oculator* is remarkable, but its peculiarities have been much exaggerated by M. LÉON DUFOUR. According to this eminent anatomist, *Chelonus* possesses neither ovary nor eggs, but instead of them, four matrices containing a great number of living embryos, or perhaps nymphs?. I was naturally very anxious to examine such an extraordinary insect, and by the kindness of Mr. FREDERICK SMITH was enabled to do so.

M. LÉON DUFOUR has correctly described the external form of the generative organs. There are four similar matrices, two on each side, and divided by a transverse constriction into two chambers, the lower chamber being the smaller of the two. The lower ends of the four matrices unite into a very short oviduct. The upper ends are gradually contracted into a narrow egg-tube. There are therefore four egg-tubes altogether; they are of very considerable length, and each pair is closely united together. At the free end they swell into an oblong terminal chamber. M. LÉON DUFOUR has given also an excellent figure of these organs. He denies, however, to these long tubes the name of egg-tube, because it is not divided into egg-chambers; but in this they only resemble the egg-tubes of other Hymenoptera, which are quite cylindrical as long as they retain their outer membrane. It is the inner membrane only which is constricted.

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* This idea is confirmed by LEUCKART, Zur Kenntniss, &c. p. 376, and HEROLD.

Secondly, M. LÉON DUFOUR found in them no eggs; but if he had examined fresh specimens with a powerful glass, he would have found in them a succession of eggs as usual, but more numerous than in most of the Hymenoptera. Each egg is formed on the normal type, and consists of a double chamber, the lower half containing the true yelk, and the upper being full of the vitelligenous cells. It appeared to me that the vitelline membrane was in these eggs developed earlier than usual; for on cutting the lower part of the egg-tubes the eggs came out, and both the germ-chamber and the yelk-chamber appeared to be enclosed in a common membrane (Plate XVIII. figs. 13 to 17). By the time, however, that the eggs had reached the matrix the constriction had disappeared, and they were now cylindrical bodies, slightly larger at one end than the other.

If, however, some of the eggs are removed from the matrix and put in water, one end of the greater number of them will gradually swell until many resemble Plate XVIII. fig. 17. This alteration in shape, however, appears to be due to endosmosis, and I never saw the eggs take any other form. I had forgotten, during my examination of them, that M. LÉON DUFOUR attributed to them "une contractilité spontanée de tissu," and my attention was not specially directed to this point, but I did not notice anything of the sort.

The swollen end of the eggs often showed minute circular marks. I never was able to see the deposition of the eggs, and cannot say therefore that the species is not ovoviviparous. The eggs, however, never showed any distinct traces of an embryo; and having explained away the appearances on which that opinion was founded, I think myself justified in concluding that *Chelonus* is oviparous like the other Hymenoptera, although the process of egg-formation certainly does offer various peculiarities.

DEVELOPMENT OF PSEUDOVA.

Coccus hesperidum.—The generative organs of *Coccus* consist of a short egg-canal, which has more or less largely developed colleterial glands, and at its upper end separates into two tubes, each about $\frac{1}{20}$th of an inch in length, and constituting the ovaries. The whole surface of these two tubes is covered with egg-tubes in all stages of development, from a mere bud to an almost mature embryo. The ovary passes gradually into the oviduct, or rather the latter can hardly be said to exist as a distinct part of the system.

The spermatheca is a pyriform gland lying between the two ovaries, and connected with them by a duct which is attached to the egg-canal between the two ovaries.

It is correctly described and figured by Dr. LEYDIG, as it appears under the action of acetic acid, but in water or syrup it is coloured yellow by its contents. These are very minute, strongly refracting, yellow granules or globules, which offer no resemblance whatever to spermatozoa.

This organ is certainly homologous with the spermatheca of other allied species.

No one has yet seen a male of *C. hesperidum*, but this presence of the spermatheca indicates that this sex does occasionally occur. Whether the spermatheca fulfills any other function, as for instance that of a secreting gland, I am unable to assert.

The colleterial glands are very small in *C. hesperidum*, but in *C. persicæ* they are very conspicuous. This difference is probably owing to the fact that the egg-shell is much more developed in the latter species than in the former.

The two tubes of which the ovary consists, and which in old specimens occasionally throw off short branches, are by no means homologous with the egg-tubes of other insects, but rather with the uterus, or ovarian cavity into which they fall, and the short egg-chamber is the true homologue of the egg-tube.

In its earliest stage the egg-follicle is a simple projection of the ovarian wall, which becomes gradually pear-shaped, and may then be seen to consist of a structureless outer membrane, a layer of epithelial cells, and three masses bounded by very delicate walls, and each containing a large nucleus (Plate XVIII. figs. 1, 2). These three masses evidently represent the vitelligenous cells of other insects, as LEUCKART* has correctly suggested; but they are often very indistinct, and in some cases were altogether undistinguishable, though acetic acid will generally make them visible. They are very seldom more than three in number; indeed I only found four in one or two specimens, and five once, out of many hundreds which I examined. Sometimes. indeed, only two or even one could be seen; but it is probable that in these instances the cells were not really absent, but were only rendered invisible by their refractive power being nearly the same as that of the surrounding medium. After the egg-tube has attained to a certain size the boundaries of these cells disappear, as seems to be sometimes the case in Hymenoptera and other insects, and the nuclei only remain visible. Each nucleus often contains a solid-looking, irregular, greenish body or nucleolus.

I am inclined to believe that these cells are originally the same as those constituting the epithelial layer, but on this point it is difficult to form a decided opinion. Their nucleolus is certainly very different, and under the action of water they become cloudy, whilst the epithelial cells remain perfectly clear.

The epithelial cells line the structureless membrane forming the outer covering of the egg-follicle. As in other insects, they are columnar in the lower egg-chamber, and very much flattened in the upper. They contain a distinct circular nucleus, whose tissue differs but little from that of the cell itself, so that they cannot be seen unless they are exactly in focus. Plate XVIII. fig. 1 represents the earliest follicle in which I have seen them; I did not not notice in this case the nuclei, but have no reason to doubt their presence. They differ very much in size in different specimens, though tolerably equal in each, except that they are generally larger in the upper chamber than in the lower. Water causes not only the epithelial cells, but also the whole upper chamber to swell considerably. They are originally round, or rather somewhat polygonal from mutual pressure, but as the follicle grows they are laterally distended so as to become

* Loc. cit. p. 366.

fusiform. At this period also they contain numerous very small greenish granules (Plate XVIII. fig. 6), which have probably been secreted by the inner surface of their walls; and as these granules cannot be distinguished from the smallest oil-globules, I presume that these also are secreted by the epithelial cells.

The columnar epithelial cells of the lower chamber are almost always visible, but the flatter ones lining the upper chamber can seldom be seen unless the specimen is put in water.

The germinal vesicle very soon makes its appearance, but it is very curious that though it is (when seen at all) very distinct indeed, yet in many instances in which I should have expected to see it, it could not be discerned. Compare for instance Plate XVIII. figs. 3 and 4; in the latter the germinal vesicle and its nucleus are quite plain, while in the former, which was taken from the same animal, was in the field of view at the same time, and is in other respects rather more developed, not a trace of germinal vesicle could be seen.

We are therefore compelled to believe, either that the germinal vesicle is in some instances absent, which is extremely improbable, or that the time of its appearance varies considerably. Of course I do not rest this conclusion upon the examination of this one specimen only, but from the comparison of a great many. In the youngest specimen in which I ever found the germinal vesicle, the egg-follicle was pyriform, ·0032 in its greatest diameter, and about ·0058 in length. The germinal vesicle was ·0008 in diameter. It varies very little in different specimens either in appearance or size, the largest being ·0009 and the smallest ·0006; neither do these differences appear to have any reference to the state of maturity of the egg-follicle.

In the former part of my paper it has been mentioned that the vitelligenous cells and the egg-cell appear in insects generally to be modifications of the epithelial cells. The same is probably the case in *Coccus*; but in spite of my anxiety to throw light on this part of my subject, I was unable to do so. In one specimen, however, a few of the cells in the lower part of the upper chamber were larger than usual, and had a more distinct nucleus. The uppermost of these cells had very much the appearance of the germinal vesicle. If, however, this change takes place rapidly, there are evidently so many chances against our detecting the transitional forms, that it seemed desirable to record anything which may possibly be a link in the chain of evidence.

I did not find in *Coccus* any distinct membranous envelope surrounding the germinal vesicle, and it is therefore probably very soon absorbed, as in the Orthoptera and many other insects.

The later history of the germinal vesicle is equally obscure, for it soon disappears from view. At first I thought it was hidden by the oil-globules; but this can hardly be the case, because I never once succeeded in making the germinal vesicle visible by pressure when I could not see it otherwise. The largest specimen of the germinal vesicle I ever saw was very faint as it was disappearing, and I never saw two cells which could have resulted from the division of the germinal vesicle. Professor LEYDIG probably failed to perceive the germinal vesicle, because he generally examined his specimens in a weak solution of acetic acid, which causes it to swell and then disappear in a few moments.

At about the same time as the germinal vesicle, the *oil-globules* make their appearance, and soon become the most conspicuous part of the egg. They are at first very minute, but in a mature egg the larger ones are as much as ·0016 in diameter, from which to a mere granule every intermediate stage may be found. These globules are of a yellowish colour, and refract light, like the oil-globules in milk. They are also affected in a similar manner by acetic acid and sulphuric ether, being dissolved and running into one another if subjected to the latter; while in the former they split at the edges, as in Plate XVI. fig. 7, and finally become marked by irregular fine lines, some of which also seem to project from the side. Neither oxalic nor tartaric acid have much effect on the oil-globules.

The epithelial cells contain almost always several yellowish-green minute globules, which appear to me to agree in every respect, except in size, with the oil-globules; and it seems to me probable therefore that these are partly secreted by that side of the epithelial cells which is turned towards the inside of the egg-follicle.

It is evidently very difficult to obtain any decisive proof upon this point, and it might rather perhaps be concluded, from the descriptions and figures of M. DE QUATREFAGES*, that the oil-globules are secreted by the germinal vesicle. I have, however, never found that they are particularly collected round the germinal vesicle, which, moreover, is sometimes apparently absent after the formation of oil-globules has commenced (Plate XVIII. fig. 3). The oil-globules may often be seen lying somewhat compressed together (so as to have one or more straight sides), without running together, which indicates, I think, that the outer surface is covered by a somewhat harder pellicle or skin, though the effect of sulphuric ether shows, if any proof was wanting, that they are not covered by any distinct membrane.

Very soon after the appearance of the first oil-globules, the basal part of the egg-follicle begins to swell up, and to be separated from the upper part by a gradually deepening constriction, so that the egg-follicle comes, as described by MM. LEYDIG and LEUCKART, to consist of two chambers, the upper one of which is spherical, and the lower fusiform. The upper chamber, though short, is at first much broader than the lower, and contains the three vitelligenous cells (*e*); the lower chamber contains the oil-globules, and the germinal vesicle is situated at the apical end of the lower chamber.

Besides the oil-globules, the lower chamber contains also some few small yelk-cells and immense numbers of very small greenish granules, resembling in many respects very minute oil-globules; and I am not, as already stated, inclined to believe that these latter are produced by the coalescence of the minute green globules.

The egg-follicle continues to grow, and the germinal vesicle remains visible for some time, the largest specimen in which I have seen it being Plate XVIII. fig. 5.

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* Ann. des Sci. Nat. 1849, p. 202.

According to Dr. LEYDIG, the constriction separating the two chambers begins after a while to become less marked, and finally disappears, the egg itself being formed in and filling both chambers, so that the contents of the upper chamber occupy the apical end of the egg, and form the commencement of the blastoderm. In this I have no doubt he is mistaken, and that M. LEUCKART's account is correct, although I too have seen some instances which might account for his mistake. These, however, are quite exceptional; and as a general rule, after the constriction is formed, the upper chamber ceases to grow larger; and as, on the contrary, the lower one continues to expand, it forms gradually a larger and larger fraction of the whole. After remaining of the same size for some time, during which the lower part has considerably increased, the upper chamber begins to diminish, and finally becomes atrophied.

According to Dr. LEYDIG, if I understand him right, the three vitelligenous cells, which he considers as germinal vesicles, divide and subdivide, carrying with them each time a portion of periplast, so that when the reunion of the two chambers takes place, the apical portion contains a number of nucleated cells, which together constitute the blastoderm.

This description is entirely erroneous, as M. LEUCKART also has mentioned. Plate XVIII. fig. 7 shows a nearly full-grown egg, in the upper chamber of which the three vitelligenous cells (*e*), larger indeed than at first, but otherwise unaltered, are plainly visible. This is no solitary instance, but is, on the contrary, the usual process. Moreover, if several large egg-follicles are placed in water, it will probably happen that the upper chamber of one, at least, will burst, and one or more of the vitelligenous cells will be pressed out. They then present a cloudy appearance, but no orifice could ever be seen in them.

After having fulfilled their functions, and when the upper chamber is becoming atrophied, the vitelligenous cells sometimes break up into small irregular balls, which become smaller and smaller, and finally disappear.

The general cavity of the body of the female *Coccus* contains an immense number of oval green cells, which are apparently of vegetable nature. They are about $\frac{1}{8000}$ in breadth and $\frac{8}{8000}$ in length. Some specimens, however, are considerably longer, and present a constriction in the middle; these are evidently undergoing transverse division. I have also found similar cells in the larvæ at various ages, as at the beginning of June and July.

Almost always at the period when the vitelligenous cells have disappeared, and sometimes earlier, a mass of these oval green cells may be found at the lower part of the upper chamber. As the upper chamber becomes atrophied, they find their way down to the front of the egg. Thus we may say, almost without a metaphor, that the germs of future disease can be detected even in the unlaid egg*. I have scarcely ever found these oval green cells in a specimen which still contained the three vitelligenous cells unaltered, but believe that I have done so in a few instances.

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* Entozoa have been found in fœtus, but not, so far as I know, in an egg which would have hatched properly. See ALLEN THOMSON's article 'Ovum,' in TODD's Cyclopædia, p. 12. It is true that M. NORDMANN has described certain parasites as being produced from the yelk of *Tergipes*, without arresting the development of the embryo. M. VOGT, however, considers, with reason, that the supposed parasites are only detached cells, and not independent animals.

These parasitic cells are present in such great numbers in the full-grown female, that there is no difficulty in accounting for the entry of some of them into the egg; but it is difficult to understand why they do not appear in other parts of the egg-follicle, and why they increase so rapidly at a particular period in the development of the egg.

Weak solutions of oxalic acid, of tartaric acid, and of sulphuric ether produced no effect upon these cells.

The mature egg, besides the oil-globules and the surrounding fluid, contains also numerous vitelline spherules, which have every appearance of being true cells, but that they contain no nucleus.

I have sometimes found a few of these spherules (or others very nearly resembling them) in egg-tubes which had not yet lost their pyriform shape; but as a general rule, even in specimens as far advanced as Plate XVIII. fig. 6, few, if any, could be perceived. Soon, however, after the germinal vesicle ceases to be visible they make their appearance, and rapidly increase in numbers, apparently at the expense of the oil-globules and surrounding fluid.

At an early period they are small, not exceeding $\frac{3}{8000}$ in diameter, but in more mature ova they vary from $\frac{2}{8000}$ to $\frac{7}{8000}$.

Weak solutions of acetic or tartaric acid dissolved the yelk-cells, or at least rendered them invisible, and left instead a flocculent mass.

In the greenish eggs of *Phryganea*, as described by ZADDACH, the colour is owing to the yelk-globules themselves. In *Coccus* this is not the case. The yelk-globules are slightly yellow, and the green hue of the eggs is at first owing to the green granules, which I have already mentioned as being perhaps only very minute oil-globules. When, however, the egg is full-grown, and the upper chamber has been absorbed, these green granules will be found to be replaced by dark-green globules, regular in size, and about $\frac{1}{8000}$ in diameter, and which appear, therefore, to be in no way represented in the yelk of *Phryganea* eggs.

The vitelline membrane is formed so gradually that it is difficult to ascertain the exact time of its formation; when, however, the egg is full-grown, it may clearly be seen to be enclosed by this membrane, which is firm in texture, quite colourless, and shows no structure.

I could see no trace of micropyle, which under the circumstances is quite natural.

The mature egg is a light green, oval mass, $\frac{25}{2000}$ in length and $\frac{18}{2000}$ in breadth. It is enclosed in an outer structureless membrane, probably the vitelline membrane, but, like STEIN*, I never could find a trace of a second, though the eggs of most insects are generally said to have double covering.

Since these eggs are hatched almost immediately after leaving the body of the mother, it is evident that they do not require so thick a shell as those which are deposited in water or in the open air, and this may perhaps account for the small size of the colleterial glands.

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* Loc. cit. p. 66.

At this period the egg-contents consist of yellowish oil-globules, colourless yelk-cells, a small quantity of fluid, the green globules which give the egg its colour, and the parasitic, oval green cells, which are congregated near the cephalic end.

Dr. LEYDIG describes the first trace of the embryo as arising at the free end of the egg, and then extending with a waved course as a clear streak across the egg. It seems to me, however, that the first trace of embryo is at the basal end of the egg, and I think that he has been led into error by supposing that this embryonic structure arises immediately from the division of the vitelligenous cells (*e*), which we have already seen is not the case. Professor LEUCKART also asserts that the thickening of the blastoderm commences at the hinder pole*.

The remainder of the egg consists, as before, of oil-globules, yelk-globules, and green globules, which latter, however, are more numerous round the embryo than elsewhere.

Gradually, however, the yelk-cells become smaller, tolerably equal in size; and as the embryo continues to enlarge, they diminish greatly, being no doubt changed in some manner into the nucleated cellular tissue of which it consists. At the time when the antennæ make their appearance the yelk-cells have almost entirely disappeared.

At this period the tissue of the embryo consists of minute cells about $\frac{1}{4000}$ in diameter, and each containing a bright central spot. When pressed out into water they often lose their distinctness, and sometimes they seem to change into green bodies, apparently somewhat cubic in shape, and separated a little from one another by a colourless, structureless substance. What conditions are necessary to this change I was unable to determine.

The small cells constituting the embryo at a very early period, when subjected to a weak solution of either tartaric or oxalic acid, took a variety of forms, more or less oval, and pointed at the narrow end. Their contents became granular. Acetic acid made the cell-wall less distinct and the contents opake, as if coagulated.

Thus we see that the different bodies, which are either contained in the egg, or have contributed to its formation, are by no means few in number. We have, first, the structureless external membrane of the egg-follicle; secondly, the cells forming the epithelium; thirdly, their nuclei; fourthly, the vitelligenous cells; then the germinal vesicle with its nucleus, the oil-globules, the periplast, in which the three last substances are contained, the yelk-cells, the parasitic vegetable cells, the green globules, the blastodermic cells and their nuclei, and, finally, the vitelline membrane.

Up to this period, and indeed until the different appendages of the embryo have become quite distinct, the egg has not altered its position, but still remains in the follicle in which it was formed; and its head, as usual in insects, is always furthest from the vulva, or rather is at the free end of the follicle.

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* Fortpflanzung und Entwickelung der Pupiparen. Halle, 1858, p. 69.

Coccus Persicæ.—On the 22nd of May I found in our garden some specimens of *C. persicæ*, which I was much surprised to find is oviparous and not nearly ovoviviparous, like *C. hesperidum*. The females were, at the period in question, rapidly approaching the term of their existence, and most of them had already laid many eggs. These latter are developed almost in a similar manner to those of the preceding species, with however some interesting differences.

The form of the ovaries and arrangement of the egg-tubes is very nearly the same in both species. The spermatheca of *C. persicæ*, however, is less firm, and is pale in colour. The colleterial glands, which, probably in consequence of that species being ovoviviparous, have become small and inconspicuous, are in the Peach-coccus large and pedunculated.

They are six in number, four large and two small, these latter being apparently attached by a short stalk to the peduncle of the large one, which is furthest from the vulva. They lie three on each side, and their ducts open into the egg-canal close together, and about half-way between the vulva, and the division of the egg-canal into two oviducts.

The internal structure is very distinct and interesting. It consists of many cells lying loose in the internal cavity, and resembling very much in form, size, and appearance the vitelligenous cells of the egg-follicle. Indeed, if one of these bodies was lying free from its attachments, I believe it would be quite impossible to decide whether it belonged to these colleterial glands or had come from an egg-follicle. I counted about six in the smaller glands, and from twenty-five to thirty in each of the large ones; they were nearly the same size in all the glands.

As might be expected, several specimens did not show this structure, but in others it was very distinct. In one specimen the glandular bodies were more numerous and smaller. In this case, however, they varied considerably in size.

The egg-follicle is tougher, and the epithelian layer is more distinct than in the preceding species. The number of vitelligenous cells is greater, varying between five and eight in each chamber, and they are also more distinct and rounder.

As might be expected, their number does not depend upon the age of the follicle. I have seen a very young one with seven cells, and have often found old ones with only five. The size sometimes varies considerably in the same follicle, and did not appear to depend upon the stage of growth.

The nucleus of the cells could almost always be distinguished. It consists of an irregularly-shaped body of a greenish colour, and presents often projections which appear to be undergoing a process of separation. Sometimes this appears to have taken place, and we find then two to four smaller and more regular nuclei.

They never contained granulations, like those described by HUXLEY, in the corresponding cells of *Aphis*.

They appear finally to break up and to be absorbed, exactly as has been already described; but it is difficult to prove this satisfactorily, as in large follicles their cell-wall becomes very indistinct.

In the early stages of egg-development there appear to be fewer oil-globules than in *C. hesperidum*, but more numerous granules. These latter are about $\frac{1}{16,000}$ in diameter, and are congregated principally round the germinal vesicle. They disappear if subjected to the action of tartaric acid.

The germinal vesicle is small, but it is so much covered by the granules that I never could obtain a satisfactory view of it, and still less of the nucleus. It is, however, smaller than the vitelligenous cells.

The production of the parasitic green cells occurs in the same manner and position as in *C. hesperidum*. Instead of being oval, however, they are shaped like a sausage. They are $\frac{4}{8000}$ in length by about $\frac{1}{7000}$ in breadth. They are, however, multiplied by division, and consequently vary in size as much as the corresponding parasite of *C. hesperidum*; and in one specimen I found them much longer than usual.

In an egg almost full-grown the contents were of two sorts; first, oil-globules from $\frac{2}{8000}$ to $\frac{6}{8000}$ in diameter, and of a whitish colour; and, secondly, small globules $\frac{1}{8000}$ to $\frac{1}{16,000}$ in diameter. These latter are rendered invisible by tartaric acid. The eggs when laid are of a brilliant white colour, $\frac{13}{1000}$ in length, and $\frac{7}{1000}$ in breadth. They generally showed the commencement of the blastodermic layer, and in some this had even extended quite across the egg.

The oil-globules are by far the most conspicuous part of the egg-contents; the remainder appears to consist of a periplastic substance.

The small granules, which at an earlier period were very numerous, have almost disappeared; and the yelk-globules, which are so conspicuous in *C. hesperidum*, were in the present species very few and of a small size.

The outer surface of the egg-shell is covered by minute rings, of which the ends somewhat overlap. These rings are from $\frac{1}{6000}$ to $\frac{1}{4000}$ in diameter, and are no doubt identical with the white substance which exudes from pores on the underside of the body. Immersion in spirits of wine for thirty-six hours had no apparent effect on them. I also placed them in dilute sulphuric acid for two hours and a half with the same result.

In June the females covered a number of eggs, besides those which were not yet deposited. I examined a good many of the former, and found that each contained an embryo consisting of a waved line of small nucleated cells, extending from one end of the egg to the other[*].

Cynips lignicola.—Although the gall insects are so common and so numerous, the female sex only has yet been found, so that the males are probably only produced at distant intervals. Except, however, in so far as the peculiar shape of the egg is concerned, the mode of egg-formation does not differ from that which occurs in other Hymenoptera.

The ovary consists of a great number of tubes, which form a verticillated bunch, and fall into a common oviduct. Each tube contains about thirteen eggs (figs. 2 and 3); in the earliest stages there is absolutely nothing to distinguish the egg-formation in this species from the type common in insects. A glance at the figures is sufficient to show this, so that it is unnecessary to enter into any detailed description.

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[*] Leydig, *loc. cit.* pl. 1. fig. 4 *i*.

Each of the pseudova, even after it has attained its full length, is surrounded by a distinct layer of nucleated cells. The walls of the tube appear to be constituted by a delicate, structureless membrane, so that I suppose it is the epithelial layer (consisting of nucleated cells) which has in some places detached itself from the outer membrane in order to form a close covering for the eggs. After the egg has arrived at maturity, this cellular layer gradually disappears.

At the lower end, in Plate XVII. fig. 2, one of the eggs is elongating and pushing its narrow end up the tube. This process appears to be somewhat rapid, at least there is always at this stage a considerable difference between the form of the lowest egg and that of the one immediately preceding it.

The egg gradually becomes longer and longer, the narrow end at the same time forcing itself up the tube; and the other eggs successively undergo the same changes, until at length the ovarian tube offers the appearance represented in fig. 3, Plate XVII. The narrow ends now all lie at the upper end of the tube, and the swollen ends at the lower.

The germinal vesicle is present, as usual in insects, but it remains visible longer than I have found to be the case in most other species, and indeed may be seen after the pseudovum has attained its mature form.

The development of the pseudova of *Solenobia lichenella* appears also to offer no great peculiarities. Plate XVIII. fig. 17, copied from LEUCKART, represents one of the egg-tubes taken from a caterpillar of *S. lichenella*, and it resembles, in all important particulars, a similar organ of any other Lepidopterous insect. The epithelial cells, the large vitelligenous cells with their nuclei, and the egg-cell itself, with its germinal vesicle, are all of the usual structure, and arranged in the ordinary manner.

Many other Lepidoptera have presented us with individual instances of parthenogenesis, in which the eggs, though fertile without impregnation, were no doubt identical with the true ova, and would have been impregnated under ordinary circumstances[*].

In the Hive-bee, also, the ovarian development of the ova and pseudova must apparently be identical, since it would appear that in normal instances, it is not decided until after the ovarian product has entered the oviduct, whether it is to be an ovum or a pseudovum, in other words, whether it is to be impregnated or not.

At the same time the sex of the future animal is determined, since, according to MM. LEUCKART and SIEBOLD, eggs always in this species produce females, and pseudova give birth to males.

We are then, I think, justified in asserting that in the present state of our knowledge no difference can be pointed out between the ovarian development of the pseudovum in insects and that of the true ovum.

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[*] M. MILLIÈRE has lately instituted a new genus, *Agerona*, which belongs to the same group of moths, and contains two new species, both of which multiply by parthenogenesis. Ann. de la Soc. Linnéenne de Lyon, 1857, vol. iv. p. 181.

Description of the Plates.

PLATE XVI.

Fig. 1. Egg-tube of *Blatta orientalis*. × 30.
Fig. 2. Egg-tube of *Æschna cyanea*. × 60.
Fig. 3. Egg-tube of *Pulex irritans*. × 60.
Fig. 4. Egg-tube of *Telephorus*. × 60.
Fig. 5. Egg-tube of *Forficula auricularia*. × 60.
Fig. 6. Terminal portion of *Forficula auricularia*. × 250.
Fig. 7. Egg-tube of *Psocus*. × 250.
Fig. 8. Egg-tube of *Panorpa communis*. × 60.
Fig. 9. Egg-tube of *Cheilosoia*. × 60.
Fig. 10. Part of egg-tube of *Nepa*, to show the yelk-ducts. × 80.

PLATE XVII.

Fig. 1. Egg-tube of *Carabus violaceus*. × 30.
Fig. 2. Egg-tube of *Cynips lignicola*. × 60.
Fig. 3. Egg-tube of *Cynips lignicola*. × 60, in a more advanced state.
Fig. 4. Egg of *Cynips lignicola*. × 60.
Fig. 5. Egg-tube of *Odynerus*. × 60.
Fig. 6. Egg-tubes of *Ophion luteum*. × 30.
Fig. 7. Egg-tubes of *Nepa cinerea*. × 60.
Fig. 8. Egg-tubes of *Eristalis tenax*. × 250.
Fig. 9. End of egg of *Cynips lignicola*. × 120.

PLATE XVIII.

Fig. 1. Very young egg-tube of *Coccus hesperidum*, showing epithelial cells. × 250.
Fig. 2. Very young egg-tube of *Coccus hesperidum*, showing vitelligenous cells. × 250.
Fig. 3. Egg-tube of *Coccus hesperidum*. × 250.
Fig. 4. Egg-tube of *Coccus hesperidum*. × 250.
Fig. 5. Egg-tube of *Coccus hesperidum*. × 250. More advanced.
Fig. 6. Egg-tube of *Coccus hesperidum*. × 250. More advanced.
Fig. 7. Egg-tube of *Coccus hesperidum*. × 250. More advanced.
Fig. 8. Egg-tube of *Coccus hesperidum*. × 250. More advanced.
Fig. 9. Egg-tube of *Coccus hesperidum*. × 250. More advanced.
Fig. 10. Egg-tube of *Coccus hesperidum*. × 250. More advanced.
Fig. 11. Egg-tube of *C. persicæ*. × 250.
Fig. 12. Very young tube of *C. persicæ*.

Fig. 13. Egg of *Chelonus oculator*, from the lower part of the egg-tube. ×60.

Fig. 14. Egg of *Chelonus oculator*, from the matrix of the egg-tube. ×60.

Fig. 15. Egg of *Chelonus oculator*, from the matrix under the action of water. ×60.

Fig. 16. Egg of *Chelonus oculator*, from the matrix under the action of water. ×60.

Fig. 17. Egg-tube of *Solenobia lichenella*. After Leuckart.

Fig. 18. Three germinal vesicles of *Pterostichus melanarius*, from three successive egg-cells. ×250.

The letters refer to the same parts in all the Plates.

m.	Muscular or outer membrane.
i.	Inner membrane.
a.	Epithelial cells.
b.	Nucleus of epithelial cells.
e.	Egg-chamber, often divided into
y.	Yelk-chamber.
z.	Germ-chamber.
t.	Terminal chamber of egg-tube.
f.	Terminal filament of egg-tube.
v.	Vitelligenous cell.
u.	Nucleus of vitelligenous cell.
s.	Egg-cell.
g.	Germinal vesicle.
h.	Macula germinativa.
o.	Oil-globules.
p.	Parasitic cells.
d.	Yelk-duct.

XIX. *On the Conic of Five-pointic Contact at any point of a Plane Curve.*

By A. Cayley, *Esq.*, *F.R.S.*

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The tangent is a line passing through two consecutive points of a plane curve, and we may in like manner consider the conic which passes through five consecutive points of a plane curve; and as there are certain singular points, viz. the points of inflexion, where three consecutive points of the curve lie in a line, so there are singular points where six consecutive points of the curve lie in a conic. In the particular case where the given curve is a cubic, the last-mentioned species of singular points have been considered by Plücker and Steiner, and in the same particular case, the theory of the conic of five-pointic contact has recently been established by Mr. Salmon. But the general case, where the curve is of any order whatever, has not, so far as I am aware, been hitherto considered;—the establishment of this theory is the object of the present memoir.

I. *Investigation of the Equation of the Conic of Five-pointic Contact.*

1. I take (X, Y, Z) as current coordinates, and I represent the equation of the given curve by

$$\Upsilon=(*\between X, Y, Z)^m=0.$$

Let (x, y, z) be the coordinates of a given point on the curve, and let $U=(*\between x, y, z)^m$ be what Υ becomes when (x, y, z) are written in the place of (X, Y, Z); we have therefore $U=0$ as a condition satisfied by the coordinates of the point in question.

2. Write for shortness

$$DU=(X\partial_x+Y\partial_y+Z\partial_z)U,$$

$$D^2U=(X\partial_x+Y\partial_y+Z\partial_z)^2U,$$

and let $\Pi=aX+bY+cZ=0$ be the equation of a line. It is easy to see that

$$D^2U-\Pi.DU=0$$

will be the equation of a conic having an ordinary (two-pointic) contact with the curve at the point (x, y, z). In fact the equation $DU=0$ is that of the tangent at the point in question, and the equation $D^2U=0$ is that of the penultimate polar (or polar conic) of the point, which conic is touched by the tangent; the assumed equation represents therefore a conic having an ordinary (two-pointic) contact with the polar conic, and therefore with the curve. It may be added that the two conics intersect besides in a pair of points, and that the line joining these, or common chord of the two conics, is the line represented by the equation $\Pi=0$; and this being so, the constants (a, b, c) of the line $\Pi=0$ can be so determined as to give rise to a five-pointic contact.

3. Consider the coordinates of a point of the curve as functions of a single variable parameter; then for the present purpose the coordinates of a point consecutive to (x, y, z) may be taken to be

$$\begin{aligned} &x+dx+\tfrac{1}{2}d^2x+\tfrac{1}{6}d^3x+\tfrac{1}{24}d^4x, \\ &y+dy+\tfrac{1}{2}d^2y+\tfrac{1}{6}d^3y+\tfrac{1}{24}d^4y, \\ &z+dz+\tfrac{1}{2}d^2z+\tfrac{1}{6}d^3z+\tfrac{1}{24}d^4z, \end{aligned}$$

values which, substituted for X, Y, Z, must satisfy the equations

$$\Upsilon=0,\ \mathrm{D}^3\mathrm{U}-\Pi.\mathrm{DU}=0.$$

4. I write for shortness

$$\begin{aligned} \partial_1&=dx\,\partial_x+dy\,\partial_y+dz\,\partial_z, \\ \partial_2&=d^2x\partial_x+d^2y\partial_y+d^2z\partial_z, \\ \partial_3&=d^3x\partial_x+d^3y\partial_y+d^3z\partial_z, \\ \partial_4&=d^4x\partial_x+d^4y\partial_y+d^4z\partial_z, \end{aligned}$$

then the consecutive value of Υ is

$$\text{exp.}\ (\partial_1+\tfrac{1}{2}\partial_2+\tfrac{1}{6}\partial_3+\tfrac{1}{24}\partial_4)\mathrm{U}$$

(Read exp. z, exponential of z, $=e^z$), which is

$$= \left.\begin{array}{l} (1+\partial_1+\tfrac{1}{2}\partial_1^2+\tfrac{1}{6}\partial_1^3+\tfrac{1}{24}\partial_1^4) \\ \times(1 \quad +\tfrac{1}{2}\partial_2 \quad +\tfrac{1}{8}\partial_2^2) \\ \times(1 \quad +\tfrac{1}{6}\partial_3 \quad) \\ \times(1 \quad +\tfrac{1}{24}\partial_4) \end{array}\right\}\mathrm{U}$$

$$= \left.\begin{array}{l} 1+\partial_1+\tfrac{1}{2}\partial_1^2+\tfrac{1}{6}\partial_1^3+\tfrac{1}{24}\partial_1^4 \\ \quad +\tfrac{1}{2}\partial_2+\tfrac{1}{2}\partial_1\partial_2+\tfrac{1}{4}\partial_1^2\partial_2 \\ \quad +\tfrac{1}{8}\partial_2^2 \\ \quad +\tfrac{1}{6}\partial_3+\tfrac{1}{6}\partial_1\partial_3 \\ \quad +\tfrac{1}{24}\partial_4 \end{array}\right\}\mathrm{U}$$

$$\begin{aligned} = &\ \mathrm{U} \\ &+\partial_1\mathrm{U} \\ &+\tfrac{1}{2}(\partial_1^2+\partial_2)\mathrm{U} \\ &+\tfrac{1}{6}(\partial_1^3+3\partial_1\partial_2+\partial_3)\mathrm{U} \\ &+\tfrac{1}{24}(\partial_1^4+6\partial_1^2\partial_2+4\partial_1\partial_3+3\partial_2^2+\partial_4)\mathrm{U}, \end{aligned}$$

the several terms of which must respectively vanish, and we have therefore

$$\begin{aligned} &\mathrm{U}=0, \\ &\partial_1\mathrm{U}=0, \\ &\partial_2\mathrm{U}=-\partial_1^2\mathrm{U}, \\ &\partial_3\mathrm{U}=-(\partial_1^3+3\partial_1\partial_2)\mathrm{U}, \\ &\partial_4\mathrm{U}=-(\partial_1^4+6\partial_1^2\partial_2+4\partial_1\partial_3+3\partial_2^2)\mathrm{U}. \end{aligned}$$

5. Next, preparing to substitute in the equation

$$D^2U-\Pi.DU=0,$$

the consecutive value of DU is

$$\begin{aligned}&(x+dx+\tfrac{1}{2}d^2x+\tfrac{1}{6}d^3x+\tfrac{1}{24}d^4x)\partial_x U+\&c\\ &=(\partial_0+\partial_1+\tfrac{1}{2}\partial_2+\tfrac{1}{6}\partial_3+\tfrac{1}{24}\partial_4)U,\end{aligned}$$

where

$$\partial_0 U=(x\partial_x+y\partial_y+z\partial_z)U=mU.$$

Reducing by the above results, the consecutive value of DU is

$$=-\tfrac{1}{2}\partial_1^2U-\tfrac{1}{6}(\partial_1^3+3\partial_1\partial_2)U-\tfrac{1}{24}(\partial_1^4+6\partial_1^2\partial_2+4\partial_1\partial_3+3\partial_2^2)U.$$

6. Hence also writing

$$\begin{aligned}P&=ax\ \ +by\ \ +cz,\\ \partial_1P&=adx\ +bdy\ +cdz,\\ \partial_2P&=ad^2x+bd^2y+cd^2z,\end{aligned}$$

the consecutive value of $-\Pi DU$ is $-(P+\partial_1P+\tfrac{1}{2}\partial_2P)$ multiplied into the consecutive value of DU, and the product is

$$\begin{aligned}&=P.\tfrac{1}{2}\partial_1^2U\\ &+P.\tfrac{1}{6}(\partial_1^3+3\partial_1\partial_2)U \qquad\qquad +\partial_1P.\tfrac{1}{2}\partial_1^2U\\ &+P.\tfrac{1}{24}(\partial_1^4+6\partial_1^2\partial_2+4\partial_1\partial_3+3\partial_2^2)U+\partial_1P.\tfrac{1}{6}(\partial_1^3+3\partial_1\partial_2)U+\tfrac{1}{2}\partial_2P.\tfrac{1}{2}\partial_1^2U.\end{aligned}$$

7. The consecutive value of D^2U is

$$=(x+dx+\tfrac{1}{2}d^2x+\tfrac{1}{6}d^3x+\tfrac{1}{24}d^4x)^2\partial_x{}^2U+\&c.$$

$$=\left.\begin{array}{l}\qquad x^2\\ +\ 2xdx\\ +\ \ xd^2x+(dx)^2\\ +\ \tfrac{1}{3}xd^3x+\ dxd^2x\\ +\tfrac{1}{12}xd^4x+\tfrac{1}{3}dxd^3x+\tfrac{1}{4}(d^2x)^2\end{array}\right\}\partial_x^2U+\&c.,$$

which is

$$=\left.\begin{array}{l}\qquad \partial_0^2\\ +\ 2\partial_0\partial_1\\ +\ \ \partial_0\partial_2+\partial_1^2\\ +\ \tfrac{1}{3}\partial_0\partial_3+\partial_1\partial_2\\ +\tfrac{1}{12}\partial_0\partial_4+\tfrac{1}{3}\partial_1\partial_3+\tfrac{1}{4}\partial_2^2\end{array}\right\}U;$$

and observing that

$$\begin{aligned}\partial_0^2\ U&=m(m-1)\ \ U,\\ \partial_0\partial_1U&=\ (m-1)\partial_1U,\\ \partial_0\partial_2U&=\ (m-1)\partial_2U,\\ \partial_0\partial_3U&=\ (m-1)\partial_3U,\\ \partial_0\partial_4U&=\ (m-1)\partial_4U,\end{aligned}$$

and reducing as before, the consecutive value of D^2U is

$$\begin{aligned}=&-(m-2)\partial_1^2U\\&-\tfrac{1}{3}[(m-1)\partial_1^3+3(m-2)\partial_1\partial_2]U\\&-\tfrac{1}{12}[(m-1)(\partial_1^4+6\partial_1^2\partial_2)+(m-2)(4\partial_1\partial_3+3\partial_2^2)]U.\end{aligned}$$

8. Substituting in the equation $D^2U-\Pi.DU=0$, we obtain as the conditions of a five-pointic contact

$$-(m-2)\partial_1^2U+P.\tfrac{1}{2}\partial_1^2U=0,$$

$$\begin{aligned}&-\tfrac{1}{3}[(m-1)\partial_1^3+3(m-2)\partial_1\partial_2]U\\&+P.\tfrac{1}{6}(\partial_1^3+3\partial_1\partial_2)U+\partial_1P.\tfrac{1}{2}\partial_1^2U=0,\end{aligned}$$

$$\begin{aligned}&-\tfrac{1}{12}[(m-1)(\partial_1^4+6\partial_1^2\partial_2)+(m-2)(4\partial_1\partial_3+3\partial_2^2)]U\\&+P.\tfrac{1}{24}[\partial_1^4+6\partial_1^2\partial_2+4\partial_1\partial_3+3\partial_2^2]U+\partial_1P.\tfrac{1}{6}[\partial_1^3+3\partial_1\partial_2]U+\tfrac{1}{2}\partial_2P.\tfrac{1}{2}\partial_1^2U=0;\end{aligned}$$

or reducing

$$P=2(m-2),$$

$$\partial_1P=\tfrac{2}{3}\,\frac{\partial_1^3U}{\partial_1^2U},$$

$$\partial_2P=\tfrac{1}{2}\,\frac{[\partial_1^4+6\partial_1^2\partial_2]U}{\partial_1^2U}-\tfrac{4}{9}\,\frac{\partial_1^3U}{\partial_1^2U}\,\frac{[\partial_1^3+3\partial_1\partial_2]U}{\partial_1^2U},$$

which are the conditions of a five-pointic contact: it is to be remarked that if only the first and second conditions are satisfied, we have a four-pointic contact, and if only the first condition is satisfied, a three-pointic contact.

9. We have to reduce the last-mentioned equations; suppose that A, B, C are the first derived functions of U, then the equation $\partial_1U=0$ may be written

$$Adx+Bdy+Cdz=0,$$

and this will be satisfied identically if

$$\begin{aligned}dx&=B\nu-C\mu,\\dy&=C\lambda-A\nu,\\dz&=A\mu-B\lambda,\end{aligned}$$

where λ, μ, ν are arbitrary multipliers, which may be taken to be constants. We have therefore $\partial_1=\mathbf{D}$, where

$$\mathbf{D}=(B\nu-C\mu)\partial_x+(C\lambda-A\nu)\partial_y+(A\mu-B\lambda)\partial_z.$$

10. The resulting expressions for ∂_1^2U, ∂_1^3U, ∂_1^4U may be exhibited in the reduced forms given by Hesse, viz. if $\vartheta=\lambda x+\mu y+\nu z$, we have

$$\begin{aligned}\partial_1^2U&=P_2U-Q_2\vartheta^2,\\\partial_1^3U&=P_3U-Q_3\vartheta^3,\\\partial_1^4U&=P_4U-Q_4\vartheta^4,\end{aligned}$$

where the values of P_2, P_3, P_4; Q_2, Q_3, Q_4 are as follows, viz. if (a, b, c, f, g, h) are the second derived functions of U, and if

$$\mathrm{H}=\begin{vmatrix} a, & h, & g \\ h, & b, & f \\ g, & f, & c \end{vmatrix}$$

be the Hessian; if, moreover,

$$\Phi=-\begin{vmatrix} & \lambda, & \mu, & \nu \\ \lambda & a, & h, & g \\ \mu & h, & b, & f \\ \nu & g, & f, & c \end{vmatrix}$$

be the bordered Hessian (we may also write $\Phi=(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\lambda, \mu, \nu)^2$, where $(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})$ are the inverse coefficients of (a, b, c, f, g, h), viz. $\mathfrak{A}=(bc-f^2)$ &c.); and finally, if for shortness we write

$$\Pi=\partial_x\Phi\,.\,\partial_\lambda\Phi+\partial_y\Phi\,.\,\partial_\mu\Phi+\partial_z\Phi\,.\,\partial_\nu\Phi,$$

$$\square=\partial_x\mathrm{H}\,.\,\partial_\lambda\Phi+\partial_y\Pi\,.\,\partial_\mu\Phi+\partial_z\mathrm{H}\,.\,\partial_\nu\Phi,$$

then we have

$$\mathrm{P}_2=\frac{m}{m-1}\quad\Phi, \qquad \mathrm{Q}_2=\frac{1}{(m-1)^2}\mathrm{H},$$

$$\mathrm{P}_3=\frac{m}{m-1}\mathbf{D}\Phi, \qquad \mathrm{Q}_3=\frac{1}{(m-1)^2}\mathbf{D}\,\mathrm{H},$$

$$\mathrm{P}_4=\frac{m}{m-1}\mathbf{D}^2\Phi-\frac{m\vartheta}{(m-1)^2}\Pi, \qquad \mathrm{Q}_4=\frac{1}{(m-1)^2}\mathbf{D}^2\mathrm{H}-\frac{\vartheta}{(m-1)^3}\square+\frac{3(m-2)}{(m-1)^3}\mathrm{H}\Phi.$$

In the present case U = 0, and we have

$$\partial_1^2\mathrm{U}=-\mathrm{Q}_2\vartheta^2,$$

$$\partial_1^3\mathrm{U}=-\mathrm{Q}_3\vartheta^2,$$

$$\partial_1^4\mathrm{U}=-\mathrm{Q}_4\vartheta^2.$$

11. Hence, substituting for Q_2 and Q_3 their values, the first and second of the equations for the five-pointic contact give

$$\mathrm{P}=2(m-2),$$

$$\partial_1\mathrm{P}=\tfrac{2}{3}\frac{\mathbf{D}\mathrm{H}}{\mathrm{H}},$$

and observing that Π is a linear function of (X, Y, Z), and consequently that P, $\partial_1\mathrm{P}$ denote simply the values which Π assumes when (x, y, z), (dx, dy, dz) are respectively substituted for (X, Y, Z), we see at once that these two conditions will be satisfied if we put

$$\Pi=\tfrac{2}{3}\frac{1}{\mathrm{H}}\,\mathrm{DH}+\Lambda.\mathrm{DU},$$

where Λ is an arbitrary constant, or, what is the same thing, an arbitrary function of (x, y, z). We have thus the general equation of a conic of four-pointic contact.

12. The above value of Π gives

$$\partial_2\mathrm{P}=\tfrac{2}{3}\frac{1}{\mathrm{H}}\partial_2\mathrm{H}+\Lambda\partial_2\mathrm{U},$$

and the third equation of the system of conditions for a five-pointic contact is therefore

$$\tfrac{2}{3}\frac{1}{H}\partial_2 H+\Lambda\partial_2 U=\tfrac{1}{3}\frac{[\partial_1^4+6\partial_1^2\partial_2]U}{\partial_1^2 U}-\tfrac{4}{9}\frac{\partial_1^3 U}{\partial_1^2 U}\frac{[\partial_1^3+3\partial_1\partial_2]U}{\partial_1^2 U},$$

which leads to the value of Λ.

13. We have in general

$$\partial_1^2 U=\frac{m}{m-1}\Phi U-\frac{1}{(m-1)^2}H\vartheta^2,$$

$$\partial_1^3 U=\frac{m}{m-1}\mathbf{D}\Phi.U-\frac{1}{(m-1)^2}\mathbf{D}H.\vartheta^2,$$

$$(\partial_1^3+3\partial_1\partial_2)U=\frac{m}{m-1}\mathbf{D}\Phi.U-\frac{1}{(m-1)^2}\mathbf{D}H.\vartheta^2,$$

$$(\partial_1^4+6\partial_1^2\partial_2)U=\frac{m}{m-1}\left(2\partial_2\Phi+\mathbf{D}^2\Phi+\frac{\vartheta}{m-1}H\right)U$$
$$-\frac{1}{(m-1)^2}\left(2\partial_2 H+\mathbf{D}^2H+\frac{\vartheta}{m-1}\square-\frac{3(m-2)}{m-1}H\Phi\right)\vartheta^2,$$

the last two of which have not yet been demonstrated. The value of $\partial_2 H$ (which, however, is not required for the present purpose) is

$$\partial_2 H=\frac{-3(m-2)}{m-1}H\Phi+\frac{\frac{1}{2}\square}{m-1}\vartheta,$$

which also is not yet demonstrated.

14. Putting $U=0$, we have

$$\partial_1^2 U=-\frac{1}{(m-1)^2}H\vartheta^2,$$

$$\partial_1^3 U=-\frac{1}{(m-1)^2}\mathbf{D}H.\vartheta^2,$$

$$(\partial_1^3+3\partial_1\partial_2)U=-\frac{1}{(m-1)^2}\mathbf{D}H.\vartheta^2,$$

$$(\partial_1^4+6\partial_1^2\partial_2)U=-\frac{1}{(m-1)^2}\left(2\partial_2 H+\mathbf{D}^2H+\frac{\vartheta}{m-1}\square-\frac{3(m-2)}{m-1}H\Phi\right)\vartheta^2,$$

and substituting,

$$\tfrac{2}{3}\frac{1}{H}\partial_2 H+\frac{1}{(m-1)^2}H\vartheta^2\Lambda$$
$$=\tfrac{1}{3}\frac{1}{H}\left(2\partial_2 H+\mathbf{D}^2H+\frac{\vartheta}{m-1}\square-\frac{3(m-2)}{m-1}H\Phi\right)-\tfrac{4}{9}\left(\frac{\mathbf{D}H}{H}\right)^2,$$

where the term involving $\partial_2 H$ disappears; the equation may be written

$$\frac{9H^3\vartheta^2}{(m-1)^2}\Lambda=3H\left(\mathbf{D}^2H+\frac{\vartheta}{m-1}\square-\frac{3(m-2)}{m-1}H\Phi\right)-4(\mathbf{D}H)^2,$$

which I will represent by

$$\frac{9H^3\vartheta^2}{(m-1)^2}\Lambda=3R_2R_4-4R_3^2,$$

the values of R_2, R_3, R_4 being

$$\left\{\begin{array}{l} R_2=H, \\ R_3=\mathbf{D}H, \\ R_4=\mathbf{D}^2H+\frac{\vartheta}{m-1}\square-\frac{3(m-2)}{m-1}H\Phi. \end{array}\right.$$

15. We have $R_2=H$, and it will be shown that

$$\begin{aligned}(m-1)^2R_3^2=&-\ 9(m-2)^2H^2\Phi \\ &+\ 3(m-2)\,H\square\vartheta \\ &-\ \Psi\vartheta^2 \\ (m-1)^2R_4=&-12(m-2)^2H\Phi \\ &+\ 4(m-2)\,\square\vartheta \\ &-\ \Omega\vartheta^2,\end{aligned}$$

where for shortness

$$\Psi=(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\partial_x H, \partial_y H, \partial_z H)^2,$$
$$\Omega=(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\partial_x, \partial_y, \partial_z)^2H,$$

and hence writing

$$9H^3\vartheta^2\Lambda=3R_2(m-1)^2R_4-4(m-1)^2R_3^2,$$

we have

$$9H^3\Lambda=-3\Omega H+4\Psi;$$

or replacing Ω, Ψ by their values,

$$\Lambda=\frac{1}{9H^3}\left\{\begin{array}{l}-3(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\partial_x, \partial_y, \partial_z)^2H.H \\ +4(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\partial_x H, \partial_y H, \partial_z H)^2\end{array}\right\},$$

and Λ having this value, the equation of the five-pointic conic is

$$D^2U-\left(\tfrac{2}{3}\frac{1}{H}DH+\Lambda DU\right)DU=0,$$

where it will be recollected that the current coordinates are (X, Y, Z), and that D denotes $X\partial_x+Y\partial_y+Z\partial_z$.

16. I remark, in passing, that the problem of finding the circle of curvature at a given point of a plane curve, is in fact that of determining the conic having with the curve at the given point a three-pointic contact, and besides passing through two given points. The equation of a conic having an ordinary contact, is

$$D^2U-\Pi DU=0,$$

where

$$\Pi=aX+bY+cZ,$$

and the condition of a three-pointic contact is

$$ax+by+cz=2(m-2).$$

Let the coordinates of the two given points be

$$(x_1, y_1, z_1),\ (x_2, y_2, z_2),$$

and let $(D^2U)_1$ &c. be the corresponding values of D^2U &c., then we have

$$aX+bY+cZ=\frac{D^2U}{DU},$$

$$ax_1+by_1+cz_1=\left(\frac{D^2U}{DU}\right)_1,$$

$$ax_2+by_2+cz_2=\left(\frac{D^2U}{DU}\right)_2;$$

and if from the four equations we eliminate a, b, c, we find

$$\begin{vmatrix} XDU, & YDU, & ZDU, & D^2U \\ x\ , & y\ , & z\ , & 2(m-2) \\ x_1\ , & y_1\ , & z_1\ , & \left(\frac{D^2U}{DU}\right)_1 \\ x_2\ , & y_2\ , & z_2\ , & \left(\frac{D^2U}{DU}\right)_2 \end{vmatrix}=0$$

for the equation of the conic in question; x, y, z being the coordinates of the point of contact, and X, Y, Z current coordinates.

II. *Demonstration of Identities assumed in the preceding section.*

Proof of the expressions for $(\partial_1^3+3\partial_1\partial_2)U$ and $(\partial_1^4+6\partial_1^2\partial_2)U$:—

17. It will be remembered that ∂_1, ∂_2 stand originally for

$$dx\,\partial_x+dy\,\partial_y+dz\,\partial_z,$$

$$d^2x\partial_x+d^2y\partial_y+d^2z\partial_z,$$

and that A, B, C being the first derived functions of U, dx, dy, dz are changed into

$$B\nu-C\mu,\quad C\lambda-A\nu,\quad A\mu-B\lambda,$$

and that the resulting value of ∂_1, viz.

$$(B\nu-C\mu)\partial_x+(C\lambda-A\nu)\partial_y+(A\mu-B\lambda)\partial_z,$$

is also represented by $\mathbf{D}$, so that $\partial_1=\mathbf{D}$. The corresponding values of d^2x, d^2y, d^2z are

$$d^2x=\nu\, dB-\mu dC,$$

$$d^2y=\lambda dC-\nu\, dA,$$

$$d^2z=\mu dA-\lambda\, dB,$$

where we have

$$dA=adx+hdy+gdz,\ \&c.,$$

in which dx, dy, dz are to be replaced by the values

$$B\nu-C\mu,\quad C\lambda-A\nu,\quad A\mu-B\lambda;$$

and ∂_2 really denotes what

$$d^2x\partial_x+d^2y\partial_y+d^2z\partial_z$$

becomes when the above values are substituted for d^2x, d^2y, d^2z. But in the expressions

∂_1^2U, $\partial_1\partial_2U$ &c., the symbols ∂_x, ∂_y, ∂_z contained in ∂_1 and ∂_2 operate only on U, and not on the variable quantities A, B, C, &c. contained in ∂_1 and ∂_2.

18. If now we treat ∂_1 as an operand, that is, perform the differentiations on the variable quantities A, B, C which enter into ∂_1, we obtain

$$\partial_1.\partial_1=\partial_2,$$

or, what is the same thing, operating on ∂_1U with ∂_1, the result is

$$\partial_1.\partial_1U=\partial_1^2U+\partial_2U=(\partial_1^2+\partial_2)U,$$

and in like manner

$$\partial_1.\partial_1^2U=(\partial_1^3+2\partial_1\partial_2)U,$$
$$\partial_1.\partial_1^3U=(\partial_1^4+3\partial_1^2\partial_2)U.$$

It is, in fact, upon these principles that HESSE's values of ∂_1^2U, ∂_1^3U &c. were obtained, and we may by means of them obtain the other expressions assumed in the preceding section.

19. In fact, starting from HESSE's equation,

$$\partial_1^2U=\frac{m}{m-1}\Phi U-\frac{1}{(m-1)^2}\mathrm{H}\vartheta^2,$$

we have

$$(\partial_1^3+2\partial_1\partial_2)U=\frac{m}{m-1}(\mathrm{U}\mathbf{D}\Phi+\Phi\mathbf{D}\mathrm{U})-\frac{1}{(m-1)^2}(\mathbf{D}\mathrm{H}.\vartheta^2+\mathrm{H}.2\vartheta\mathbf{D}\vartheta).$$

But we have identically $\mathbf{D}\mathrm{U}=0$, $\mathbf{D}\vartheta=0$, and this equation becomes therefore

$$(\partial_1^3+2\partial_1\partial_2)U=\frac{m}{m-1}\mathrm{U}\mathbf{D}\Phi-\frac{1}{(m-1)^2}\mathbf{D}\mathrm{H}.\vartheta^2.$$

But this is precisely HESSE's value of ∂_1^3U, or we have $\partial_1\partial_2U=0$, and therefore

$$(\partial_1^3+3\partial_1\partial_2)U=\partial_1^3U=\frac{m}{m-1}\mathrm{U}\mathbf{D}\Phi-\frac{1}{(m-1)^2}\mathbf{D}\mathrm{H}.\vartheta^2.$$

20. In like manner, starting from the expression of ∂_1^3U, we have

$$(\partial_1^4+3\partial_1^2\partial_2)U=\frac{m}{m-1}(\mathbf{D}\mathrm{U}.\mathbf{D}\Phi+\mathrm{U}\mathbf{D}.\mathbf{D}\Phi)-\frac{1}{(m-1)^2}(\mathbf{D}\mathrm{H}.2\vartheta\mathbf{D}\vartheta+\vartheta^2\mathbf{D}.\mathbf{D}\mathrm{H});$$

or since $\mathbf{D}\mathrm{U}$ and $\mathbf{D}\vartheta$ vanish identically, and the values of $\mathbf{D}.\mathbf{D}\Phi$, $\mathbf{D}.\mathbf{D}\mathrm{H}$ are $\partial_2\Phi+\mathbf{D}^2\Phi$, $\partial_2\mathrm{H}+\mathbf{D}^2\mathrm{H}$, we have

$$(\partial_1^4+3\partial_1^2\partial_2)U=\frac{m}{m-1}\mathrm{U}(\partial_2\Phi+\mathbf{D}^2\Phi)-\frac{1}{(m-1)^2}\vartheta^2(\partial_2\mathrm{H}+\mathbf{D}^2\mathrm{H});$$

and if from the double of this equation we subtract HESSE's equation,

$$\partial_1^4U=\frac{m}{m-1}\mathrm{U}\left(\mathbf{D}^2\Phi-\frac{\vartheta}{m-1}\Pi\right)-\frac{1}{(m-1)^2}\vartheta^2\left(\mathbf{D}^2\mathrm{H}-\frac{\vartheta\Box}{m-1}+\frac{3(m-2)}{m-1}\mathrm{H}\Phi\right),$$

we find the required relation,

$$(\partial_1^4+6\partial_1^2\partial_2)U=\frac{m}{m-1}\left(2\partial_2\Phi+\mathbf{D}^2\Phi+\frac{\vartheta}{m-1}\Pi\right)\mathrm{U}$$
$$-\frac{1}{(m-1)^2}\left(2\partial_2\mathrm{H}+\mathbf{D}^2\mathrm{H}+\frac{\vartheta\Box}{m-1}-\frac{3(m-2)}{m-1}\mathrm{H}\Phi\right)\vartheta^2.$$

Proof of the expression for $\partial_2\Pi$:—

21. We have

$$\partial_2\Pi=\partial_a\Pi(\nu d\mathrm{B}-\mu d\mathrm{C})+\partial_b\Pi(\lambda d\mathrm{C}-\nu d\mathrm{B})+\partial_c\Pi(\mu d\mathrm{A}-\lambda d\mathrm{B}),$$

where

$$\begin{aligned} d\mathrm{A}&=a\,dx+h\,dy+g\,dz,\\ d\mathrm{B}&=h\,dx+b\,dy+f\,dz,\\ d\mathrm{C}&=g\,dx+f\,dy+c\,dz,\end{aligned}$$

in which dx, dy, dz are to be replaced by their values; we have therefore

$$\begin{aligned}\partial_2\Pi=\partial_a\Pi\Big\{\quad&\nu[(b\mathrm{C}-f\mathrm{B})\lambda+(f\mathrm{A}-h\mathrm{C})\mu+(h\mathrm{B}-b\mathrm{A})\nu]\\ &-\mu[(f\mathrm{C}-c\mathrm{B})\lambda+(c\mathrm{A}-g\mathrm{C})\mu+(g\mathrm{B}-f\mathrm{A})\nu]\Big\}\\ +\&\text{c.},\end{aligned}$$

where the coefficient of $\partial_a\Pi$ is

$$\begin{aligned}&0\lambda^2+(g\mathrm{C}-c\mathrm{A})\mu^2+(h\mathrm{B}-b\mathrm{A})\nu^2\\ &+(2f\mathrm{A}-g\mathrm{B}-h\mathrm{C})\mu\nu+(b\mathrm{C}-f\mathrm{B})\nu\lambda+(c\mathrm{B}-f\mathrm{C})\lambda\mu;\end{aligned}$$

or, since we have

$$\begin{aligned}(m-1)\mathrm{A}&=ax+hy+gz,\\ (m-1)\mathrm{B}&=hx+by+fz,\\ (m-1)\mathrm{C}&=gx+fy+cz,\end{aligned}$$

the coefficient, omitting the factor $\frac{1}{m-1}$, which will be afterwards restored, is

$$\begin{aligned}&0\lambda^2\\ &+[\,g(gx+fy+cz)-c(ax+hy+gz)]\mu^2\\ &+[\,h(hx+by+fz)-b(ax+hy+gz)]\nu^2\\ &+[2f(ax+hy+gz)-g(hx+by+fz)-h(gx+fy+cz)]\mu\nu\\ &+[\,b(gx+fy+cz)-f(hx+by+fz)]\nu\lambda\\ &+[\,c(hx+fy+cz)-f(gx+fy+cz)]\lambda\mu\\ =\;&0\lambda^2\\ &+(-\mathfrak{B}x+\mathfrak{H}y)\mu^2\\ &+(-\mathfrak{C}x+\mathfrak{G}z)\nu^2\\ &+(-2\mathfrak{F}x+\mathfrak{G}y+\mathfrak{H}z)\mu\nu\\ &+(-\;\mathfrak{G}x+\qquad\mathfrak{A}z)\nu\lambda\\ &+(-\;\mathfrak{H}x+\mathfrak{A}y\qquad)\lambda\mu,\end{aligned}$$

which is equal to

$$\begin{aligned}&-(\mathfrak{A}\lambda^2+\mathfrak{B}\mu^2+\mathfrak{C}\nu^2+2\mathfrak{F}\mu\nu+2\mathfrak{G}\nu\lambda+2\mathfrak{H}\lambda\mu)x\\ &+(\mathfrak{A}\lambda+\mathrm{H}\mu+\mathfrak{G}\nu)(\lambda x+\mu y+\nu z).\end{aligned}$$

The coefficients of $\partial_y H$, ∂H_z have a similar form; and uniting the three terms, observing that $x\partial_x H + y\partial_y H + z\partial_z H$ is equal to $3(m-2)H$, and attending to the values of Φ, $\square$, ϑ, we have, restoring the omitted factor $\frac{1}{m-1}$,

$$\partial_z H = \frac{-3(m-2)}{m-1} H\Phi + \frac{\frac{1}{2}\square}{m-1}\vartheta.$$

Proof of the expressions for $(\mathbf{D}H)^2$ and $\mathbf{D}^2 H$:—

22. These are obtained (for the particular case $m=4$, which makes but little difference) in Mr. SALMON's 'Higher Plane Curves,' pp. 88 and 89, and I merely reproduce his investigation; we have

$$(\mathbf{D}H)^2 = \{(B\nu - C\mu)\partial_x H + (C\lambda - A\nu)\partial_y H + (A\mu - B\lambda)\partial_z H\}^2,$$

or, what is the same thing,

$$(\mathbf{D}H)^2 = \{\lambda(C\partial_y - B\partial_z)H + \mu(A\partial_z - C\partial_x)H + \nu(B\partial_x - A\partial_y)H\}^2;$$

and if we consider first the term which contains λ^2, the coefficient is

$$\{(C\partial_y - B\partial_z)H\}^2.$$

Now making use of the equations

$$(m-1)A = ax + hy + gz,$$
$$(m-1)B = hx + by + fz,$$
$$(m-1)C = gx + fy + cz,$$

and

$$m(m-1)U = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0,$$

we have

$$\begin{aligned}
(m-1)^2C^2 &= (gx+fy+cz)^2 - c(ax^2+by^2+cz^2+2fyz+2gzx+2hxy)\\
&= -\mathfrak{B}x^2 + 2\mathfrak{H}xy - \mathfrak{A}y^2,\\
(m-1)^2BC &= (hx+by+fz)(gx+fy+cz) - f(ax^2+by^2+cz^2+2fyz+2gzx+2hxy)\\
&= \mathfrak{F}x^2 - \mathfrak{G}xy - \mathfrak{H}xz + \mathfrak{A}yz,\\
(m-1)^2B^2 &= (hx+by+fz)^2 - b(ax^2+by^2+cz^2+2fyz+2gzx+2hxy)\\
&= -\mathfrak{C}x^2 + 2\mathfrak{G}xz - \mathfrak{A}z^2;
\end{aligned}$$

and hence

$$\begin{aligned}
(m-1)^2\big((C\partial_y - B\partial_z)H\big)^2 = &\quad (-\mathfrak{B}x^2 + \quad 2\mathfrak{H}xy \quad - \mathfrak{A}y^2)(\partial_y H)^2\\
&+ 2(-\mathfrak{F}x^2 + \mathfrak{G}xy + \mathfrak{H}xz - \mathfrak{A}yz)\partial_y H . \partial_z H\\
&+ \ (-\mathfrak{C}x^2 + \quad 2\mathfrak{G}xz \quad - \mathfrak{A}z^2)(\partial_z H)^2\\
= &- x^2(\mathfrak{B}, \mathfrak{F}, \mathfrak{C})(\partial_y H, \partial_z H)^2\\
&+ 2x(y\partial_y H + z\partial_z H)(\mathfrak{H}\partial_y H + \mathfrak{G}\partial_z H)\\
&- \ \mathfrak{A}(y\partial_y H + z\partial_z H)^2,
\end{aligned}$$

and

$$y\partial_y H + z\partial_z H = 3(m-2)H - x\partial_x H,$$

so that the term is

$$
\begin{aligned}
&-x^2(\mathfrak{B},\mathfrak{F},\mathfrak{C})(\partial_y H,\partial_z H)^2\\
&+2x\big(3(m-2)H-x\partial_x H\big)(\mathfrak{H}\partial_y H+\mathfrak{G}\partial_z H)\\
&-\mathfrak{A}\big(3(m-2)H-x\partial_x H\big)^2;
\end{aligned}
$$

or reducing

$$
\begin{aligned}
(m-1)^2\{(C\partial_y-B\partial_z)H\}^2&\\
=-x^2(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H})&(\partial_x H,\partial_y H,\partial_z H)^2\\
+6(m-2)Hx(\mathfrak{A}\partial_x H&+\mathfrak{H}\partial_y H+\mathfrak{G}\partial_z H)\\
-9(m-2)^2\mathfrak{A}H^2&.
\end{aligned}
$$

23. The other terms may be obtained in a similar manner; and it is easy to see that, collecting all the terms, the sum will be

$$
\begin{aligned}
&-(\lambda x+\mu y+\nu z)^2(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H})(\partial_x H,\partial_y H,\partial_z H)^2\\
&+6(m-2)H(\lambda x+\mu y+\nu z)\{(\mathfrak{A}\lambda+\mathfrak{H}\mu+\mathfrak{G}\nu)\partial_x H+(\mathfrak{H}\lambda+\mathfrak{B}\mu+\mathfrak{F}\nu)\partial_y H+(\mathfrak{G}\lambda+\mathfrak{F}\mu+\mathfrak{C}\nu)\partial_z H\\
&-9(m-2)^2H^2(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H})(\lambda,\mu,\nu)^2;
\end{aligned}
$$

or, attending to the signification of the symbols $\mathfrak{Z}$, Φ, $\square$, Ψ, we have

$$
\begin{aligned}
(m-1)^2(\mathbf{D}H)^2=&-9(m-2)^2H^2\Phi\\
&+3(m-2)H\square\mathfrak{Z}\\
&-\Psi\mathfrak{Z}^2.
\end{aligned}
$$

24. Next,

$$\mathbf{D}^2H=\big((B\nu-C\mu)\partial_x+(C\lambda-A\nu)\partial_y+(A\mu-B\lambda)\partial_z\big)^2H,$$

or, what is the same thing,

$$\mathbf{D}^2H=\{\lambda(C\partial_y-B\partial_z)+\mu(A\partial_z-C\partial_x)+\nu(B\partial_x-A\partial_y)\}^2H;$$

and if we attend first to the term which contains λ^2, the coefficient is

$$((C\partial_y-B\partial_z)^2H.$$

Now substituting for C^2, CB, B^2 as before, we have

$$
\begin{aligned}
(m-1)^2(C\partial_y-B\partial_z)^2H=&\quad(-\mathfrak{B}x^2+2\mathfrak{H}xy\quad-\mathfrak{A}y^2)\partial_y^2H\\
&+2(-\mathfrak{F}x^2+\mathfrak{G}xy+\mathfrak{H}xz-\mathfrak{A}yz)\partial_y\partial_z H\\
&+\ (-\mathfrak{C}x^2+2\mathfrak{G}xz\quad-\mathfrak{A}z^2)\partial_z^2H\\
=&-x^2(\mathfrak{B},\mathfrak{F},\mathfrak{C})(\partial_y,\partial_z)^2H\\
&+2x(y\partial_y+z\partial_z)(\mathfrak{H}\partial_y+\mathfrak{G}\partial_z)H\\
&-\mathfrak{A}(y\partial_y+z\partial_z)^2H,
\end{aligned}
$$

where it is to be observed that the symbols of differentiation affect H only. We have

$$
\begin{aligned}
(y\partial_y+z\partial_z)^2H=&(x\partial_x+y\partial_y+z\partial_z)^2H\\
&-2(x\partial_x+y\partial_y+z\partial_z)x\partial_x H\\
&+\ (x\partial_x)^2H,
\end{aligned}
$$

where H is a *homogeneous function of the degree* $3m-6$; $x\partial_x H$, *since the* x *is not* affected by the differentiation, must be treated as of the degree $3m-7$, and $(x\partial_x)^2 H$, for the like reason, stands for $x^2\partial_x^2 H$; we have

$$\begin{aligned}(y\partial_y+z\partial_z)^2 H=&(3m-6)(3m-7)H\\ &-2(3m-7)x\partial_x H\\ &+x^2\partial_x^2 H.\end{aligned}$$

In like manner,

$$\begin{aligned}&(y\partial_y+z\partial_z)(\mathfrak{H}\partial_y+\mathfrak{G}\partial_z)H\\ &\quad=(x\partial_x+y\partial_y+z\partial_z)(\mathfrak{H}\partial_y+\mathfrak{G}\partial_z)H-x\partial_x(\mathfrak{H}\partial_y+\mathfrak{G}\partial_z)H\\ &\quad=(3m-7)(\mathfrak{H}\partial_y+\mathfrak{G}\partial_z)H-x\partial_x(\mathfrak{H}\partial_y+\mathfrak{G}\partial_z)H;\end{aligned}$$

and hence

$$\begin{aligned}&(m-1)^2(C\partial_y-B\partial_z)^2 H\\ &\quad=-x^2(\mathfrak{B},\ \mathfrak{F},\ \mathfrak{C}\between\partial_y,\ \partial_z)^2 H\\ &\qquad+2x[(3m-7)(\mathfrak{H}\partial_y+\mathfrak{G}\partial_z)H-x\partial_x(\mathfrak{H}\partial_y+\mathfrak{G}\partial_z)H]\\ &\qquad-\mathfrak{A}[(3m-6)(3m-7)H-2(3m-7)x\partial_x H+x^2\partial_x^2 H]\\ &\quad=-x^2(\mathfrak{A},\ \mathfrak{B},\ \mathfrak{C},\ \mathfrak{F},\ \mathfrak{G},\ \mathfrak{H}\between\partial_x,\ \partial_y,\ \partial_z)^2 H\\ &\qquad+2(3m-7)x(\mathfrak{A}\partial_x+\mathfrak{H}\partial_y+\mathfrak{G}\partial_z)H\\ &\qquad-(3m-6)(3m-7)\mathfrak{A}H.\end{aligned}$$

25. The other terms are formed in a similar manner; and collecting all the terms, we have

$$\begin{aligned}(m-1)^2\mathfrak{D}^2 H=&-(3m-6)(3m-7)(\mathfrak{A},\ \mathfrak{B},\ \mathfrak{C},\ \mathfrak{F},\ \mathfrak{G},\ \mathfrak{H}\between\lambda,\ \mu,\ \nu)^2 H\\ &+2(3m-7)(\lambda x+\mu y+\nu z)((\mathfrak{A}\lambda+\mathfrak{H}\mu+\mathfrak{G}\nu)\partial_x H+(\mathfrak{H}\lambda+\mathfrak{B}\mu+\mathfrak{F}\nu)\partial_y H+(\mathfrak{G}\lambda+\mathfrak{F}\mu+\mathfrak{C}\nu)\partial_z H)\\ &-(\lambda x+\mu y+\nu z)^2(\mathfrak{A},\ \mathfrak{B},\ \mathfrak{C},\ \mathfrak{F},\ \mathfrak{G},\ \mathfrak{H}\between\partial_x,\ \partial_y,\ \partial_z)^2 H,\end{aligned}$$

or, attending to the signification of the symbols ϑ, Φ, $\Box$, Ω, this is

$$\begin{aligned}(m-1)^2\mathfrak{D}^2 H=&-(3m-6)(3m-7)H\Phi\\ &+(3m-7)\vartheta\Box\\ &-\Omega\vartheta^2.\end{aligned}$$

Proof of the expressions for $(m-1)^2 R_3^2$, $(m-1)^2 R_4$:—

26. We have

$$\begin{aligned}(m-1)^2 R_3^2=&(m-1)^2(\mathfrak{D}H)^2\\ =&-9(m-2)^2H^2\Phi\\ &+3(m-2)H\Box\vartheta\\ &-\Psi\vartheta^2,\end{aligned}$$

$$\begin{aligned}(m-1)^2 R_4=&(m-1)^2\mathfrak{D}^2 H+(m-1)\vartheta\Box-(3m-6)(m-1)\Phi H\\ =&-12(m-2)^2 H\Phi\\ &+4(m-2)\Box\vartheta\\ &-\Omega\vartheta^2,\end{aligned}$$

which are the expressions required.

Proof of corresponding expressions for $(m-1)^6Q_3^2$, $(m-1)^4Q_4$:—

27. We have

$$\begin{aligned}(m-1)^6Q_3^2&=(m-1)^2(\mathbf{D}\mathrm{H})^2\\&=-9(m-2)^2\mathrm{H}^2\Phi\\&\quad+3(m-2)\mathrm{H}\square\vartheta\\&\quad-\Psi\vartheta\end{aligned}$$

$$\begin{aligned}(m-1)^4Q_4&=(m-1)^2\mathbf{D}^2\mathrm{H}-(m-1)\vartheta\square+(3m-6)(m-1)\mathrm{H}\Phi\\&=-6(m-2)(m-3)\mathrm{H}\Phi\\&\quad+2(m-3)\square\vartheta\\&\quad-\Omega\vartheta^2.\end{aligned}$$

To which I join

$$(m-1)^2Q_2=\mathrm{H}.$$

28. We have consequently

$$\begin{aligned}(m-1)^6(3Q_2Q_4-Q_3^2)=&-9(m-2)(m-4)\mathrm{H}^2\Phi\\&+3(m-4)\mathrm{H}\square\vartheta\\&+\ (-3\Omega\mathrm{H}+\Psi)\vartheta^2,\end{aligned}$$

and

$$(m-1)^6\big(3(m-2)Q_2Q_4-2(m-3)Q_3^2\big)=\big(-3(m-2)\Omega\mathrm{H}+2(m-3)\Psi\big)\vartheta^2,$$

which for $m=4$ become

$$729(3Q_2Q_4-Q_3^2)=(-3\Omega\mathrm{H}+\Psi)\vartheta^2.$$

29. In the case $m=4$, we have HESSE's theorem, that the equation $3Q_2Q_4-Q_3=0$ gives a curve of the 14th order, which passes through the points of contact of the double tangents, viz. substituting for Ω, Ψ their values, the equation of this curve is

$$\begin{aligned}&-3\mathrm{H}(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H})(\partial_x\ ,\partial_y\ ,\partial_z)^2\mathrm{H}\\&+\quad(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H})(\partial_x\mathrm{H},\partial_y\mathrm{H},\partial_z\mathrm{H})^2=0.\end{aligned}$$

I have added these remarks for the sake of pointing out the striking resemblance of the expressions which occur in the double-tangent problem for $m=4$, and in the present theory of the five-pointic conic for any value whatever of m. It has not hitherto been shown what the expressions $3(m-2)Q_2Q_4-2(m-3)Q_3^2$ and $-3(m-2)\Omega\mathrm{H}+2(m-3)\Psi$ respectively denote, except in the particular case $m=4$.

III. *Application of the Formulæ to the Cubic.*

30. I shall apply the formula for the five-pointic conic to a cubic; to avoid confusion to a numerical factor, I write U′, H′ in the place of U, H, so that we have

$$\Delta=\frac{1}{9\mathrm{H}'^3}\left\{\begin{aligned}&-3(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H})(\partial_x\ ,\partial_y\ ,\partial_z)^2\mathrm{H}'.\mathrm{H}'\\&+4(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H})(\partial_x\mathrm{H}',\partial_y\mathrm{H}',\partial_z\mathrm{H}')^2\end{aligned}\right\},$$

and then the equation of the five-pointic conic is

$$D^2U'-\tfrac{2}{3}\left(\frac{1}{H'}DH'+\Lambda DU'\right)DU'=0.$$

I take as the equation of the cubic,

$$U=x^3+y^3+z^3+6lxyz=0\,;$$

the formulæ Table No. 70 of my Third Memoir on Quantics*, putting H for HU, give

$$H=l^2(x^3+y^3+z^3)-(1+2l^3)xyz.$$

Hence writing

$$U'=\tfrac{1}{6}(x^3+y^3+z^3+6lxyz),$$

the first derived functions are

$$\tfrac{1}{2}(x^2+2lyz),\ \tfrac{1}{2}(y^2+2lzx),\ \tfrac{1}{2}(z^2+2lxy)\,;$$

the second derived functions, or (a, b, c, f, g, h), are (x, y, z, lx, ly, lz), whence $H'=-H$; the inverse coefficients $(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})$ are

$$(yz-l^2x^2,\ zx-l^2y^2,\ xy-l^2z^2,\ l^2yz-lx^2,\ l^2zx-ly^2,\ l^2xy-lz^2)\,;$$

and putting $U'=\tfrac{1}{6}U$ and $H'=-H$, we have

$$\Lambda=-\frac{1}{9H^3}\left\{\begin{array}{l}-3(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\partial_x\ \ , \partial_y\ \ , \partial_z)^2H\,.\,H\\ +4(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\partial_xH, \partial_yH, \partial_zH)^2\end{array}\right\}\,;$$

and the equation of the five-pointic conic is

$$D^2U-\left(\tfrac{2}{3}\frac{1}{H}DH+\tfrac{1}{6}\Lambda DU\right)DU=0.$$

31. We have

$$\begin{aligned}&(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\partial_x, \partial_y, \partial_z)^2H\\ &=\quad (yz-l^2x^2\ ).6l^2x\\ &\quad+\ (zx-l^2y^2\ ).6l^2y\\ &\quad+\ (xy-l^2z^2\ ).6l^2z\\ &\quad+2(l^2yz-lx^2).-(1+2l^3)x\\ &\quad+2(l^2zx-ly^2).-(1+2l^3)y\\ &\quad+2(l^2xy-lz^2).-(1+2l^3)z,\end{aligned}$$

which is

$$\begin{aligned}&=\ 18l^2xyz-6l^4(x^3+y^3+z^3)\\ &\quad-6l^2(1+2l^3)xyz+2l(1+2l^3)(x^3+y^3+z^3)\\ &=\ (12l^2-12l^5)xyz+(2l-2l^4)(x^3+y^3+z^3)\\ &=\ 2(l-l^4)(x^3+y^3+z^3+6lxyz),\end{aligned}$$

or we have

$$(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\partial_x, \partial_y, \partial_z)^2H=-2S.U,$$

where S is the quartinvariant (see the Table No. 70). For the present purpose $U=0$, and consequently

$$(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H})(\partial_x, \partial_y, \partial_z)^2H=0.$$

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* Philosophical Transactions, vol. cxlvi. (1856), pp. 627–647.

32. Next,

$$\begin{aligned}
&(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H} \between \partial_x H, \partial_y H, \partial_z H)\\
&= \quad (yz-l^2x^2)\,[3l^2x^2-(1+2l^3)yz]^2\\
&\quad + (zx-l^2y^2)\,[3l^2y^2-(1+2l^3)zx]^2\\
&\quad + (xy-l^2z^2)\,[3l^2z^2-(1+2l^3)xy]^2\\
&\quad +2l^2yz(-lx^2)[3l^2y^2-(1+2l^3)zx][3l^2z^2-(1+2l^3)xy]\\
&\quad +2(l^2zx-ly^2)[3l^2z^2-(1+2l^3)xy][3l^2x^2-(1+2l^3)yz]\\
&\quad +2(l^2xy-lz^2)[3l^2x^2-(1+2l^3)yz][3l^2y^2-(1+2l^3)zx],
\end{aligned}$$

the first three lines of which are

$$\begin{aligned}
&9l^4xyz(x^3+y^3+z^3)-18l^2(1+2l^3)x^2y^2z^2+(1+2l^3)^2(y^3z^3+z^3x^3+x^3y^3)\\
&-9l^6(x^6+y^6+z^6)+6l^4(1+2l^3)xyz(x^3+y^3+z^3)-3l^2(1+2l^3)^2x^2y^2z^2;
\end{aligned}$$

or collecting and reducing,

$$\begin{aligned}
&(-9l^6 &&)(x^6+y^6+z^6)\\
&+(1+4l^3+4l^6 &&)(y^3z^3+z^3x^3+x^3y^3)\\
&+(15l^4+12l^7 &&)(x^3+y^3+z^3)xyz\\
&+(-21l^2-48l^5-12l^8) && x^2y^2z^2;
\end{aligned}$$

the second three lines are

$$\begin{aligned}
&18l^6(y^3z^3+z^3x^3+x^3y^3)-12l^4(1+2l^3)xyz(x^3+y^3+z^3)+6l^2(1+2l^3)^2x^2y^2z^2\\
&-54l^5x^2y^2z^2+12l^3(1+2l^3)(y^3z^3+z^3x^3+x^3y^3)-2l(1+2l^3)^2xyz(x^3+y^3+z^3);
\end{aligned}$$

or collecting and reducing,

$$\begin{aligned}
&(12l^3+42l^6 &&)(y^3z^3+z^3x^3+x^3y^3)\\
&+(-2l-20l^4-32l^7 &&)(x^3+y^3+z^3)xyz\\
&+(6l^2-30l^5+24l^8) && x^2y^2z^2.
\end{aligned}$$

Hence in the first part replacing the top line by

$$-9l^6(x^3+y^3+z^3)^2+18l^6(y^3z^3+z^3x^3+x^3y^3),$$

and uniting the two parts, we find

$$\begin{aligned}
&(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H} \between \partial_x H, \partial_y H, \partial_z H)^2\\
&= \quad (1+8l^3)^2(y^3z^3+z^3x^3+x^3y^3)\\
&\quad +(-9l^6 \quad)(x^3+y^3+z^3)^2\\
&\quad +(-2l-5l^4-20l^7 \quad)(x^3+y^3+z^3)xyz\\
&\quad +(-15l^2-78l^5+12l^8)\,x^2y^2z^2;
\end{aligned}$$

and referring to the Table No. 70, and writing Θ for ΘU, we have

$$(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H} \between \partial_x H, \partial_y H, \partial_z H)^2=\Theta,$$

where Θ is the first of the three functions which may be chosen to represent the octicovariant of the cubic.

33. We have thus

$$\Lambda = -\tfrac{4}{9}\frac{\Theta}{H^3},$$

and thence

$$D^2U - \left(\tfrac{2}{3}\frac{1}{H}DH - \tfrac{2}{27}\frac{\Theta}{H^3}DU\right)DU = 0$$

as the equation of the five-pointic conic: the investigation has been conducted by means of the canonical form of the equation of the cubic, but the form of the result shows that it applies to the equation of the cubic in any form whatever.

34. If, however, we continue to represent the cubic by the canonical equation

$$x^3 + y^3 + z^3 + 6lxyz = 0,$$

the result may be further reduced. We have, putting $U=0$, or writing $x^3+y^3+z^3=-6lxyz$.

$$H = -(1+8l^3)xyz;$$

moreover, putting $U=0$, the Table No. 70 gives

$$\Theta = (1+8l^3)^2(y^3z^3+z^3x^3+x^3y^3) - 3l^2H^2;$$

or substituting for H the last-mentioned value, and putting for shortness

$$Q = y^3z^3 + z^3x^3 + x^3y^3 - 3l^2x^2y^2z^2,$$

we have

$$\Theta = (1+8l^3)^2Q;$$

and with these values of H and Q, the equation of the five-pointic conic is

$$D^2U + \left(\tfrac{2}{3}\frac{1}{(1+8l^3)xyz}DH - \tfrac{2}{27}\frac{Q}{(1+8l^3)x^3y^3z^3}DU\right)DU = 0,$$

where

$$\begin{aligned} DU &= 3\{(x^2+2lyz)X + (y^2+2lzx)Y + (z^2+2lxy)Z\} \\ D^2U &= 6\{(X^2+2lYZ)x + (Y^2+2lZX)y + (Z^2+2lXY)z\}, \end{aligned}$$

or, as it will be convenient to write it

$$= 6(x, y, z, lx, ly, lz \between X, Y, Z)^2,$$

$$DH = \big(3l^2x^2-(1+2l^3)yz\big)X + \big(3l^2y^2-(1+2l^3)zx\big)Y + \big(3l^2z^2-(1+2l^3)xy\big)Z;$$

whence, finally, the equation of the five-pointic conic of the cubic

$$X^3 + Y^3 + Z^3 + 6lXYZ = 0$$

at the point (x, y, z) is

$$\begin{aligned} &9(1+8l^3)x^3y^3z^3(x, y, z, lx, ly, lz \between X, Y, Z)^2 \\ &\quad + \{(x^2+2lyz)X + (y^2+2lzx)Y + (z^2+2lxy)Z\} \times \\ &\left\{\begin{array}{l} 3x^3y^3z^3\Big[\big(3l^2x^2-(1+2l^3)yz\big)X + \big(3l^2y^2-(1+2l^3)zx\big)Y + \big(3l^2z^2-(1+2l^3)xy\big)Z\Big] \\ \quad -Q\Big[(x^2+2lyz)X + (y^2+2lzx)Y + (z^2+2lxy)Z\Big] \end{array}\right\}, \end{aligned}$$

a result which I had previously obtained by a special method.

35. But the expression may be exhibited in a different form by a transformation

suggested by a geometrical theorem of Mr. SALMON'S. In fact the tangent at the point (x, y, z) meets the cubic in the tangential of this point, and the coordinates of the tangential are $x(y^3-z^3)+y(z^3-x^3)$, $z(x^3-y^3)$. Calling these ξ, η, ζ, the equation of the tangent to the cubic at the tangential is

$$(\xi^2+2l\eta\zeta)\mathrm{X}+(\eta^2+2l\zeta\xi)\mathrm{Y}+(\zeta^2+2l\xi\eta)\mathrm{Z}=0.$$

Now we have identically,

$$\begin{aligned}
&3x^2y^2z^2\Big\{\big(3l^2x^2-(1+2l^3)yz\big)\mathrm{X}+\big(3l^2y^2-(1+2l^3)yz\big)\mathrm{Y}+\big(3l^2z^2-(1+2l^3)zx\big)\mathrm{Z}\Big\}\\
&\quad -\mathrm{Q}\{(x^2+2lyz)\mathrm{X}+(y^2+2lzx)\mathrm{Y}+(z^2+2lxy)\mathrm{Z}\}\\
&= \quad \{(\xi^2+2l\eta\zeta)\mathrm{X}+(\eta^2+2l\zeta\xi)\mathrm{Y}+(\zeta^2+2l\xi\eta)\mathrm{Z}\}\\
&\quad +\mathrm{U}\{x^2(-y^3-z^3+2lyz)\mathrm{X}+y^2(-z^3-x^3+2lzx)\mathrm{Y}+z^2(-x^3-y^3+2lxy)\mathrm{Z}\}.
\end{aligned}$$

In fact this relation will be true if only

$$\begin{aligned}
&3x^2y^2z^2\big(3l^2x^2-(1+2l^3)yz\big)-\mathrm{Q}(x^2+2lyz)-(\xi^2+2l\eta\zeta)\\
&\quad =\mathrm{U}x^2(-y^3-z^3+2lyz).
\end{aligned}$$

And substituting for ξ, η, ζ and Q their values, the left-hand side is

$$\begin{aligned}
&3x^2y^2z^2\big(3l^2x^2-(1+2l^3)yz\big)\\
&-(x^2+2lyz)(y^3z^3+z^3x^3+x^3y^3-3l^2x^2y^2z^2)\\
&-x^2(y^3-z^3)^2\\
&-2lyz(z^3-x^3)(x^3-y^3)\,;
\end{aligned}$$

and expanding and reducing, the result is

$$x^2\big(-(y^3+z^3)^2-x^3(y^3+z^3)\big)+2lyz\big(-2x^3(y^3+z^3)+x^6\big)+12l^2x^4y^2z^2\,;$$

whence, dividing by x^2, the equation becomes

$$\begin{aligned}
&-(y^3+z^3)(x^3+y^3+z^3)+2lxyz\big(x^3-2(y^3+z^3)\big)+12l^2x^2y^2z^2\\
&\qquad =(-y^3-z^3+2lyz)(x^3+y^3+z^3+6lxyz),
\end{aligned}$$

which is identically true.

36. Hence in the identical equation putting $\mathrm{U}=0$, we see that the equation of the five-pointic conic may be written

$$\begin{aligned}
&9(1+8l^3)x^2y^2z^2(x, y, z, lx, ly, lz)(\mathrm{X}, \mathrm{Y}, \mathrm{Z})^2\\
&+\{(x^2+2lyz)\mathrm{X}+(y^2+2lzx)\mathrm{Y}+(z^2+2lxy)\mathrm{Z}\}\times\\
&\quad\{(\xi^2+2l\eta\zeta)\mathrm{X}+(\eta^2+2l\zeta\xi)\mathrm{Y}+(\zeta^2+2l\xi\eta)\mathrm{Z}\}\\
&=0,
\end{aligned}$$

where ξ, η, ζ stand for $x(y^3-z^3)$, $y(z^3-x^3)$, $z(x^3-y^3)$, the coordinates of the tangential of the given point, and which puts in evidence the geometrical theorem above referred to, viz.

Theorem.—The common chord of the five-pointic conic and the polar conic is the tangent to the cubic at the tangential of the given point.

37. The five-pointic conic meets the cubic in the point of contact, considered as five coincident points, and in a remaining sixth point or point of simple intersection. The process by which I originally obtained the equation of the five-pointic conic, led also to the equation of the line joining the point of contact with the point of simple intersection: the equation of this line is

$$9x^3y^3z^3\left\{\begin{array}{l} \quad Xx\big((1+8l^3)x^4+(4l+41l^4)x^2yz+(-2l^2+2l^5)y^2z^2\big) \\ +Yy\big((1+8l^3)y^4+(4l+41l^4)y^2zx+(-2l^2+2l^5)z^2x^2\big) \\ +Zz\,\big((1+8l^3)z^4+(4l+41l^4)z^2xy+(-2l^2+2l^5)x^2y^2\big) \end{array}\right\}$$

$$-6Qx^2y^2z^2\big\{X\big(3l^2x^2-(1+2l^3)yz\big)+Y\big(3l^2y^2-(1+2l^3)zx\big)+Z\big(3l^2z^2-(1+2l^3)xy\big)\big\}$$

$$+\ Q^2\quad \{X(x^2+2lyz)+Y(y^2+2lzx)+Z(z^2+2lxy)\}=0.$$

38. If the conic meet the cubic in six coincident points, that is, if the point of contact be a singular point of the kind already spoken of, or, as we may term it, a sextactic point, then the last-mentioned line must coincide with the tangent at the point. Represent for a moment the equation of the line by

$$AX+BY+CZ=0,$$

then this line is to coincide with the line

$$(x^2+2lyz)X+(y^2+2lzx)Y+(z^2+2lxy)Z=0,$$

or we must have

$$B(z^2+2lxy)-C(y^2+2lzx)=0,$$

$$C(x^2+2lyz)-A(z^2+2lxy)=0,$$

$$A(y^2+2lzx)-B(x^2+2lyz)=0,$$

which must be equivalent to a single condition. The terms of A, B, C, which contain

$$x^2+2lyz,\ y^2+2lzx,\ z^2+2lxy$$

respectively, may, it is clear, be omitted, and omitting also a factor $3x^2y^2z^2$, we may write

$$A=3x^2yz\big((1+8l^3)x^4+(4l+41l^4)x^2yz+(-2l^2+2l^5)y^2z^2\big)-2Q\big(3l^2x^2-(1+2l^3)yz\big),$$

$$B=3xy^2z\big((1+8l^3)y^4+(4l+41l^4)y^2zx+(-2l^2+2l^5)z^2x^2\big)-2Q\big(3l^2y^2-(1+2l^3)zx\big),$$

and the like value for C. The last of the three equations is

$$3xyz\left\{\begin{array}{l} \quad (y^2+2lzx)\big((1+8l^3)x^5+(4l+41l^4)x^3yz+(-2l^2+2l^5)xy^2z^2\big) \\ -(x^2+2lyz)\big((1+8l^3)y^5+(4l+41l^4)y^3zx+(-2l^2+2l^5)yz^2x^2\big) \end{array}\right\}$$

$$-2Q\big\{\ (y^2+2lzx)\big(3l^2x^2-(1+2l^3)yz\big)-(x^2+2lyz)\big(3l^2y^2-(1+2l^3)zx\big)\big\}$$

$$=0,$$

where the function on the left hand is

$$=3xyz\left\{\begin{array}{l} \quad (1+8l^3)\big(x^5y^2-x^2y^5+2l(x^6z-y^6z)\big) \\ +(4l+41l^4)2l(x^4yz^2\ -xy^4z^2) \\ +(-2l^2+2l^5)(xy^4z^2\ -x^4yz^2) \end{array}\right\}$$

$$-2Q\ \{-(1+2l^3)(y^3z-x^3z)\ +6l^3(x^3z-y^3z)\},$$

or, what is the same thing, throwing out the factor z, it is

$$=\{3(1+8l^3)x^3y^3+6l(1+8l^3)xyz(x^3+y^3)+30l^2(1+8l^3)x^2y^2z^2\}(x^3-y^3)$$
$$-2Q(1+8l^3)(x^3-y^3);$$

or throwing out the factor $(1+8l^3)$ and substituting for Q its value, it is

$$=\{3x^3y^3+6lxyz(x^3+y^3)+30l^2x^2y^2z^2-2(y^3z^3+z^3x^3+x^3y^3-3l^2x^2y^2z^2)\}(x^3-y^3).$$

The first factor, reducing by the equation $x^3+y^3+z^3+6lxyz$, is

$$=3x^3y^3-(x^3+y^3)(x^3+y^3+z^3)+(x^3+y^3+z^3)^2-2(y^3z^3+z^3x^3+x^3y^3)$$
$$=3x^3y^3+z^3(x^3+y^3+z^3)-2(y^3z^3+z^3x^3+x^3y^3)$$
$$=z^6-z^3(x^3+y^3)+x^3y^3$$
$$=(z^3-x^3)(z^3-y^3).$$

39. Hence putting for the moment

$$M=(y^3-z^3)(z^3-x^3)(x^3-y^3),$$

it appears that the last of the three equations is $Mz=0$; the first and second are of course $Mx=0$ and $My=0$, and the required condition is $M=0$, that is,

$$(y^3-z^3)(z^3-x^3)(x^3-y^3)=0,$$

the equation which, combined with the equation of the curve

$$x^3+y^3+z^3+6lxyz=0,$$

gives the sextactic points. There are consequently twenty-seven such points, and it is at once seen that these are the points of contact of the tangents to the cubic from the points of inflexion, or, what is the same thing, that the twenty-seven sextactic points form nine groups of three each, such that the three points of a group have for their common tangential one of the nine points of inflexion. In fact, let ω be a cube root (real or imaginary) of unity, the three sextactic points of one of the groups will be given by

$$\begin{cases} x-\omega y=0, \\ x^3+y^3+z^3+6lxyz=0. \end{cases}$$

Now consider the tangential of any one of those points, its coordinates are

$$x_1=x(y^3-z^3),$$
$$y_1=y(z^3-x^3),$$
$$z_1=z(x^3-y^3);$$

or, reducing by the equation $x-\omega y=0$, $x_1=\omega y(x^3-z^3)$, $y_1=-y(x^3-z^3)$, $z_1=0$, or, what is the same thing, $x_1+\omega y_1=0$, $z_1=0$; that is, the point (x_1, y_1, z_1) is one of the points of inflexion. This is the construction of the sextactic points obtained by Plücker and Steiner.

40. Reverting to the equation of the line joining the point of contact of the five-pointic conic with the point of simple intersection, this meets the cubic in a third point, and Mr. Salmon has shown that this third point is in fact the second tangential (tangential of the tangential) of the point of contact, or, what is the same thing—

Theorem.—The point of simple intersection of the cubic and the five-pointic conic is the third point of intersection with the cubic, of the line joining the point of contact with the second tangential of this point.

41. I have not sought to verify this theorem by my formulæ. I remark, that combining it with the before-mentioned theorem, the five-pointic conic is completely determined as follows; viz.—

Theorem.—The five-pointic conic touches the conic at the point of contact (two conditions); it passes through the two points in which the polar conic is intersected by the tangent to the cubic at the tangential of the point of contact (two conditions); and it passes through the point which is the third point of intersection with the cubic of the line joining the point of contact with its second tangential.

42. The construction for the point of simple intersection leads at once to that for the sextactic points; in fact, consider a point having for its tangential a point of inflexion: a point of inflexion is its own tangential, and the second tangential of the first-mentioned point is therefore the point of inflexion: the line joining the point with the second tangent is therefore the tangent at the point, and the point of simple intersection coincides with the point itself, that is, the point in question is a sextactic point.

43. I represent the equation of the five-pointic conic by

$$(a, b, c, f, g, h \between X, Y, Z)^2=0\,;$$

the value of a is

$$\begin{aligned}&=9(1+8l^3)x^4y^3z^3\\&\quad+3x^2y^2z^2\big(3l^2x^2-(1+2l^3)yz\big)(x^2+2lyz)\\&\quad-Q(x^2+2lyz)^2,\end{aligned}$$

in which equation

$$Q=y^3z^3+z^3x^3+x^3y^3-3l^2x^2y^2z^2\,;$$

or reducing by the equation $x^3+y^3+z^3+6lxyz=0$,

$$=y^3z^3+x^3(-x^3-6lxyz)-3l^2x^2y^2z^2,$$

that is,

$$-Q=x^6+6lx^4yz+3l^2x^2y^2z^2-y^3z^3,$$

and we have

$$\begin{aligned}a&=9(1+8l^3)x^4y^3z^3\\&\quad+3x^2y^2z^2\big(3l^2x^4+(-1+4l^3)x^2yz-2l(1+2l^3)y^2z^2\big)\\&\quad+(x^6+6lx^4yz+3l^2x^2y^2z^2-y^3z^3)(x^4+4lx^2yz+4l^2y^2z^2).\end{aligned}$$

We have in like manner

$$\begin{aligned}2f&=18(1+8l^3)lx^4y^3z^3\\&\quad+3x^2y^2z^2\Big\{\big(3l^2y^2-(1+2l^3)zx\big)(z^2+2lxy)+\big(3l^2z^2-(1+2l^3)xy\big)(y^2+2lzx)\Big\}\\&\quad-2Q(y^2+2lzx)(z^2+2lxy)\,;\end{aligned}$$

the coefficient of $3x^2y^2z^2$ in the second line is

$$=6l^2y^2z^2+(-1+4l^3)x(y^3+z^3)-4l(1+2l^3)x^2yz\,;$$

or reducing by the equation of the curve,

$$=(1-4l^3)x^4+(2l-32l^4)x^2yz+6l^2y^2z^2.$$

And the coefficient of $-2Q$ is

$$=y^2z^2+2lx(y^3+z^3)+4l^2x^2yz;$$

or reducing by the equation of the curve,

$$=-2lx^4-8l^2x^2yz+y^2z^2.$$

And we have

$$\begin{aligned}2f=&18l(1+8l^3)x^4y^3z^3\\&+3x^2y^2z^2\big((1-4l^3)x^4+(2l-32l^4)x^2yz+6l^2y^2z^2\big)\\&+2(x^6+6lx^4yz+3l^2x^2y^2z^2-y^3z^3)(-2lx^4-8l^2x^2yz+y^2z^2).\end{aligned}$$

Reducing the expressions of a and $2f$, we find for the coefficients (a, b, c, f, g, h),

$$\left\{\begin{aligned}a=&\quad x^{10}+10lx^7yz+40l^2x^6y^2z^2+(5+120l^3)x^4y^3z^3-10lx^2y^4z^4-4l^2y^5z^5,\\&\cdot\\2f=&-4lx^{10}-40l^2x^8yz+(5-120l^3)x^6y^2z^2+40lx^4y^3z^3+8l^2x^2y^4z^4-2y^5z^5,\\&\vdots\end{aligned}\right.$$

which gives the completely developed form of the equation of the five-pointic conic.

44. I investigate the coordinates of the point of simple intersection of the cubic and the five-pointic conic as follows: the equations of the two curves are

$$\begin{aligned}X^3+Y^3+Z^3+6lXYZ&=0,\\(a, b, c, f, g, h\between X, Y, Z)^2&=0;\end{aligned}$$

or if we write

$$\begin{aligned}\alpha&=1, & A&=c,\\\gamma&=6lXY, & B&=2(gX+fY),\\\delta&=X^3+Y^3, & C&=aX^2+2hXY+bY^2,\end{aligned}$$

then the two equations are

$$\begin{aligned}\alpha Z^3+\gamma Z+\delta&=0,\\AZ^2+BZ+C&=0,\end{aligned}$$

and the result of the elimination of Z will be

$$(\alpha Z_1^3+\gamma Z_1+\delta)(\alpha Z_2^3+\gamma Z_2+\delta)=0,$$

where Z_1, Z_2 are the roots of the equation $AZ^2+BZ+C=0$; that is, we have

$$\left.\begin{aligned}&\alpha^2\ C^3\\&+\alpha\gamma\ C(B^2-2AC)\\&+\alpha\delta(-B^3+2ABC)\\&+\gamma^2\ CA^2\\&-\gamma\delta\ BA^2\\&+\delta^2\ A^3\end{aligned}\right\}=0.$$

And substituting for A, B, C their values, but attending only to the terms which involve X^6 and Y^6, the result is

$$(a^3+c^3-8g^3+6acg)X^6+\cdots+(b^3+c^3-8f^3+6bcf)Y^6=0.$$

45. But the result of the elimination must obviously be

$$(Xy-Yx)^6(Xy_1-Yx_1)=0,$$

if (x_1, y_1, z_1) are the coordinates of the point of simple intersection. Comparing the two results, and forming the analogous third equation, we may write

$$\begin{aligned} x^5x_1&=b^3+c^3-8f^3+6bcf,\\ y^5y_1&=c^3+a^3-8g^3+6cag,\\ z^5z_1&=a^3+b^3-8h^3+6abh, \end{aligned}$$

where the value of x^5x_1 may also be written $(b+c-2f)(b+c\omega-2\omega^2f)(b+c\omega^2-2\omega f)$, ω being an imaginary cube root of unity, and so for the other two terms. The factors of x^5x_1 might be calculated from the identical equation

$$\begin{aligned} bY^2+2fYZ+cZ^2=9(1+8l^3)x^3y^3z^3(yY^2+zZ^2+2lxYZ)\\ +\{(y^2+2lzx)Y+(z^2+2lxy)Z\}\times\\ \left\{\begin{aligned} &3x^2y^2z^2\left[\left(3l^2y^2-(1+2l^3)zx\right)Y+\left(3l^2z^2-(1+2l^3)xy\right)Z\right]\\ &\quad -Q[(y^2+2lzx)Y+(z^2+2lxy)Z] \end{aligned}\right\}. \end{aligned}$$

I remark, that putting $x=0$, we have

$$bY^2+2fYZ+cZ^2=-y^3z^3(y^2Y+z^2Z)^2,$$

and hence writing 1 for Y, and -1, $-\omega$, $-\omega^2$ for Z, we have

$$b+c-2f=-y^3z^3(y^2-z^2)^2,\quad b+c\omega-2\omega^2f=-y^3z^3(y^2-\omega^4z^2)^2,\quad b+c\omega^2-2\omega f=-y^3z^3(y^2-\omega^2z^2)^2$$

and hence the product of the three factors is $-y^9z^9(y^2-z^2)^2(y^2-\omega^2z^2)^2(y^2-\omega^4z^2)^2$, which is equal to $-y^9z^9(y^3-z^3)^2(y^3+x^3)^2$, which vanishes in virtue of the assumed equation $x=0$. This shows that the function $b^3+c^3-8f^3+6bcf$ contains the factor x. I have not verified *à posteriori*, but I assume it to be true, that it contains in fact the factor x^5, and consequently that the expressions for x_1, y_1, z_1 are rational and integral functions of (x, y, z) of the degree 25, and containing respectively the factors x, y, z.

46. In the theory of the cubic, a point which depends linearly upon a given point may be termed a derivative of such point. According to a very beautiful theorem of Professor SYLVESTER'S, the coordinates of a derivative point are necessarily rational and integral functions *of a square degree* of the coordinates (x, y, z) of the given point; and moreover, there is but one derivative point having its coordinates of any given square degree m^2, or, as we may express it, only one derivative point of the degree m^2. The successive tangentials are derivative points of the degrees 4, 16, 64, &c.; the third point of intersection with the cubic, of the line joining two derivative points of the degrees m^2 and n^2 respectively, is a derivative point of the degree $(m\pm n)^2$. Thus the third point of intersection with the cubic, of the line joining the given point with its second tan-

gential, is a derivative point of the degree $(4\pm1)^2$, and it is easy to see that the degree is not 9; it is therefore 25. The point of simple intersection of the five-pointic conic is a derivative point of the degree 25; it is therefore, according to Professor SYLVESTER's general theory, identical with the point given by the former construction; this agrees with the before-mentioned theorem of Mr. SALMON.

IV. *Independent investigation for the Cubic.*

47. The following is, in substance, the method by which I first obtained the equation of the five-pointic conic, for the cubic

$$X^3+Y^3+Z^3+6lXYZ=0.$$

Write for shortness

$$\begin{aligned}
U &= x^3+y^3+z^3+6lxyz,\\
V &=(x^2+2lyz)X+(y^2+2lzx)Y+(z^2+2lxy)Z,\\
W &=(X^2+2lYZ)x+(Y^2+2lXZ)y+(Z^2+2lXY)z,\\
\Upsilon &=X^3+Y^3+Z^3+6lXYZ,\\
P &=ax+by+cz,\\
\Pi &=aX+bY+cZ.
\end{aligned}$$

Then X, Y, Z being current coordinates, and x, y, z the coordinates of a point of the cubic (so that $U=0$), the equation of the cubic will be

$$\Upsilon=0,$$

and the equation of a conic having with it an ordinary (two-pointic) contact at the point (x, y, z), will be*

$$2W-\Pi V=0.$$

48. Now imagine from the point of contact lines drawn to the other four intersections of the two curves; in the case of the five-pointic conic, three of these lines will coincide with the tangent $V=0$, and the remaining line will be the line joining the point of contact with the point of simple intersection. The equations of the lines in question can be found by JOACHIMSTHAL's theorem, viz. if (x, y, z) be the coordinates of a given point, and (X, Y, Z) current coordinates, then if in the equations of any two curves we substitute for the coordinates, $\lambda x+\mu X$, $\lambda y+\mu Y$, $\lambda z+\mu Z$, and between the equations so obtained eliminate λ, μ, the resulting equation will be that of the lines drawn from the point (x, y, z) to the points of intersection of the two curves. The point (x, y, z) is any point whatever, and it may therefore be a point of intersection, or, as in the present instance, a point of contact of the two curves; the only difference is, that in either case the degree of each equation as regards (λ, μ) is reduced by unity, and the degree of the resulting equation in X, Y, Z is also reduced by unity: in the case of a point of simple intersection this is the only reduction; but in the case of a point of contact, the result-

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[*] I have introduced the factor 2, to make this correspond with the form $D^2U-\Pi DU=0$, in the case in question, $m=2$.

ing equation contains the equation of the tangent as a factor, and rejecting this factor, the reduction in degree is two units.

49. Applying the method to the two equations, $\Upsilon=0$, $2W-\Pi V=0$, and substituting therein for the original current coordinates X, Y, Z the values $\lambda x+\mu X$, $\lambda y+\mu Y$, $\lambda z+\mu Z$, the equations become

$$\lambda^3U+3\lambda^2\mu V+3\lambda\mu^2W+\mu^3\Upsilon=0,$$
$$2(\lambda^2U+2\lambda\mu V+\mu^2W)-(\lambda P+\mu\Pi)(\lambda U+\mu V)=0;$$

or writing $U=0$, and omitting from each equation the factor μ, the equations become

$$\lambda^2 3V+\lambda\mu.3W+\mu^2\Upsilon \quad =0,$$
$$\lambda(4-P)V+\mu(2W-\Pi V)=0;$$

and putting in the first equation $\lambda=2W-\Pi V$, $\mu=-(4-P)V$, the result of the elimination contains the factor V, rejecting which it becomes

$$3(2W-\Pi V)^2-3(4-P)(2W-\Pi V)W+(4-P)^2V\Upsilon=0,$$

which is of the fourth degree in (X, Y, Z), as it should be, and represents therefore the lines drawn from the point of contact to the other four points of intersection of the conic and cubic.

50. The equation may be written

$$-3(2W-\Pi V)\big((2-P)W+\Pi V\big)+(4-P)^2V\Upsilon=0,$$

and we obtain at once the condition that this may contain the factor V, viz. this condition is

$$P=2;$$

and if this be satisfied the conic will have a three-pointic conic, and there will be three other points of intersection. And writing $P=2$, and throwing out the factor V, we find

$$3\Pi^2V-6\Pi W+4\Upsilon=0$$

for the equation of the lines from the point of contact to the three points of intersection. And we have now to determine Π so that the function on the left hand may divide by V^2.

51. I simplify my original method by the use of a theorem of Mr. SALMON'S, viz. writing

$$\begin{array}{ll|ll}
\Upsilon & =X^3+Y^3+Z^3+6lXYZ, & \mathfrak{H} & =l^2(X^3+Y^3+Z^3)-(1+2l^3)XYZ, \\
W & =(X^2+2lYZ)x+\cdots \quad , & \mathfrak{H}_1 & =\big(3l^2X^2-(1+2l^3)YZ \quad \big)x+\cdots, \\
V & =(x^2+2lyz \quad)X+\cdots \quad , & H_1 & =\big(3l^2x^2-(1+2l^3)yz \quad \big)X+\cdots, \\
U & =x^3+y^3+z^3+6lxyz \quad , & H & =l^2(x^3+y^3+z^3)-(1+2l^3)xyz \quad ,
\end{array}$$

we have identically

$$\Upsilon H-U\mathfrak{H}=WH_1-V\mathfrak{H}_1,$$

and in the present case, since $U=0$,

$$\Upsilon H=WH_1-V\mathfrak{H}_1.$$

Hence, multiplying by H and substituting this value of ϒH, the equation becomes

$$3H\Pi^2V-6H\Pi W+4WH_1-4V\mathfrak{P}_1=0,$$

or, as we may write it,

$$V(3H\Pi^2-4\mathfrak{P}_1)+2W(2H_1-3H\Pi)=0;$$

and we can at once make the equation divide by V, viz. by assuming

$$\Pi=\tfrac{2}{3}\frac{H_1}{H}+\tfrac{1}{2}\Lambda V,$$

where Λ is arbitrary. We have thus a four-pointic contact. And substituting for Π, and throwing out the factor V, the equation becomes

$$3H\left(\tfrac{2}{3}\frac{H_1}{H}+\tfrac{1}{2}\Lambda V\right)^2-4\mathfrak{P}_1+2W(\tfrac{3}{2}H\Lambda)=0;$$

or reducing,

$$16(H_1^2-3H\mathfrak{P}_1)-36H^2\Lambda W+24HH_1\Lambda V+9H^2\Lambda^2V^2=0,$$

which is the equation of the lines drawn from the point of contact to the remaining two points of intersection.

52. I write for greater convenience

$$\Lambda=-\tfrac{4}{9}\frac{\Theta}{H^3};$$

Θ being as yet indeterminate, the equation is thus reduced to

$$9H^3\{H(H_1^2-3H\mathfrak{P}_1)+\Theta W\}-6H^2H_1\Theta V+\Theta^2V^2=0.$$

And we have then to determine Θ so that the left-hand side may divide by V; or, what is the same thing, we must determine Θ so that

$$H(H_1^2-3H\mathfrak{P}_1)+\Theta W$$

may divide by V. This implies the existence of an identical equation,

$$H(H_1^2-3H\mathfrak{P}_1)+\Theta W=MU+NV,$$

which for $U=0$ would give the decomposition in question; but I have not investigated the values of M and N. I assume at the outset $U=0$, and putting, as before,

$$Q=y^3z^3+z^3x^3+x^3y^3-3l^3x^2y^2z^2,$$

and writing also

$$\begin{aligned}S=\ &Xx[(1+8l^3)x^4+(4l+41l^4)x^2yz+(-2l^2+2l^5)y^2z^2]\\&+Yy[(1+8l^3)y^4+(4l+41l^4)y^2zx+(-2l^2+2l^5)z^2x^2]\\&+Zz[(1+8l^3)z^4+(4l+41l^4)z^2xy+(-2l^2+2l^5)x^2y^2],\end{aligned}$$

I remark that for $U=0$ we have

$$-xyzH_1^2+VS+(1+8l^3)(QW-3x^2y^2z^2\mathfrak{P}_1)=0,$$

an equation which, observing that

$$H=-(1+8l^3)xyz,$$

and assuming also

$$\Theta=(1+8l^3)^2Q,$$

may be written

$$H(H_1^2-3H\mathfrak{P}_1)+\Theta W=-(1+8l^3)VS,$$

which gives the required decomposition, so that Θ having this value, the conic will have a five-pointic contact. Reducing by the last equation, and throwing out the factor V, we find

$$-9H^3(1+8l^3)V-6H^2\Theta H_1+\Theta^2V=0$$

for the equation of the line joining the point of contact with the point of simple intersection. And if in this equation we write $H=-(1+8l^3)xyz$, and $\Theta=(1+8l^3)^2Q$, we obtain, finally, for the equation of the line in question,

$$9x^3y^3z^3S-6x^2y^2z^2QH_1+Q^2V=0,$$

which is the before-mentioned result.

53. It only remains to verify the assumed equation

$$-xyzH_1^2+VS+(1+8l^3)(QW-3x^2y^2z^2\mathfrak{P}_1)=0.$$

We may write

$$-Q=x^6+6lx^4yz+3l^2x^2y^2z^2-y^3z^3,$$

and then observing that

$$W=\quad xX^2+\cdot\cdot+\quad 2lx\ \ YZ+\cdot\cdot\quad ,$$

$$\mathfrak{P}_1=3l^2xX^2+\cdot\cdot-(1+2l^3)xYZ-\cdot\cdot\quad ,$$

we find at once

$$(1+8l^3)(QW-3x^2y^2z^2\mathfrak{P}_1)=$$

$$-(1+8l^3)\left\{\begin{array}{l}(x^7+6lx^5yz+12l^2x^3y^2z^2\quad -xy^3z^3)X^2\\ \quad\vdots\\ +(2lx^7+12l^2x^5yz-3x^3y^2z^2-2lxy^3z^3)YZ\\ \quad\vdots\end{array}\right.$$

Next writing

$$H_1=\ \big(3l^2x^2-(1+2l^3)yz\big)X+\cdot\cdot,$$

$$V\ =\ (x^2+2lyz)X+\cdot\cdot,$$

$$S\ =x\{(1+8l^3)x^4+(4l+41l^4)x^2yz+(-2l^2+2l^5)y^2z^2\}X+\cdot\cdot,$$

and forming the expression for

$$-xyzH_1^2+VS,$$

the coefficient of X^2 is

$$-xyz\qquad\{3l^2x^2-(1+2l^3)yz\}^2$$

$$+(x^2+2lyz)x\{(1+8l^3)x^4+(4l+41l^4)x^2yz+(-2l^2+2l^5)y^2z^2\},$$

which is

$$=(1+8l^3)(x^7+6lx^5yz+12l^2x^3y^2z^2-xy^3z^3);$$

the coefficient of YZ is

$$-2xyz\big(3l^2y^2-(1+2l^3)zx\big)\big(3l^2z^2-(1+2l^3)xy\big)$$

$$+(y^2+2lzx)z\{(1+8l^3)z^4+(4l+41l^4)z^2xy+(-2l^2+2l^5)x^2y^2\}$$

$$+(z^2+2lxy)y\{(1+8l^3)y^4+(4l+41l^4)y^2zx+(-2l^2+2l^5)x^2y^2\},$$

which is

$$=(1+8l^3)\{y^3z^3(y^3+z^3)+8lxy^3z^3+12l^2x^2yz(y^3+z^3)+2lx(y^6+z^6)-2x^3y^2z^2\};$$

or substituting for y^3+z^3 and y^6+z^6 the values $-x^3-6lxyz$ and $(x^3+6lxyz)^2-2y^3z^3$ respectively, this is

$$(1+8l^3)(2lx^7+12l^2x^5yz-3x^3y^2z^2-2lxy^3z^3).$$

We have thus

$$-xyz\mathrm{H}_1^2+\mathrm{VS}$$

$$=(1+8l^3)\left\{\begin{array}{l}(x^7+6lx^5yz+12l^2x^3yz-xy^3z^3)\mathrm{X}^2\\ \cdot\\ \cdot\\ +(2lx^7+12l^2x^5yz-3x^3y^2z^2-2lxy^3z^3)\mathrm{YZ}\\ \cdot\\ \cdot\end{array}\right.$$

and the equation

$$-xyz\mathrm{H}_1^2+\mathrm{VS}+(1+8l^3)(\mathrm{QW}-3x^2y^2z^2\mathfrak{P}_1)=0$$

is thus verified.

Addition.—The foregoing memoir was communicated to Mr. SALMON, and I am indebted to him for two notes, containing the extension to a curve of any order, of the preceding investigation for the case of a cubic; I reproduce this extension in the following section.

V. *Extension of the last preceding method to a curve of any order.*

54. Consider the curve of the m-th order $\Upsilon=0$, and in the place of the coordinates write $\lambda x+\mu\mathrm{X}$, $\lambda y+\mu\mathrm{Y}$, $\lambda z+\mu\mathrm{Z}$, where, as before, (x, y, z) are the coordinates of the point of the curve, and X, Y, Z current coordinates; the term involving λ^m vanishes, and dividing out the factor μ the equation becomes

$$\lambda^{m-1}\mathrm{DU}+\tfrac{1}{2}\lambda^{m-2}\mu\mathrm{D}^2\mathrm{U}+\tfrac{1}{6}\lambda^{m-3}\mu^2\mathrm{D}^3\mathrm{U}+\tfrac{1}{24}\lambda^{m-4}\mu^3\mathrm{D}^4\mathrm{U}+\&\text{c.}=0.$$

Making the like substitution in $\mathrm{D}^2\mathrm{U}-\Pi\mathrm{DU}=0$, the assumed equation of the five-pointic conic, the factor μ divides out and the equation becomes

$$2\lambda(m-1)\mathrm{DU}+\mu\mathrm{D}^2\mathrm{U}-(\lambda\mathrm{P}+\mu\Pi)\mathrm{D}^2\mathrm{U}=0,$$

or, what is the same thing,

$$\lambda\mathrm{DU}\big(2(m-1)-\mathrm{P}\big)+\mu(\mathrm{D}^2\mathrm{U}-\Pi\mathrm{DU})=0;$$

and if from the two equations we eliminate λ, μ, the result, throwing out the factor DU, is

$$(\mathrm{D}^2\mathrm{U}-\Pi\mathrm{DU})^{m-1}-\tfrac{1}{2}\big(2(m-1)-\mathrm{P}\big)\mathrm{D}^2\mathrm{U}(\mathrm{D}^2\mathrm{U}-\Pi\mathrm{DU})^{m-2}+\&\text{c.}=0,$$

where all the terms after the second contain the factor DU; the condition in order that the equation may divide by DU, is consequently $2(m-1)-\mathrm{P}=2$, or $\mathrm{P}=2(m-2)$, the condition of a three-pointic contact. Substituting this value, and dividing by DU, the equation becomes

$$-\Pi(\mathrm{D}^2\mathrm{U}-\Pi\mathrm{DU})^{m-2}+\tfrac{2}{3}\mathrm{D}^3\mathrm{U}(\mathrm{D}^2\mathrm{U}-\Pi\mathrm{DU})^{m-3}-\tfrac{1}{3}\mathrm{D}^4\mathrm{U}.\mathrm{DU}(\mathrm{D}^2\mathrm{U}-\Pi\mathrm{DU})^{m-4}+\&\text{c.}=0,$$

which will be divisible by DU if $-\Pi D^2U+\frac{2}{3}D^3U$ is divisible by DU, and the condition for this is found to be, as before,

$$\Pi=\tfrac{2}{3}\frac{1}{H}DH+\Lambda DU,$$

where Λ is arbitrary; we have thus the conditions of a four-pointic contact.

55. Substituting this value of Π, we see that $D^3U-\frac{1}{H}DH.D^2U$ divides by DU, viz. there exists an identical equation,

$$D^3U-\frac{1}{H}DH.D^2U=IU+J.DU;$$

and hence if $U=0$,

$$\frac{1}{DU}\left(D^3U-\frac{1}{H}DH.D^2U\right)=J,$$

where J is a quadric function of (X, Y, Z). I do not know the general form of this function, but Mr. SALMON has obtained a result which may be generalized as follows, viz. writing for X, Y, Z the values $B\nu-C\mu$, $C\lambda-A\nu$, $A\mu-B\lambda$ (where, as before, A, B, C are the first derived functions of U and λ, μ, ν are arbitrary), the expression for J is

$$J=\frac{1}{DU}\left(D^3U-\frac{1}{H}DH.D^2U\right)=\frac{3(m-2)}{(m-1)}\Phi-\frac{1}{(m-1)\Pi}\square\vartheta;$$

a formula which will be presently useful.

56. The foregoing equation may be written

$$(D^2U)^{m-3}(-\Pi D^2U+\tfrac{2}{3}D^3U)$$
$$+(D^2U)^{m-4}DU\{(m-2)\Pi^2D^2U-\tfrac{2}{3}(m-3)\Pi D^3U-\tfrac{1}{3}D^4U\}+\&c.(DU)^2\ldots=0;$$

and the term $-\Pi D^2U+\frac{2}{3}D^3U$ is equal to

$$\tfrac{2}{3}\left(D^3U-\frac{1}{H}DH.D^2U\right)-\Lambda DU.D^2U=\tfrac{2}{3}JDU-\Lambda DU.D^2U.$$

Substituting this value the equation divides by DU, and throwing out this factor it becomes

$$(D^2U)^{m-3}(\tfrac{2}{3}J-\Lambda D^2U)$$
$$+(D^2U)^{m-4}\{(m-2)\Pi^2D^2U-\tfrac{2}{3}(m-3)\Pi D^3U-\tfrac{1}{3}D^4U\}+\&c.DU=0;$$

or observing that $\frac{2}{3}\Pi D^3U=\Pi^2D^2U+$ term containing DU, this may be written

$$(D^2U)^{m-3}(\tfrac{2}{3}J-\Lambda D^2U)+(D^2U)^{m-4}(\Pi^2D^2U-\tfrac{1}{3}D^4U)+\&c.DU=0.$$

57. If the equation divides by DU we shall have a five-pointic contact; the condition for this is that

$$-\Lambda(D^2U)^2+\tfrac{2}{3}JD^2U+\Pi^2D^2U-\tfrac{1}{3}D^4U$$

may divide by DU, or more simply that

$$-\Lambda(D^2U)^2+\tfrac{2}{3}JD^2U+\tfrac{4}{9}\left(\frac{1}{H}DH\right)^2D^2U-\tfrac{1}{3}D^4U$$

may divide by DU, or, what is the same thing, the function in question must vanish in virtue of the substitution of the values $B\nu-C\mu$, $C\lambda-A\nu$, $A\mu-B\lambda$ in the place of X, Y, Z.

The expression for J has just been given; we have besides

$$\left(\frac{1}{H}DH\right)^2=\frac{(m-1)^4}{H^2}Q_3^2,\ D^4U=-Q_4\vartheta^2,$$

where the values of Q_3^2, Q_4 are given (*ante*, No. 27); we have thus

$$58.\quad \frac{1}{DU}\left(D^3U-\frac{1}{H}DH.D^2U\right)=\frac{3(m-2)}{m-1}\Phi-\frac{1}{(m-1)H}\square\vartheta,$$

$$\frac{4}{9}\left(\frac{1}{H}DH\right)^2 \qquad =-\frac{4(m-2)^2}{(m-1)^2}\Phi+\frac{4}{3}\,\frac{(m-2)^2}{(m-1)^2}\,\frac{\square}{H}\vartheta-\frac{1}{9}\,\frac{1}{(m-1)^2}\,\frac{1}{H^2}\Psi\vartheta^2,$$

$$-\frac{1}{3}D^4U \qquad =\vartheta^2\left\{-\frac{2(m-2)(m-3)}{(m-1)^4}\Phi H+\frac{4}{3}\,\frac{(m-3)}{(m-1)^4}\square\vartheta-\frac{1}{3}\,\frac{1}{(m-1)^4}\Omega\vartheta^2\right\}.$$

and

$$D^2U=-\frac{1}{(m-1)^2}H\vartheta^2,$$

whence

$$\Lambda\frac{1}{(m-1)^4}H^2\vartheta^4=$$

$$\begin{aligned}
&-\frac{2}{3}\,\frac{1}{(m-1)^2}H\vartheta^2\left\{\frac{3(m-2)}{m-1}\Phi-\frac{1}{(m-1)H}\square\vartheta\right\}\\
&-\frac{1}{(m-1)^2}H\vartheta^2\left\{\frac{-4(m-2)^2}{(m-1)^2}\Phi+\frac{4}{3}\,\frac{(m-2)}{(m-1)^2H}\square\vartheta-\frac{1}{9}\,\frac{1}{(m-1)^2}\,\frac{1}{H^2}\Psi\vartheta^2\right\}\\
&\qquad+\vartheta^2\left\{-\frac{2(m-2)(m-3)}{(m-1)^4}\Phi H+\frac{4}{3}\,\frac{(m-3)}{(m-1)^4}\square\vartheta-\frac{1}{3}\,\frac{1}{(m-1)^4}\Omega\vartheta^2\right\}\\
&\qquad=\frac{1}{9(m-1)^4H}(4\Psi-3\Omega H),
\end{aligned}$$

and consequently

$$\Lambda=\frac{1}{9H^3}(4\Psi-3\Omega H),$$

which agrees with the result before obtained, and thus the present method gives the complete solution of the problem.

XX. THE BAKERIAN LECTURE.—*Researches on Organo-metallic Bodies.*—Fourth Memoir.

By E. FRANKLAND, *Ph.D., F.R.S., Lecturer on Chemistry at St. Bartholomew's Hospital.*

Received February 17,—Read March 3, 1859.

IN a former memoir I described the production of a series of organic compounds containing the metal tin in combination with the radicals methyl, ethyl, and amyl. These bodies were formed by the action of light or heat upon the iodides of methyl, ethyl, and amyl, respectively placed in contact with strips of tinfoil. My attention was at that time especially directed to the compound formed by the union of tin with ethyl, and to which the name of stanethyl was given. The iodide of stanethyl ($Sn\,C_4H_5I$) was prepared by exposing iodide of ethyl to light or heat in the presence of tinfoil; and by acting with zinc upon an aqueous solution of this iodide of stanethyl or of the chloride of the same body, stanethyl itself (C_4H_5Sn) was obtained.

In accordance with a theory of the constitution of all organo-metallic bodies which I then proposed, the above tin compounds were respectively represented as the analogues of the protiodide and biniodide of tin; thus—

$$\underbrace{SnI}_{\text{Stannous iodide.}} \qquad \underbrace{Sn(C_4H_5)}_{\text{Stannous ethide.}}$$

$$\underbrace{Sn\left\{\begin{matrix}I\\I\end{matrix}\right.}_{\text{Stannic iodide.}} \qquad \underbrace{Sn\left\{\begin{matrix}(C_4H_5)\\I\end{matrix}\right.}_{\text{Stannic ethiodide.}}$$

It is evident that the application of this view to the above bodies would receive additional support if the second equivalent of iodine in the stannic iodide could be replaced by ethyl, or some other analogous organic group. In the memoir already alluded to, I mentioned that in studying the behaviour of stanethyl under the influence of heat, evidence was obtained of the existence of this very compound—*stannic ethide*, or *binethide of tin*, as I then named it. This body obviously bears the same relation to stannic iodide, as stanethyl bears to stannous iodide.

$$\underbrace{Sn\left\{\begin{matrix}I\\I\end{matrix}\right.}_{\text{Stannic iodide.}} \qquad \underbrace{Sn\left\{\begin{matrix}C_4H_5\\C_4H_5\end{matrix}\right.}_{\text{Stannic ethide.}}$$

Although there could be little doubt of the formation of stannic ethide by heating stanethyl to 150° C., yet I could not succeed in obtaining the former body in a state of purity from this source. It occurred to me, however, that stannic ethiodide would probably be easily converted into stannic ethide by bringing it into contact with zincethyl; and a preliminary experiment completely realized this expectation. The results of this reaction, together with its extension to other analogous organo-metallic bodies, are described in the following pages.

I. *Action of Zincethyl upon Iodide of Stanethyl.*

About two ounces of crystals of iodide of stanethyl were gradually added to a strong solution of zincethyl in ether. The crystals rapidly dissolved, with a moderate evolution of heat, and the liquid finally assumed a syrupy consistence. Care was taken to preserve an excess of zincethyl, by stopping the addition of the iodide before the liquid ceased to effervesce with water. The syrupy liquid was now submitted to distillation. It began to boil at 70° C.; but the thermometer rapidly rose to 180°, between which temperature and 200° C. the greater part of the product passed over, solid iodide of zinc containing a little zincethyl being left in the retort. The distillate on being washed with water effervesced strongly, depositing oxide of zinc, which was dissolved on the addition of a little acetic acid. The dense ethereal liquid was now separated from the supernatant aqueous stratum, and placed over chloride of calcium for forty-eight hours. On being distilled, the thermometer soon became stationary at 181° C., and the distillate passing at this temperature was collected apart.

Submitted to analysis it yielded the following results:—

I. ·2185 grm., burnt with oxide of copper and free oxygen, yielded ·3295 grm. carbonic acid and ·1708 grm. water.

II. ·2338 grm. gave ·3518 grm. carbonic acid and ·1845 grm. water.

III. ·0992 grm., placed in a stream of oxygen, and the vapour thus mixed with excess of oxygen passed through an ignited tube, gave ·0618 grm. binoxide of tin.

These results correspond closely with the formula of stannic ethide—

$$\mathrm{Sn}\begin{cases}C_4H_5\\ C_4H_5,\end{cases}$$

as is seen from the following comparison:—

	Calculated.		Found.			
			I.	II.	III.	Mean.
C_8 .	48	41·09	41·09	40·92	——	41·01
H_{10}	10	8·56	8·64	8·31	——	8·48
Sn	58·82	50·35	——	——	50·28	50·28
	116·82	100·00				99·77

The following equation therefore expresses the action of zincethyl upon iodide of stanethyl:—

$$\left.\begin{matrix}\mathrm{Sn}(C_4H_5)\,\mathrm{I}\\ \mathrm{Zn}(C_4H_5)\end{matrix}\right\}=\begin{cases}\mathrm{Sn}(C_4H_5)_2\\ \mathrm{Zn\,I}\end{cases}$$

Stannic ethide, or binethide of tin, is a limpid colourless liquid even at −18° C., possessing a very faint ethereal odour, resembling that of the oxide of stanethyl, and a slight metallic, though not unpleasant taste. Its specific gravity is 1·187 at 23° C. A determination of the specific gravity of its vapour by GAY-LUSSAC's method gave the following results:—

Weight of stannic ethide	·1127 grm.
Observed volume of vapour	25·0 cub. cent.
Temperature of oil-bath	193°·3 C.
Height of barometer	762·0 mm.
Height of inner column of mercury	217·2 mm.
Height of column of oil	275·0 mm.

From these numbers the specific gravity of the vapour was calculated to be 8·021. The vapour of stannic ethide thus consists of one volume of tin vapour and four volumes of ethyl, the five volumes being condensed to two.

1 vol. Tin vapour	4·05367
4 vols. Ethyl	4·00780
2 vols. Stannic ethide vapour	=8·06147
Found	8·021

Stannic ethide boils at 181° C., and distils unchanged, thus differing from stannous ethyl, which decomposes at 150° C., chiefly into metallic tin and stannic ethide,—a reaction calling to mind the behaviour of stannous oxide when boiled with a caustic alkali. Stannic ethide is highly inflammable, burning with a lurid flame fringed with deep blue, and evolving white fumes of stannic oxide. In oxygen it burns much more brilliantly, with a red light fringed with blue.

It was important to ascertain the deportment of stannic ethide with negative elements, since, if it were found to be capable of direct combination, its analogy to inorganic stannic compounds would be to a great extent disproved. Like zincethyl, however, stannic ethide is incapable of combining with any other element without the expulsion of at least an equivalent amount of the ethyl it contains.

Treated with iodine, the latter dissolves with a deep red colour, which gradually disappears; and if the addition of iodine be continued until decoloration be no longer effected, the resulting liquid, on being submitted to distillation, is found to consist of iodide of ethyl, which distils over, and an iodine salt, possessing the insupportably pungent odour of one of the products of the action of tin upon iodide of ethyl at 160° C., and described by MM. CAHOURS and RICHE as iodide of distannous ethyl, $\left(Sn_2(C_4H_5)_2I\right)$. The iodine salt appears, in fact, to be either identical with this body, or to consist of stannic *iodotriethide*, $\left(Sn_2(C_4H_5)_3I\right)$*.

---

* Whilst engaged with these experiments, Mr. BUCKTON announced the formation of stannic ethide (Proceedings of the Royal Society, vol. ix. p. 315), and at the same time mentioned his intention to study the salts formed by the action of iodine, bromine, &c. upon that body; I have therefore not prosecuted the inquiry further in this direction.

Stannic ethide does not decompose water, and is not acted upon by strong aqueous hydrochloric acid in the cold. When, however, the two liquids are mixed over mercury and exposed to a heat of 80° or 90° C., bubbles of gas are slowly evolved; but it requires from twelve to eighteen hours to complete the reaction. Treated in this way, ·2268 grm. stannic ethide yielded 75 cub. cent. of gas at 8° C. and 209·8 mm. mercurial pressure, equivalent to 20·11 cub. cent. at 0° C. and 760 mm. pressure.

The gas, freed from hydrochloric acid and submitted to analysis, yielded the following results:—

I.

	Pressure.	Temperature.
Gas used	249·6 mm.	13°·1 C.
After action of SO_3, HO	249·0	13°·1
After action of SO_3	248·4	13°·1

II.

Oxygen used for combustion	432·3	12°·0
After admission of combustible gas	366·4	12°·0
After explosion	375·6	12°·0
After absorption of carbonic acid	302·3	12°·0

III.

Oxygen used for combustion	322·1	10°·7
After admission of combustible gas	357·0	10°·7
After explosion	270·0	10°·7
After absorption of carbonic acid	197·0	10°·7

No. I. shows the gas to be free from ether vapour, chloride of ethyl, and ethylene.

Nos. II. and III. prove it to possess the composition and specific gravity of hydride of ethyl, one volume of which requires for its combustion 3·5 volumes of oxygen, and generates two volumes of carbonic acid:

	Vol. of combustible gas.		Vol. of O consumed.		Vol. of CO_2 generated.
II.	37·1	:	130·0	:	73·3
	1	:	3·50	:	1·98
III.	34·9	:	125·1	:	73·0
	1	:	3·58	:	2·09

This result, taken in connexion with the volume of hydride of ethyl disengaged from a known weight of stannic ethide, indicates the following to be the reaction between that body and hydrochloric acid:

$$\left.\begin{matrix} 2Sn(C_4H_5)_2 \\ HCl \end{matrix}\right\} = \left\{\begin{matrix} C_4H_5H \\ Sn_2(C_4H_5)_3Cl. \end{matrix}\right.$$

This reaction requires the expulsion of 12·39 per cent. of ethyl in the form of hydride of ethyl, whereas in the above experiment ·2268 grm. of stannic ethide gave ·0271 grm.

hydride of ethyl, equivalent to ·0262 grm. of ethyl, or 11·55 per cent. The analysis of the salt formed is still required to corroborate this result.

II. *Action of Zincmethyl upon Iodide of Stanethyl.*

The successful substitution of the second equivalent of iodine in stannic iodide by ethyl, led me to hope that it might in like manner be replaced by methyl, amyl, or any other homologous radical, thus forming a stannic compound containing two radicals of different composition. To establish this point, I submitted iodide of stanethyl to the action of zincmethyl. About three ounces of the crystallized iodide were gradually added to a solution of zincmethyl in ether, care being taken to preserve an excess of zincmethyl; considerable heat was evolved, and the vessel in which the reaction was performed required to be plunged into cold water. On distilling the product, it began to boil at 40° C.; and the whole of the volatile portion passed over below 140°, leaving iodide of zinc in the retort. The distillate, on being treated with dilute acetic acid, effervesced strongly, owing to the presence of zincmethyl; at the same time a dense layer of an ethereal liquid separated. This latter, after being washed with six or eight times its bulk of water, was dried over chloride of calcium. On rectification some ether came over at 65°; but the thermometer soon rose to 143°, between which temperature and 148° C. a large portion distilled. The last drops came over at 150° C. The section which distilled between 143° and 148° was taken for the following experiments:—

I. ·2985 grm., burnt with oxide of copper and free oxygen, gave ·3823 grm. carbonic acid and ·2097 grm. water.

II. ·2888 grm. gave ·3733 grm. carbonic acid and ·2120 grm. water.

III. ·0678 grm., decomposed in a sealed tube with nitric acid, gave ·0495 grm. binoxide of tin.

These results indicate the formula

$$\mathrm{Sn}\left\{\begin{matrix}\mathrm{C_2H_3}\\ \mathrm{C_4H_5}.\end{matrix}\right.$$

			I.	II.	III.	Mean.
C_6 . . .	36	35·01	34·92	35·26	——	35·09
H_8 . .	8	7·78	7·81	8·15	——	7·98
Sn . . .	58·82	57·21	——	——	57·38	57·38
	102·82	100·00				100·45

The action of zincmethyl upon iodide of stanethyl may therefore be thus expressed:—

$$\left.\begin{matrix}\mathrm{Sn}\left\{\begin{matrix}\mathrm{C_4H_5}\\ \mathrm{I}\end{matrix}\right.\\ \mathrm{Zn(C_2H_3)}\end{matrix}\right\}=\left\{\begin{matrix}\mathrm{Sn}\left\{\begin{matrix}\mathrm{C_2H_3}\\ \mathrm{C_4H_5}\end{matrix}\right.\\ \mathrm{ZnI}\end{matrix}\right.$$

The new body thus formed, and for which I propose the name *stannic ethylomethide*, is a colourless, limpid liquid undistinguishable in appearance from stannic ethide. It possesses, like the latter, a very faint ethereal odour and a slightly metallic taste. Its specific gravity is 1·2319 at 19° C. It does not solidify at −13° C. Stannic ethylomethide boils between 144° C. and 146° C. A determination of its vapour-density gave the following resuls:—

Weight of stannic ethylomethide	·1967 grm.
Observed volume of vapour	44·2 cub. cent.
Temperature of oil-bath	199°·0 C.
Height of barometer	754·4 mm.
Height of inner column of mercury	113·0 mm.
Height of column of oil	275·0 mm.

From these data the specific gravity of the vapour was calculated to be 6·838. Stannic ethylomethide vapour therefore consists of one volume of tin vapour, two volumes of ethyl, and two volumes of methyl, the five volumes condensed to two:—

1 vol. Tin vapour	4·05367
2 vols. Ethyl	2·00390
2 vols. Methyl	1·03652
2 vols. Stannic ethylomethide vapour	7·09409
Found	6·838

Stannic ethylomethide is easily inflammable, exhibiting the same phenomena on combustion in air and oxygen as stannic ethide. It manifests also the same deportment as the latter with chlorine, iodine, and bromine. Its combination with these elements is always attended with the expulsion of methyl. Stannic ethylomethide dissolves iodine, assuming a magnificent crimson colour, which disappears with extreme slowness unless heat be applied to the liquid; when, however, action has in this way once been set up, it goes on with considerable rapidity, even in the cold. About two ounces of stannic ethylomethide were treated with iodine until the liquid no longer became decolorized. The excess of iodine was then removed by agitation with mercury, and the product submitted to distillation. It began to boil at a low temperature; and the distillate had the odour, specific gravity, and all the properties of iodide of methyl. The boiling-point of the residual liquid now rapidly rose to 207° C., whilst very little of it passed over. The distillation was then interrupted, and the straw-coloured liquid remaining in the retort was submitted to analysis. It yielded the following results:—

I. ·4916 grm., burnt with oxide of copper, 3 inches of metallic copper being placed in front of the combustion tube, gave ·2888 grm. carbonic acid and ·1493 grm. water.

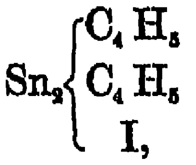
II. ·5620 grm. gave ·3278 grm. carbonic acid and ·1761 grm. water.

III. ·5487 grm. gave ·3203 grm. carbonic acid and ·1696 grm. water.

IV. ·6502 grm., decomposed with alcoholic potash, the solution then acidified with nitric acid, and precipitated with nitrate of silver, gave ·5146 grm. iodide of silver.

V. ·5217 grm., heated with strong nitric acid, the solution evaporated to dryness and ignited, gave ·2591 grm. binoxide of tin.

These numbers closely correspond with those calculated from the formula of iodide of distanethyl—

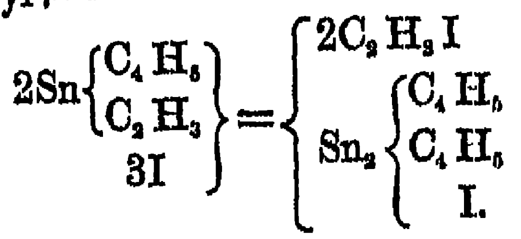


$$Sn_2\left\{\begin{array}{l}C_4H_5\\C_4H_5\\I,\end{array}\right.$$

as is obvious from the following comparison:—

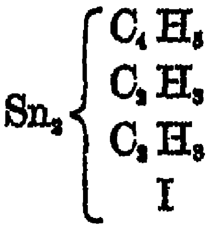
	Calculated.		Found. I.	II.	III.	IV.	V.	Mean.
C_8 . . .	48	15·87	16·02	15·88	15·92	——	——	15·94
H_{10} . .	10	3·31	3·37	3·48	3·43	——	——	3·42
Sn_2 . .	117·64	38·89	——	——	——	——	39·03	39·03
I . . .	126·84	41·93	——	——	——	42·63	——	42·63
	302·48	100·00						101·02

The foregoing numbers, taken in connexion with the production of iodide of methyl, prove that the action of iodine upon stannic ethylomethide converts two equivalents of the latter into one equivalent of iodide of distanethyl, with the separation of two equivalents of iodide of methyl:—



$$2Sn\left\{\begin{array}{l}C_4H_5\\C_2H_3\end{array}\right.\left.\begin{array}{c}\\ \\3I\end{array}\right\}=\left\{\begin{array}{l}2C_2H_3I\\Sn_2\left\{\begin{array}{l}C_4H_5\\C_4H_5\\I.\end{array}\right.\end{array}\right.$$

It might have been expected that a stannic compound of the form



$$Sn_2\left\{\begin{array}{l}C_4H_5\\C_2H_3\\C_2H_3\\I\end{array}\right.$$

would have been produced, differing empirically from iodide of distanethyl only by containing one additional equivalent of hydrogen; but the formula of the former body requires 15·81 per cent. of carbon and 3·62 per cent. of hydrogen; and although these numbers are not altogether incompatible with the results of the above analyses, yet the fact of the production of iodide of methyl in the reaction is utterly irreconcileable with the formation of such a compound. Iodide of distanethyl has already been examined both by M. Löwig and by MM. Cahours and Riche; but as the descriptions of these chemists are so little in harmony with each other, it is impossible to make use of them for the identification of the body obtained in the above reaction. It appears, however, to agree in properties with the iodide of distanethyl prepared by MM. Cahours and Riche.

Iodide of distanethyl, produced by the action of iodine upon stannic ethylomethide, is a light straw-coloured, somewhat oily liquid, which does not solidify on exposure to a cold of −13° C., and possesses an extremely pungent and intolerable odour, resembling oil of mustard; the inhalation even of a very minute amount of its vapour being highly irritating to the respiratory organs. Its specific gravity at 15° C. is 2·0329. At 208° C. it enters into ebullition, but cannot be distilled without decomposition; the thermometer rapidly rises, and at 230° C. a copious deposition of iodide of tin takes place. It would be interesting to study the action of zincethyl upon this body.

Stannic ethylomethide is more readily acted upon by aqueous hydrochloric acid than the corresponding ethyl compound; a copious evolution of gas takes place, and a crystallizable salt is formed. Submitted to analysis, the gas yielded the following results:—

I. It did not contain any gases absorbable by anhydrous sulphuric acid.

II. Treated with an equal volume of alcohol it was partially absorbed:

	Pressure.	Temperature.
Gas used	193·0 mm.	7°·5 C.
After absorption by alcohol	34·0	7°·5

III. The combustion of the gas with oxygen gave the following results:—

a.

	Pressure.	Temperature.
Oxygen used for combustion	391·9 mm.	11°·7 C.
After admission of combustible gas. . .	422·1	11°·7
After explosion	349·5	11°·7
After absorption of carbonic acid	295·7	11°·7

b.

	Pressure.	Temperature.
Oxygen used for combustion	337·1 mm.	7°·7 C.
After admission of combustible gas . . .	359·6	7°·7
After explosion	306·5	7°·7
After absorption of carbonic acid	265·3	7°·7

The two combustions, *a* and *b*, which were made with portions of gas collected in two different operations, show the gas to be a mixture of the hydrides of ethyl and methyl; and this result is confirmed by determination No. II.,—hydride of ethyl being soluble in alcohol, whilst hydride of methyl is insoluble. The results of analyses Nos. II. and III. may be thus stated:—

	II.		III.	Mean.
	a.	*b.*		
Hydride of ethyl	78·14	83·11	82·38	81·21
Hydride of methyl . . .	21·86	16·89	17·62	18·79
	100·00	100·00	100·00	100·00

In a separate experiment it was found that ·1641 grm. of stannic ethylomethide yielded with hydrochloric acid, 17·81 cub. centimetres of the mixed hydrides at 0° C., and 760 mm. pressure,—an amount which indicates the displacement of exactly one-fourth of the positive groups in the stannic ethylomethide, as was the case in the corresponding reaction with stannic ethide. But the composition of the mixed gases does not permit of any safe conclusion being drawn as to the exact nature of the reaction. It is obvious, however, that it is the ethyl group chiefly which is removed by the hydracid. The analysis of the salt formed at the same time would no doubt supply the necessary evidence.

III. *Action of Zincethyl upon Iodide of Mercurymethyl.*

The formation of stannic ethylomethide, in the manner just described, encouraged me to attempt a similar reaction in the case of the iodide of mercurymethyl,—a body which is formed under precisely the same conditions as iodide of stanethyl, and to which I assigned the formula $Hg\left\{\begin{matrix}C_2H_3\\ I\end{matrix}\right.$, regarding it as related to biniodide of mercury in the same manner as iodide of stanethyl is connected with biniodide of tin. Mr. BUCKTON'S announcement of the formation of mercuric ethide $\left(\text{which from my point of view must be regarded as } Hg\left\{\begin{matrix}C_4H_5\\ C_4H_5\end{matrix}\right.\right)$ by an analogous reaction, tended also to strengthen the hope that a mercuric ethylomethide might be thus obtained.

When iodide of mercurymethyl is added to pure zincethyl, there is scarcely any action perceptible beyond the solution of the mercury compound; nevertheless, after the lapse of a few hours, a copious deposit of iodide of zinc takes place. The product obtained in this manner from two ounces of the iodide of mercurymethyl was submitted to distillation: it began to boil at 98° C.; the thermometer remained stationary for some time at 120° C., and then suddenly rose to 158° C., at which temperature nearly the whole of the remaining product passed over. The distillate possessed in a high degree the powerful and unmistakeable odour of zincmethyl. It was washed with water and dilute hydrochloric acid, dried over chloride of calcium, and rectified in an oil-bath. It began to boil at 149° C., but the thermometer rose almost immediately to 155°, between which point and 157° C. nearly the whole of the remaining liquid passed over. The last drops distilled at 165° C. The portion distilling between 155° and 157° C. was received apart and used in the following determinations:—

I. ·4590 grm., burnt with oxide of copper and free oxygen, the mercury being collected in a drawn-out portion of the combustion-tube projecting from the furnace, gave ·3044 grm. carbonic acid, ·1930 grm. water, and ·3569 grm. mercury.

II. ·4979 grm. gave ·3337 grm. carbonic acid and ·2101 grm. water.

III. ·6338 grm. gave ·4918 grm. mercury.

These results correspond with the formula of mercuric ethide,—

$$Hg\left\{\begin{matrix}C_4H_5\\ C_4H_5\end{matrix}\right.$$

	Calculated.		Found.			
			I.	II.	III.	Mean.
C_8 . . .	48	18·60	18·59	18·28	——	18·44
H_{10} . . .	10	3·88	4·20	4·21	——	4·20
Hg . . .	200	77·52	77·75	——	77·59	77·67
	258	100·00	100·54			100·31

The body perfectly agrees in all its properties with the mercuric ethyl obtained by Mr. Buckton; and to the remarks of that accurate observer respecting this compound I have not one word to add.

The entire absence of mercuric methide and the presence of zincmethyl amongst the products of the action of zincethyl upon iodide of mercurymethyl, point to the following equation as representing the reaction:—

$$\left.\begin{array}{r}Hg(C_2H_3)I\\ 2Zn(C_4H_5)\end{array}\right\}=\left\{\begin{array}{l}Hg(C_4H_5)_2\\ Zn(C_2H_3)\\ ZnI.\end{array}\right.$$

This result indicates a mobility in the organic groups contained in these compounds which could scarcely have been expected.

IV. *Action of Zincmethyl upon Chloride of Mercuryethyl.*

Although the above reaction failed to produce mercuric ethylomethide, it was still possible that this body might be formed by acting upon a mercuryethyl compound with zincmethyl. For this purpose the iodide of mercuryethyl naturally suggested itself; but as I found this body difficult to prepare in large quantity by the action of diffused light upon iodide of ethyl in presence of mercury, I selected the chloride of mercuryethyl, which was prepared by the following modification of the process suggested by Mr. Buckton.

To an ethereal solution of zincethyl, dried and powdered bichloride of mercury was added until the zincethyl was as nearly as possible all decomposed. The liquid had then separated into two layers,—an upper one nearly solid, consisting of a saturated solution of chloride of zinc in ether, and a lower and more voluminous one, composed of nearly pure and limpid mercuric ethide. The latter, which can easily be separated from the tenacious solution of chloride of zinc, was washed with dilute acetic acid to remove a trace of zincethyl which it contained, and then dissolved in fifteen or twenty times its bulk of alcohol. This alcoholic solution was now poured upon a quantity of powdered bichloride of mercury, rather less than sufficient to convert the mercuric ethyl into chloride of mercurous ethyl, and, the whole being heated to the boiling-point, the liquor was filtered whilst hot; on cooling, it deposited a copious crop of magnificent satiny crystals of chloride of murcurous ethyl, perfectly free from bichloride of mercury; and the mother-liquor being repeatedly returned to the flask containing the undissolved chloride of mercurous ethyl, the whole of the latter was finally obtained in a pure and crystalline form. In this way the zincethyl from a single digester-charge produced upwards of eight ounces of the chloride.

About five ounces of chloride of mercurous ethyl, dried *in vacuo* over sulphuric acid, were added to four ounces of a strong ethereal solution of zincmethyl. Considerable heat was evolved; and after forty-eight hours the product was distilled. It began to boil at 60° C. The thermometer was stationary for some time at 128°, and finally rose to 140° C., when a mixture of zincethyl and zincmethyl distilled over, whilst a permanent gas was at the same time evolved. The distillate was washed with weak acetic acid and dried over chloride of calcium. On rectification, a considerable proportion distilled between 127° and 137° C., and was collected apart. The last few drops came over at 156° C. Repeated rectifications of the product, boiling between 127° and 137° C., did not serve to isolate any portion of the distillate having a fixed boiling-point; on the contrary, it was evident that the range of the temperature of distillation became wider each time the operation was repeated. A section boiling between 127° and 133° C. yielded, on analysis, 13·68 per cent. of carbon, whilst another section, boiling between 141° and 143° C., gave 16·71 per cent. of carbon. The formula $Hg\left\{\begin{matrix}C_4H_5\\C_2H_3\end{matrix}\right.$ requires 14·75 per cent. of carbon. According to Mr. BUCKTON, mercuric methide boils at 96° C., and mercuric ethide at 159° C.; consequently mercuric ethylomethide might be expected to boil at about 128° C. It is more than probable that mercuric ethylomethide was formed in the above reaction; but subsequent distillations gradually transformed it, more or less perfectly, into a mixture of mercuric ethide and mercuric methide.

$$2Hg\left\{\begin{matrix}C_4H_5\\C_2H_3\end{matrix}\right\}=\left\{\begin{matrix}Hg\left\{\begin{matrix}C_4H_5\\C_4H_5\end{matrix}\right.\\Hg\left\{\begin{matrix}C_2H_3\\C_2H_3.\end{matrix}\right.\end{matrix}\right.$$

V. *Action of Zinc upon a Mixture of the Iodides of Ethyl and Methyl.*

In a former memoir* I pointed out that the vapour-volume of zincethyl allows that compound to be represented by the formula $\left.\begin{matrix}C_4H_5\\C_4H_5\end{matrix}\right\}Zn_2$. This formula would be more firmly established if we could succeed in combining zinc with two radicals of different composition; the iodides of methyl and ethyl, mixed with an equal volume of anhydrous ether, were therefore submitted to the action of zinc at 100° C. in a copper digester, in the hope of obtaining the body $Zn_2\left\{\begin{matrix}C_2H_3\\C_4H_5\end{matrix}\right.$. In eighteen hours, the decomposition of the iodides being complete, the product was distilled from the digester at a temperature not exceeding 150°. In order to prevent as far as possible the formation of any zincethyl, which would be more difficult than zincmethyl to separate from the intermediate body, if the latter were produced, iodide of methyl in slight excess over the equivalent quantity, was employed. On rectification the product began to boil at 38°, ether and zincmethyl distilling over; the thermometer then gradually and uniformly rose to 120° C., at which temperature the remainder of the product, which was considerable, and consisted of pure zincethyl, distilled over. No evidence whatever was obtained of the existence of an intermediate compound containing both ethyl and methyl.

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* Transactions of the Royal Society for 1855, p. 266.

VI. *Zincmethyl.*

The experiments detailed in the foregoing pages requiring the use of considerable quantities of zincmethyl, my attention was directed to the preparation of this body in much larger quantities than could be obtained by the operations in sealed glass tubes, described in my previous papers on this compound. I found that the preparation of a strong ethereal solution of zincmethyl succeeded most satisfactorily in the copper digester* used for the production of large quantities of the corresponding ethyl compound; in fact, the decomposition by zinc of an ethereal solution of iodide of methyl is much more quickly and perfectly effected than that of a similar solution of iodide of ethyl. Heated to 100° in the copper digester, three ounces by measure of iodide of methyl mixed with two ounces of anhydrous ether were perfectly decomposed in six hours. On opening the digester, a very small quantity only of gas escaped,—the whole of the iodide having been transformed into zincmethyl. On heating the apparatus in an oil-bath, the distillation began at 90°, and was complete before the thermometer reached 140° C. Compared with an operation for zincethyl with the same amount of materials, a very large product was obtained. Rectified, it began to boil at 35°; the thermometer gradually rose to 48°, between which point and 51° nearly the whole of the remaining and larger portion of the liquid came over. The product obtained between the two latter temperatures possessed the intolerable odour of zincmethyl, was spontaneously inflammable to the last degree, and, in short, had all the properties of zincmethyl; on analysis, however, it was found to contain considerably more carbon and hydrogen than is required by the formula C_2H_3Zn, whilst the last portion of the distillate, boiling between 51° and 57°, yielded results on combustion agreeing closely with the formula

$$2\left(\left.\begin{matrix}C_2H_3\\C_2H_3\end{matrix}\right\}Zn_2\right)+\left.\begin{matrix}C_4H_5\\C_4H_5\end{matrix}\right\}O_2.$$

·2380 grm. gave ·3174 grm. carbonic acid and ·1711 grm. water.

	Calculated.		Found.
C_8 . .	48	36·35	36·37
H_{11} . .	11	8·33	7·99
Zn_2 . .	65·04	49·26	
O . .	8	6·06	
	132·04	100·00	

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* Philosophical Transactions for 1855, p. 260.

A determination of the specific gravity of the vapour of this body gave the following results:—

Weight of substance	·1808 grm.
Observed volume of vapour . . .	67·0 cub. cent.
Temperature of oil-bath	124°·5 C.
Height of barometer	756·0 mm.
Height of inner column of mercury	38·0 mm.
Height of column of oil	275·0 mm.
Specific gravity of vapour	=3·1215

This number does not correspond with that which ought to have been obtained with a *compound* of the above formula, unless we adopt the exceedingly improbable assumption that it contains two volumes of zincmethyl vapour united with one volume of ether vapour *without* condensation. On the other hand, it accords closely with the specific gravity of the vapour of a *mixture* of zincmethyl and ether in the above proportions, as seen from the following calculation:—

Four volumes of zincmethyl vapour	6·5672
Two volumes of ether vapour	2·5567
	3)9·1239
Two volumes of mixed vapour	3·0413
Found	3·1215

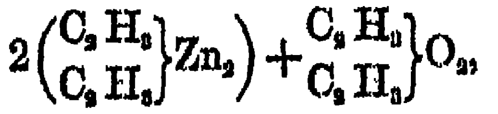
Without at present offering any decided opinion as to the nature of this body, I may state that in repeated operations with large quantities of materials, I have entirely failed in obtaining pure zincmethyl by acting with zinc upon a mixture of ether and iodide of methyl.

Repeated attempts to produce pure zincmethyl from zinc and iodide of methyl alone in the copper digester were also unsuccessful, although this method generally succeeds in small glass tubes. Iodide of methyl is not attacked by zinc at 100° C., even after long digestion; an admixture of 3 or 4 per cent. of ether is, however, quite sufficient to determine the action at this temperature; but with so small a proportion of ether the whole of the iodide is never decomposed, and it is impossible to effectually separate either this or the ether from the zincmethyl produced. When the digester containing zinc and pure iodide of methyl is heated to any temperature from 120° to 200° C., the iodide is decomposed; but at the lower temperatures the decomposition is never complete, whilst at the higher ones little or no zincmethyl is produced, the methyl being resolved into gases. This anomaly in the results obtained from the same materials heated in a copper digester and in glass tubes, is doubtless due to the difference in the conditions obtaining in the two cases. In a glass tube half-immersed in a heated oil-bath, a distillation of the internal liquid is constantly going on, the vapour formed in the lower portion of the tube being condensed in the upper part exposed to the cooling influence of the air, and flowing over an extensive surface of zinc in its descent. But in a digester of thick copper, the different parts of the vessel are, owing to the high conductivity of the metal, maintained at so uniform a temperature as to prevent any such distillation and circulation of the liquid from taking place.

Regarding the body described above as a mere mixture of zincmethyl and ether, incapable of being separated on account of the close proximity of their boiling-points, I anticipated a more successful result by mixing the iodide of methyl with methylic ether instead of vinic ether. As methylic ether boils, according to BERTHELOT, at −21° C., it was thought that no such difficulty of separation could arise; the bodies employed would then, in fact, be exactly homologous with those so successfully used in the preparation of pure zincethyl on the large scale. It was found that iodide of methyl, mixed with oxide of methyl, was readily attacked by zinc, even at the temperature of the water-bath; the following experiment was therefore made.

About three ounces by measure of iodide of methyl were placed in the copper digester with the usual quantity of zinc; oxide of methyl, well dried by passing over chloride of calcium, was forced into the digester by a compression pump[*] until it was thought that the volume of liquified ether would be about equal to that of the iodide employed. The digester was then heated in a water-bath for three days. The product obtained on distillation was very rich in zincmethyl; it contained, however, traces of undecomposed iodide of methyl; but this was easily got rid of by the addition of a few grains of sodium, which, forming sodiummethyl, reacted upon the iodide of methyl in the manner I have already pointed out[†].

On rectification, the greater portion of the liquid thus obtained distilled at 43°, a small residuum only coming over between 43° and 48°. Both these portions yielded on analysis the same results, viz. 29·59—29·54 per cent. of carbon, and 7·11—7·10 per cent. of hydrogen. These numbers do not agree with those required for zincmethyl (viz. 25·25 carbon and 6·31 hydrogen); they approach, however, to those which would be yielded by two equivalents of zincmethyl and one equivalent of oxide of methyl,



$$2\left(\left.\begin{matrix}C_2H_3\\C_2H_3\end{matrix}\right\}Zn_2\right)+\left.\begin{matrix}C_2H_3\\C_2H_3\end{matrix}\right\}O_2,$$

which require 30·49 per cent. of carbon and 7·62 per cent. of hydrogen. This result is therefore exactly homologous with that obtained by the admixture of vinic ether with the iodide of methyl.

In conclusion, after an expenditure of many pounds of iodide of methyl, I have been unable to obtain even the smallest quantity of pure zincmethyl by the use of a copper digester. On the other hand, in ethereal solution, a much larger product is obtained than in the corresponding preparation of zincethyl, owing, no doubt, to the lower temperature at which the crystalline compound of zincmethyl and iodide of zinc is broken up.

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[*] Philosophical Transactions, vol. cxlvii. p. 61.

[†] Proceedings of the Royal Society, vol. ix. p. 345.

I have, in fact, ascertained that a considerable amount of the total product of zincethyl, obtained in an operation in the copper digester, is decomposed during the subsequent distillation,—one of the products of decomposition being metallic zinc in a finely divided state, which is always found in large quantity interspersed throughout the iodide of zinc left in the digester. The temperature required to expel the ethereal solution of zincmethyl from the digester is less than 140° C., whilst zincethyl cannot be wholly expelled below 190°, or even 200° C.

XXI. *On the Isolation of the Organo-Metals, Mercuric, Stannic, and Plumbic Ethyls; and Observations on some of their Derivatives.*—Second Memoir.

By GEORGE BOWDLER BUCKTON, *Esq.*, *F.R.S.*, *F.L.S.*, *F.C.S.*

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SEVERAL years have elapsed since BUNSEN brought before the notice of chemists the interesting substance known as kakodyl. His elaborate memoir indicated, for the first time, the existence of a class of bodies now recognized as the organo-metals, the members of which, through the labours of FRANKLAND, LÖWIG, WÖHLER and others, have proved to be very numerous.

The series indeed ranges over a wide field, and on the one hand comprises organo-combinations with electro-negative metals, such as arsenic and antimony, and on the other hand embraces elements nearly allied to the noble metals, such as mercury and lead.

Mr. WANKLYN's recent discovery of sodiumethyl shows also that the members of this class extend to the extreme limits of the electro-positive metals.

In a paper which the Royal Society has honoured me by publishing[*], I described some preliminary experiments, undertaken with reference to testing the analogies of some of the organo-metals containing mercury, lead, and tin. At the same time more than one method was shown by which their isolation could be effected. In the present memoir I have endeavoured to fill up the sketch then offered, and have added some considerations, supported by experiment, which it is hoped may prove of value in the interpretation of some of the more complex and less known individuals of the group.

Mercuric Ethyl.

Action of Zincethyl on Iodide of Mercurous Ethyl.—Although diffused daylight causes the decomposition of iodide of ethyl in presence of mercury, the action is slow, and requires many days' exposure before any considerable quantity of iodide of mercurous ethyl is formed. On this account attempts were made to obtain the salt by employing strong heat in sealed tubes.

The digestion was first made at a temperature of 130° C., and continued for four hours; but an examination proved that little or no change had taken place in the materials. The same tubes were afterwards heated for twelve hours from 160° to 170°, at the expiration of which time, much iodide of mercury had formed, in fine scarlet crystals. On cutting the tubes the internal pressure of gas was considerable, but no notable quantity of the required iodide was present.

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[*] Philosophical Transactions for 1858, p. 163.

From these experiments light would appear to be an almost necessary agent in the reaction.

It has been already shown that the radical mercuric methyl may be prepared through the reducing action of cyanide of potassium, the change being represented by the equation

$$Hg_2\,C_2\,H_3\,I+K\,Cy=HgC_2\,H_3+KI+Hg+Cy.$$

The cyanogen principally occurs in the form of paracyanogen.

If the same process be adopted towards the corresponding ethyl compound, the quantity obtained is very trifling; almost the whole of the ethyl is disintegrated by the high temperature into gaseous products. A far more efficient and generally applicable method presented itself, from a consideration of the powerful and well-marked reactions of zincethyl on water, and on hydrochloric acid. There seemed to be well-grounded reasons for believing that interesting decompositions would result from the contact of zincethyl with various iodides and chlorides, of both the organo and true metals.

The radical mercuric ethyl may be very conveniently formed by introducing zincethyl into a retort, into which a slow stream of coal-gas is driven through the beak. Dry and powdered iodide of mercurous ethyl is then added, in small quantities at a time, through the neck, and incorporated by stirring. As soon as the zincethyl is saturated, which may be known by the stirring rod moistened with it ceasing to fume on exposure to the air, distillation may be commenced, and continued until gases alone pass over.

The product consists of a heavy liquid, mixed generally with a little ether. It is well again to distil with a slight excess of zincethyl, so as to ensure entire decomposition of the iodide. If this be neglected, the purification of the radical by rectification will be almost impossible.

The distillate should be washed with dilute hydrochloric acid, to remove oxide of zinc, and afterwards well agitated with water. After the ether has been removed by rectification, a liquid remains which possesses a boiling-point between 158° and 160° C.

A portion thus prepared was analysed, and gave the following composition:—

I. 0·5305 grm., burnt with oxide of copper, gave 0·3630 grm. carbonic acid and 0·1780 grm. water.

II. 1·0457 grm., burnt with oxide of copper, gave 0·8078 grm. mercury.

These numbers very closely point to the formula

$$Hg\,C_4\,H_5.$$

On comparing experiment with theory, we have in 100 parts—

	Theory.		Experiment.
1 equiv. of Mercury	100	77·53	77·24
4 equivs. of Carbon	24	18·60	18·66
5 equivs. of Hydrogen	5	3·87	3·72
	129	100·00	

Mercuric ethyl is a perfectly colourless liquid, as limpid as water, and possessed of a faint ethereal odour. Its specific gravity is 2·46. It burns with a smoky flame, more luminous than that of mercuric methyl, and disengages at the same time much mercurial vapour. It is very insoluble in water, less so in alcohol, but readily soluble in ether.

Towards concentrated sulphuric and hydrochloric acids, mercuric ethyl follows the deportment of its homologue, mercuric methyl. When the action is assisted by a gentle heat, hydride of ethyl is disengaged, and the crystalline sulphate or chloride of mercurous ethyl is produced in colourless plates. The radical bursts into spontaneous ignition when poured into a jar of chlorine gas, and the mercury is principally converted into chloride. With bromine and iodine also the action is violent; but if the action be conducted under water, ethyl gas or its products of decomposition are eliminated.

With a freezing mixture to moderate the reaction, the nascent ethyl gas unites with excess of bromine; and by distillation, bromide of ethyl may be obtained.

These changes may be represented by the equations—

$$4(\underbrace{Hg\,C_4H_5)}_{\text{Mercuric ethyl.}}+H_2S_2O_8=2\underbrace{(Hg_2C_4H_5)SO_4}_{\text{Sulphate of mercurous ethyl.}}+2(\underbrace{C_4H_5H}_{\text{Hydride of ethyl.}})$$

$$2(\underbrace{Hg\,C_4H_5)}_{\text{Mercuric ethyl.}}+2Br=\underbrace{Hg_2C_4H_5Br}_{\text{Bromide of mercurous ethyl.}}+\underbrace{C_4H_5Br}_{\text{Bromide of ethyl.}}$$

If the vapour of mercuric ethyl be heated to about 226°, it is suddenly decomposed with a slight explosion, into mercury and an inflammable gas. This gas has not been analysed; but from reasons deduced from the behaviour of sodium towards this radical, there seems to be little doubt of its being a mixture of ethylene and hydride of ethyl, the results of a splitting up of a double molecule of ethyl.

From this tendency to decomposition at a high temperature, considerable difficulty was experienced in obtaining accurate numbers for the vapour-density of mercuric ethyl. In one experiment the apparatus was shattered by employing heat to fill the glass balloon with liquid, the mixture with air causing a loud detonation. This difficulty was easily overcome by inverting and dipping the mouth of the balloon in liquid, partially exhausting the air, and then causing the liquid to rise by atmospheric pressure. It is also necessary to draw the neck of the balloon very fine; otherwise the heat during sealing will infallibly cause a decomposition throughout the whole of the vapour.

	Grammes.	Pressure.	Temperature.
Weight of globe filled with vapour .	16·0895	767·60 mm.	194° C.
Weight of globe filled with air . .	15·3530	766·48 mm.	14° C.
P=	0·7365		

Capacity of the globe =V=118·75 cubic centimetres.

Volume of mercury entering the globe=117·25 cubic centimetres.

Difference=v= 1·50

n_t=weight of one cubic centimetre of air at 14° C.=0·001230.

$n_{t'}$=weight of one cubic centimetre of air at 194° C.=0·000755.

By placing these numerical values in MÜLLER's formula, we have

$$\left.\begin{matrix}\text{Specific gravity}\\ \text{of the vapour}\end{matrix}\right\}=\frac{P+Vm_{\prime}}{(V-v)n_{\prime}}=\frac{0{\cdot}88256}{0{\cdot}08852}=9{\cdot}97.$$

On dividing the equivalent weight, represented by the formula $Hg\,C_4\,H_5$, by the experimental density, we obtain the number $\frac{129}{9{\cdot}97}=12{\cdot}94$, which points sufficiently near to a condensation to two volumes. From the equivalent weight divided by the more accurate number 14·46, we find the theoretical density of mercuric ethyl to be $\frac{129}{14{\cdot}46}=8{\cdot}92$.

The vapour-volume, therefore, coincides with that of zincethyl; and accordingly, to bring it into correspondence with four volumes, the formula should be doubled. If we consider the great difference in the two boiling-points of mercuric methyl, which equals 96°, and mercuric ethyl, which equals 160°, it appears more probable that the increment of 64° represents a greater difference in composition than is represented by one term, $C_2\,H_2$.

Experiment perhaps has not yet satisfied this interesting question of the doubled formula. It was anticipated that a proof of its correctness would be shown by the reaction of zincethyl on iodide of mercurous methyl. The two bodies unite readily, and a rise in temperature indicates chemical action; but distillation of the product yields a liquid possessing no fixed boiling-point, the thermometer ranging between 60° and 160° C.

By fractional distillation a partial separation was effected into the two radicals, mercuric methyl and mercuric ethyl.

The anticipated and true decompositions were respectively,—

$$2\left\{\begin{matrix}Hg\,C_2\,H_3\\ Hg\,I\end{matrix}\right\}+\left\{\begin{matrix}Zn\,C_4\,H_5\\ Zn\,C_4\,H_5\end{matrix}\right\}=2\left\{\begin{matrix}Hg\,C_2\,H_3\\ Hg\,C_4\,H_5\end{matrix}\right\}+2Zn\,I.$$

$$2\left\{\begin{matrix}Hg\,C_2\,H_3\\ Hg\,I\end{matrix}\right\}+\left\{\begin{matrix}Zn\,C_4\,H_5\\ Zn\,C_4\,H_5\end{matrix}\right\}=2Hg\,C_2\,H_3+2Hg\,C_4\,H_5+2Zn\,I.$$

Notwithstanding the failure of proof in this experiment, the existence of a radical compounded of two molecules of mercury, one of methyl, and one of ethyl, cannot be considered as definitively settled in a negative.

Action of Sodium on Mercuric Ethyl.

The electro-negative character of the group $C_{2n}\,H_{2n+1}$, in the class of organo-metals to which zincethyl belongs, may now perhaps be considered as established; yet there would appear to be some importance in knowing the deportment of sodium towards ethyl, when combined with a metal like mercury. Mr. WANKLYN has shown how sodium, being more electro-positive than zinc, displaces that metal from zincethyl. If mercury should be positive to ethyl in a similar organo-metal, the sodium in like manner should displace the mercury and form sodiumethyl.

Sodium well freed from rock oil, was cut with clean surfaces, and introduced together with mercuric ethyl into a glass tube. As scarcely any action was observed, the tube was sealed without heating the contents. After the lapse of five or six hours, a voluminous grey and spongy-looking mass was formed, which, from circumstances happening, was not immediately examined. The tube was left undisturbed in a cupboard for a week, at the expiration of which time, it suddenly burst and scattered the contents, which spontaneously ignited, and burnt so strongly, that some neighbouring woodwork was set on fire.

In another experiment one part of sodium was placed in a tube with five parts of mercuric ethyl. The mouth of the tube was drawn at the blowpipe into a capillary tube, the point of which was placed under a jar over mercury. If the temperature be low, gas is given off very slowly, and appears to cease when the grey sponge is formed. If, however, a gentle heat be applied, a powerful rush of gas is produced, which again slackens on cooling the apparatus. The first portion of gas was allowed to escape, as being mixed with atmospheric air. It did not fume, but gave a white flame when ignited. Bottles were filled with the gas both at the commencement and towards the close of the disengagement, when the action became more moderate. On a supposition that the gas might contain ethylene, portions were subjected, in a graduated tube, to fuming sulphuric acid, introduced by means of a coke ball. The acid fumes were afterwards removed by a ball of fused potash.

	First sample.		Second sample.
	I.	II.	III.
Corrected volume of gas	74·14	74·24	73·24
After absorption by $H\,SO_4$ &c. . .	48·63	48·63	10·56
Volume of ethylene	25·51	25·61	62·68
Gas transferred.			
Corrected volume	45·19	42·23	
+Oxygen taken	238·71	260·23	
+Air taken	411·10	416·80	
After explosion	292·60	308·10	
After absorption by KO	206·38	225·98	
Total of oxygen taken	229·66	250·80	
Total of oxygen burnt	159·53	148·59	
Carbonic acid formed	86·22	82·12	

One volume of gas therefore consumed in first experiment . . . . $\left.\right\}\frac{159·53}{45·19}=3·53$ vols. of oxygen,

and produced . . . . $\left\{\frac{86·22}{45·19}=1·91\right.$ vol. of carbonic acid.

One volume of gas also consumed in second experiment . . . . $\left.\right\}\frac{148·59}{42·23}=3·51$ vols. of oxygen,

and produced . . . . $\left\{\frac{82·12}{42·23}=1·96\right.$ vol. of carbonic acid.

These eudiometric results indicate that this gas must be either methyl or its isomer. hydride of ethyl, both of which satisfy the conditions of the above experiments. They may be distinguished, however, by their different behaviour towards chlorine gas. The gas under examination, when exposed to diffused daylight, absorbed a large quantity of chlorine, and after some hours crystals of sesquichloride of carbon made their appearance, thus affording a clear proof of the gas being hydride of ethyl.

A comparison of the first and last samples of gas taken for analysis, shows that the ratio of the gases ethylene and hydride of ethyl is not constant at all phases of the experiment.

	First sample.		Second sample.
Ethylene	0·655	0·654	0·856
Hydride of ethyl . . .	0·345	0·346	0·144
1 volume	1·000	1·000	1·000

No doubt these gases are produced by the decomposition of a double molecule of ethyl; yet from the circumstance that little or no gas is disengaged at ordinary temperatures after the spongy matter has been formed, it is probable that the grey body is a double compound with sodiumethyl, or at all events contains that body. All attempts to discover the real nature of this grey material have as yet failed. Its component parts are evidently in a very unstable condition of equilibrium; for the body is liable to explosion, and often without an assignable cause. In one experiment, a loud detonation followed the dropping of a few lumps of the substance into alcohol. On repeating the experiment, the entire contents of the tube, containing the grey matter, exploded violently, and, as it were, sympathetically, leaving behind scarcely a vestige of the glass. This phenomenon was perhaps caused by the detonation of that portion of the body which touched the alcohol, transmitting the shock through the air to the remainder.

The grey material loses its explosive properties after it has been cautiously, but rather strongly heated; yet even then, after it is cold, it becomes incandescent on freely admitting air to its surface.

From these experiments it is concluded that ethyl, and doubtless methyl, in these organo-metals is still negative to mercury, and therefore we should not suppose, with Mr. **WANKLYN**, that mercury, copper, and platinum would displace ethyl from sodiumethyl. The foregoing reactions would rather tend to show that sodiumethyl is formed from sodium and mercuric ethyl, and it is only by heat that that body is broken up,

$$\mathrm{Hg\,C_4H_5+Na=Na\,C_4H_5+Hg};$$

and by heat,

$$\mathrm{2Na\,C_4H_5+2Hg=2Hg\,Na+C_4H_4+C_4H_5\,H}.$$

The mercury is not supposed to be active in the last decomposition, or in any way to determine it; but being present, it simply forms an amalgam with the liberated sodium.

Action of Zincethyl on Chloride of Mercury.

Having thus effected the reduction of an organo-metallic salt through zincethyl, it was important to see how far an ordinary iodide or chloride would follow the same deportment. Corrosive sublimate produces much heat when mixed with zincethyl, so much so as to render it necessary to cool the retort by water or other means, to prevent waste of the zincethyl.

After breaking up the cake of chloride of zinc which is formed, the radical may be obtained in considerable quantity by distillation, and washing the product with dilute acid, as before described. Excess of mercuric chloride, however, must be carefully avoided, as chloride of mercurous ethyl would be produced according to the equations,

$$\mathrm{Zn\,C_4\,H_5+Hg\,Cl=Hg\,C_4\,H_5+Zn\,Cl};$$

by addition of mercuric chloride,

$$\mathrm{Hg\,C_4\,H_5+Hg\,Cl=Hg_2\,C_4\,H_5\,Cl}.$$

Although this process gives certain and good results, it will not be found so economical in practice as that previously noticed. Iodide of mercurous ethyl is readily made; and by its employment it is very obvious that a less consumption of zincethyl is required.

Action of Zincethyl on Chlorides of Silver, Copper, and Platinum.

Dry powdered chloride of silver strongly reacts on zincethyl, turning quite black and liberating much gas. The chloride, however, seems to be incapable of decomposing the whole of the zinc radical, even when in excess and heat is applied. The clear liquid was removed by a pipette and treated with water, which extracted simply chloride of zinc, and isolated no silver compound. The black matter in the retort contained a mixture of chloride and metallic silver.

In another experiment anhydrous ether was employed instead of water, under the supposition that a solid silver compound might be present, which was soluble in ether. The only reaction, however, seemed to be,

$$\mathrm{Zn\,C_4\,H_5+Ag\,Cl=Zn\,Cl+Ag+C_4\,H_5}.$$

Results have also been negative with reference to the formation of radicals containing copper or platinum.

Protochloride of copper immediately loses its green colour on mixing with zincethyl, a circumstance very probably due to a loss of traces of water. No combination is effected; and on exposure to the air, the black powder speedily recovers its green hue.

Well-dried protochloride of platinum decomposes zincethyl into chloride of zinc and gaseous matter. The metal is reduced in the form of platinum black.

Stannic Diethyl.

Much of the uncertainty which attaches to the compounds styled by LÖWIG, in his laborious researches, "ethyl-stanethyl," "aceto-stanethyl," &c., is doubtless due to the method he adopted for forming the iodides of stanethyl and stibethyl, from which salts all the bodies described by him appear to have been obtained.

The plan adopted in the present memoir has proceeded rather in an opposite direction,—the wish being, first to construct the pure radical, and then to form from it the various salts, according to the mode usually adopted in the case of an ordinary metal.

The iodide of stanethyl employed was prepared by Dr. FRANKLAND's process. Sealed tubes containing iodide of ethyl and excess of tinfoil were heated in a strong iron digester to 150°—160° C. After about ten hours' action, the contents of the tubes were transferred to a retort and heated to 120°, to remove all traces of undecomposed iodide of ethyl.

Action of Zincethyl on Iodide of Stanethyl.

If the above-mentioned iodide, in a melted state, be added to a quantity of zincethyl in a retort surrounded by cold water, a solid mass is formed, which is again in great part dissolved by agitation and aid of the heat generated. Distillation may be then commenced, and carried on until gases alone come over. The dry residue consists only of chloride of zinc and a little protiodide of tin.

It will not be found economical to saturate the zincethyl at one operation, on account of the caking of the materials, which renders the fracture of the retort very probable; independently of which, the strong heat then necessary partly decomposes the radical when formed. The distillate last obtained should, however, always contain a slight excess of zincethyl; otherwise there is no certainty of removing traces of an iodide, the nature of which will be presently noticed, and which imparts a pungent odour to the radical. By agitation with dilute acid and water, two strata of liquids are produced, the lower of which was fractioned with the thermometer.

At first a little ether appeared, after which the boiling-point ran up quickly to 170°, between which and 180° by far the larger quantity distilled over. The fraction taken between 176° and 180° C. was considered pure, and set apart for examination. It was very combustible, and gave, when ignited, a lurid flame with fine blue edges, and disengaged at the same time a thick cloud of tin oxide.

I. 0·2885 grm. of substance gave, when burnt, 0·4265 grm. carbonic acid and 0·2213 grm. water.

II. 0·3117 grm. of substance gave 0·4672 grm. carbonic acid and 0·2455 grm. water.

These numbers correspond to the per-centage composition—

	I.	II.
Carbon	40·31	40·81
Hydrogen	8·52	8·72

and also to the formula

$$\mathrm{Sn\,C_8\,H_{10}}, \text{ or } \mathrm{Sn(C_4\,H_5)_2}.$$

The numbers required by theory and obtained by experiment are—

	Theory.		Mean of experiment.
1 equiv. of Tin . .	59	50·43	——
8 equivs. of Carbon .	48	41·02	40·56
10 equivs. of Hydrogen	10	8·55	8·62
	117	100·00	

This radical differs therefore from stannic ethyl, in that it contains double its quantity of ethyl. Accordingly I propose to call it stannic diethyl.

It is a perfectly colourless, limpid, and almost inodorous liquid, very stable, and having a specific gravity of 1·192. It is very insoluble in water, and but slightly so in alcohol. Ether takes it up freely. Cold concentrated acids do not immediately attack stannic diethyl; but when heated, they disengage gases and form salts, which differ in constitution according to the prolongation of the action. Hydrochloric acid converts the substance into an uncrystallizable chloride, insoluble in water, and possessed of an exceedingly pungent and irritating odour. It is only after long digestion that a crystalline body is formed.

With iodine and bromine the action is very violent, and produces hissing as when one drops hot metal into water. Explosion ensues if the two bodies are mixed in any quantity.

An experiment was conducted under water, the bromine being added to the stannic diethyl in such quantity that a slight but permanent red tint showed that it was in excess. An oily liquid was obtained which possessed the characteristic odour, but which, when exposed to the air, shot into a mass of crystals. These were pressed in paper, and placed for some hours under the air-pump before they were analysed.

0·3405 grm. of substance gave 0·3770 grm. bromide of silver, which nearly accords with the number required for bromide of stanethyl, $Sn\,C_4\,H_5\,Br$.

	Theory.	Experiment.
Bromine . .	47·07	46·51

The powerful odour of this substance, however, left some reason for doubt as to its exact composition. The salts of stanethyl have been, at different times, stated by FRANKLAND and LÖWIG as odorous and inodorous. Further experiment, therefore, appeared desirable before definitively fixing the formula.

The remaining portion of the bromide was dissolved in weak alcohol, and aqueous potash added. An abundant white precipitate was formed, which, after being well washed, was converted by acid into a chloride. By gentle evaporation, fine hard and almost inodorous crystals were obtained, which differed from the bromide in being easily and entirely soluble in water. The irritating odorous substance was by this treatment entirely transferred to the alkaline solution. This chloride was dried over sulphuric acid and analysed.

0·5070 grm. substance gave 0·3600 grm. carbonic acid and 0·1977 grm. water. This result places beyond a doubt the composition of the salt, and moreover confirms the last analysis of the bromide.

Chloride of stanethyl, $Sn\,C_4\,H_5\,Cl$, requires—

	Theory.		Experiment.
1 equiv. of Tin . . .	59·0	47·77	——
4 equivs. of Carbon .	24·0	19·43	19·34
5 equivs. of Hydrogen	5·0	4·05	4·32
1 equiv. of Chlorine .	35·5	28·75	——
	123·5	100·00	

It is immaterial whether the bromine be added to the radical, or the radical to the bromine. In both cases the salt of the same radical is formed. If, however, bromine be placed under water, and the vessel be carefully cooled, evolution of gas may be almost entirely prevented. In this case ethyl in its nascent state is presented to free bromine, and bromide of ethyl is produced, which may be readily detected by distillation.

The reaction is in strict accordance with that set forth when describing the mercuric radicals.

STRECKER has lately shown*, with much probability, that some of the organo-stannic bodies prepared by LÖWIG may be referred to double salts of comparatively simple formula, the analogues of which may be found in the inorganic oxyiodides and oxychlorides of tin. From the method employed by LÖWIG in forming the stanethyls, viz. by distilling iodide of ethyl with an alloy of tin and sodium disseminated through sand, it may be easily supposed that a partial reduction only of the iodides of stanethyl is effected.

The following experiments prove that various bodies are formed when zincethyl and bichloride of tin react on each other in different proportions, their constitution varying with their more or less complete reduction by the zincethyl. Bichloride of tin produces great heat when dropped into zincethyl. If the tin-salt be in excess, an impure chloride of stanethyl is formed; if the zincethyl be in excess, the radical stannic diethyl is formed; whilst, lastly, if the zincethyl be present in quantity insufficient for the last reaction, that is, less than sufficient to remove the whole of the chlorine, the chief product is an oily body, which possesses the pungent and irritating odour before alluded to. This pungency, which produces continued sneezing, entirely disappears on adding zincethyl to saturation. It is insoluble in water, inflammable, and is clearly a stanethyl chloride. In its chief properties and behaviour it accords with the liquid described by LÖWIG, and named by him "chloride of methylo-stanethyl," the formula of which is $Sn_3\,C_{15}\,H_{15}\,Cl$.

Experiments were then commenced under an impression that the substance in question might prove to be a double compound of stannic diethyl and chloride of stannic ethyl, or, failing that, the liquid might result from a union of three molecules of stannic diethyl with one of bichloride of tin.

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* LIEBIG'S Annalen, cv. p. 306.

One of the following expressions, it was thought, might show the synthesis of the chloride:—

$$\mathrm{Sn(C_4\,H_5)_2+Sn\,C_4\,H_5\,Cl=Sn_2\,(C_4\,H_5)_3\,Cl},$$

or else

$$\mathrm{3Sn(C_4\,H_5)_2+Sn\,Cl_2=2[Sn_2\,(C_4\,H_5)_3\,Cl]}.$$

To test the first equation, pure inodorous crystals of chloride of stanethyl were added to an excess of stannic diethyl, and kept a short time at the boiling-point. Cold water was then added, and the whole was again raised to ebullition. The aqueous solution was then poured off, and solution of potash added, when an abundant precipitate of oxide of stanethyl showed that no double salt had been effected with the chloride of stanethyl. The portion insoluble in water proved to be the unchanged radical, and did not exhibit the characteristic odour of the oily chloride, which thus could have been readily detected. This negative answer to the first equation showed that the substance was no simple mixture, but a definite compound.

A more satisfactory result was obtained when bichloride of tin was made to act on stannic diethyl, in the proportion of three parts of the former to four parts of the latter. The mixture produced a heat considerably greater than could be borne by the hand, and the liquid partook of a clear brown shade. When exposed to the air it gave slight fumes of vapour, causing much lachrymation.

When distilled, ebullition commenced at about 200°; but no fixed boiling-point could be observed, the thermometer slowly rising up to 220°, at which temperature the whole had volatilized, leaving a mere trace of solid matter in the retort.

The distillate was divided into two portions. That below 210° C. was of an oily consistence; but above that point the portion solidified, on cooling, into a mass of crystals. These were strongly pressed between bibulous paper, and analysed:—

I. 0·4170 grm. of substance gave 0·3440 grm. carbonic acid and 0·1825 grm. water.

II. 0·5138 grm. of substance gave 0·5035 grm. chloride of silver.

These numbers lead most nearly to a formula

$$\mathrm{Sn_7\,C_{32}\,H_{40}\,Cl_6},$$

which at first seems rather improbable, but which becomes perhaps more intelligible if grouped as a double salt.

$$\mathrm{5Sn\,C_4\,H_5\,Cl,\ Sn_2(C_4\,H_5)_3\,Cl}$$

requires the following theoretical values:—

	Theory.		Experiment.
7 equivs. of Tin . . .	413	48·15	——
32 equivs. of Carbon .	192	22·37	24·40
40 equivs. of Hydrogen .	40	4·66	4·84
6 equivs. of Chlorine .	213	24·82	24·20
	858	100·00	

Although experiment gives too large a proportion of both carbon and hydrogen for the ascribed formulæ, the salt still less accords with chloride of stanethyl,—from which also the crystals differ, both in form and a sort of fatty consistence.

From the circumstance that no steady boiling-point could be obtained from that portion of the distillate which came over below 210° C., and which refused to crystallize, it appeared to be no pure chemical compound, but simply a mixture of an oily body and the salt last analysed. This belief was confirmed by noticing its deportment towards aqueous potash, which precipitated a considerable quantity of white oxide of stanethyl. The solution, on the other hand, with hydrochloric acid liberated the above-mentioned odorous oil. To isolate this last, the whole was mixed in a mortar with rather strong aqueous potash, and the magma washed in a funnel plugged with asbestos. The washing was continued with weak potash until the rinsings were nearly inodorous.

The liquid was then distilled, a good condensing apparatus being attached. At first white pungent fumes passed over with the aqueous vapour, and collected in the receiver in the form of oily drops, possessed of great causticity and powerful basic properties. This product, in fact, is the aqueous solution of the oxide of an ethylated base containing tin, from which definite compounds may be obtained with sulphuric, hydrochloric, and other acids.

As the sulphate is but little soluble in water, it seemed well adapted for fixing the constituents of the base. A portion of the aqueous solution of the oxide was therefore nearly neutralized with sulphuric acid, and heated to about 80° C., when an opake colourless and crystalline salt appeared, which was dried *in vacuo* over sulphuric acid, and analysed.

I. 0·4061 grm. of substance gave 0·4790 grm. carbonic acid and 0·2470 grm. water.

II. 0·5080 grm. of substance gave 0·5223 grm. carbonic acid and 0·2716 grm. water.

III. 0·5529 grm. of substance gave 0·2593 grm. sulphate of barium.

Corresponding to

	I.	II.	III.
Carbon	28·01	28·03	—
Hydrogen	5·88	5·94	—
Sulphur	—	—	6·44

These results point to the formula

$$Sn_2 C_{12} H_{15} SO_4, \text{ or } Sn_2 (C_4 H_5)_3 SO_4,$$

which requires

	Theory.		Mean of experiment.
2 equivs. of Tin	118	46·64	—
12 equivs. of Carbon	72	28·45	28·02
15 equivs. of Hydrogen	15	5·92	5·91
1 equiv. of Sulphur	16	6·34	6·44
4 equivs. of Oxygen	32	12·65	—
	253	100·00	

This substance is identical with the sulphate of Löwig's base, methylo-stanethyl. I would propose a name which has a reference to the true constitution of the salts, and call it sulphate of distanno-triethyl.

This sulphate differs in many respects from the ordinary sulphate of stanethyl. The former is inflammable, and crystallizes either in fine needles or shining grains, which under a low magnifying power present the form of well-defined octahedra. Sulphate of stanethyl, on the other hand, is not combustible, and appears under the microscope in small, somewhat roundish irregular plates. The sulphate of distanno-triethyl, moreover, shows, in a remarkable manner, the unusual character of greater insolubility in hot than in cold water. A clear, cold, saturated solution becomes semisolid when raised to a temperature somewhat short of boiling. The salt is very volatile, and attacks the nose vigorously.

I have not succeeded in isolating the corresponding radical, most probably from the reason that its salts exchange with zincethyl their acid molecule for an additional ethyl atom, and thus are converted into two equivalents of stannic diethyl. Although the affinities of these sesquiethylated compounds are remarkably strong, resisting, as we have seen, the action of potash, there may be some hesitation in assuming with certainty the existence of the radical itself. All the salts may be viewed simply as double compounds; and if future investigation proves them to be such, no exceptional case would arise against viewing them as formed on the type of the tin oxides.

Sesquioxide of tin does not appear to be known either as an artificial or natural product. The bromide and iodide of the sesquiethylated tin base is always one of the products of the action of those elements upon stannic diethyl.

Oxide of distanno-triethyl may be conveniently formed from the sulphate by boiling the salt in strong aqueous potash. If the latter be not in much excess, the oxide is liberated as an oily body, heavier than water, and which solidifies when cold. It is moderately soluble in water, and may be distilled, as has been seen, from aqueous potash. It has a powerful caustic taste, and when left in contact with the skin, produces painful blisters. The aqueous solution precipitates the oxide, on addition of common salt. These properties confirm Löwig's experiments, and very generally accord with the properties of the oxide of methylo-stanethyl.

Occasionally it happens that, from an irregularity in the temperature employed for causing iodide of ethyl to act upon metallic tin, a yellow oily body is formed, and that in sufficient quantity to prevent the crystallization of the iodide of stanethyl. This body has a penetrating odour, and appears to be identical with that obtained by Riche and Cahours. If this impure iodide be treated with aqueous potash, it will be remarked that a comparatively small quantity of oxide of stanethyl falls. The aqueous solution, on the other hand, by distillation yields an alkaline liquid, which furnishes with sulphuric acid a salt with all the properties of sulphate of distanno-triethyl. No doubt therefore remains that the oil is the corresponding iodide of the formula $Sn_2(C_4H_5)_3I$. By separation of biniodide of tin, which is to be found as one of the products of the reaction, the occurrence of this iodide may be explained,

$$3Sn+3C_4H_5I=Sn_2(C_4H_5)_3I+SnI_2.$$

Action of Zincethyl on Protochloride of Tin.

To obtain this tin-salt perfectly anhydrous, metallic tin was distilled at a gentle heat with corrosive sublimate. The chloride was powdered, and by little and little introduced into a tube containing zincethyl, which almost immediately became of a yellow colour. After agitating the contents of the tube with anhydrous ether, the solution was pipetted off and allowed to evaporate spontaneously. In this manner a small quantity of stanethyl, SnC_4H_5, was obtained; but by far the larger quantity remained combined with zincethyl, from which it could not be separated by water, without causing a deposit of metallic tin at the expense of the radical.

Dr. FRANKLAND has before noticed that this radical cannot be distilled without decomposition into tin and a clear liquid. There can be no reasonable doubt of this body being stannic diethyl.

$$ZnC_4H_5+SnCl=\underbrace{SnC_4H_5+ZnCl}_{\text{Double compound?}}$$

by heat,

$$2SnC_4H_5=Sn(C_4H_5)_2+Sn.$$

This method offers no advantage over that hitherto employed for obtaining the organo-metal, viz. by decomposing the chloride with a strip of zinc.

The several reactions of zincethyl on bichloride of tin may be thus represented, step by step. The tin-salt being added to the zincethyl, we have

$$\begin{aligned}
&\text{I.}\quad SnCl_2+2ZnC_4H_5=Sn(C_4H_5)_2+2ZnCl.\\
&\text{II.}\quad 2SnCl_2+3ZnC_4H_5=Sn_2(C_4H_5)_3Cl+3ZnCl.\\
&\text{III.}\quad SnCl_2+ZnC_4H_5=SnC_4H_5Cl+ZnCl.
\end{aligned}$$

It is worthy of remark how prone the metal tin is to assimilate the double equivalent of ethyl, just as we see ordinary oxide of tin passes into binoxide. Salts both of stanethyl and distanno-triethyl readily convert themselves into stannic diethyl.

Plumbic Diethyl.

A certain relation has been long recognized between the metals silver, lead, and mercury. This similarity extends to their equivalent weights, the constitution of their salts, and various other characters.

Notwithstanding the sluggish action of iodide of ethyl towards lead, even at high temperatures, there seemed to be no reason to doubt success in preparing the ethylated radical by one of the processes already detailed. LÖWIG has indeed obtained through iodide of ethyl and an alloy of sodium and lead, an iodide analogous to a sesquioxide, in which ethyl plays the part of oxygen. The production of this salt appears to be attended with some difficulty. I have tried LÖWIG's process, but have found the action of the alloy very slow, even when brought to the boiling-point of iodide of ethyl. Neither bromide nor iodide of ethyl showed any tendency to combine with granulated lead when exposed to a clear April sun for one week, neither did the bodies unite under pressure and a temperature of 170° C. The bromide in the last case was quite unchanged; but the iodide gave crystalline plates of iodide of lead and gaseous matter.

Action of Zincethyl on Chloride of Lead.

Chloride of lead turns black immediately it comes in contact with zincethyl. It sinks in the liquid, and forms a crust which is so hard that it will be found more convenient to operate in a bottle than in a retort. In the former vessel the chloride may be incorporated with the zincethyl without fear of fracture.

As soon as the chloride of lead ceases to change colour, the vessel is placed in a water-bath and heated to 100° C., after which the liquid, when cold, may be decanted. This liquid fumes slightly in the air from the presence of zincethyl, which cannot be wholly saturated even when much chloride of lead is in excess. A compound of the lead radical and zincethyl is thus obtained, which may be treated with very dilute hydrochloric acid and then washed with water. The mass of chloride of zinc in the first bottle may be made to furnish an additional quantity of the radical, by agitation with anhydrous ether.

Considerable difficulty was experienced in freeing this radical entirely from ether. When distilled, the thermometer rises rapidly to nearly 204°, at about which point the vapour suddenly decomposes with a slight explosion and deposit of metallic lead.

This difficulty was overcome by conducting the distillation under reduced atmospheric pressure, in a simple condenser a sketch of which is annexed. (See p. 433.)

The apparatus was found very conveniently to permit of frequent change of the receiver, without disturbing the tightness of the various joints. When the barometer stood at 30·5 inches, the radical passed over without the least decomposition at a temperature of 152° C., under a pressure of about 7·5 inches of mercury, thus depressing the boiling-point in a remarkable degree.

In filling the bulbs for combustion, the precaution was taken of employing the air-pump; and before proceeding to burn the substance, it was found necessary again to empty the bulbs by exhaustion.

I. 0·3615 grm. of substance gave 0·3928 grm. carbonic acid and 0·2000 grm. water.

II. 0·3116 grm. of substance gave 0·3400 grm. carbonic acid and 0·1775 grm. water.

The composition in 100 parts accordingly is—

	I.	II.
Carbon	29·62	29·74
Hydrogen	6·14	6·32

the formula,

$$Pb\,C_8\,H_{10}, \text{ or } Pb\,(C_4\,H_5)_2$$

	Theory.		Mean of experiment.
1 equiv. of Lead	103·5	64·09	——
8 equivs. of Carbon . . .	48·0	29·72	29·68
10 equivs. of Hydrogen . .	10·0	6·19	6·23
	161·5	100·00	

Plumbic diethyl is a colourless and limped fluid with a faint odour, very like that of the other radicals described. The specific gravity is 1·62. It is soluble in ether, but insoluble in water. It burns brightly, with abundant fumes of oxide of lead. The flame is somewhat differently coloured from that of the tin radical. The latter has an orange flame, with blue edges, whilst the former has a distinct but faint green tinge, with a border less luminous.

It reacts violently on iodine and bromine, so much so as to convert the salts, which doubtless are first formed, into bromide and iodide of lead. When poured into chlorine it bursts, like the tin radical, into flame.

Chloride of Diplumbic Triethyl.

With concentrated hydrochloric acid the organo-metal yields a crystalline salt with evolution of hydride of ethyl, but by this method it is not easy entirely to convert the radical into chloride.

The chloride is more conveniently formed by passing excess of hydrochloric acid gas into a tube containing plumbic diethyl moistened with aqueous acid. After a brisk effervescence, a solid mass is formed, which may be freely exposed to the air until acid fumes cease to rise. From a solution in ether the crystals may be obtained pure by spontaneous evaporation.

I. 0·6400 grm. of substance gave 0·5088 grm. carbonic acid and 0·2640 grm. water.
II. 0·7077 grm. of substance gave 0·3240 grm. chloride of silver.
III. 0·5447 grm. of substance gave 0·4985 grm. sulphate of lead.

The calculations for 100 parts, when contrasted with theory, are—

	Theory.		Experiment.
2 equivs. of Lead	207·0	62·85	62·51
12 equivs. of Carbon	72·0	21·85	21·69
15 equivs. of Hydrogen	15·0	4·55	4·57
1 equiv. of Chlorine	35·5	10·75	11·32
	329·5	100·00	100·09

This salt accordingly is the chloride of diplumbic triethyl, having the formula

$$Pb_2 C_{12} H_{15} Cl, \text{ or } Pb_2 (C_4 H_5)_3 Cl.$$

It may be obtained in long colourless crystals, which have a strong penetrating odour, very similar to that of the corresponding body in the tin series. It fuses between watch-

Fig. 1.

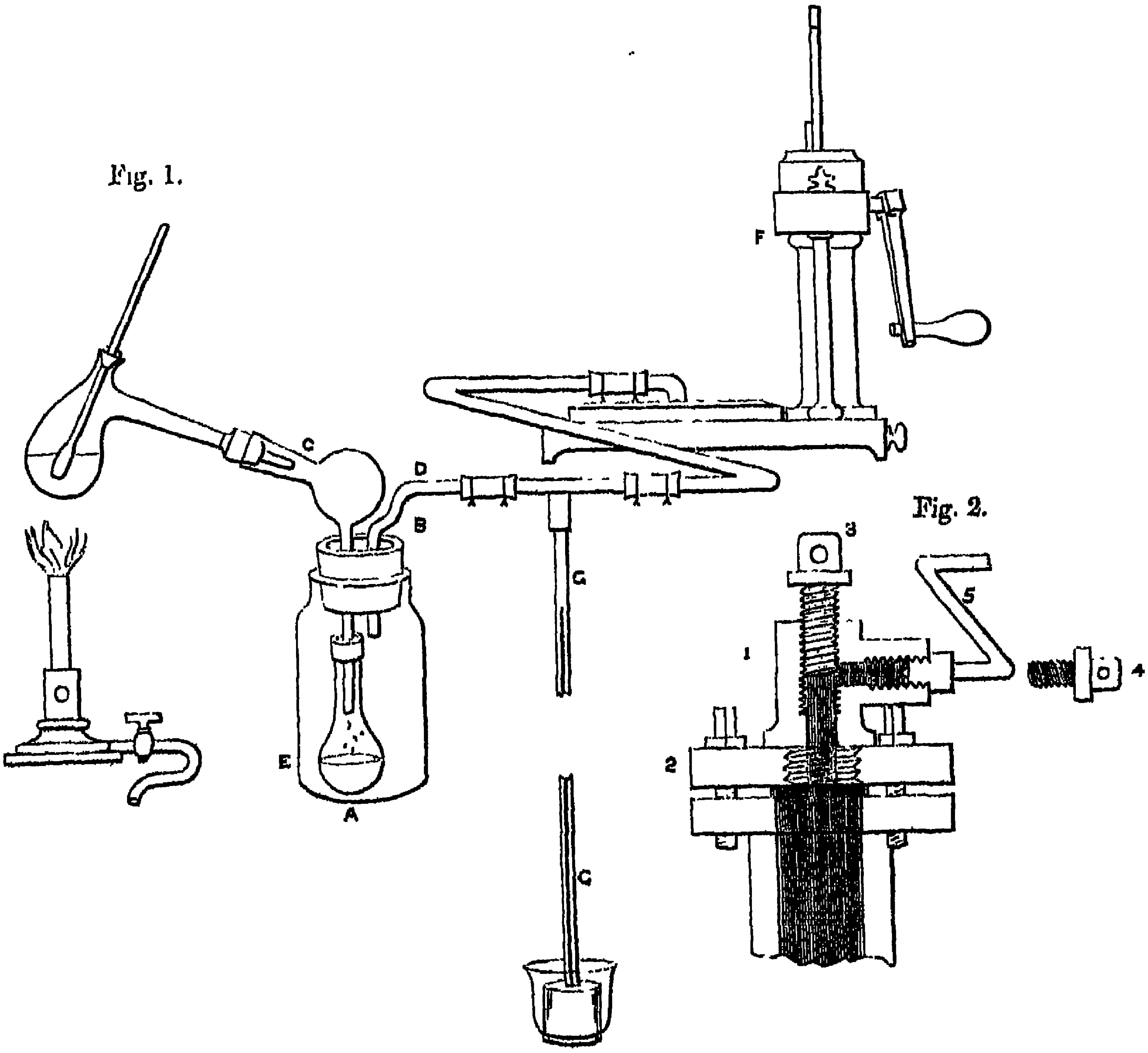

Fig. 2.

Fig. 1—Apparatus for distilling Plumbic Diethyl, &c.

A. A wide-mouthed bottle.

B. A well-ground stopper formed from a piece of stout glass tube, 2 inches in diameter. The stopper is ground rather taper to prevent adhesion.

C A small quilled condenser.

D. Tube connected by caoutchouc with mercurial gauge.

G. Mercurial gauge.

F. Air-pump.

The tubes C and D are cemented into the stopper B by plaster of Paris and a layer of resinous cement to close the pores.

E. Small receiver capable of being changed when needful.

Fig. 2.—Modification of Dr. **Frankland**'s apparatus for preventing loss by frothing when making zincethyl.

1. Wrought-iron head screwed into upper plate of digester 2; 3, 4 screws with nuts and leaden washers for closing apparatus during the formation of zincethyl.

When distillation is to be performed, the screw 4 is removed and the delivery tube 5 attached. 3 is then partly unscrewed; and if frothing takes place, it passes through 5 into the retort without loss.

glasses, and ignites at a comparatively low temperature, with the characteristic lead flame.

When heated with concentrated sulphuric acid, the salt blackens, and is converted into sulphate of lead. After addition of a little nitric acid, this carbonaceous deposit is removed, and the sulphate may be used for a quantitative determination of the lead.

Sulphate of Diplumbic Triethyl

May be formed by adding dilute sulphuric acid to a warm concentrated solution of the chloride. On account of its greater insolubility, it crystallizes out on cooling, in the form of fine asbestos-like needles. These may be purified by solution in alcohol, in which the sulphate is more soluble than in water. From alcohol the crystals may be obtained of considerable size.

0·5055 grm. of substance gave 0·3850 grm. carbonic acid and 0·2035 grm. water, leading to the formula

$$Pb_2C_{12}H_{15}SO_4, \text{ or } Pb_2(C_4H_5)_3SO_4,$$

which requires the following values:—

	Theory.		Experiment.
2 equivs. of Lead	207	60·52	——
12 equivs. of Carbon	72	21·05	20·77
15 equivs. of Hydrogen . . .	15	4·38	4·47
1 equiv. of Sulphur	16	4·64	——
4 equivs. of Oxygen	32	9·41	——
	342	100·00	

It would appear, therefore, that from two molecules of the organo-metal, one equivalent of ethyl only is displaced by the negative chlorine, or the compound group SO_4. Hitherto my attempts to produce a lead base richer in ethyl have failed. Experiment shows that if chlorine be passed over the sesquiethylated chloride, the decomposition proceeds beyond the removal of the additional equivalent of ethyl; in fact the salt enters into combustion and is entirely destroyed. The aqueous solution of the chloride appears also to be unaffected by a stream of chlorine gas. Chloride of lead is not thrown down.

Oxide of Diplumbic Triethyl.

The alkalies do not immediately form precipitates in the aqueous solutions of the salts of this base; but when the solid chloride is gently heated with strong potash, an oily body sinks without liberation of oxide of lead. The same body may be obtained more conveniently through the agency of oxide of silver. The oil which forms on cooling a crystalline mass, is oxide of diplumbic triethyl. It readily volatilizes in white pungent fumes, which provoke sneezing. It strongly blues litmus paper, and attracts carbonic acid from the air with production of a crystalline salt, which is the carbonate of the base. The various sesquiethylated plumbic salts may be readily obtained from both the oxide and carbonate here described.

Doubtless, from the difficulty hitherto experienced in procuring these plumbic salts, no verification of their properties appears to have been made since the appearance of Löwig's researches above alluded to. On this account, perhaps, less apology may be thought necessary for bringing the foregoing remarks before the attention of the Royal Society.

As a broad field yet remains open for inquiry, I purpose resuming the subject of the organo-metals, but in a somewhat different direction.

February 28th, 1859.

XXII. *Further Researches on the Grey Substance of the Spinal Cord.*

By J. LOCKHART CLARKE, *Esq.*, *F.R.S.*

Received June 17,—Read June 17, 1858.

SINCE the appearance of my second series of investigations on the structure of the Spinal Cord, certain modifications in the method I employ have enabled me to carry my inquiries to a still greater extent, and in many respects beyond the limits of what was actually known on this important subject. By this modified method the elementary nerve-tissues undergo less alteration from their natural state; for the most minute nerve-cells and their processes, as well as the finest fibres, are permanently preserved, and display a sharpness of outline unattainable by any other mode of preparation employed at the present day; while sections the one-twelfth of an inch in thickness may be rendered perfectly transparent[*].

Many of the older anatomists, from BARTHOLINUS downwards, had observed that the grey substance of the spinal cord is softer, more delicate and more vascular than the surrounding white columns; but ROLANDO was the first to point out a diversity in its structure. He observed, chiefly in quadrupeds, that on each side the posterior third of the grey crescent consists of a peculiar cineritious substance, which presents a different aspect from that which forms its two anterior thirds: it is different in colour, darker, and less red[†]. ROLANDO, however, assigned too large a space to this "new substance," which does not comprise so much as the posterior third of the grey crescent, but forms only a comparatively narrow and curved lamina or band around the extremity of each cornu, and, when viewed in a thin section by transmitted light, is found to be actually much paler and more transparent than the rest of the grey crescent. That this lamina is the part indicated by ROLANDO appears evident on examining his plates[‡].

Now I propose, also, to make a primary but different division of each posterior cornu into two parts, namely, the *caput* cornu and the *cervix* cornu*. The *caput* consists of the broad or expanded extremity of the cornu, and is separated from the *cervix* by an imaginary line drawn across from the opposite anterior extremities of the arched lamina of *gelatinous substance*. In some regions of the cord a line of somewhat paler substance partially marks the separation. The *cervix* comprehends the remaining portion of the cornu as far forwards as the level of the central canal. This distinction is grounded on the facts,—1, that in ascending the *medulla oblongata*, the part which I designate the *caput* cornu is thrown aside from the rest, as a distinct mass which is traversed in succession by the vagus and glossopharyngeal nerves, and then becomes the principal nucleus of the sensory root of the trifacial; and 2, that, independently of the *gelatinous substance* which surrounds it, it differs in structure from the *cervix*, or remaining part of the cornu, which, in the *medulla oblongata*, supplies the grey substance of the *posterior pyramid* and *restiform body*†. This distinction is not only consistent with anatomical facts, but will be the means, I think, of adding clearness and precision to the description of these parts.

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[*] See Appendix.

[†] "Non è molto difficile il vedere nel midollo spinale di bue, di maiale e di pecora, che le corna posteriori della sostanza cinericcia, che in questi si mostrano molto più spesse e più grosse che nell' uomo, sono formate in gran parte da una sostanza cinericcia particolare, che presenta un aspetto diverso da quello, che si osserva nella porzione che forma i due terzi anteriori della mezza luna. Questa nuova sostanza è più gelatinosa, ciò che fa sì che prende eziandio un colore diverso, e che in generale è meno rossigno, e di un colore più oscuro." *Ricerche Anatomiche sulla Struttura del Midollo Spinale* (con figure), 1824, Torino, p. 60. He states that it was only in quadrupeds that he succeeded in distinctly seeing this new substance, partly because in them the spinal cord may be obtained in that state of freshness which is more favourable for observation, and partly because this substance is found there in greater abundance; but that, nevertheless, in Man, unequivocal traces of it may be seen in the lumbar region.

[‡] Some authors, however, seem to think that the *gelatinous substance* comprises a larger portion of each

The *caput cornu posterioris* varies in form in different regions of the cord. In the upper part of the cervical region it is long and pointed, or pyramidal; broader and more rounded in the lumbar enlargement and in the conus medullaris; and again pointed, or somewhat pyramidal, through the whole of the dorsal region in Man (see fig. 12, Plate XXV.; fig. 15, Plate XXI.; fig. 17, Plate XXIII.; figs. 5 & 6, Plate XXI.). From its border a network of blood-vessels, accompanied by nerve-fibres, extends outwards through the posterior and lateral columns (see fig. 1, Plate XIX.), and in the lumbar region especially is very conspicuous around its pointed extremity, at the bottom of the lateral fissure, where it divides the white columns into a number of separate fasciculi of different shapes and sizes.

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posterior cornu,—an opinion which is perhaps founded on the statement of ROLANDO, that it forms the posterior *third* of the grey crescent.

* In my memoir on the Medulla Oblongata I have written "caput *cornûs* posterioris." As I have heard that this has been objected to by an "eminent classic," on the ground that the noun *cornu* is indeclinable in the singular number, I may here take the opportunity of informing any one who has made or may make a similar objection, that the expression is perfectly grammatical and correct; for there is another and more ancient form of the same noun, which makes both the nominative and genitive singular to end in *us*. This ancient form may be found in several of the Latin classics—in CELSUS, CICERO, LUCAN, and the older PLINY. In PLINY it occurs frequently: see *Historia Naturalis*, lib. viii. 29; lib. x. 90; lib. xiii. 51 and 52; lib. xxviii. 42, 46, and 64. I find that it has also been used by METZLER in his dissertation *De medullæ spinalis avium textura*. Dorpat, 1855. I employed it on account of the distinctive character of its genitive; but as it appears necessary at times to use the words *cornu* and *cornua*, because they are so generally known, I shall confine myself to the more modern form of the noun.

† See the author's "Medulla Oblongata," Philosophical Transactions, Part I. 1858, figs. 12, 15, 16, 19, and 28. The grey tubercle of ROLANDO, which reaches the surface on each side of the *medulla oblongata*, is generally considered to consist of the *gelatinous substance*, but in fact it includes the whole dilated extremity of the posterior cornu, or caput cornu.

Structure of Caput Cornu Posterioris.

The *caput cornu* may be seen even with the naked eye to consist of two different portions: 1, an outer and comparatively transparent portion—the *gelatinous substance*; 2, an inner and more opake portion, or base.

1. The *outer* portion or *gelatinous substance* consists of—

A. Nerve-fibres.
B. Nerve-cells.
C. Blood-vessels.
D. Connective tissue with nuclei.

A. *The nerve-fibres are transverse, longitudinal, and oblique.*—The *transverse* fibres, proceeding from different parts of the grey substance, run backwards across the *gelatinous substance* in a kind of radiating series of bundles, and in planes at right angles to the axis of the cord (fig. 1, Plate XIX.). The bundles near the middle line (as seen in a transverse section) are straighter and more parallel than the rest, but partly join or cross each other near the posterior border of the *gelatinous substance*, from which they run out through the posterior columns as radiating fibres and posterior roots of the nerves. The bundles on each side take a curved direction towards the middle line, but before they pass out, their fibres describe a series of arcs of different lengths within the margin of the *gelatinous substance*: along the inner side of the latter there is always a large bundle which subdivides and runs out towards the posterior median fissure; and from the outer side of the *gelatinous substance* a smaller number of fibres proceed through the posterior part of the lateral column. The primitive fibres composing these bundles, as I showed on a former occasion, are not grey fibres, but tubules of small average size, the larger kind possessing double contours. By far the greater number vary from about the $\frac{1}{8000}$th to the $\frac{1}{20,000}$th of an inch in breadth, but are intermixed with others of about the $\frac{1}{1700}$th of an inch in diameter. The marginal fibres within the posterior border of the *gelatinous* substance are more often of the latter kind.

The *longitudinal* fibres of the *gelatinous substance* are very numerous, and resemble the finer tubules of the transverse bundles[*].

The oblique fibres, which are found in different planes, are in various degrees intermediate in direction between the two former sets, and, as I formerly stated, are apparently continuous on the one hand with the longitudinal, and on the other with some of the fibres of the posterior roots[†].

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[*] These fibres were more fully described in my first communication, Philosophical Transactions, 1851, Part II. p. 610.

[†] In his first treatise on the Spinal Cord, STILLING described all the fibres of the grey substance as *grey* fibres, but in his recent work they are shown to be tubules. He seems to think that I was not acquainted with the fibres which take an oblique course through the gelatinous substance. It is true that I gave no separate description of them, but in the explanation of fig. 2, Plate XX., Philosophical Transactions, 1851, Part II., when speaking of the bundles which project into the substantia gelatinosa from the grey substance, I observed that "this appearance is caused by oblique sections of bundles of nerves and blood-vessels which

B. *Cells of the Gelatinous Substance**.—The cells of the gelatinous substance, in regard to size, may be described as (*a*) large, (*b*) small, and (*c*) intermediate.

(*a*) The *large* cells are almost exclusively found amongst the semicircular fibres which run within the external border of the *substantia gelatinosa*, and never extend beyond the middle of its depth (fig. 1, Plate XIX., and fig. 47, Plate XXII.). They are more or less oval, pyriform, fusiform, crescentic, triangular, or otherwise irregular, and contain distinct nuclei. They all give off processes from two to five or six in number. The *fusiform* and *oval* variety are elongated transversely, longitudinally, and in planes intermediate or oblique. Sometimes a *fusiform* cell tapers at each end into a broad process, which attenuates in its course, and either remains single or subdivides into two or more; or it bifurcates at once into two processes at each extremity or only at one. In a transverse section of the cord, some of these processes appear to terminate in one or more of the semicircular or marginal fibres; some run backwards through the posterior columns, or forwards across the *gelatinous substance*; others pursue a longitudinal course with the *fibres* of the *gelatinous substance*, or along the verge of the posterior columns. From a cell of another shape three or more processes may be seen to extend in different directions—backwards, forwards, laterally, obliquely, or longitudinally. Frequently a *triangular* cell, resting on the verge of the gelatinous substance, sends a process right and left, and a third through the posterior column along the side of a bundle of the poste-

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enter the grey substance in a more or less longitudinal direction, or with various degrees of obliquity." And in my second communication, Philosophical Transactions, 1858, p. 358, it is stated, "I believe that they (the longitudinal fibres of the gelatinous substance) are all derived from the fine fibres of the posterior roots," which must therefore cross the gelatinous substance in an oblique direction.

* The cells of the gelatinous substance have been described by Kölliker, Schröder van der Kolk, and by Stilling in his late work. In my first communication I only incidentally mentioned them in the following passage: "All (the nerve-cells of the grey substance), except those peculiar to the gelatinous substance, have remarkably delicate processes issuing from their sides." Upon this passage Stilling takes the opportunity of remarking, that by my peculiar method of preparation the sections are rendered too transparent, so that the finer elementary structures escape observation. This objection is without sufficient foundation. In the first place, I described the gelatinous substance, as I then stated, from examination of the *fresh* cord, treated with acetic acid, and not by means of my "peculiar method" of preparation, which, however, shows the cells and processes in the most perfect manner; secondly, to see those preparations to advantage, the illumination must be adapted to their degrees of transparency. But they may be made of all degrees of transparency and of thickness up to the one-twelfth of an inch. If Stilling has given but a hasty trial to this method and only messed with a little turpentine and Canada balsam, he is not prepared to offer an opinion worth notice. I may add that the few of my preparations that have found their way to the Continent are some of the first that I made, and are very inferior to those I have since made, especially by my modified method. The great thickness, also, of some of these preparations renders them exceedingly valuable for tracing the course of fibres, which must often be inevitably cut across in the very thin sections required by other methods. This is evident in Stilling's large wall-diagram, where the grey substance, particularly of the cervix cornu posterioris, appears chopped up like chaff; so that it is impossible to form any idea of the destination of the fibres. It may not be out of place to state that all the observations described in this communication have been carefully repeated on sections of the cord hardened simply in chromic acid, and without further preparation.

rior roots, or through a fissure containing a blood-vessel; or two contiguous bundles of the posterior roots, as they pass out from the gelatinous substance, are accompanied by the processes of a *crescentic* cell, which occupies the curve between them (see fig. 1, Plate XIX.). And beyond the *point* of the cornu, a variable number of others, of different shapes and sizes, may be seen amongst the *network* of blood-vessels and fibres, which is formed, as already stated, around separate fasciculi of the posterior and lateral columns. They occupy the angular interspaces or partially embrace the fasciculi, and send between them their processes, which contribute to form the network (*b*, *b*, fig. 1).

(*b*) The *small* cells abound in every part of the *gelatinous substance*. They are round or oval, somewhat pyriform, fusiform, or angular, and give off two processes or more. Fig. 46, Plate XXII. *a*, *b*, *c*, represents them magnified 350 diameters. They vary considerably in size, and many are nearly as minute as blood-disks*. The largest of them contain distinct nuclei. Near the border of the posterior columns, the oval and fusiform variety, like those of the first set, are elongated transversely, longitudinally, and obliquely; but through the rest of the gelatinous substance the longer axes of many, as well as their processes, are parallel to the antero-posterior fibres that form the posterior roots of the nerves, with which they are apparently continuous (fig. 1, Plate XIX.).

(*c*) The cells of *intermediate* size, like the first set, which they closely resemble in shape, are found chiefly near the verge of the posterior columns. Some are so exceedingly fusiform that they appear like gradual dilatations in the course of the fibres, but still they are true cells.

There can be no doubt whatever that a great number of the *smallest* cells of the gelatinous substance belong to the connective tissue. This tissue extends as a fine network from the border of the grey substance, through the white columns as far as the surface of the cord, where it forms a layer of variable thickness, in which its fibres run circumferentially, and return at intervals to rejoin the network in the white substance. Now in very delicate preparations which I lately made of the spinal cord of the Calf between three and four months old, I discovered that in every part of the *white columns* the connective tissue between the primitive nerve-fibres abounds with both *nuclei* and small *nucleated cells*, many of which are quite equal in size and similar in character to numbers of those which are found in the gelatinous substance; but on account of their position, some of them are more diversified in shape. They are more or less round or oval, pyriform or club-shaped, angular, fusiform or crescentic, and lie in the interspaces between the nerve-fibres, sending out fine processes in different directions to join the network of connective tissue. Frequently a crescentic cell half encircles a primitive nerve-fibre, and forms a swelling or knot around one side of its sheath. Fig. 48, Plate XXII. represents the appearance of some of these nucleated cells and free nuclei. There is no difference in structure between them and the smallest of those in the grey substance; indeed from this substance an *uninterrupted* layer of cells and nuclei, of an apparently identical character, may be traced through the *white columns* between the fibres, as far as the surface of the cord. The nuclei are more or less round or oval, and contain a number of delicate granules. In a free state (see fig. 48 *e* and fig. 46 *c*) they abound in every part of both the grey and white substance, but sometimes they are partially or wholly surrounded by a kind of shaggy, flocculent, or finely filamentous mass (fig. 48, below the letter *e*), while in other instances they are attached to the surrounding parts only by a few fine fibres. Now there is reason to believe that this flocculent or filamentous substance is really the remains of the cell in process of development into the surrounding tissue and sheath of the nerve-fibre; for on examining the cord of an adult animal, I found that the *nucleated cells* had disappeared, while in the midst of the filamentous tissue between the nerve-fibres, the same kind of *nuclei* were still present*.

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* Care must be taken not to confound these minute cells with the cut ends of small blood-vessels: a mistake not at all unlikely to occur where they are not very distinct and well-defined.

These observations, then, render it apparently impossible to point out the exact distinction between the connective and the true nerve-tissue, and *might* suggest the question, whether there is any actual and essential difference between them, or whether the connective tissue of the cord be intermediate in its nature, passing on the one hand into *nerve*-tissue, and on the other into pia mater.

2. The *anterior* or *inner* and more *opake* portion of the caput cornu is continuous with the *grey substance* of the *cervix*, and surrounded behind and on each side by the arched lamina of the *substantia gelatinosa*, with which it varies in shape at different regions of the cord (figs. 5 to 17, Plates XXI. XXV. & XXIII.). In some parts of the cervical and in the dorsal region, as seen in a transverse section, it projects into the gelatinous substance in the form of a cone, while in other regions its posterior border is more or less angular or rounded. Its elementary structure, in addition to blood-vessels and connective tissue, consists of—

a. Longitudinal fibres.
b. Transverse and oblique fibres.
c. Cells.

a. The longitudinal fibres are collected into bundles, and are the principal cause of the opacity of this portion of the *caput* cornu. They are broader and coarser than the fibres of the *gelatinous substance*, which, however, they immediately adjoin. Sometimes, as in the neck, they form round the extremity of the cone a kind of arched band or roof, of the same shape and about the same depth as the *substantia gelatinosa*, which rests upon it*; while in the cervical and lumbar *enlargements* they also extend, but in smaller numbers, through deeper parts of the cone. The bundles themselves are of different sizes and not exactly parallel, even in the same plane, but intersect each other at acute angles, and extend obliquely, at intervals, into the *gelatinous substance*; so that in a transverse section some of their divided extremities are more or less oval or elongated, and give to the posterior border of the cone, or lamina, an irregular or jagged outline, like the worn extremity of a brush (see fig. 2, Plate XX. D, on left side).

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* I find that precisely similar nuclei are abundantly scattered throughout both the white and grey substance of the cerebrum and cerebellum. In the convolutions of the cerebellum, the nuclei, which in a dense layer form the internal portion of the grey substance, and into which the fibres of the white substance first penetrate, are for the most part a little different from those in the immediate neighbourhood. They are *generally* round or globular, are more uniform in size, and mostly contain a distinct and clear circular or globular nucleolus (as in fig. 46, Plate XXII., below the letter *a*), surrounded by fine granules. Nuclei like these, but frequently of larger size and sometimes containing two globular nucleoli, are very numerous in the *gelatinous substance* (see the two above *a*, fig. 46), but are not peculiar to this substance. They are also found in all parts of the grey substance of the cerebrum.

b. The transverse fibres are continuous with the posterior roots of the nerves and longitudinal columns, and cross each other and the longitudinal fibres in a great variety of ways. Some of the roots, on reaching the base of the cone or *caput cornu*, return through it in loops to the posterior columns (see fig. 2, Plate XX. P, on right side); some become continuous with the longitudinal bundles, particularly on the outer side; others cross them obliquely in opposite directions and run out on the one side through the lateral, and on the other through the posterior, columns; while the rest proceed forwards through the cervix cornu, and in part escape through the posterior commissure to the opposite half of the cord.

The oblique fibres, as may be inferred from the descriptions just given, are only continuations of the transverse and longitudinal, at different angles and in different planes.

c. Most of the cells are of the smallest and intermediate size. The majority are oval and fusiform—sometimes to a remarkable degree, and lie with their longer axes and their processes in the direction chiefly of the oblique and horizontal nerve-roots, but also in the direction of the longitudinal fibres, with which they appear to be continuous.

Amongst these, a few larger cells, from three to five in a transverse section of moderate thickness, are scattered at unequal intervals. They are more or less oval or fusiform, or irregular in shape, and give off from three to five processes, or perhaps more. Frequently one of these cells is found nearly as far back as the verge of the gelatinous substance; but they mostly lie deeper. Sometimes they bifurcate around longitudinal bundles, which are probably accompanied by *some* of their processes, while the *others* may be traced to a surprising distance and in different directions, backwards, forwards, and laterally to the white columns.

Having given the structure of the *caput* cornu posterioris, I shall next describe the *cervix*, or rest of the posterior grey substance, as it appears in the dorsal region, and then proceed to trace it both upwards and downwards, through the remaining length of the cord. In the dorsal region I shall describe it in considerable detail, and illustrate it by drawings of the greatest possible exactness, not only on account of its great interest and apparent importance, but in order that we may recognize its parts as the *same* during the *changes* which they undergo in passing through other regions.

About the middle of the dorsal region in the spinal cord of the higher Vertebrata, the inner sides of the posterior cornua are in close contact along the middle line, or are separated only by blood-vessels which descend from the median fissure; so that together they form but a single mass, along the posterior part of which the *gelatinous substance* is uninterruptedly continued from side to side[*]. Fig. 2, Plate XX. represents a transverse section of the grey substance in the dorsal region of the Calf, magnified 60 diameters. The *caput* cornu posterioris, on each side, would be marked off by an imaginary line extending across from the *antero-lateral* extremity of the *gelatinous substance* to the bottom of the posterior median fissure (*l*); while the *cervix* is included between this line and another drawn nearly horizontally across from the anterior border of the transverse commissure which arches over the pellucid space (II) surrounding the central canal.

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* This layer of fibres I pointed out in my first communication on the Spinal Cord, Philosophical Transactions, 1851, Part II. p. 611.

The inner or median half of the *cervix* is occupied by a remarkable longitudinal column, which in this part of the dorsal region is nearly cylindrical or oval. These bodies I described in my first communication, and named them the *posterior vesicular columns—columnæ vesiculosæ posteriores*; but their highly interesting structure and connexions have induced me to examine them again with more attention.

Each *posterior vesicular column* (M, M, fig. 2, Plate XX.) consists of a somewhat dark cylinder of fibres interspersed and surrounded by cells and their processes. The fibres are in great part derived from the posterior roots of the nerves, and are much finer than those of the white columns. They are more or less longitudinal, transverse, and oblique, and interlace each other in bundles in the most intricate manner, as may be seen in a longitudinal section.

The cells in the cylinder or opake portion of the column are oval, pyriform, or more or less fusiform, and variously stellate (see also Plate XXII. figs. 49 & 50). They differ considerably in size; some are very small, but the largest are quite equal to those of the anterior cornu. Their *number*, also, varies in different sections of the same thickness, and even in the opposite columns of the same section. Sometimes only a few small cells, with perhaps a single large one, appear at irregular intervals; and occasionally they are all absent. Their processes extend *transversely*, *obliquely*, and *longitudinally*: *transversely* they intersect the cylinder in various ways (M, M, Plate XX. fig. 2), in a kind of network formed by their subdivision, and in irregular curves, which sometimes suddenly change their course to become *longitudinal*, and sometimes escape *directly* to surrounding parts, or after running for some distance round the cylinder. In sections made *longitudinally* all the cells are seen to be elongated in that direction (Plate XXII. fig. 50), and are frequently quite fusiform; it would therefore appear that each cell is prolonged in different planes. They lie imbedded in the plexus of bundles, and branch into many processes, which run *transversely* backwards and forwards, *obliquely* and *longitudinally*. Sometimes longitudinal processes extend to a surprising distance, and then become transverse, and often again longitudinal.

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[*] In the Rodentia—at least in the *Guinea-pig*, *Rat*, and *Rabbit*—the posterior cornua, as in Man, remain separate in all regions of the cord. It is curious, also, that in these animals the *caput* cornu is remarkably large in the *dorsal*, but particularly in the *cervical* region, and is thronged, as indeed is every other part of the grey substance, with small cells.

Around the whole of the cylinder, and partly encroaching on its border, other cells are disposed, that contribute to form, by their *collective* processes, an irregular circle of fibres; but these *individually*, after describing parts of the circle, run off tangentially in different directions (fig. 2, Plate XX.). On the outer side of the cylinder they cross each other obliquely from before backwards and *vice versâ*, extending in opposite directions to the anterior and posterior cornua. They also contribute with the posterior roots to form the transverse commissure behind the central canal. The cells are of all sizes, and mostly fusiform—often to a remarkable degree; but some are triangular and stellate. Many are curved and elongated circumferentially; others are inclined at a tangent, and send off their processes, on the one hand, within or around the circle, and, on the other, to distant parts. Sometimes an irregular or stellate cell sends two of its processes round the circle, one longitudinally or obliquely, and the rest in other directions, but mostly towards the anterior parts, or towards the lateral column.

When the posterior roots of the nerves are traced inwards, they are found to be most intimately connected with all parts of the *posterior vesicular columns*. One large bundle is generally observed proceeding from near the middle of the caput cornu to the outer side of the cylinder, where its fibres diverge or separate: some of them partly enclose it, apparently in connexion with the cell-processes; while the rest interlace within it, or form a series of tortuous curves, as may be seen best in a longitudinal section; or after running *longitudinally* backwards and forwards, in a kind of looped or zigzag course, escape into the lateral white column[*]. Another large bundle (P) invariably enters along the inner side of the caput cornu near the posterior median fissure; some of its fibres return backwards towards the point and outer side of the gelatinous substance, partly to form loops with other roots, and partly to escape through the lateral white columns; some interlace through the cylinder; some sweep round on its outer side and join both the posterior and anterior commissure, while others are prolonged forwards through the lateral portion of the cervix cornu.

A band of fibres from the posterior transverse commissure, after curving round the front of the vesicular cylinder, runs nearly horizontally outwards (as seen in a transverse section) to the lateral border of the grey substance and lateral column (see figs. 2, 3 & 4). These fibres, which are more conspicuous at the upper and lower parts of the dorsal region, are joined by others, which proceed from the inner side of the cylinder, and from a *different plane*; for at this point they are cut across in the section, and terminate abruptly in divided extremities[†]. The *lateral* portion of the *grey substance* which is traversed by this band, and near the level of the canal, may be considered as intermediate between the anterior and posterior cornua. At the border it forms a small and more or less pyramidal projection (F, figs. 2 & 4, Plate XX.; and figs. 3, 5, 6, Plate XXI.), which is very transparent, and in colour resembles the *gelatinous substance*. In some parts of the dorsal region, as we shall presently see, it extends inwards as a somewhat conical layer, and becomes less transparent towards the point, which reaches nearly to the front of the posterior vesicular column (fig. 3, Plate XXI.; and fig. 4, Plate XX.). This tract, which I intend to follow through other regions of the cord, was first pointed out by myself in 1851*. I shall call it, on account of its position, the *tractus intermedio-lateralis*. It consists for the most part of oval, fusiform, or pyriform and triangular cells, which are somewhat smaller and of more uniform size than those of the anterior cornu. Figs. 51 and 52, Plate XXII., represent them magnified 220 diameters. In the *middle* of the dorsal region they are less numerous than in some other parts, and found only near the verge of the grey substance. They are elongated as well transversely as longitudinally,—*transversely* both in a lateral and antero-posterior direction, in which they send their processes, on the one hand, to the transverse commissure and the lateral column, and on the other, to the anterior and posterior cornua. Sometimes a few lie beyond the border of the grey substance, where their processes accompany nerve-fibres and blood-vessels which radiate through the lateral column (fig. 51 *b*, Plate XXII.; and fig. 2, Plate XX. F).

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[*] It is difficult, however, to determine whether these tortuous and zigzag fibres are directly continuous with the posterior roots, or with the processes of the cells, for their diameters are often considerable.

[†] This appearance is observable chiefly in the upper and lower parts of the dorsal region, but particularly the latter; it is also to be seen in some degree in the cervical and lumbar enlargements; in the upper part of the latter it is very conspicuous. At the point where these fibres are divided in the transverse section, that is, at the inner side of the *posterior vesicular column*, and a little behind the spinal canal, they appear to be continuous with the tortuous and zigzag fibres seen in a longitudinal section, as above described.

The outer or lateral portion of the *cervix* cornu posterioris (Plate XX. fig. 2, *n*, and corresponding part on the opposite side of the figure), between the *tractus intermedio-lateralis* and the *caput* cornu, is composed of fibres and cells. The fibres descend from the roots of the nerves, and run partly outwards to the lateral column, and partly forwards to the anterior cornu, passing through the *tractus intermedio-lateralis* (F). The cells, which are often of considerable size, are mostly fusiform in the same direction as the fibres, and taper into processes of remarkable length, which seem to be continuous with them. But *some* of the cells are stellate or irregular in shape, and send out their processes in all directions. These are generally observed near the outer border of the posterior vesicular column, with which they are partly connected by their processes, and near the *edge* of the *cervix* and the *base* of the *caput* cornu, where *some* of their processes contribute with blood-vessels and fibres to form a network around several longitudinal bundles, with which *others* are *probably* continuous. At N (fig. 2) are three of these longitudinal bundles, the most anterior of which is partially embraced by a multipolar cell, and the most posterior by another of a crescentic form. The outer cells of the posterior vesicular column are so gradually intermixed with this lateral layer, that it seems almost impossible to determine their exact limits.

In receding from the middle of the dorsal region towards either extremity of the cord, the posterior cornua gradually separate from behind forwards, receiving between them the deep encroaching layers of the posterior columns, until in the middle of the cervical and lumbar enlargements they are joined only by a narrow band of the transverse commissure, immediately behind the canal. During these changes, the posterior vesicular columns undergo certain modifications in form, size, and structure. As they approach the cervical enlargement, the cylinder or opake portion of each is gradually reduced in size, but the whole inner half of the cervix is still occupied by the surrounding layer of cells. Sometimes the cylinder is intersected, and divided into smaller portions, by fibres of the transverse commissure, in continuity with the processes of some of its own cells, which are more or less intermixed with those of the surrounding layer: other fibres of the transverse commissure, in their passage outwards, enclose it in front and behind, and after partly reuniting on its outer side and running backwards through the caput cornu, where they are also continuous with cells, they diverge and traverse the gelatinous substance as posterior roots of the nerves. Some of those fibres which enclose it in front, in company with processes or fibres from the inner side of the vesicular column, proceed, as already stated, with a gentle curve, or in a slightly serpentine course, to the lateral border of the grey substance, where they are partly continuous with the cells of the *tractus intermedio-lateralis* (F, figs. 2 & 4, Plate XX.), while the rest escape through the lateral column. Behind, and at the sides of, the canal, the fibres of the transverse commissure are connected with many small oval and fusiform cells,—sometimes with others of larger size,—and are separated at intervals by blood-vessels which run longitudinally and obliquely and communicate with one of considerable size, which proceeds transversely, with the commissure, to the lateral grey substance, where it divides into numerous branches. In the middle of the cervical enlargement, the circumscribed cylinder of the posterior vesicular column has entirely disappeared, but the whole inner half of the cervix cornu is filled with cells, and contains a somewhat dark and imperfectly-defined mass, which is traversed by the fibres of the transverse commissure and interlaced by the posterior roots. In the Ox many of these cells are very much branched, and send out their processes in all directions: some are fusiform in various degrees; others are apparently riband-shaped, and sometimes not much broader than the processes into which they gradually taper.

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* Philosophical Transactions, Part II. p. 613.

In the upper part of the cervical region, near the origin of the third pair of nerves, a darker and more defined mass (M, fig. 12, Plate XXV.) reappears at the base of the cervix cornu. It is composed of cells both large and small, and of bundles of the posterior roots which interlace amongst them. This mass is not distinctly circumscribed like that of the posterior vesicular column in the dorsal region, but is somewhat triangular, with one of its angles directed towards the point of the posterior cornu, another towards the transverse commissure, and the third obliquely forwards and outwards towards the antero-lateral column. It gradually diminishes upwards, and disappears near the first pair of nerves.

The *tractus intermedio-lateralis* is larger at the *upper part* than in the *middle* of the dorsal region. On the one hand it projects further into the lateral column, and on the other tapers inwards across the grey substance to near the front of the *vesicular cylinder* (see F, Plate XX. fig. 4; Plate XXI. fig. 3). *Transversely* its *outermost* or marginal vesicles lie, for the most part, with their longer axes and processes in an *antero-posterior* direction, as in fig. 2, Plate XX.; while those more internal are elongated in the direction of the transverse commissure, with the fibres of which, as already stated, their processes are continuous. As it ascends, however, through the cervical enlargement, it gradually disappears; but, as in the dorsal region, the lateral portion of the grey substance is still traversed, from behind forwards, by numerous fibres apparently in connexion with branched and with very elongated cells of all sizes, amongst which are scattered a few that resemble those of the *tractus intermedio-lateralis*. Through this lateral portion of the grey substance the *lowest* roots of the *spinal-accessory* nerve bend *forwards* to the cells of the *anterior cornu*.

In the region of the upper cervical nerves, there reappears a vesicular tract in the same position as the *tractus intermedio-lateralis*, and composed of the same kind of cells, which are elongated in a lateral direction and send their processes, on the one hand, outwards through the lateral column, and on the other, inwards to join the fibres of the transverse commissure behind the central canal (see F, fig. 11, Plate XXI.; and fig. 12, Plate XXV.). It is traversed by the roots of the spinal-accessory nerve (*r*) as they bend forwards on their way to the anterior cornu.

In *descending* the cord from the *dorsal* to the *lumbar* region, the posterior grey substance undergoes a series of changes nearly similar to those which are observed in *ascending* to the cervical enlargement. The posterior cornua become gradually more separate, or thrown aside from each other, in a direction obliquely backwards. At the *upper* part of the *lumbar* enlargement, the posterior vesicular columns are decidedly larger than in any other region of the cord (M, fig. 13, Plate XXI.). In a transverse section each presents the appearance of a dark oval mass, lying along the whole inner half of the cervix cornu, the border of which it renders convex against the deep strata of the posterior column. Its larger cells are more numerous than in the dorsal region. They do not form in the centre of the column a circumscribed group, but lie scattered irregularly through the whole inner half of the cervix, which their processes traverse in different directions and envelope in different planes. Many of the oval variety are elongated in the direction of the cornu, and their processes are continuous, on the one hand, with the transverse commissure, and on the other, with the posterior roots of the nerves. Some stretch along the convex border of the cervix; and sometimes a crescentic or triangular cell, in the same situation, sends one of its processes through the posterior column with a bundle of radiating fibres. Part of the fibres of the transverse commissure run out in front of the mass, as already stated, in company with processes or fibres which proceed from its inner side; of these some are continuous with the cells of the *tractus intermedio-lateralis*, while others escape through the lateral column.

The *tractus intermedio-lateralis* is prominent at the border of the grey substance, between the anterior and posterior cornua, but its cells are not so numerous as in the upper *dorsal* region.

Through the remaining half of the lumbar enlargement, the *posterior vesicular columns* are gradually less encircled and enveloped by the processes of their cells, which also diminish in number, but are still traversed by the divergent fibres of the transverse commissure and by a plexus of the posterior roots which sweep round on their outer sides. In the Ox two or more of these cells are often very large and wonderfully branched into six or seven processes, which radiate in all directions. I think there can be no reason to doubt that they form a part of the *posterior vesicular column* which has just been traced from above.

In descending through the lower portion of the lumbar enlargement, the border of the grey substance between the posterior cornua is gradually drawn backwards, so that in the same proportion the space behind the canal, containing the transverse commissure, becomes deeper (compare fig. 15, Plate XXI., with figs. 16 & 17, Plate XXIII.). Some of the posterior roots of the nerves cross the cornu from without inwards to the side of the *anterior* commissure, and a large number converge to form the *posterior* commissure behind the canal. From the inner side of the caput cornu several large bundles sweep round through the cervix, where they interlace with the others and separate in different directions. Some of them descend to the anterior grey substance; others curve inwards to the side of the canal; and amongst the plexus which they form with the transverse commissure are a few scattered cells, which may be regarded, I think, as the remains of the posterior vesicular column. These cells are scattered in front as far as the border of a peculiar group which, near the level of the second pair of sacral nerves, begins to make its appearance on each side and a little behind the spinal canal. This group was first pointed out by myself in 1851, and considered as the commencement of the posterior vesicular column. I think, however, that STILLING is right in considering it as a distinct group, although it is reached by the posterior roots; but, as I shall presently show, its connexions with the anterior roots are more direct and intimate than in the case of the *posterior vesicular column*. It is more or less round or oval, but not entirely isolated from the surrounding cells, nor circumscribed by fibres. It increases in size through the level of the second pair of sacral nerves, below which it gradually disappears. The cells are oval, fusiform, irregular in shape, or stellate, and all of the larger kind. Some of their processes cross over to the opposite side behind the canal; others run out towards the lateral column; some join the anterior and posterior roots, and others form a system of longitudinal fibres, the cut extremities of which, in a transverse section, are seen scattered amongst the group. The oval and fusiform cells are in part elongated transversely towards the anterior roots, but chiefly in a lateral direction, and a continuous layer or tract of these elongated cells may be frequently seen to extend from the outer side of the group to near the border of the grey substance, in company or in connexion with fibres which proceed in the same direction, and escape through the lateral column. But the *principal part* of the group is most intimately connected with the *anterior roots* of the nerves; for a large number of these roots, frequently side by side, run directly backwards to reach it, through the middle and inner part of the cornu: some of them are seen to be continuous with the cells, especially with those which are elongated towards them; others wind about them in a series of curves, which, like the processes themselves, are frequently divided in a transverse section*.

In the spinal cord of Man, the form of the grey substance differs in some respects from that of Mammalia. Throughout the whole of the dorsal region, the posterior cornua stand completely apart, and are joined only at their bases by a narrow transverse commissure. Fig. 5, Plate XXI. represents a transverse section of the entire grey substance about the middle of this region. Each posterior vesicular column (M) occupies nearly the whole inner half of the cervix cornu. It is more or less oval, and, in structure and appearance, closely resembles that of Mammalia in the upper parts of the lumbar enlargement. Its large cells are more numerous than in the dorsal region of Mammalia. They are not accumulated near the centre, but scattered at unequal intervals to the very border of the cervix, from which some of their processes run out with blood-vessels and fibres to the posterior white column. Most of the oval variety are elongated in the direction of the cornu, and are continuous by their processes with the transverse commissure, on the one hand, and on the other, with the posterior roots of the nerves, which also interlace through the mass and sweep round on its outer side.

Towards the upper end of the dorsal region the dark oval masses diminish in size, as may be seen in fig. 6, Plate XXI. They are frequently divided or broken into smaller portions by bundles of the posterior roots in their course to the transverse commissure, and in connexion with some of their cells. In ascending through the cervical enlargement they continue to decrease, and at length disappear; but the spaces which they occupied along the inner halves of the cornua are still interspersed with a multitude of cells and traversed by the posterior roots, which also sweep round as usual, on their outer sides, in bundles of considerable thickness (see figs. 8 & 9, Plate XXI.). The cells, however, are very much smaller than in the dorsal region; the majority are scarcely larger than those in the middle of the gelatinous substance; but a few of superior size are unequally scattered amongst them. Many are oval, round, and fusiform, especially in the direction of the fibres; others, particularly the larger kind, and nearer the centre of the cervix, are triangular or irregular in shape, and send out their processes on all sides.

Above the cervical enlargement, the dark masses present nearly the same appearance as in Mammalia; but they are rather paler, and the cells they contain are of smaller size.

The *tractus intermedio-lateralis*, also, in Man, closely resembles that of Mammalia, and contains cells of the same kind, but rather smaller (figs. 51 & 52, Plate XXII.). In the upper part of the cervical region, a similar but somewhat larger tract reappears in the same situation, and projects in the same way into the lateral column (see fig. 11, Pl. XXI. F). It increases in ascending to the third pair of nerves (fig. 12, Plate XXV. F), where the form of the entire grey substance presents a very striking resemblance to that in the upper part of the dorsal region (compare with fig. 6, Plate XXI.). This tract is traversed by several roots of the *spinal-accessory* nerve, in their course forwards to the *anterior* cornu, and contributes, with the edge of the posterior cornu, to form a beautiful network (N, N, N) in the lateral column, through which the nerve enters. Its cells are triangular, oval, and fusiform in different directions: some of them are elongated in the direction of the transverse commissure and towards the front of the posterior vesicular column; others extend outwards with radiating fibres through fissures of the lateral column. There is reason, therefore, to believe that this tract forms a part of the *tractus intermedio-lateralis.* In the Sheep and Ox, and probably in all Mammalia, a *peculiar* group of cells, which is traversed by the roots of the spinal-accessory nerve, is found in the same situation; and this group, in ascending the medulla oblongata, retires inwards to the space behind the canal, and there contributes to form the nucleus which gives origin to the highest roots of the nerve. It has also been seen that the cells of the *tractus intermedio-lateralis* are elongated with their processes in a longitudinal direction, and reached by both the *posterior* and *anterior roots* of the *spinal* nerves, and perhaps by the *spinal-accessory*; that the latter nerve extends *forwards* to the cells of the *anterior cornu,* which also send some of their processes *longitudinally,* and are reached by the *posterior* roots. Moreover, I have in another memoir shown that, while *one* portion of the *upper* roots of the *spinal-accessory* nerve and *one* portion of the *vagus roots* proceed *inwards to their respective nuclei behind the canal, other* portions of *both bend forwards to the vesicular network into which the anterior cornu has become resolved*[*]. Again, I have shown, in the same memoir, that some of the roots of the *trifacial nerve descend longitudinally through the caput cornu,* between the transverse roots of the *vagus*; in which course they are probably brought into connexion with the *respiratory centres,* and perhaps also, like the *vagus,* with the anterior grey substance of the medulla. These extensive and intimate connexions seem to afford an explanation of the mechanism by which impressions made on the vagus and on the incident fibres of the trifacial and spinal nerves, may call into action the whole class of respiratory muscles; and if the tract which I have just described in the upper part of the cervical region be continuous, as it probably is, with the *tractus intermedio-lateralis,* which is reached by the dorsal nerves supplying the intercostal and other respiratory muscles of the trunk, the explanation in question will be still more complete[†].

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* As there are no traces of these groups of cells in the *human* cord, and I have not found them in the *Guinea-pig*, it is probable that they supply the nerves of the *tail*.

I shall now briefly trace the posterior vesicular columns of the human cord from the dorsal region downwards. At the upper part of the lumbar region, as in Mammalia, they are larger than in any other portion of the cord (see fig. 13, M, Plate XXI.). They have the same form and dark appearance as in the dorsal region, but contain more large cells. As they descend, however, the dark masses diminish, and are frequently broken into parts; their larger cells, also, are gradually reduced in number, but others of a smaller kind are scattered through the spaces which they occupied. In the middle of this region, the lumbar (fig. 15, Plate XXI. M), there is still, near the centre of the cervix cornu, a somewhat dark and irregular mass, which in a moderately thin section may be very distinctly seen even with the naked eye. Each of these masses is interlaced and surrounded, as usual, on its outer side by bundles of the posterior roots, and traversed by the transverse commissure. They present no traces of the large, strongly-marked cells which are found higher up, but, when examined under a sufficiently high magnifying power, they are seen to be interspersed and surrounded by cells of a smaller and more delicate kind, like those contained in the corresponding part of the cervical region. In descending through the conus medullaris, the posterior cornua, by the retreat of the border between them, become gradually shortened and approach each other towards the middle line, so that the grey substance on the inner side of each *cervix* is brought nearer the central canal (compare figs. 16 & 17, Plate XXIII. M). This grey substance, which, I think, must be considered as the remains of the posterior vesicular column, is still interspersed with a multitude of small and delicate cells, which lie amongst the fibres of the transverse commissure and posterior roots of the nerves. But there is scarcely any trace of the peculiar group which is found in Mammalia on each side, and a little behind, the canal. Sometimes, however, two or three cells rather larger than the rest may be seen at this spot in connexion with a few fibres of the anterior roots, but they are not distinguishable as a separate group.

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[*] See the author's "Medulla Oblongata," Phil. Trans. Part I. 1858, pp. 252, 258, figs. 19 and 36.

[†] Both in this communication and that on the Medulla Oblongata, I have abstained from drawing any physiological conclusions, except on this one occasion, where so *many different* facts required to be brought together while fresh in the memory of the reader.

In my first description of the posterior vesicular columns, it was stated that in the Calf they commence small at the lower extremity of the cord, increase in size in lumbar and cervical enlargements, and terminate at the upper part in the medulla oblongata. This statement, though in many respects true, must be modified by the descriptions above given. It is true that they are actually largest, and contain the greatest number of large cells in the lumbar region, but only at its upper part; it is also true that both in the Calf and Sheep the large cells scattered through the inner half of the cervix cornu are at least as numerous in the lower part of the cervical enlargement as in the dorsal region. STILLING, however, while agreeing in the main with my first description of these columns and of their connexion with the posterior roots, denies their existence at the upper part of the cervical enlargement; for he does not consider the cells scattered through the inner half of the cervix cornu as part of the vesicular columns; and in the two lower thirds of the lumbar enlargement, he denies the existence of cells altogether: under low powers of the microscope they are certainly not visible*.

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* In discussing the causes which have led to this difference of opinion between himself and me, he adopts a very singular kind of argument. 1. He *assumes* that I made the sections thicker at one part of the cord than at another. 2. He *assumes* that I *squeezed* cells, lying at different depths, into one layer, in consequence of the *necessary* pressure resulting from the employment of Canada balsam and thin covering glass. The Canada balsam which I always use is fluid; *everyone* covers his preparations with thin glass; and if, as he asserts, my preparations are already too transparent, there is no need of pressure.

I was desirous to know whether the posterior vesicular columns are present in the spinal cord of the lower Vertebrata. I found that in Birds, as in the larger Mammalia, the posterior cornua, near the middle of the dorsal region, are united along their inner borders, and form but a single mass, around which the *gelatinous substance* extends without interruption. Each on its outer side, where the *caput* cornu joins the *cervix*, is very much constricted, but again dilates as it extends forwards to be continuous with the anterior cornu, so that the entire grey substance has nearly the form of an hour-glass or goblet (see fig. 32, Plate XXIII.). Numerous cells of various shapes and of moderate size are scattered on each side and behind the canal, and are traversed by the posterior roots proceeding through the constriction; but there are no dark masses at all corresponding to those of the posterior vesicular columns. In receding from the dorsal region, both upwards and downwards, the posterior cornua gradually separate, and the anterior cornua become enlarged. In Mammalia, and especially in Man, it has been seen that, through the middle of the cervical and lumbar enlargements, the cells on the inner side of the cervix cornu are reduced in size. In Birds, on the contrary, they are much larger than in any other region. This is particularly the case in the lumbar enlargement, especially near its middle, where they form a distinct group, which is traversed and surrounded by the posterior roots of the nerves (see M, figs. 34, 35, and 36, Plate XXIII.). They are mostly oval and round, with large nuclei, and send out their processes in different directions. In the corresponding part of the cervical region the larger cells are less numerous, but are mingled with others of a smaller kind.

In Reptiles, it is only in the *conus medullaris* that the posterior cornua are united into a single mass. In a transverse section they present no dark masses resembling those of the posterior vesicular columns, but only a few cells, of various shapes, at their bases. A distinct stratum, however, of small fusiform cells, in connexion with the fibres of the posterior roots of the nerves, extends diagonally from the point of each cornu to the posterior commissure, which is also interspersed with numerous nuclei, like those of the epithelium which lines the canal. In the cervical and lumbar enlargement, the cells, as well as the stratum which they form, are larger than in the dorsal region. (See figs. 44 & 45, Plate XXV.).

On the Grey Substance of the Filum Terminale.

In the *conus medullaris* of the *human* cord, as soon as the posterior cornua have united at their inner sides in a single mass (fig. 19, Plate XXIII.), the *gelatinous substance* (*g*) becomes continuous from side to side in a curve with its concavity backwards, in which the posterior columns (A) are included *; but, as it descends through the *filum*

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* The continuity of the gelatinous substance across the middle line was asserted in my first communication, and has since been found in birds by METZLER, 'De Medullæ spinalis Avium Textura,' p. 14 (1855). STILLING, however, in his late work, denies the fact, and maintains "that the middle portion is *grey*, and not *gelatinous*, substance" ("das Mittelstück ist graue Substanz, und nicht gelatinöse"); and adds that my method of preparation rendered these two substances too transparent to perceive any difference between

terminale, it rises to the surface in the middle line and extends directly across (see fig. 20, Plate XXIII.); meanwhile the posterior columns (A, A) become gradually shallower and at length disappear, *except* at the *sides*, so that the *gelatinous substance* behind is covered only by the pia mater (*m*) of the cord, with which it is immediately connected. The deep posterior strata of the lateral columns obliterate it at the sides, by encroaching in bundles on the grey substance which forms a continuous network around them. Still lower down (see fig. 21, Plate XXIII.) the canal is prolonged backwards as far as the gelatinous substance, and dilates in a lateral direction behind the dark bundles (D), which in the preceding sections occupied the interior of each caput cornu. (See explanation of Plates.)

In Mammalia a series of nearly similar changes may be observed, except that the posterior columns are not interrupted in the middle line.

Fig. 22, Plate XXIV. represents a transverse section of the *conus medullaris* of the Ox, near the third pair of coccygeal nerves, magnified 60 diameters. Multitudes of fibres are observed to radiate from the grey substance and form a beautiful network through the white columns. From the extremities of the anterior cornua considerable offsets of the grey substance are prolonged through the anterior columns, rejoining each other around masses of longitudinal fibres, and reaching the surface to be continuous with the layer of connective tissue. Amongst those offsets, and even *in* the masses of longitudinal fibres, several large cells are scattered very nearly to the surface of the cord; some of their processes *appear* to reach the connective tissue at the surface.

Through the lower part of the conus medullaris the canal continues to enlarge. It nearly reaches the anterior fissure, and extends backwards to the *gelatinous substance*, in front of which it dilates in a lateral direction (see fig. 23, Plate XXIV.). The grey substance, as it descends through the *conus*, gradually decreases in quantity, and is encroached upon more and more by the antero-lateral columns, around the bundles of which it radiates outwards as a network. Numerous small oval and fusiform cells, with one or two others of superior size, are scattered through it. The filum terminale, in its natural state, appears to be a nearly cylindrical tubule, but, when hardened in spirit or chromic acid, is more or less flattened at the sides, so that the canal becomes compressed and sometimes completely collapsed (see figs. 24 & 25, Plate XXIV.). The grey substance is at length reduced to a mere border or fringe (fig. 25), from the edges of which a series of tapering processes (*a*, *a*) extend through the white substance to the pia mater at the surface. The larger kind of cells have entirely disappeared, but a number of small fusiform cells, or nuclei, resembling those of the connective tissue, are observed in continuity with fibres which run along it and form arches from one process of the grey substance to another, as they pass out to the white columns.

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them ("CLARKE's Preparations-methode zeigte ihm aber beide Substanzen zu hell, als dass er einen Unterschied hätte finden können"); but, if he had taken the trouble to read the explanation of the figure in question, he would have seen that it was drawn from a section of the conus medullaris, treated *only by acetic acid*. The continuity of the gelatinous substance is so apparent at this part, both in a fresh state and when hardened with chromic acid, that STILLING's denial of the fact seems to me unaccountable.

Of the Epithelium of the Central Canal.

In the Ox, the *epithelium* which lines the central canal consists not only of *columnar* or cylindrical cells—as I first described them*, and as they have since been described by other observers—but also of cells which are more or less *fusiform*. Fig. 53, Plate XXII. (*a*) is an exact representation of some of their forms, magnified 400 diameters. In the *columnar* variety (the first three on the left of *a*) the nucleus is large, sometimes spheroidal, but generally oval, and contains apparently from two to five large, brilliant, globular granules or nucleoli, which powerfully refract the light. Between this and the verge of the canal the free ciliated end of the cell is broad and short like the end of a cylindrical brush, and *appears* to be made up of granular fibres; but in the *fusiform* epithelium (the four on the right of *a*), the corresponding free portion consists only of a long and narrow kind of stalk extending from the tapering end of the cell, which latter in this case is similar to what forms the *nucleus* of the *columnar* variety, and contains several brilliant granules, which are frequently arranged in a row. Between these two kinds of epithelium there are different grades of transition. They are all beautifully packed in close apposition, so that the convexity of each is applied to the concavity of those which surround it, and by reason of its peculiar shape, it exactly occupies the intervening space (see fig. 53 *b*, Plate XXII.). In the *human* spinal cord, the canal is often completely filled up by what would appear to be the *débris* of the epithelium; for nothing is to be seen but a confused heap of *nuclei*, which are here mostly large and round: but sometimes in the midst of this heap there remains a small opening or canal, which, strange to say, is still lined or surrounded at its *margin* by the *usual regular* layer of columnar cells; and what is still more curious, I occasionally find, particularly in the cervical region, *two* such secondary canals, each lined in the ordinary way (see fig. 55, Plate XXII.).

The cilia of this epithelium are much coarser and less numerous than those in the larynx and trachea (fig. 53 *b*). In the *conus medullaris* they may be very well seen around the entire circumference of the canal, but in other regions of the cord they are very sparingly found. If this be the result of accident in manipulation, it is difficult to say why they are always preserved in the one place and not in the others.

The light-coloured space immediately surrounding the canal is interspersed with nuclei or small cells. Some of these are round or oval, finely granular, and exactly resemble those in the connective tissue of the white columns (see fig. 46, Plate XXII. *c*); others are similar to the *nuclei* of the epithelium, and have the same kind of large brilliant granules, or nucleoli, but are themselves rather smaller. These latter nuclei, which are frequently elongated in different directions, are connected with the processes of the epithelium (see fig. 53, Plate XXII. *b*), and also with the fine fibres which surround the canal, and which are precisely similar to the fibres of connective tissue at the circumference of the cord †. They are not, however, confined to the neighbourhood of the canal, for they may be found in different parts of the grey substance, and in the posterior fissure. The only difference between them and the *nuclei* found in the *white columns* and *pia mater*, is, that in the *latter* cases the *granules* or *nucleoli* are *smaller*; but in the Tortoise I have found that nuclei which, in every minute particular, *exactly* resemble those of the epithelium, may be traced continuously from the edge of the canal through the *whole* of the grey substance and the *white* columns of the cord*. Indeed, the spinal canal appears only to be the part where they are more thickly accumulated in layers; and even there they are not enclosed in cells, but are connected only by a variable number of fibres with the verge of the canal (see fig. 54, Plate XXII. G, and explanation of Plate). There is good reason, then, for believing that both the epithelium and the small cells or nuclei by which it is surrounded, are allied to the connective tissue.

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* Philosophical Transactions, 1851, p. 614.

† In 1851, I described the light-coloured part immediately surrounding the canal, as composed of fila-

The outer ends of the epithelial cells taper into delicate fibres, of which those proceeding from the columnar variety are the thickest (fig. 53, Plate XXII.). These fibres radiate in all directions, and either run side by side, or cross each other in various ways and subdivide to join in an intricate network. For a considerable distance around the canal they may be frequently seen, as already stated, to be continuous with the small cells or nuclei above described. Some of them run directly backwards across the posterior commissure and along the posterior median fissure; some extend forwards across the *anterior* commissure to the inner edges of the anterior *columns*, and may be traced in part to blood-vessels and pia mater contained in the median sulcus; while others run off in a lateral direction and are lost in the grey substance. In the *filum terminale*, where this substance, as already remarked, is reduced to a narrow fringe (figs. 24 & 25, Plate XXIV.), their actual destination may often be very satisfactorily traced by means of my method of preparation. Here they may be seen, sometimes at right angles, and sometimes more obliquely to the axis of the cord, to pass *through* the *grey substance* and then through the *white columns* to join the connective tissue, every part of which is interspersed with nuclei like those in continuity with the fibres *in the central fringe*, which *here* evidently consists of the same kind of tissue as the light-coloured space surrounding the canal in other regions of the cord.

HANNOVER regards the cells which line the cerebral ventricles as true *nerve-cells*, and BIDDER takes the same view of those round the spinal canal; while STILLING considers them as epithelium, but, nevertheless, believes that the fibres which they give off form elementary parts of the primitive *nerve*-fibres and *nerve*-cells, with both of which, according to him, they are directly continuous. He professes to have seen the peripheral ends of two epithelial-cells unite with each other after a shorter or longer course, and then enter a *nerve*-cell; or the process of a *nerve*-cell divides into two or three branches, which end in two or three epithelium-cells†. By the most careful examina-

mentous or connective tissue, in opposition to STILLING, who considered it to consist of *grey nerve-fibres*, and called it the circular *commissure*, but now calls it the "substantia gelatinosa centralis" (see Philosophical Transactions, Part II. p. 614, 1851).

* I have found the same in the Cat.

† "[illegible] sieht Man die peripherischen Enden zweier Epithelialcylinder in kürzerer oder längerer

tion of some hundreds of preparations, I have never been able to perceive that the epithelial-processes are connected with any other than the *small* cells or nuclei which I have already described. I have sometimes seen the process of a *large nerve*-cell extend close up to the epithelium, but I have generally succeeded in tracing it round the canal to the *opposite side* of the cord*. If the processes of the epithelial-cells were directly continuous with, and formed elementary parts of, *nerve*-cells and *nerve*-fibres, we might reasonably expect to find the number of the former always in proportion to that of the latter; but the very reverse is the case; for, as we have just seen, in the *filum terminale*, where both *nerve*-cells and *nerve*-fibres have *entirely disappeared*, the *canal* is much *larger*, and the *epithelial-cells* are consequently much more numerous than in any other region; while, as I have already shown, their processes may be traced through the surrounding white substance as far as the surface of the cord.

I have nothing to add to the description which I formerly gave of the anterior roots of the spinal nerves, except that in the *conus medullaris* I constantly find that some of the fibres of these roots terminate *in loops* round the group of cells in the anterior cornu; but whether they be really *nerve*-fibres I have not quite satisfied myself. Sometimes they form loops between two different bundles of roots, and sometimes a fibre from one bundle returns in a loop to the inner or outer side of the anterior column. But in the same region of the cord I have seen the processes of the nerve-cells extend so frequently into the anterior roots, that there can be no doubt that some of the latter arise from them†. In the cervical and lumbar regions of the Tortoise, but especially in the lumbar enlargement, the course and connexions of the processes proceeding from the cells of the anterior cornua are extremely interesting. Fig. 44, Plate XXV. represents a transverse section of one lateral half of the grey substance, through the middle of the lumbar enlargement. From the whole of its lateral border numerous large bundles of fibres proceed outwards and form with each other a beautiful network between fasciculi of the antero-lateral column. Some of these fibres converge from the central parts of the grey substance; the rest proceed from the roots of the nerves, and from the cells of both the posterior and anterior cornua, but particularly from the latter. The large cells of each anterior cornu form a considerable group; they are angular, sigmoid, crescentic and fusiform, and *elongated* to an *extraordinary* degree in a direction obliquely backwards. The processes from their anterior extremities are continuous with the anterior

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Entfernung vom Mittelstück mit einander verschmelzen, und in eine Nervenzelle übergehen, resp., den Fortsatz einer Nervenzelle sich dichotomisch oft trichotomisch theilen und in zwei oder drei Epithelialcylindern endigen." "Es ist mir aber sehr wahrscheinlich, dass diese Ausläufer oder peripherischen Enden der Epithelialzellen als Elementarröhrchen der betreffenden Primitivnervenfaser oder Nervenzelle, in welche sie eintreten, bestehen."—Neue Untersuchungen, Erste Lieferung, p. 11 (1857).

* In the coccygeal region of the cord, processes from the nerve-cells in the anterior cornu may be very distinctly seen to cross both in *front* and *behind* the canal to the *opposite* side.

† I never, as SCHRÖDER VAN DER KOLK asserts, *denied* the connexion of nerve-fibres with processes of cells, but simply maintained that the occasional extension of these processes into nerve-roots was not a sufficient proof of such a connexion; for they *might* be distributed to blood-vessels.

roots of the nerves, or escape through the anterior columns; while those from their posterior ends extend backwards to different distances, and form a continuous stratum of fibres along the lateral margin of the grey substance: some of them join the fibres which radiate from the centre, and contribute in succession to form the bundles which escape as a network through the antero-lateral column; others extend as far as the *posterior* cornu, and seem to be continuous with the *posterior* roots which escape through the postero-lateral fissure. The rest of the posterior roots partly descend towards the centre of the anterior cornu, and partly cross the posterior cornu diagonally in connexion with a chain of cells, to be continuous with the transverse commissure. In the cervical enlargement (fig. 45), the cells of the anterior cornu are more numerous and more diversified in shape; some of them send processes towards the anterior commissure, but a great number have no apparent connexion with it.

In my first communication on the *spinal cord*, I stated that the posterior roots of the spinal nerves are attached *exclusively* to the *posterior* white columns. I have since found that the same assertion was made by ROLANDO: he says, "tutti i fili nervosi delle radici posteriori se staccano esclusivamente dai cordoni posteriori del gran funicolo spinale; ciò che può essere comprovato in vario e differenti maniere*." STILLING denies the fact, and maintains that in the lumbar region a few fibres traverse the posterior part of the antero-lateral column. I have again carefully gone over the ground, in the cord both of Man and different animals, and find that my statement is correct. STILLING has probably confounded nerve-fibres with the blood-vessels which traverse the posterior part of the antero-lateral column much in the way that he represents.

APPENDIX.

[Added during the printing of the Paper.]

Convinced of the superiority and value, in many respects, of my method of anatomical research, I will here describe it more fully, together with the modifications which I have lately introduced in some stages of the process, and the precautions necessary to be observed in order to ensure success.

The structure or part intended for examination should be as fresh as possible, and cut into portions as small as is compatible with the end in view. These portions I formerly hardened by means of a mixture of one part of spirit of wine and three parts of water, which at the end of twenty-four hours was replaced by a fresh mixture of equal parts of spirit and water, and this again after the same interval was replaced by pure spirit, which ought to be renewed every five or six days. At the end of ten to fourteen days the medulla is sufficiently hard for making sections, which are then subjected to the following process for the purpose of inducing transparency. The sections are first placed in a mixture of one part of strong acetic acid, and four, five, or six of spirit, for a period varying from two or three to ten minutes, according to their thickness. They are then washed in pure spirit, after which, if thin, they are floated on the surface of spirit of turpentine, where they remain until they are quite, or nearly transparent, when they are removed to glass slides on which a little Canada balsam has been previously dropped. If now examined under the microscope, they frequently show but little or no traces of cells or fibres,—a circumstance which seems to have *at first* caused Schröder van der Kolk and some others to abandon the method; but if the sections be set aside for some time and treated occasionally with a little turpentine and Canada balsam, the cells and fibres reappear and present a beautiful appearance. Before they are finally covered with thin glass, they should be examined at intervals by the microscope. If the sections be *thick*, I find it best to place them in a shallow vessel, the bottom of which is kept simply wet with turpentine, which can therefore ascend from below while the spirit evaporates from their *upper* surfaces; for the *principle* of the method is this,—to replace the spirit by turpentine, and this by Canada balsam without *drying* the sections. The method at first presents some difficulties, and practice is necessary to ensure complete success. Experience, also, will suggest, according to circumstances, many little deviations from the exact rules here given, which to a certain extent must be considered as general.

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* *Op. cit.* p. 82.

For the last three years I have used chromic acid instead of spirit in the process of hardening. This is one of the modifications mentioned in my memoir "On the Medulla Oblongata"[*]. The medulla of Man and the higher Mammalia is steeped in a solution of one part of crystallized chromic acid in 200 parts of water, for two or three weeks, and then kept in a solution of about one part of bichromate of potash in 100 or 200 parts of water[†]. Spirit is used to wet the knife in making the sections, which are first placed in spirit for a few minutes, and then (with or without the previous use of acetic acid) transferred to the turpentine and Canada balsam, as before.

Lenhossék, Gerlach, and quite recently Schröder van der Kolk, have adopted this method of rendering sections transparent. Lenhossék uses spirit for hardening the medulla, with some slight modification in other stages of the process. An entire series of very beautiful preparations of the cord and medulla oblongata have been purchased of him by the Royal College of Surgeons of London. Gerlach uses bichromate of potash in the process of hardening; then acetic acid, spirit, and Canada balsam[‡]. He does not *mention* turpentine; but if this be dispensed with, the Canada balsam must be very thin, and the section must be placed on its surface to allow of the evaporation of the spirit, which will not mix with the balsam, but in contact with it becomes turbid and opake.

In his Essay on the Spinal Cord published in 1854, Schröder van der Kolk says that my (second) method, "which appears to have been extremely successful," did not succeed with him; "the preparation became only partially clear, and the ganglionic cells, from which the fat seemed to be removed, were rendered indistinct*." It is perfectly clear that he had not sufficient practice in the process. He says again, in speaking of LENHOSSÉK, "he follows CLARKE's method with turpentine, which, *according to my experience*, cannot lead to correct results, as the spinal marrow is too much altered by that agent†." After criticising every other method, he describes his own, which consists in the use of concentrated chloride of calcium, and which, according to his experience, "deserves to be preferred to all the others." But now we find that he has abandoned it, and adopted *my* method in conjunction with the colouring process recommended by GERLACH; for in a supplementary paragraph to the English translation which has just been issued by the Sydenham Society (1859), he states that after some trials he finds this mode of making preparations the most satisfactory, and in many respects still better than with *chloride of calcium*. The section, after having been coloured with solution of carmine and washed in spirit, is laid in a watch-glass and allowed to dry a little, after which, "a couple of drops of spirit of turpentine are added. If it is now left for twenty-four or forty-eight hours without a covering glass, but protected from the dust, the spirit evaporates, and the object becomes perfectly clear. It is now covered with Canada balsam and a glass‡." The colouring with carmine is often very useful, but I think it rather interferes with the sharpness of the fibres. I have long used, occasionally, a colouring fluid known to painters and wood-stainers by the name of *archel*. When carmine is used, the solution should be previously filtered, otherwise a deposit or crust is apt to form on the object; and for the same reason, the section, after having been coloured, should be washed in water before it is placed in the spirit, which readily precipitates the carmine.

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[*] 1857. Philosophical Transactions for 1858, Part I.

[†] For the Rodentia, Birds, Reptiles, and Fishes, it is necessary to use the solution much weaker—about one part to 600 of water, and gradually increase the strength at the end of a week,

[‡] Mikroskopische Studien, p. 2, 1858, Erlangen.

STILLING has given a full, but somewhat inconsistent and incorrect criticism on my method. He concludes by saying, "While it is allowed that by the employment of CLARKE's method many truths indeed in reference to the minute structure of the spinal cord may be brought to light, and that CLARKE's labours must be said to be quite trustworthy (recht anerkennenswerthe), it must nevertheless be observed that the condition of certain textures, such as the elementary structure of the primitive nerve-fibres and the nerve-cells, are so altered by it, that this method in many respects must be considered as an obstacle to the more exact kind of investigation§."

Now, although I am not so blindly prejudiced as to maintain that preparations made by this method are in every respect perfect, I do maintain that it produces but little alteration from the natural appearance of the nerve-cells, and that almost the only structure that appears to suffer more than from chromic acid alone is the white substance of the nerve-fibre; and even this may be perfectly preserved if the medulla is placed first in a weak solution of chromic acid and then in a solution which is much stronger. Fig. 1, Plate XIX. was drawn from a preparation made in this way, and of which it is as exact a copy as possible. But even the *loss* of the *white substance* would be of no consequence for tracing the *course* of the nerve-fibres, the axis-cylinders of which in these preparations are rendered unusually strong and distinct. And if I wish to investigate the *natural* structure of the nerve-fibres and cells, I select a perfectly fresh and unaltered specimen from an animal just killed (as I stated in my first communication to the Royal Society, 1851); for I never think of trusting to any kind of preparation, or even to chromic acid, which STILLING uses, and which coagulates the contents of the nerve-fibres and alters their appearance after a short maceration. Not that I reject other methods; for I find sections of the medulla, simply hardened in chromic acid, of great service, and use them largely, particularly for drawing figures under low powers, and sometimes for examination under higher powers, when I have been in some doubt as to the nature of the tissue. In fact, I employ whatever means appears most suitable for the occasion and most likely to lead me to the truth. But I still contend, that for clearness, sharpness of outline and fine definition under the higher powers,—for the advantage of obtaining *thick* sections of *great transparency*, and for the durability and unchangeableness of the preparations when properly made, I know of no method that will bear any comparison with this. To confirm this opinion in one respect, I may mention the fact, that although STILLING, in his recent work, has described and represented the spiral structure of the fibres of the white columns of the *Calf* under a power of 1100 diameters, he has failed to detect the nucleated cells and nuclei which are so beautifully seen by means of my method under a power of 350 diameters.

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* Translation of Sydenham Society, 1859, p. 30.

† Ibid. p. 28.

‡ *Loc. cit.* p. 33.

§ Neue Untersuch., 5 Lief. p. 1071.

EXPLANATION OF THE PLATES.

Fig. 1, Plate XIX. represents a transverse section of the *caput cornu posterioris* and part of the *cervix* cornu, with a portion of the surrounding posterior and lateral white columns: from the middle of the cervical enlargement of the Ox; drawn from a preparation magnified 50 diameters:—A, A, A, part of the posterior white column; B, B, part of the lateral column: the small circles and dark spots represent the cut ends of longitudinal and oblique fibres. These columns are traversed by a beautiful network of blood-vessels and nerve-fibres radiating from all parts of the grey substance. At *b*, *b* (the peak of the cornu) may be seen extending outwards the processes of nerve-cells situated in the network between bundles of the white column. The *darker* portion of the *caput* cornu consists principally of numerous longitudinal and oblique bundles of fibres, with blood-vessels, and of the posterior roots of the nerves, with cells chiefly of the small and intermediate size; the *lighter* space behind and at the sides is the *gelatinous substance*, studded with small cells, and having a stratum of *large* cells and marginal nerve-fibres along its posterior border next the posterior column, into which some of the processes of the cells may be seen to extend. C, C, part of the *cervix* cornu, containing some large cells, and crossed in different directions by the posterior roots of the nerves, and by fibres which radiate to the white columns. At the junction of the *cervix* and *caput* cornu, near the right border of the grey substance, are five large bundles of longitudinal fibres, interlaced by transverse fibres, and more or less encircled by large elongated nerve-cells, which send some of their processes longitudinally.

Fig. 2, Plate XX. represents a transverse section of the grey substance from the middle of the dorsal region of the Ox; drawn from a preparation magnified 60 diameters:—D, D, posterior cornua, the points of which are cut off; on the left side, the letter D is in the darker portion of the *caput* cornu. The nerve-cells are shown chiefly on the right side. *g*, the gelatinous substance; E, E, anterior cornua; F, tractus intermedio-lateralis of right side; two of its cells are seen extending into the lateral column towards the letter F; G, central canal, lined with epithelium, from which fibres radiate in connexion with small cells and nuclei, through the surrounding transparent space, H; behind H is the posterior transverse commissure, the fibres of which are in connexion with fusiform cells, and partly extend outwards to the tractus intermedio-lateralis, F; I, two bundles of longitudinal fibres, cut off from the anterior columns by transverse fibres of the anterior commissure, K, K; L, anterior median fissure; *l*, posterior median fissure; P, P, bundles of posterior nerve-roots; M, M, cylinders of the *posterior vesicular columns*; N, three bundles of longitudinal fibres near the margin of the grey substance, and two of which are seen to be partially embraced by nerve-cells; T, fibres of the posterior commissure extending outwards; *n*, outer side of cervix cornu posterioris of left side, where there is a decussation between the fibres of the posterior roots, in connexion with the posterior vesicular columns and the posterior commissure; o', anterior roots of the nerves.

Fig. 3, Plate XXI. Transverse section of the grey substance of the Ox, at the upper part of the dorsal region, where the posterior cornua are beginning to separate from each other in the middle line:—F, tractus intermedio-lateralis, extending inwards towards the posterior transverse commissure, and the front of the posterior vesicular column.

Fig. 4, Plate XX. A similar section, higher up, near the lower end of the cervical enlargement.

Figs. 5 to 21 are transverse sections of the grey substance of the spinal cord, from a Woman aged 33 years.

Fig. 5, Plate XXI. Section near the middle of the dorsal region:—F, tractus intermedio-lateralis; M, posterior vesicular column.

Fig. 6, Plate XXI. Another, at the upper part of the dorsal region.

Fig. 7, Plate XXI. A similar section through the lower part of the cervical enlargement; the tractus intermedio-lateralis, F, rests on the lateral projection of the anterior cornu, behind a group of large vesicles, and is connected by a network with the *cervix* cornu posterioris.

Figs. 8 & 9, Plate XXI. Sections of the lateral half of the grey substance, successively nearer the middle of the cervical enlargement:—N, large bundles of longitudinal fibres near the lateral edge of the grey substance.

Fig. 10, Plate XXI. Lateral half of the grey substance, through the middle of the cervical enlargement: the tractus intermedio-lateralis is no longer visible, but the lateral projection of the anterior cornu, on which it rested, is still connected by a network with the side of the cervix cornu, which is pierced by several bundles of longitudinal fibres, N.

Fig. 11, Plate XXI. A similar section of the grey substance at the fourth cervical nerves:—N, longitudinal bundles of the lateral column, enclosed in the meshes of a network proceeding from the grey substance, and containing small cells; through this network the roots of the spinal-accessory nerve proceed from behind forwards to the *tractus intermedio-lateralis*, F, and to the cells of the anterior cornu; P, posterior roots of the spinal nerves; M, posterior vesicular column traversed by bundles of the posterior roots, and by the transverse commissure.

Fig. 12, Plate XXV. Similar section at the upper part of the third cervical nerves, magnified 60 diameters:—N, N, N, network proceeding from the side of the posterior cornu and tractus intermedio-lateralis, containing small cells and enclosing bundles of fibres of the lateral column, between which the roots of the spinal-accessory nerve proceed forwards to the *tractus intermedio-lateralis*, F, and to the anterior cornu; the extremity, and particularly the outer portion, of the anterior cornu is pierced by numerous longitudinal bundles (represented by the dark round or oval masses), between which the fibres of the anterior roots (O′) cross each other in an intricate plexus; amongst this plexus are several large nerve-cells, which partially embrace the bundles and send some of their processes longitudinally.

Fig. 13, Plate XXI. Transverse section of one lateral half of the grey substance at the upper part of the lumbar enlargement; the posterior vesicular column, M, is larger than in any other region.

Fig. 14, Plate XXI. A similar section, nearer the middle of the lumbar enlargement.

Fig. 15, Plate XXI. Another section, through the middle of the same enlargement: the posterior vesicular column, M, is composed almost entirely of small cells; the lateral part of the cervix cornu is pierced by several bundles of longitudinal fibres.

Fig. 16, Plate XXIII. A similar section, through the lower part of the lumbar enlargement.

Fig. 17, Plate XXIII. Section through the upper part of the conus medullaris.

Figs. 18, 19, 20 & 21, Plate XXIII. Sections through the remaining portion of the conus medullaris.

Fig. 19. The *gelatinous substance*, *g*, is continuous across the middle line beneath the posterior columns, A; D, opake portion of the caput cornu; N, bundles of longitudinal fibres.

Fig. 20. The posterior white columns (A, A) are wanting in the middle line, and terminate by a thin edge on each side of the gelatinous substance (*g*), which rises to the surface in a straight transverse layer, covered only by pia mater, *m*; D, are the dark bundles which in the preceding figures formed the opake portion of the caput cornu; similar, but less numerous, bundles are scattered through the rest of the grey substance anteriorly.

Fig. 21. The spinal canal has extended backwards and sideways in the form of T; its lateral extension is through the *gelatinous* substance, which thus becomes continuous with the space previously surrounding the central canal, while the part corresponding to the opake portion of the caput cornu (D) is now in front and at the side of the canal. The remains of the gelatinous substance are covered and permeated by a quantity of pia mater, *m'*; indeed they now seem to consist of little or nothing else; this pia mater is continuous with that surrounding the cord, *m'*. E, the anterior grey substance, forming a network between longitudinal bundles of fibres.

ig. 22, Plate XXIV. Transverse section of the *conus medullaris* of the Ox, at the first pair of coccygeal nerves; drawn from a preparation magnified 60 diameters:—A, A, posterior columns; B, B', antero-lateral columns. From the grey substance a beautiful network of nerve-fibres with blood-vessels extends through the white, especially the antero-lateral, columns (B, B'), and encloses in its meshes a multitude of longitudinal bundles. From the extremities of the anterior cornua, considerable offsets of the grey substance extend through the anterior white columns, which are interspersed with large nerve-cells nearly to the surface of the cord. Fibres from around the canal, joined by others proceeding from the epithelium, run forwards along the sides of the anterior median fissure to the circumference. In the posterior cornu are many large bundles of longitudinal fibres; one of these is partially encircled by a cell; several other longitudinal bundles are seen amongst the fibres of the posterior commissure.

ig. 23, Plate XXIV. Transverse section through the lower part of the *conus medullaris* of the Ox; drawn from a preparation magnified 60 diameters:—A, A, posterior white columns; B, B, antero-lateral columns; G, central canal; *a*, *a*, *a*, posterior grey substance; E, anterior grey substance: the whole of the grey substance, on each side of the canal (which is here very large), is pierced by a multitude of longitudinal bundles of fibres of different shapes and sizes (represented by the dark masses), and is interspersed with a number of *small* cells; a small group of *large* cells is found at its anterior extremity on each side.

Fig. 24, Plate XXIV. A similar section, near the end of the *conus medullaris* of the Ox. It appears to be altered in shape by some accidental cause; for in its natural state the canal is nearly of the same form as in fig. 23:—A, A, posterior white columns; B, B, antero-lateral columns. The grey substance around the canal is reduced in quantity, and appears as a kind of network enclosing a multitude of longitudinal bundles; it is also interspersed with small cells, but contains only a few of *a larger kind*, near its anterior extremity.

Fig. 25, Plate XXIV. Transverse section of the grey substance of the *filum terminale*, or filiform extremity of the cord, magnified 60 diameters. The canal is collapsed laterally, but is naturally very large, and the epithelium is consequently abundant. The grey substance surrounding it is reduced to a mere fringe, pierced here and there by small longitudinal bundles, and interspersed with small cells or nuclei:—*a*, *a*, processes of pia mater, and blood-vessels radiating from the border of the grey substance through the white columns towards the surface.

Figs. 26 to 36, Plate XXIII. represent transverse sections of the spinal cord of the domestic Fowl.

Fig. 26. Section at the first pair of cervical nerves.

Fig. 27. Section in the middle of the neck.

Fig. 28. Section at the upper end of the cervical enlargement.

Fig. 29. Section through the middle of the cervical enlargement.

Figs. 30 & 31. Sections through the lower half of the cervical enlargement.

Fig. 32. Section through the middle of the dorsal region.

Figs. 33, 34 & 35. Sections through the upper half of the sacral enlargement.

Fig. 36. Section through the middle of the sacral enlargement.

In fig. 34 the posterior median fissure (*l*) begins to widen towards the surface, so as to form the commencement of the sacral ventricle; and through it the pia mater descends and expands into a mass which surrounds the canal, and replaces the posterior transverse commissure. In fig. 35 is seen an extension of the same changes. In fig. 36 the posterior columns, with the grey substance, are thrown widely apart, so that the posterior median fissure dilates into the broadest portion of the ventricle, which is filled by a large mass of the pia mater (*m*). The lateral halves of the grey substance are consequently united only by the anterior commissure (K), in which there is a very evident decussation. Some of the decussating fibres may be traced into the cervix cornu posterioris. METZLER, whose observations are in general in accordance with my own, states, however, that a real decussation is found only in the *sacral* enlargement, and in the upper part of the cervical region;

but I have certainly found a decided but slighter decussation in the *cervical enlargement.*

Figs. 37 to 45 represent transverse sections of the spinal cord of the Tortoise.

Fig. 37, Plate XXII. Section at the first pair of cervical nerves. A network (N) is prolonged from the sides of the grey substance into the antero-lateral columns, enclosing bundles of their longitudinal fibres.

Fig. 38. Section through the upper part of the cervical enlargement.

Fig. 39. Section through the middle of the cervical enlargement.

Fig. 40. Section through the middle of the dorsal region.

Fig. 41. Section through the middle of the lumbar enlargement.

Figs. 42 & 43. Sections through the lower part of the lumbar enlargement.

Fig. 44, Plate XXV. Transverse section of one lateral half of the grey substance, through the middle of the lumbar enlargement, of the Tortoise, magnified 80 diameters:—D, posterior cornu; E, anterior cornu; O′, anterior roots of the nerves; P, posterior roots; B, B′, antero-lateral column, divided into separate fasciculi by a beautiful network, formed by nerve-fibres and processes of nerve-cells from the grey substance.

Fig. 45, Plate XXV. Similar section through the middle of the cervical enlargement of the Tortoise, magnified 80 diameters. The cells of the anterior cornu present a more varied shape than in the preceding figure; and the network in the antero-lateral column is neither so considerable nor so uniform as in the last figure.

Fig. 46, Plate XXII. Cells from the *gelatinous* substance of the Ox, magnified 350 diameters:—*a*, *b*, cells of small and intermediate size; *c*, round, oval, and fusiform nuclei of connective tissue.

Fig. 47, Plate XXII. Large cells from near the posterior border of the same, magnified 220 diameters.

Fig. 48, Plate XXII. Nucleated cells and free nuclei from the white columns of the spinal cord of a Calf, between three and four months old, magnified 420 diameters:—*a*, cut end of a nerve-fibre, showing a kind of spiral or concentric arrangement around the axis-cylinder; *b*, *b*, nucleated cells partially encircling the nerve-fibres, and attached to their sheaths; *c*, *c*, separate nucleated cells, some with ramifying processes; *d*, two nucleated cells from a longitudinal section; *e*, free granular nuclei.

Fig. 49, Plate XXII. Large cells from a *transverse* section of the *posterior* vesicular columns of the Ox, magnified 220 diameters:—*a*, one of the processes running in a nearly longitudinal direction.

Fig. 50, Plate XXII. Large cell from a *longitudinal* section of the posterior vesicular columns, magnified 220 diameters:—*a*, process extending towards the posterior nerve-roots; *b*, another extending towards the anterior cornu.

Fig. 51, Plate XXII. Transverse section of the human *tractus intermedio-lateralis*, from near the middle of the dorsal region, magnified 220 diameters:—*c*, the inner portion, adjoining the grey substance between the anterior and posterior cornua (see fig. 6, F, Plate XXI.); *b*, the free extremity extending transversely across the lateral column; *a*, *a*, *a*, fasciculi of longitudinal fibres of the lateral column. The tractus is interspersed with cells, from some of which the processes escape with nerve-fibres into the lateral column; while others are continuous with nerve-fibres proceeding from the transverse commissure and other deep parts of the grey substance. At its inner end (*c*), some of the cells with their processes are elongated towards the anterior and posterior cornua (compare fig. 6, Plate XXI.).

Fig. 52, Plate XXII. Cells from a transverse section of the *tractus intermedio-lateralis* of the Ox, magnified 220 diameters.

Fig. 53, Plate XXII. Epithelial-cells from the spinal canal of the Ox, magnified 400 diameters:—*a*, isolated cells (the letter is placed over their free, ciliated end); *b*, same cells packed together *in situ*, and bearing cilia, which project into the canal.

Fig. 54, Plate XXII. Epithelium from the spinal canal of the Tortoise:—*a*, *b*, separate cells, magnified 400 diameters; G, part of spinal canal, with the cells disposed around it, and a little less magnified; *c*, free nuclei from the white columns: magnified 400 diameters.

Fig. 55, Plate XXII. Human spinal canal from the cervical region. Its outer parts are filled up with a heap of epithelial-nuclei, in the midst of which are two secondary canals, surrounded by the usual regular layer.

Fig. 56, Plate XXII. Longitudinal section of part of the *tractus intermedio-lateralis*, magnified about 200 diameters. The cells are elongated longitudinally at the verge of the lateral column, the fibres of which are accompanied by their processes.

XXIII. *Researches into the Nature of the Involuntary Muscular Tissue of the Urinary Bladder.* *By* GEORGE VINER ELLIS, *Professor of Anatomy in University College, London. Communicated by Dr.* SHARPEY, *Sec. R.S.*

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DURING the last few years anatomists have considered the muscular substance of the bladder to be composed of elongated contractile fibre-cells, each with a nucleus in it, which possess free ends, and overlap at their pointed extremities without being united or joined together. This notion of the cell-structure originated with Professor KÖLLIKER; and it has since received very general acceptance[*]. From the correctness of that opinion I am led to dissent by my researches; and I purpose to show in this communication to the Royal Society, that both the involuntary muscular tissue of the bladder and the voluntary muscular substance in other parts of the human body have a like composition.

In a former paper to the Society (in June 1856)[†] I made the announcement that the views now brought forward of the muscular structure of the bladder were applicable to the involuntary muscular tissue in general; but as my declaration was received with doubt, I determined to withhold its publication until I had been able to repeat my microscopical observations. Before this time I hoped to have completed the task imposed on myself, but occupation has left me leisure enough to examine thoroughly only the muscular structure of the urinary bladder. As my idea is confirmed by the result of the second examination of that viscus, I submit this paper with greater confidence to the consideration of the Society; and I hope to furnish at a future time additional researches into the nature of the involuntary muscular tissue in other parts of the body.

After a lengthened inquiry, I am emboldened to entertain opinions at variance with the doctrine received at the present day respecting the cell-nature of the muscular substance of the urinary bladder, and to conclude that this material is composed of lengthened fibres with fixed and tendinous end-attachments. According to this view, the muscular wall of the bladder and the voluntary muscles will be formed of similar threads or fibres. This resemblance between the two can be best demonstrated by considering the arrangement of the fibres when in mass, the characters of the same when separate, together with the length, end-attachments, and the structure of the component fibres in each kind of muscular substance.

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[*] Professor KÖLLIKER, in his "Treatise on the Nature of the Involuntary Muscular Tissue in general," says of that in the urinary bladder, "Die Elemente der Muskeln der Blase sind überall die bekannten Faserzellen. In den gelbröthlichen Bündeln, die den Detrusor und Sphincter vesicæ, sowie die schiefen Fasern zusammensetzen, sind dieselben äusserst zierlich spindelförmig und oft wellenförmig auslaufend 0·05—0·07‴ lang, 0·002—0·004‴ breit, mit charakteristischen Kernen von 0·006—0·009‴ Länge."—Zeitschrift für Wissenschaftliche Zoologie, Erst. Band, S. 64. Leipz. 1848.

[†] See the Abstract in the 'Proceedings' of the Royal Society, vol. viii. p. 212.

Arrangement of the Muscular Fibres.

In both kinds of muscular tissue (voluntary and involuntary) the fibres are arranged after the same plan.

In the voluntary muscles the bundles of fibres communicate freely with each other, producing a net-like arrangement with meshes, as in Plate XXVI. fig. 2, which represents this plexiform condition in the sartorius muscle of the thigh. This interweaving of the fibres of contiguous bundles existed in all the muscles submitted to examination, and muscles varying in length, breadth, and thickness were designedly selected for comparison.

In the involuntary muscular layer of the bladder the fibres form fasciculi with an angular outline on a cross section, and are surrounded each by a strong fibrous sheath, as in the voluntary muscle. All fibres in the same stratum have the same general direction, either circular or longitudinal, and those of contiguous bundles intercommunicate often, as in Plate XXVI. fig. 1, and give rise to a web with meshes. Not only are the adjacent fasciculi of the same stratum united in the manner specified, but the different strata in the wall of the bladder are joined at innumerable points,—the longitudinal fibres of one layer becoming transverse in another*. The interchanging bands unite in one compact whole the vesical strata; and the same must be cut through before the more superficial can be detached from a deeper muscular layer. The meshes produced by the interweaving of the fibres are smaller than in the voluntary muscle, and those in the deeper parts are closer or finer than the corresponding intervals in the superficial layer. In consequence of this intimate interchange amongst the bundles, a fibre cannot be detached so readily or for so great a distance in the involuntary, as in the voluntary muscular tissue.

Each bundle of fibres of the bladder is marked at short intervals by a tendinous intersection more or less complete, as is the rectus abdominis muscle of the human body. See Plate XXVI. fig. 1, which represents some of the superficial tendinous points dissected out. These small tendons are best recognized in a contracted viscus with rather red fibres; and though they are present throughout the whole muscular substance, they escape detection by the naked eye in the deeper layers in consequence of the general paleness of the texture. In some of the muscular fasciculi the tendons exist only on the surface, involving a few of the fleshy fibres of a fasciculus; whilst in other bundles, and at other spots of the same bundle, they extend through the whole thickness. Where the larger bundles of fibres communicate, there is usually a tendon to be found.

The tendinous points may be placed at tolerably regular or at different distances from each other, the closeness varying from $\frac{1}{10}$th to $\frac{1}{2}$ an inch. When a fasciculus is free for some distance from the usual communicating bundles, the tendons are about $\frac{1}{4}$th of an inch apart. In length they average $\frac{1}{12}$th of an inch (varying from $\frac{1}{20}$th to $\frac{1}{10}$th); and their breadth, which is not quite so great as that of the bundles of fibres bearing them, ranges, according to the difference in size of the bundles, from $\frac{1}{20}$th to $\frac{1}{10}$th of an inch.

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* Transactions of the Royal Medico-Chirurgical Society for 1856, vol. xxxix. p. 328.

On inspection with the naked eye or with a low magnifying power, the tendons may be seen to be formed of white threads, which project unequal distances amongst the fleshy fibres, each receiving a certain number of the fibres, and do not therefore give a defined transverse line across the muscular fasciculus. Under a low power of the microscope, the fleshy fibres, when traced onwards, may be observed to advance into the tendinous mass, and, becoming smaller and fewer in number, to leave a spot where only the common fibrous texture serves as the bond of union between the fleshy bundles entering the tendon at opposite ends. Commonly, however, some fleshy fibres will be seen extending all along the object under examination, in consequence of their reaching different distances in the tendon. How the muscular is connected with the fibrous tissue will be subsequently described.

The tendons above noticed resemble those in the voluntary rectus abdominis muscle in their structure, though, from being placed at unequal distances on the fasciculi of the fibres, they do not form a transverse band as in that muscle; and they are also like them in their use, serving as fixed points for the contracting fleshy fibres.

Similar tendinous points amongst the fleshy bundles of the œsophagus were described in my paper of 1856: these reach entirely or only partly through the bundles of fibres, as in the bladder; they measure in length $\frac{1}{20}$th to $\frac{1}{10}$th of an inch, but in width only about $\frac{1}{80}$th of an inch; and they are distant $\frac{1}{2}$ to $1\frac{1}{2}$ inch from each other. In these both the voluntary and the involuntary fibres of the gullet are collected.

Characters of the Muscular Fibres.

When some fibres of a fasciculus of the bladder have been freed from the surrounding sheath and separated carefully from the rest, they will present the under-mentioned characters with an object-glass of $\frac{1}{8}$th of an inch focus.

Form.—The fibres are slender rounded strings, like the fibres of the voluntary muscle, and present at intervals corpuscles or nucleus-like bodies. If the object under examination is but little disturbed in the attempts to detach it with needles, the pale fibres will appear generally of uniform width. With transmitted light the surface is distinctly granulous or longitudinally striated, and without a dark limiting border. On a cross fracture the end is either solid and dotted, or jagged with separate points, as in a fibre of the voluntary muscle under similar conditions. Not uncommonly many of the fibres under inspection are contracted here and there, and wavy at the narrowed part, as in Plate XXVI. fig. 3 *a*; so that, supposing a fibre to be broken into pieces, the fragments would produce objects with pointed ends and nucleus-like bodies (fig. 3 *b*), resembling the muscular fibre-cells of Professor KÖLLIKER. When a fibre is contracted, it is like a string of beads in its outline.

Variations in the characters above given may be produced by the manipulation employed in the preparation of the microscopic object: thus if the piece of glass covering the object is pressed down too much, the fibres will be easily flattened in consequence of their great softness. Should a fibre be broken across, it may be bent or twisted, or may differ in width at intervals, narrow wavy parts alternating with larger; or it may be shortened with slightly waved edges and transverse darkened lines; and its free end or ends may be pointed and round, or flattened and irregular like the end of a broken piece of string.

Size.—The fibres vary greatly in diameter in the two kinds of muscular substance.

In the voluntary orbicularis oris muscle of the human body, the smallest fibres measure $\frac{1}{2333}$rd, and the largest $\frac{1}{700}$th of an inch across; and three other sizes can be recognized between those extremes.

In the involuntary fibres of the bladder the smallest measure $\frac{1}{7000}$th, and the largest $\frac{1}{3500}$th of an inch in diameter, whilst the average width is $\frac{1}{5000}$th of an inch. Most preservative fluids diminish the width of the fibres.

Corpuscles.—Elongated granular bodies exist in all muscular tissue; and in the involuntary muscle, as in the bladder for instance, they have been described by Professor KÖLLIKER as the nuclei (Stäbchenförmige Kerne) of his so-called muscular fibre-cells*.

On the addition of acetic acid to voluntary muscle, two kinds of corpuscles, the one oval, the other fusiform, come into sight†. The oval kind, fewest in number, are very pale, with fine granular contents: they are situate within the sarcolemma of the fibres, and flow therefrom with the disintegrated mass on the action of caustic potass; they measure about $\frac{1}{1400}$th of an inch in length, and $\frac{1}{4000}$th in breadth. The fusiform bodies are commonly pointed at each end, and have a yellowish-green colour, with coarse granular contents and a very dark outline, whilst from one or both points a line of fine granules extends for a short distance. In width they vary from $\frac{1}{20,000}$th to $\frac{1}{14,000}$th of an inch, and in length from $\frac{1}{1400}$th to $\frac{1}{700}$th of an inch. They have a linear arrangement along the fibres, and are at variable distances from each other. Oftentimes they are twisted or subdivided.

Usually only one kind of corpuscle, that corresponding with the fusiform of the voluntary muscle, can be recognized in the bladder; but occasionally others, answering to the oval form above described both in their characters and position, will be observed.

The fusiform corpuscles of the muscular substance of the bladder (the rod-shaped nuclei of cells of Professor Kölliker) should be examined without acetic acid, or with only a very weak mixture of it and water, as when the acid is strong it destroys the characters of the object under the microscope*. These bodies (Plate XXVII. fig. 4) have the same general form, position on the fibres, and appearance as the corresponding bodies in voluntary muscle, and from each pointed end a faint line of granules may be often traced if weak acid has been used; but their contents are more coarsely granular, and they are somewhat larger and better marked, measuring across from $\frac{1}{14,000}$th to $\frac{1}{4000}$th of an inch, and along from $\frac{1}{1400}$th to $\frac{1}{400}$th of an inch.

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* "Zweitens besitzt jede Faserzelle ohne irgend eine Ausnahme einen blassen, in vielen Fällen erst nach Essigsäurezusatz sichtbaren oder wenigstens deutlicher hervortretenden, seltener durch die Säure erblassenden Kern, dessen Grösse und Gestalt sehr eigenthümlich und bezeichnend sind."—Zeitschrift für Wissenschaftliche Zoologie, 1848.

† Reference was made to these in my former paper to the Royal Society.

The number of the corpuscles on a single involuntary fibre of the bladder will depend on the length of the fibre, and their distance apart is greater than in the voluntary muscle. There is not any bulging of the fibre opposite the seat of the granular bodies (see Plate XXVII. fig. 4 *a*). On account of the softness of the muscular fibres and their frequent interweaving, a single fibre can be separated far enough from its fellows to show more than one corpuscle on it only rarely, and with considerable difficulty. With care, however, I have succeeded several times in detaching a piece of a fibre long enough to have two corpuscles, but only once, as in fig. 4 *a*, a fragment of a length sufficient to possess three corpuscles.

The distance of the corpuscles from one another varies in the same fibre, as well as in different fibres. In the fibre with three large corpuscles (fig. 4 *a*), the middle one is $\frac{1}{233}$rd of an inch from one corpuscle, and $\frac{1}{350}$th of an inch from the other. In five other fibres, each with two large corpuscles, of which measurements were taken, in no two was the interval the same, for it varied from $\frac{1}{280}$th to $\frac{1}{93}$rd of an inch, but the average was $\frac{1}{215}$th of an inch.

Some small and imperfect corpuscles, as in fig. 4 *a*, are often seen between the larger and more complete bodies; and occasionally two larger ones are found near together, as if one was cleft in two.

The measurements above given of the distance of the corpuscles from each other, were made on the fibres of a bladder that was very much contracted, of which fig. 1, Plate XXVI. is a representation drawn of the natural size; and it is therefore to be inferred that the distance would be increased in fibres in an uncontracted state, or lengthened in the distended condition of that viscus. Possibly, too, my seeing two and three corpuscles on a fibre may be due to the approximation of those bodies by the shortening of the intercorpuscular spaces in the contracted fibres. In this same small, but very healthy-looking and muscular bladder, some of the fibres presented a coarse granulous appearance, as in fig. 5, Plate XXVII., either through the whole length of the field of the microscope, or only at intervals: this condition was very like the granulous state often seen in voluntary muscular fibre.

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* Little knowledge of these bodies or of the fibres can be obtained by adding acid to a mass of the muscular substance, for the acid takes away all indication of the separateness of the fibres, and allows underlying corpuscles to appear through the superficial and transparent tissue.

Length of the Muscular Fibres.

Actual measurement under the microscope of an entire fibre from the wall of the bladder has been found to be impossible, in consequence of its frequent interweavings with its fellows rendering it inseparable. The length will correspond, it is to be supposed, with the distance between two of the tendinous points into which the fibres are gathered, just as is the case in the voluntary rectus abdominis muscle. Thus the length would vary from $\frac{1}{10}$th to $\frac{1}{2}$ an inch, as the intervals between the tendons, in the greater number of the fibres; whilst in others it may extend over one tendon, as do some fibres before they reach their point of insertion. Considering the length to be determined by the tendinous intersections, it must be borne in mind that this will be greatly influenced by the contracted or relaxed state of the bladder.

By parity of reasoning, the length of the fibres of the œsophagus, where like tendinous points exist, may be estimated.

End-attachment of the Fibres.

The fibres of both the voluntary muscular tissue and the involuntary muscular substance of the bladder have similar end-attachments, by means of fibrous tissue or tendon, as before said.

A voluntary fibre, when about to end in tendon, is surrounded by fibrous tissue, and soon afterwards its component threads or fibrils may be seen to be collected into bundles of varying length and size, which separate slightly as they are continued onwards, like the roots of a tree entering the soil. Each bundle of fibrils is invested with a separate sheath of fibrous tissue, and the aggregate of all the sheaths forms the little tendinous band which is prolonged from each fibre to make up the common tendon of insertion. It appears as if each fibril or thread in the bundle possessed its proper tendinous appendage. The muscular bundles gradually cease to be distinguishable amongst the fibrous tissue; but the central reach further than the circumferential, so that a fibre may appear to have a pointed ending if the surrounding pieces have been broken off in the preparation of the microscopic object. In what way the fibrous tissue is united with the muscular fibre I have not been able to ascertain.

The involuntary fibres of the bladder end in tendinous tissue, like the fibres of the voluntary muscle; but, from their minuteness and softness, I have experienced great difficulty in isolating one, about to cease, from its fellows. Supposing the end of a fibre to be separated, it will be found to terminate as in the voluntary muscle: viz. fibrous tissue is arranged around it, like the long hairs in a horse's tail, whilst further on the fibre appears to divide into pieces, each having its appertaining sheath of fibrous tissue; and lastly, the muscular tissue ceases to be discernible in the fibrous band connecting it with the common tendon (see Plate XXVII. fig. 8). Examination of the fibres entering a tendinous mass does not afford so satisfactory a view of the disposition of the terminal parts as in the case of a detached fibre; but even then they can be observed to divide, and diminishing in size, with oftentimes wavy ends, gradually to cease amongst the surrounding fibrous tissue, as in fig. 7 of the same Plate.

Structure of the Muscular Fibres.

The characters of the fibres of the bladder above detailed do not lend support to the doctrine of the cell-structure, as explained by Professor KÖLLIKER, but rather lead to the supposition that in construction, as in other respects, the two kinds of muscular tissue have something in common; and after careful examination I am induced to believe that the ultimate composition of the two is not dissimilar. To me the fibres seem to be made up of very minute threads—the ultimate fibrils—as in the voluntary muscle. Indications of this composition are manifested by longitudinal marking, both with and without the action of dilute nitric acid*, and by the separation of the component parts when a fibre is about to end in its tendon, as before described. It is not possible to detach at pleasure one of the ultimate fibrils making up a fibre, as in the voluntary muscular tissue, and possibly from some difference in their uniting medium; but partial accidental separation of them in small bundles, about $\frac{1}{14,000}$th of an inch wide, is not very unfrequent.

I have not succeeded in distinguishing any sheath or sarcolemma to the fibres of the bladder; but by the action of dilute nitric acid the fibres present oftentimes a rough and uneven appearance, as if some encasing material was partly removed.

There seems to be also a further resemblance in structure between the involuntary and the voluntary muscle.

In the voluntary muscular fibre the ultimate fibril is composed, as is well known, of small elongated dark pieces—the sarcous elements, which are united by a transparent material, and give rise by their arrangement to the transverse markings or striæ of the fibre.

On viewing a clean and separate fibre of the bladder with a magnifying power of 600 diameters, it may sometimes appear fibrillated, as in Plate XXVII. fig. 4 *a*, but at other times a fibre will have a granular or dotted surface; just as there may be a difference in the surface-marking of the voluntary fibre, the longitudinal striation being most conspicuous in one specimen and the transverse marking in another specimen. In the large and somewhat flattened fibres, the dark spots, though of small size, are lengthened rather than rounded, and have a linear arrangement in a clear matrix (see fig. 6), though only a small portion of the line of dots will be in focus at the same time, in consequence of the high magnifying power employed. These dots seem to me to indicate the existence of small opake masses—sarcous elements of the involuntary muscular tissue of the bladder, which are the representatives of the sarcous elements of the voluntary fibre. By their arrangement they occasion the granular appearance of the surface under a low magnifying power[*].

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* Dilute nitric acid, 20 per cent. strong, has been recommended as suitable for showing the cell-structure of the involuntary muscular substance; but if the microscopist desires to see fibres as they are here described, he must refrain from using that mixture. By the action of the acid the fibres are softened so that they will tear readily in any direction, and are deprived of the tendons attaching their ends.

In the involuntary muscular tissue now described, as well as in the voluntary fibre, the ultimate fibrils will be composed, then, of similar minute particles or "sarcous elements;" but these are so arranged as to produce transverse striæ in the fibre of voluntary muscle, and a dotted condition in the fibre of the involuntary muscle of the bladder.

Explanation of the Plates.

PLATE XXVI.

Fig. 1. This drawing shows the back of the human bladder with the prostate and seminal appendages, together with the urinary canals. The bladder was very fleshy and much contracted, and is here depicted of the natural size. The greater part of the drawing is in outline; but a portion on one side (about the middle) is finished to display the intercommunications of the fleshy bundles, and the tendinous intersections on those bundles for the insertion of the muscular fibres.

Fig. 2. Represents, once enlarged, a fasciculus of fibres from the sartorius muscle of the thigh. Two of the secondary bundles have been unravelled to show the interweaving between smaller contiguous sets of fibres, as in the involuntary muscle; and with greater care a more minute separation might be made. It serves to demonstrate that the muscle is a web-like mass, instead of a collection of long parallel fibres which are fixed only at the extremities of the muscle.

Fig. 3. The figure marked by *a* is copied from a bundle of muscular fibres of the bladder which had been immersed in dilute nitric acid, after the manner recommended to bring into view the cell-structure of the tissue. Bulgings alternate with narrowed and twisted parts on the fibres,—a condition which is not uncommon. Granular masses, some being evidently the remains of corpuscles, are seen here and there.

The letter *b* points to the second figure, exemplifying how fragments of such fibres would resemble the cells of Professor Kölliker, by being wide at the middle, pointed at the ends, and possessing a corpuscle or nucleus. These two fragments, and others like them, were in another part of the field of the microscope.

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[*] Mr. Bowman seems to have thought it probable that similarity of structure might exist in the two kinds of muscle. He says, "Occasionally these granules are arranged in a linear series for some distance. This condition is probably an approximation towards the structure of the striped fibre, for I have observed the granules to be about the size of the sarcous elements of the voluntary muscles above described."—Cyclop. of Anat. and Physiol., article "Muscle and Muscular Action."

PLATE XXVII.

Fig. 4. The chief characters of the involuntary fibres of the bladder have been here portrayed, viz. the inextricable interlacement, the striated appearance of the surface, the corpuscles, and the slender form with nearly uniform width. The fibre referred to by the letter *a* is furnished with three large corpuscles, and with one small, apparently imperfect body.

Fig. 5. A granular state of the involuntary fibres is here indicated. A similar condition may be seen more frequently in voluntary muscular fibres. It has not the appearance of fatty degeneration.

Fig. 6. Parts of two muscular fibres of the bladder, viewed through an object-glass of $\frac{1}{12}$th of an inch focus, are drawn in this figure. The dotted and striated surface-markings, illustrating the structure, are very difficult to recognize and to picture. Oblique is better than direct light for examining the object, as the latter causes the dots to appear like lines. Shading the mirror of the microscope at a distance is sometimes more serviceable for seeing what is described, than either of the before-mentioned modes of illumination.

Fig. 7. Is a representation of the ending of a bundle of the muscular fibres of the bladder in its tendon. Diminishing in size, the fibres penetrate a certain distance without losing their characters; but they then give place to bundles of fibrous tissue, which become undiscernible as distinct bands amongst the mass of surrounding fibrous tissue. A like arrangement exists at the opposite side of the tendon. Corpuscles can be recognized on the fibres near their insertion in the tendon.

Fig. 8. Exhibits the ending of a single fibre of involuntary muscle of the bladder, magnified 600 times. The terminal division of the fibre, and the wavy fibrous tissue encasing the whole, are delineated in detail. The offset of fibrous tissue is prolonged finally to the common tendon of insertion. In the preparation from which the drawing was taken, the solid part of the fibre was not quite so long as it is here made.

XXIV. *On the Anatomy of* Victoria regia.—Part II.

By Arthur Henfrey, *F.R.S.*, *F.L.S.*, *Professor of Botany in King's College, London.*

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In a contribution to the Philosophical Transactions of this Society, published in 1852 (Part II. p. 289), I gave an account of the internal anatomy of the stem or rhizome of *Victoria regia*. I then stated that I regarded the arrangement of the vascular structures as essentially agreeing with that typical in Monocotyledons, and I directed attention to certain points of less importance, some of which appeared to strengthen the general argument in question, while others were opposed to it. Subsequently I have had an opportunity of studying the anatomy and development of the leaves, roots, and flowers of *Victoria*, and of observing some interesting facts which have not been clearly made out in any of the various memoirs which have appeared on this subject. I have also been led to a reconsideration of the subject of the anatomy of the stem by a criticism which appeared in the 'Flora Indica' (p. 236), the distinguished authors of which, in their advocacy of the opinion that the Order Nymphæaceæ is really Dicotyledonous, question the agreement which I have declared to exist between the structure of the stems of *Victoria* and of Monocotyledons.

In the present paper, which may be regarded as a continuation and completion of the former, I shall first deal with the debated question of the structure of the rhizome, and afterwards give an account of the development of the appendicular organs, including the flower, and of the phenomena presented in the germination of the seed. For the absolute determination of the question whether a stem were organized on the Monocotyledonous or the Dicotyledonous type, it would be requisite first of all to define clearly what conditions we regard as typical in the two classes. This is no easy matter in the present state of our knowledge. The old terms Endogenous and Exogenous growth are founded on no corresponding contrast in the organization of the plants to which they refer. Exogenous growth is a reality, but endogenous growth is a figment of the imagination. The general distinctions between the stems of the two great classes of Angiospermous Flowering Plants depend upon a different mode of arrangement of the fibro-vascular bundles, and, usually, in addition to this, upon a different mode of arrangement of the elementary organs in the fibro-vascular bundles. These are the only essential differences; the formation of annual rings is by no means a constant phenomenon in perennial Dicotyledons. It offers, however, a negative character for the Monocotyledons, since, from the construction and arrangement of their fibro-vascular bundles, they cannot produce annual (or periodical) rings of the same kind as those of the Dicotyledons. The layers formed in such exceptional instances as *Yucca*, *Dracæna*, and analogous stems are totally different from those of the layers of Dicotyledons, being formed of entirely new fibro-vascular cords quite independent of those of the inner substance of the stem, while the successive layers of the stems of Dicotyledons are ordinarily horizontal developments from the cambium layers of the pre-existing bundles.

Many Dicotyledonous rhizomes, although perennial, form no distinct annual rings, and it is common to find the older parts of these thinner than the younger; the reverse of what is ordinarily seen in trees of this class. But in those cases even where there are no annual rings, it is almost always possible to distinguish the typical arrangement of the bundles of the yearling shoot in a single circle surrounding the pith, with processes of cellular tissue forming medullary rays, running out between them to the rind or bark. This circular arrangement is seen most distinctly when the stem has the internodes developed, but it is evident even when they are little elongated. The arrangement of the fibro-vascular bundles in a single circle depends upon the fact, that the leaves of Dicotyledons are supplied by a comparatively small number (3—5—7) of fibro-vascular bundles which are developed collaterally, and in their course through the few internodes which they traverse, remain side by side, separated by plates of cellular tissue (medullary rays) which are interrupted at more or less distant intervals by collateral anastomoses of the bundles. The bundles supplying the leaves are formed in the bud, and their earliest vessels constitute the sheath of the central pith; as the leaf is carried out from the *punctum vegetationis*, no new bundles are developed, but the fibro-vascular bundles are increased in diameter by the development of wood-cells and ducts at their peripheral surface, and in most cases at the sides, thus encroaching on the medullary rays.

In Monocotyledons it is usual to find the nascent leaf embracing with its base a much greater segment of the circumference of the growing apex of the stem than in Dicotyledons; often it extends all round, so as to appear like, or actually to form a tube, as we see even in the full-grown state of the Grasses and Sedges. Into this wide base run generally a great number of separate, small, fibro-vascular bundles, as is seen very clearly in broad parallel-veined leaves, such as those of the Flag. And when a petiole is formed, the bundles do not unite collaterally as in the petioles of Dicotyledons, but pass out separately, as they run in the stem. Again, as the leaf is pushed outward in the bud, towards the circumference of the stem, the earlier bundles receive no peripheral additions, but in place of this new and distinct bundles are formed on the outer side of the old ones. Further, the new bundles formed for the succeeding leaves are not placed between these older ones, but are formed within and cross to the outside of them, passing down in the peripheral region; those of the next leaf follow in the same way; and when, as is often the case, the bundles traverse a great number of internodes (20—50), a vast number of them must necessarily appear in a cross section of the inferior part of the stem. These bundles contain no provision for enlargement in subsequent seasons of growth: the delicate elongated cells called by VON MOHL *vasa propria*, and compared by him with the cambium structure of Dicotyledonous bundles, are necessarily incapable of a development similar to that of the cambium region just referred to, inasmuch as they are closed in all round by the other perfectly-formed elements of the fibro-vascular bundle.

The lower extremities of the bundles of Monocotyledons, moreover, do not anastomose collaterally below in the inner part of the stem, as those of Dicotyledons do; they run more or less obliquely outwards until they reach the region where the central substance of the stem joins the rind, at which point they usually branch and anastomose with their fellows above, below, and around, sometimes forming an intricate fibro-vascular plexus. The cellular tissue in the vicinity of this region, forming the division between the rind and the medullary substance, partakes in some measure of the characters of the cambium region formed by the collective bundles of a Dicotyledonous stem, since it is here that adventitious roots first arise; and here also are produced those peculiar fibrous layers of increase which give so anomalous a character to the stems of *Yucca*, *Dracæna*, &c. But it must be observed that there exists this essential difference, that the cambium layer is the outer, horizontally developed region (or outer side) of the Dicotyledonous bundle, while this pseudo-cambium region of Monocotyledons is connected only with the inferior extremities of the closed fibro-vascular bundles.

Applying these considerations to the structure presented by the stem of *Victoria*, we find unmistakeable resemblance to the typical structure of Monocotyledons in the arrangement of its bundles, and the entire absence of that kind of regularity which produces an annular appearance of the fibro-vascular structure in a cross section; the great number and the isolation of the bundles, and the absence of a central medullary region are especially Monocotyledonous; and the relative position of the bundles in their course is closely in agreement. The chief difference consists in the collection of the bundles together into a kind of cord where they run out from the central region into the petiole—which arises from the comparatively narrow base of the leaf at its origin in the *punctum vegetationis*,—and in the existence of numerous bundles connected with the roots running up and down and around the stem in the region immediately within the rind. The former of these points indicates an agreement with the Dicotyledonous type; the latter departs less from the Monocotyledonous type than from the Dicotyledonous.

The general agreement of structure with that typical in Monocotyledons may be admitted without our assuming that this should largely influence us in determining the systematic position of the plant it occurs in. The general resemblance of the leaves of Monocotyledons with dictyogenous venation to those of Dicotyledons, is an example of strongly marked Monocotyledonous plants exhibiting a resemblance in certain characters to Dicotyledons. The weight attributable to such cases can only be estimated after a careful examination of the various modifications they present, and of collateral circumstances.

At first sight the annular arrangement of the fibro-vascular bundles around a central pith appears a very important systematic character, since it is extremely rare to miss it, even in the most irregularly developed rhizomes of Dicotyledons, where little or no production of annual rings takes place. On the other hand, we know of no case of its occurrence in Monocotyledons. But although the 'scattered' arrangement is so rare in Dicotyledonous stems, instances of it do actually occur; and one case recently made known by M. VAUPELL[*] is especially interesting, as it indicates that we must be very cautious in regarding it as a systematic character. In the rhizomes of the genus *Primula* the fibro-vascular bundles usually stand in a circle around the pith, as in *P. vulgaris*, *sinensis*, and others; but in *P. Auricula* the bundles are scattered generally through the parenchymatous structure without any appearance of a distinction between pith and wood, essentially therefore, although in a more simple form, exhibiting the same aberration as *Victoria*. The existence of this character in certain species only of a clearly-marked genus would be fatal to its use as a systematic character, did we not know that no single character in natural classification can be regarded as absolute[†]. If, however, its importance be admitted, it becomes necessary to inquire how far it is to be allowed to prevail, and what attending circumstances must be looked to in estimating it.

In the present case, when we pass from the arrangement of the fibro-vascular bundles to the structure of the bundles themselves, a most important qualifying circumstance becomes evident. The horizontal and perpendicular sections given with my former memoir[‡], show that the fibro-vascular bundles consist of a central group of spiral vessels surrounded by delicate elongated cells, forming a kind of cambium tissue: there is no wood here, nor anything comparable to the so-called liber-fibres of the bundles of ordinary Monocotyledons. The vascular bundles are reduced to their simplest elements,—a few vessels enclosed by a cambium region. In this state they agree with the rudimentary condition of the Dicotyledonous vascular bundle, which is especially characterized by its peripheral region being composed of cambium, giving it the power of growth; while in the bundles of Monocotyledons the cambium tissue is, as a rule, found in the middle part, with the older woody structure external to it, forming a fixed boundary, precluding further growth.

Considering the rudimentary condition of the vascular bundles of the stem, we should perhaps be rash in drawing any positive conclusion from their structure; yet the evidence, such as it is, derivable from this is in favour of the reference of *Victoria* to Dicotyledons.

A far more decided indication in the same direction is afforded by the plan of arrangement of the vascular structures in the roots. The typical structure of the root of a Monocotyledon is that of a cylinder of parenchymatous tissue containing a central woody axis, which axis, when cut across, exhibits a centre composed of more or less lignified prosenchymatous cells, surrounded by vessels or ducts, the orifices of which form a kind of ring at the point where the central axis joins the cellular rind. This structure prevails in all the Monocotyledonous roots I have examined*.

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[*] Untersuch. über das peripher. Wachsthum d. Gefässbdl. der Dicotyled. Rhizome: Leipsic, 1855.

[†] The so-called 'anomalous' stems of Nyctaginaceæ, Chenopodiaceæ, Menispermaceæ, &c., present conditions which greatly increase the difficulty of laying down a typical character for the stems of Dicotyledons; [illegible] researches at present in progress on these structures do not lead to our regarding them as really inter[illegible] between the Dicotyledonous and Monocotyledonous.

[‡] [illegible] Transactions, 1852, Plate XX. figs. 10 and 13.

In Dicotyledons, on the contrary, the woody substance of the root, like that of the stem, always has the vascular elements in the central region and the woody tissue outside it; in fact the root of a Dicotyledon corresponds essentially with the stem, omitting the central pith and the liber-structure of the bark. In *Victoria* the cross section of a root (Plate XXXI. fig. 53) shows that the centre is occupied by the vascular structure, the large open ends of the vessels appearing scattered in a central cylindrical region, which is bounded externally by a circle of compact, small-celled cellular tissue, which may be compared with the woody portions of the vascular bundles of ordinary Dicotyledons. Perpendicular sections made through the region where the roots pass out from the stem, show that the bundles of vascular tissue destined for the root become collected together into a solid cord before entering it, and they then occupy its axis; while similar sections of the origin of Monocotyledonous roots show that a hollow cylinder of vascular tissue arises from the 'fibrous layer' of the stem and passes out in this cylindric form, its centre being occupied by tissue passing gradually from parenchymatous into prosenchymatous, but directly continuous with the central region of the stem. The structure of the root of *Victoria* is therefore according to the Dicotyledonous type.

The structure and germination of the seeds of Nymphæaceæ have long attracted the notice of systematic botanists, and very different views have been entertained on this subject. *Victoria* has been observed in reference to these matters, especially by Trécul, Planchon, and the authors of the 'Flora Indica.' My own observations are, however, at variance in several respects with those of all these observers, more particularly in regard to the phenomena presented in germination.

In the ripe seed we find the embryo, enclosed in its special sac (amnion), lodged in a cavity excavated in the perisperm (Plate XXIX. fig. 19). It is of a depressed conical form, the flat base exhibiting a small 'boss' or blunt elevation in the centre, which is the radicle, and this of course looks towards the surface of the seed. The position of the radicle is visible through the amniotic coat (fig. 20 *a*), but it is not until this is removed that the Dicotyledonous structure of the embryo is clearly seen. We then find the conical embryo (fig. 21) presenting a vertical groove running over the summit and down each side, dividing it into two halves, the two cotyledons; and the groove ends in a kind of notch at each side of the base, next the radicle. Separating the cotyledons in the line of the commissure (fig. 21 *d*, *e*), we perceive that they are excavated internally, so as to form, when applied together, a somewhat globular or very convex lenticular case, enclosing the plumule. The plumule, placed centrally and directly over the radicular papilla, is a fleshy body, somewhat compressed in the direction parallel to the commissure of the cotyledons, and presents in its upper part two irregular lobes, standing opposite to each other and decussating with the cotyledons (*f*). These are described by TRÉCUL as the primordial leaves.

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* A more particular account of this character is contained in a former paper read in 1847 before the British Association, and published in the 'Annals of Natural History,' Series 2, vol. i. p. 180.

In germination the hard seed-coat is opened by the separation of a little lid or cap at the point opposite the radicle, and from the orifice thus formed emerges the sprouting structures of the young plant. In the accounts generally given, it is stated that the cotyledons remain in the seed, and the radicle and plumule emerge together, the latter elongating into a kind of stalk between the cotyledons and the first leaf. I cannot reconcile this account with the facts which I have met with. The cotyledons certainly remain within the seed-coats, and, as it appeared to me, soon undergo decomposition; but I believe that, in addition to this, the first leaf of the plumule (the larger of the two lobes visible in the embryo (fig. 21 *f*) previously to germination) also remains within the seed-coats. In the specimen I examined the radicle had elongated but slightly, pushing off the lid or operculum, which remained sticking upon its extremity (figs. 23 *r*, 24 A, 25 *r*, *o*); at the side of the radicle emerged a stalk-like process (fig. 25, *t* 1), which at a little distance from the seed exhibited a blunt lateral protuberance (fig. 25 *sl*), and there became bent at right angles; beyond the bend a long stalk-like portion succeeded (*t* 2), at the end of which the regular stem with undeveloped internodes commenced its growth (fig. 22). On extracting the germinating plant from the seed-coats (fig. 24), the stalk-like process coming out at the side of the radicle was found to arise side by side with the radicle from a fleshy mass (fig. 25 *pl*), which appears to be the plumule, and not the cotyledons, as usually described. The radicle is directly continuous with the main body of this fleshy mass, which seems to correspond with the larger lobe of the plumule of the embryo (fig. 21 *f*). The stalk above referred to (fig. 25, *t* 1) seems to be an internode developed immediately above the first, undeveloped, leaf (fig. 25 *pl*), which remains in the seed, merely becoming larger and more solid in texture. The protuberance at the angular bend (fig. 25 *sl*) is probably the rudimentary second leaf (the smaller lobe seen on the plumule, fig. 21 *f*), indicating the end of the first internode and the commencement of the second. The first, shorter, internodial portion (fig. 25, *t* 1) is not cylindrical, but flat on one side, and twisted upon its axis, by which means the second internode comes to have an ascending direction; it was about ⅛th of an inch long. The second internode (*t* 2) was about 1 inch long. It appears from TRÉCUL's account that an adventitious root is sometimes developed at the node where the bend takes place, but this was not the case in my specimen. I had not an opportunity of observing the earliest stages of development of the true stem, which commences its formation in the summit of the second internode, but the specimen which I examined bore all its leaves, and therefore left no uncertainty as to their conditions. TRÉCUL says the leaves are alternate, but that the first two have no stipules, and the first no adventitious rootlets at its base. According to the account in the 'Flora Indica,' the first two leaves are opposite. In my plant the leaves of the rudimentary stem had assumed the spiral arrangement characteristic of the full-grown rhizome, and even the very first penetrated a stipule, like all the succeeding, but as the earliest leaf is rudimentary in form, so its stipule is a mere scale. TRÉCUL's error arose from his mistake about the position of the stipule, which does not sheath its own petiole, but is always *axillary* to the leaf, and envelopes the next succeeding leaf. The first leaf is necessarily naked outside.

The petioles of these early leaves already exhibit the peculiar thickened base so highly developed on the full-grown rhizome; and the earlier adventitious roots break out from the interior of these thickened petiole-bases, like the bundles of roots on the adult plant (fig. 22). On the base of the petiole of the first leaf I found two rootlets; the petiole of the second leaf emitted three; those of the third, fourth, and fifth each four rootlets.

The structure and development of the rudimentary stem on which these leaves and rootlets are borne, agree very closely with that of germinating Monocotyledons,—for instance, with that of the Grasses; the vertical section (Plate XXX. fig. 26) shows this, representing the scattered arrangement and crossings of the vascular bundles, and the absence of any pith. The vascular bundles of the rootlets (*r r*) are directly continuous with those of the stem; but, as in the full-grown rhizome, the vessels occupy the absolute centre of the root, according to the Dicotyledonous type, and do not form a ring round a central cellular axis, as in the roots of Monocotyledons.

The mode of germination here described appears somewhat anomalous; but if, as the facts appear to indicate, the general arrangement of the axial organs approximates to the Monocotyledonous rather than the Dicotyledonous type, it is permissible to seek among the germinating seeds of Monocotyledons for parallels, in spite of the presence of two distinct cotyledons here. The point of view I should feel most inclined to take is, that while two cotyledons do actually exist, these become dissolved in germination, the primordial leaf of the embryo assumes the usual characters of the single cotyledon of such Monocotyledonous embryos as those of Palms, its radicle and petiolar portion (*tigelle*) emerging from the orifice in the seed-coats, and giving origin, between the radicle and the base of the first leaf, to an elongated internode upon which the second leaf is seated*. The chief difference would be that in the Palm embryo the bud is enclosed in the interior of the '*tigelle*' of the cotyledon, while in the Victoria the bud is naked on the side of the '*tigelle*,' opposite the primordial leaf (or third cotyledon), so that it need not break out from the interior of the elongated '*tigelle*,' as in the Palm.

The spiral arrangement of the leaves of the rhizome, with their bunches of roots at the base of the petioles, was described in my former paper, and attention was at the same time directed to the peculiarity of the position of the flower-stalks forming a spiral series midway between the turns of the spirals of the leaves, with each peduncle removed to one side of the axil of its subtending leaf. An examination of the terminal bud of the rhizome rendered the relations of these parts clear to me. The leaves are arranged according to the formula $\frac{13}{21}$, *i. e.* each succeeding leaf stands $\frac{13}{21}$ parts of the circumference of the stem to one side of its predecessor; hence thirteen turns are made round the stem before a leaf appears immediately over any given one taken as a starting-point, and this leaf is the twenty-second in order of development from that taken as zero. In this arrangement numerous secondary spirals are formed according to the well-known laws of phyllotaxy. The mode of succession of the leaves is illustrated by the figure representing the upper end of the rhizome viewed from above (Plate XXVIII. fig. 4).

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* Since the above was written, I have endeavoured in vain to obtain more specimens of germinating seeds of *Victoria*. I have, however, observed in seedlings of a *Nymphæa* the conditions described by TRÉCUL and the authors of the 'Flora Indica,' namely, the cotyledons, soldered together, remaining in the seed-coats, while the radicle and *tigelle* being pushed out as a simple process, soon formed an 'elbow,' at which point the radicle and adventitious roots came off and the second internode rose up to form the tuft of leaves. My *Victoria* may hence have been abnormally developed. GRISEBACH some years ago (Annals of Natural History, i. 6) described the germination of *Limnanthemum* from dried specimens; in that case he supposed one of the cotyledons to remain in the seed and the other to emerge. The appearances presented by his drawings approach very much those exhibited by germinating Nymphæaceæ, which is one more argument for those who incline to believe in a relationship between the latter order and the *Menyantheæ*.

In the *punctum vegetationis* the origin of the leaves and peduncles was readily seen with a moderate magnifying power (Plate XXIX. fig. 5; XXX. fig. 27). Each leaf arises as a conical papilla, very soon presenting three lobes, the middle one being the blade of the leaf, and the two lateral the two points of the double axillary stipule (fig. 27 *l*; *s*, *s*, *b*). Very soon the blade of the leaf becomes distinctly bounded below by a constricted portion, and a ridge arises running across its inner surface from one stipule to the other; as the parts increase, this ridge rises up and forms the membranous axillary portion of the double stipule standing inside the petiole. The petiole soon becomes distinguishable, and as it elongates it removes the blade more and more from the axillary stipule. The gradual evolution of the blade of the leaf, with its curious inrolled vernation, was traced, but demands no special attention, except the remark that the general plan of vernation consists of the rolling in of the edges towards the centre; but there is a complication in the details from a wrinkled or 'corrugated' condition of the parenchyma in the areolæ between the anastomosing ribs. In the specimen I examined, which was taken up covered with flower-buds, these latter were found of every size, diminishing to a conical papilla in the *punctum vegetationis* (Plate XXX. fig. 27), but only the very youngest that could be distinguished stood in the axil of its subtending leaf. Where this existed the nascent organs were in absolute contact. As soon as they become at all separated by the expansion of the apex of the stem on which they arise, the flower-bud is carried away from the axil and takes its position, where it permanently remains, in the axil of the right-hand half of the double stipule (Plate XXIX. fig. 5, *f* 1, *f* 2; Plate XXX. fig. 27). This is the real explanation of the apparently anomalous position of the flowers referred to in my former paper.

The stipules deserve some notice, both from their size and their condition. As they enlarge with the development of the growing axis, they do not become defined below by a constricted base, but extend on each side of the petiole and form a sheathing structure, which reaches at first nearly all round the stem (Plate XXVIII. fig. 4), and ultimately about one-half or one-third of its circumference. This broad base is continuous below with a rounded projection of the cortical structure of the stem, and it is opposite these projections that the transverse plates of solid tissue are found that form the framework of the excavated structure of the rind*. As the stipules enlarge, they overlap their neighbours to some extent. The left-hand half of each double stipule lies over the outside of the right-hand half of the one older than itself, and its right-hand half is in turn overlapped by the left-hand half of its successor; as this right-hand half has the flower-bud in its axil, the latter becomes furnished with a double covering, which, as the stipules converge inward above, completely encloses and protects the flower-bud until it is pushed out by the elongation of the peduncle. The general character of the stipules is shown in the drawings. Each apparently axillary stipule is distinctly double, and the two halves are, when mature, connected by a membranous plate running across the axil, which base is quite unconnected with the base of the petiole (Plate XXVIII. fig. 3). Each half of the compound stipule has a very distinct midrib, which forms an angular projection in the earlier, more folded state of the organ. It was stated in the former paper that the spiral-fibrous structure soon disappears in the ducts and vessels of the leaf-stalks. This is not the case in the stipules, which exhibit perfect vessels when fully developed.

To this paper are appended some illustrations of the elementary structure of the petioles and of the arrangement of the tissues, which do not require further notice here (Plate XXXII. figs. 59–61).

The external aspect of the stem is rendered somewhat peculiar by the mode of arrangement of the roots, which arise in dense tufts or bundles at the bases of the petioles. The manner in which they emerge from the stem is highly curious, and unlike any other case with which I am acquainted.

The roots are adventitious, and arise in the peripheral part of the central substance of the stem, just below where the fibro-vascular bundles run out to the petioles. No trace of the roots is visible externally until the leaves, especially the petioles, have acquired considerable dimensions (Plate XXVIII. fig. 4); then it is observed that the cortical substance of the stem is protruded so as to form a kind of knob at the base of the leaf-stalk; or perhaps it may be best described by comparing it with a highly-protruding *pulvinus* under the point of origin of the leaf-stalk (Plate XXIX. fig. 8). Ultimately a vertical fissure is perceived in the protuberance, the sides of which gradually separate, with rounded edges like lips, leaving an elongated elliptical slit, at the bottom of which are seen the points of the nascent roots (figs. 11, 12). The roots, densely crowded, make their way out, the oldest at the lowest part, and they are given out in such rapid succession that they form a close bunch. As they emerge and expand, they convert the vertical slit into a large rounded bordered orifice (figs. 15, 16). During the subsequent growth of the roots, the protuberance keeps firm and forms a large projecting knob; but their mutual pressure gives the bases of the roots an angular form, so that when they become detached on the decaying parts of the stem, their ends form the six-sided areolæ shown in the illustration of my former paper.

It has been noticed above that even the earliest roots, the bundles of which consist of only two to four rootlets, break out in the same way from the bases of the petioles of the germinating plant (Plate XXIX. fig. 22).

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* Philosophical Transactions, 1852, Plate XIX. *ff.*

Unlike the petioles, the roots retain the proper character of their vascular structures, and in these, as above noticed, they afford a further evidence of the systematic position of the plant. In the fibro-vascular cord running up the centre of the roots, the spiral vessels are collected in the centre of the bundle, as in Dicotyledons (Plate XXIX. fig. 17, Plate XXXI. fig. 53), whereas in Monocotyledons they stand at the circumference of the bundles. The arrangement of air-chambers, so remarkable in all parts of *Victoria*, is especially elegant in the roots (Plates XXXI. & XXXII. figs. 51–55).

The development of the flower was traced from the earliest period at which the organs could be distinguished. In the youngest bud (Plate XXX. fig. 27), about $\frac{1}{30}$th of an inch in diameter, the peduncle was surrounded by a circle of nascent foliar organs, which did not then cover-in the growing apex. Seen from above, this circle was found to consist of five organs (fig. 28), that is, of the four sepals and the first petal. They stood distinctly in a spiral order (fig. 29), in the succession as regards age indicated in the figure. Within these were a few rounded papillæ continuing the spiral, but soon lost in the rounded *punctum vegetationis* of the bud. This bud showed clearly that the primary arrangement of the organs here is a spiral, and that this spiral is continued from the imbricated calyx into the corolla, where the arrangement is on a complex phyllotactic type.

In the next bud (fig. 30), the four sepals had grown up so as to enclose the younger organs, and they now occupied the whole circumference of the receptacle. In the inside of them appeared nascent petals, and at this time the whole centre was occupied by minute papillæform rudiments of organs. Fig. 31 shows the succeeding stages; and when we come to fig. 32, the centre of the receptacle appears naked,—the first step toward the production of the peculiar structure on which the floral envelopes and stamens are ultimately elevated.

When the bud is about $\frac{1}{10}$th of an inch in diameter (fig. 33), the centre of the receptacle exhibits a dome-shaped smooth surface bounded by a raised rim, on which are supported the stamens and petals, bounded externally by the calyx. In the next bud (fig. 34) the centre of the receptacle had risen up in a conical form, and the internal surface of the annular ridge was marked by fine grooves, indicating the formation of the free edges of the carpels.

It is unnecessary to describe at length the details of the successively older buds, as their important difference is clearly shown in the drawings accompanying this paper (Plates XXX. & XXXI. figs. 34–50). It will be sufficient to notice two or three points.

The carpels are never completely free from the receptacle, except at the edges which form the stigmatic lines, which lines are carried up on the internal surface of the ring surrounding the ovary, to the 'horns' which stand inside the stamens. The 'horns' appear to consist of the points of the carpellary leaves. As regards the cavities of the ovary, they at first point somewhat obliquely upwards, but gradually, by the development of the centre of the receptacle into the columella, they become horizontal, and the horizontal stigmatic lines must be regarded as the ventral sutures of the carpels. The upper portion of the carpels, adherent to the annularly raised part of the receptacle, is developed in a remarkable manner. The central columella is originally conical, but it soon exhibits a constricted base, tapering downward in a reversed conical form. The inner surface of the ring, as it were, moulds itself in this, and sends in a ridge-like process, which fits into the hollow at the constricted base of the columella. Afterwards, as the receptacle expands, this ridge, pointing downwards, is drawn away from its original position, but still continues to grow; the stigmatic lines are seen first running horizontally out over the cells of the ovary, next turning inwards, then curving round the ridge upwards, until ultimately they are lost at the base of the circle of 'horns' bordering the annularly raised receptacle.

From the foregoing account it is evident that the inferior position of the ovary of *Victoria* is only apparent, and that the structure consists of a circle of numerous distinct carpels collaterally united and imbedded in an enlarged receptacle which runs up in a circle around them, carrying up the stamens and floral envelopes, and likewise rising into a columella in the centre, where its expansion carries up to some extent the bases of the carpels and brings them into an oblique and almost horizontal position. This removes any difficulty which might arise in considering the relation of this genus with others where the ovary is distinctly superior.

Many other particulars of less consequence were observed, which for the most part have been correctly explained by preceding observers; such are the structure of the orifices found in the leaves, called by PLANCHON 'stomatodes,' the form of the pollen, &c. For an account of these we refer to M. TRÉCUL's memoir*.

In conclusion, I must briefly allude to the bearing of the facts contained in the present paper upon the systematic position of the Nymphæaceæ. Notwithstanding that the anatomical character of the roots adds one more Dicotyledonous feature to the character of these plants, I am more inclined than ever to regard the structure of the rhizome as conforming to the general type of the stem of Monocotyledons rather than to that of Dicotyledons, among which I have found nothing at all like it. But I do not consider that this character alone can decide the position of the Order, the majority of whose characters are, on the contrary, clearly Dicotyledonous; it merely brings it into the position of a link between the two classes, which are by no means so markedly separated from each other as is often supposed from the mode of their arrangement in Systematic works. A further attempt at the settlement of the true affinities of this Order does not come within the scope of the present paper.

January 1859.

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* Ann. des Sc. Nat. sér. 3, Botanique, vol. i. p. 145.

Explanation of the Plates.

PLATE XXVIII.

Fig. 1. Side view (reduced) of the rhizome of *Victoria regia*, with the older leaves and flowers cut off. This drawing shows the bundles of roots coming out from the bases of the petioles, and the young leaves emerging from the large sheathing stipules.

Fig. 2. Portion from the centre of fig. 1, showing that the growing point is covered up by the large sheathing stipules: the stipule s is axillary, and belongs to the leaf of which p is the petiole; f is the base of a flower-stalk axillary to the leaf of which p'' is the petiole, and whose stipule is removed.

Fig. 3. A sketch showing the axillary position of the stipule and its double nature, having a rib or keel on each side of the petiole.

Fig. 4. View of the summit of the rhizome, with the leaves, stipules and peduncles cut off close to their origin; showing the arrangement of the organs, the mode in which the stipules ensheath the succeeding parts, and the gradual evolution of the bundles of roots from the bases of the petioles.

PLATE XXIX.

Fig. 5. Magnified view of the *punctum vegetationis* of the rhizome:—f^1, f^2, flower-buds, shifted into the axil of the right-hand half of the stipules; s^1 s^1, the base of the sheathing stipule of an outer (removed) leaf; s^2 s^2, the base of another removed leaf; s^3 s^3, the stipulary portions, and l, the blade of the next leaf, within which are several other still more rudimentary leaves,—the central one still a simple papilla.

Fig. 6. A young leaf, magnified (*ns*, of the natural size).

Fig. 7. Another, more advanced, also magnified.

Fig. 8. A more advanced leaf, with its axillary two-keeled stipule and swollen base.

Fig. 9. Side view of a similar leaf and stipule.

Fig. 10. Terminal bud enveloped in stipules, with all the leaves older than fig. 8 removed; showing the relative degree of development of the peduncle and flower-bud f.

Fig. 11. Base of a petiole about the same age as in fig. 9, with the early appearance of the fissure whence the roots emerge; the petiole at this time $\frac{1}{8}$th of an inch in diameter.

Fig. 12. More advanced stage; petiole now half an inch in diameter, and the blade of the leaf 2 inches long.

Fig. 13. Side view of a more advanced stage.

Fig. 14. Side view of the blade of the leaf in fig. 8.

Fig. 15. Base of a petiole and bunch of young roots, natural size, side view.

Fig. 16. The same, seen from above.

Fig. 17. Perpendicular section through the base of a petiole, as in fig. 15; showing the course of the vascular bundles of the leaves (*vl*) and those of the roots (*vr*).

Fig. 18. A seed, natural size.

Fig. 19. Vertical section of a seed, showing the embryo (*e*) lodged in an excavation of the perisperm.

Fig. 20. Embryo (magnified) removed from the seed, but still enclosed in its amniotic sac:—*a*, from below; *b*, side view; *c*, vertical section, showing the amniotic coat and the cotyledons, with the plumule in the middle. The figure *ns* indicates the natural size of *a*.

Fig. 21. Embryo (magnified) with the amniotic coat removed, showing the distinctly Dicotyledonous structure:—*a*, side view; *b*, from above; *c*, from below, with the central boss-like radicle; *d*, cotyledons separated to show the plumule; *e*, one cotyledon only turned down; *f*, plumule laid bare in side view.

Fig. 22. Germinating plant, with the petioles, of the leaves 3, 4, 5, 6: 2 is the scar of the removed second leaf, with a minute scale-like axillary stipule remaining; the remains of the first leaf would appear on the other side of the stem.

Fig. 23. The seed, with the base of the germinating plant emerging from it:—*r*, radicle; *t*, stem (*tigelle*); nat. size.

Fig. 24. Front (A) and back (B) view of the base of the germinating plant, magnified:—*pl*, the portion contained within the coats of the seed in fig. 23, and apparently corresponding to *f* in fig. 21.

Fig. 25. Another and more enlarged view of the same structure:—*r*, radicle; *o*, operculum; *pl*, primordial leaf; *t* 1, first internode; *sl*, second (rudimentary) leaf; *t* 2, second internode.

PLATE XXX.

Fig. 26. Perpendicular section of the young stem represented by fig. 22; showing the pseudo-Monocotyledonous arrangement of the vascular bundles, except in the rootlets *r r*, where they occupy the absolute axis.

Fig. 27. *Punctum vegetationis* of the rhizome, highly magnified:—*l*, a rudimentary leaf turned back, showing the relative development of the blade *b*, and the stipulary portions *s*, *s*,—the petiole not yet developed; *f*, rudimentary flower-bud.

Fig. 28. The flower-bud *f* in fig. 27.

Fig. 29. Diagram of the arrangement of the four sepals and first petal, showing it to be a continuous spiral.

Fig. 30. Magnified perpendicular section of a flower-bud rather older. In this and the following figure the sketches marked *ns* indicate the natural size.

Fig. 31. A somewhat more advanced flower-bud:—*a*, seen from above, with the sepals turned back; *b*, perpendicular section; *c*, perpendicular section of one rather older.

Figs. 32–35. Successively more advanced stages.

Fig. 36. Perpendicular section, as in fig. 35, with the columella cut out to show the carpels.

Fig. 37. Perpendicular section of the elevated receptacle in fig. 36.

Fig. 38. Horizontal section of fig. 37 at ×, showing the carpels open internally.

Fig. 39. An older bud.

Fig. 40. Cross section of its ovarian portion, showing the carpellary cells closed.

Figs. 41–43. Successive stages.

PLATE XXXI.

Figs. 44, 45. Successive stages; 45 *b*, cross section.

Fig. 46. Magnified fragment of *b* in fig. 45.

Fig 47. Perpendicular section of a fully-developed flower-bud, showing only one-half.

Fig. 48. Magnified horizontal section of fig. 47, at *a*.

Fig. 49. Section at *b*.

Fig. 50. Section at *c*.

Fig. 51. Cross section of a young root.

Fig. 52. Magnified view of a portion of ditto.

Fig. 53. Centre of ditto, more magnified.

PLATE XXXII.

Fig. 54. Spongy cortical parenchyma, as in figs. 52 and 53, magnified 50 diameters.

Fig. 55. Perpendicular view of one of the septa in fig. 54, magnified 50 diameters.

Fig. 56. Perpendicular section of the origin of a root:—*vb*, central vascular cord; *l*, large lacunæ.

Fig. 57. Portion of a vascular bundle, near the origin of a root-bundle: magnified 200 diameters.

Fig. 58. Perpendicular section of *l* in fig. 56, showing that the lacunæ of the roots do not communicate with the air-cavities in the stem, and that the walls are thickened by a sort of cuticle: magnified 100 diameters.

Fig. 59. Cross section of a petiole.

Fig. 60. The same, magnified: the white spaces are air-canals; the shaded spaces fibrous bundles, composed of dense cellular tissue.

[illegible] Highly magnified fragment enclosed in the square space *a* of fig. 60.

[illegible] Cross section of a flower-stalk.

[illegible] The same, magnified.

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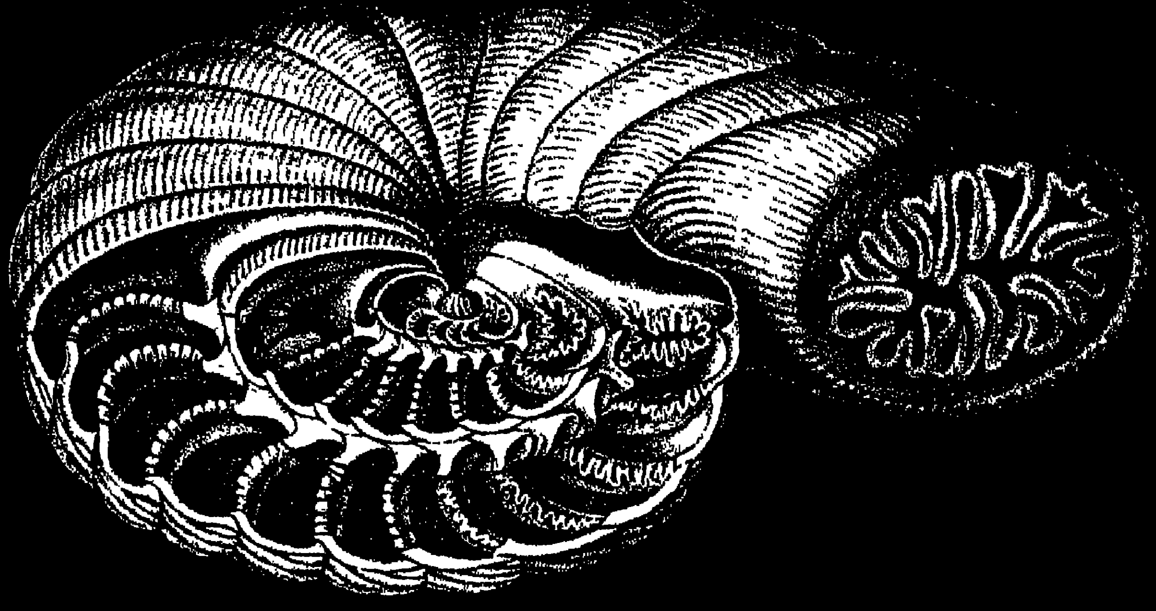

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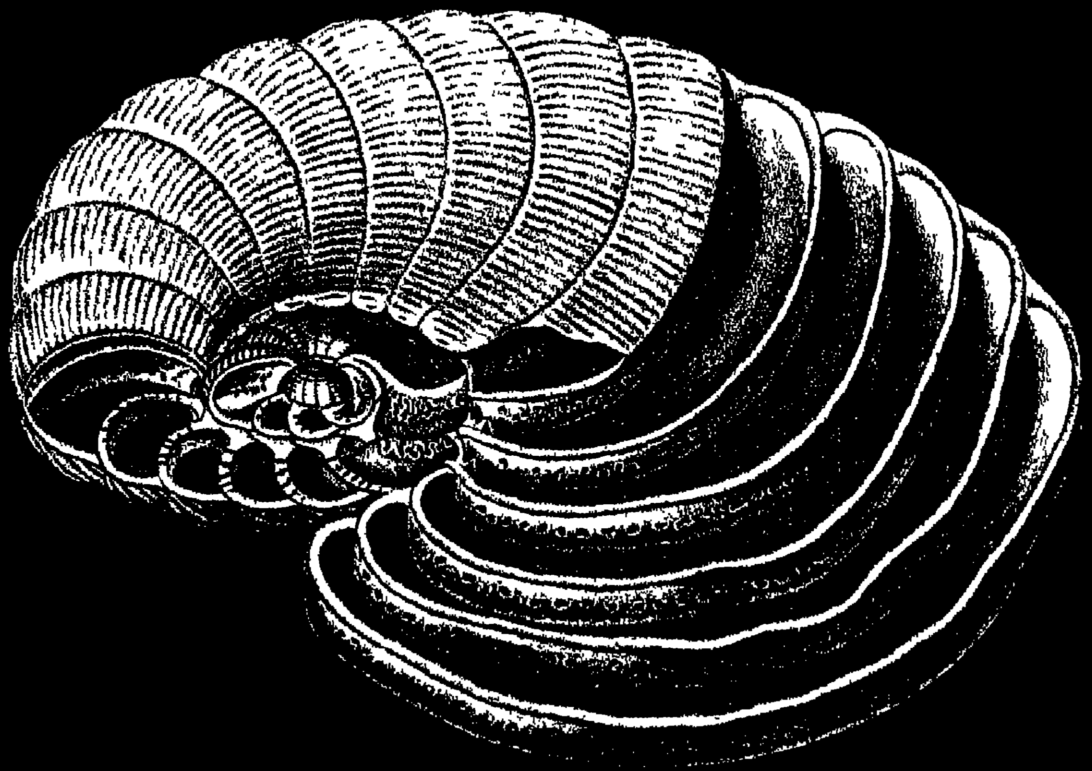

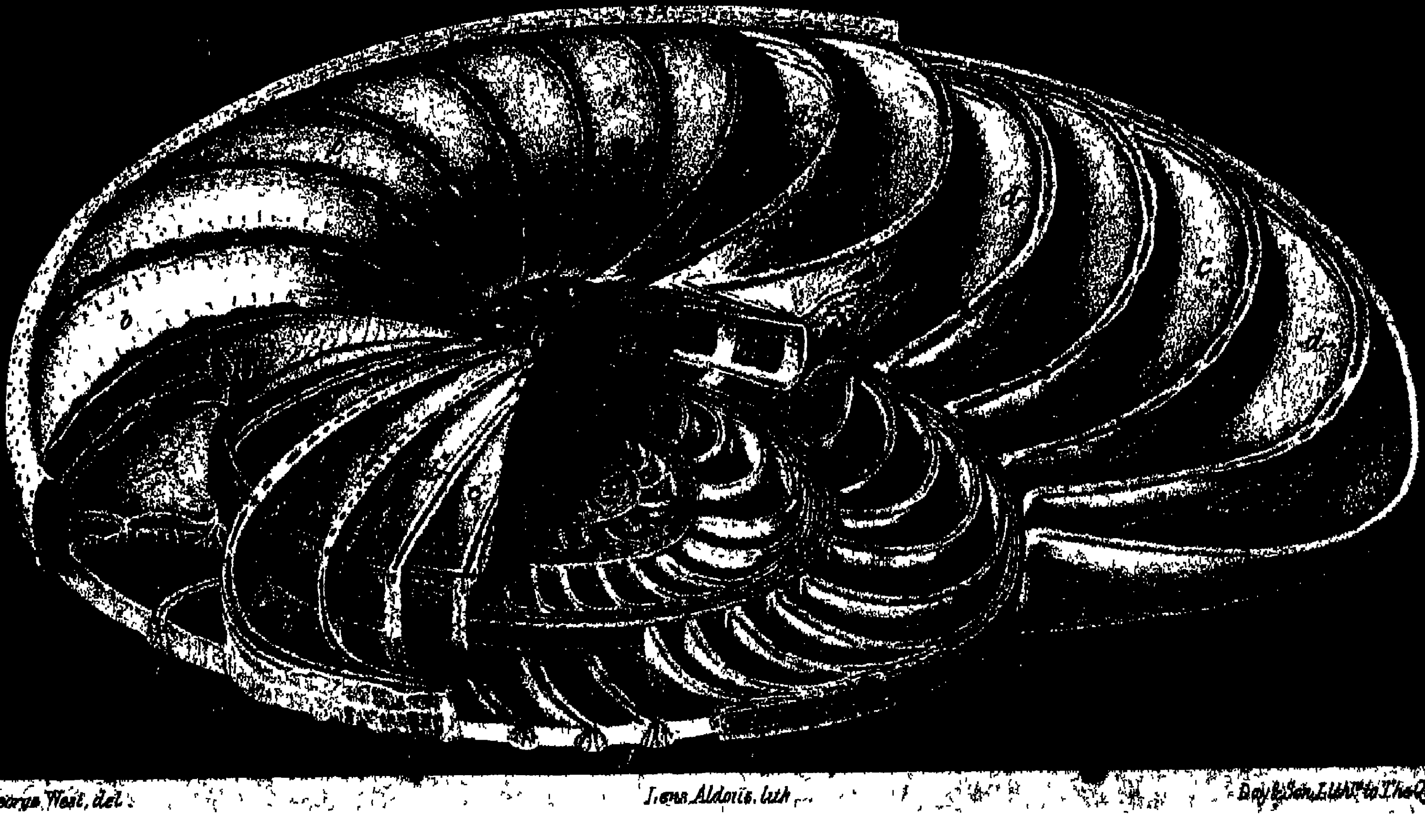

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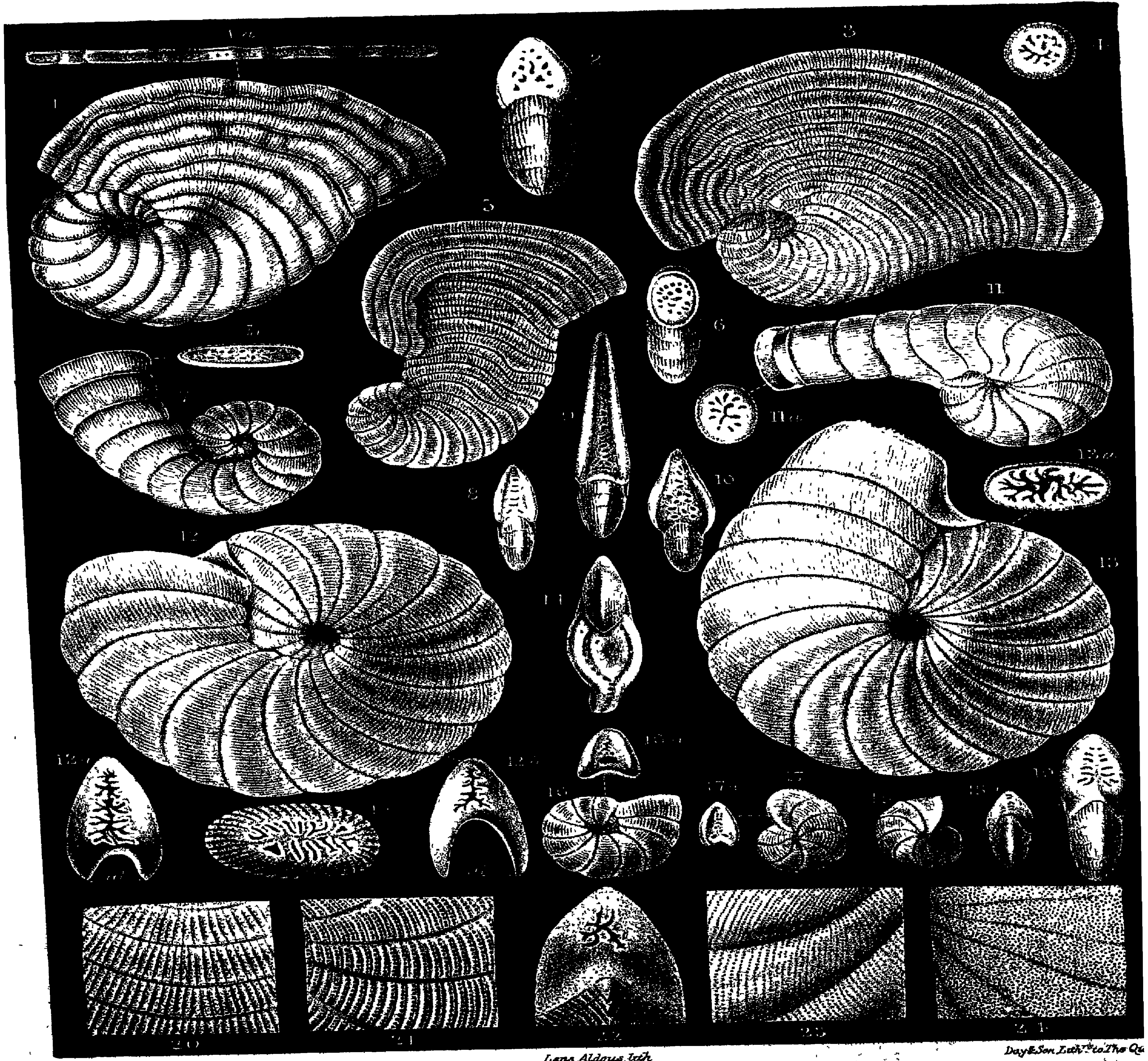

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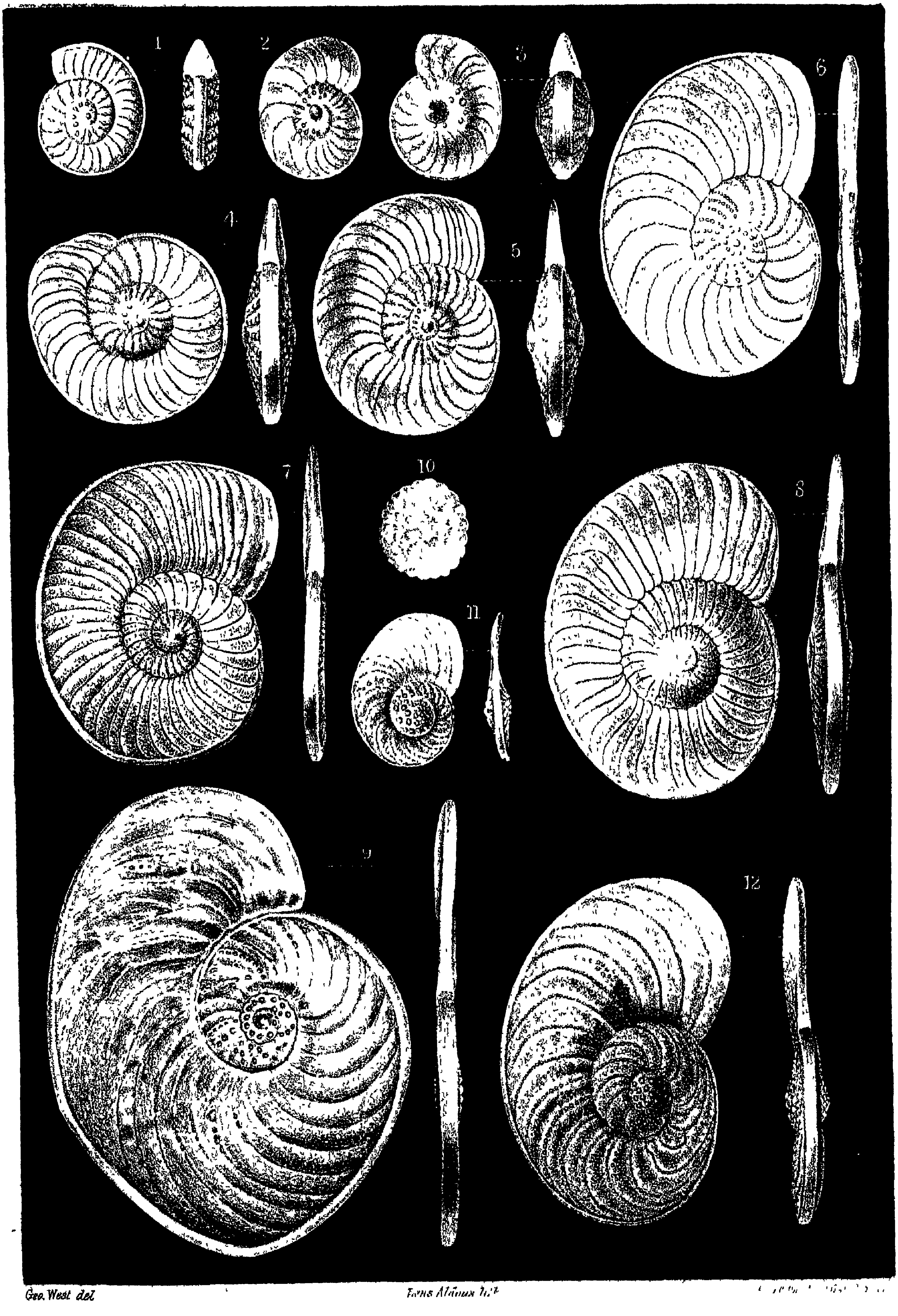

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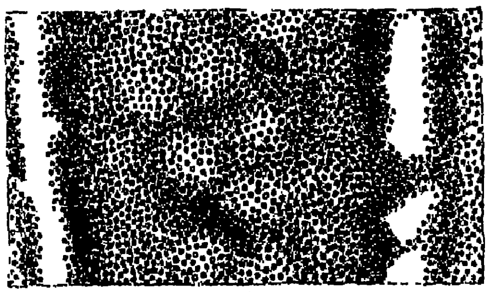

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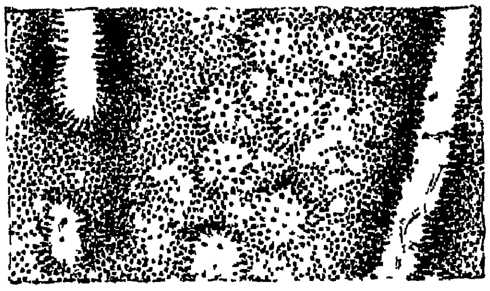

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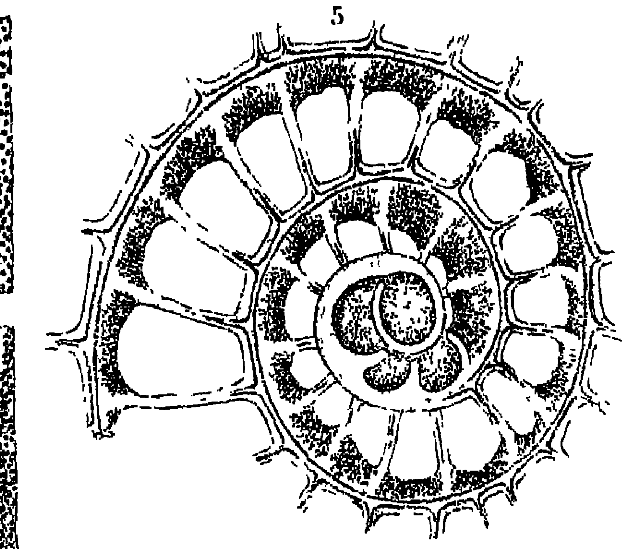

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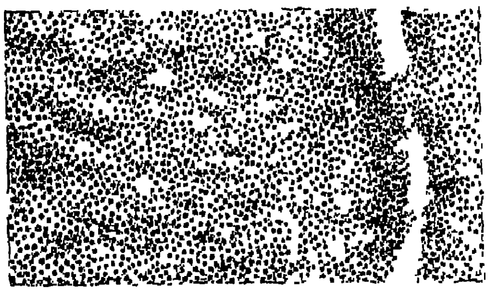

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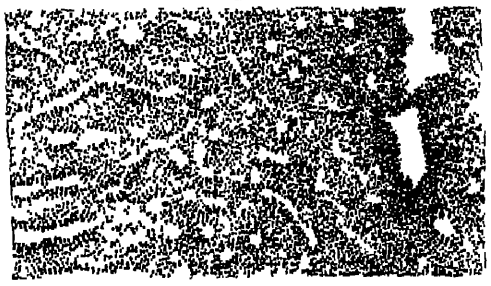

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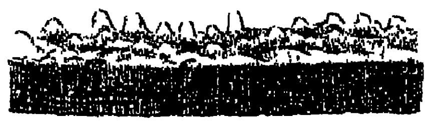

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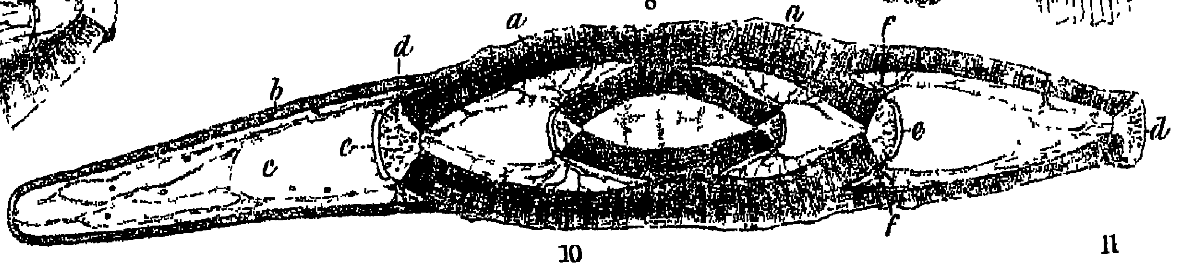

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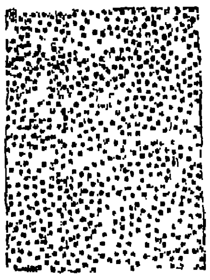

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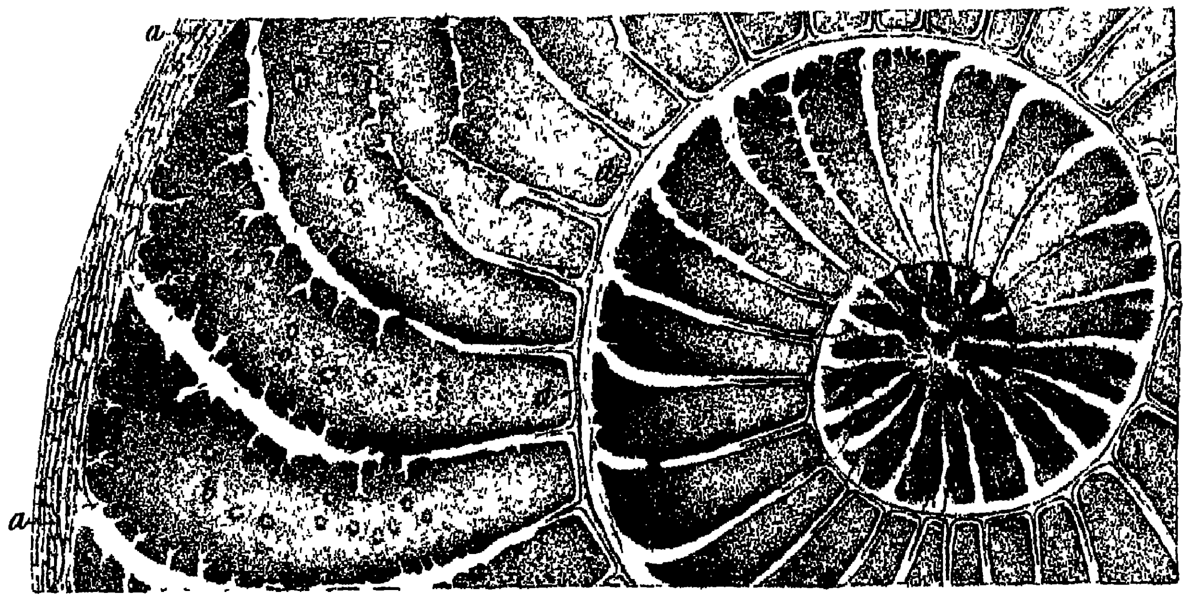

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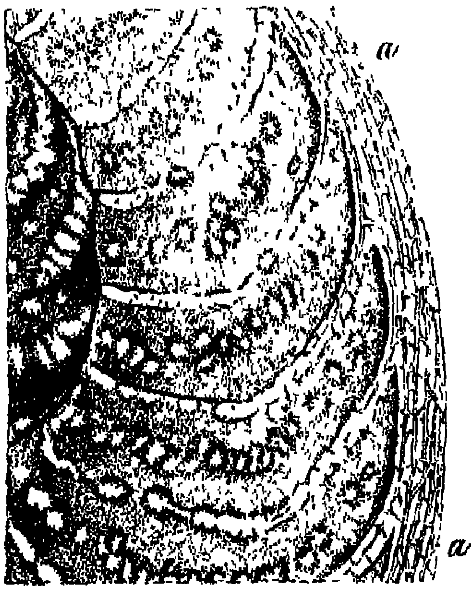

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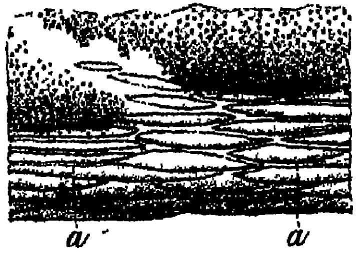

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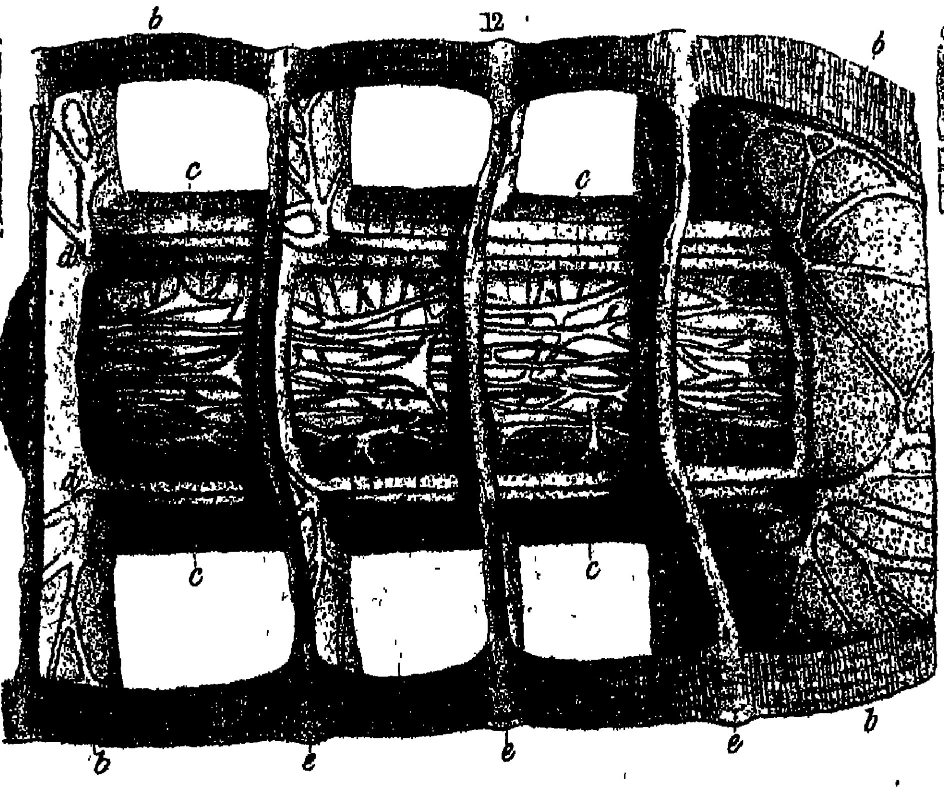

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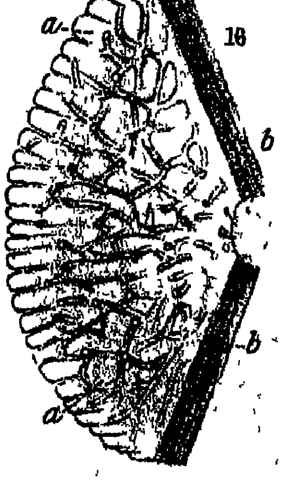
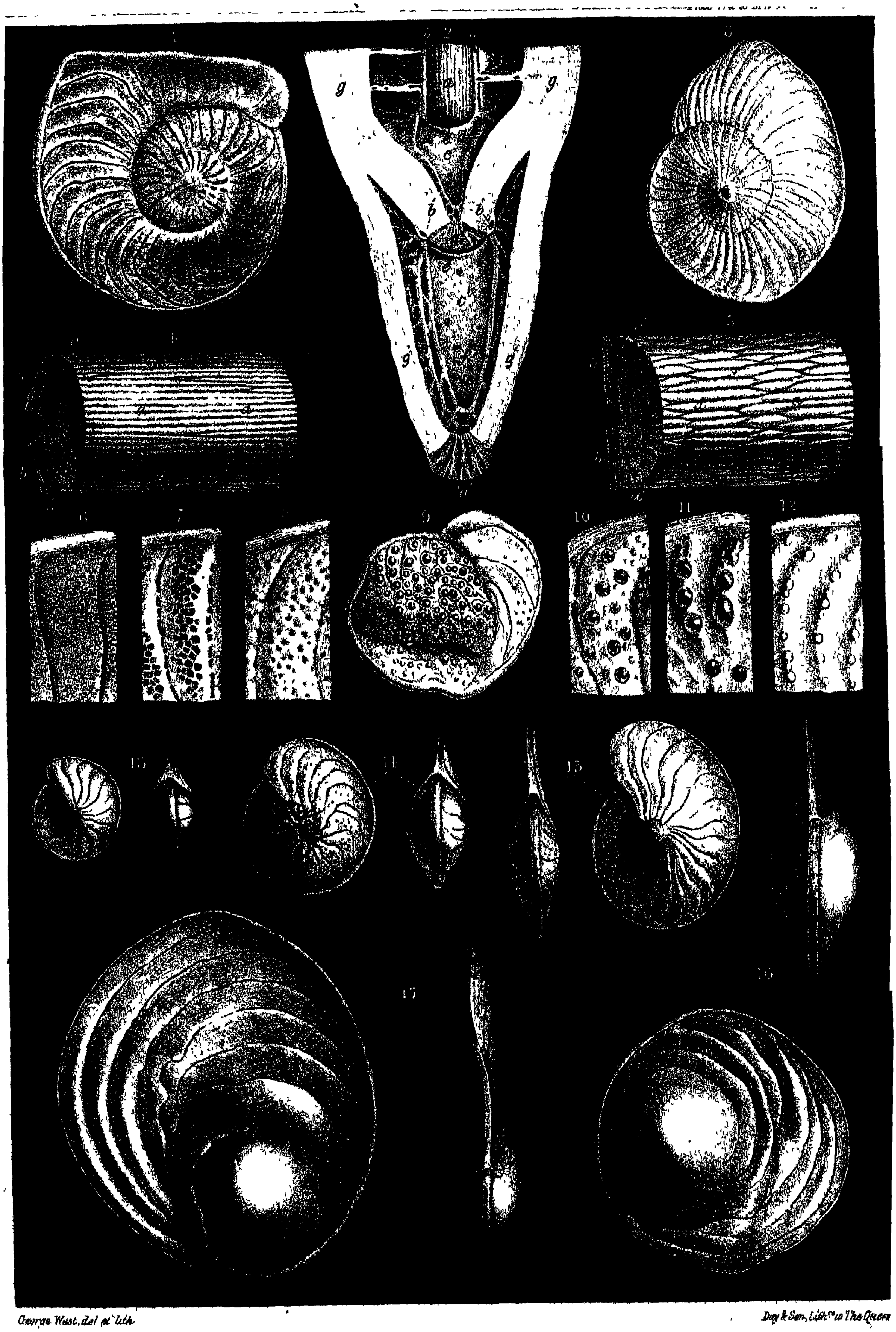


Tosy del. et lith

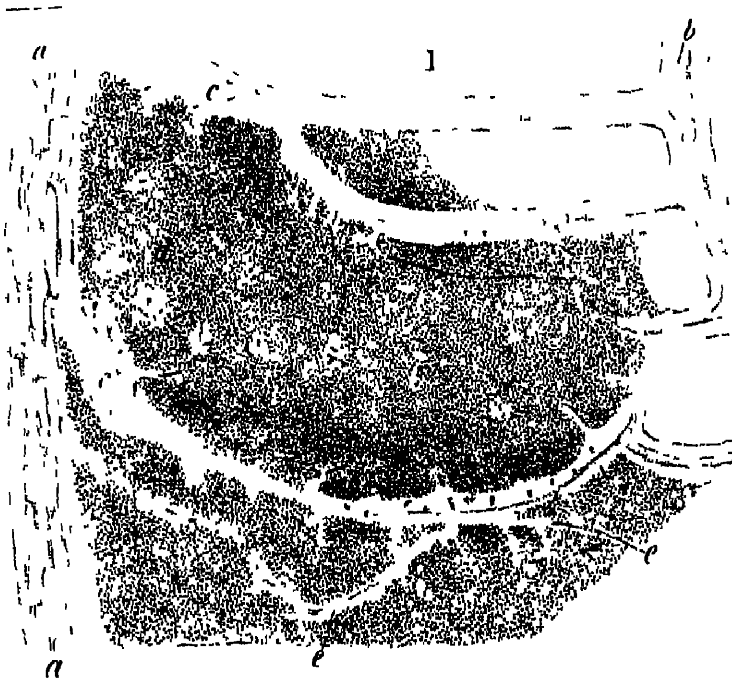
Day & Son, Lith^{rs} to Q



George West, del et lith.

Day & Son, Lith^rs to The Queen

1



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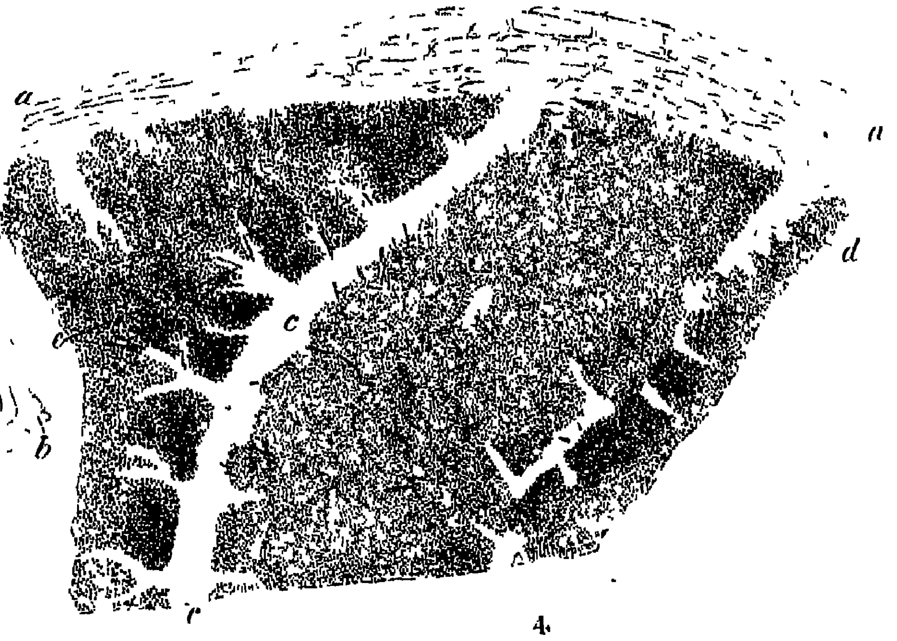

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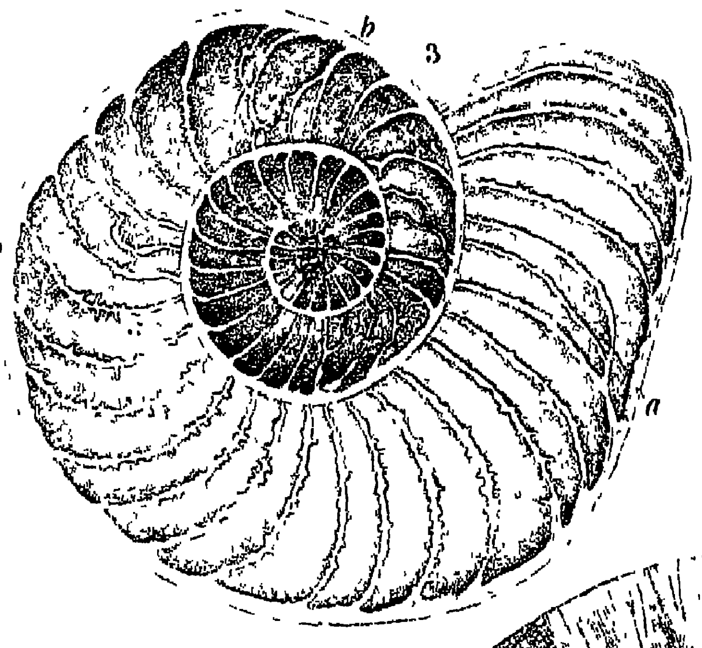

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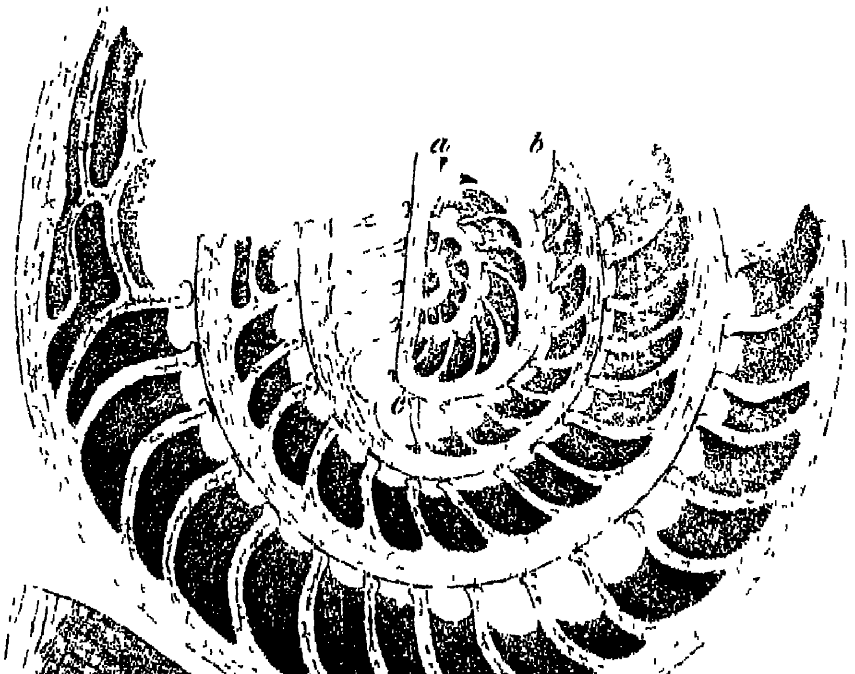

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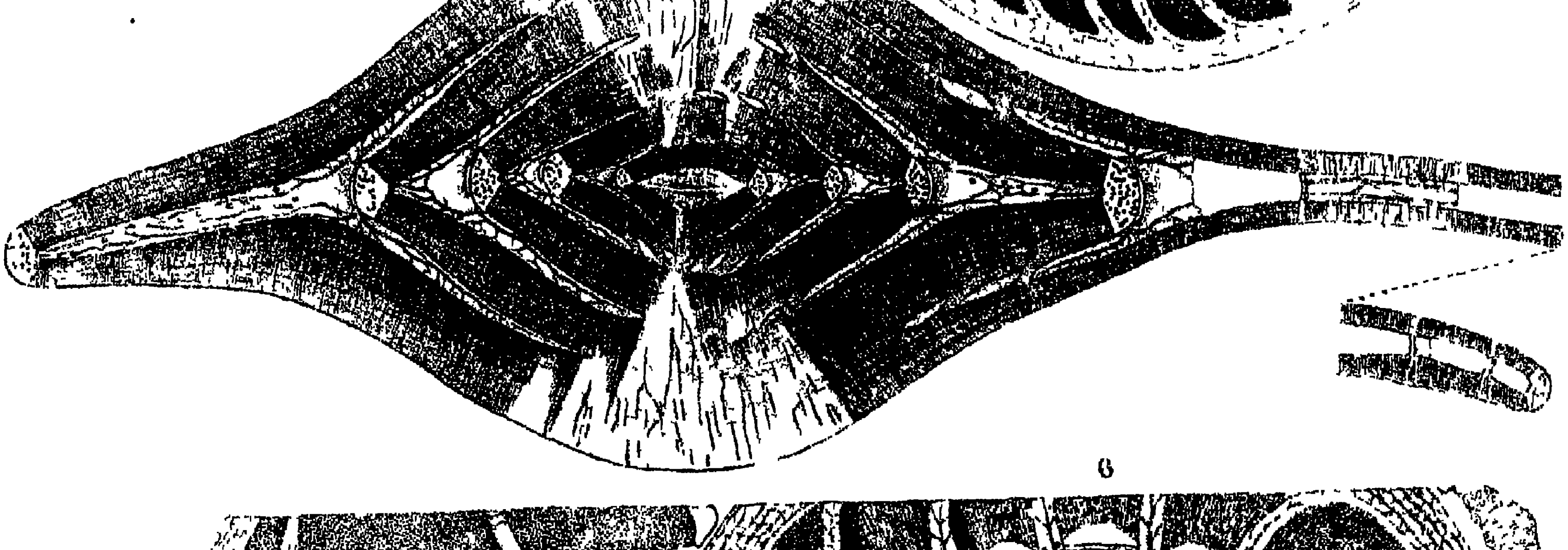

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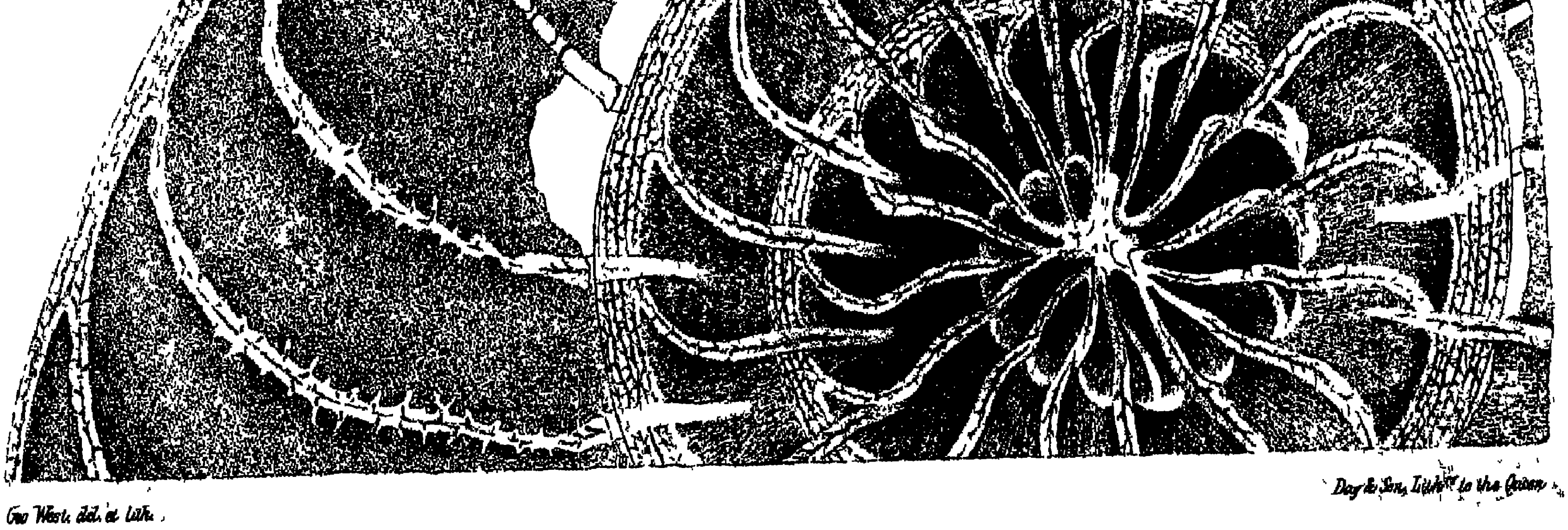
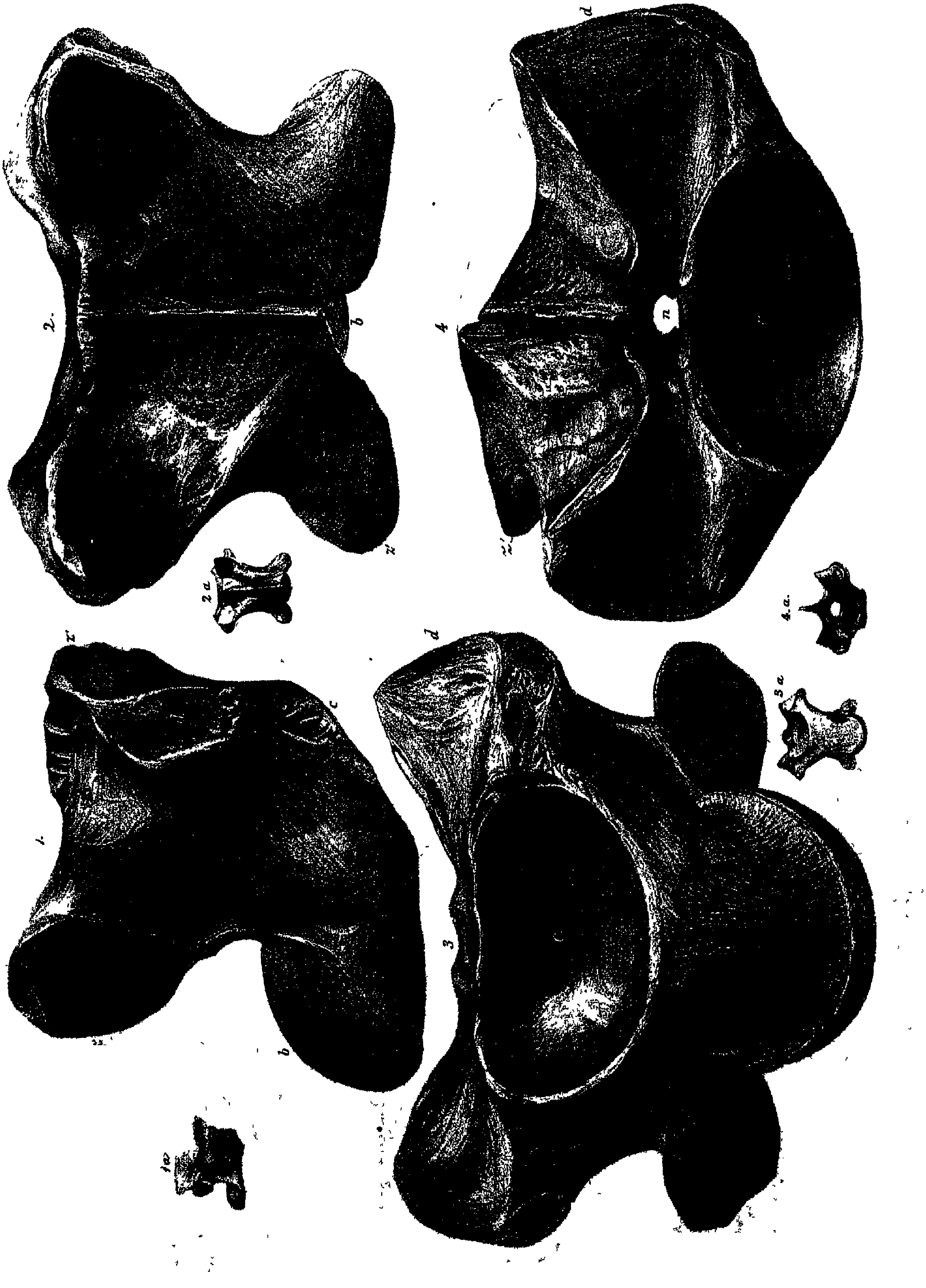
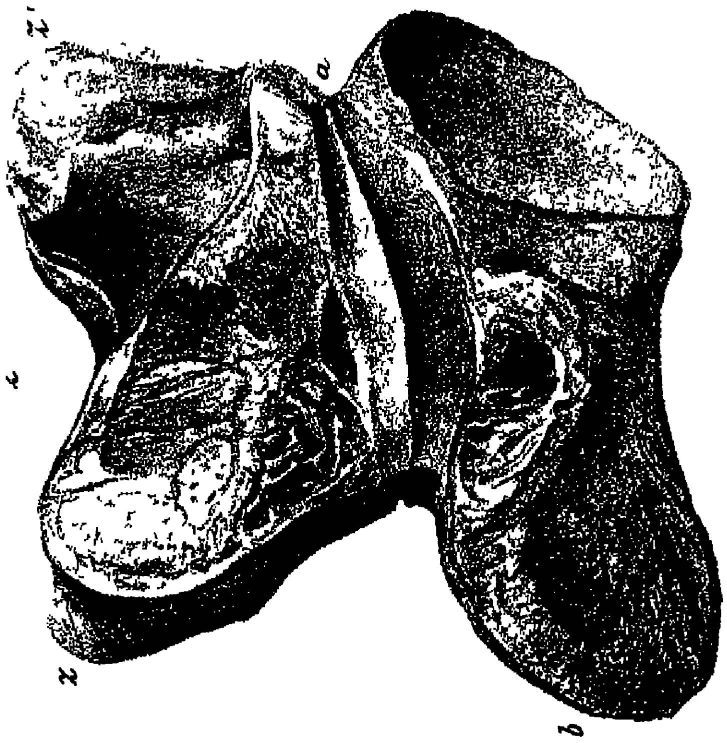


Geo West del. et lith.

Day & Son, Lithrs to the Queen.

Phil. Trans. MDCCCLVIII. Plate VII.





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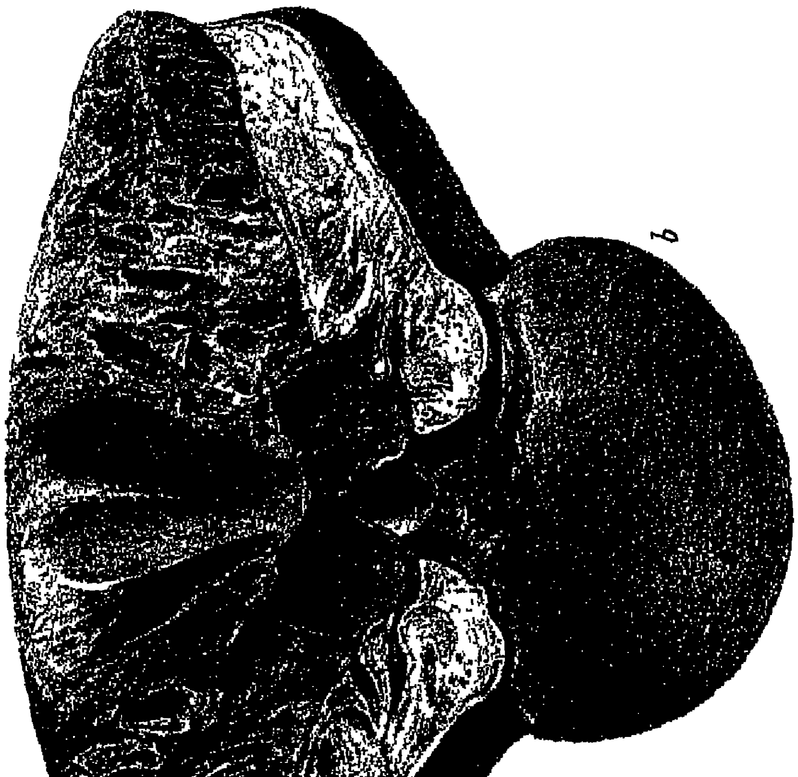

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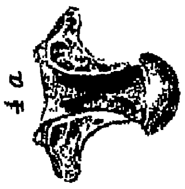

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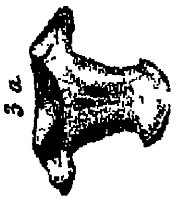

1 a

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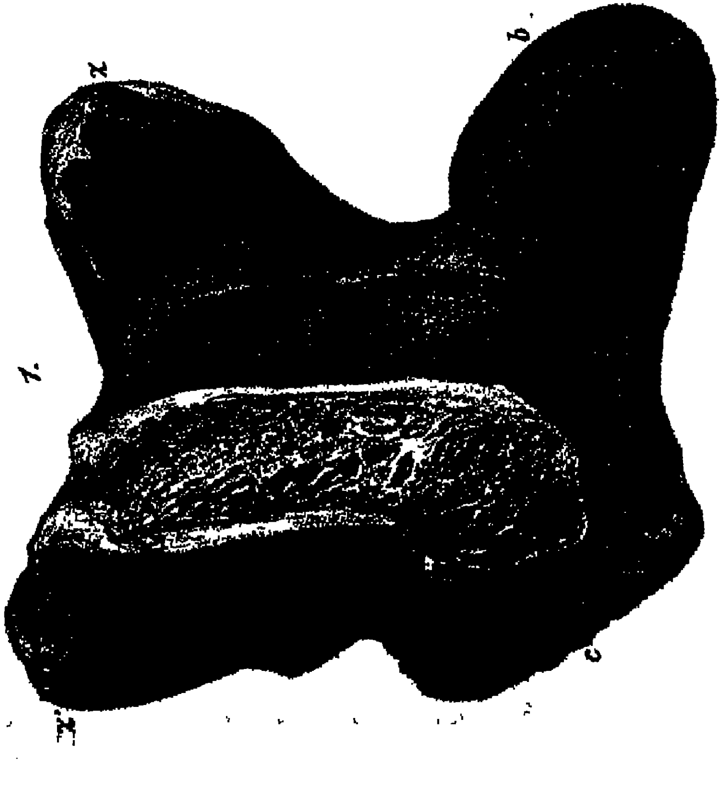

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FIG 10.

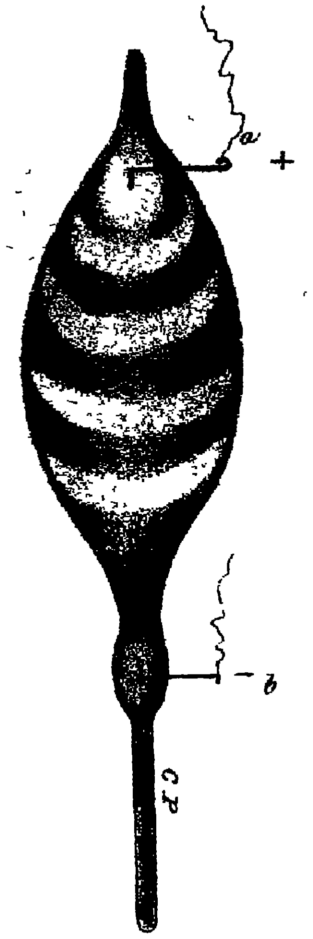


FIG 12.

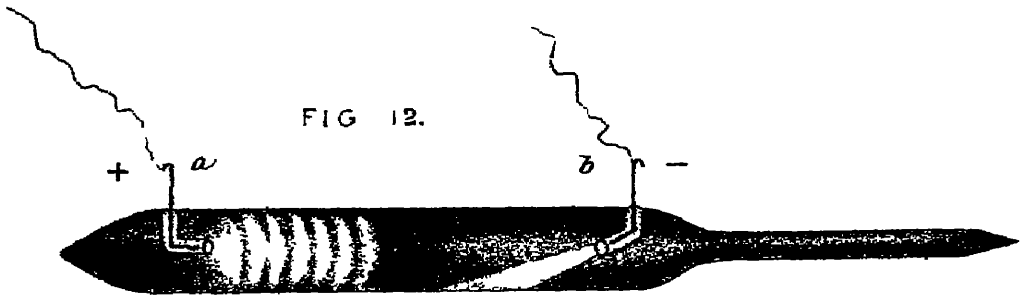


FIG. 13.

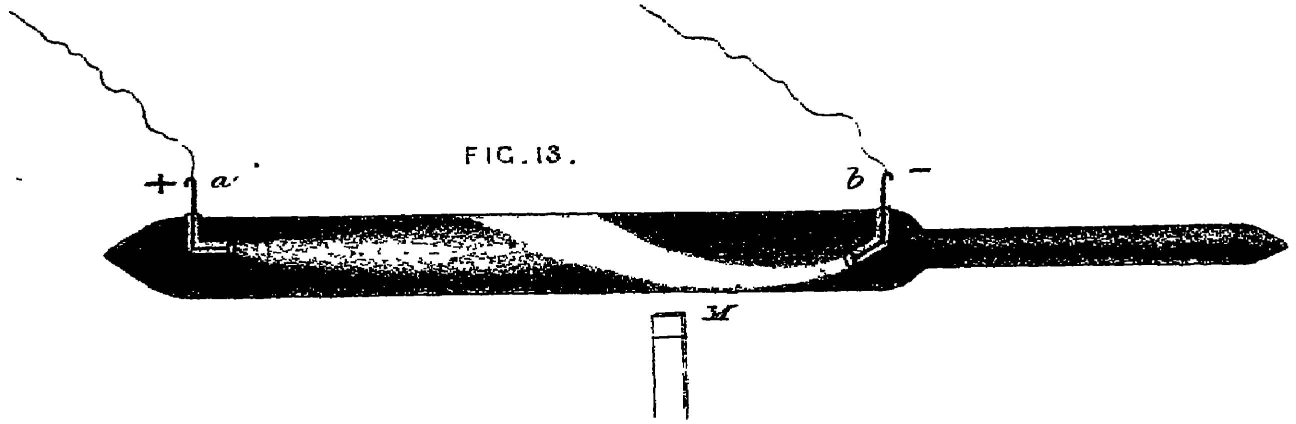
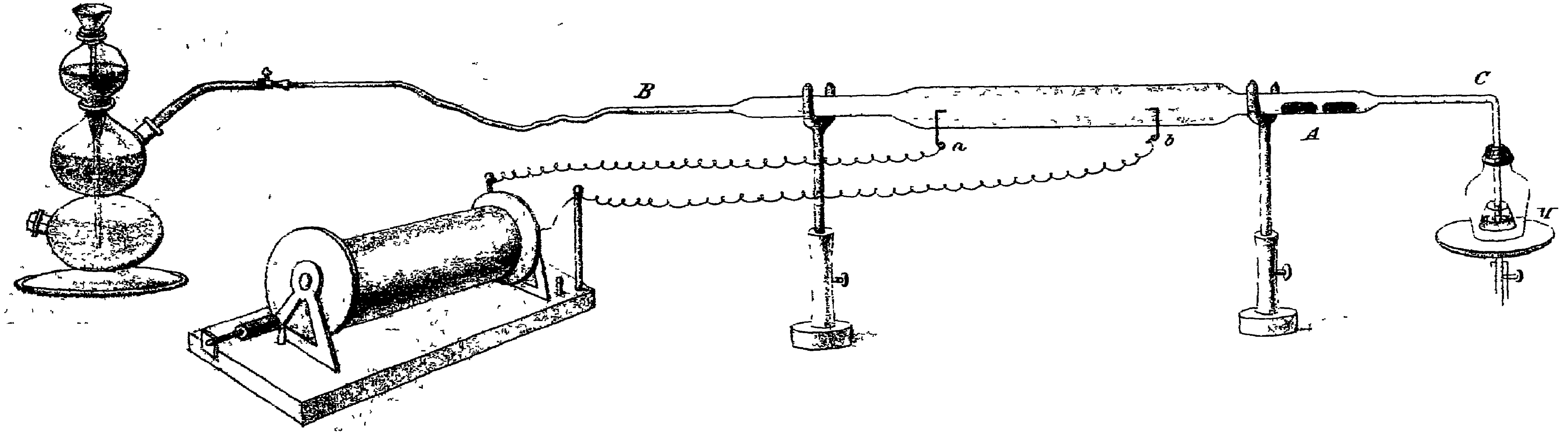
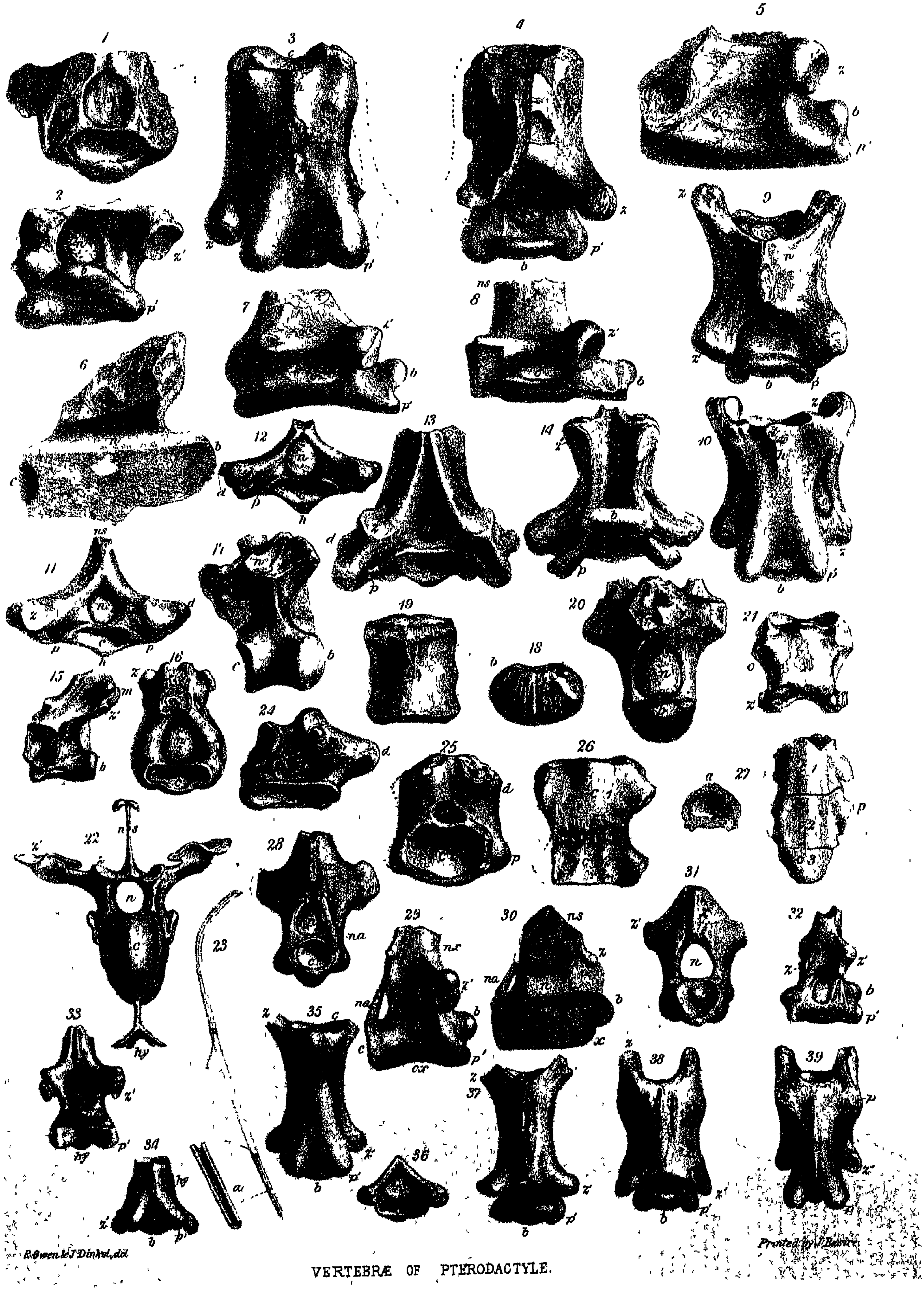


FIG 8



J BASIRE LITH

Phil. Trans MDCCCLIX Plate X



R. Owen & J. Dinkel, del.

Printed by J. Basire.

VERTEBRÆ OF PTERODACTYLE.

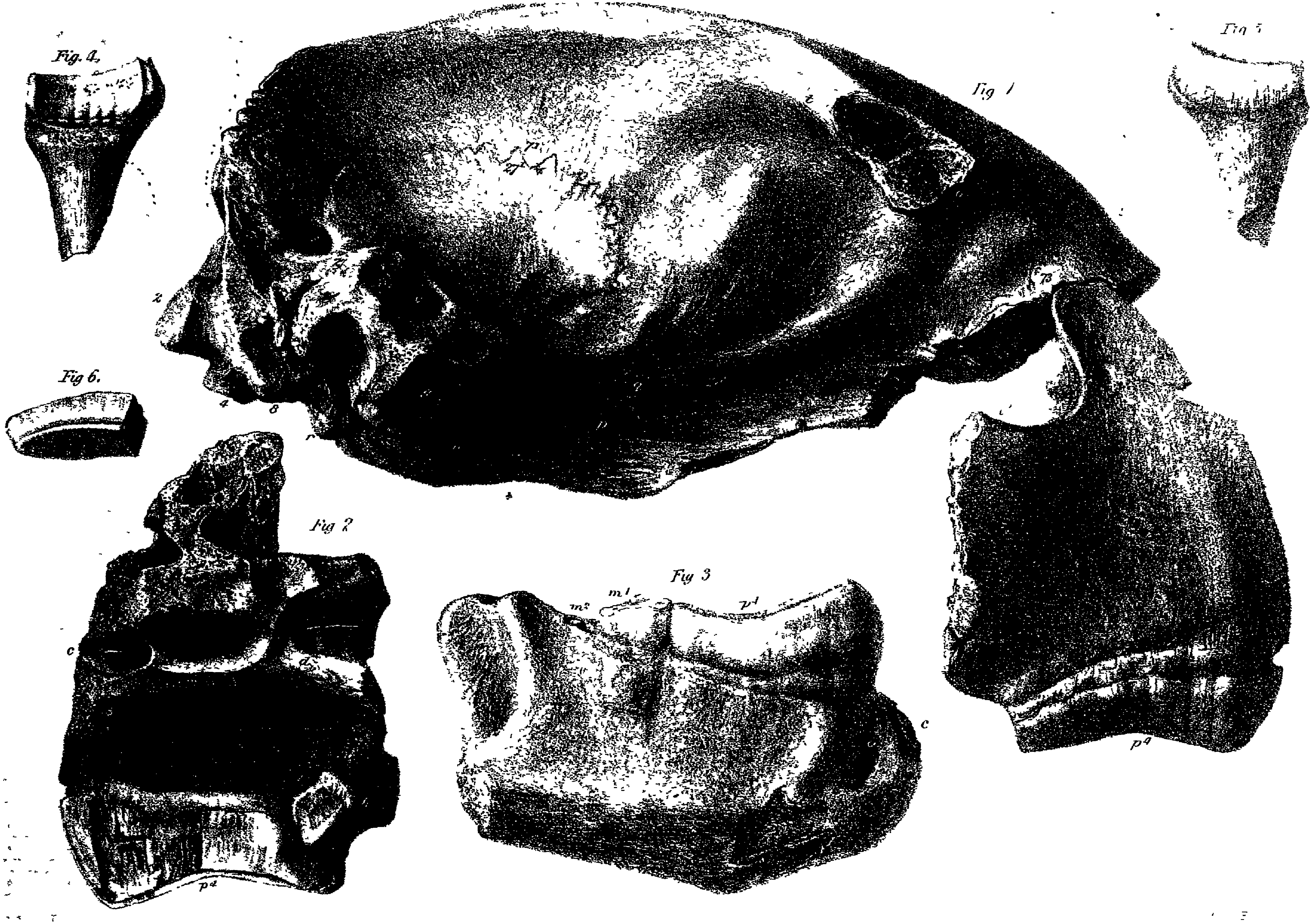


Fig. 4.
Fig 6.
Fig 1
Fig 5
Fig 2
Fig 3
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8
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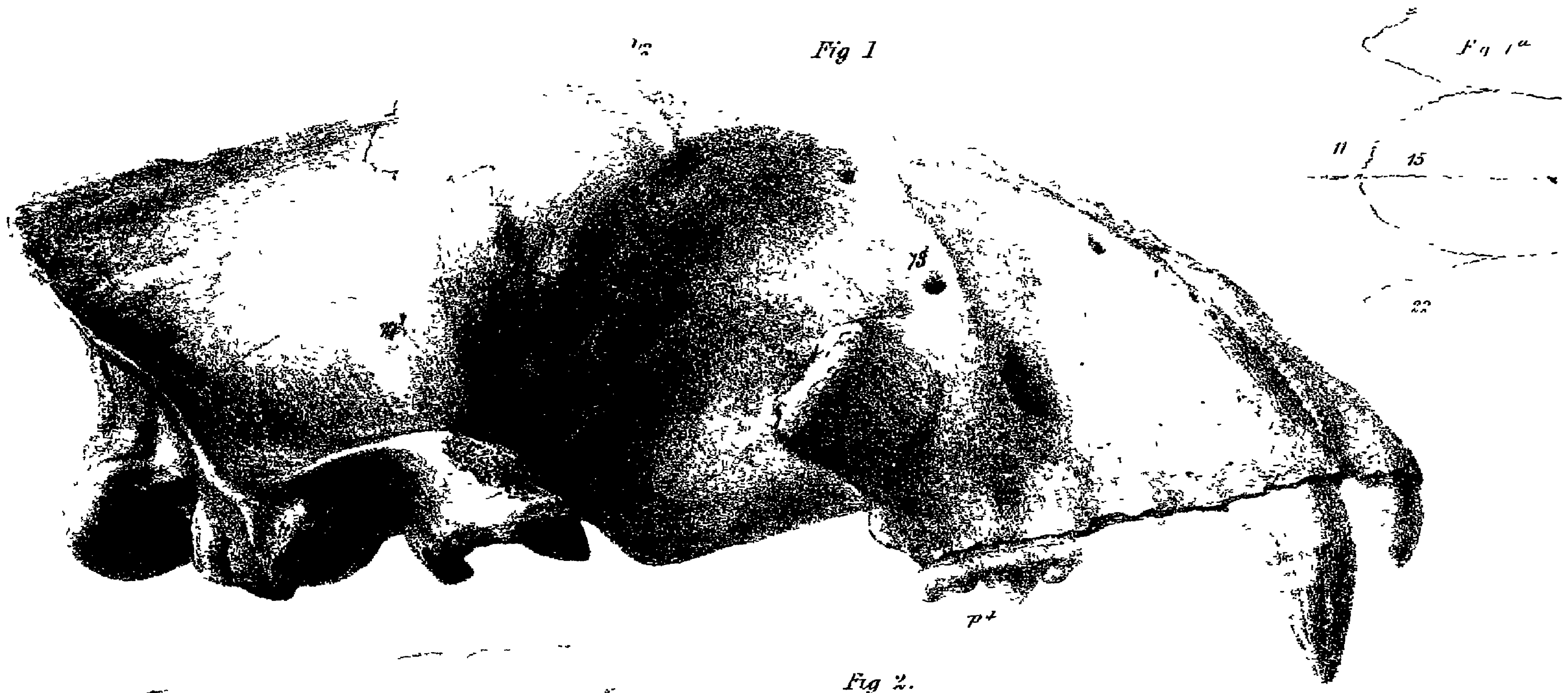
73
c'
m²
m¹
p¹
c
c
d
p⁴
p⁴

Fig 1.



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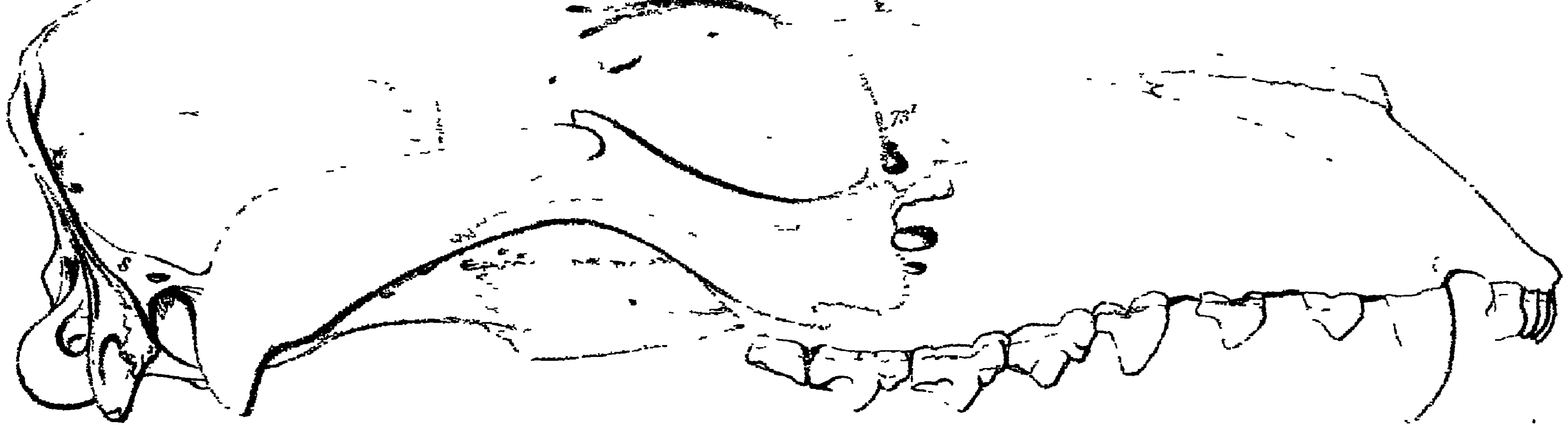
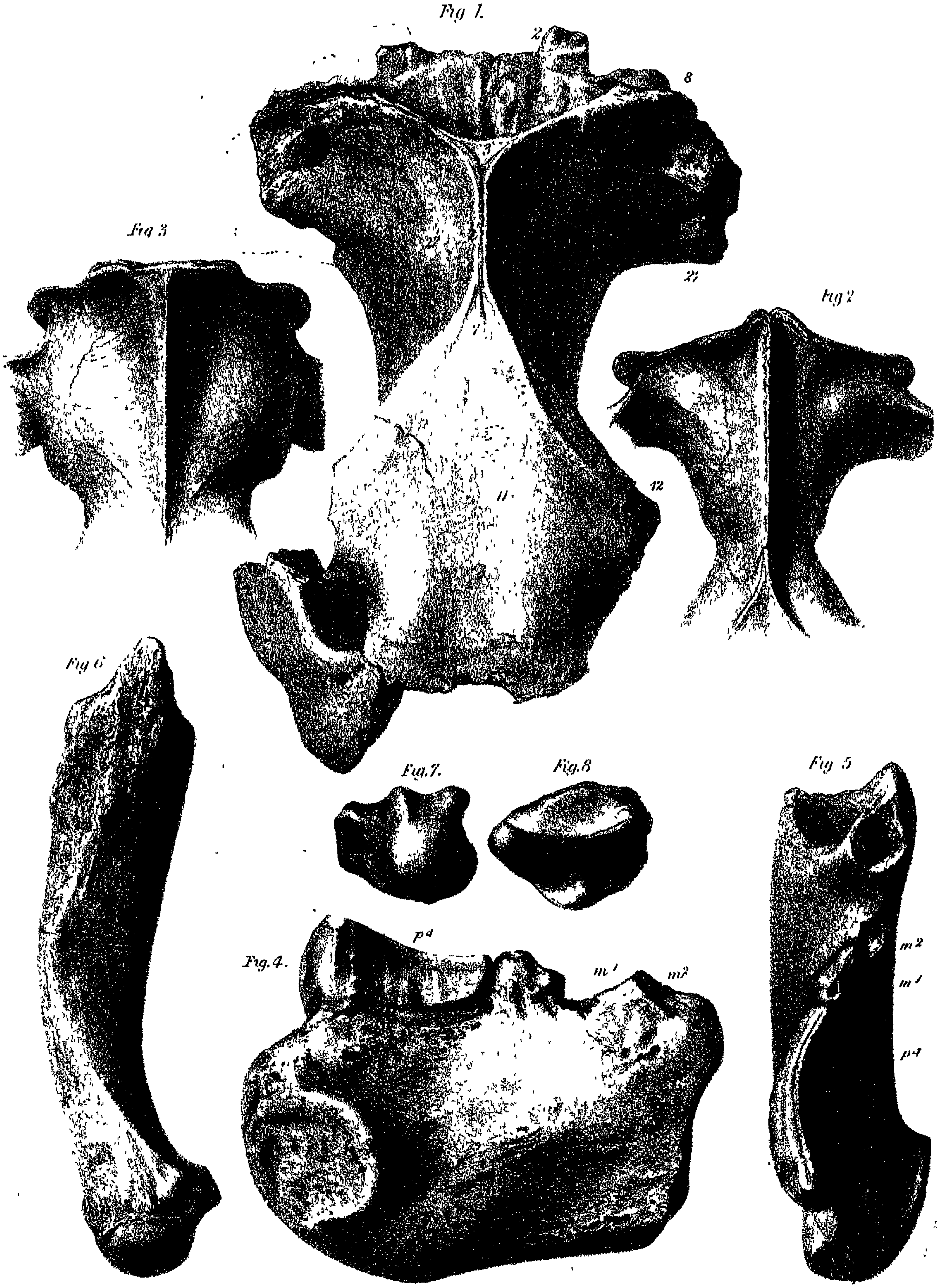


Phil. Trans MDCCCLIX Plate XIII



J. Dinkel del

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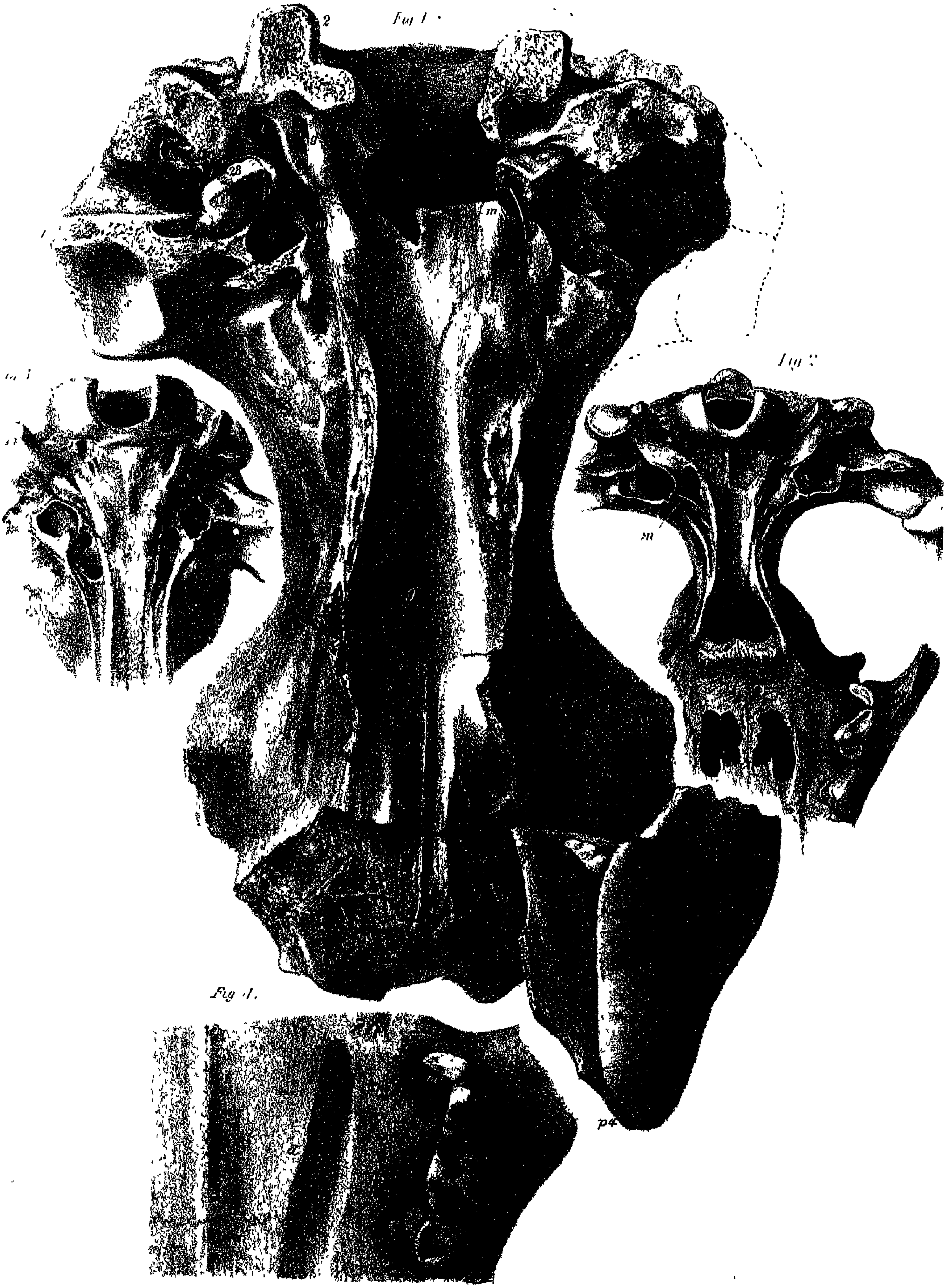
Phil. Trans. MDCCCLIX. Plate XIV.



J. Dinkel del.

Printed by J. Basire

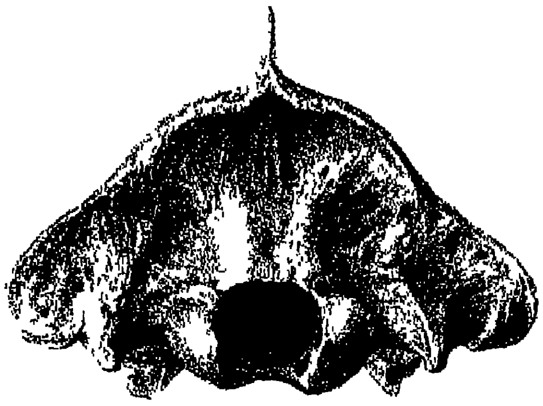


Fig 2

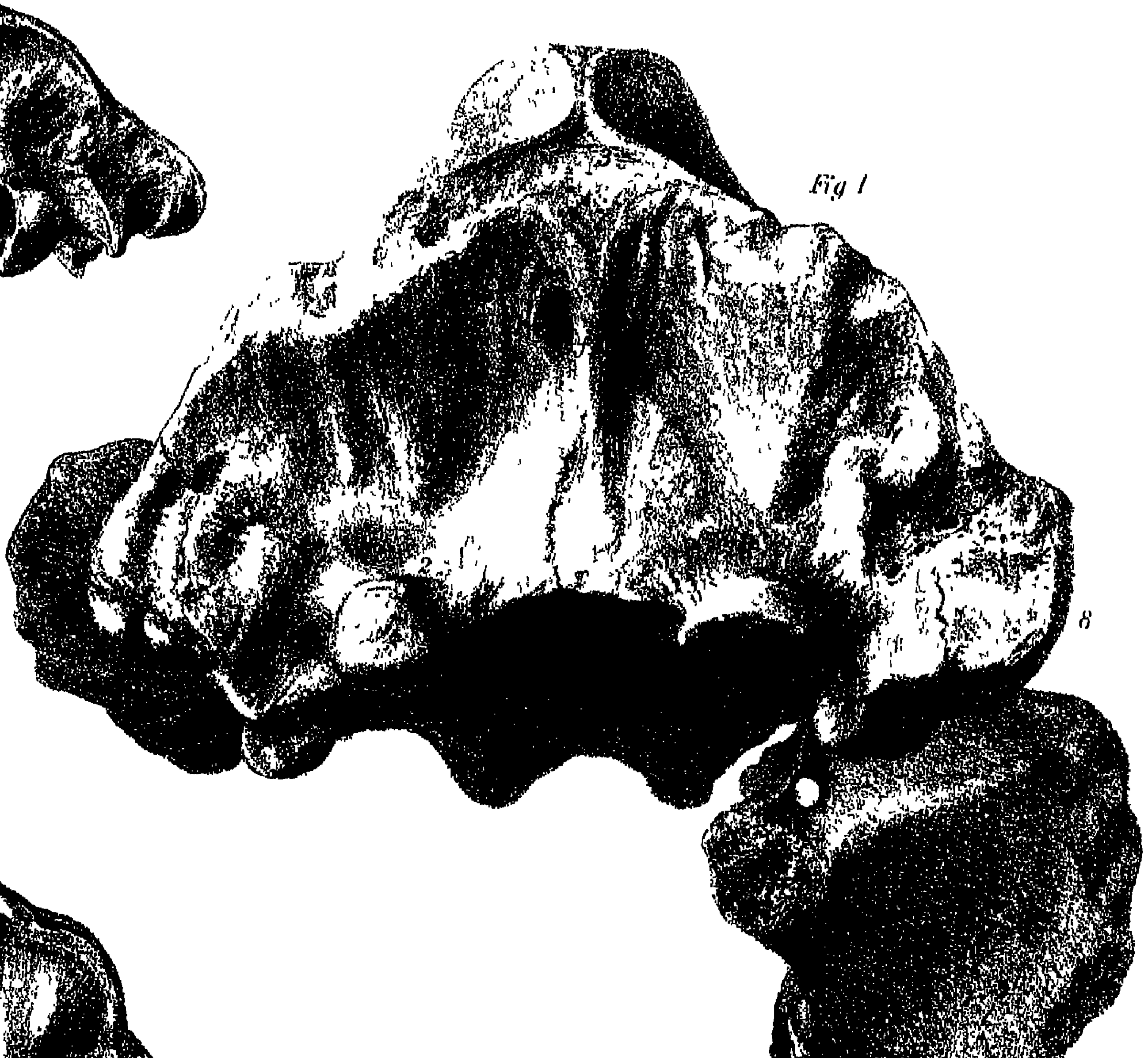


Fig 1

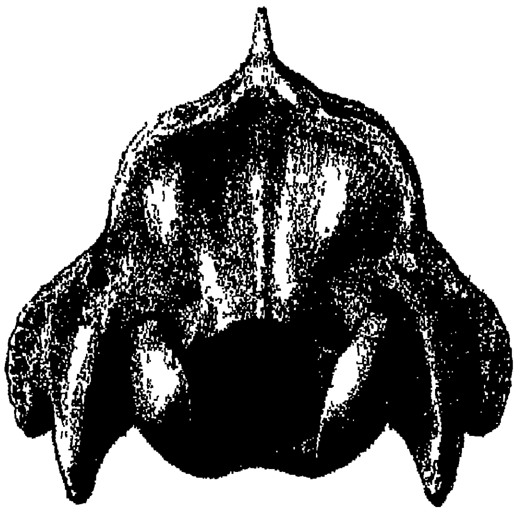


Fig 3.

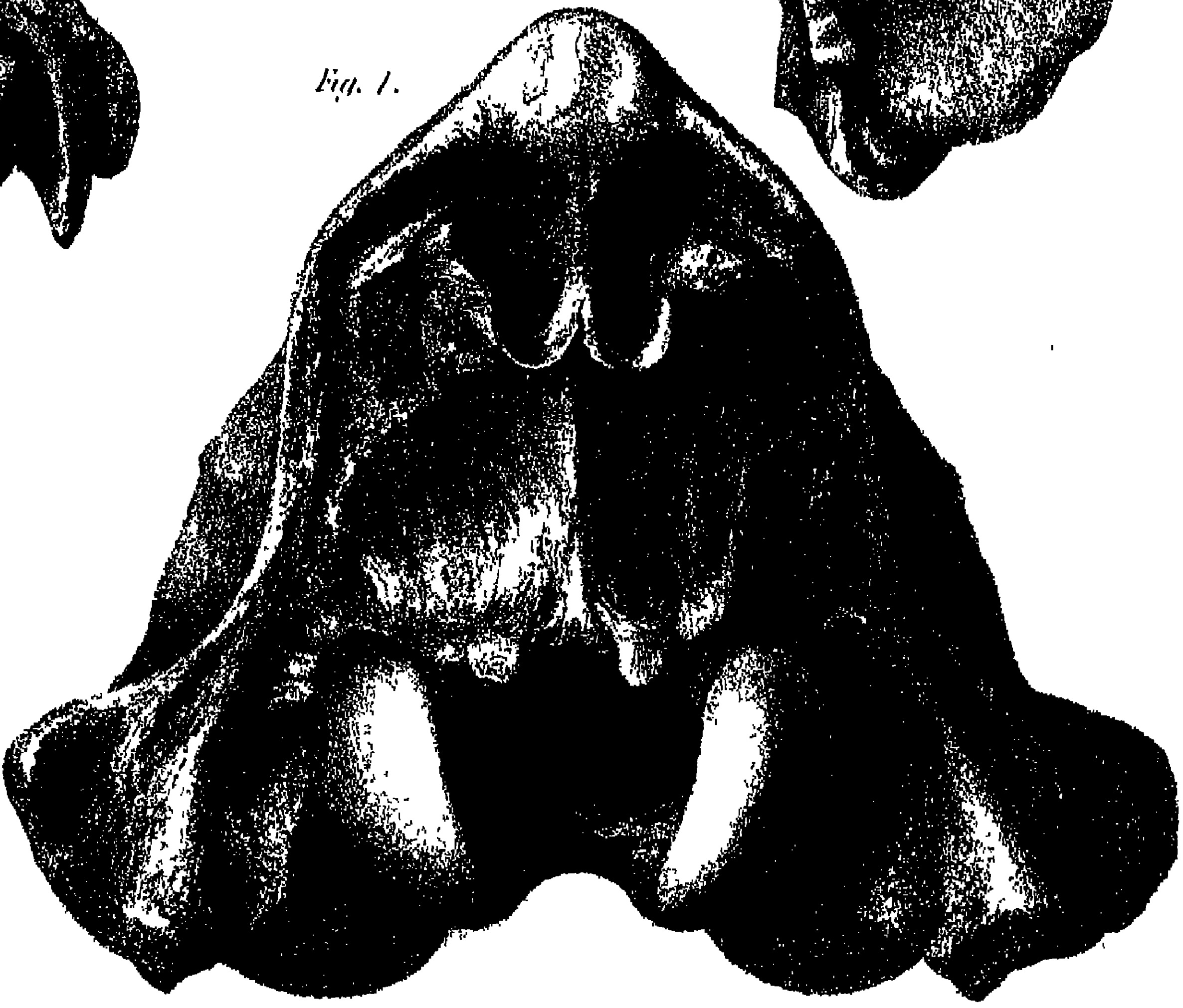


Fig. 1.

J. Dinkel del.

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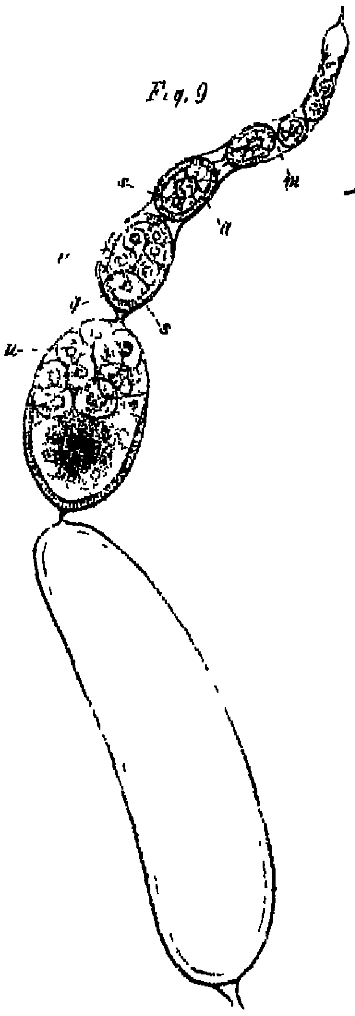
Fig. 9



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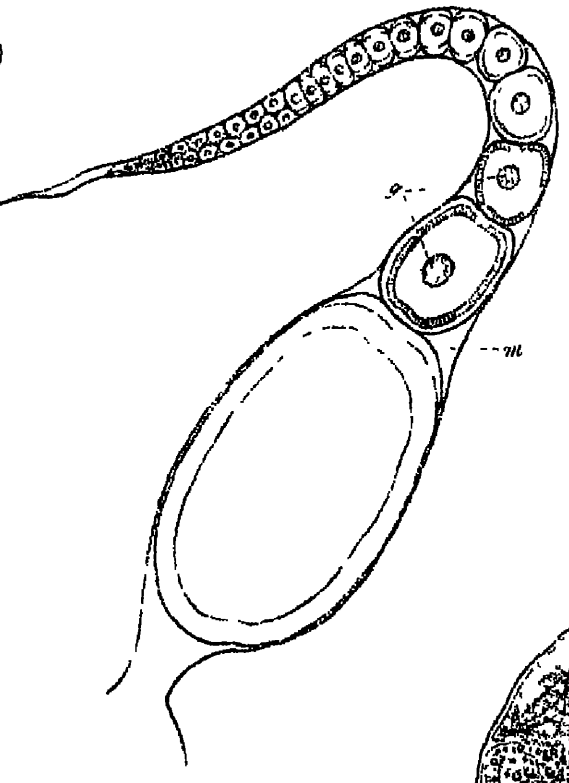


Fig. 1

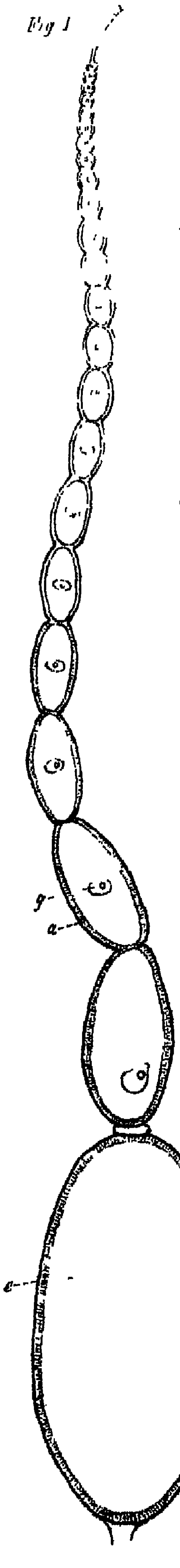
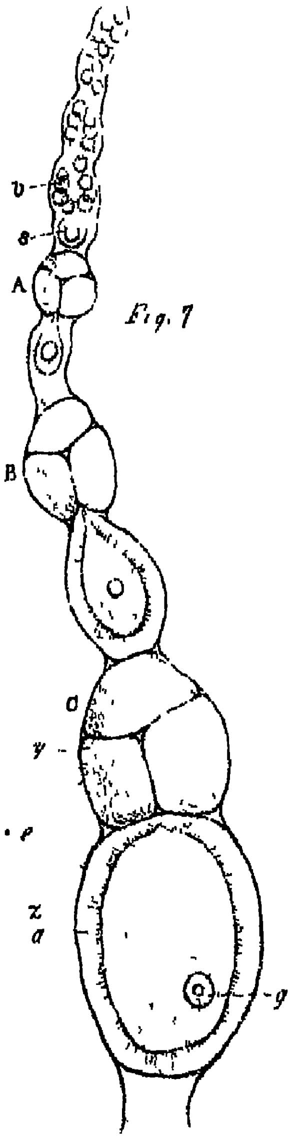


Fig. 7



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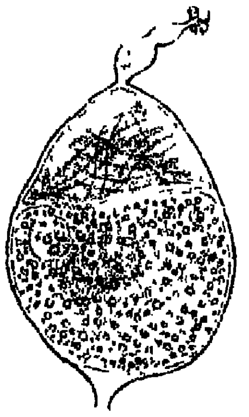

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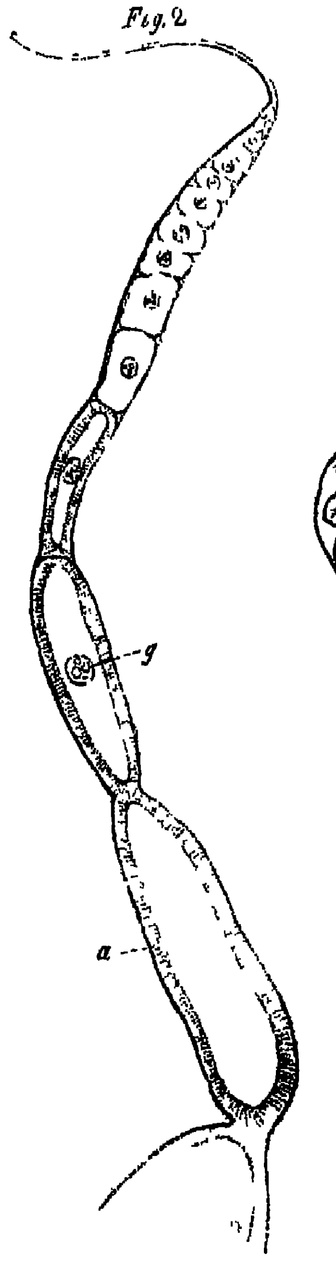

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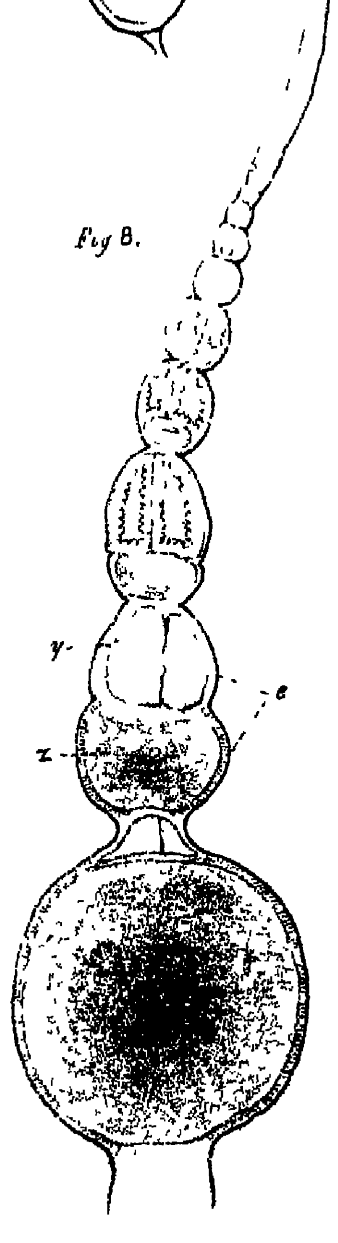

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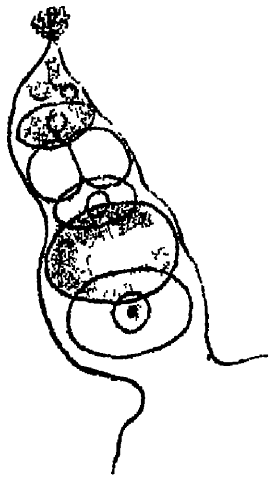

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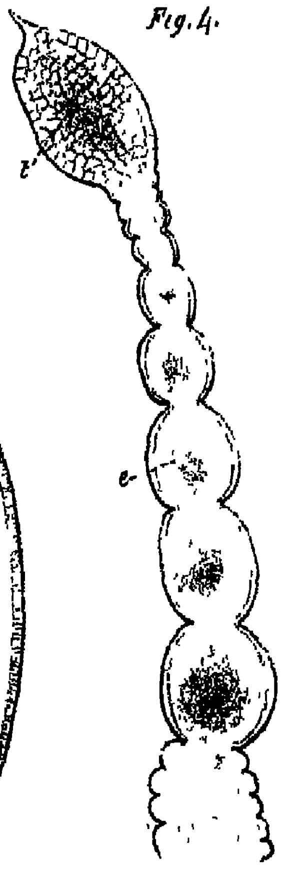

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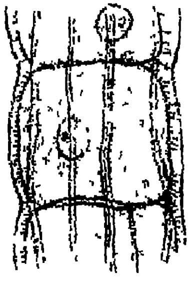
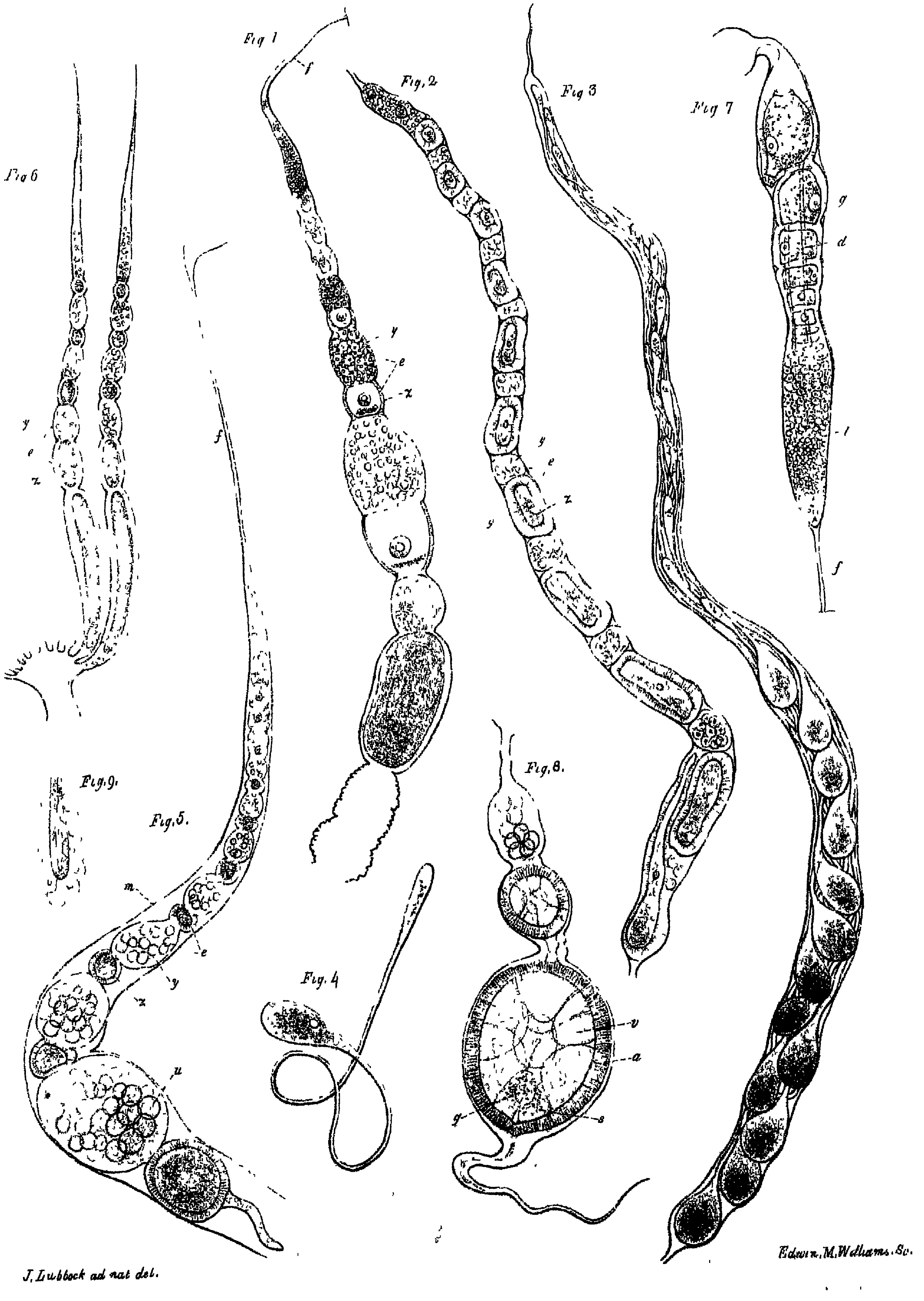


J. Lubbock ad nat del

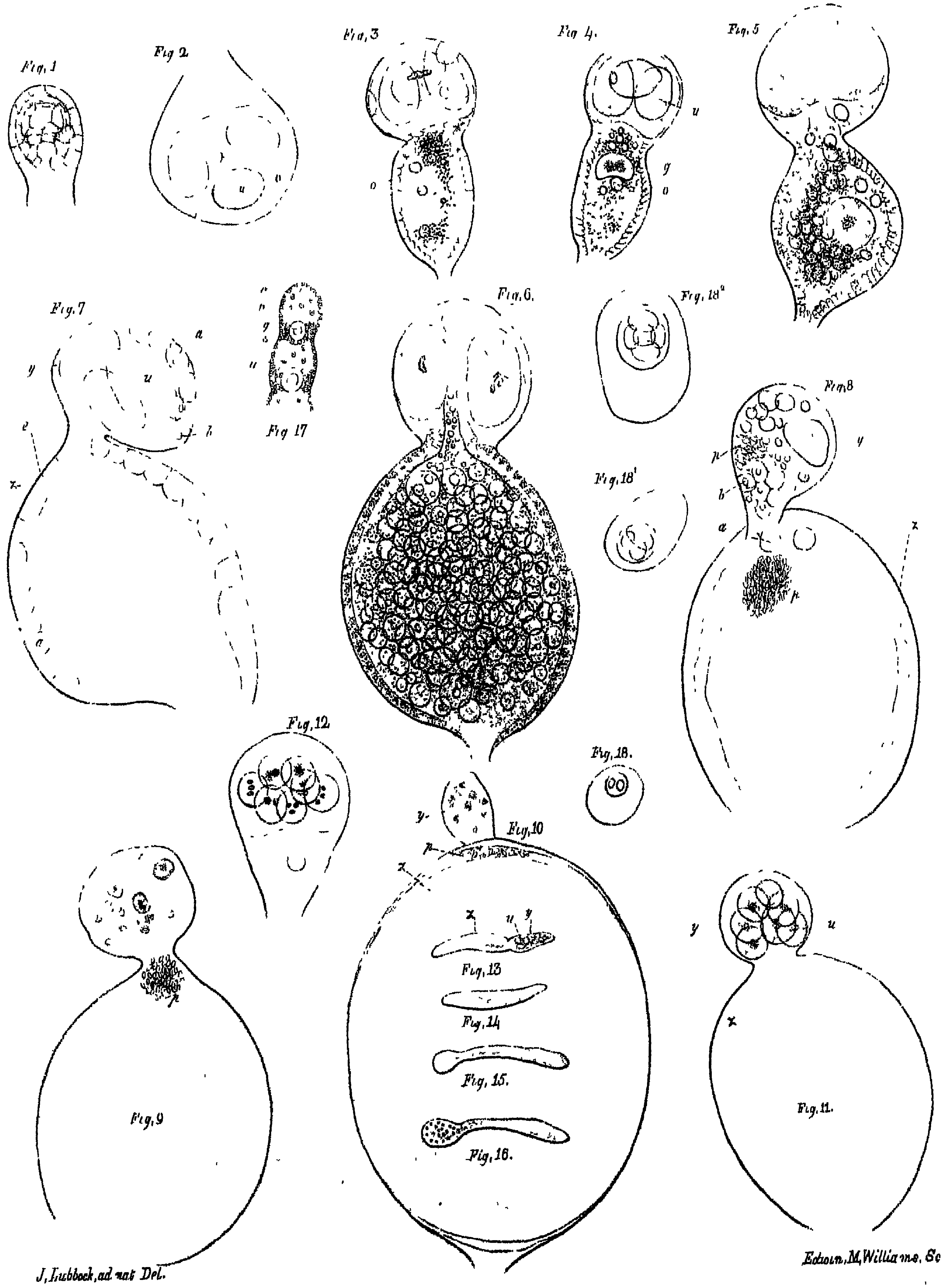
Edwin M. Williams, Sc

Phil. Trans. MDCCCLIX. Plate XVII.



J. Lubbock ad nat. del.

Edwin M. Williams Sc.



J. Lubbock, ad nat. Del.

Edwin M. Williams, Sc.

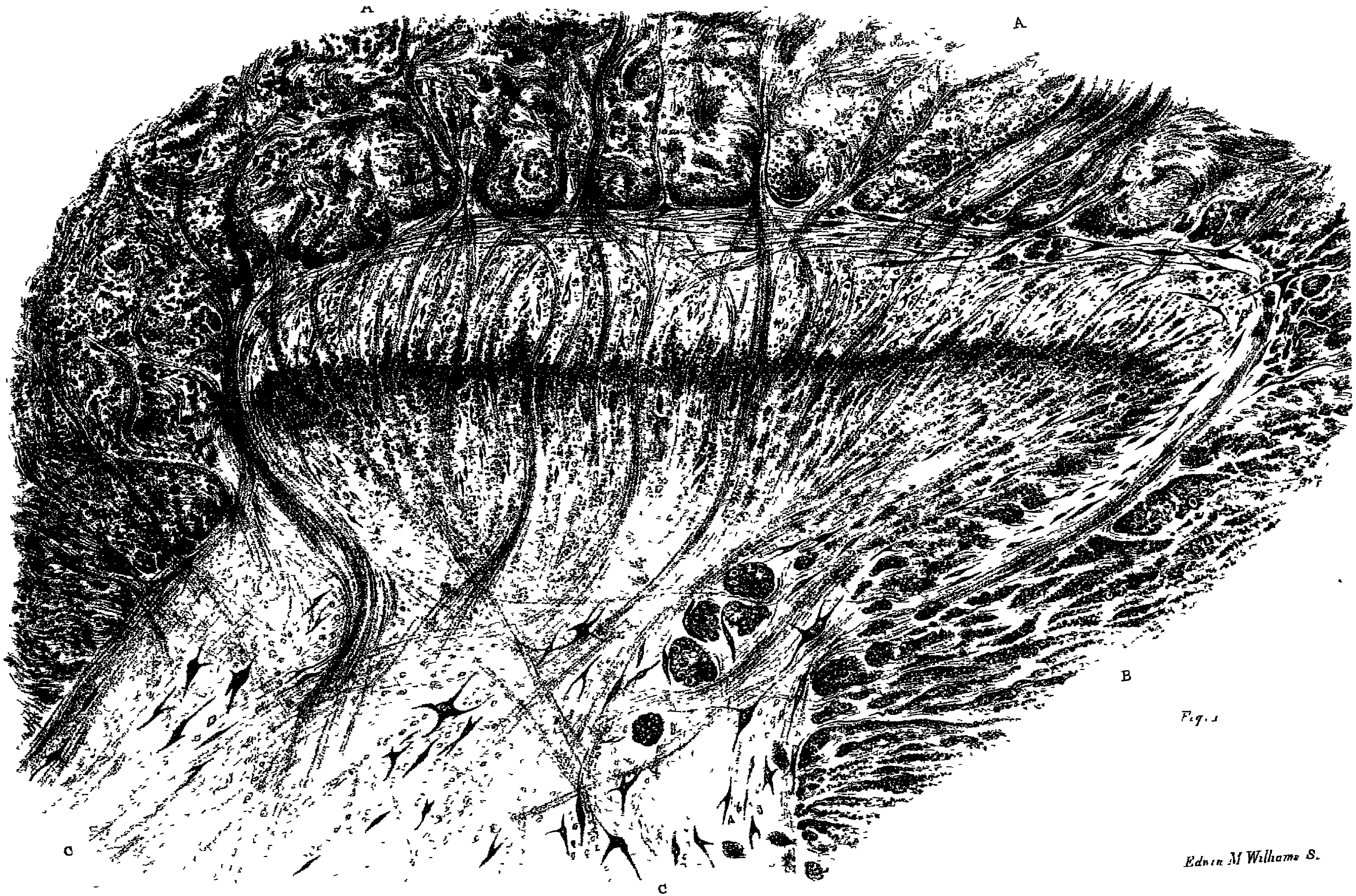
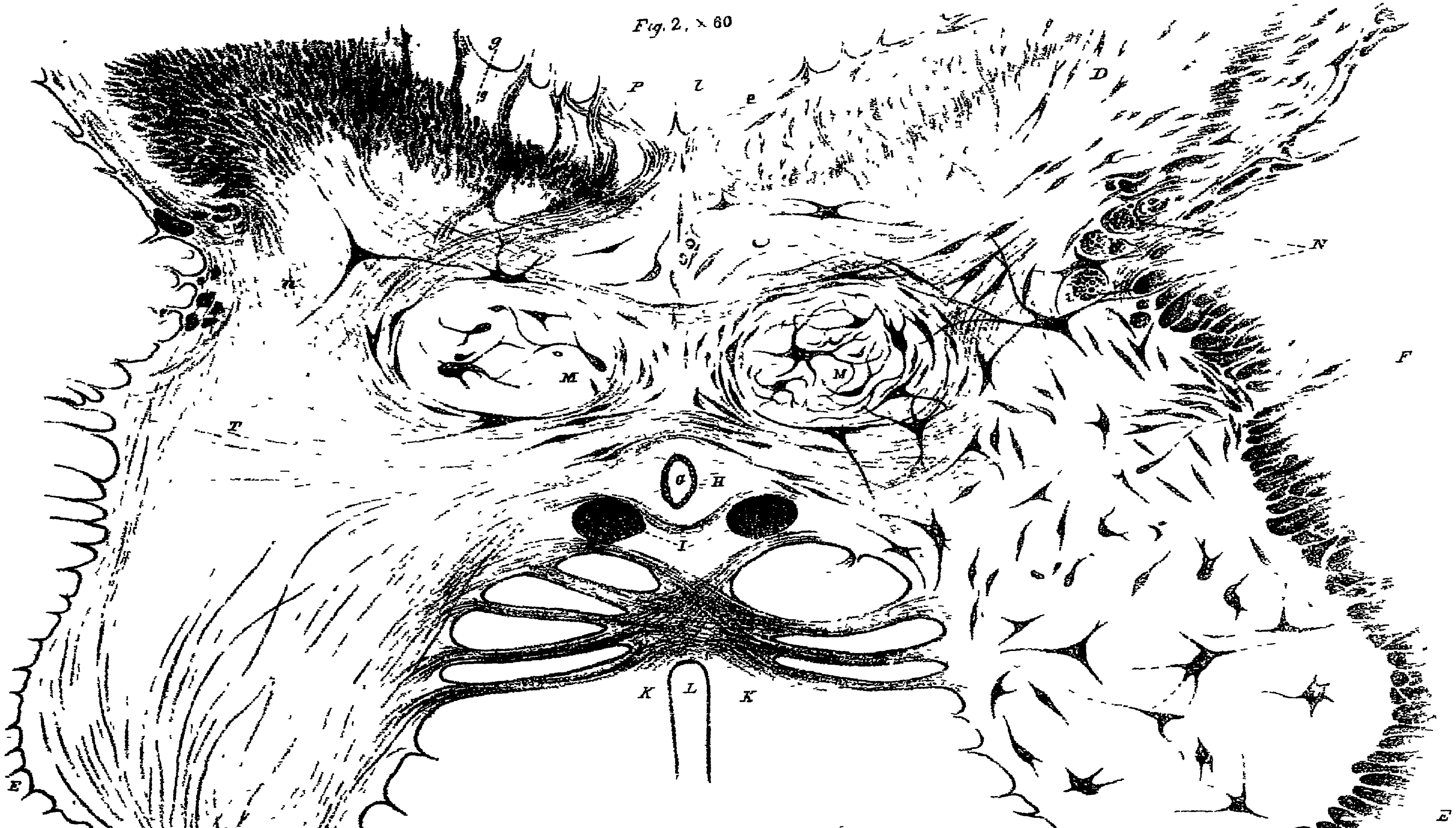


Fig. 1

J.L. Clarke & G West Del

Edwin M Williams S.

Fig. 2, × 60



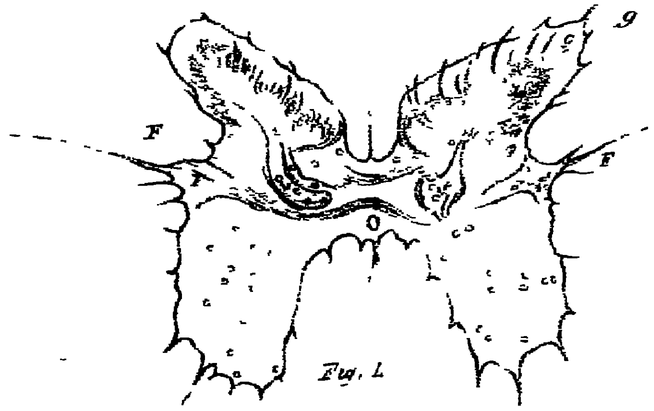
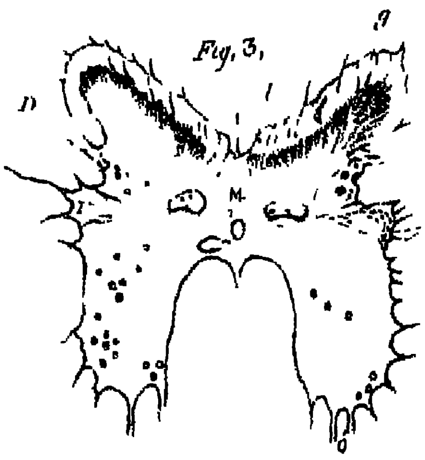


Fig. 1.

J. L. Clarke Del.

Fig. 3.



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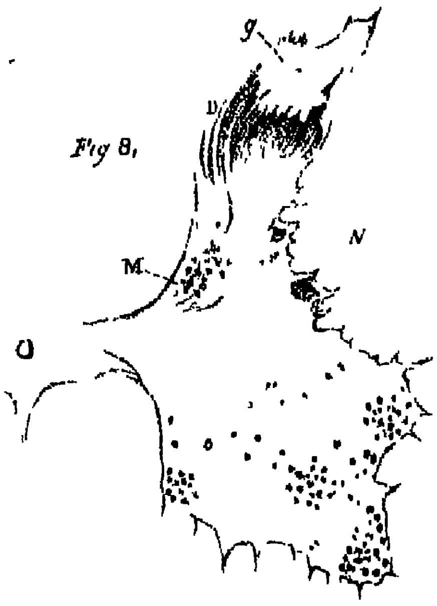

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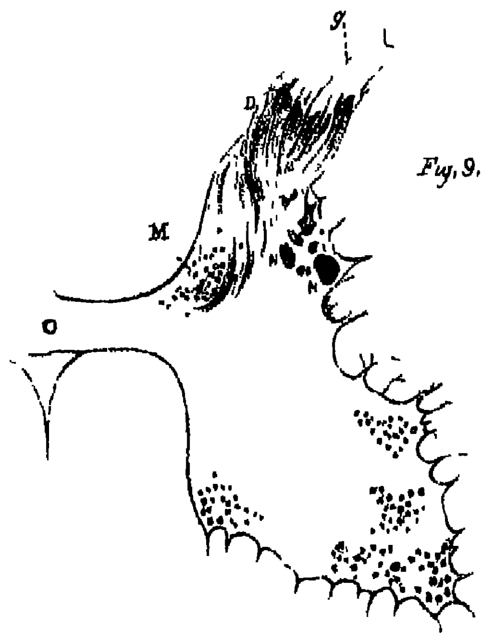

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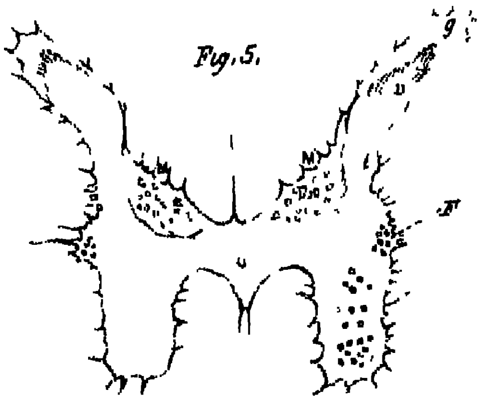
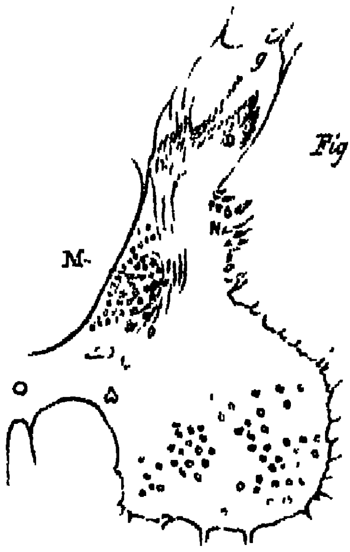


Fig. 14.



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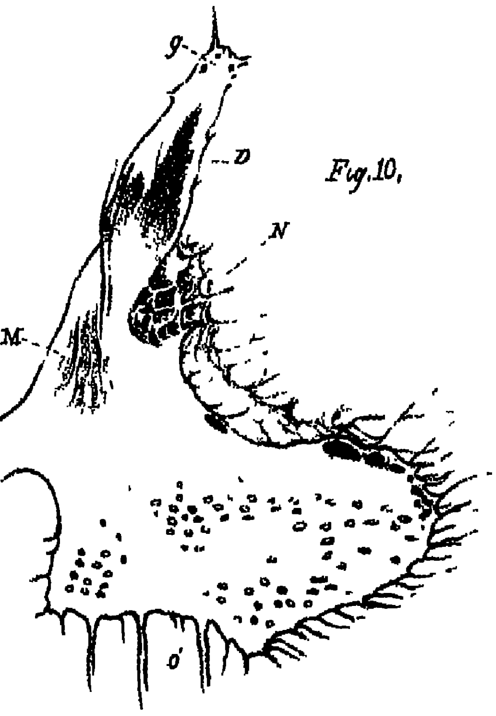

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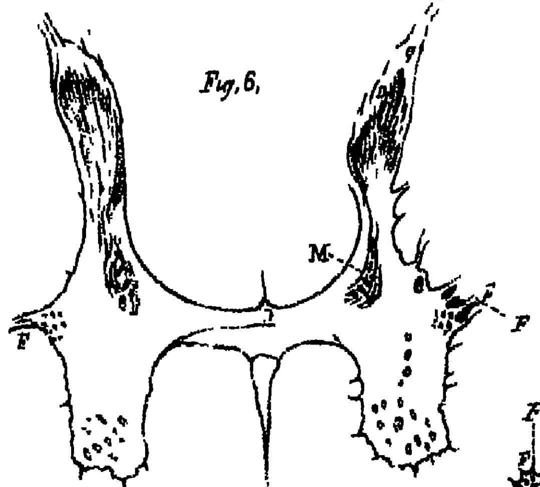

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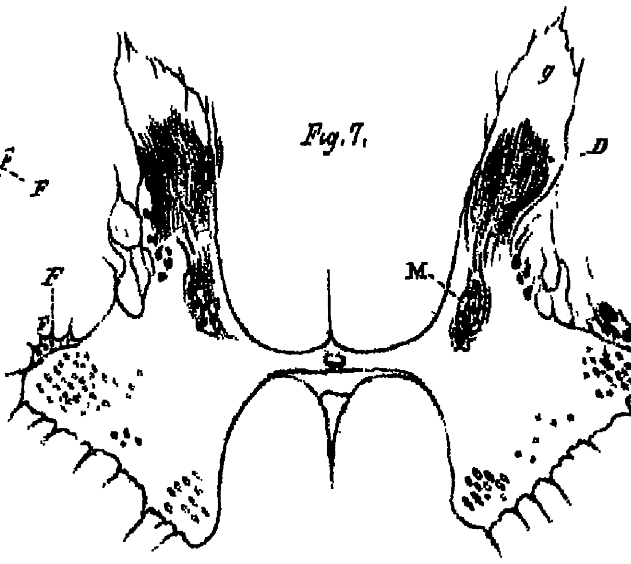

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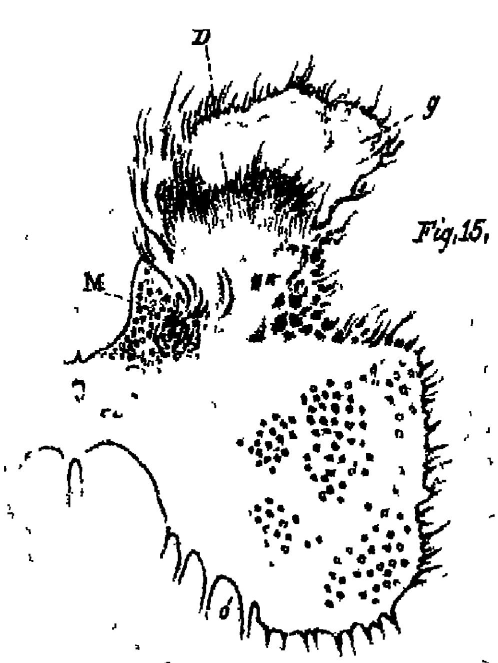

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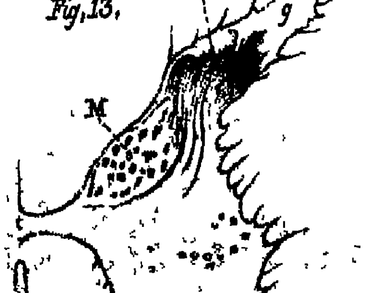

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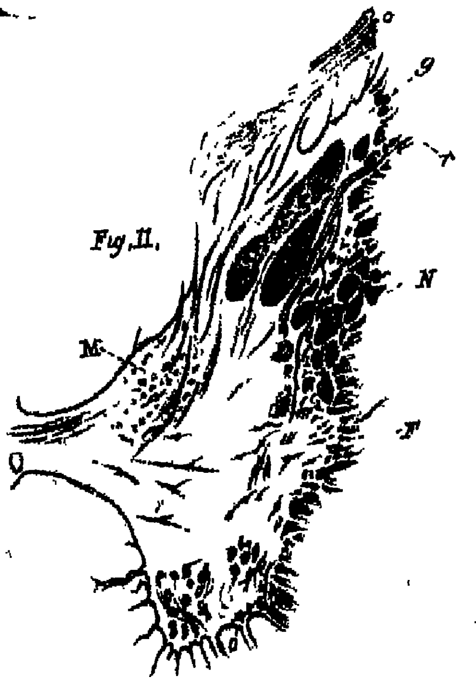
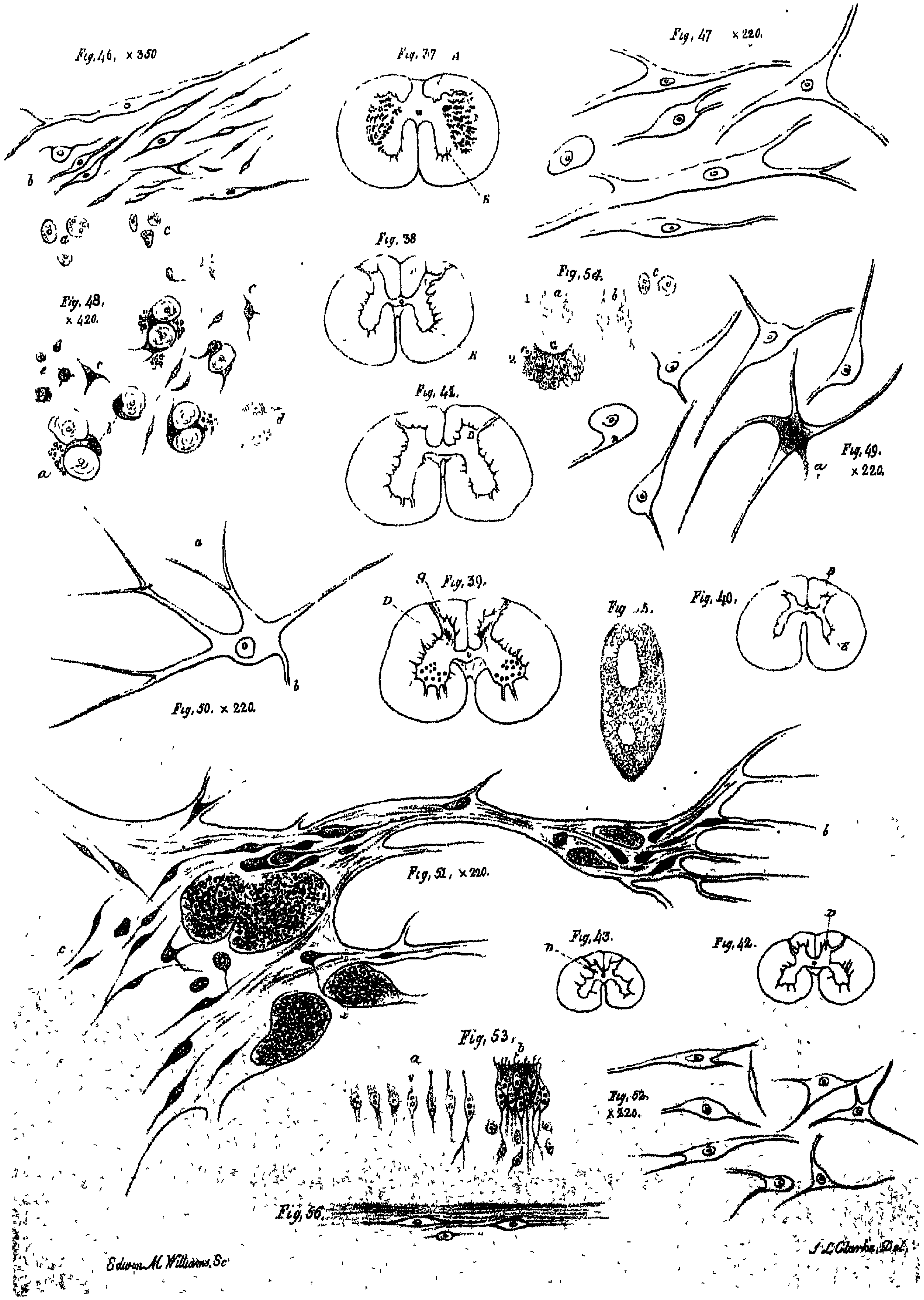


J. L. Clarke, Del.

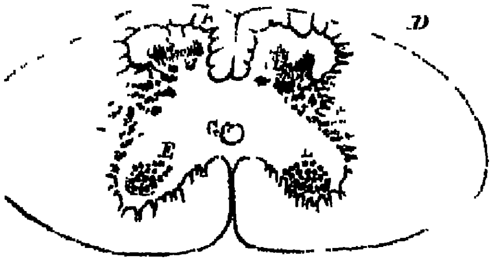
Edwin M. Williams, Sc.



Edwin M. Williams, Sc

J. L. Clarke, Del.

Fig. 26.



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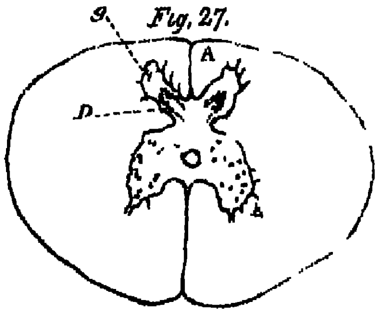


Fig. 28.

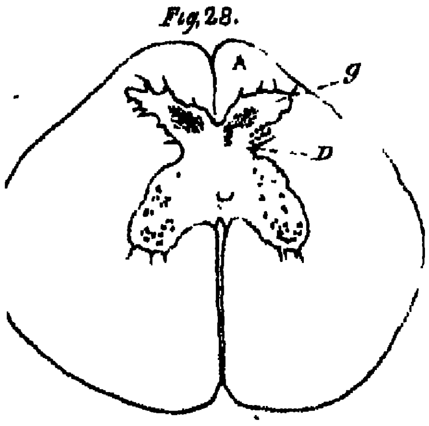
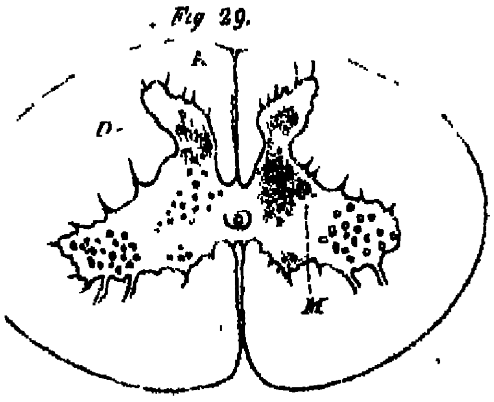


Fig. 29.



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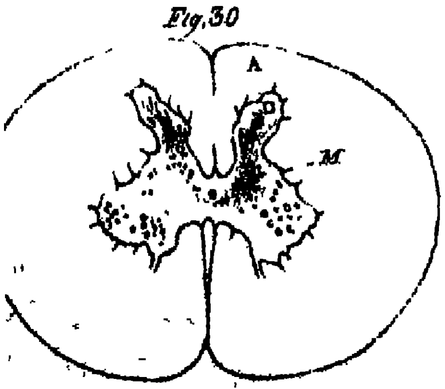

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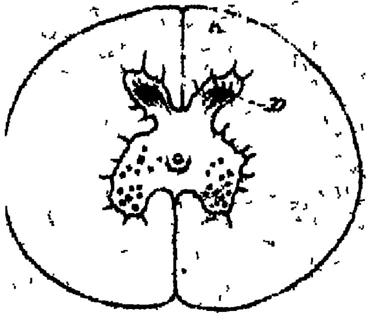

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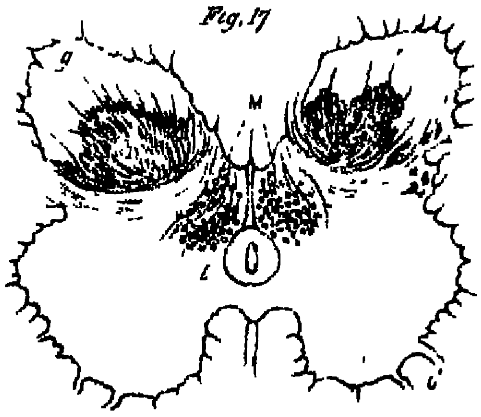

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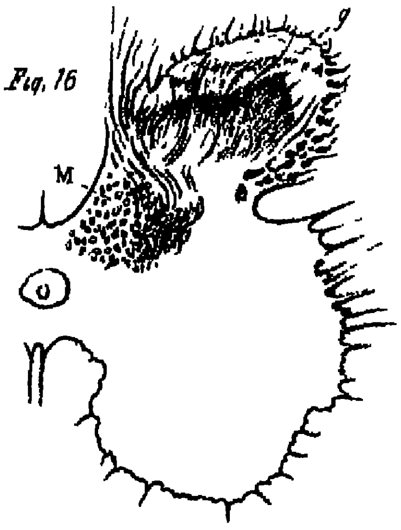

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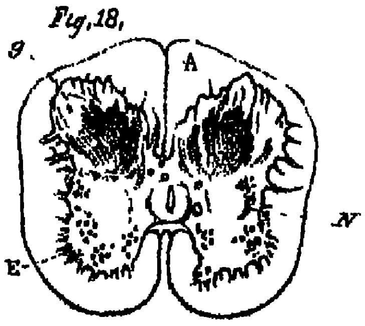

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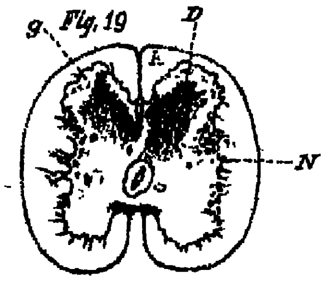

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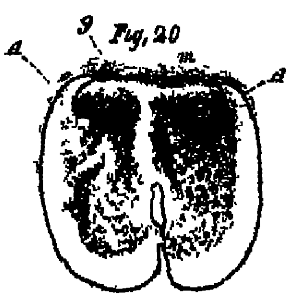

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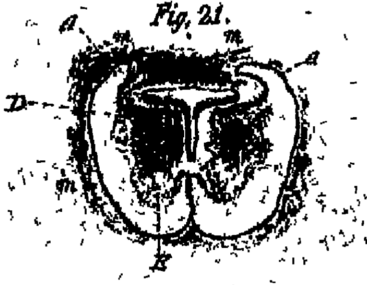

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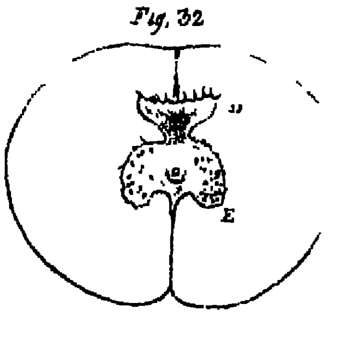

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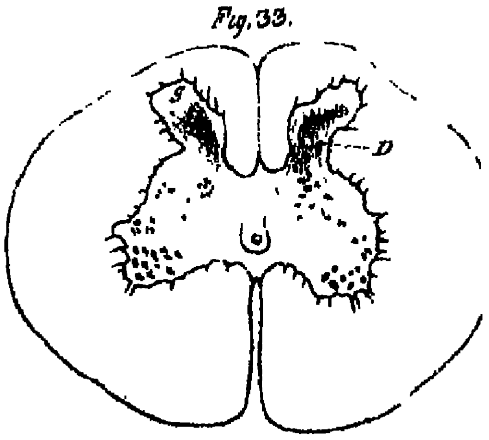

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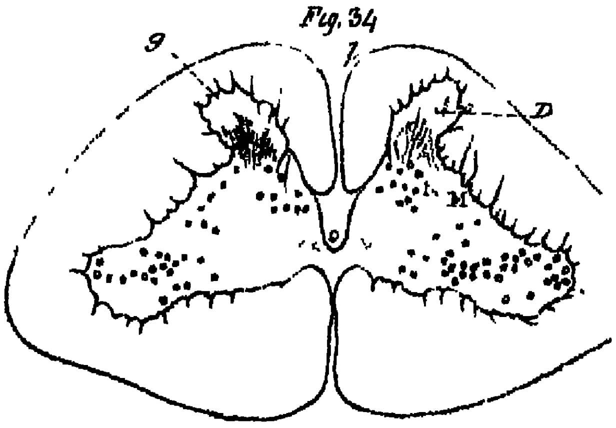

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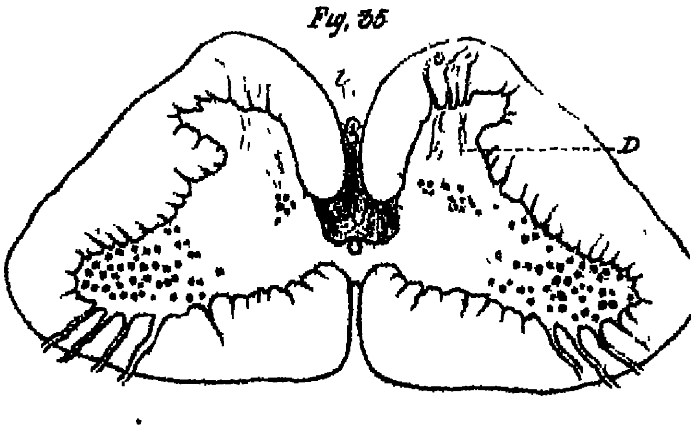

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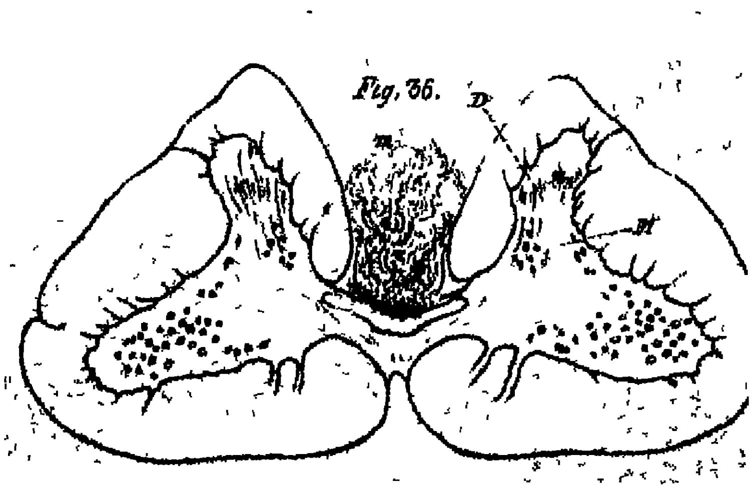

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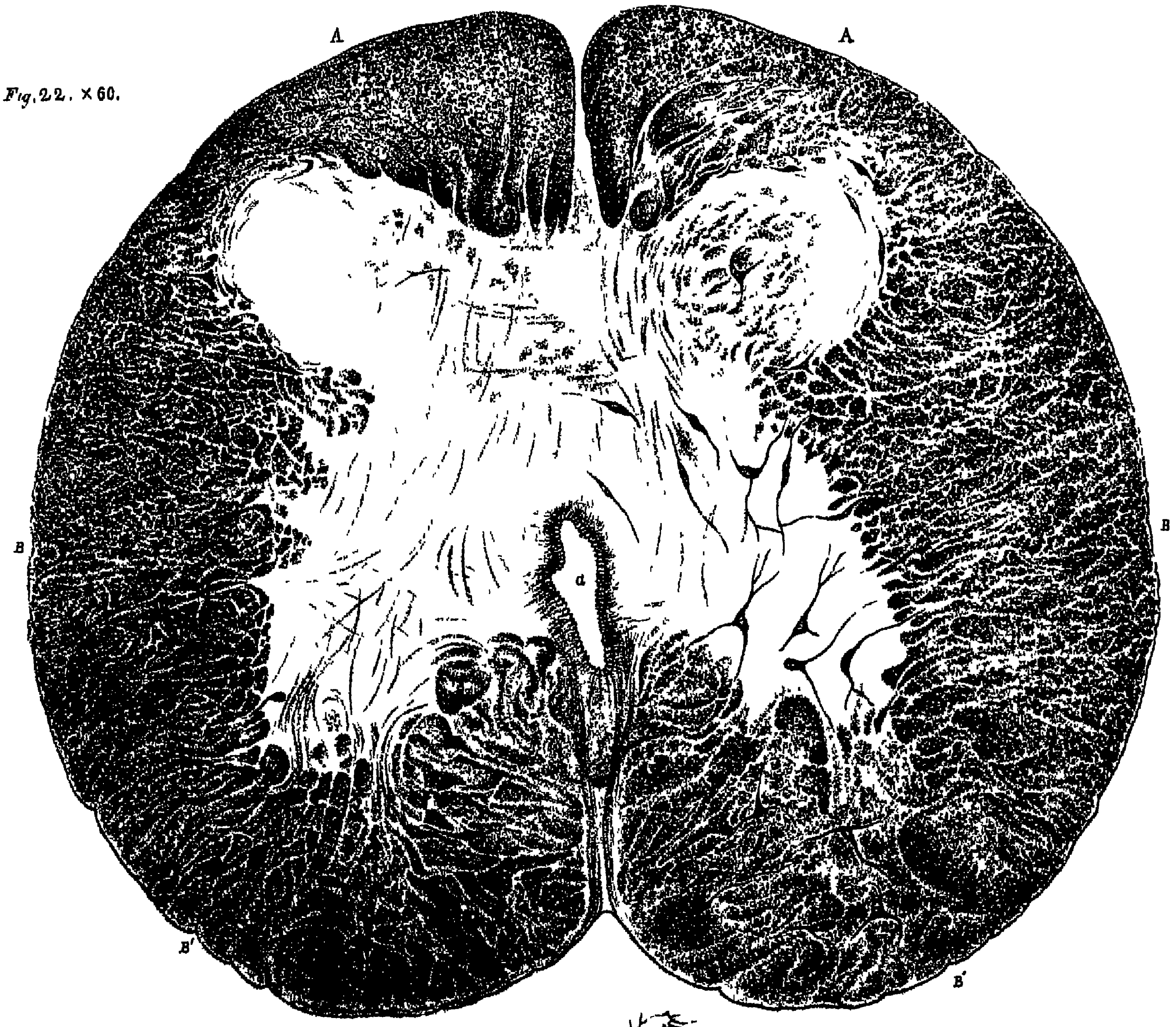
Fig. 22. × 60.



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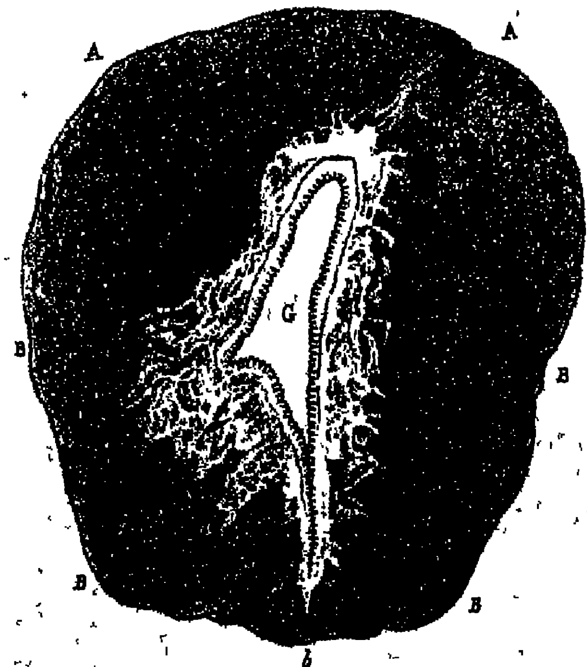


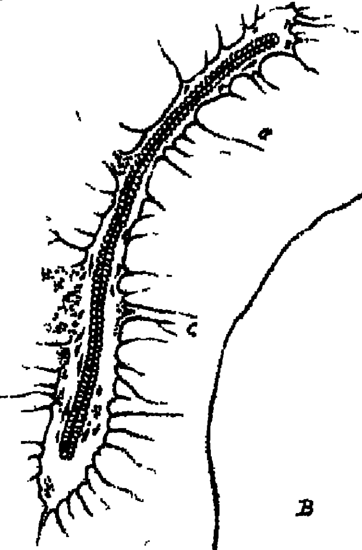
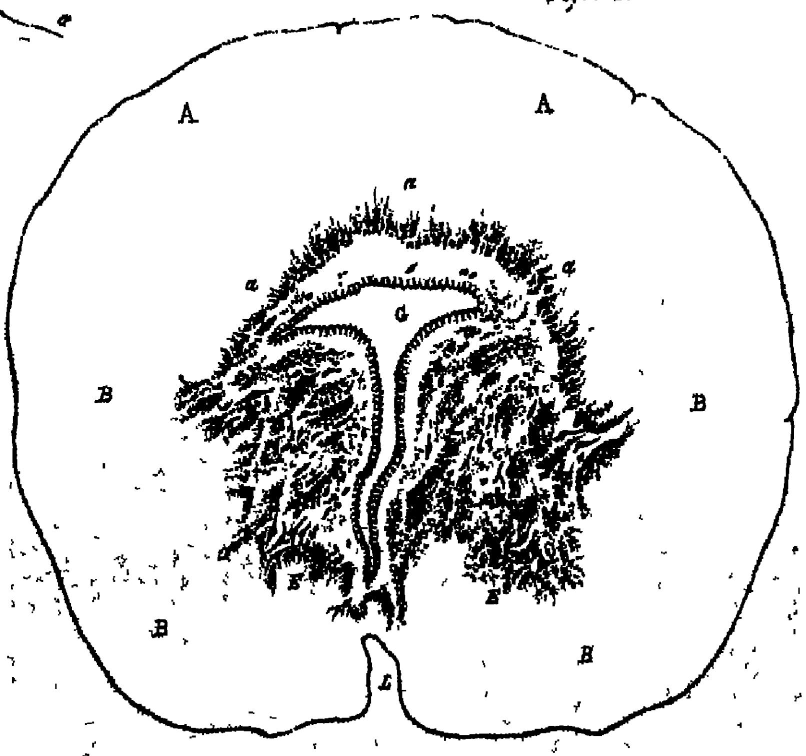


Fig. 25.
× 60.

Fig. 23. × 60.



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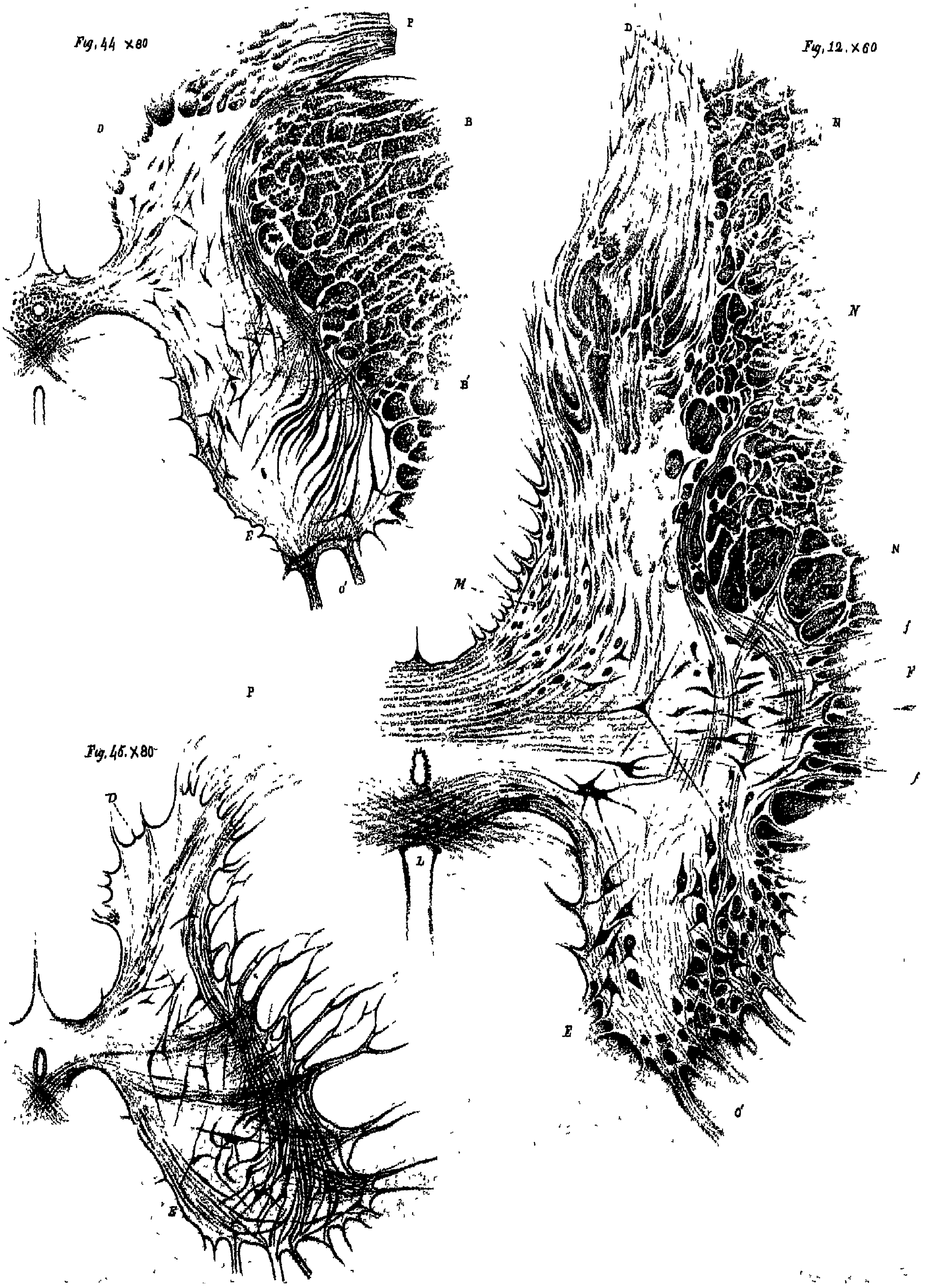

Fig. 44 ×80

Fig. 12 ×60

Fig. 46 ×80

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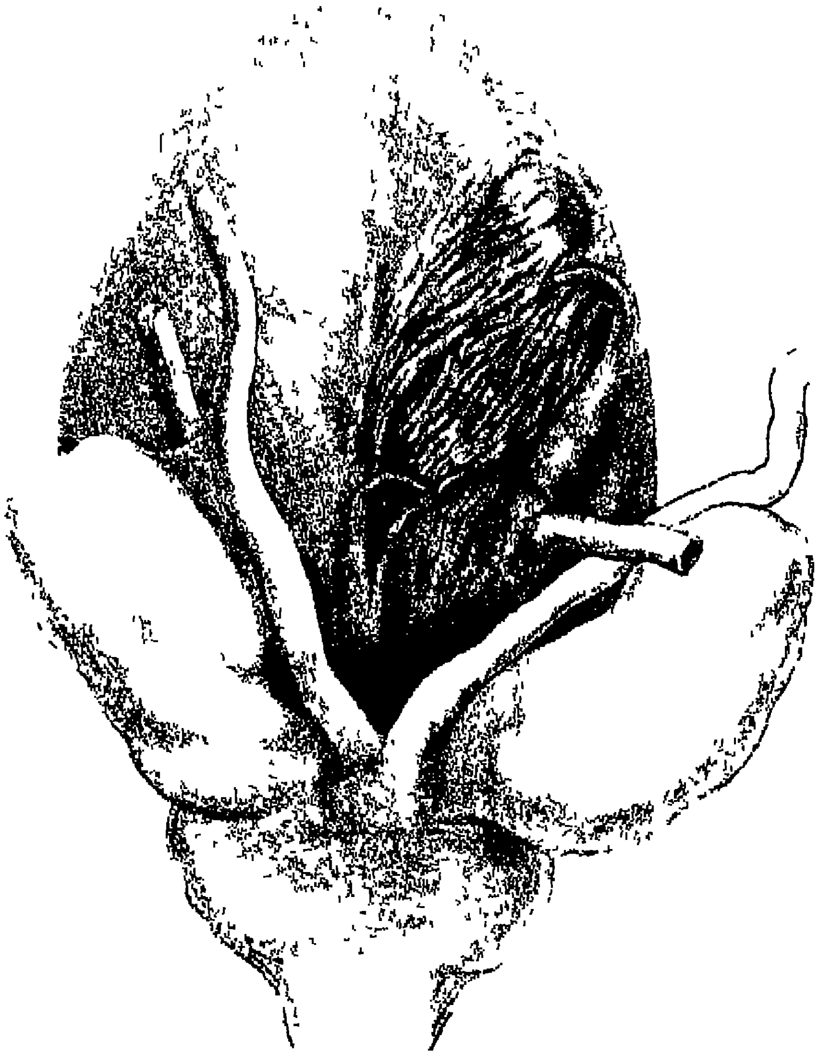
Edwin M. Williams, Sc.



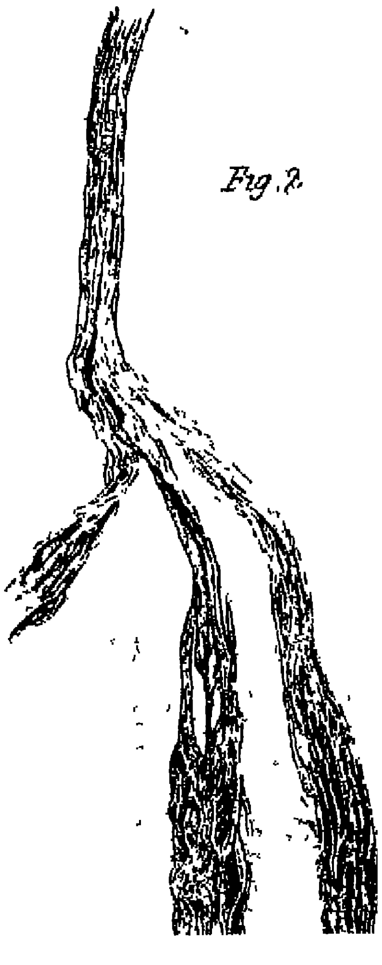


Fig. 2

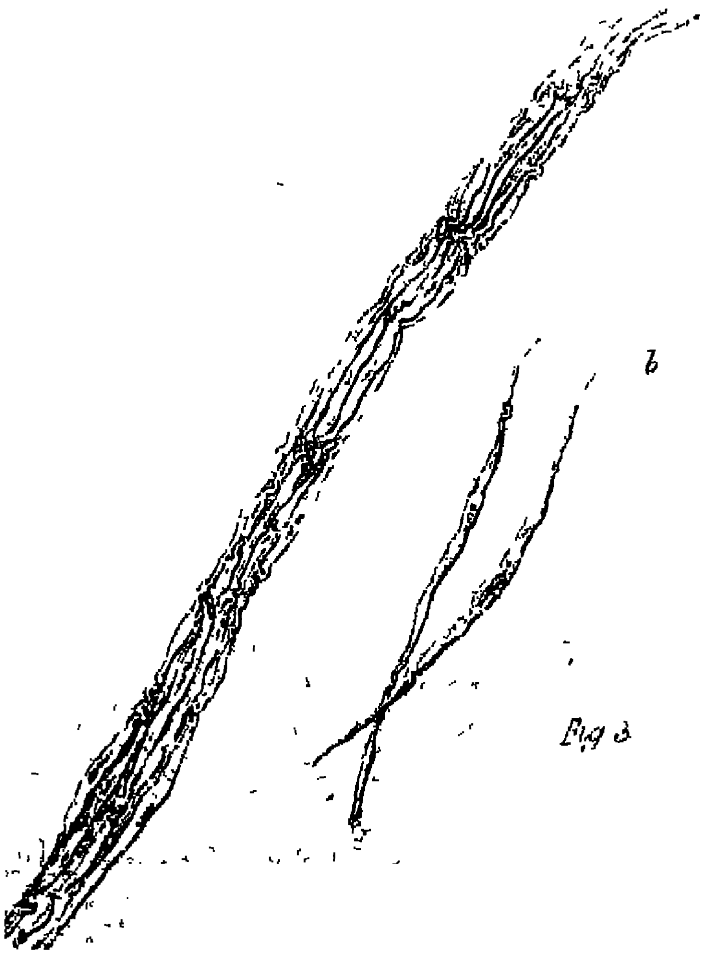
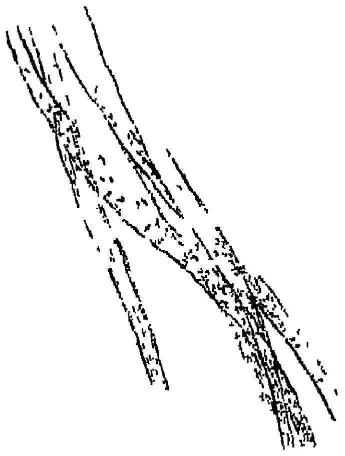


Fig. 3

Fig 5



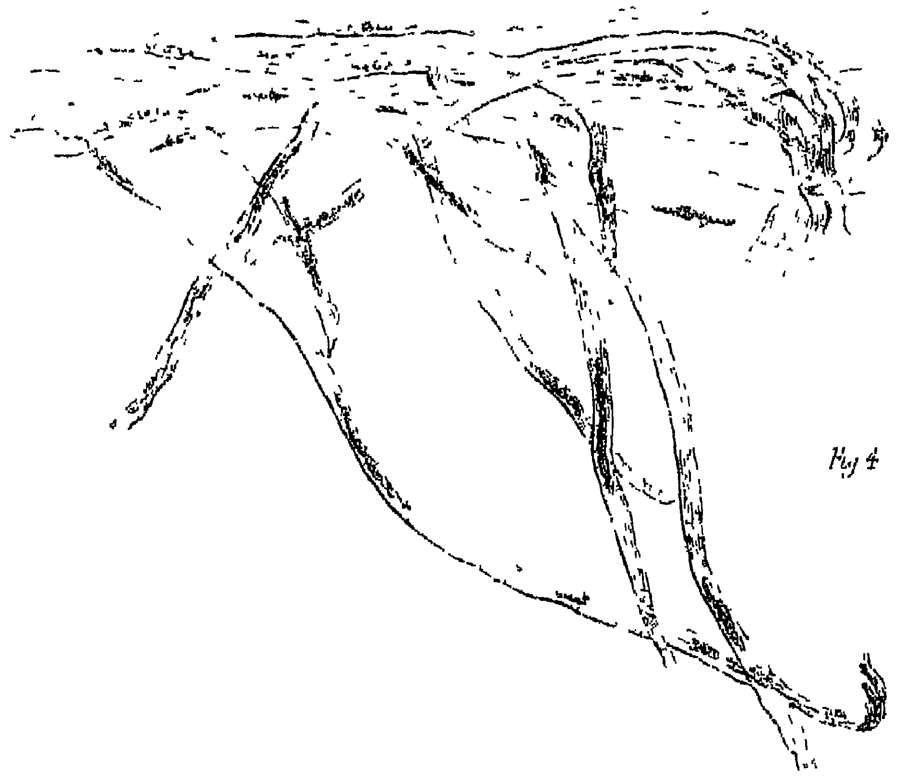
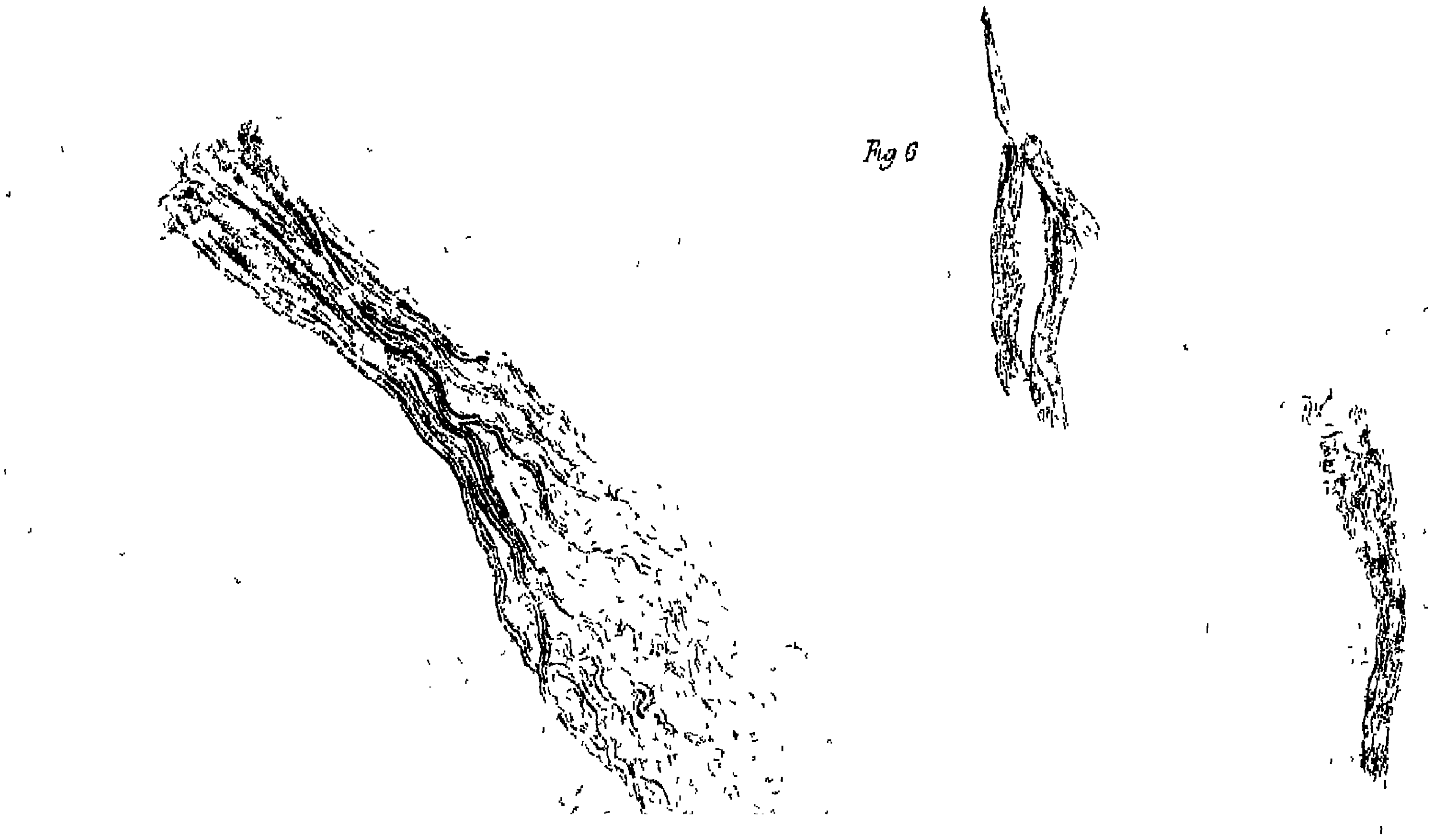


Fig 4

Fig 6



Wm West imp.

Fig. 1 $\frac{1}{3}$

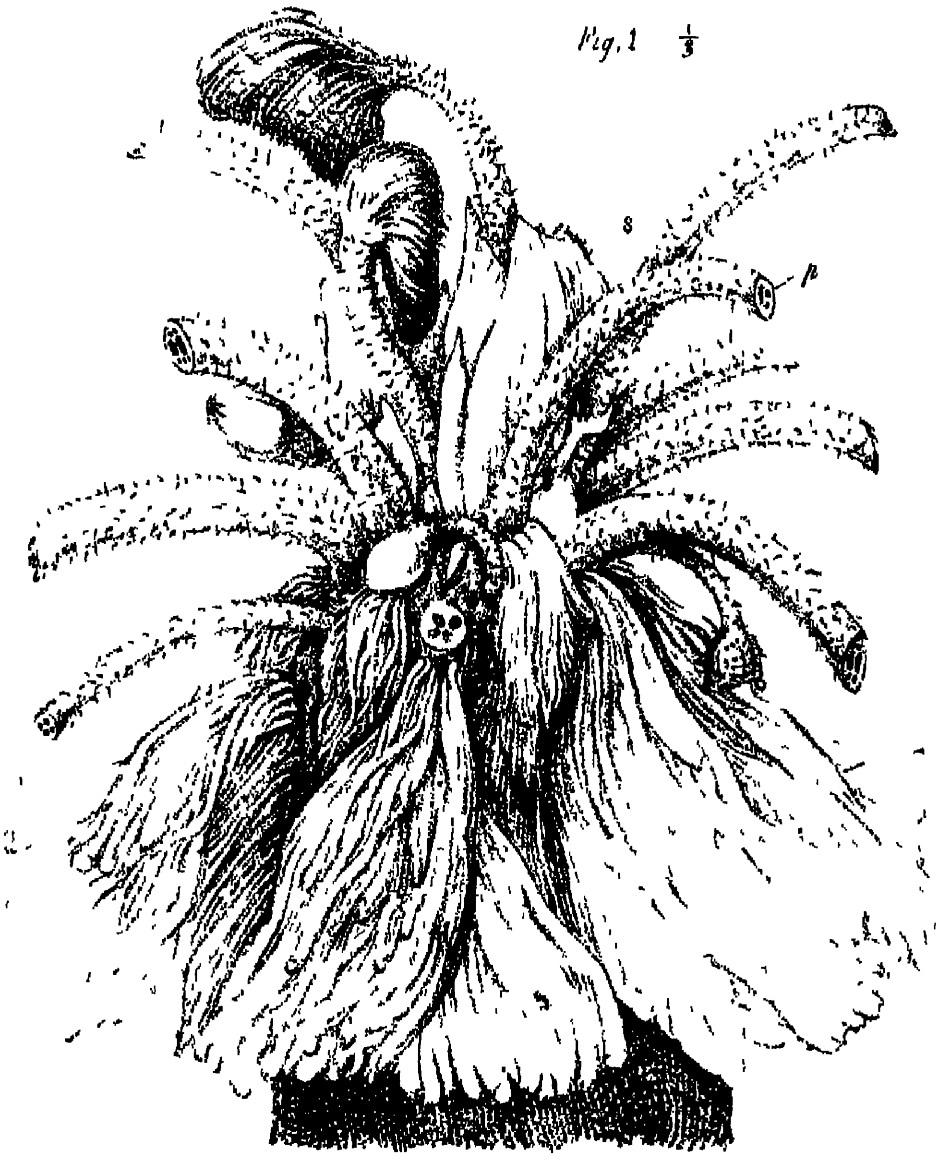
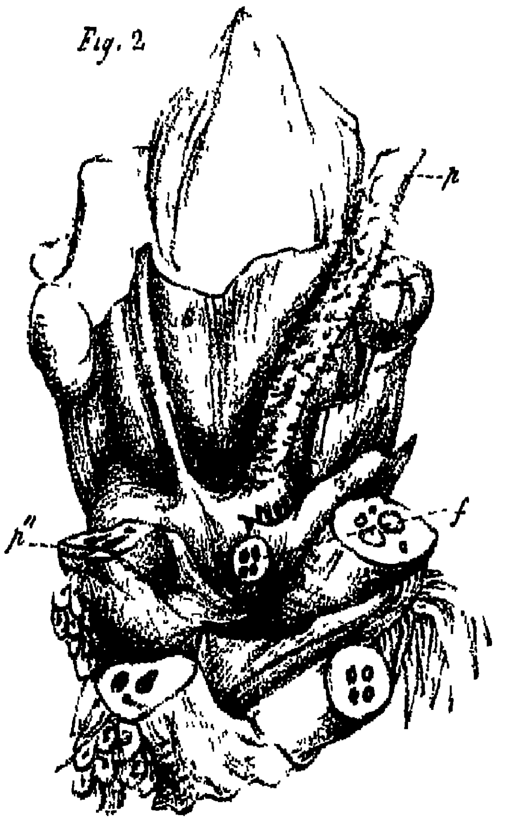


Fig. 2



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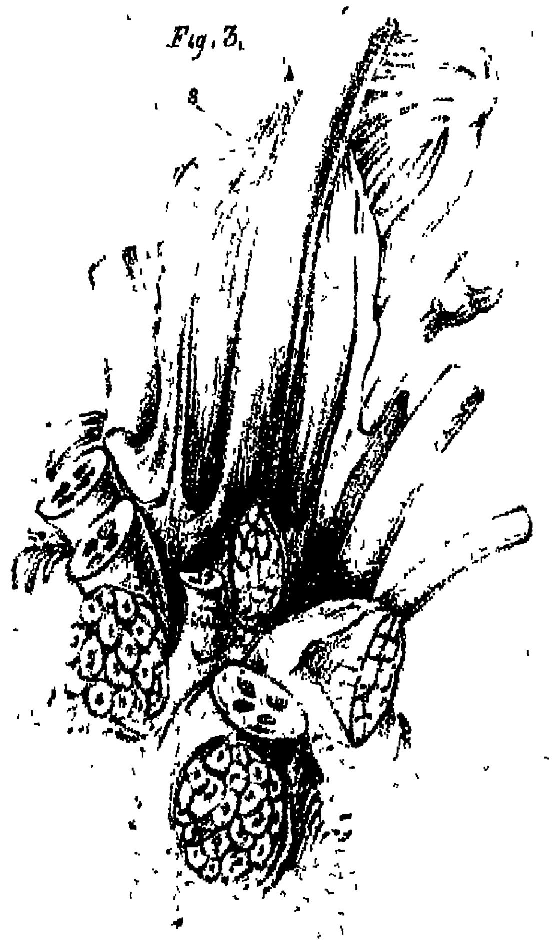

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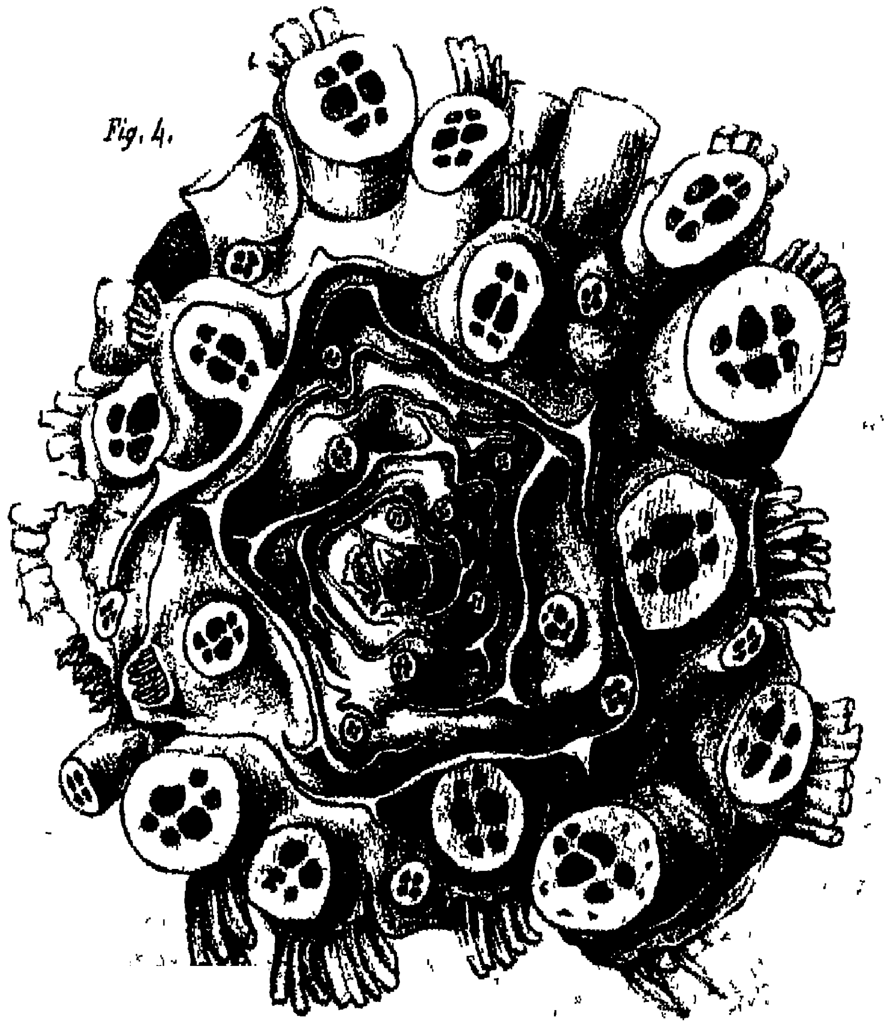

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Fig. 5. ×12

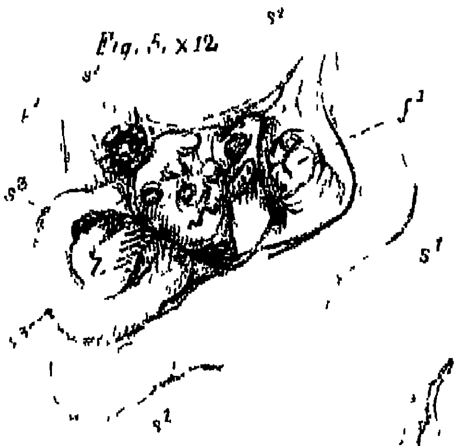
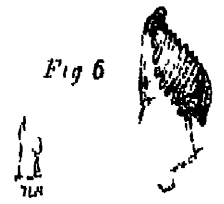


Fig. 6



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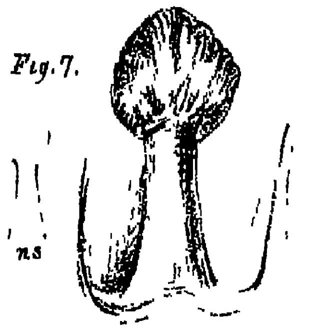

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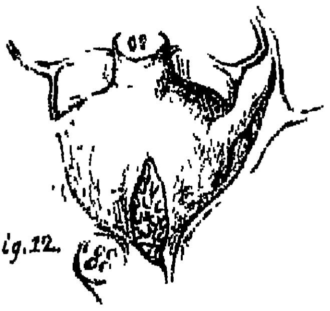

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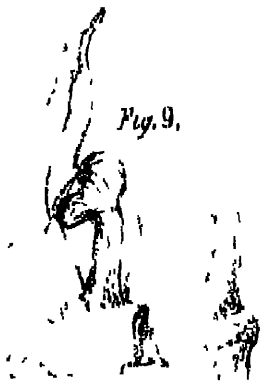

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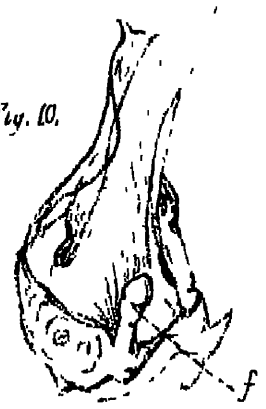

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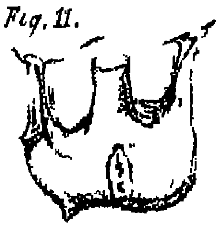

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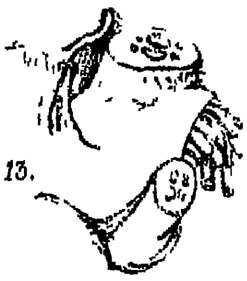

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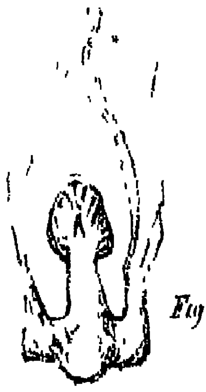

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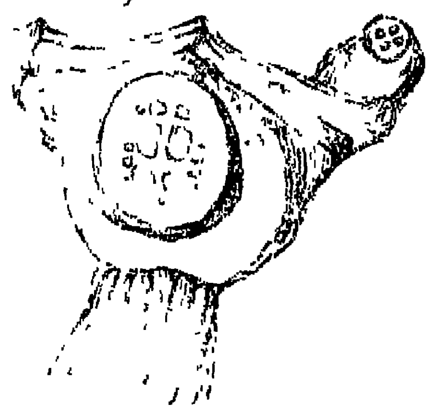

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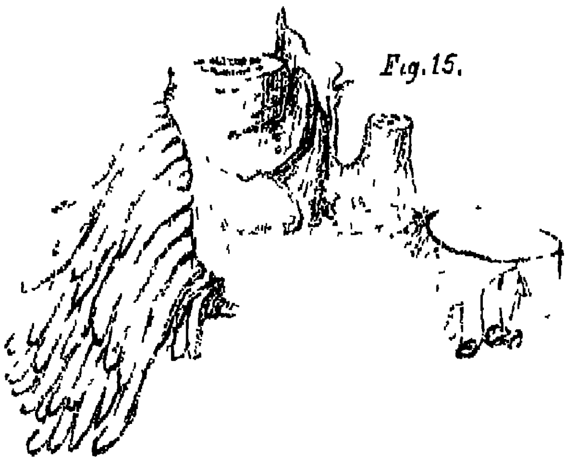

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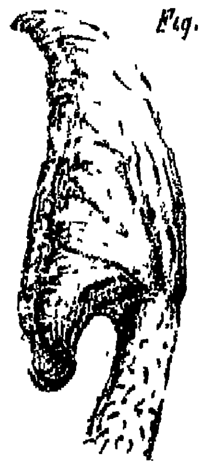
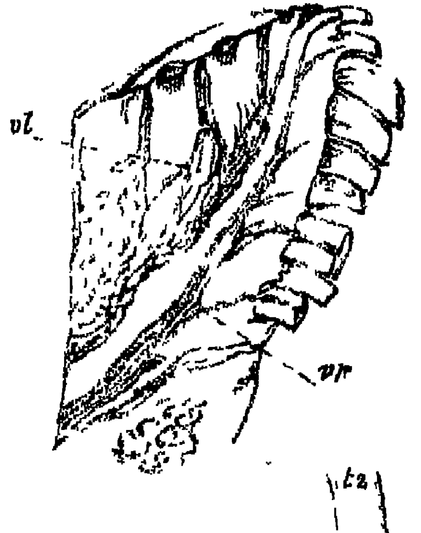


Fig. 18.

Fig. 17.



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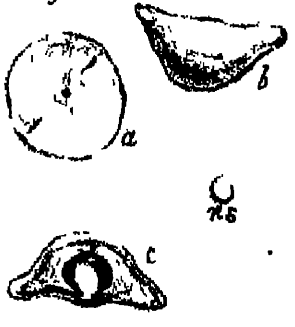

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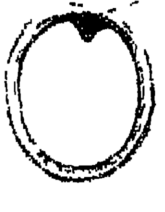

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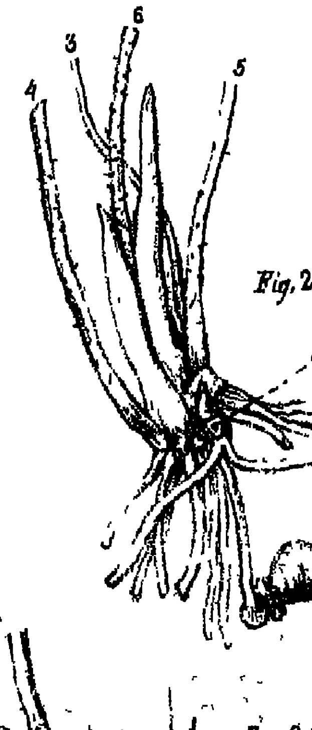

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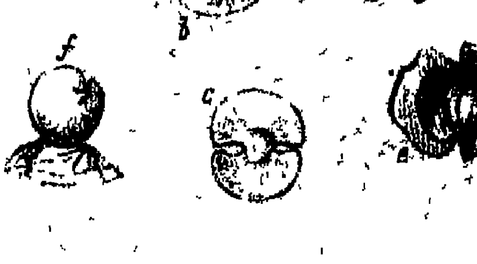

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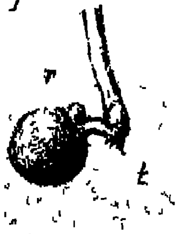

Fig. 24. ×3.

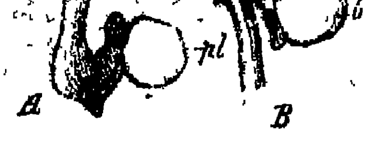

Fig. 25. ×8

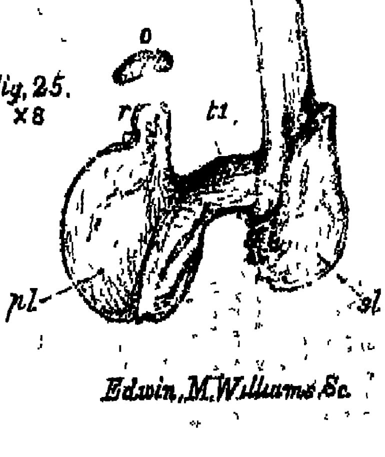

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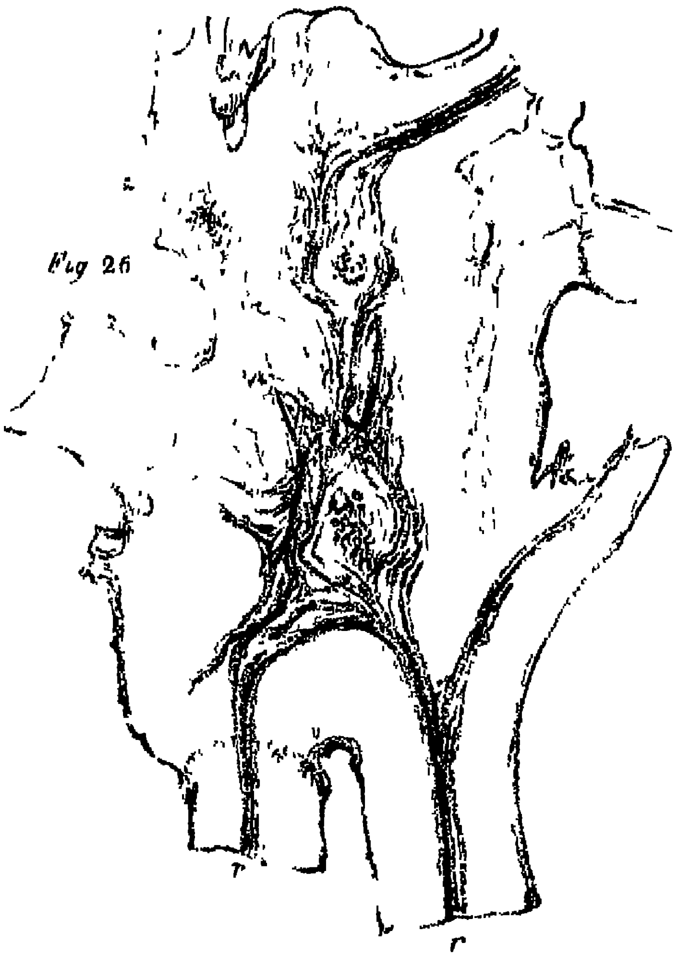
Fig. 26



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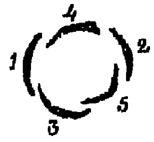


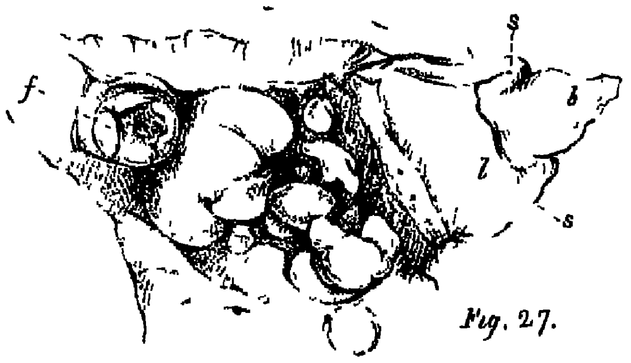
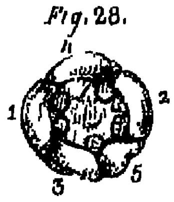


Fig. 27.

Fig. 28.



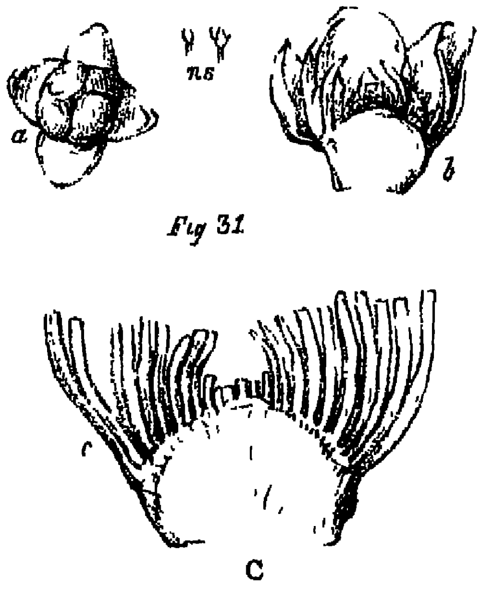
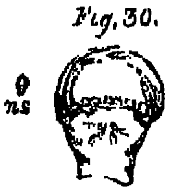


Fig 31

Fig. 30.



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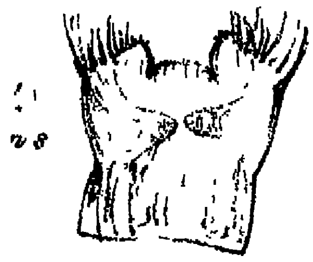

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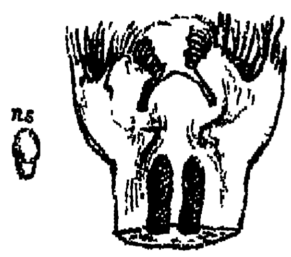

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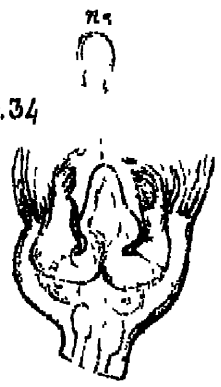

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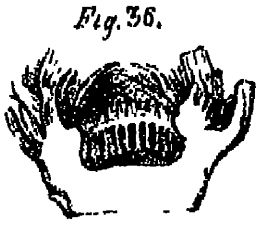


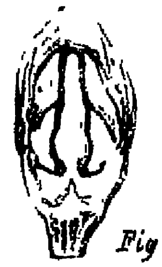
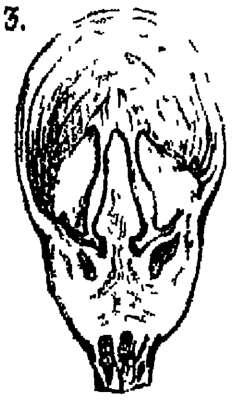


Fig 41

Fig. 43.



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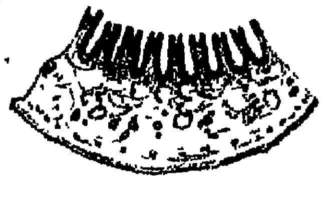

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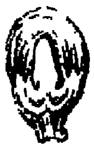

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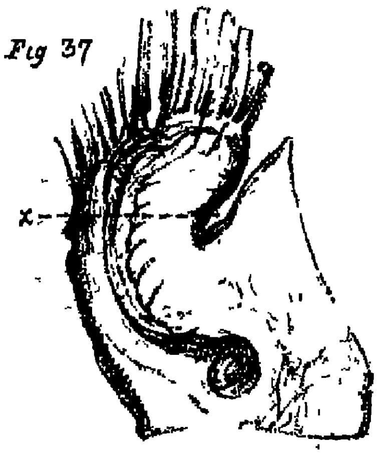

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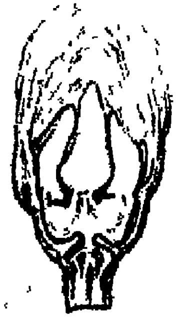

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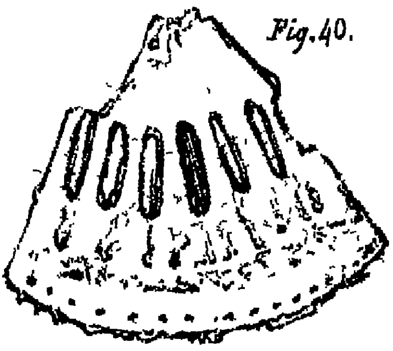

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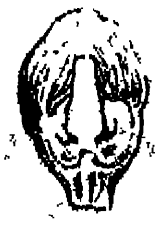

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Fig. 44.

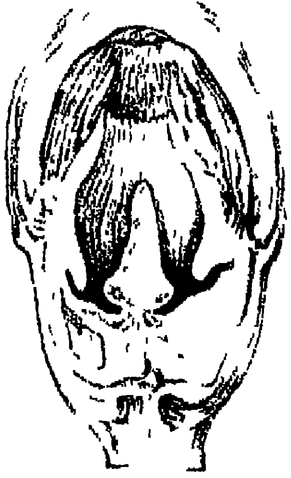
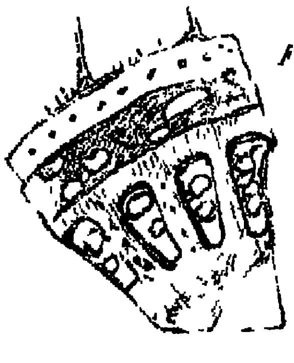


Fig. 46, ×3



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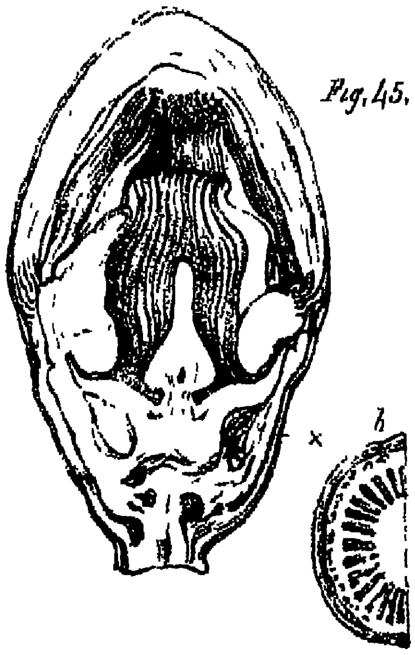

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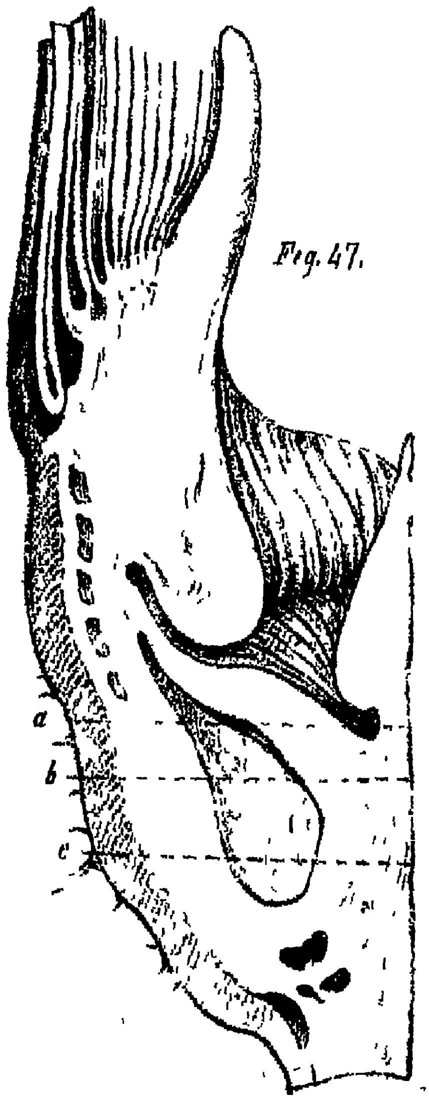

Fig. 49. ×4

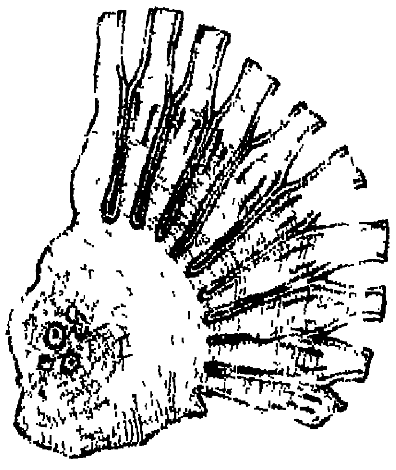

Fig. 48, ×4

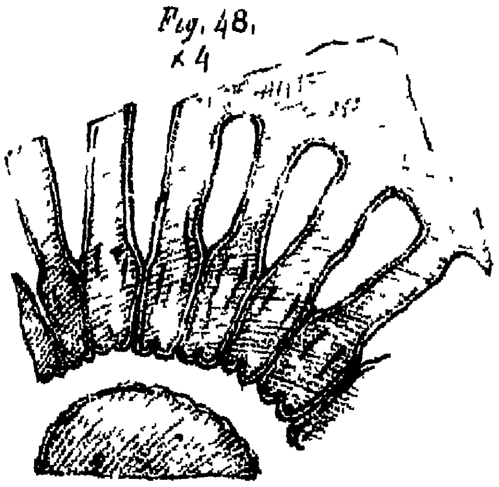

Fig. 52. ×10.

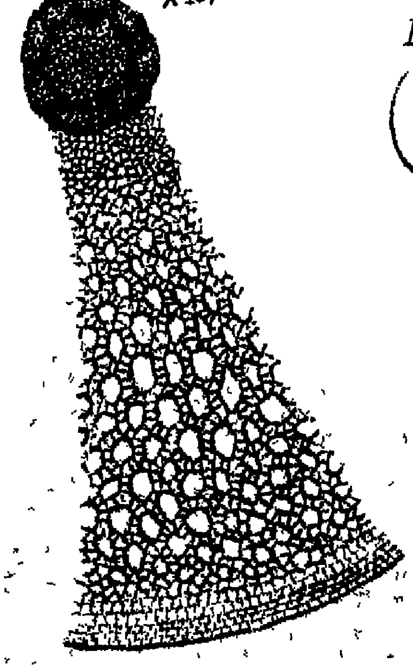

Fig. 51.

Fig. 50. ×4

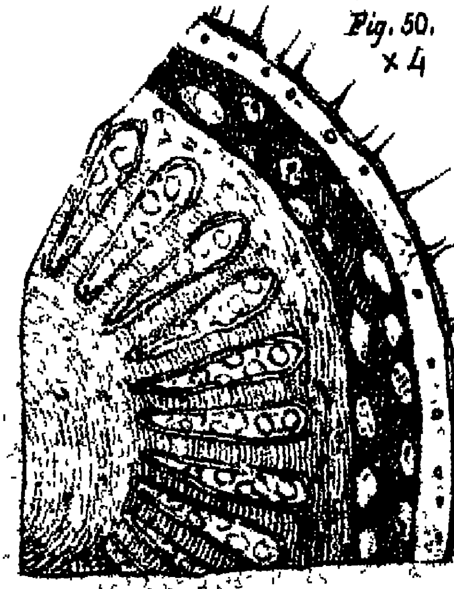

Fig. 53. ×20.

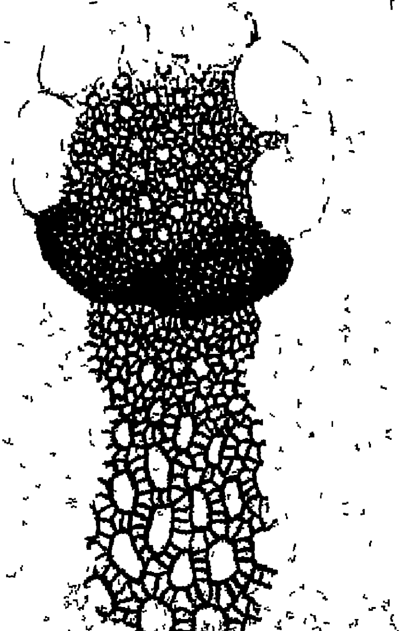

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Fig. 57 × 200

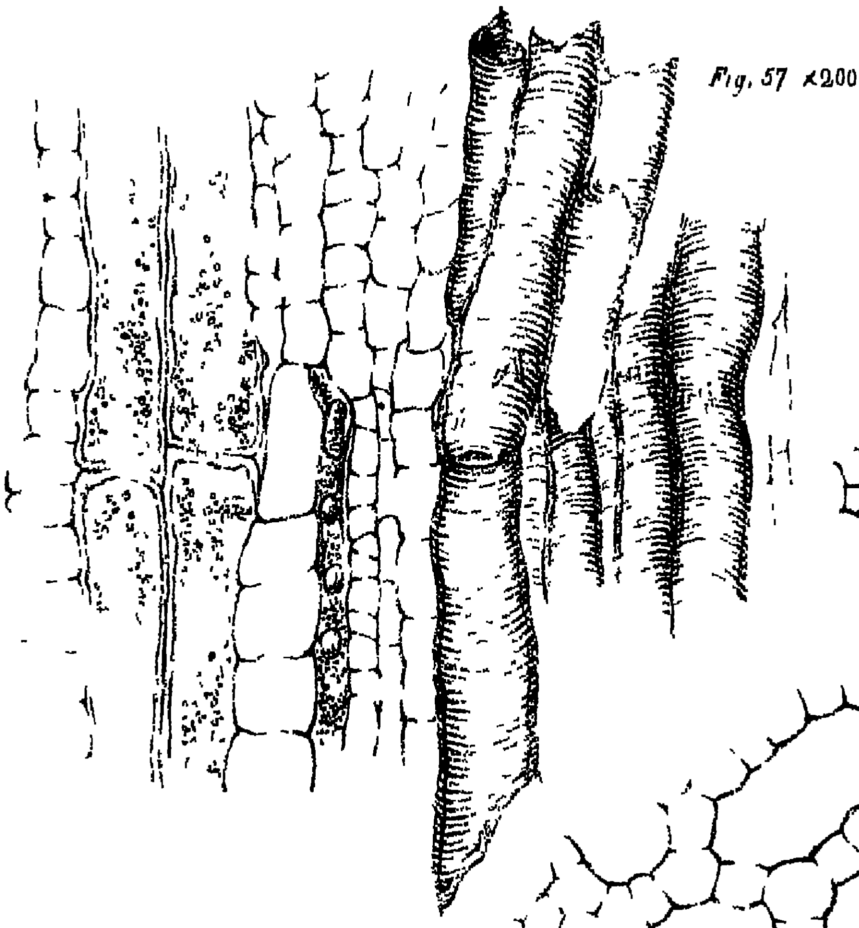
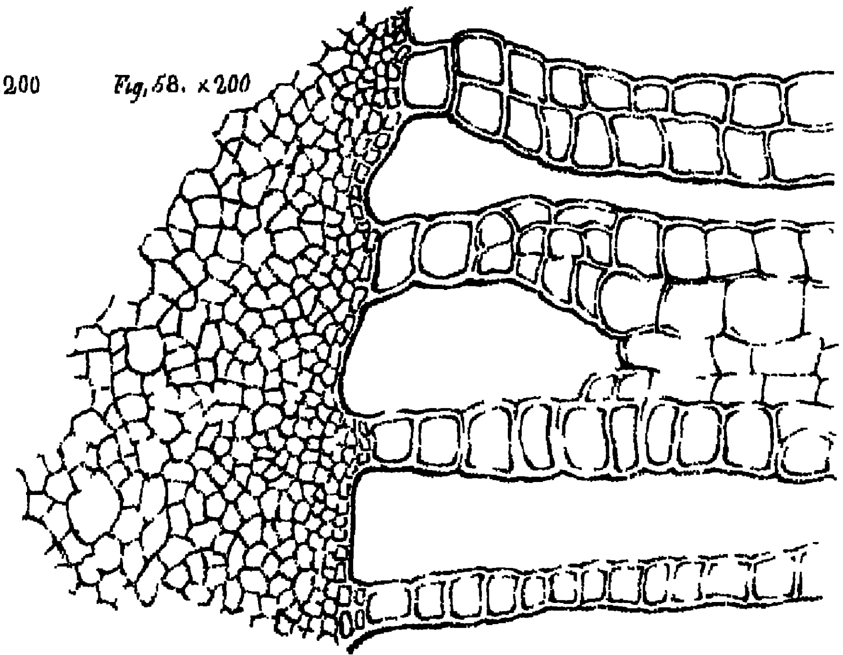


Fig. 58. × 200



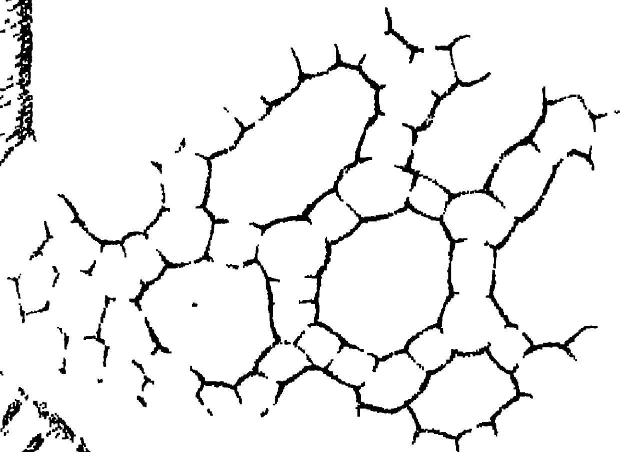


Fig. 54. × 50

Fig. 56

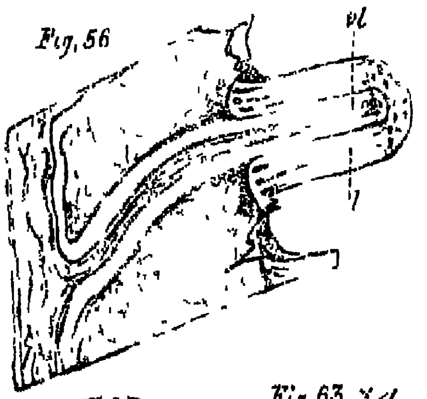
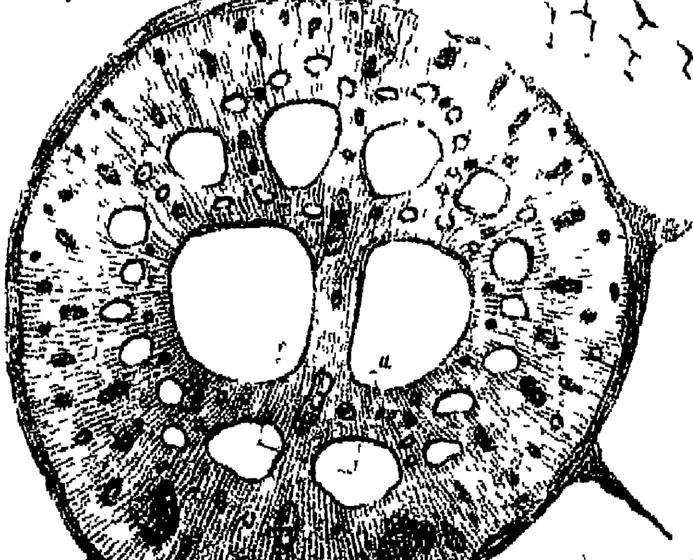


Fig. 60 × 4



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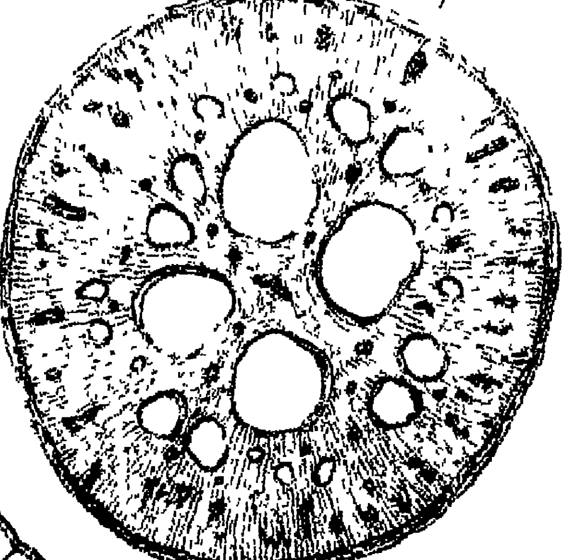

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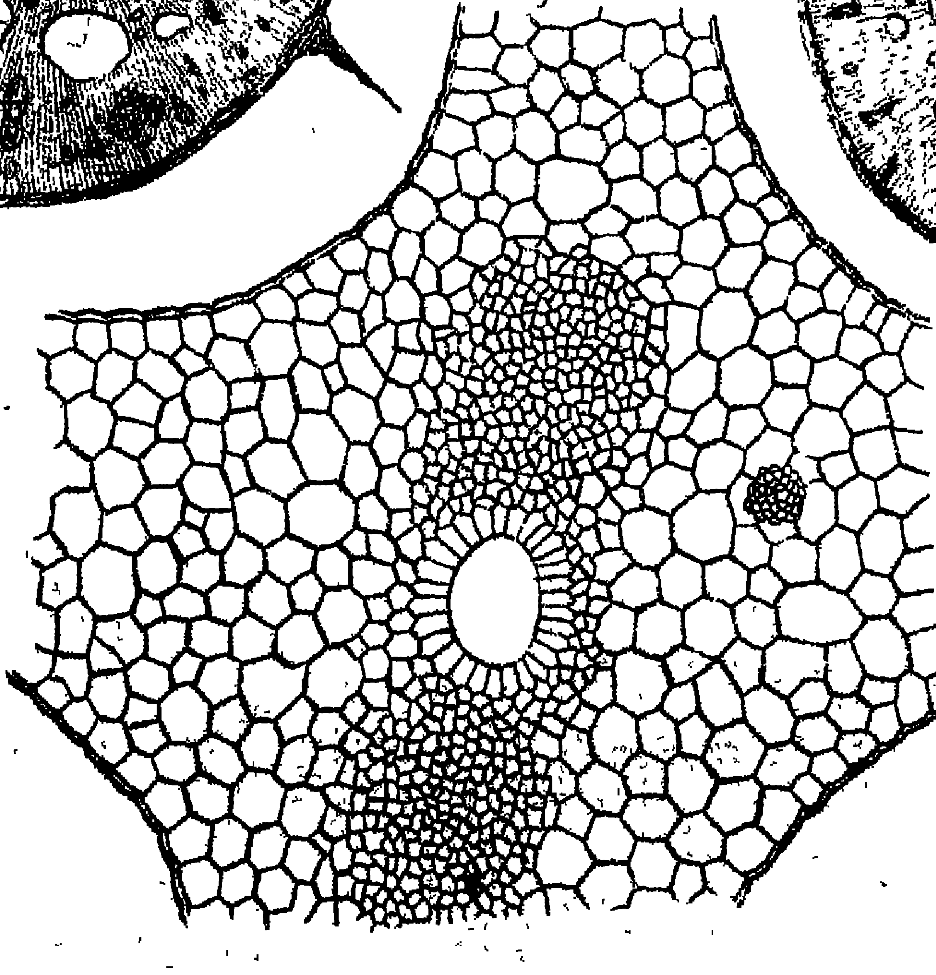

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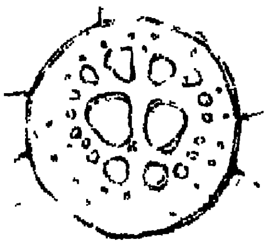
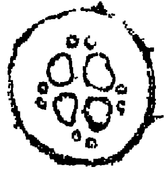


Fig. 62. n. s.



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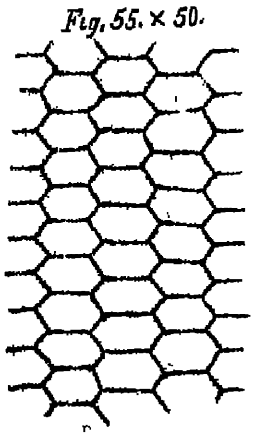

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The COPLEY MEDAL to Professor WILHELM EDUARD WEBER, of Göttingen, Foreign Member of the Royal Society, for the investigations contained in his "Maasbestimmungen," and other Researches in Electricity, Magnetism, Acoustics, &c.

A ROYAL MEDAL to Mr. GEORGE BENTHAM, F.L.S., for his important contributions to the advancement of Systematic and Descriptive Botany.

A ROYAL MEDAL to Mr. ARTHUR CAYLEY, F.R.S., for his Mathematical Papers published in the Philosophical Transactions, and in various English and Foreign Journals.

The BAKERIAN LECTURE was delivered by Dr. FRANKLAND, F.R.S., and entitled "Researches on Organo-Metallic Bodies."—Fourth Memoir.

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A KNOWLEDGE of the quantitative relation of the organs or parts, and of the ultimate and proximate composition, of animal bodies, is of great interest in many points of view. More or less accurate conceptions on these subjects, are essential to the Chemical

Physiologist, in determining the relations of the system, to the matters ingested as food; nor is such knowledge an unimportant element in studying the changes which the latter undergo, and the offices they subserve, in their passage through the body. Especially, is a knowledge of the general composition of the animals slaughtered as human food, of great importance in the application of Chemistry and Physiology to Dietetics. This, indeed, is a branch of applied physiology, so to speak, which, owing in great part to the attention drawn to it by the labours of Boussingault, Mulder, and Liebig, from fifteen to twenty years ago, may be said to have entered upon a new era about that period. It is, moreover, daily gaining ground, both with the Physician and the Economist. To the Farmer, too, who is engaged in producing animal food for the consumption of the community at large, it is very desirable to know something of the chemical relations of the substance so produced and sold, to the constituents expended in producing it. In other words, he should possess some data for determining—what is the probable proportion of the consumed food, or of its several constituents, which he recovers in the form of *meat*?—how much he may calculate as *manure*?—and how much as *expenditure* or *loss* by the feeding process?

It is obvious, that these comprehensive factors involved in the great question of animal nutrition, may be sought, individually, or collectively, and in various ways. For valuable contributions on special points, we are indebted to Dulong and Despretz, to Allen and Pepys, to Dumas and Milne-Edwards, to Andral and Gavarret, to Regnault and Reiset, to Coathupe, Scharling, Vierordt, Marchand, Becquerel, Lecanu, Chossat, Bischoff, Persoz, and others. For the study of the subject in its more collective form, we are indebted, more particularly, to Lining, to Dalton, to Boussingault, to Liebig, to Playfair, R. D. Thomson, Payen, Valentin, Simon, Bidder and Schmidt, Barral, and Lehmann. So far as the animals of the farm are concerned, the labours of Boussingault, E. Wolff, Ritthausen, and Henneberg, stand prominent for persevering experimental research; whilst to Liebig we owe much for the stimulus given, and the discussion incited, by his generalizations on various branches of animal chemistry. To Lehmann again, independently of his own original researches, we are indebted for a systematic review of the labours of others; and we are glad to have the sanction of one who has ably executed the task herein implied, to the importance, under the existing conditions of our knowledge, of the *statistical* method of inquiry. Thus, he says—

"As long as zoo-chemistry and the theory of the juices continue to occupy their present subordinate position, the only method by which the foundation necessary to an exact investigation can be obtained is that which we may term the statistical. Liebig, Boussingault, and Valentin have indeed, with a more correct view of what was required, attempted to compare the final effects of the whole with the material substrata supplied to the organism. We cannot, it is true, arrive at any conclusion regarding the working of the process itself by a mere juxtaposition and quantitative comparison of the ingesta and excreta of the animal organism, any more than we can judge of the causes and course of diseases by the number of fatal cases recorded; but such experiments furnish us with certain general results which serve as guides to further investigations*."

The statistical method itself, may, however, be very variously carried out. It may be sought to determine the several separate factors coincidently upon one and the same animal, placed under given conditions. Or, they may be investigated, either one by one, or coincidently, upon a large number of animals, so as to exclude, as far as possible, the influence of special circumstances, or of individual peculiarities. We have found it necessary to have recourse to each of these modes of operation.

Our more immediate branch of the subject on the present occasion—that of *Animal Composition*—obviously constitutes but an item in a general inquiry into the chemical statistics of animal nutrition. Such an inquiry, looking at it from an *agricultural* point of view, we have considered as involving the following distinct objects of research:—

1st. The amount of food, or its several constituents, consumed *in relation to a given weight of animal within a given time.*

2nd. The relation of the *gross increase in live-weight*, to the amount of food, or its constituents, consumed.

3rd. The comparative development of the different *organs*, or *parts*, of fattening animals;—their final *ultimate* and *proximate composition*;—and the probable composition of their *gross increase of live-weight*, during the feeding process.

4th. The composition of the *solid* and *liquid excrements*—that is, the *manure*—in relation to that of the food consumed.

5th. The *loss* or *expenditure* of constituents, by respiration, and by the cutaneous exhalations—that is, in the mere sustenance of the living meat and manure-making machine.

The general plan of experimenting adopted may be briefly described as follows:—

Some hundreds of animals, Oxen, Sheep, and Pigs, were supplied, for many weeks consecutively, with given quantities of food of known composition. The weights of the animals themselves were also taken, both at the beginning and at the end of the experiments. The data were thus provided for ascertaining the amounts of food, or of certain of its constituents, which were consumed in relation to a *given weight of animal within a given time*, or which were required to yield a given amount of *gross increase in live-weight*. Most of the results relating to these first two branches of the main inquiry, have been published in detail elsewhere†; and we shall have to adduce only a condensed summary of them, when we come to apply the more special results of the present Paper.

To ascertain the *relations*, and the *tendency of development*, of the *different parts of the system*, the weights of the organs, and parts—also of several hundred animals—

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* Cavendish Society's translation of LEHMANN's Physiological Chemistry, vol. i. p. 14.

† Journal of the Royal Agricultural Society of England, vol. x. part 1; vol. xii. part 2; vol. xiii. part 1; vol. xiv. part 2; vol. xvi. part 1. Report of the British Association for the Advancement of Science, for 1852; and 1854.

were determined. It is one of the objects of the present Paper to treat of the *summaries* of the results so obtained; and the details will be given for reference in the Appendix.

To determine the *ultimate composition*, and in a sense the *proximate composition* also, of Oxen, Sheep, and Pigs, and to acquire the data in such manner that they might serve to estimate the probable composition of their *increase* whilst fattening, was a labour obviously too great to be undertaken with a large number of such animals. A few individuals only, of each of the above descriptions of animal, but in different conditions of maturity, were therefore selected for the purpose. It is to the *methods*, and to the *results*, of the analysis of the animals so selected, and to the application of the data thus arrived at, that we shall have chiefly to address ourselves on the present occasion.

Ten animals were submitted to analysis. Those taken were—

1. A Fat Calf;—of the Durham breed; 9 or 10 weeks old; taken from the dam feeding upon grass; killed September 12, 1849.
2. A Half-fat Ox;—Aberdeen breed; about 4 years old; had been fed on fattening food, but had grown rather than fattened; killed November 14, 1849.
3. A moderately Fat Ox;—Aberdeen breed; about 4 years old; fed on fattening food; killed October 30, 1849.
4. A Fat Lamb;—Hampshire Down; about 6 months old; killed August 17, 1849.
5. A Store* Sheep;—Hampshire Down; about a year old; killed February 28, 1850.
6. A Half-fat old Sheep;—Hampshire Down Ewe; 3½ years old; killed May 3, 1849.
7. A Fat Sheep;—Hampshire Down; 1¼ year old; killed May 7, 1849.
8. A very Fat Sheep;—Hampshire Down; 1¾ year old; killed December 13, 1848.
9. A Store* Pig; killed May 12, 1850.
10. A Fat Pig; same litter as last; fed on fattening food for 10 weeks; killed July 18, 1850.

The still remaining points of the main inquiry are:—first, as to the composition of the *solid* and *liquid excreta*, in regard to which we have collected much experimental data, which must form the subject of a separate Report:—and, secondly, as to the proportion of the food-constituents *expended* or *lost* by the respiratory and other processes. The latter amount is obviously the complementary quantity making up the constituents consumed, those assimilated being estimated, and those voided in the solid and liquid form determined experimentally, and the sum deducted from the whole amount of the solid and liquid ingesta.

After this brief outline of the scope of the main inquiry, of which the subject of the present Paper constitutes but a branch, it will be sufficiently understood, that it was chiefly with a view to the agricultural bearings of the results, and to their connexion with collateral investigations, that the researches now to be recorded were planned and executed. So enormous, indeed, has been the labour necessarily expended in so determining the ultimate composition of several animals as to serve the special purposes prominently in view, that it was found quite out of the question to comprehend in the inquiry much that otherwise would have been desirable. Attempts were in fact made, to associate with ourselves a Veterinary Physiologist, but the undertaking was found to be impracticable. Still, it is hoped, that the analytical labour of several years devoted to such a subject, though it may lack the special direction of either the Physician or the Physiologist, may yet supply important facts to both. We have, then, only faithfully to record the manner and circumstances of attaining our results, leaving it to others to determine, to what they are, and to what they are not, applicable, beyond that to which we ourselves apply them.

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* The term "store" is applied to animals not yet put upon fattening food.

SECTION II.—METHODS OF EXPERIMENTING, ANALYSIS, ETC.

The following is an outline of the plan adopted, in determining the actual, and proportional weights, of the organs and parts of the numerous animals operated upon with that view, and in determining the composition of the ten animals which were further analysed.

1st. *Determination of the Original or Fresh-weights of the Entire Bodies, and of the Internal Organs, and other separated Parts, of Calves, Oxen, Lambs, Sheep, and Pigs.*

After being fasted eighteen to twenty-four hours, the weight of the animal was taken, immediately before being killed. This weight, is that which is afterwards spoken of as the "*Fasted-weight*," or the "*Fasted Live-weight*." It is in relation to this *Fasted Live-weight*, that the *percentages* of the organs, or parts, or of the several constituents of the animals, are calculated.

The animal being killed, and the blood then flowing carefully collected and weighed, the different organs and parts were separated by the butcher in his ordinary way; but with more than usual care, and with particular attention to uniformity in the manipulations. The weight of each part was taken as rapidly as possible after separation, in order that the amount of loss to be attributed to evaporation might be reduced to the minimum. The weights so taken, constitute the *Actual Fresh-weights*; and when these are calculated to the *Fasted Live-weight* as 100, they are called the *Percentage Fresh-weights*. In the Tables, however, for the sake of convenience, the different parts are classified, in the butcher's way, into "*Carcass*," and "*Offal*."

In Calves and Oxen, Lambs and Sheep, the "Offal" includes the head, the feet, and the skin, as well as the whole of the internal organs or parts, excepting the kidneys, and the fat in which they are embedded. The "Carcass," in the case of these animals, comprises, therefore, the whole of the skeleton (excluding the head and feet), with the whole of the muscles, membranes, vessels, and fat, attaching to it; also the kidneys and the fat surrounding them. The so-called "Offal" of the *Pig*, however, does not include either the head, the feet, or the skin. In its case these parts are weighed with the Carcass; and its Offal consists, therefore, only of the internal organs and parts, excluding the kidneys and kidney-fat. Such a classification of the parts of the animals, into *Carcass* and *Offal*, is, of course, to a certain extent, arbitrary. But, whilst its adoption will not interfere with the study of the results in their more physiological or scientific bearings, it will much facilitate the perception of their practical and economic application.

Fresh-weights, as here described, both actual and percentage, have been determined in the cases of 18 Calves, Heifers, and Bullocks, 249 Sheep, and 59 Pigs. Attention will be called to a summary of these results further on in the course of our Paper; and the details will be given for reference in the Appendix (Tables XV.—LXIV. pp. 594—677).

So much for the separation, the determination, and the calculation, of the weights of the *fresh* matters. We have next to describe the further treatment of the fresh matters themselves, in the case of the ten animals submitted to further analysis.

2nd. *Determination of the Water, and Crude Dry Substance, in the Animals Analysed.*

Half of the Carcass, and the whole of every separated organ or part comprised in the Offal, were in each case operated upon. The half-carcass was separated into—

(*a*) Flesh and fat:
(*b*) Bones:
(*c*) Kidney and kidney-fat.

After being cut up, as required, these Carcass parts separately, and each of the separate internal organs or parts constituting the Offal, were put into a large water-bath, and were arranged in it in such a manner, that the fat which melted from any one, or any number of the parts, could be collected and weighed separately at pleasure, in vessels placed under them for that purpose. After being kept in the bath at a temperature of 212° FAHR., for several days, those parts which still retained a large quantity of fat were tied up in a dried and weighed canvass-cloth, and squeezed in a screw-press. The further fat so obtained, was added to that obtained from the same parts by melting only. The whole of the separated fat, after a little further exposure to the heat of the water-bath, was then weighed; and this, together with the amount retained by the press and cloth, which were weighed both before and after the operation, constitutes the weight of the *melted and expressed fat*, as given in the Tables. The remaining *crude dry substance*, generally, still retained a considerable amount of fat. But, excluding the bones, which had to be first otherwise broken, all these dried parts were now in such a state as to allow of being ground through a steel mill, made for the purpose, into a coarse but manageable powder. This, then, is the *crude dry substance*, excluding melted and expressed fat.

3rd. *Treatment of the Crude Dry Substance.*

Accurately calculated and weighed *proportional* parts of the whole of the respective *crude dry matters* were taken as follows:—

(*a*) For the determination of the *Ash* of each separated part.
(*b*) For a mixed sample of *entire Carcass Parts*, for analysis as such.
(*c*) For a mixed sample of *entire Offal Parts.*
(*d*) For a mixed sample of the *Entire Animal*;—that is, including both Carcass and Offal parts (but of course excluding the melted and expressed fat).

The remaining portion of the *crude dry substance* of each separated part is preserved as such.

4th. *Determination of the Mineral Matter, or Ash.*

The proportional part of each separate *crude dry substance* weighed out for *ash*, was burnt in a platinum dish, 10 inches long and 5 inches wide, placed in a cast-iron muffle heated by coke. The weight of the ash of each separate part being taken, *proportional* parts of the respective ashes were weighed out, and mixtures made, as follows*:—

(*a*) Of the ashes of all *Carcass* parts.
(*b*) Of the ashes of all *Offal* parts.
(*c*) Of the ashes of *all* parts; that is, of the *Entire Animal.*

The remaining ash of each separate part is preserved as such.

5th. *Determination of the Fat not separated by melting and expression, and which still remained therefore in the separate, and proportionally mixed samples, of the crude dry substance.*

This remaining Fat was determined by extraction with ether. A portion, amounting generally to about 4 or 5 grammes, of the *crude dry substance*, was weighed in a small porcelain capsule, and the hygroscopic water determined by drying in a water-bath at 212°. The re-dried substance was then transferred into a small flask, and the capsule rinsed into it with ether. More ether being added, and the flask lightly closed, the mixture was digested for some time, and then thrown upon a weighed filter. The filter and its contents were first washed with the ethereal rinsings of the flask, and finally with pure ether, until the filtrate no longer stained bibulous paper. The ethereal filtrate, containing the dissolved fatty matter, was collected in a small balanced flask, the ether distilled off over a water-bath, and the flask with its fatty contents, after being fully dried at 212°, was re-weighed. The filter, containing the matters insoluble in ether, was also dried and re-weighed. The hygroscopic water, the fat, and the matters insoluble in ether, were thus all estimated upon the same weighed quantity of crude dry substance. A very good control was therefore obtained of the accuracy of the fat determinations. Moreover, two or more analyses were always made of each specimen. The results of these Fat determinations will be found tabulated in detail in the Appendix (Table XIII. p. 592).

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* The statement of the method of preparing the ashes of the *collective* parts given in the text, applies to those of all the animals operated upon, excepting the two pigs. Of these, the collective carcass, the collective offal, and the entire animal ashes, were made by burning, at one process, carefully made mixtures of the proportional parts of the respective crude dry matters.

6th. *Determination of the Nitrogen.*

The *nitrogen* was determined by burning with soda-lime, and collecting and weighing as the double chloride of platinum and ammonium, in the usual way. The crude dry matters in which the nitrogen was determined, were, for the most part, as under*:—

(*a*) Mixed Carcass parts, without bones.
(*b*) Mixed Carcass Bones.
(*c*) Mixed Offal parts, including bones, if any.
(*d*) Hair or Wool.
(*e*) Mixed parts of the Entire Animal, excluding hair or wool.

It will be obvious, that, owing to the heterogeneous character of these mixed specimens, very great care was requisite, both in the preparation of the mixtures themselves, and in taking the weighed quantities for analysis. Duplicate nitrogen determinations were always made, and the individual results are given in the Appendix (Table XIV. p. 593).

7th. *Constituents of the Ash determined.*

The substances determined under this head were—

(*a*) Phosphoric acid.
(*b*) Potash and Soda.
(*c*) Lime and Magnesia.
(*d*) Matter insoluble in acid.

As the results of the analyses of the animal-ashes must form the subject of a separate Paper, the description of the methods adopted need not be given here.

The experimental results obtained by the methods above described are given for reference, in full detail, in the Appendix. The discussion, *seriatim*, of the voluminous data thus provided, would tend rather to embarrass than to facilitate the perception of the main and more prominent facts and conclusions, which the investigation, as a whole, is calculated to establish. We shall seek, therefore, to bring these to view by reference to a series of *Summary Tables* only. The first point to be illustrated, is the *percentage composition of the ten animals actually analysed.* This is shown in Tables I. to VII. inclusive; in which we have recorded the percentages—of *mineral matter*, of *fat*, of *nitrogen* or *nitrogenous compounds*, of *total dry substance*, and of *water*—in certain classified parts, and in the entire fasted live-weight, of each of the ten animals.

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* The exceptions were the cases of the two Pigs. In these, the sample of *mixed carcass parts*, upon which the nitrogen was determined, *included* all the bones, excepting those of the head and feet; a separate mixed sample was made of the head and feet, including both soft and bony parts, excepting the tongue and brains, which latter went with the *mixed offal parts* in the samples prepared for nitrogen determinations. The mixed samples of the offal parts of the Pigs did not therefore contain bone, as did those in the cases of the other animals.

SECTION III.—THE MINERAL MATTER—IN CERTAIN SEPARATED PARTS, AND IN THE ENTIRE BODIES, OF 10 ANIMALS ANALYSED.

In Table I. is given the percentage of *mineral matter* in the fresh carcass, in the fresh total offal parts, and in the fasted live-weight, of each of the ten animals; there is at the same time shown, the distribution of the mineral matter in certain classified parts of these collective weights. But, as some of the points of general interest can be more easily studied by reference to the results given in a still more summary form, they are so arranged in Table II. In this second Table, the only subdivision of the carcass, the offal, and the entire animal, respectively, is into*—

(*a*) Flesh, membrane, &c., or soft parts:
(*b*) Bones:
(*c*) Hair or wool (if any).

It has already been explained, that in all the animals, except Pigs, the so-called "*carcass*," which comprises the most important edible portions, is generally understood to include the whole of the frame (excepting the head and feet) with the flesh and fat adhering to it, and, in addition, the kidneys, and the fat in which they are imbedded. Thus, the head and feet, the skin, and the whole of the internal organs or parts, except the kidneys, constitute the "*offal.*" In Pigs, on the contrary, the head, feet, and skin are weighed with the Carcass. For the sake of the better comparison of the composition of pigs with that of other animals, the constituents of their head and feet are, in our Tables, included with those of the offal parts; but for reference, so far as the mineral matter is concerned, the percentages for the pigs are given, at the foot of Table I., on the assumption that the head and feet, as in usual practice, are included with the carcass.

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* For the *actual amount* of mineral matter, or ash, due to each organ or part, as separated in the original dissection of the animals, see Tables I. to X. inclusive in the Appendix (pp 580—589); and for the *percentage amount* in the separate parts, see Appendix, Table XII. (p. 591)

Table I.—Percentages of Mineral Matter in Ten Animals.

1st. In Fresh Carcass.—2nd. In Fresh Offal (equal Sum of Parts excluding Contents of Stomachs and Intestines).—3rd. In Entire Animal (Fasted Live-weight—including therefore Contents of Stomachs and Intestines).

Description of animal.	Percentages in fresh Carcass.				Percentages in fresh Offal.											Percentages in fasted live-weight of Entire Animal.															
										From bones.						From Carcass parts.				From Offal parts.											
																									From bones.						
	From flesh, membrane, &c.	From kidney and kidney-fat membrane.	From bones.	From total carcass parts.	From blood.	From thoracic viscera: lungs and windpipe, heart and aorta, thymus gland, &c.	From abdominal viscera, &c.: stomachs, intestines, spleen, liver, bladder, pancreas, throat glands, diaphragm, fat membrane, &c.	From head-flesh, brains, and tongue	From tail-flesh.	Tail.	Head.	Legs and feet.	From skin or pelt.	From hair or wool.	From total offal parts.	From flesh, membrane, &c.	From kidney and kidney-fat membrane.	From bones.	From total carcass parts.	From blood.	From thoracic viscera: lungs and windpipe, heart and aorta, thymus gland, &c.	From abdominal viscera, &c.: stomachs, intestines, spleen, liver, bladder, pancreas, throat glands, diaphragm, fat membrane, &c.	From head-flesh, brains, and tongue	From tail-flesh.	Tail.	Head.	Legs and feet.	From skin or pelt.	From hair or wool.	From total offal parts.	From total parts.
Fat calf	0·771	0·004	3·707	4·482	0·193	0·299		0·074	0·003	0·026	1·357	1·249	0·170	0·014	3·415	0·479	0·002	2·301	2·782	0·058	0·089		0·022	0·001	0·008	0·418	0·372	0·051	0 004	1·018	3·800
Half-fat ox	0·795	0·009	4·760	5·564	0·157	0·052	0·246	0·084	0·004	0·043	1·964	1·274	0·191	0·036	4·051	0·515	0·006	3·082	3·603	0·041	0·014	0·065	0·022	0·001	0·011	0·514	0·334	0·050	0·009	1·061	4·664
Fat ox................	0·597	0·010	3·953	4·560	0·120	0·254		0·063	0·002	0·035	1·554	1·177	0·175	0·021	3·401	0·395	0·007	2·617	3·019	0·032	0·067		0·017	0·001	0·009	0·412	0·312	0·046	0 005	0·901	3·920
Fat lamb...	0·473	0·004	3·155	3·632	0·116	0·275		0·071	.....	......	1·203	0·263	0·276	0·244‡	2 448	0·283	0·002	1·888	2·173	0 036	0·086		0·022	..	.....	0·375	0·082	0·086	0·076‡	0·763	2·936
Store sheep	1·246	0·008	3·106	4·360	0·098	0·054	0·182	0·058	......	......	0·917	0·240	0·242	0 401‡	2·187	0·664	0·004	1·657	2·325	0·036	0·021	0·070	0·022	.....	...	0 351	0·092	0·093	0·154‡	0·839	3·164
Half-fat old sheep	0·631	0·002	3·501	4·134	0·103	0·237		0·051	. ....	......	1·462	0·185	0·156	0·522‡	2·716	0·238	0·001	1·875	2·214	0·036	0·084		0·018	. .	.	0·516	0·066	0·055	0·184‡	0·959	3·173
Fat sheep	0·464	0·012	2·973	3·449	0·075	0·247		0·052	......	.. ...	0·976	0·207	0·211	0·556‡	2·324	0·266	0·007	1·709	1·982	0·027	0·088		0·018	. ..	.. ...	0·349	0 074	0 076	0·197‡	0·829	2 811
Extra fat sheep ...	0·255	0·070	2·447	2 772	0·116	0·175			......	......	0·961	0·214	0·233	1·942‡	3·641	0·161	0·044	1·543	1·748	0·037	0·055			.. .	. .	0·305	0·068	0·074	0·616‡	1·155	2 903
Store pig*	0·636	0·010	1·926	2·572	0·153	0·067	0·308	0·150	0·007		1·766	0·600	......	0·022	3·073	0 422	0·007	1·279	1·708	0·048	0·021	0·096	0·047	0·002		0·552	0·188	. . .	0·007	0 961	2·669
Fat pig*	0·269	0·009	1·120	1·398	0·208	0·060	0·278	0·134	0·007		1·591	0·667	.....	0·025	2·970	0·204	0·007	0·851	1·062	0·041	0·012	0·055	0·027	0·001		0·314	0·132	.. ..	0·005	0·587	1·649
Means	0·614	0·014	3·065	3·692	0·133	0·347			. . ..		1·378	0·608	.. ..	0·378	3 023	0 373	0·009	1·880	2·262	0 039	0 103			. ..		0 410	0·172	. .	0·126	0 907	3·169
Store pig†	0·605	0·009	2·669	3·253	0·218	0·095	0·438	0·049	.... .	......	......	......	......	0·031	0 831	0·458	0·007	2·021	2 486	0·048	0 021	0·096	0 011	..	.	. .	.	. .	0·007	0·183	2·669
Fat pig†	0·275	0·008	1·568	1·851	0·317	0·092	0·125	0·028	......	......	......	.. ...	..	0·033	0·900	0·227	0·007	1·298	1·532	0 041	0·012	0 055	0·004	. .	. .	. .	...	.....	0 005	0·117	1 649

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* For comparison with the other animals, the ashes of the head and feet of the pigs are here included in the *offal.*

† Here the ashes of the head and feet of the pigs are included with the *carcass,* they generally being sold with it, and not with the offal, as in the case of the other animals.

‡ These ashes, especially those of the extra-fat sheep, are too high, owing to adventitious matter retained by the wool. The statements of ash from total offal parts will, of course. be too high, from the same cause.

TABLE II.—Summary of Percentages of MINERAL MATTER in Ten Animals.

1st. In Fresh Carcass.—2nd. In Fresh Offal (equal Sum of Parts excluding Contents of Stomachs and Intestines).—3rd. In Entire Animal (Fasted Live-weight—including therefore Contents of Stomachs and Intestines).

Description of animal	Percentages in fresh Carcass			Percentages in fresh Offal				Percentages in fasted live-weight of Entire Animal.							Final Summary		
								From Carcass parts		From Offal parts.					Per cent in Entire Animal		
	From flesh, membrane, &c.	From bones.	From total carcass.	From flesh, membrane, &c.	From bones	From hair or wool	From total offal	From flesh, membrane, &c.	From bones.	From flesh, membrane, &c.	From bones	From hair or wool	From total soft parts	From total bones	From carcass parts	From offal parts.	From all parts.
Fat calf	0·775	3·707	4·482	0·739	2·662	0·014	3·415	0·481	2·301	0·221	0·798	0·004	0·706	3·094	2·782	1·018	3·800
Half-fat ox	0·804	4·760	5·564	0·734	3·281	0·036	4·051	0·521	3·082	0·193	0·859	0·009	0·723	3·941	3·603	1·061	4·664
Fat ox	0·607	3·953	4·560	0·614	2·766	0·021	3·401	0·402	2·617	0·163	0·733	0·005	0·570	3·350	3·019	0·901	3·920
Fat lamb	0·477	3·155	3·632	0·738	1·466	0·244*	2·448	0·285	1·888	0·230	0·457	0·076*	0·591	2·345	2·173	0·763	2·936
Store sheep	1·254	3·106	4·360	0·629	1·157	0·401*	2·187	0·668	1·657	0·242	0·443	0·154*	1·064	2·100	2·325	0·839	3·164
Half-fat old sheep	0·633	3·501	4·134	0·547	1·647	0·522*	2·716	0·339	1·875	0·193	0·582	0·184*	0·716	2·457	2·214	0·959	3·173
Fat sheep	0·476	2·973	3·449	0·585	1·183	0·556*	2·324	0·273	1·709	0·209	0·423	0·197*	0·679	2·132	1·982	0·829	2·811
Extra fat sheep	0·325	2·447	2·772	0·524	1·175	1·942*	3·641	0·205	1·543	0·166	0·373	0·616*	0·987	1·916	1·748	1·155	2·903
Store pig	0·646	1·926	2·572	0·678	2·373	0·022	3·073	0·429	1·279	0·212	0·742	0·007	0·648	2·021	1·708	0·961	2·669
Fat pig	0·278	1·120	1·398	0·680	2·265	0·025	2·970	0·211	0·851	0·135	0·447	0·005	0·351	1·298	1·062	0·587	1·649
Means of all	0·627	3·065	3·692	0·647	1·998	0·378	3·023	0·382	1·880	0·196	0·585	0·126	0·704	2·465	2·262	0·907	3·169
Means of the 8, excluding store sheep and store pig	0·547	3·202	3·749	0·645	2·056	0·420	3·121	0·340	1·983	0·189	0·583	0·137	0·665	2·567	2·323	0·909	3·232
Means of the 6, excluding the store and half-fat animals	0·490	2·892	3·382	0·647	1·919	0·467	3·033	0·310	1·818	0·187	0·538	0·151	0·647	2·356	2·128	0·876	3·003

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* These ashes, especially those of the extra-fat sheep, are too high, owing to adventitious matter retained by the wool. The statements of ash from the total offal parts, will, of course, be *too high*, from the same cause.

Looking first to the percentage of mineral matter, obtained by incineration, in the so-called Carcass, it is seen, as was to be expected, that the *bones* yielded by far the larger portion of it. The total *soft parts*, indeed, in most cases, provided only about one-fifth to one-seventh as much mineral matter as the solid framework of bones. The proportion of the whole mineral matter which was obtained from the soft parts, was, as well with bullocks, sheep, and pigs, the less the more matured the animal—that is, the fatter; and it was much the greater in the lean condition. Thus, in the *store* or *lean* sheep, and also in the *store* pig, the soft parts of the Carcass yielded somewhat more than a third as much ash as the bones. On the other hand, in the *fattest* sheep there was less than one-seventh, and in the *fat* pig only a fourth as much mineral matter derivable from the soft parts as from the bones. That is to say, it is with the *nitrogenous matters* of the soft parts, that the constituents yielding ash on incineration are shown to be associated. In what chemical relation the several ash-constituents stand to these nitrogenous matters, it is not within the scope of the present inquiry to determine. But, it is probable, that at any rate some of them (for example, the sulphuric, and perhaps the phosphoric acid also) are, in a greater or less degree, products of the incineration. This remark may apply to the ashes of other parts also. To what extent there may be a loss of certain mineral constituents, when, as is usually the case, it is sought to collect the whole by the process of incineration, depends in part on the mineral composition of the substance, but also on the conduct of the incineration itself. The ashes here in question were, however, prepared with great care, and at as low a temperature as possible, so as at least to avoid all unnecessary source of loss. It seemed desirable to call attention to these points, that in assuming, as is usually done, that the total mineral matter of animal and vegetable products is represented by the amount of ash they leave on burning, no more of exactitude should be attributed to the assumption than is really due.

To proceed: in Table II. the total ash of the Offal parts is divided into—that of the soft parts exclusive of hair or wool, that of the bones, and that of the hair or wool. On the amount of ash indicated in the hair or wool, no great stress must be laid; for, as will be easily understood, it was almost impossible to free these parts from adventitious dirt, which would leave a considerable incombustible residue. As in the Carcass, so in the Offal (which included the head, feet, and tail), by far the larger proportion of the ash was due to the bones. The portion of the total percentage of ash in the Offal due to the soft parts, is generally as great as in the Carcass. A part is doubtless attributable to debris of the food not entirely removed by washing from the alimentary cavities and passages. The blood too yielded a not inconsiderable quantity; and this portion may be looked upon as not belonging to the fixed substance of the body, or to those portions of the Offal which are, as the rule, appropriated as human food. The blood of the Pig is, however, frequently appropriated to this purpose. After making some allowance on these heads, there is still, at least in the fattened animals, about as high a percentage of mineral matter in the collective soft parts of the Offal as in those of the Carcass. It will be seen further on, how good an index is the percentage of mineral matter, to the amount of the fixed nitrogenous compounds of the total body, or its parts.

In the third main Division of Table II., the amounts of ash yielded by the same collective parts of the Carcass and of the Offal, are calculated to 100 *of the entire animal* (fasted live-weight), instead of to 100 of the Carcass, or of the Offal respectively. From this arrangement of the results we learn, that, on the average, the ash yielded by the total soft parts of the body, amounted to considerably less than 1 per cent. of its entire or live-weight. That yielded by the bones, on the other hand, amounted always to more than 1, generally to more than 2, and in some cases to from 3 to 4 per cent. of the whole body. Thus, the bones of the Pigs yielded ash amounting to 2 per cent. and under, of the weight of the animal; those of the Sheep to from 2 to 2½ per cent.; and those of the Calf and Bullocks to, from 3 to nearly 4 per cent., of the live-weight.

It is worthy of remark, that in the fat Pig the ash due to the soft or more edible portions of the Carcass, amounted to only 0·211 per cent. of the whole weight of the animal; the ash from the soft Carcass parts, of the three fatter Sheep to, from 0·205 to 0·339 of the live-weight; and that from the same parts in the fat Bullock to only 0·402 per cent. of its whole weight. Again, the per cent. in the entire animal of ash due to bones, is 3·35 per cent. in the fat Bullock, against 3·04 per cent. in the half-fat one. There is among the Sheep, in like manner (if we exclude the half-fat one), a diminution in the entire animal in the amount of ash from bones, from 2·10 per cent. in the store animal, to 1·916 per cent. in the very fat one. In the Pig, the proportion in the entire animal of bony structure, as indicated by the amount of ash of bone yielded, declines still more markedly as the animal fattens. Thus, the ash of the total bones of the store Pig amounted to 2·021 per cent. of the entire animal, and that from the total bones of the fat Pig to only 1·298 per cent. of its live-weight.

Turning now to the last division, or "Final Summary," of Table II., the decline in the percentage of mineral matter in the *entire animal*, as it fattens, is very clearly brought to view. This decline is indicated not only in the amount of ash derived from all parts of the body collectively, but in that from both total Carcass, and total Offal parts, taken separately. The exception is the case of the Offal parts of the fattest Sheep; but the large amount of ash in this instance was due to the great quantity of adventitious matter not removed by washing from the wool. Calling attention to the actual figures in some of the other cases, it is seen, that, whilst the half-fat Bullock yielded, from all parts, 4·664 per cent. of mineral matter or ash, the fatter Bullock gave only 3·92 per cent. Again, the whole body of the store Sheep yielded 3·164 per cent. of ash, and that of the fat Sheep only 2·811 per cent. And lastly, whilst the whole of the store Pig gave 2·669 per cent. of mineral matter or ash, that of the fat one gave only 1·649 per cent.

From the results as to *mineral matter* as a whole, it would appear, that during the [illegible] process, neither the accumulation of mineral matter in the bony structure, nor that [illegible] soft parts, takes place commensurately with the increase of some other constituents of the body. It will presently be seen which are the constituents that accumulate most rapidly under that process.

SECTION IV.—THE FAT—IN CERTAIN SEPARATED PARTS, AND IN THE ENTIRE BODIES, OF 10 ANIMALS ANALYSED.

When speaking on the subject of method, it was explained, that a large portion of the Fat of the animals analysed, was obtained from their different parts by *melting and expression*, the remainder being determined by *extraction with ether*. In the melting and expression manipulations, the parts were generally classed as under:—

(*a*) Kidneys, with the fat surrounding them:
(*b*) Other carcass parts, including bones:
(*c*) Head, and miscellaneous offal parts:
(*d*) Heart with its accompanying fat:
(*e*) Caul or omentum fat:
(*f*) Mesenteric or intestinal fat.

The determinations of Fat *by ether* were made upon the dry residues after the melting and expression, which, for the purpose, were mixed or classified to represent—

(*a*) Total carcass parts, including bone:
(*b*) Head, and other offal parts, including bone:
(*c*) Hair or wool.

Separate Fat determinations, by ether, were also made upon a mixture representing *the whole carcass and offal parts together*, excluding only hair or wool, and of course the previously melted and expressed fat. The actual results of the melting and expression experiments will be found in Tables I. to X. inclusive, in the Appendix. The ether determinations are given in Table XIII. of the Appendix. From the data thus provided, have been calculated the percentages of *melted* and *expressed*, of *ether-extracted*, or of *total* Fat (and its distribution), in the Carcass, in the Offal, and in the Entire Fasted Live-weight, of the several animals. These particulars are given in a somewhat detailed form in Table III., and in a more summary one in Table IV., which now follow.

TABLE III.—Percentages of FAT (by Melting, by Expression, and by Extraction with Ether), in Ten Animals.

1st. In Fresh Carcass.—2nd. In Fresh Offal (equal Sum of Parts excluding Contents of Stomachs and Intestines).—3rd. In Entire Animal (Fasted Live-weight, including therefore Contents of Stomachs and Intestines).

Description of animal.	Percentages in fresh Carcass.				Percentages in fresh Offal.									Percentages in fasted live-weight of Entire Animal (including contents of stomachs and intestines).													
	By melting and expression.		By ether.	From total carcass parts, by melting, expression, and ether.	By melting and expression.					By ether.			From total offal parts, by melting, expression, and ether.	From Carcass parts.				From Offal parts.									From Total Parts, by melting, expression, and ether.
														By melting and expression.		By ether.	From total carcass parts, by melting, expression, and ether.	By melting and expression.					By ether.			From total offal parts, by melting, expression, and ether.	
	From around kidneys.	From other carcass parts.	From mixed carcass parts.		From head.	From miscellaneous parts.	From heart.	From caul or omentum.	Mesenteric or intestinal.	From wool.	From head and feet.	From other offal parts.		From around kidneys.	From other carcass parts.	From mixed carcass parts.		From head.	From miscellaneous parts.	From heart.	From caul or omentum.	Mesenteric or intestinal.	From wool.	From head and feet.	From other offal parts.		
Fat calf	2·57	10·0	4·00	16·6	3·59		7·11			...	3·86		14·6	1·59	6·2	2·48	10·3	1·07		2·12			·	1·15		4·34	14·6
Half-fat ox.........	3·07	12·0	7·51	22·6	1·88		0·88	4·63	4·66	....	3·68		15·7	1·99	7·8	4·86	14·6	0·49		0·23	1·21	1 22		0·96		4·11	18·7
Fat ox	5·44	24·5	4·87	34·8	3·87		1·69	7·93	8·79	....	3·98		26·3	3·60	16·2	3·23	23·1	1·03		0·45	2·10	2·33	. ..	1·05		6·96	30·0
Fat lamb.........	6·21	25·6	5·03	36·9	1·04		13·86			0·66	4·58		20·1	3·71	15·3	3·01	22·1	0·33		4·32			0·21	1·43		6 29	28·3
Store sheep	1·69	17·6	4·54	23·8	2·87		0·43	5·14	3·08	0·92	3·66		16·1	0·90	9·4	2·42	12·7	1·10		0·17	1·97	1·18	0·35	1·40		6·17	18·8
Half-fat old sheep...	3·60	20·8	7·38	31·8	1·59		1·11	6·51	5·69	0·79	2·78		18·5	1·93	10 9	3·90	16·7	0·56		0·39	2·30	2·01	0·28	0·98		6·52	23·2
Fat sheep	8·38	33·8	3·20	45·4	2·55		13·19		6·57	1·17	2·95		26·4	4·81	19·4	1·84	26·1	0·91		4·71		2·34	0·42	1 05		9·48	35·5
Extra fat sheep	49·7		5·38	55·1	3·12			19·50	7·41	1·04	3·40		34·5	31·34		3·39	34·7	0·99			6·19	2·35	0·33	1·08		10·94	45·7
Store pig	1·67	22·5	3·91	28·1	9·60			2·12		... ..	*1·56	1·69	15·0	1·11	15·0	2·60	18·7	3·00			0·67		..	*0·49	0·53	4·69	23·4
Fat pig	4·82	42·4	2·78	49·6	9·05	1·84		8·35		......	*2·00	1·54	22·8	3·28	32·2	2·11	37·6	1·79	0·36		1·64		. .	*0·40	0·31	4·50	42·1
Means	29·6		4·86	34·4	......	......	......	.....	.....	0·92	3·57		21·0	18·7		2·98	21·7	. ..	..				0·32	1·08		6·39	28 1

* In practice, the head and feet of the Pig generally go with the Carcass: but here they are classed with the Offal parts, for the sake of comparison with the other animals.

TABLE IV.—Summary of Percentages of TOTAL FAT (by Melting, Expression, and Ether) in Ten Animals.

1st. In Fresh Carcass.—2nd. In Fresh Offal (equal Sum of Parts excluding Contents of Stomachs and Intestines).—3rd. In Entire Animal (Fasted Live-weight—including therefore Contents of Stomachs and Intestines).

[The mean percentage of Fat from Total Parts in the Entire Animal is given—1st, by addition of the amounts in the separate items; 2nd, with the ether-determinations made direct on a mixture of Entire Animal crude-dry-matters (except wool, with the amount in wool added).]

Description of animal.	Percentages in fresh Carcass.	Percentages in fresh Offal (excluding contents of stomachs and intestines).	Percentages in fasted live-weight of Entire Animal.			
			From Carcass parts.	From Offal parts.	From Total Parts.	
					By addition of items.	By other determinations direct on mixture of entire animal dry matter (except wool) and woolfat added.
Fat calf	16·6	14·6	10·3	4·34	14·6	14·8
Half-fat ox	22·6	15·7	14·6	4·12	18·7	19·1
Fat ox	34·8	26·3	23·1	6·96	30·0	30·1
Fat lamb	36·9	20·1	22·1	6·28	28·3	28·5
Store sheep	23·8	16·1	12·7	6·18	18·9	18·7
Half-fat old sheep	31·3	18·5	16·7	6·52	23·2	23·5
Fat sheep	45·4	26·4	26·1	9·43	35·5	35·6
Extra fat sheep	55·1	34·5	34·7	10·94	45·7	45·8
Store pig	28·1	15·0	18·7	4·68	23·3	23·3
Fat pig	49·6	22·8	37·6	4·50	42·1	42·2
Means of all	34·4	21·0	21·7	6·40	28·0	28·2
Means of the 8, excluding store sheep and store pig	36·5	22·3	23·2	6·64	29·8	29·9
Means of the 6, excluding the store and half-fat animals	39·7	24·1	25·6	7·08	32·7	32·8

Taking first the percentage of Fat in the fresh Carcass yielded by melting and expression, it is seen, that the amount around the kidneys increases very considerably as the animal fattens. In fact, practically, the judgment of the butcher on first disembowelling an animal is perhaps more influenced by the fatness of the kidneys than by any other single point. The figures show (Table III.), that, whilst the Carcass of the half-fat Ox afforded 3·07 per cent. of fat, by melting and expression, from around the kidneys, that of the fatter, but only moderately fat Ox, gave 5·44 per cent. of such fat. Of the Sheep, the Carcass of the store animal gave only 1·69 per cent. of melted and expressed kidney fat; that of the half-fat old one 3·60 per cent.; and that of the moderately fat one 3·88 per cent. Again, the percentage of melted and expressed kidney fat in the Carcass of the store Pig was 1·67, against 4·32 per cent. in that of a Pig in every respect similar, excepting that, from the time the former was slaughtered, it was fed on fattening food during a period of ten weeks. It must be remembered, that these amounts of fat obtained by melting and expression are exclusive of a not immaterial quantity afterwards extracted by ether, from the *mixed* Carcass parts, in which the kidneys and kidney-fat expressed residue were included.

The increase in the proportion of *Fat* in the other Carcass parts, during the period in which the animal is currently said to be *fattening*, is, as indicated by the amount obtainable from them by melting and expression, equally striking. But time and space will be saved, if we illustrate this accumulation by special reference to only the *total* amount of fat from *all* Carcass parts, whether obtained by melting, expression, or ether. Referring the reader, then, to the first three columns in Table III., to see how the total amount of Carcass fat is made up, we will proceed to draw our illustrations as to the total amount itself, from the fourth column of the same Table—or, as there repeated without the detail, in column 1 of Table IV. It is seen, that the Carcass of the half-fat Ox contained 22·6 per cent., that of the store Sheep 23·8, and that of the store Pig 28·1 per cent. of pure fat*. The Carcass of even a *fat* Calf, on the other hand, gave only 16·6 per cent. of total fat; which, however, is quite consistent with the current notion, that veal is leaner than any other of our ordinary meats. Of the Carcasses better representing the average condition of butcher's meat of good quality, that of the moderately fat Ox yielded 34·8 per cent. of pure fat; that of the fat Lamb 36·9; that of a moderately fat Sheep 45·4; that of a very fat Sheep 55·1; and that of an only moderately fattened Pig 49·6 per cent.

It is thus seen, that the *animal food*, of reputed high quality, as sold by the butcher, and to which such a *highly nitrogenous* character is generally attributed, will probably consist of *fat* to the extent of from one-third, to one-half, or even more, of the total fresh-weight of the Carcasses. To this point we shall have to recur, when taking a review of the *collective composition* of the animals; and also when we come to the application of the results, and to a consideration of the general conclusions to be drawn from them. But it may be here remarked in passing, in reference to the percentages of Fat above enumerated, that they would have been even somewhat higher in the condition of the Carcasses as weighed out by the butcher; for, in the Tables, the percentages are calculated in relation to the weight of the fresh carcass taken as soon as possible after the animal was killed; but between this condition and that in which the meat is generally sold to the consumer, there may be a loss in weight of even several per cent. by the evaporation of water.

From the want of a strict uniformity in classifying the several parts of the Offal in the different animals, for the determination of the Fat they contained, a detailed comparison of its amount in the corresponding parts in the different animals is not so practicable as might be wished. But, since only a small proportion of the fat from the Offal parts is consumed as human food, such a comparison is of the less importance in a practical point of view. Still, there are here some points worthy of notice, as indicating the accumulation of Fat internally as the animal matures.

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* The fat obtained by *melting* and *expression* contained but very immaterial amounts of foreign substance; [illegible] independently of other considerations, the composition of the final residue afforded confirmation of the [illegible] *determinations* very closely indicated the complementary amounts of the originally existing [illegible].

The percentage in the total Offal parts of fat obtained by melting and expression from the caul or omentum, was, in the half-fat Ox, 4·63, and in the moderately fat Ox, 7·93. In the Offal of the store Sheep the percentage of melted and expressed fat from the same source was 5·14, and in that of the very fat Sheep it was 19·5. Another item of fat, which is a considerable index to the fattening character and maturity of an animal—but which, with frequently a portion of the omentum fat also, is generally employed for tallow, and therefore not as food—is the mesenteric or intestinal fat. This also is seen to increase as the animals fatten; though those breeds which have the greater tendency to fatten on the outer frame or Carcass, have the less aptitude to do so around the internal organs. To go to the figures, it is seen, that the intestinal fat of the half-fat Ox amounted to 4·66 per cent. of the total Offal parts, and that of the fatter Ox to nearly double, or 8·79 per cent. The intestinal fat of the store Sheep amounted to 3·08, that of the half-fat Sheep to 5·69, that of the fat Sheep to 6·57, and that of the very fat Sheep to 7·41 per cent. of the collective Offal parts. The Offal of the store Pig again, yielded only 2·12 per cent. of its weight of melted fat from the intestinal regions (including the so-called caul-fat), and that of the fat Pig 8·35 per cent. It need hardly be remarked, that it is only in a practical or economic point of view, that any comparisons can be drawn between animals differing so essentially in their characters and habits, as the Pig and the Ruminant. But, whilst speaking of the amounts of fat deposited around the internal organs in the two cases, it may not be out of place to call to mind how much more concentrated, so far as digestible matter is concerned, is the food of the Pig than that of the bulky-feeding Ruminant, and that, in conformity with this, the alimentary cavities and passages constitute, collectively, a much less proportion of the bulk and weight of the animal in the former, than in the latter.

It is seen that nearly 1 per cent. of the collective Offal parts of the Sheep is fatty matter contained in the wool.

Of *total fat* obtainable by melting, expression, and ether, the collective Offal parts yielded only from one-half to two-thirds as high a percentage as the collective Carcass or more universally edible parts. Even in the Offal, however, the fat, in the cases of the fattened animals (excluding the calf and lamb), amounted to about one-fourth of the total Offal, as, for example, in the fat Bullock, the fat Sheep, and the fat Pig, and to more than one-third in the case of the very fat Sheep. Of the probable proportions of the fat of the *carcass* and of the *offal*, respectively, which are, on the average, consumed as human food, and of the relation of this consumed fat to the nitrogenous substance taken with it, we shall have to speak further on.

Let us now turn from the percentages of Fat in the collective Carcass, or collective Offal parts, respectively, to the amounts derived from the same sources, calculated in relation to the *entire* or *fasted live-weight* of the animals taken as 100. Looking to the percentages as so calculated, and which are given in the more detailed form in Table III., it is obvious that the *relation* of the [illegible], comparing one animal with another, in regard to the fat of any corresponding part or parts, will be nearly the same as in the comparisons already drawn, wherein the percentages were given in relation to the total Carcass, or total Offal parts, separately. The actual figures are of course less when taking the larger weight—that of the Entire Animal—as the divisor in the calculation; but the relation of the figures representing any given part or set of parts of one animal compared with another, will only differ from that in the forms of calculation already considered, in so far as the proportion, in the Entire Live-weight, of the contents of stomachs and intestines (included in the live-weight) is different in the different animals. Some space may be saved, therefore, by leaving to the reader the study of the further details in Table III., and proceeding to call attention to the more summary view of the percentages of fat in the Entire Animals, as given in columns 3, 4, 5, and 6, of Table IV.

In column 3 of Table IV., the percentages in the Entire or Fasted Live-weight of the animals, of fat due to total Carcass parts, are given; and in column 4, those yielded by the total Offal parts. Taking the average of the ten animals, more than three-fourths of their total fat belonged to their Carcass, or more valuable edible parts. The proportion of the whole fat due to the Carcass parts is seen to be generally greater the "riper," or more matured, the animal. The proportion of the whole fat, due to carcass and offal respectively, is about the same in both the half-fat and the moderately fat Ox. In the Sheep, however, the proportion of the whole fat due to carcass is much greater in all the more mature animals than in the one in the store condition. In the latter, there was only about twice as much of the total fat coming from the carcass as from the offal; whereas, taking the average of the four fatter animals—the fat Lamb, the half-fat Sheep, the fat Sheep, and the very fat Sheep—there was three times as much of the total fat due to the carcass as to the offal parts. In even the *store* Pig there was four times as much of the total fat of the body, in the Carcass, as in the Offal parts; and in the fat Pig there were from eight to nine times as much fat from the Carcass as from the Offal parts. The general conclusion would seem to be then, that, perhaps on the average, three-fourths or more of the total fat of a slaughtered animal, in good condition, will belong to the *carcass parts*—that is to say, to those parts which may be reckoned as almost entirely devoted, in some form or other, as human food.

It has been already seen in Table III., and the column illustrating the fact is repeated in Table IV., that the percentage of fat in the collective Carcass parts of the fattened animals amounted to from one-third to one-half, or even more, of the total weight of the Carcass. We will now consider what was the proportion of the *whole animal* at the time of its being slaughtered, which was nearly, if not quite, *pure fat.*

In the fifth column of Table IV., the percentage of total fat in the Fasted Live-weight of the several animals has been calculated by the addition of the respective items recorded in Table III. In the sixth and last column of Table IV., are given the percentages of total fat in the Live-weight of the ten animals, obtained, for the purposes of control, by another method. In this check column No. 6, the amount of fat obtained by melting and expression is calculated by the addition of the amounts thus obtained from the several parts, as in the other case; but, the fat remaining in the different expressed parts, is now obtained by ether determinations made on proportionally mixed samples of *all* the parts of the animal, Carcass and Offal together, the wool only excepted; the fat in the wool itself being extracted separately, and its amount taken into the calculation. A glance at the two columns (5 and 6) will show that there is never half, and in only one or two cases, one-fourth of 1 per cent. of variation between the results obtained by the two different methods.

The striking fact appears, that, of the *whole body* (fasted live-weight), 45¾ per cent. of the very fat Sheep, and 42 per cent. of the moderately fat Pig, were *dry fatty matter*. Of the moderately fat Sheep 35½ per cent., of the moderately fat Bullock 30 per cent., and of the fat Lamb 28½ per cent. were dry fat. The half-fat old Sheep contained 23¼, and the half-fat Bullock 18¾ per cent. of fat. The *store* Sheep even contained 18¾, and the *store* Pig 23⅓ per cent. Of the Calf, on the other hand, the entire body, though professedly fat, yielded only 14¾ per cent. of its weight of dry fatty substance.

It may, perhaps, from these data be concluded, that fattened Oxen of good quality, will, on the average, consist of *fat*, to the amount of nearly one-third of their whole weight; moderately fattened Sheep, to rather more; and moderately fattened Pigs, to more still; whilst, probably, fat Bacon-Pigs will frequently contain fat to the amount of one-half of their whole live-weight.

SECTION V.—THE NITROGEN—IN CERTAIN SEPARATED PARTS, AND IN THE ENTIRE BODIES, OF 10 ANIMALS ANALYSED.

It has been shown, how large is the proportion of the whole body, of some of the animals of most importance as human food, which is *fatty matter*. It has been further seen, that the proportion of fat in the collective Carcass parts, that is those which are the more exclusively appropriated to food purposes, is still greater than in the whole body. The next point of interest is as to the proportion, in the whole body or certain collective parts, of *nitrogen* and the thence calculated amount of protein or other nitrogenous compounds, a class of constituents, the comparative predominance of which is generally supposed so prominently to characterize our *animal food*.

The *nitrogen* was determined upon mixed samples of the crude dry matters remaining after the removal of fat by melting and expression, as follow:—

(*a*) Of all soft parts of the carcass:
(*b*) Of carcass bones:
(*c*) Of offal, soft parts and bones together (excluding hair or wool):
(*d*) Of all parts, carcass and offal together (excluding hair or wool).

It was also determined upon the hair or wool separately. The exceptions to the above arrangement were, that, in the cases of the Pigs, the mixed sample of Carcass included both soft parts and bones, that of the Offal was without bones, and a separate mixture was made of the head and feet, soft parts and bones together.

The actual experimental determinations of nitrogen, in the expressed nitrogenous residues, mixed as above described, are given in Table XIV. in the Appendix. The calculated *mean* results are given in a detailed form in Table V., and in a summary one in Table VI., which now follow.

TABLE V.—Mean Percentages of NITROGEN in Ten Animals.

1st. In Fresh Carcass.—2nd. In Fresh Offal (equal Sum of Parts excluding Contents of Stomachs and Intestines).—3rd. In Entire Animal (Fasted Live-weight, including therefore Contents of Stomachs and Intestines).

Description of animal.	Percentages in fresh Carcass.			Percentages in fresh Offal.				Percentages in fasted live-weight of Entire Animal.							
								From Carcass parts.			From Offal parts.				
	From fleshy and membranous parts.	From bones.	From total carcass parts.	From hair or wool.	From head and feet.	From other offal parts.	From total offal parts.	From fleshy and membranous parts	From bones.	From total carcass parts	From hair or wool.	From head and feet (excluding tongue and brains).	From other offal parts.	From total offal parts.	From Total Parts.
Fat calf	2·121	0·487	2·608	0·142	2·670		2·812	1·316	0·302	1·618	0·042	0·796		0·838	2·456
Half-fat ox	2·331	0·462	2·793	0·242	3·194		3·436	1·509	0·299	1·808	0·063	0·837		0·900	2·708
Fat ox	1·919	0·432	2·351	0·216	2·656		2·872	1·271	0·286	1·557	0·057	0·704		0·761	2·318
Fat lamb	1·371	0·341	1·712	1·099	1·923		3·022	0·820	0·204	1·024	0·343	0·600		0·943	1·967
Store sheep	1·896	0·454	2·350	1·193	1·727		2·920	1·012	0·242	1·254	0·457	0·662		1·119	2·373
Half-fat old sheep	1·925	0·364	2·289	1·344	1·584		2·928	1·031	0·195	1·226	0·475	0·559		1·034	2·260
Fat sheep	1·467	0·324	1·791	1·129	1·480		2·609	0·843	0·186	1·029	0·403	0·528		0·931	1·960
Extra fat sheep	1·136	0·261	1·397	1·220	1·554		2·774	0·716	0·164	0·880	0·387	0·493		0·880	1·760
Store pig*	2·319		2·319	......	0·757*	1·415	2·172	1·541		1·541	......	0·237*	0·442	0·679	2·220
Fat pig*	1·712		1·712	......	0·800*	1·513	2·313	1·300		1·300	......	0·158*	0·299	0·457	1·757
Means of all	2·132		2·132	0·659	2·127		2·786	1·324		1·324	0·223	0·631		0·854	2·178
Means of the 8, excluding store sheep and store pig	2·082		2·082	0·674	2·172		2·846	1·305		1·305	0·221	0·622		0·843	2·148
Means of the 6, excluding the store and half-fat animals	1·928		1·928	0·634	2·099		2·734	1·235		1·235	0·205	0·596		0·802	2·036

* For comparison with the other animals, the nitrogen of the *head* and *feet* of the pigs is here included in the Offal, but in practice these parts generally go with the Carcass.

TABLE VI.—Summary of Mean Percentages of NITROGEN in Ten Animals.

1st. In Fresh Carcass.—2nd. In Fresh Offal (equal Sum of Parts excluding Contents of Stomachs and Intestines).—3rd. In Entire Animal (Fasted Live-weight, including therefore Contents of Stomachs and Intestines).

[The mean percentage of Nitrogen from Total Parts in the Entire Animal is given—1st, by addition of the amounts in the separate items—2nd, by direct determination on the mixture of all crude dry parts (except wool, the amount in wool being added by calculation)—3rd, by calculation, deducting fat and mineral matter from total dry, and dividing by 6·3 = an average per cent. of nitrogen, of 15·873, in the nitrogenous substance.]

			Per cent. in Entire Animal (fasted live-weight).				
					From Total Parts.		
Description of animal.	Per cent. in fresh Carcass.	Per cent. in fresh Offal (excluding contents of stomachs and intestines).	From carcass parts.	From offal parts.	1. By addition.	2. By direct determinations on hair or wool separately, and on a mixture of all other parts collectively.	3. By deducting fat and mineral matter, from total dry, and dividing by 6·3.
Fat calf	2·608	2·812	1·618	0·838	2·456	2·471	2·421
Half-fat ox	2·793	3·436	1·808	0·900	2·708	2·781	2·635
Fat ox	2·351	2·872	1·557	0·761	2 318	2·333	2·304
Fat lamb	1·712	3·022	1·024	0·943	1·967	1·974	1·949
Store sheep	2·350	2·920	1·254	1·119	2·373	2·380	2·353
Half-fat old sheep	2·289	2·928	1·226	1·034	2·260	2·267 2·282*	2·226
Fat sheep	1·791	2·609	1·029	0·931	1·960	1·947 2·035*	1·941
Extra fat sheep	1·397	2·774	0·880	0·880	1·760	1·814 1·747*	1·736
Store pig	2·319	2·172	1·541	0·679	2·220	2·196	2·180
Fat pig	1·712	2·313	1·300	0·457	1·757	1·773	1·725
Means of all	2·132	2·786	1·324	0·854	2·178	2·194	2·147
Means of the 8, excluding store sheep and store pig	2·082	2·845	1·305	0·834	2·148	2·170	2·117
Means of the 6, excluding the store and half-fat animals	1·928	2·733	1·235	0·802	2·036	2·052	2·013

These Tables of the percentages of actual *nitrogen* (V. and VI.), will enable us conveniently to compare the relative nitrogenous percentage of one animal, or its several parts, with that of another, and also, the duplicate results obtained by different methods of experimentation, or calculation, as the case may be. The amounts of *nitrogenous compounds* which the amounts of the nitrogen itself are supposed to represent, will be better considered further on, when we shall have before us, side by side, and at one view, the percentages in the ten animals or their collective parts, of the several *classes of constituents* of which the whole is made up, namely—mineral matter, dry nitrogenous compound, fatty matter, and the total dry substance and the complementary water.

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* In these cases, two sets of determinations were made, at different times.

When speaking of the *mineral matter*, it was found that there was four or five times as much in the total bones, as in the total soft parts of the Carcasses. A reference to Table V. shows, that the amount of *nitrogen* was, on the other hand, four or five times as great in the soft parts of the Carcasses, as in the hard or bony parts. Still it would result, that, whenever no nutriment was reclaimed from the bones, one-fifth or one-sixth of the total nitrogenous substance of the Carcasses would be lost to human food.

Comparing first the percentage of nitrogen in the different *carcasses*, it is seen greatly to decrease with the progress of the animal from the store to the fat condition. It will be seen further on, that the fattening or maturing is accompanied by a considerable diminution in the percentage of *water* in the body. The dry matter accumulated consists, however, in a much greater proportion of fatty substance, than of nitrogenous compound. Indeed, it would seem probable that, necessarily, the larger the amount of the nitrogenous compounds, the larger the amount of water required for their proper hydration, for the purposes they subserve in the system.

To go to the figures, Table V. shows, that whilst the entire *carcass* of the half-fat Ox contained 2·793 per cent. of nitrogen, that of the moderately fat Ox contained about one-sixth less, or 2·351 per cent. Of the Sheep, the carcass of the store animal contained 2·350 per cent., that of the half-fat one 2·289 per cent., that of the fat one 1·791 per cent., and that of the very fat one only 1·397 per cent. of nitrogen. Again, whilst the carcass of the store Pig contained 2·319 per cent. of nitrogen, that of the moderately fatted one contained only 1·712 per cent. Lastly, the carcass of the fat Calf, which yielded a less proportion of fat than that of any of the other animals, contained of nitrogen on the other hand, a higher percentage than that of any but the half-fat Ox, namely, 2·608 per cent. This, again, is perfectly consistent with the reputed relatively lean character of veal.

Turning to the percentage of nitrogen in the collective *offal* parts (exclusive of contents of stomachs and intestines), it is seen at a glance, to be in every instance excepting that of the store Pig, higher than in the collective Carcass. In the Lamb, and in the four Sheep, however, more than one-third of the nitrogen of the offal is contained in the wool. Deducting this, the percentage of nitrogen in their collective other Offal parts would be less than in their collective Carcass parts.

In the right-hand portion of Table V., the amounts of nitrogen in the respective parts are calculated so as to show their percentage in the *total* or *fasted live-weight* of the animals, instead of in the Carcass or the Offal taken separately. Particular attention need only be called, however, to the more summary view of the percentage of nitrogen in the *entire animal*, as given in Table VI. The third column of this Table shows that amount of the total percentage of nitrogen in the Entire Animal which is contributed by the Carcass parts, and the fourth column that due to the total Offal parts. The last three columns of the Table give the percentage of nitrogen in the Entire Animals due to all parts together, both Carcass and Offal; but determined or calculated by three different methods. Before considering the actual composition of the animals, in regard to nitrogen, as shown in these three concluding columns, it will be interesting to consider the degree of agreement in the percentages obtained by the three different methods referred to.

In the first of the three concluding columns (Table VI.), headed "*By Addition*," the figures there recorded are obtained by the addition of the percentages afforded by the separate items or parts, the details of which are given in Table V. The percentages so obtained, are, in fact, the result of duplicate nitrogen determinations made on each of *four* separate parts or mixed samples, for each animal. The next column, headed—"By direct determinations on hair or wool separately, and on a mixture of all other parts collectively"—is obtained, as the description indicates, by duplicate determinations on *two* series of parts only. The last column, on the other hand, is obtained entirely by *calculation*, as a check upon the percentages of nitrogen made by direct experimental determination. The method of calculation is as follows:—From the percentage of the crude dry substance, remaining after the removal of most of the fat by melting and expression, the fat afterwards extracted by ether is deducted. From the result so obtained, is next deducted the amount of the mineral matter. The remainder—the water, the fat, and the mineral matter, being thus all excluded—consists, of course, of nitrogenous compounds of some kind or other. With the view of founding an estimate as to the probable amount of nitrogen contained in the mixed nitrogenous matter of entire animal bodies, upon a basis of something like specific and detailed facts, we have in vain endeavoured to find sufficient published data for estimating the probable relative proportions in the body of albumen, fibrin, (*quasi*) gelatin, or chondrin, &c. In the absence of any appropriate data on this subject, we have assumed, of necessity somewhat arbitrarily, 6·3 as probably the nearest round number applicable as a divisor of the crude nitrogenous substance of the animal bodies in question, to reduce it to nitrogen. This number, 6·3, supposes an average percentage of nitrogen in the mixed nitrogenous compounds, of 15·873. This is slightly higher than in either albumen or fibrin; considerably higher than in chondrin; but on the other hand, considerably lower than in gelatin*. It is probably, therefore, as good a figure as could be taken under the circumstances, as some confirmation of nitrogen determinations made upon such heterogeneous matters, and of the propriety of their application to the objects we have in view.

If we are to assume, that the direct nitrogen determinations are nearer the truth than the calculated estimates, it would appear that the collective nitrogenous compounds of the whole body, in the cases in question, had a rather higher percentage of nitrogen than that represented by our number 6·3—namely, 15·873. For, although the discrepancy in the percentages of nitrogen obtained by the different methods is invariably within the range of the second decimal place, the percentage indicated by the method of calculation merely, is in every instance somewhat lower than that by the direct experimental *determinations*. It would be easy to suggest several sources of probable inaccuracy, in thus assuming a percentage of nitrogen in the collective nitrogenous compounds of the body determined as a whole by the deduction of the amounts of other matters. For instance; it may be a question—how far the mineral matters determined by incineration, and deducted by calculation, contain the oxidated sulphur and phosphorus of the nitrogenous compounds themselves?—what were the relative proportions of the different nitrogenous compounds in the collective mixture of all of them?—and so on. Any consideration, of the interference with strict accuracy, of such sources of error, is, with our present main objects, immaterial.

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* M. Boussingault formerly adopted 15·0 per cent. of nitrogen (=6·666), in his calculations of nitrogenous *vegetable* compounds from the amounts of nitrogen; but he has more recently adopted 16·0 per cent. of nitrogen (=6·25).

Looking to the figures obtained by the three different methods, we are free to confess, that the correspondence between them is such as we had scarcely hoped to attain. We accept it as a proof of success in a difficult and extremely laborious undertaking, such as gives us more confidence in our final results, and in the conclusions derived from them, than at the commencement we had at all anticipated. It will be easily understood, that, although the difficulty of getting, for analysis, perfectly proportional and uniform mixtures, of such heterogeneous matters as those in question, must be very great, yet, that success on this point will depend only upon the amount of care and labour devoted to it. Feeling that so much depended upon these, we were, particularly in the preparation of the samples, not sparing in their exercise.

So far as the nitrogen determinations themselves are concerned, we are very glad to have this opportunity of stating, that they were, for the most part, made by Mr. F. A. MANNING. The degree of credit due to Mr. MANNING for this labour, will be best ascertained by an examination of the duplicate determinations given in Table XIV. in the Appendix, and of the coincidence of the final results, by the different methods, as given in Table VI. now under consideration. In this examination the extremely heterogeneous character of the substances operated upon should not be forgotten.

It has already been remarked, that the difference between the percentage of nitrogen indicated for the Entire Animals, obtained on the one hand by direct determinations, and on the other by calculation merely, was, pretty uniformly, within the limits of the second decimal place; and further, that the calculated percentage was invariably somewhat lower than the experimental ones. Comparing with each other the two differently obtained experimental results, the difference between them is always within the same limit. In the majority of cases, the difference, even in the second decimal place, amounts to very few units. The actual figures, in these three columns, may then be taken as pretty closely indicating the real percentages of nitrogen in the Entire Bodies operated upon. They furnish, therefore, a concise view of the differences in this respect, between the different animals, according to their description or condition.

It is remarkable, that, of the beef-yielding animals, the whole body of the half-fat Ox contained scarcely 2¾ per cent. of nitrogen, and that of the moderately fat one only about 2¼ per cent. The fat Calf contained a rather larger proportion of nitrogen than the fat Ox; namely, nearly 2½ per cent.

The entire body of the fat Lamb contained less than 2 per cent. of nitrogen. The store Sheep contained less than 2·4 per cent. of nitrogen; the half-fat old Sheep 2¼ per cent.; the moderately fat Sheep not quite 2 per cent.; and the very fat Sheep scarcely more than 1¾ per cent.

The store Pig contained about 2⅛ per cent. of nitrogen; and the moderately fat one, only about 1¾ per cent.

The striking fact of there being so small a percentage of nitrogen in the bodies of the animals which we feed to supply our meat-diet, is one of great interest and importance. On the one hand, as will be fully illustrated further on, the proportion of the nitrogen consumed in the fattening food, which remains stored up in the animal, and is sent to market as meat, is extremely small; and on the other, as already alluded to, a considerable proportion of the nitrogen which really is retained by the animals is not appropriated as human food; whilst, of that which is so appropriated, a considerable portion will exist in the form of gelatin and chondrin-yielding matters, the value of which as food is, to say the least, questioned.

Before leaving the Tables showing the percentage of *nitrogen* in the different animals and their several parts, attention may be recalled to the illustrations of the latter point, afforded in the lines of *mean percentages*, given at the foot of Table VI. It has already been noticed, that the collective Offal parts of the animals contained a higher percentage of nitrogen than the collective Carcass parts. But it is seen, that the actual amount in the *entire body*, of nitrogen accumulated in the Carcass parts, is still about three-fifths of the whole. It was shown, that about one-fifth of this is due to the bones. It results, that there is left, in round numbers, only about *half* of the entire nitrogen of the body associated with the *soft edible parts of the carcass*. As to the proportion of the two-fifths of the total nitrogen in the body contained in the Offal parts, which will probably, on the average, be consumed as human food, some observations will be made further on.

SECTION VI.—SUMMARY OF THE COMPOSITION OF THE TEN ANIMALS ANALYSED:—*Mineral Matter, Dry Nitrogenous Compounds, Fat, Total Dry Substance, and Water.*

Having considered the percentages of mineral matter, of fat, and of nitrogen, *individually*, in the different animals and their respective parts, it will be well now to take a summary view of their *collective* composition as deducible from the data thus provided. In Table VII., therefore, which now follows, are given side by side, at one view, the percentages in the Carcass, in the Offal, and in the Entire Animal respectively, of—

1st. Mineral matter (ash):
2nd. Total dry nitrogenous compounds (by deduction of other constituents):
3rd. Total fat (by melting, expression, and extraction by ether):
4th. Total dry substance (sum of the mineral, nitrogenous, and fat):
5th. Water (the complementary quantity to the total dry):

And, in the Entire Animal, the percentage of the whole due to contents of Stomachs and Intestines is also given.

TABLE VII.—Summary of the Composition of the Ten Animals;—showing the Percentages of Mineral Matter, Dry Nitrogenous Compounds, Fat, Total Dry Substance, and Water.

1st. In Fresh Carcass.—2nd. In Fresh Offal (equal Sum of Parts excluding Contents of Stomachs and Intestines).—3rd. In Entire Animal (Fasted Live-weight, including therefore the weight of Contents of Stomachs and intestines).

Description of animal.	Per cent. in Carcass.					Per cent. in Offal (sum of parts excluding contents of stomachs and intestines).					Per cent. in Entire Animal (fasted live-weight).					
	Mineral matter.	Dry nitrogenous compounds.	Fat.	Total dry substance.	Water.	Mineral matter.	Dry nitrogenous compounds.	Fat.	Total dry substance.	Water.	Mineral matter.	Dry nitrogenous compounds.	Fat.	Total dry substance.	Contents of stomachs and intestines (in moist state).	Water.
Fat calf	4·48	16·6	16·6	37·7	62·3	3·41	17·1	14·6	35·1	64·9	3·80	15·2	14·8	33·8	3·17	63·0
Half-fat ox	5·56	17·8	22·6	46·0	54·0	4·05	20·6	15·7	40·4	59·6	4·66	16·6	19·1	40·3	8·19	51·5
Fat ox	4·56	15·0	34·8	54·4	45·6	3·40	17·5	26·3	47·2	52·8	3·92	14·5	30·1	48·5	5·98	45·5
Fat lamb	3·63	10·9	36·9	51·4	48·6	2·45	18·9	20·1	41·5	58·5	2·94	12·3	28·5	43·7	8·54	47·8
Store sheep	4·36	14·5	23·8	42·7	57·3	2·19	18·0	16·1	36·3	63·7	3·16	14·8	18·7	36·7	6·00	57·3
Half-fat old sheep	4·13	14·9	31·3	50·3	49·7	2·72	17·7	18·5	38·9	61·1	3·17	14·0	23·5	40·7	9·05	50·2
Fat sheep	3·45	11·5	45·4	60·3	39·7	2·32	16·1	26·4	44·8	55·2	2·81	12·2	35·6	50·6	6·02	43·4
Extra fat sheep	2·77	9·1	55·1	67·0	33·0	3·64	16·8	34·5	54·9	45·1	2·90	10·9	45·8	59·6	5·18	35·2
Store pig	2·57	14·0	28·1	44·7	55·3	3·07	14·0	15·0	32·1	67·9	2·67	13·7	23·3	39·7	5·22	55·1
Fat pig	1·40	10·5	49·5	61·4	38·6	2·97	14·8	22·8	40·6	59·4	1·65	10·9	42·2	54·7	3·97	41·3
Means of all	3·69	13·5	34·4	51·6	48·4	3·02	17·2	21·0	41·2	58·8	3·17	13·5	28·2	44·9	6·13	49·0
Means of 8; namely, of the half-fat, fat, and very fat animals	3·75	13·3	36·5	53·6	46·4	3·12	17·4	22·4	42·9	57·1	3·23	13·3	29·9	46·4	6·26	47·3
Means of 6; namely, of the fat, and very fat animals	3·38	12·3	39·7	55·4	44·6	3·03	16·9	24·1	44·0	56·0	3·00	12·7	32·8	48·5	5·48	46·0

Looking more particularly to the first Division in the Table (VII.), which shows the collective composition of the *carcasses*, and comparing one animal with another, there is seen to be a general disposition to a rise or fall in the percentage of mineral matter, with the rise or fall in that of the nitrogenous compounds. In fact, all the results tend to show a prominent connexion between the amount of the mineral matters, and that of the nitrogenous constituents of the body.

Next comparing the relative proportions of *fat*, and of *nitrogenous compounds*, in the different Carcasses, it is seen, that, in every instance excepting that of the Calf (in which case the percentages of nitrogenous substance and of fat were equal), there was considerably more of dry fat than of dry nitrogenous compounds. In the Carcass even of the *store* or *lean* Sheep, there was more than one and a half time as much fat as nitrogenous substance. In that of the *store* or *lean* Pig there was twice as much. In the Carcass of the half-fat Ox, there was one-fourth more fat than nitrogenous matter; and in that of the half-fat Sheep there was more than twice as much. Of the fatter animals, the Carcass of the fat Ox contained twice and one-third as much dry fat as nitrogenous substance; that of the fat Sheep four times, and that of the very fat Sheep even six times as much. Lastly, in the Carcass of the moderately fat Pig, there was nearly five times as much fatty matter as nitrogenous compounds.

From these results, it may perhaps be safely inferred, that in Carcasses of *beef* of reputed good condition, there will be seldom less than twice as much, and frequently nearly three times as much dry fat as dry nitrogenous substance. In the Carcasses of *sheep* we should conclude, that the fat would generally amount to more than three, and frequently to four or even more times as much as the nitrogenous matter. Finally, it may be estimated that in the Carcasses of *pigs killed for fresh pork*, there will be seldom as little as four times as much fat as nitrogenous compounds; whilst, in those *fed for curing*, the fat will generally be in a higher proportion still.

The *fat* of the bones bears but a small proportion to that of the whole Carcass. As has been seen, however, perhaps one-fifth of the whole *nitrogen* of the Carcasses will be contained in the bones, and not included therefore in the more currently edible portions. It results, that, provided the whole of the fat of the soft parts of the Carcass be consumed as human food, its proportion to the amount of the consumed nitrogenous substances will, on the average of such consumption, be still greater than the actual composition of entire Carcasses would indicate.

The question here arises, what proportion of the fat of our slaughtered animals is probably, on the average, actually consumed as human food in one form or another? We have instituted numerous inquiries on this point; and we cannot do better than quote the opinion given by an experimenter and writer of great observation and experience in such matters. Mr. JOHN EWART, of Newcastle, writes in answer to our inquiry for his opinion as follows:—

"As to beasts. The *opening* or the thick vein of fat that is presented and protrudes from the internal cavity on the first opening of the carcass, and also the *crook of the* *reed* or the fat on the *reticulum* or second stomach is reserved for suet, whilst the mesentery or web of fat which holds the intestines is sent to the tallow-melter. The proportion of the internal fat spoken of as reserved for suet may amount to about one-fourth of the whole of the loose or offal fat yielded by the beast; but the demand for suet not being at all times equal to the quantity reserved for such in addition to the real suet or the fat on the inside of the loins and covering the kidneys, a portion of the loose fat reserved for suet is sometimes added to the tallow, by which the quantity of offal fat of beasts used as human food will be reduced to one-fifth, or twenty per cent. of its entire quantity, and which must be understood as in addition to the whole of the carcass fat. Before dismissing this portion of the subject, I may remark, that the suet in the victualling of shipping always consists of loose or offal fat, some being taken for that purpose even when entire carcasses are purchased.

"In sheep, although a portion of loose fat is generally reserved for suet, yet there being a portion of the carcass fat, when redundant—which is very freqnently the case—sent to the tallow-melter, the whole of the carcass fat, but without any addition of offal fat, will represent, very nearly, the quantity used as human food from this description of stock.

"I am fully aware that the disposition of the fat yielded by the kinds of stock already spoken of is not uniformly alike, and exactly that stated above in all localities, but I do not think that the variation is such as to affect the statement as an average for England. In Ireland and Scotland the consumption of flesh is insignificant, when compared to that in England.

"The whole of the fat of both calves and lambs is eaten, or at least used—sometimes also with the addition of sheep's offal fat—in the process of cooking their flesh.

"The mesentery in swine is very small, and that of small animals slaughtered for fresh pork is entirely used in the process of cooking their edible offals, such as the liver, &c. Hog's lard is the melted *leaf* or the layer of fat which accumulates in and lines the internal cavity in this description of stock, together with the melted mesentery of large animals slaughtered for being cured. The principal uses of hog's lard are as a substitute for butter by the pastry-cooks and others, and in adulterating that article by unprincipled dealers. Another use of the fat of swine in the form of lard is in the preparation of ointments by apothecaries, and in that of pomatum—particularly in the kind sold as bear's grease—by the perfumers."

With these statements, our information, derived from other sources, leads us to concur. It would appear, then, that as an average, we may assume:—that the whole of the Carcass fat, and about one-fifth of the Offal fat, of beasts, will be consumed as human food; that, of Sheep, an amount equal to the whole of their Carcass fat, but without reckoning any from their Offal parts, will be so consumed;—and that, of the Pig, an amount equal to the whole of its Carcass fat, which is in greater proportion than in the other animals, and [illegible] a part of its Offal fat also, will be consumed as food. So far then as *carcasses* are concerned, the conclusion would seem to be fully borne out, that the proportion of the consumed fat to the consumed nitrogenous substance, will, on the average, be greater than that indicated by the relation of the total fat to the total nitrogenous matter in the Carcasses of fattened animals.

Still confining attention for the present to the composition of the *carcasses*, the Table (VII.) shows, that, whilst the percentages of both mineral matter and nitrogenous substance decrease, as the animals mature, that of the fat, on the other hand, very considerably increases. Indeed, the increase in the percentage of fat is much more than equivalent to the collective decrease in that of the other solid matters: that is to say, as the animal matures, the percentage in its Carcass, of *total dry substance* (and especially of fat), much increases. There is then, of course, a corresponding diminution in the proportion of the water. Thus, in the Carcasses of the leaner animals, there were from 54 to 62 per cent. of water; namely, 62¼ per cent. in that of the Calf; 57⅓ per cent. in that of the store Sheep; 55⅓ per cent. in that of the store Pig; and 54 per cent. in that of the half-fat Ox. The Carcasses of all the other animals contained less than 50 per cent., and those of the fattest less than 40 per cent. of water. That of the moderately fattened Ox contained 45½ per cent.; that of the fat Lamb 48⅔ per cent.; that of the half-fat Sheep 49⅔ per cent.; that of the fat Sheep 39⅔; and that of the very fat Sheep scarcely one-third of its weight, or 33 per cent. only, of water. Lastly, in the Carcass of the moderately fattened Pig, there were 38½ per cent. of water. It may be remarked, that these particular Carcasses, in the condition in which they would have been sold by the butcher, would perhaps have contained 1 to 2 per cent. less water than is indicated in the Table. For, between the condition in which these Carcasses were weighed, namely, as soon as possible after killing, and that in which the meat is usually sold to the consumer, it would probably have lost 1 or 2 per cent. of water by evaporation. On the other hand, as the bones contain a higher percentage of dry matter than the collective soft parts, the percentage in these edible soft parts will be somewhat lower than in the entire Carcass including bones. The actual fresh and dry weights of the bones of the different animals analysed will be found in Tables I. to X., in the Appendix; and the percentages of dry matter in the bones in Table XI., also in the Appendix. It may be here observed, however, that the proportion of bone was much less in the Sheep than in the Oxen, and much less in the Pigs than in the Sheep. It was, too, in all cases less, the fatter the animal. The percentage of dry matter in the bone increased, however, as the animal matured; and it was higher in that of the Oxen than in that of the Sheep; and higher in that of the Sheep than in that of the Pigs. For example: the percentage of bone in the Carcass of the fat Ox was 11·8, in that of the fat Sheep 8·9, and in that of the fat Pig 4·6. The percentage of dry matter in the Carcass bones of the fat Ox and fat Sheep was from 73 to 74, and in those of the fat Pig only 61·7. From the large proportion of bone, and the high percentage of dry matter in the bone, of the Carcass of the fat Ox, the percentage of dry matter in the soft parts would be about 2½ lower than in the entire Carcass with bones. In the same way, the percentage of dry matter in the soft Carcass parts of the fat Sheep would be 59·1, instead of 60·35, as in the entire

Carcass. In the fat Pig, however, owing to the much smaller proportion of the bone, and the percentage of dry matter in the bone being so nearly the same as that in the total Carcass, the percentage of dry substance in the soft parts will not differ materially from that in the whole Carcass.

From the whole of the data adduced on the point, it may perhaps be safely concluded, that the average of Carcass *beef*, in well-fattened condition, will contain 50 per cent., or rather more of dry substance; that the average of properly fattened *mutton* will contain rather more dry matter than beef, say, 55 to 60 per cent.; that the Carcasses of Pigs killed for *fresh pork* will be rather drier than those of mutton; whilst the sides of Pigs killed for *curing* will, no doubt, be drier still. *Lamb* carcasses would seem to contain a smaller proportion of total dry substance than either moderately fattened beef, mutton, or pork. Their proportion of bone is also comparatively high. Lastly, *veal* appears to be the most watery of all. The Carcass of the Calf experimented upon, though the animal was considered to be well-fattened, contained only 37¾ per cent. of dry substance; its proportion of bone was also higher than in any of the other cases.

Turning to the second division of Table VII., which shows the composition of the *collective offal parts* (excluding contents of stomachs and intestines), the figures do not show such a uniform tendency to a diminution in the percentage of mineral matter coincidently with that of the nitrogenous substance, as was observed in the case of the Carcasses. But, as already referred to, the percentage of mineral matter in the *collective* Offal parts (and it is this which is here under notice), does not represent only the mineral matter properly associated with the other constituents of the parts, but includes a quantity of adventitious matter adhering to the pelt, hair, or wool, of the animals.

It is seen that the percentage of dry nitrogenous substance is, in every case, greater, and that of the fat very much less, in the collective Offal, than in the collective Carcass parts. In Oxen and Sheep, the pelt, hair or wool, hoofs, and for the most part stomachs and intestines, as well as some other nitrogenous parts of the Offal, will not be consumed as human food. The parts that will as a rule, or at least frequently be so consumed, are the head flesh with tongue and brains, the heart, the liver, the pancreas, the spleen, the diaphragm, and sometimes the lungs. Calculation leads us to estimate that the nitrogenous substance of these parts will in these animals (beasts and sheep) amount to about one-sixth of the whole nitrogenous matter of their collective Offals. This portion of edible nitrogenous substance from the Offal parts must, therefore, be added to the amount estimated as eaten from the Carcass, when considering the proportion of the whole nitrogen of the slaughtered animals which is appropriated as human food. Calculation further shows, however, that in the cases of Oxen and Sheep, the whole of the nitrogenous matter reclaimed as food from the Offal parts will fall short of the amount contained in the bones of the Carcass. So nearly, however, will these quantities balance one another (especially if a portion of the gelatine from the carcass bones be considered as eaten), that the total nitrogen of the Carcass parts, excluding any from the Offal, may perhaps be taken as little exceeding the average proportion of the whole nitrogen of the bodies of these animals which will be consumed as human food. In the case of Pigs, a larger proportion of the total nitrogenous parts of the Offal will be consumed; an amount indeed more than equivalent to that in the bones of the Carcass, supposed to be not consumed. It results in fact upon the whole, that there would appear to be a larger proportion of the total nitrogen of the body of the Pig consumed, than of that of the other animals. But, as has been seen in the Pig, the percentage in the body of that total nitrogen is less, and the percentage of the fat greater than in the other cases. The Offal parts enumerated as eaten, are of themselves associated with very little fat; so that, such food, which is chiefly used by the poorer classes, would be highly nitrogenous, if not combined with extraneous fatty matter. In cooking, it is generally so combined. Moreover, the classes which consume the most of the internal organs of slaughtered animals, are also those which consume the larger proportion of Pig-meat, in which the proportion of the fat to nitrogenous substances is higher than in any other description.

It is observable, that with their larger percentage of nitrogenous substance and less percentage of fat, the collective Offal parts have invariably a less percentage of total dry substance, and therefore a larger proportion of water, than the collective Carcass parts.

The next point to consider is the collective composition of the *entire animal*, as it stands at the time of being slaughtered. The information on this head will be found in the third and last division of Table VII.

The marked diminution in the percentage of *mineral matter* as the animal fattens, which was observed in reference to the composition of the Carcasses, is clearly illustrated in that of the Entire Animals, notwithstanding the inclusion here of the Offal ash, in which was contained the incombustible impurity of the hair or wool.

We should judge from the figures, that from 3½ to 4 per cent. (according to breed and condition) of the standing fasted weight of a fattened Ox will be mineral matter. The proportion in Sheep appears to be less. Excluding the adventitious matter of the wool, it would probably be often as little as 2½ and seldom more than 3 per cent. of the fasted weight. In Pigs, the proportion of mineral matter is still less. We should gather, that in a well-fattened animal of good breed, it would amount to only 1½ per cent., or even less, of its standing fasted live-weight. In a young unfattened Pig, there were 2·67 per cent. of mineral matter; but in an animal of a worse breed, or in a leaner condition still, we should judge that there might be 3 per cent. As an average estimate of the mineral matter in *store* animals, sold off or brought on the farm, we should be disposed to adopt 4½ to 5 per cent. of their live-weight for Bullocks, 3 to 3½ per cent. for Sheep, and 2½ to 3 per cent. for Pigs. As an average estimate for the mineral matter in *fattened* animals so far as the data at command enable us to form an opinion, we should take 3½ to 4 per cent. of their live-weight for Calves and Bullocks, 2½ to 2¾ per cent. for Sheep and Lambs, and 1¼ to 1¾ per cent. for Pigs.

Of total *nitrogenous compounds*, as well as total mineral matter, the beef-yielding animals contain in parallel conditions, rather more than Sheep, and Sheep rather more than Pigs. Of the standing fasted live-weight of the moderately fat Ox, there were, including bones, pelt, and internal organs, &c., only 14¼ per cent. of dry nitrogenous compounds. The fat Sheep contained only 12¼ per cent.; the very fat one not quite 11 per cent.; and the moderately fattened Pig about the same, namely, 10·9 per cent. The store animals contained from 2 to 3 per cent. more total dry nitrogenous substance than the moderately fat ones.

Of the standing live-weight of the animals, the *fat* obviously constitutes the most prominent item in the dry or solid matter. In the half-fat Ox there was nearly as much fat as nitrogenous substance and mineral matter put together; in the store Sheep there was more of fat than other solid matter; in the half-fat Sheep the proportion of fat to other matters was larger still; and in the store Pig it was larger than in the half-fat Sheep.

In the fat Calf alone was the total fat less than the total nitrogenous substance of the body. Of the other animals fit for the butcher, the fat Ox and fat Lamb consisted of fat to the amount of about 30 per cent. of their live-weight, which was nearly twice as much as the total amount of the remaining solid matters, and more than twice as much as the total nitrogenous substance only. The fat Sheep contained 35½ per cent. of fat, or nearly three times as much as of nitrogenous substance. The very fat Sheep yielded 45¾ per cent. of fat, and less than one-fourth as much of nitrogenous substance. Lastly, the fat Pig, with 42¼ per cent. of its entire body consisting of fat, had just about one-fourth as much, or 10·9 per cent., of dry nitrogenous substance. Taking the mean composition of the six animals assumed to be fit for the butcher—namely, the fat Calf, the fat Ox, the fat Lamb, the fat Sheep, the very fat Sheep, and the fat Pig—we have in round numbers, 3 per cent. of mineral matter, 12¼ per cent. of nitrogenous compounds (dry), and 33 per cent. of fat, in their fasted live-weight. The proportion to one another, and the proportion to the whole amount of each contained in the different descriptions of animal, in which the *fat*, and the *nitrogenous substance*, respectively, will probably be consumed as human food, has already been considered in some detail in the proper place. To these points reference will again be made, when calling attention at the end of our Paper to the application of the experimental results as a whole, and to the general conclusions to be drawn from them.

All the experimental evidence which has been adduced conspires to show, that the so-called "*fattening*" of animals for the butcher, is properly so designated. Even the so-called "*store*" or "*lean*" animals are seen to contain as much, or more, of dry fatty substance, than of dry nitrogenous compounds. After the feeding or fattening process, the percentage of the collective dry substance of the body was considerably increased; whilst fatty matter had accumulated in much larger proportion than the nitrogenous compounds. It is obvious, therefore, that in the *increase itself* of the fattening animal, the proportion of fat to the nitrogenous substance of growth, must be *greater* than in the total or standing live-weight of the animal. In other words, the composition of the *increase* in weight of a fattening animal, must show a less percentage of nitrogenous substance, and a higher one both of fat and total dry substance, than that of the whole body of the slaughtered animal. With the decrease in the proportion of bone, moreover, as well as the small accumulation of soft nitrogenous parts, we should also expect the percentage of mineral matter in the *increase* to be very small.

SECTION VII.—ESTIMATED COMPOSITION OF THE INCREASE IN WEIGHT OF FATTENING ANIMALS.

The first and most obvious application of the data provided in the preceding sections, is, to employ them as a means of estimating the composition of the *increase in weight* of an animal whilst passing from one given point of progress to another—as distinguished from the actual composition of the entire body, or its several parts, as furnished by analysis at any one fixed period. So far as the analysed fat Pig is concerned, the result of such a calculation has been already given elsewhere*. It will be interesting, however, to extend the application to numbers of such animals, and also to the equally, or even more important animals of the farm—Oxen and Sheep.

It is obvious, that provided we know the exact composition of an animal when it weighs any given weight, say 100 lbs.—and again, when, after fattening, it has reached another weight, say 150 lbs.—nothing would be easier than to calculate the actual and the percentage composition of the 50 lbs. that has been gained. By deducting the amount of the respective constituents in the 100 lbs. weight, from the amount of the same in the 150 lbs., we should at once ascertain the *actual* amount of each in the 50 lbs. of *increase*. The calculation of the *percentage* composition of the *increase* would then of course be a very simple matter. The practical difficulty obviously rests on the fact, that we cannot know the exact composition of a fattened animal at the time it was put upon fattening food, or when it had reached any given previous weight.

In the case of the store and fat Pigs which were analysed, the two animals selected for experiment were of the same breed and age—indeed of the same litter; of very nearly equal weights; and, so far as competent judges could decide, as nearly as possible alike in all other particulars. One of these animals was killed at once in the *store* condition, and its composition determined. Of the other, the exact increase in weight from this store or lean to the fat condition, as well as the amount and the composition of the food it consumed in gaining it, is known; as also is its composition in the *fattened* state. The application of the data in the manner above supposed, is likely therefore to lead to a pretty trustworthy estimate of the composition of the increase of this particular fattening Pig.

Unfortunately, equally parallel data are not available for calculating the composition of the increase of the other fattened animals analysed. This is the more to be regretted, since, from the results of the Pigs it would appear, that data of this kind, if obtained under duly considered circumstances, are much more directly applicable to the determination of the composition of increase, than we had pre-supposed would be the case. In illustration of the inapplicability of the data provided in regard to the other descriptions of animal, to determine the composition of the *increase* from one condition to another—supposing this to be represented by the difference in weight between the animal which was analysed in the lean, and that which was analysed in the fat condition—it may be observed, that owing to the larger frame, and growing rather than fattening character, of the half-fat Ox analysed, compared with the fatter one, it approached so nearly the actual weight of the latter, as to indicate (if the difference in weight were alone taken as the measure) a very small amount of actual increase; whilst the difference in the composition of the two animals was very considerable. In fact, in the case supposed, the so-calculated *total or gross increase* would be less than the estimated gain in *dry substance of increase alone*: that is to say, it would appear that there had been an actual displacement of water, and replacement of it by a corresponding amount of dry substance. It may be said that the displacement of water, and the replacement of it by fat, in the fattening animal, or in other words a greater increase in *dry substance* than in the *gross live-weight*, is not impossible. The consideration of the results relating to the Pigs, as well as the tendency of observation, comparing animals of this description with others, would, however, militate against such an assumption. For similar reasons to those alluded to in regard to the two Bullocks, the difference in weight between the Sheep analysed in the respective conditions of fatness, cannot be taken as representing the amount of gross increase in weight in passing from the one standard condition to the other. Instead, therefore, of taking the live-weights of the individual animals actually analysed, as the data upon which to calculate the composition of the increase from one condition to another, it will be more appropriate to adopt the known live-weights of considerable numbers of animals, taken first in a store or lean, and afterwards in a fatter condition.

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* Report of the British Association for the Advancement of Science for 1852.

So far as *oxen* are concerned, we take for our illustrations the best experiments on record with which we are acquainted, that show, so far as can be judged, a progression comparable with that implied in the change from the condition of the "half-fat" to that of the "fat Ox" analysed.

In regard to *sheep*, we take the data supplied by published experiments of our own*. In some of these, considerable numbers of animals of different breeds were fed upon similar food; whilst in others, animals otherwise comparable were fed upon different foods.

Lastly, the compositions of the store and of the fat *pig* analysed, have, respectively, been applied to the weights store, and the weights fat, of numbers of pigs fed experimentally—the amounts of whose gross increase in live-weight, together with the particulars of the constituents consumed in food to produce it, have already been published elsewhere†. For the sake of comparison, by the side of the estimates of the compo-

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[*] Journal of the Royal Agricultural Society of England, vol. x. part 1; vol. xii. part 2; vol. xiii. part 1; and vol. xvi. part 1.

[†] Journal of the Royal Agricultural Society of England, vol. xiv. part 2; Reports of the British Association for the Advancement of Science, for 1852 and 1854.

sition of the increase of fattening Pigs so obtained, is adduced that indicated by the direct application of the analytical results to determine the composition of the increase from the store to the fat condition, in the case of the two Pigs actually analysed, which provide the data for application to the other cases.

Table VIII. shows the estimated *percentage composition of the increase* of fattening *oxen.*

Table IX. shows the estimated *percentage composition of the increase* of fattening *sheep.*

Table X. shows the estimated *percentage composition of the increase* of fattening *pigs.*

In each case the *original* and *final* weights, and the *increase* in weight, of the animals are given. The composition to be applied to each in the calculation is also stated. There is always added some description of the food consumed. When at command, as in the case of most of the Sheep, and all the Pigs, the amount of certain constituents of the food which were consumed to produce a given amount of increase, are also given, by the side of the estimated composition of that increase.

TABLE VIII.—Showing the Estimated Percentage Composition of the Increase in Weight of fattening BULLOCKS and HEIFERS.

Note.—*Original* Weight, taken at the Composition of the "Half-fat Ox," analysed.
Final Weight, taken at the Composition of the "Fat Ox," analysed.

General particulars of the experiments.										Calculated percentages in *Increase.*			
Authority.	Description of animal.	Number of animals.	Duration of experiment.		Description of fattening food.	Actual weights (fresh) in lbs.			Increase upon 100 original weight.	Mineral matter (ash).	Nitrogenous compounds (dry).	Fat.	Total dry substance.
			wks.	days.		Original.	Final.	Increase.					
Mr. TEMPLETON* ...	Heifers.	12	18	6	Swedish turnips, hay, and oat straw.	12124	15274	3150	26·0	1·05	6·51	72·5	80·0
Hon. Capt. GREY†...	Bullocks.	50	23¼	0	Oilcake, bean-meal, and turnips.	54796	71470	16674	30·4	1·47	7·68	66·3	75·4
Hon. Capt. GREY†...	Bullocks.	36	26¼	0	Oilcake, bean-meal, and turnips.	41188	54530	13342	32·4	1·62	8·10	64·1	73·8
Average for the 98 animals										1·47	7·69	66·2	75·4

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* Journal of the Royal Agricultural Society of England, vol. xvi. pp. 168–9.

† Gardeners' Chronicle and Agricultural Gazette, pp. 715 and 782 (1852).

Table IX.—Showing the Calculated Percentage Composition

General particulars of the experiments.

Breed.	Number of animals.	Duration.		Description of fattening food.		Actual weights (fresh) in lbs.		
				Given in limited quantities.	Given *ad libitum*.	Original.	Final.	Increase.
Class I.†—*Original* weight taken at the composition of the "store sheep" analysed.								
		wks.	days.					
Cotswolds	46	19	5	Oilcake and clover hay	Swedish turnips.	5511	8439¾	2928¼
Leicesters	40	20	0	Oilcake and clover hay	Swedish turnips.	4053	5835¼	1782¼
Cross-bred wethers...	40	20	0	Oilcake and clover hay	Swedish turnips.	3804	5584	1780
Cross-bred ewes......	40	20	0	Oilcake and clover hay	Swedish turnips.	3650	5350	1700
Hants downs	40	26	0	Oilcake and clover hay	Swedish turnips.	4538	7322¾	2784¾
Sussex downs.........	40	26	0	Oilcake and clover hay	Swedish turnips.	3520	5629	2109
Class II.—*Original* weight taken at the composition of the "fat sheep" analysed.								
Cotswolds	6	34	6	Oilcake and clover hay	Grass, turnips, &c. in the field.	1037	1472	435
Leicesters	7	34	4	Oilcake and clover hay		948	1367	419
Cross-bred wethers...	8	34	4	Oilcake and clover hay		1041	1490	449
Cross-bred ewes......	8	34	4	Oilcake and clover hay		1006	1457	451
Hants downs	8	31	5	Oilcake and clover hay		1411	1897	486
Sussex downs.........	8	31	5	Oilcake and clover hay		1045	1428	383
Class III. (Series 1‡).—*Original* weight taken at the *mean* composition of the "store" and the								
Hants downs	5	13	6	Oilcake	Swedish turnips.	558	688	130
Hants downs	5	13	6	Oats.............................	Swedish turnips.	548	689$\frac{5}{10}$	141$\frac{5}{10}$
Hants downs	5	11	6	Clover chaff....................	Swedish turnips.	558$\frac{5}{10}$	714	155$\frac{5}{10}$
Class IV. (Series 2‡).—*Original* weight taken at the *mean* composition of the "store" and the								
Hants downs	5	19	1	Oilcake	Clover chaff......	607	759$\frac{5}{10}$	152$\frac{5}{10}$
Hants downs	5	19	1	Linseed	Clover chaff......	607	750	143
Hants downs	5	19	1	Barley	Clover chaff......	602	741	139
Hants downs	5	19	1	Malt..............................	Clover chaff......	602	723	121
Class V. (Series 4‡).—*Original* weight taken at the composition of the "store sheep" *plus* *Final* weight taken at the composi-								
Hants downs	4	10	0	Barley (ground)	Mangolds.........	519	600	81
Hants downs	5	10	0	Malt (ground) and malt dust	Mangolds.........	653	758	105
Hants downs	4	10	0	Barley (ground) and *steeped*	Mangolds.........	536	637$\frac{5}{10}$	101$\frac{5}{10}$
Hants downs	4	10	0	Malt (ground and *steeped*) and malt dust	Mangolds.........	560	638	78
Hants downs	5	10	0	Malt (ground) and malt dust	Mangolds.........	666	774	108

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* The amounts of "*mineral matter*" are too high, owing to the adventitious matters retained by the

† Journal of the Royal Agricultural Society of England, vol. xii. part 2; vol. xiii. part 1; and vol. xvi.

of the *Increase in Weight* of fattening SHEEP.

			Consumed to produce 100 *Increase* in live-weight.				Calculated Composition of 100 *Increase* in live-weight.			
Non-nitrogenous substance, to 1 nitrogenous substance, in food.	Increase upon 100 original weight.	Per cent. carcass in fasted live-weight.	Mineral matter (ash).	Nitrogenous compounds (dry).	Non-nitrogenous substance.	Total dry substance.	Mineral matter (ash*).	Nitrogenous compounds (dry).	Non-nitrogenous substance (fat).	Total dry substance.
Final weight taken at the composition of the "fat sheep" analysed.										
3·51	53·1	59·6	53·7	166	582	802	2·14	7·34	67·5	77·0
3·31	44·0	57·2	63·7	187	619	870	2·01	6·34	74·2	82·5
3·31	46·8	58·0	63·6	186	616	866	2·06	6·70	71·8	80·6
3·30	46·6	58·6	63·1	185	610	858	2·05	6·67	72·0	80·7
3·28	61·4	59·5	65·6	187	613	866	2·23	8·01	63·2	73·5
3·26	59·9	58·9	67·2	190	620	877	2·22	7·90	63·9	74·0
					Means …………		2·12	7·16	68·8	78·0
Final weight taken at the composition of the "extra fat sheep" analysed.										
……	39·5	64·1	……	……	……	……	3·13	7·86	70·0	81·0
……	41·0	64·6	……	……	……	……	3·13	8·02	68·7	79·9
……	40·2	64·8	……	……	……	……	3·09	7·95	69·3	80·4
……	42·1	64·3	……	……	……	……	3·10	8·07	68·5	79·6
……	33·2	63·2	……	……	……	……	3·17	7·18	75·3	85·6
……	34·0	63·2	……	……	……	……	3·13	7·41	73·5	84·1
					Means …………		3·12	7·75	70·9	81·8
"fat sheep" analysed. *Final* weight taken at the composition of the "fat sheep" analysed.										
3·89	23·3	56·6	48·1	167	650	865	2·00	6·69	72·0	80·7
6·71	25·8	56·5	37·0	102	684	823	2·12	7·21	68·5	77·8
7·21	27·8	53·3	55·0	102	736	893	2·19	7·59	66·1	75·7
					Means …………		2·10	7·16	68·8	78·1
"fat sheep" analysed. *Final* weight taken at the composition of the "fat sheep" analysed.										
3·44	25·1	56·6	124	321	1103	1548	2·10	7·08	69·4	78·5
3·96	23·6	57·5	116	289	1144	1549	2·10	6·71	71·6	80·3
5·40	23·1	58·5	115	235	1269	1619	2·01	6·62	72·4	81·0
5·48	20·1	59·2	130	266	1458	1854	1·90	5·78	77·8	85·4
					Means …………		2·03	6·55	72·8	81·3
two-thirds of the difference between the "store" and "fat sheep" analysed. tion of the "fat sheep" analysed.										
6·20	15·6	58·3	55·3	118	732	905	2·10	6·67	71·8	80·6
6·10	16·1	57·9	52·0	111	677	840	2·10	6·86	70·7	79·7
6·03	18·9	57·1	58·8	121	730	909	2·17	7·68	65·4	75·3
6·04	13·9	58·6	65·0	136	822	1023	1·92	5·90	76·1	84·2
6·11	16·2	55·9	58·9	127	776	962	2·04	6·94	70·5	79·4
					Means …………		2·07	6·81	70·9	79·8
					General means of all …………		2·34	7·18	70·4	79·9

wool; the numbers for "Class II." will be the most excessive from this cause.

part 1. ‡ Journal of the Royal Agricultural Society of England, vol. x. part 1.

TABLE X.—Showing the Calculated Percentage Com-

Note.—In all cases, *Original* Weight taken at the Composition of the "Store Pig"

General particulars of the experiments.								
Pens.	No. of animals.	Duration.	Description of fattening food.		Actual weights (fresh), in lbs.			Total non-nitrogenous substance, to 1 nitrogenous substance, in food.
			Given in limited quantity.	Given *ad libitum*.	Original.	Final.	Increase.	
						The "Store" and "Fat		
		wks. dys.						
*	1	......	....	....	......	......	......	......
	1	10 0	None	Bran one part, bean and lentil meal two parts, and barley meal three parts	103	191	88	3·57
						Series 1. (For further particulars, see Journal of the		
1	3	8 0	None	Bean and lentil meal......	440	743	303	1·99
2	3		Indian meal	Bean and lentil meal......	422	758	336	2·43
4	3		Indian meal and bran...	Bean and lentil meal......	427	679	252	2·91
5	3		None	Indian meal	431	652	221	6·61
6	3		Bean and lentil meal ...	Indian meal	445	743	298	4·65
7	3		Bran	Indian meal	415	724	309	5·69
8	3		Bean and lentil meal, and bran	Indian meal	432	779	347	4·26
12	3		None	Bean and lentil meal, Indian meal and bran, each *ad libitum*	429	685	256	3·28
	24			Means	3441	5763	2322	3·48
						Series 2. (For further particulars, see Journal of the		
1	3	8 0	None	Bean and lentil meal......	433	628	195	2·17
2	3		Barley meal..............	Bean and lentil meal......	446	730	284	2·72
3	3		Bran......................	Bean and lentil meal......	405	647	242	2·29
4	3		Barley meal and bran...	Bean and lentil meal......	431	671	240	3·04
5	3		None	Barley meal	448	739	291	6·02
6	3		Bean and lentil meal ...	Barley meal	428	679	251	3·87
7	3		Bran	Barley meal	426	703	277	5·71
8	3		Bean and lentil meal, and bran	Barley meal	419	606	187	3·71
9 and 10	6		None	Mixture of one part bran, two parts barley meal, and three parts bean and lentil meal ..	841	1377	536	3·10
11 and 12	6		None	Mixture of one part bran, two parts bean and lentil meal, and three parts barley meal	827	1444	617	3·66
	36			Means	5104	8224	3120	3·37
						Series 3. (For further particulars, see Journal of the		
1	4	8 0	Dried cod-fish	Bran and Indian meal (equal parts)	632	955	323	3·13
2	4		Dried cod-fish	Indian meal	647	1036	389	3·80
	8			Means	1279	1991	712	3·37
						Series 4. (For further particulars, see Report of the British		
1	3	10 0	Lentil meal and bran ...	Sugar	286	533	247	4·06
2	3		Lentil meal and bran ...	Starch	285	533	248	4·06
3	3		Lentil meal and bran ...	Sugar and starch	281	555	272	4·71
4	3		None....................	Lentils, bran, sugar, starch, each *ad libitum*	292	604	312	3·90
	12			Means	1144	2223	1079	4·17
				General means ...	......	......	......	......

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* These figures are somewhat corrected from those given in the Report of the British Association for the as follows:—Mineral matter 0·43, nitrogen 1·88 (equal about 8·88

position of the *Increase in Weight* of Fattening Pigs.

analysed. *Final* Weight taken at the Composition of the "Fat Pig" analysed.

		Consumed to produce 100 *Increase* in live-weight.					Calculated Composition of 100 *Increase* in live-weight.			
Increase upon 100 original weight.	Per cent. carcass in fasted live-weight.	Mineral matter (ash).	Nitrogenous compounds (dry).	Total non-nitrogenous substance (including fat).	Fatty matter.	Total dry substance	Mineral matter (ash).	Nitrogenous compounds (dry).	Non-nitrogenous substance (fat)	Total dry substance.
Pig" actually analysed.										
...... 85·4	75·7 82·8	19·9	100	358	15·6	478	0·53*	7·76*	63·1*	71·4*
Royal Agricultural Society of England, vol. xiv. part 2.)										
68·9	81·9	24·1	138	275	11·2	437	0·16	6·73	69·6	76·5
79·6	83·0	19·2	114	278	13·8	412	0·36	7·29	65·9	73·6
59·0	82·2	24·2	120	351	20·5	496	—0·07	6·03	74·2	80·1
51·3	85·4	17·3	57	378	26·3	452	—0·36	5·29	79·0	84·0
67·0	84·4	10·2	72	337	21·7	420	0·10	6·61	70·4	77·1
74·5	83·7	11·0	58	333	22·5	401	0·26	7·02	67·5	74·9
80·3	83·5	13·8	73	309	21·4	396	0·37	7·32	65·7	73·4
59·7	83·9	17·7	107	350	20·4	474	—0·04	6·05	73·9	79·8
67·5	83·5	17·0	93	323	19·5	433	0·09	6·54	70·8	77·4
Royal Agricultural Society of England, vol. xiv. part 2.)										
45·0	......	20·8	146	317	10·5	484	—0·66	4·56	84·1	88·0
63·7	......	21·3	137	374	12·5	533	0·03	6·37	71·9	78·3
59·7	......	25·4	152	348	13·5	525	—0·04	6·07	73·8	79·8
55·7	......	22·1	125	378	14·8	525	—0·17	5·71	76·1	81·7
64·9	......	12·2	64	385	12·4	461	0·07	6·46	71·2	77·8
58·6	......	15·0	91	352	12·0	459	—0·08	5·98	74·4	80·3
65·0	......	14·7	66	378	14·1	460	0·07	6·46	71·3	77·8
44·6	......	14·8	100	372	14·6	491	—0·64	4·49	84·4	88·2
63·7	......	21·0	113	351	14·5	486	0·06	6·38	71·8	78·3
74·6	......	17·4	87	320	13·1	425	0·27	7·05	67·4	74·8
61·1	......	18·7	105	354	13·3	478	—0·10	5·95	74·6	80·5
Royal Agricultural Society of England, vol. xiv. part 2.)										
51·1	84·6	34·9	104	326	25·1	464	—0·37	5·26	79·1	84·1
60·1	87·3	19·7	75	287	20·9	382	—0·05	6·12	73·6	79·7
55·7	86·0	26·6	90	303	22·8	419	—0·21	5·69	76·3	81·8
Association for the Advancement of Science for 1854.)										
86·4	83·1	15·8	81	330	......	427	0·48	7·53	64·1	72·1
87·0	80·1	15·2	81	329	......	425	0·48	7·58	63·9	72·0
96·8	81·7	14·5	74	351	......	439	0·58	7·98	62·0	70·6
106·8	80·8	14·7	82	320	......	417	0·70	8·17	59·9	68·8
94·3	81·4	15·0	80	332	......	427	0·56	7·81	62·5	70·9
......	......	......	......	......	......	......	0·06	6·44	71·5	78·0

Advancement of Science for 1852, where the composition of the increase of this analysed "fat pig" is given nitrogenous compounds), 68·4 fat, and 71·8 total dry substance.

It is obvious, that the correctness of the indications of these Tables will entirely depend upon the appropriateness of the composition of the animals actually analysed in different conditions of maturity, to represent that of the animals (in their respective conditions), to which the direct analytical data are to be applied. The results must, therefore, be only looked upon as approximations; though, so far as we believe, the data now supplied constitute the most reliable basis for estimates of this kind at present at command Indeed, in corroboration of the probable general correctness of the indications, it may be remarked, that, as fattened animals are seen to contain a much larger proportion of dry substance than leaner ones, and as their dry substance contains a larger proportion of fat, and less of nitrogenous and mineral matters, it is clear, that the *increase itself* must contain a higher percentage of total dry substance and of fatty matter, and less of nitrogenous and mineral matters, than the entire body of the fattened animal. The estimates of the composition of *increase* recorded in the Tables agree in fact very well with what we might anticipate; and they are, with some slight qualifications, in the main consistent with the *direct* results arrived at, as to the composition of the increase of the individual fattened *pig.* It should be remarked that the compositions of the animals analysed, which form the basis of these estimates of the composition of increase, are given (see Table VII.) on the *fasted* live-weight, so as to eliminate as far as possible the influence of the variable amount of contents of stomachs and intestines The *percentages* thus calculated to the *fasted live-weight*, are, however, for the purposes of the Tables now under consideration, applied to the original and final weights of animals, in the *un*fasted condition. The tendency of the correction due on this head, would be slightly to *reduce* our estimated percentages of *fat*, and of *total dry matter*, and slightly to *raise* those for the *nitrogenous* and *mineral matters.*

With regard to *oxen*, it was difficult to find the record of experiments, in which the animals had been fed over a period of time sufficiently long to represent a change in condition equal to that assumed between the Bullock taken as "half-fat," and that analysed as "fat." As is seen, in Table VIII., the calculation has, in all, been made for ninety-eight animals; twelve of which were fed for nearly nineteen weeks; fifty for 23¾ weeks, and thirty-six for 26¾ weeks. It is probable that the estimate is the most nearly correct for the thirty-six animals, whose period under experiment was the longest, and whose proportion of increase upon 100 of their original weight was the highest. The mean of all the ninety-eight animals gives for the *composition of the increase* 75·4 per cent. of total dry substance, of which 66·2 was fat, 7·67 dry nitrogenous compounds, and 1·47 mineral matter. These figures may, perhaps, be taken as pretty nearly representing the average composition of the increase over the concluding period of half a year or more, of animals well fed on fattening food, and brought at last to a fair condition of maturity and fatness. In passing a judgment as to the probable direction of their error, we should say, that the fat and total dry matter are more likely to be given somewhat too high, and the nitrogenous matter somewhat too low.

[illegible] the composition of the increase has been calculated for 348 animals, in lots of never less than four, and in some cases forty or more. These animals were all carefully selected for the purposes of experiment; their weights were accurately taken at the different periods; and, in most cases, the amount and the composition of the food they consumed were determined. The compositions applied in the calculations to the *original* and *final* weights respectively, of the different lots, are adopted, or deduced, from those of the Sheep actually analysed, according to the reputed condition of the animals at the commencement and the conclusion of the feeding experiments. In Class I. (see Table IX.), large numbers of animals were fed for a considerable period of time, from a fair "*store*," to a fair "*fat*" condition. Accordingly, the composition of the "store Sheep" analysed, is applied to the original weights, and that of the "fat Sheep" to their final weights. In Class II. the animals were fed from the "*fat*" to a "*very fat*" condition. In these cases, therefore, the original weights are calculated at the composition of the "fat Sheep," and the final weights at that of the "extra-fat Sheep." In Classes III. and IV. the animals were taken in a partially fattened condition, and fed to that of moderate fatness. The per cent. of *carcass in fasted live-weight* as given in the Table, shows pretty well the comparative *final* condition of the different lots; and this was obviously not very widely different, in Classes I., III., IV., and V. In Class III., however, the period of the feeding experiment was comparatively short; and in Class IV., though the period was longer, the food was not so well adapted; so that, in both these cases, the proportion of *increase to the original weight* is seen to average only about half as much as in Class I. Owing to the circumstances here enumerated, the composition of the *original* weights of the Sheep of Classes III. and IV. is taken at the *mean* between that of the "store," and that of the "fat Sheep," analysed; and that of their final weights at the composition of the "fat Sheep" itself. From considerations of a similar nature, in Class V. the original weights are taken at a composition between that of the "store" and that of the "fat Sheep"—but supposed to be *two-thirds*, instead of only one-half, advanced towards the fatter state. The final weights are, as in most of the cases, taken at the composition of the "fat Sheep" analysed.

It will not, of course, for a moment be supposed, that the differences indicated in Table IX., between the composition of the increase of the animals of *different breeds*, or those fed on *different foods*, are really to be attributed to the variations in those conditions as there described. It is enough to claim, that the results, as a whole, give us the best indication of the probable composition of the increase of fattening Sheep, at present at command.

To go to the figures (see Table IX.), the increase of fattening *Sheep* appeared to contain from 2 to 3 per cent. of *mineral matter*. Either of these estimates is, however, undoubtedly too high. The error is due to the amount of adventitious mineral matter in the ash of the wool, as before referred to, which happened to be the greater in that of the fatter animals. It was still thought better to record the numbers in the Table as calculation gave them, as it would thus be seen (with the explanation given) *below* what amounts the truth must really fall. In fact, excluding altogether from the calculations the ash of the wool, the percentage of mineral matter in the increase would appear to be certainly under 2, and sometimes under 1½ per cent., for the cases of the fattening Sheep given in the Table.

The average estimated percentage of *nitrogenous compounds* in the increase of the fattening Bullocks, is 7·69; that for the fattening Sheep is, as would be expected, somewhat less, namely, 7·13. The average estimated percentage of *fat* in the increase of the Oxen is 66·2; whilst that in the Sheep is 70·4. The *direction* of the difference is, here again, that which would be anticipated. Finally, the estimated percentage of *total dry substance* in the increase of the Oxen, was 75·4; and that for the Sheep, was 79·9. Granting that the estimates for the composition of the increase of the Sheep, like those for that of the Oxen, are more probably too high than too low, still there can be little doubt, that, under at all comparable conditions, the increase of the fattening Sheep would contain a somewhat less proportion of nitrogenous matter, and a somewhat larger one of both fat and total dry substance, than that of Oxen. On the other hand, common observation would lead to the supposition, that the increase of the fattening Pig would be less nitrogenous, and contain both more fat and more total dry substance, than that of the Sheep.

In Table X. are given the calculated estimates of the composition of the *increase* of about eighty fattening Pigs—divided into lots of three, four, or six animals each. The composition of the increase of the analysed "fat Pig" (given at the top of the Table for the sake of comparison with the other estimates), shows 0·53* per cent. of mineral matter, 7·76 per cent. of nitrogenous compounds, 63·1 per cent. of fat, and in all, 71·4 per cent. of total dry substance. Against these numbers, which undoubtedly represent the truth very closely for the particular case in question, the average of all the other estimates in the Table gives 0·06 per cent. of mineral matter, 6·44 per cent. of nitrogenous compounds, 71·5 per cent. of fat, and 78·0 per cent. of total dry substance. We have then, in the average of the estimated composition of the increase of these numerous fattening Pigs, rather less mineral matter and nitrogenous compounds, and several per cent. more fat and total dry substance, than in that of the single analysed "fat Pig." Most of the animals, the composition of whose increase is thus estimated, were, however, in a somewhat further advanced condition, both at the commencement and the conclusion of the experiment, than the single analysed "fat Pig." Some evidence of this is to be found in the relation of the "original" weights, and of the *percentages of carcass in fasted live-weight*, as recorded in the Table. It would therefore in all probability be really the case, that in the average of the instances brought under the calculation, the *increase* would contain a less proportion of both mineral and nitrogenous matter, and a larger one of both fat and total dry substance, than that of the single analysed Pig. With regard to the amount of mineral matter in the increase, attention should be called to the fact, that, according to the figures in the Table, it was always very small; whilst, in many cases, there was *apparently* no increase whatever, but even a loss of mineral matter during the fattening process. From the general character and habits of the animal, and its known tendency to fatten rather than to grow, we should indeed anticipate that the bony frame-work, which is the chief storehouse of mineral matter, would develop proportionally much less in the fattening Pig, than in either fattening Sheep or Oxen. Still, it would be hardly safe to assume, upon the evidence of the analysis of two animals alone, that there would frequently be an actual reduction of the total mineral matter of the body, during the fattening period. The alternative is to suppose, that the analysed fattened Pig was of rather lighter frame, than should have been, for strict comparison with the analysed leaner one.

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* These figures are somewhat corrected from those given in the Report of the British Association for the Advancement of Science for 1852, where the composition of the increase of this analysed "fat pig" is given as follows:—mineral matter 0·48, nitrogen 1·88 (equal about 8·88 nitrogenous compounds), 68 4 fat, and 71·8 total dry substance.

The following is a Summary of these numerous estimates of *the composition of the increase of fattening oxen, sheep, and pigs*:—

TABLE XI.—Summary of the Estimated Composition of the *Increase* of fattening Oxen, Sheep, and Pigs.

Cases.	Calculated composition of 100 *Increase* whilst fattening.			
	Mineral matter.	Nitrogenous compounds (dry).	Fat.	Total dry substance.
Average for 98 Oxen	1·47	7·69	66·2	75·4
Average for 348 Sheep	2·34*	7·13	70·4	79·9
Average for 80 Pigs	0·06	6·44	71·5	78·0
The analysed fat Pig	0·53	7·76	63·1	71·4
Mean	1·10	7·26	67·8	76·2

It would appear, that we may probably estimate the *increase* in weight of *liberally fed Oxen*, over six months or more of the final fattening period, to contain from 70 to 75 per cent. of its weight of total dry substance. Of this, by far the larger proportion, say 60 to 65 parts, will be fat; 7 to 8 parts will be nitrogenous substance; and 1 to 1½ part mineral matter.

On the same plan of calculation, the final *increase* of *well-fed Sheep*, fattening during several months, will probably consist of 75 per cent., or more, of total dry substance; of which 65 to 70 parts will be fat; 7 to 8 parts nitrogenous compounds; and (making allowance for the error in the ash of the wool) perhaps about 1½ part of mineral matter.

The *increase* of *Pigs fed for fresh pork*, during the final two or three months on fattening food, may be taken at about 70 to 75 per cent. total dry substance, 63 to 68 per cent. of fat, 6 to 8 per cent. nitrogenous substance, and considerably less than 1 per cent. of mineral matter. The increase over the last few months of high feeding, of *Pigs fed for curing*, will however contain *higher* percentages of both fat and total dry substance, and lower ones of both nitrogenous compounds and mineral matter, than that of the more moderately fattened animal.

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* Probably from 0·5 to 1·0 per cent. too high; owing to the amount of adventitious matters in the wool of the sheep analysed—particularly the fatter ones.

From the whole of the evidence the striking fact appears, that about three-fourths of the gross increase in live-weight of animals "feeding" for the butcher, will be dry or solid matter of some kind. About two-thirds of the gross increase will be pure fat. Only about 7 or 8 per cent. of the gross increase, and scarcely more than one-tenth of its total dry substance, will be nitrogenous compounds. Lastly, such increase may frequently contain less than 1, and seldom more than 1½ per cent. of mineral matter.

SECTION VIII.—RELATION OF THE CONSTITUENTS STORED UP IN THE INCREASE, TO THOSE CONSUMED IN THE FOOD, BY FATTENING ANIMALS.

1. *Amounts of Mineral Matter, Nitrogenous Compounds, Non-nitrogenous substance, and Total Dry Substance, stored up in Increase, for* 100 *of each, consumed in Food.*

Having now arrived at approximate estimates of the *composition of the increase* accumulated by certain animals, during the final fattening period, it will be interesting to consider the probable *relation of the constituents so stored up in the increase, to those consumed in the food which produced it.* In the cases of most of the Sheep, and of all the Pigs, to which Tables IX. and X. respectively refer, the amounts of certain of the most important constituents of the food, which were consumed to produce a given weight of the increase whose composition is there estimated, had previously been determined, and are, for the sake of reference, recorded in the Tables, by the side of the estimated composition of the increase itself which was due to their consumption: that is to say, by the side of the estimated amounts of mineral matter, of nitrogenous compounds, of non-nitrogenous organic substance, and of total dry substance, respectively, contained in 100 lbs. of the increase in live-weight, there is recorded in the Tables, the amount of each of these consumed in the production of that 100 lbs. of increase. We have thus the easy means of estimating the proportion of each of these classes of constituents stored up in the increase, for 100 parts of the same consumed in the fattening food. The results of such a calculation are given in Table XII. for the different lots of Sheep, and in Table XIII. for the different lots of Pigs.

TABLE XII.—Showing the Estimated Proportion of certain constituents stored up in the *Increase of Weight* of Fattening SHEEP, for 100 of each, consumed in food.

General particulars of the experiments.					Amount of each class of constituents stored up in increase for 100 of the same consumed in food.			
Breed.	Number of animals.	Duration.	Description of fattening food.		Mineral matter (ash) *.	Nitrogenous compounds (dry).	Non-nitrogenous substance.	Total dry substance.
			Given in limited quantity.	Given *ad libitum*.				
Class I. (For data, see Class I. Table IX.)								
		wks days.						
Cotswolds	46	19 5	Oilcake and clover hay	Swedish turnips	3·98	4·43	11·6	9·60
Leicesters	40	20 0	Oilcake and clover hay	Swedish turnips	3·15	3·30	12·0	9·48
Cross-bred wethers	40	20 0	Oilcake and clover hay	Swedish turnips	3·24	3·60	11·6	9·31
Cross-bred ewes	40	20 0	Oilcake and clover hay	Swedish turnips	3·25	3·60	11·8	9·40
Hants downs	40	26 0	Oilcake and clover hay	Swedish turnips	3·40	4·28	10·3	8·49
Sussex downs..	40	26 0	Oilcake and clover hay	Swedish turnips	3·30	4·16	10·3	8·14
				Means	3·39	3·91	11·3	9·12
Class III. (For data, see Class III. Table IX.)								
Hants downs	5	13 6	Oilcake	Swedish turnips	4·16	4·01	11·1	9·33
Hants downs	5	13 6	Oats	Swedish turnips	5·73	7·07	10·0	9·45
Hants downs	5	13 6	Clover chaff.	Swedish turnips	3·98	7·44	9·0	8·19
				Means . ..	4·62	6·17	10·0	9·00
Class IV. (For data, see Class IV. Table IX.)								
Hants downs	5	19 1	Oilcake	Clover chaff......	1·00	2·20	6·3	5·07
Hants downs	5	19 1	Linseed	Clover chaff......	1·81	2·32	6·2	5·19
Hants downs	5	19 1	Barley	Clover chaff......	1·75	2·82	5·7	5·00
Hants downs	5	19 1	Malt.................................	Clover chaff......	1·46	2·17	5·3	4·61
				Means	1·68	2·38	5·0	4·97
Class V. (For data, see Class V. Table IX.)								
Hants downs	4	10 0	Barley (ground)	Mangolds.........	3·80	5·65	9·8	8·91
Hants downs	5	10 0	Malt (ground) and malt dust...	Mangolds.........	4·04	6·18	10·4	9·49
Hants downs	4	10 0	Barley (ground and *steeped*) ...	Mangolds.........	3·72	6·35	8·0	8·28
Hants downs	4	10 0	Malt (ground and *steeped*) and malt dust	Mangolds.........	2·95	4·34	9·3	8·23
Hants downs	5	10 0	Malt (ground) and malt dust...	Mangolds.........	3·46	5·46	9·1	8·25
				Means	3·59	5·60	9·5	8·63
				General means...	3·27	4·41	9·4	8·06

* The amounts of "*mineral matter*" are too high, owing to the adventitious matters retained by the wool.

TABLE XIII.—Showing the Estimated Proportion of certain constituents stored up in the *Increase of Weight* of Fattening PIGS, for 100 of each, consumed in food.

General particulars of the experiments.					Amount of each class of constituents stored up in increase for 100 of the same consumed in food.				
Pens.	Number of animals.	Duration.	Description of fattening food. Given in limited quantities.	Description of fattening food. Given *ad libitum*.	Mineral matter (ash).	Nitrogenous compounds (dry).	Non-nitrogenous substance.	Total dry substance.	Fat.
The "Fat Pig" analysed.									
	1	wks. days. 10 0	None	Bran one part, bean and lentil meal two parts, and barley meal three parts	2·66	7·76	17·6	14·9	405
Series 1[*] (For *data*, see Series 1. Table X.).									
1	3	8 0	None	Bean and lentil meal	0·68	4·88	25·3	17·5	621
2	3		Indian meal . ..	Bean and lentil meal	1·86	6·39	23·7	17·0	477
4	3		Indian meal and bran	Bean and lentil meal	−0·33	5·02	21·1	16·1	362
5	3		None	Indian meal	−2·09	9·28	20·9	18·6	300
6	3		Bean and lentil meal	Indian meal	0·90	9·18	20·9	18·4	324
7	3		Bran	Indian meal	2·35	12·10	20·3	18·7	300
8	3	......	Bean and lentil meal, and bran	Indian meal	2·71	10·03	21·3	18·5	307
12	3		None	Bean and lentil meal, Indian meal and bran, each *ad libitum*	−0·22	5·65	21·1	16·8	362
				Means	0·74	7·82	21·8	17·8	382
Series 2[*] (For *data*, see Series 2. Table X.).									
1	3	8 0	None	Bean and lentil meal	−3·20	3·12	26·5	18·2	801
2	3		Barley meal..........	Bean and lentil meal	0·16	4·65	19·2	14·7	575
3	3		Bran	Bean and lentil meal	−0·16	3·09	21·2	15·2	547
4	3		Barley meal and bran	Bean and lentil meal	−0·75	4·57	20·1	15·6	514
5	3		None	Barley meal..	0·56	10·09	18·5	16·9	574
6	3		Bean and lentil meal	Barley meal............	−0·53	6·57	21·1	17·5	620
7	3		Bran	Barley meal..	0·49	9·79	18·9	16·9	506
8	3		Bean and lentil meal, and bran	Barley meal..	−4·33	4·49	22·7	18·0	578
9 and 10	3	......	None	Mixture of one part bran, two parts barley meal, and three parts bean and lentil meal	0·27	5·65	20·4	16·1	405
11 and 12	3	......	None	Mixture of one part bran, two parts bean and lentil meal, and three parts barley meal	1·58	8·10	21·1	17·6	515
				Means	−0·59	6·10	21·0	16·7	572
Series 3[*] (For *data*, see Series 3. Table X.).									
1	4	8 0	Dried cod-fish........	Bran and Indian meal (equal parts)	−1·06	5·06	24·3	18·1	315
2	4		Dried cod-fish........	Indian meal..........................	−0·26	8·16	25·6	20·9	352
				Means	−0·66	6·61	24·9	19·5	333
Series 4† (For data, see Series 4. Table X.).									
1	3	10 0	Lentil meal and bran	Sugar	3·07	9·30	19·4	16·9	
2	3		Lentil meal and bran	Starch............................	3·18	9·36	19·4	16·0	
3	3		Lentil meal and bran	Sugar and starch	4·06	10·78	17·7	16·1	
4	3		None	Lentils, bran, sugar, starch, each *ad libitum*	4·80	9·06	18·7	16·5	
				Means	3·78	9·85	18·8	16·6	
				General means	0·58	7·84	21·2	17·3	472

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[*] Journal of the Royal Agricultural Society of England, vol. xiv. part 2.

† Report of the British Association for the Advancement of Science for 1854.

It will be observed, by reference to the columns in Tables IX. and X. respectively, which show the proportion of the total *non-nitrogenous* to the total *nitrogenous* constituents of the food, that it was in some cases nearly double as much as in others. It might be urged, therefore, that it was quite irrelevant to apply one and the same composition to the final weights of animals fattened on foods differing so widely in this respect. It is not denied, that, other things being equal, a highly nitrogenous food may give some tendency to a greater proportion of increase in frame and flesh; but all observation would lead to the conclusion, that, at least with animals fattening under ordinary conditions, this would happen but in a very limited degree; in fact, by no means in anything like a numerical proportion to the increased relation of the nitrogenous to the non-nitrogenous constituents of the food. It has been found, indeed, that as our current fattening food-stuffs go, the increase in weight is more in proportion to the amount of digestible *non-nitrogenous*, or *total dry organic substance*, than to that of the nitrogenous compounds consumed. And, although with a high proportion of available *non*-nitrogenous matter in the food there is a somewhat less tendency to increase in frame, and a greater one to fatten, yet animals which have been fed on very highly nitrogenous food, though as a rule they have appeared to grow somewhat more, have nevertheless frequently been extremely fat. Upon the whole then it is concluded, that the relation of the nitrogenous matter to the fat, in the *increase* of the fattening animal, is by no means increased in the degree which might be expected, by a considerable increase in the proportion of the nitrogenous to the non-nitrogenous compounds in the food. The proportion of the nitrogenous matters in the increase is, there is little doubt, much more affected by the age and habits of the animal than by the proportion (if not below a certain limit) of the nitrogenous constituents in the food. From these considerations, and owing to the comparatively small proportion of the several constituents of the food actually stored up and retained in the increase, any error arising from adopting the same composition for the final weights of animals fattened on very various foods, will be immaterial in forming general and average estimates of the proportion of the constituents stored up in the increase, to those consumed in the food. With these explanations then, and calling attention to the reservations which they obviously imply, we adopt as they stand, for the basis of our calculations, the records of constituents actually consumed, and the estimates of the composition of the increase produced as given in Tables IX. and X. respectively, and proceed at once to consider the indications so obtained.

From Table XII. it is seen, that taking the average of the numerous experiments with Sheep, rather more than 3 per cent. of the *total mineral matter* consumed in the fattening food would appear to be retained in the increase. Assuming the due correction made for the extraneous mineral matter in the wool of the fat animals analysed, the average of the cases in question would show rather less than 3 per cent. of the mineral matter consumed, to be stored up in the increase. In Class IV. dry food alone was given, and such as contained a large proportion of mineral matter to digestible organic substance. In this case, therefore, the proportion of the consumed mineral matter which appears to be stored up in increase, is relatively very small—namely, only 1·08 per cent. The other Classes, however, in which there was a limited proportion of dry food, and the remainder consisted of succulent roots, much more nearly represent the usual conditions of the food of fattening Sheep. Upon the whole, it may be concluded, as an average estimate for Sheep fattening for the butcher on good mixed diet of dry and succulent food, that they will certainly not carry off more, and perhaps frequently less, than 3 per cent. of the consumed mineral matter. Were it not indeed that Sheep are now generally fattened when still young and growing, the proportion of the mineral matter consumed which would be retained during the so-called fattening period, would probably be extremely small. In fact, it can hardly be greater, on the average, than above supposed, taking the whole period of existence of the animal. But it is obvious, that the proportion will depend much more on the character of the food, as to the quantitative relation of its mineral matter to its available organic substance, than upon any other circumstance. At any rate, the proportion of the mineral matter consumed by either store or fattened animals, which is sent off the farm in their bodies, is comparatively small; and from the percentage indicated in the *live-weight* of the animals in the different conditions, as given in Table VII., the annual exhaustion of the farm from the sale of known weights of animals is a matter of easy calculation.

Table XII. shows the estimated proportion of the total *nitrogenous compounds* retained in the increase of the fattening Sheep, to have been, on the average, less than 5 per cent. of that consumed in the food. Assuming a liberal mixed diet of succulent roots and dry food, it is probable that when the latter consists chiefly of pulse, oilcake, or other highly nitrogenous matter, the proportion of the nitrogen consumed which will be carried off in the increase of animal, will be less than 5, and perhaps even less than 4 per cent. On the other hand, when the dry food consists to any great extent of cereal grain or other food containing a comparatively low percentage of nitrogen, it is probable that more than 5 per cent. of the total nitrogen consumed will be carried off in increase. It will be observed, that on either supposition, the proportion of nitrogen expired, perspired, or voided, will be considerably more than 90 per cent., whilst it may be more than 95 per cent., of the total nitrogen consumed by the fattening Sheep.

It appears that for 100 parts of *non*-nitrogenous substance consumed in food, there were on the average (excluding Class IV.) about 10 stored up in the fattening sheep, in the form of *fat itself*.

For 100 of *total dry substance* of the food, about 8 or 9 of dry substance would appear to be stored up in the increase of the animal. It will be remembered, that in the dry substance of the food of the Sheep, there is, compared with that of the Pig, a considerably larger amount of indigestible woody fibre. There is, therefore, a larger proportion of the consumed food necessarily at once effete.

Table XIII., which relates to the *Pigs*, would show that there is probably fully twice as much dry substance stored up for 100 consumed, as in the case of Sheep. The average of *all* the estimates shows 17·3 per cent. of the consumed dry substance stored up in the increase, against about 15 per cent. in the case of the *individual analysed fat pig.* As just stated, there were, in the case of the Sheep, only about 9 parts of dry substance stored up in increase, for 100 total dry substance consumed.

For 100 of *non*-nitrogenous constituents of food, the Pigs would seem to have stored up 20 or more of *fat*, whilst the Sheep yielded only half that amount.

Of *nitrogenous* compounds again, it would appear, according to the estimates, that there was on the average about one and a half time as much of the whole consumed stored up in the increase, as in the case of Sheep. The *average* of the estimates for the Pigs, shows 7·34 per cent. of the consumed nitrogen stored up, against 7·6 per cent. in the case of the *analysed fat pig.* The greater the proportion of pulse in the fattening food of the Pig, the smaller will be the *proportion* of the whole nitrogen consumed, which will be stored up in the increase. And on the other hand, the larger the proportion of cereal, with its comparatively small percentage of nitrogen, the larger will be the *proportion* of the whole carried off in the increase in weight of the animals. The evidence at command would lead to the belief, however, that there is almost uniformly less than 10 per cent., and sometimes perhaps as little as 5 per cent. of the nitrogen of the food of the *fattening pig* carried off in its increase.

It has already been pointed out, how small, in all probability, was the percentage of *mineral matter* in the increase of the rapidly fattening Pig. Reasons were given for supposing, however, that our estimates might show it to be lower than really was the case. There is little use therefore in examining at all closely results which are based upon those doubtful estimates. Moreover as the mineral matter in the food varies very much indeed in its proportion to those constituents which prominently rule the amount and character of the increase, the proportion of the mineral matter consumed by the fattening Pig, as well as the fattening Sheep, which will be stored up in the increase, will be much more variable than that of the other constituents. It is sufficient to say, that in the case of fattening Pigs at least, the proportion of the consumed mineral matter which will not be reclaimed in the manure is almost immaterial.

Finally, in regard to the results of Table XIII., it appears that for every 100 parts of *fatty matter* in the food there were probably, on the average (depending on the character of the food), 400 to 500 parts of *fat* stored up in the increase of the animal. It is obvious, therefore, that there was a *formation* of fat in the animal body, from some other constituent or constituents of the food. To this point we shall recur presently.

2. *Amounts of Mineral Matter, Nitrogenous Compounds, Fat, and Total Dry Substance stored up in Increase, and of matter expired, perspired, or voided, for* 100 *of Collective Dry Substance consumed in Food.*

Having by means of Tables XII. and XIII. shown the probable proportion of each of certain constituents of the food of fattening Sheep and Pigs, which will be stored up in the increase of the fattening animals for 100 of the same consumed in the food, it will be well to follow up the illustration by showing, on the same basis of calculation, how much of the several constituents would be stored up in the increase *for* 100 *of the collective dry substance of the foods consumed*; and lastly, how much of the whole would be *expired, perspired*, or *voided*. These particulars are shown in Table XIV. for the different sets of Sheep, and in Table XV. for the different sets of Pigs.

TABLE XIV.—Showing the Final Distribution of the Constituents of the Food consumed by fattening SHEEP.

General particulars of the experiment.					100 dry matter of Food gave—				
					Stored up in Increase.				
			Description of fattening food.						
Breed.	Number of animals.	Duration.	Given in limited quantity.	Given *ad libitum*.	Mineral matter (ash*).	Nitrogenous compounds (dry).	Non-nitrogenous substance (fat).	Total dry Increase.	Expired, perspired, or voided.
Class I. (For data, see Class I. Table IX.)									
		wks. days.							
Cotswolds	46	19 5	Oilcake and clover hay	Swedish turnips	0·26	0·92	8·41	9·60	90·40
Leicesters	40	20 0	Oilcake and clover hay	Swedish turnips	0·23	0·73	8·53	9·48	90·52
Cross-bred wethers...	40	20 0	Oilcake and clover hay	Swedish turnips	0·21	0·77	8·29	9·31	90·69
Cross-bred ewes......	40	20 0	Oilcake and clover hay	Swedish turnips	0·24	0·78	8·39	9·41	90·59
Hants downs	40	26 0	Oilcake and clover hay	Swedish turnips	0·26	0·93	7·30	8·49	91·51
Sussex downs..........	40	26 0	Oilcake and clover hay	Swedish turnips	0·25	0·90	7·29	8·44	91·56
				Means	0·25	0·84	8·03	9·12	90·88
Class III. (For data, see Class III. Table IX.)									
Hants downs	5	13 6	Oilcake	Swedish turnips	0·23	0·77	8·32	9·31	90·69
Hants downs	5	13 6	Oats..	Swedish turnips	0·25	0·88	8·32	9·45	90·55
Hants downs	5	13 6	Clover chaff	Swedish turnips	0·24	0·85	7·40	8·49	91·51
				Means	0·24	0·83	8·01	9·08	90·92
Class IV. (For data, see Class IV. Table IX.)									
Hants downs	5	19 1	Oilcake	Clover chaff.....	0·13	0·46	4·48	5·07	94·93
Hants downs	5	19 1	Linseed	Clover chaff.....	0·14	0·43	4·62	5·19	94·81
Hants downs	5	19 1	Barley	Clover chaff.....	0·12	0·41	4·47	5·00	95·00
Hants downs	5	19 1	Malt...........	Clover chaff ...	0·10	0·31	4·20	4·61	95·39
				Means	0·12	0·40	4·44	4·97	95·03
Class V. (For data, see Class V. Table IX.)									
Hants downs	4	10 0	Barley (ground)	Mangolds.........	0·24	0·74	7·93	8·91	91·09
Hants downs	5	10 0	Malt (ground) and malt dust...	Mangolds	0·25	0·82	8·42	9·49	90·51
Hants downs	4	10 0	Barley (ground and *steeped*) ...	Mangolds...... ..	0·24	0·84	7·20	8·28	91·72
Hants downs	4	10 0	Malt (ground and *steeped*) and malt dust	Mangolds.........	0·19	0·58	7·45	8·23	91·77
Hants downs	5	10 0	Malt (ground) and malt dust ..	Mangolds.........	0·21	0·72	7·33	8·25	91·75
				Means	0·23	0·74	7·66	8·63	91·37
				General means..............	0·21	0·72	7·18	8·06	91·94

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* The estimated amounts of mineral matter are too high, owing to the adventitious matter retained by the wool.

Table XV.—Showing the Final Distribution of the Constituents of the Food consumed by fattening Pigs.

General particulars of the experiment.					100 dry matter of Food gave—				
			Description of fattening food.		Stored up in Increase.				Expired, perspired, or voided.
Pens.	Number of animals.	Duration.	Given in limited quantity.	Given *ad libitum*.	Mineral matter (ash).	Nitrogenous compounds (dry).	Non-nitrogenous substance (fat).	Total dry Increase.	
				The "Fat Pig" analysed.					
	1	wks. days. 10 0	None	Bran one part, bean and lentil meal two parts, and barley meal three parts	0·11	1·62	13·20	14·94	85·06
				Series 1 (For data, see Series 1. Table X.).					
1	3	8 0	None	Bean and lentil meal	0·04	1·54	15·93	17·51	82·49
2	3		Indian meal..	Bean and lentil meal	0·09	1·77	16·00	17·86	82·14
4	3		Indian meal and bran	Bean and lentil meal	−0·01	1·21	14·95	16·15	83·85
5	3		None	Indian meal.......	−0·08	1·17	17·48	18 58	81·42
6	3		Bean and lentil meal	Indian meal...........................	0·02	1·57	16·76	18 35	81·65
7	3		Bran	Indian meal...........................	0·07	1·75	16·86	18·68	81·32
8	3		Bean and lentil meal, and bran	Indian meal...........................	0·09	1·85	16·59	18·53	81·47
12	3		None	Bean and lentil meal, Indian meal and bran, each *ad libitum*	−0·01	1·27	15·59	16·84	83·16
				Means	0 03	1·51	16·27	17·81	82·19
				Series 2 (For data, see Series 2. Table X.).					
1	3	8 0	None	Bean and lentil meal	−0·13	0·94	17·37	18·18	81·82
2	3		Barley meal..........	Bean and lentil meal	0 01	1·19	13·49	14·69	85·31
3	3		Bran	Bean and lentil meal	−0·01	1·15	14 06	15·20	84·80
4	3		Barley meal and bran	Bean and lentil meal	−0·03	1·09	14·50	15·56	84·44
5	3		None	Barley meal............................	0·02	1·40	15·45	16·87	83 13
6	3		Bean and lentil meal	Barley meal............................	−0·02	1·30	16·21	17·49	82·51
7	3		Bran	Barley meal............................	0·01	1·40	15·50	16·91	83·09
8	3		Bean and lentil meal, and bran	Barley meal............................	−0·13	0·91	17·18	17·96	82·04
9 and 10	6		None	Mixture, one part bran, two parts barley meal, and three parts bean and lentil meal	0·01	1·31	14·77	16·11	83·89
11 and 12	6		None	Mixture, one part bran, two parts bean and lentil meal, and three parts barley meal	0·06	1·66	15·88	17·60	82·40
				Means	−0·02	1·28	15·44	16·66	83·34
				Series 3 (For data, see Series 3. Table X.).					
1	4	8 0	Dried cod-fish.........	Bran and Indian meal (equal parts)	−0·08	1·13	17·05	18·12	81·88
2	4		Dried cod-fish.........	Indian meal..............................	−0·01	1·60	19·27	20·86	79·14
				Means	−0·04	1·36	18·16	19·49	80 51
				Series 4 (For data, see Series 4. Table X.).					
1	3	10 0	Lentil meal and bran	Sugar	0·11	1·76	15·01	16·88	83·12
2	3		Lentil meal and bran	Starch	0·11	1·78	15·04	16·94	83·06
3	3		Lentil meal and bran	Sugar and starch	0·13	1·82	14·13	16·08	83·92
4	3		None	Lentils, bran, sugar, starch, each *ad libitum*	0·19	1·96	14·36	16·50	83·50
				Means	0·13	1·83	14·63	16·60	83·40
				General means........	0·02	1·44	15·81	17·27	82·73

As already explained, in the cases to which these and the preceding Tables relating to Increase refer, the amounts and composition of the foods consumed to produce a given amount of increase. were determined by actual experiment; and the composition of the increase so produced, is deduced from that of the animals that were analysed. Thus, in Table IX. for the Sheep, and in Table X. for the Pigs, are given the experimentally determined amounts of dry substance, &c., consumed to produce 100 lbs. of increase in live-weight, and the estimated amounts of certain constituents in that 100 lbs. of increase. It is obvious, therefore, that we have an easy means of calculating the amount of the respective constituents stored up in increase, for 100 of dry matter consumed. The sum of these makes up *the total dry matter in increase for* 100 *dry matter in food*; and the difference between this dry matter in increase and that in food represents the amount *expired, perspired, or voided.* With these observations, the mode of construction of Tables XIV. and XV. will be sufficiently intelligible.

It was seen (in Table XII.), that in the Sheep there was probably an average of about 9 parts dry substance fixed as increase for 100 consumed in food. Table XIV. shows (taking the cases in which the food was of the most usual description) that about 8 parts out of the 9 of dry increase were non-nitrogenous substance—that is *fat.* It results then, that for 100 of dry substance in food, there would be little more than 1 part fixed in increase in other forms than fat;—that is, as nitrogenous and mineral matters put together. According to the Table there were only, in Class I., 0·84, in Class III. 0·83, and in Class V. 0·74 part of nitrogenous substance retained in the increase of the animals for 100 of collective dry substance in their food. The corresponding amounts of mineral matter fixed were, on the same basis of calculation, for Class I. 0·25, for Class III. 0·24, and for Class V. 0·23. But if due allowance were made for the excess in the estimate of the mineral matters in the increase, as before noticed, the average amount of them stored up for 100 of dry food consumed, would, in the cases in question, be about 0·2.

Taking the average of the cases in which the Sheep were fed upon food of a nature fairly representing that of the animals liberally fed for the butcher, it is assumed then, that for 100 parts of dry matter of such food, only about 9 parts were stored up as increase. There remained, therefore, 91 parts *expired, perspired, or voided.* In the food of Sheep (and of oxen also), the proportion of so-called "woody fibre" is very much greater than in that of Pigs. With the former, therefore, there will be a larger proportion of indigestible matter voided than with the latter; and, as will presently be seen, with the larger proportion of digestible or assimilable matter in the food of the fattening Pig, there is at the same time a less proportion of the consumed dry substance expired, perspired, or voided.

At the head of the other results in Table XV. are given the amounts of the main classes of constituents stored up in increase for 100 of collective dry matter in food in the case of the *analysed* "fat Pig." From the circumstances under which the data were obtained in this particular instance, it may be assumed that the figures exceedingly closely represent the actual facts. The indication is that, for 100 of dry matter of food consumed by this single fattening Pig, there were produced 14·94 parts of dry substance of increase. Of these 14·94 parts of total dry increase, 13·2 were fat, 1·62 nitrogenous compounds, and 0·11 mineral matter. Against these numbers we have, taking *the average of all the other estimates* (twenty-four in number, and comprising 80 animals), 17·27 total dry increase for 100 of dry food; of which 15·81 are estimated as fat, 1·44 nitrogenous substance, and an insignificant amount of mineral matter. It is admitted that the estimates in Table XIV. relating to the Sheep, show a higher proportion of mineral to other constituents, stored up, than was probably the fact. On the other hand, the estimates of assimilated mineral matter in the case of the Pigs are probably in error in the other direction. It is true, that Pigs, though young, if put upon highly fattening food will grow comparatively little in frame, whilst Sheep, fed as they now generally are at a comparatively early age, will develop more of hard bony structure. It would be expected, therefore, that the proportion of mineral matter in the increase of fattening Sheep would be greater than that in fattening Pigs. Indeed, Table VII. shows the percentage of mineral matter, in the total carcasses, to be more than twice as much in the fattened Sheep analysed, as in the fat Pig. In the case of both Sheep and Oxen moreover, there seems to be a striking parallelism in the proportion of the mineral to the nitrogenous matters of growth; whilst with the Pig, not only is the actual amount of mineral matter much less, but its proportion to the nitrogenous matters seems to decrease as the animals fatten. Thus, looking to the composition of the *carcasses* alone, in which there would be no error in the mineral matter as when the hair or wool with its extraneous dirt is brought into the calculation, it is found that the amount of mineral matter to 1 of nitrogenous substance, was—in the lean Ox 0·31, in the fat Ox 0·30, in the store Sheep 0·30, in the fat Sheep 0·30, and in the very fat Sheep 0·30. On the other hand, in the carcass of the store Pig, the proportion of mineral matter to 1 of nitrogenous substance was 0·183, and in that of the fat Pig it was less still, or 0·133. From these considerations it is obvious, that the amount of mineral matter in the *increase* of the fattening Pig, will be much less both in proportion to the total increase itself, and to the coincidently accumulated nitrogenous compounds, than in that of the Sheep. The distinctions which the Tables relating to the composition of increase show, between the two descriptions of animal in this respect, are then, without doubt, correct in the main;—that is to say, at least in their *direction*, though probably *not* in the *degree* which the actual figures indicate. It is pretty certain that the estimates of mineral matter in the increase of the Sheep are somewhat too high; and unless it be admitted as probable, that Pigs rapidly increasing in weight under the fattening process, may sometimes not only not fix any mineral matter whatever, but even lose some of that already fixed, it must be concluded, that the Pig killed and analysed as fat, had too small a proportion of bony structure to be strictly comparable with the one analysed in the leaner state.

It will be obvious, from the very nature of the subject, that these estimates of the composition of *increase*, must only be taken as applicable for any general purposes, after due regard to the various qualifying circumstances which have been pointed out.

It will be remembered, that in the average of the cases in which the Sheep had been fed upon a liberal mixed diet of dry food and succulent roots—admittedly favourable conditions for their increase—they gave only about 9 per cent. of dry increase, for 100 dry substance of food. The average of the 24 lots of Pigs (80 animals), shows on the other hand, nearly double as much, or 17·27 parts of dry increase, for 100 of dry food consumed. The yield of fat, and of nitrogenous compounds, of which these 17·27 parts are chiefly made up, is of course higher in a corresponding degree. When it is borne in mind, however, that the natural fattening food of the Pig consists chiefly of ripened seeds containing little indigestible woody fibre, or immatured vegetable products, and that that of the Sheep contains a large proportion of woody fibre, and also much of the less highly elaborated vegetable compounds, it will not appear surprising, that 100 parts of the dry substance of the food of the Pig should yield so much more of dry animal increase, than 100 parts of that of the Sheep. It results, of course, that of the fattening food of the Pig, a less proportion of the dry substance than of that of Sheep, will be *expired*, *perspired*, or *voided*. In the case of the Sheep, it was assumed, as the average of the cases wherein the food was of the most favourable kind, that about 91 per cent. of the dry substance consumed were in some form expired, perspired, or voided. In the case of the single analysed Pig, only 85 parts were expired, perspired, or voided, for 100 of dry matter consumed in food. And, taking the average of the twenty-four lots comprising the eighty animals, calculation shows only 82·7 parts of collective dry substance expired, perspired, or voided, for 100 consumed in the food.

In speaking of the proportion of dry substance stored up in increase, for a given amount consumed in food, it will not for a moment be assumed, that it is herein implied that the relation of the ultimate elements is the same in the dry matter assimilated and fixed, and in that given off in the various forms from the system. The very various amounts, respectively of mineral matter, of nitrogenous compounds, and of non-nitrogenous substance (fat), stored up for 100 of each consumed (as shown in Tables XII. and XIII.), give some means of judging how different must be the ultimate composition of the gross dry matter fixed in the body, from that of the matters of the food eliminated from it. It is not within the scope and object of the present Paper, to give any further indication of the composition of the matters collectively given off from the body in relation to those taken as food, than is implied in the figures in the Tables just referred to, which show the amounts of certain constituents stored up for a given amount consumed—the complementary quantity being of course that which is expired, perspired, or voided. Still less is it to our present purpose, to show the proportion of the different constituents of the matters collectively given out from the body, which will be respectively exhaled by the lungs, perspired by the skin, or voided in the liquid or the solid form. There is, however, one point in connection with the difference between the ultimate composition of the dry substance of increase, and that of the compounds of the food which produced it, which may be here appropriately illustrated; this is, the relation of the *fat in the increase*, to the *fat and other matters in the food*, which yielded it.

3. *Relation of the Fat stored up in the Increase, to the ready-formed Fat, and other Constituents, consumed in the Food; &c.*

The amount of *fat* in the food of the different lots of Sheep which have served in the foregoing illustrations, was not determined, so that the relation of that estimated as stored up in increase, to that *ready-formed* in the food, cannot be shown in their case.

In the majority of the experiments with the Pigs, the amount of *ready-formed fat* in the food was determined. The amount stored up in the increase, has also been legitimately deduced from experimental evidence. The results show, as already noticed, that there were on the average, between 400 and 500 parts of Fat stored up in the increase, for 100 of Fatty matter consumed in food. In the case of the analysed fat Pig, there were 405 parts of Fat stored up for 100 consumed. The result in this instance was obtained in as direct a manner as the nature of the question will admit of, and it may be taken as representing the truth very closely. The average of the other experiments shows 472 parts of Fat in increase, for 100 ready-formed in food. Nor is there much reason to doubt the general accuracy of this latter indication. Upon the whole, it is obvious, that a large proportion of the Fat of the fattening animal is *produced* from *other constituents than Fat* in the food. Attention has elsewhere been called to the evidence of this, afforded in the instance of the analysed fat Pig*. It was shown that in its case rather more than three-fourths of the Fat of the increase gained on the fattening food, must have been formed in the body *from other constituents*; and it was pointed out, that if the *produced* Fat were due to the *Starch* of the food, it would require about 2½ parts of that substance, to yield 1 part of Fat. On this supposition, it is obvious, that a much larger proportion of the non-nitrogenous constituents of the food, will directly contribute to the non-nitrogenous substance of the increase (fat), than is represented by the total amount of the Fat itself, stored up. It is equally obvious, that the proportion of the total dry substance of the food consumed, which has (if we may draw such a distinction) directly contributed to the dry matter of increase, including the produced Fat, will be much greater than that indicated by the total amount of the dry substance of the increase. The proportion which is expired, perspired, or voided, without having, in the sense implied, directly contributed to increase, will, of course, in a complementary degree, be less than the total amount represented as expired, perspired, or voided.

To illustrate, numerically, the points above alluded to, there are shown, in Table XVI., for the analysed fat Pig, and for most of the sets of Pigs before under consideration—the amount of Fat stored up in the increase for 100 of dry matter of food consumed; the proportion of Fat already formed in the food; the amount that must have been *produced* from other compounds; the amount of Starch that would be required if the produced Fat were formed from it; the proportion of the total dry matter consumed, which would be thus required directly to contribute to the fixed increase; and lastly, the proportion that would be expired, perspired, or voided, without thus directly contributing to the fixed increase.

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* Report of the British Association for the Advancement of Science for 1852.

TABLE XVI.—Showing—the Amount of Fat stored up in the Increase of Fattening Pigs, for 100 of Dry Matter of Food consumed; the Proportion of Fat already formed in the Food; the Amount of Fat that must have been *produced* from other Compounds; the Amount of Starch that would be required if the produced Fat were formed from it; the Proportion of Total Dry Matter consumed, which would thus be required directly to contribute to the fixed Increase, &c.

General particulars of the experiment.					100 dry matter of food gave—								
Pens.	No. of animals.	Duration.	Description of fattening food.		Total dry increase.	Total fat in increase.	Fat already formed in the food.	Fat produced from starch, &c.	Starch required for the produced fat.	Ready-formed fat, and starch, in the food, contributing to the total fat in increase.	Nitrogenous and mineral matter, fixed in the increase.	Summary.	
			Given in limited quantity.	Given *ad libitum*.								Total dry matter of food fixed or directly contributing to increase.	Expired, perspired, or voided, without directly contributing to the fixed increase.
The "Fat Pig" analysed.													
	1	wks. dys. 10 0	None	Bran one part, bean and lentil meal two parts, and barley meal three parts...	14·94	13·20	3·26	9·94	24·85	28·11	1·73	29·84	70·16
Series 1 (For data, see Series 1, Tables X., XIII., and XV.).													
1	3	8 0	None	Bean and lentil meal	17·51	15·93	2 89	13·04	32·60	35·49	1·58	37·07	62·93
2	3		Indian meal	Bean and lentil meal	17·86	16·00	3·66	12·34	30·85	34·51	1·86	36·37	63·63
4	3		Indian meal and bran	Bean and lentil meal	16·15	14·95	4·59	10·36	25·90	30·49	1·20	31·69	68·31
5	3		None	Indian meal	18·58	17·48	6·15	11·33	28·32	34·47	1·09	35·56	64·44
6	3		Bean and lentil meal	Indian meal	18·35	16·76	5·43	11·33	28 32	33·75	1 59	35·34	64 66
7	3		Bran	Indian meal	18·68	16·83	6·31	10·52	26 30	32·61	1·82	34·43	65·57
8	3		Bean and lentil meal, and bran	Indian meal	18·53	16 59	5·64	10·95	27·37	33·01	1·94	34·95	65·05
12	3		None	Bean and lentil meal, Indian meal and bran, each *ad libitum*	16·84	15·59	4·65	10·94	27·35	32·00	1·26	33·26	66·74
				Means	17·81	16 27	4·92	11·35	28·38	33·29	1·54	34·83	65·17
Series 2 (For data, see Series 2, Tables X., XIII., and XV.).													
1	3	8 0	None	Bean and lentil meal	18·18	17·37	2·40	14·97	37·42	39·82	0·81	40·63	59·37
2	3		Barley meal	Bean and lentil meal	14·69	13·49	2·55	10·94	27·35	29·90	1 20	31·10	68·90
3	3		Bran	Bean and lentil meal	15·20	14 06	2·85	11·21	28·02	30 87	1·14	32·01	67·99
4	3		Barley meal and bran	Bean and lentil meal	15 56	14·50	3·08	11·42	28·55	31·63	1·06	32·69	67·31
5	3		None	Barley meal	16·87	15·45	2·83	12 62	31·55	34·38	1·42	35·80	64·20
6	3		Bean and lentil meal	Barley meal	17·49	16·21	2·81	13·40	33·50	36 31	1·28	37·59	62·41
7	3		Bran	Barley meal	16·91	15·50	3·27	12·23	30·57	33·84	1·41	35·25	64·75
8	3		Bean and lentil meal, and bran	Barley meal	17·96	17·18	3·16	14·02	35·05	38·21	0·78	38 99	61·01
9 and 10	6		None	Mixture of one part bran, two parts barley meal, and three parts bean and lentil meal	16·11	14·77	2·99	11·78	29·45	32·44	1·32	33·76	66·24
11 and 12	6		None	Mixture of one part bran, two parts bean and lentil meal, and three parts barley meal	17·60	15·88	3·08	12 80	32·00	35 08	1·72	36 80	63·20
				Means	16·66	15 44	2 90	12 54	31 35	34·25	1·21	35·46	64·54
Series 3 (For data, see Series 3, Tables X., XIII., and XV.).													
1	4	8 0	Dried cod-fish	Bran and Indian meal (equal parts)	18·12	17·05	5·40	11·65	29·12	34·52	1·05	35·57	64 43
2	4		Dried cod-fish	Indian meal	20·86	19·27	5·48	13·79	34·47	39 95	1·59	41·54	58·46
				Means	19·49	18·16	5·44	12 72	31·79	37·23	1·32	38 55	61·45
				General means	17·40	16·04	3·96	12·08	30·20	34·16	1·36	35·52	64·48

Both practical and chemical considerations seem to indicate that Fat may be *produced* in the animal body, by the transformation within it of *nitrogenous* compounds. But it seems probable, that at least the main source of the produced Fat will be the *non*-nitrogenous constituents of the food. Of these, particularly in the fattening food of Pigs, the most prominent item is *starch*. It seemed desirable, therefore, to adopt this substance as the basis of the illustration of the probable amount of the constituents involved in the formation of the *produced* Fat, in the experiments in question.

The question arises, how much Starch will be required for the production of a given amount of Fat? At present but little is known as to the relative proportions in which the different Fats exist in different animals. Nor are chemists agreed as to the formulæ to be given to the several natural animal Fats. It would only be a doubtful refinement, therefore, to adopt for our purpose the exact rational formula given for any one of the more important fatty bodies, and from it to calculate, in equivalents, the amount of Starch required to produce an equivalent of the Fat, and also the number of equivalents of the collateral products. It is better to adopt an average *percentage* composition merely; and for want of more exact data, we take the mean of the three most important animal fats—namely, tri-stearine, tri-margarine, and tri-oleine. This gives, in round numbers, 77 per cent. of carbon, 12 per cent. of hydrogen, and 11 per cent. of oxygen for the *crude mixed fats*. It may be mentioned, however, that tri-oleine is stated to be in larger proportion to the other fats in Pigs, than in either Sheep or Oxen. Assuming the oxygen which is eliminated in the *formation* of Fat from Starch, to go off with a portion of its hydrogen in the form of water, and the remainder with carbon in the form of carbonic acid, it would require as a *minimum*, 2·45 parts by weight of Starch to contribute to the formation of one part by weight of the mixed Fats + the collaterally formed water and carbonic acid. If the stearine predominated, this mode of calculation would show the amount of Starch required to be rather higher, and if the oleine, rather lower than 2·45 for 1 of the Fat. As the above number is the *lowest* amount of Starch which would, in the manner supposed, yield 1 part of the mixed Fats of the percentage composition above assumed, we may adopt the convenient round number 2·5 as the amount of Starch probably on the average required for the formation of 1 part of the mixed Fats of the body, when these have their source in that substance. This number then (2·5), is that by which we multiply, for the purposes of the Table, the amount of the estimated Fat in the increase of the Pigs, over and above the *ready-formed fat* they consumed in their food, to ascertain the amount of the dry substance in the food (if in the form of starch), required for the production of that amount of Fat which could not have been directly derived from the food as such, and must therefore have been *formed* within the body of the animal. Whilst adopting the mode of calculation here described, as usefully, and sufficiently closely, illustrating the point in question, it may be remarked in passing, that when Fat is formed from the *nitrogenous* compounds in the body, a *less* amount of dry substance of the food would then be required for the formation of a given amount of Fat, than when it is produced from Starch. On the other hand, if *Sugar* were the source of the Fat, a rather *larger* quantity than of Starch would be required. Of the *pectine* bodies, again, which enter so largely into the roots which frequently constitute a large proportion of the fattening food of sheep and oxen, the quantity required would, on the same mode of calculation, be still more than of Sugar.

On a former occasion, it was shown, that according to the mode of estimation here supposed, the 15 parts of *dry solid increase* yielded during the fattening process by the analysed fat Pig for 100 parts of dry matter of food consumed, would have required for its formation about 30 parts of the *dry substance of the food consumed.* The actual figures relating to this single animal are given in Table XVI., at the head of the respective columns which refer to the numerous lots of Pigs, the Fat in whose food was determined by analysis, and that in their increase estimated.

For 100 dry matter in food, the dry matter in the increase of the analysed fat Pig was 14·94, and that taking the average of all the other cases in which the Fat in the food was determined was 17·40. Of these amounts of total dry substance assimilated, 13·2 in the case of the single fat Pig, and 16·04 in the average of the other lots, are estimated as *Fat.*

Of the 13·2 parts of Fat stored up in the increase of the single animal, 3·26 only (provided the whole supplied had been taken up) could have been derived from the Fat in the food. At least 9·94 parts must, therefore, have been formed in the body of the animal from some other constituent or constituents. If the constituent in question were primarily *Starch*, it would, on our basis of calculation, require 24·8 parts of dry Starch for the formation of the 9·94 parts of *produced Fat.* Of ready-formed Fat in the food, and Starch, thus contributing to the formation of Fat, taken together, there would therefore be 28·11 parts out of 100 of dry matter of food consumed, directly engaged in the storing up in the body, of the 13·2 parts of Fat. If we add to this, the 1·73 part of nitrogenous and mineral matters at the same time fixed in the increase, we have 29·84 parts out of the 100 of dry matter of food consumed, directly contributing, in the sense supposed, to the production of the 14·94 parts only, of dry increase. In the particular sense here implied, therefore, there would be only 70·16 parts of the 100 of dry matter of the food expired, perspired, or voided, without thus directly contributing to increase; instead of 85·06 parts, which is the difference between the 100 of dry matter in food, and the 14·94 only, of dry substance actually stored up.

Following the same line of illustration for the average result of all the other experiments cited, it appears that for 16·04 parts of Fat stored up in increase, for 100 of dry matter of food consumed, only 3·96 parts could have been derived from *ready-formed fatty matter supplied in the food.* At least 12·08 parts must, therefore, have been *formed from other substances.* If from Starch, it would require, at the rate of 2·5 parts Starch for 1 of Fat, 30·2 parts of that substance for the formation of the 12·08 parts of the *produced fat.* The ready-formed Fat, and the Starch, together thus contributing to the 16·04 parts of Fat in the increase, would amount to 34·16 parts of the 100 of dry food consumed. There were, further, 1·36 part of nitrogenous and mineral matters assimilated. In all therefore, 35·52 parts out of 100 of gross dry matter of food, contributed in this comparatively direct manner, to the formation of the 17·4 parts of gross dry increase.

In the case of the single animal, therefore, the indication is, that, owing to the large proportion of the stored-up Fat which must have been actually formed within the body, it would require, if the source of the produced Fat were Starch, 29·84 parts of dry substance out of every 100 consumed in food, to minister in this direct manner to the production of only 14·94 parts of dry animal increase. Owing to the same circumstance it is, that, on the average of the other instances, 35·52 parts out of 100 of dry substance consumed may, in the same manner, be estimated as directly engaged in the storing up of only 17·4 parts of dry increase. It is worthy of remark, that in thus assuming *Starch* to have been the source of the produced Fat, and in adopting its numerical equivalent for that purpose as above described, the resulting figures, in both cases, show almost exactly twice as much of dry substance of food thus more directly contributing to the formation of increase, as there was of dry substance in the increase which was produced. In the case of Pigs fed on good food, it would appear that about one-third of the whole dry substance consumed may be so devoted. About two-thirds, therefore, will, if at all, only in a less direct manner, contribute to the production of increase. A large proportion will serve, more or less directly, for respiration only, or for the supply of material for the transformations constantly going on in the body independently of any increase in weight. And, besides the matters voided as indigestible, and necessarily effete, a larger or smaller quantity, according to the excess of the food, will pass off unused and comparatively unchanged.

As before stated, as the particular foods upon which the experimental *sheep* were fed had not their amounts of Fat determined, similar estimates cannot be made in regard to them as to the *pigs*. From a general knowledge, however, of the character of the fattening food of both Oxen and Sheep, considered in relation to the amount of increase it yields, and to the probable composition of that increase, there cannot be any doubt that in their case, as well as that of Pigs, a large amount of Fat will frequently be *formed* in the body from other constituents of the food. But the food of Oxen and Sheep, compared with that of the Pig, contains a large proportion of indigestible woody fibre; and it has been seen, that in the case of Sheep, there was only about half as much dry increase produced for 100 of dry matter of food consumed, as in the case of Pigs. The proportion of *Fat* in the dry increase of the highly fed Sheep, for 100 of dry matter of food consumed, is also only half as great as in the case of the Pig. Its food, moreover, is frequently much more oleaginous. It would appear, then, that on the average, there will not only be less Fat *formed* by the Sheep for a given amount of dry matter consumed, but there will be a far less proportion of the consumed dry matter of its food appropriated in the direct production, so to speak, of the total dry increase. On the other hand, as before remarked, in the food of Oxen and Sheep, there will be a less proportion of Starch, and a larger one of Pectine bodies, than in that of Pigs. And so far as Pectine, rather than Starch, may serve for the formation of Fat, the amount of the dry substance of the food required directly to contribute to the increase, will be somewhat the greater.

From the whole of the foregoing considerations bearing upon the relation of the constituents of increase to those of the food consumed to produce it, it appears, that a large proportion of the *Fat*, of which the increase of the so-called fattening animals so largely consists, may be *formed in the body from other compounds of the food.* Of the *nitrogenous* compounds, on the other hand, it is probable that frequently as little, and even less than 5 per cent. of the whole consumed, will be found finally stored up in the increase of the animal. Of the *mineral* matter of the food, a less proportion still than of the nitrogenous compounds, will, especially in the case of Pigs, be thus retained in the increase.

It is not the province of the present Paper, nor are the facts applicable to such a purpose, to consider the chemical and physiological changes undergone, or the offices subserved, by the—say 95 per cent. of the consumed nitrogenous compounds in their passage through the system. But, it may be remarked, that from the form in which a large proportion of them leaves the body, it is to be concluded that they must have entered into its fluids, if not its solid structures, and therein been subjected to oxidation and transformation. That this must serve some essential purpose, even in the processes of fattening animals subject to little muscular movement, there cannot be a doubt*. It is indeed certain, that if the animals are to store up as much as they can do of matters not containing nitrogen, a very large amount of nitrogen must pass through the body, compared with that which is finally retained in the increase. That this apparently excessive supply of nitrogenous compounds, independently of any mere influence on the activity of the functions or processes of the body, may itself yield up the elements for the formation of *Fat*, is highly probable.

Since it is found that by far the larger proportion of the solid increase of so-called fattening animals is really *Fat* itself—since it is probable that at least a great part of the Fat formed in the body is normally derived from Starch and other *non*-nitrogenous constituents of the food,—and since the current fattening foods contain so very much more of nitrogen than is eventually retained in the increase—it cannot be surprising, that the tendency of the results of all careful feeding experiments should be to show, that the limit of applicability of the estimate of the comparative value of foods, according to their *percentage of nitrogenous compounds*, is in practice very easily reached. Practically, indeed, the amount of increase is much more frequently dependent on the proportion in the food, of the digestible and assimilable *non*-nitrogenous compounds, than on that of the *nitrogenous* ones. In fact, when we reflect upon what we already know of the relations of the constituents of the animal body to those taken into it as food—thanks more particularly to MÜLDER, to BOUSSINGAULT, and to LIEBIG—and when we further consider the facts now adduced as to the *Composition of Increase*, it would seem little else than a truism to say, that as our fattening food-stuffs go, their comparative values, *as such**, are not determinable by their percentage of nitrogenous compounds. In the absence of sufficient direct evidence, such as we have endeavoured to supply, as to the probable composition of the increase of animals feeding for the butcher, an opposite opinion has generally been maintained. A consideration of the essentialness of the nitrogenous compounds of food, for the formation of the most important animal structures, has doubtless had much to do with determining the view in question; and it would seem, that keeping this point very prominently in view, it has been assumed, without the requisite experimental data, that these essential nitrogenous compounds were generally relatively deficient in our current foods. It would be more nearly true to say, that the digestible and assimilable *non*-nitrogenous constituents are generally in defect relatively to the digestible and assimilable nitrogenous compounds in our foods.

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* We have found in the case of Pigs, that by far the larger portion of the nitrogen consumed in the fattening food, passed off in the form of *Urea*. This was the case with animals kept almost entirely without movement; and it was equally so, whether the food contained the proportion of *nitrogenous to non-nitrogenous* constituents, as in the Cereal grains; or the much higher amount and proportion of the former, as in Leguminous seeds.

The comparative values of food-stuffs are, however, not to be unconditionally determined by their percentage of either of these equally important classes of constituents. It has, it is true, been frequently maintained, that a certain relation of the one class of constituents to the other, varying according to circumstances, is essential in a truly rational diet. But the practical bearings of the principle, seem to have been lost sight of by some of those who have the most prominently insisted upon it in its abstract form, as soon as they came to estimate, according to analysis, the comparative values of different foods.

The records of the numerous ultimate analyses of foods which have been hitherto made, are nevertheless of high value and interest in a statistical point of view. But now possessing them, as the basis of certain general estimates, the next desideratum is —to examine more closely into the nature and condition of the proximate compounds of food-stuffs—to distinguish those which are digestible and assimilable, from those which are not so—to determine the comparative values of the comparable or mutually replaceable portions (both intrinsically and according to the varying exigencies of the system)—and above all, to fix our standards of comparative value with more of reference to direct experimental evidence on the point, and to existing knowledge of the composition of animal bodies, than has been hitherto usual or even possible.

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* As, however, the *manure* from highly nitrogenous foods is the most valuable, it frequently becomes, in this point of view, the interest of the farmer—provided the character be in other respects equal—to purchase and use those having the higher amounts of nitrogen.

SECTION IX.—AVERAGE ACTUAL WEIGHTS, AND AVERAGE PERCENTAGE PROPORTIONS IN THE ENTIRE BODIES, OF THE INDIVIDUAL ORGANS AND OTHER SEPARATED PARTS, OF ANIMALS OF DIFFERENT DESCRIPTIONS, AND IN DIFFERENT CONDITIONS OF GROWTH AND FATNESS.

Hitherto, we have endeavoured to illustrate, by means of a large amount of laboriously accumulated experimental data, the actual and comparative gross composition of certain collective portions, and of the entire bodies, of animals of different descriptions, and in different stages of growth and fatness. By the aid of the information so derived, we have sought to estimate the probable composition of the *Increase* of the animals whilst fattening, and to show the relation of certain important constituents of the increase, to those in the food consumed. The results arrived at, under these heads, comprise the most important which the inquiry can furnish, so far as its application to Agriculture and Dietetics is concerned. It seems desirable, however, at least to provide some materials for the study of the question of the feeding of animals, from a somewhat more Physiological point of view. The data acquired with this view, relate to the *actual weights*, and the *proportion in the entire body*, of the individual organs, and certain more arbitrarily separated parts. In the selection of subjects in which to determine these points, it was sought—both to take a sufficient number, to secure pretty fair average results for the different descriptions of animal—and, as far as possible, to provide the means of tracing *the tendency of the relative development* of the different parts, as the animals grew and fattened.

In all, between 300 and 400 animals—Bullocks, Sheep, and Pigs—have been operated upon. The plan was, to determine the *live-weights* of the animals just before being slaughtered; and as soon as possible afterwards (so as to lessen the error arising from evaporation), the weights of their *carcasses*, of *each of the internal organs*, and of *some other separated parts*. The results for each of the individual animals—both the actual weights, and the calculated percentages in the entire body—are given for reference in Tables XV. to LXIV. inclusive, in the Appendix. Of these, Tables XVII., XVIII., XIX., XX., XXI. and XXII., which now follow, are Summaries; and in them the results will be found in sufficient detail to bring to view the few main points, to which alone, special attention will be directed.

In Table XVII. are given the *mean actual weights*, and in Table XX. the *mean percentages* in the entire body, of the different organs and parts of 2 Calves, 2 Heifers, and 14 Bullocks. Among these, are included the calf and the 2 bullocks selected and killed for analysis. The remainder were slaughtered for ordinary purposes; and were taken without any special selection, so as to afford fair average results. The data relating to these animals are not particularly calculated (as those referring to the Sheep will be found to be) to illustrate the comparative characters at different stages of growth and fatness. By the side of the mean, or average results, however, are given those (both actual and percentage), for the individual Calf, the "Half-fat Bullock," and the "Fat Bullock," which were selected for analysis.

In Tables XVIII. and XXI. are arranged, respectively, the *mean actual weights*, and the *mean percentages* in the entire animal, of the individual organs, &c., of 249 Sheep, divided into 5 Classes, according to age, condition of maturity and fatness, and mode of feeding. Thus, there are given, the average results of:—

5 Sheep, each of a different Breed, which were killed in the *store* condition, in order to provide a standard with which to compare the others;

100 Sheep, comprising a number from six different Breeds, all fed upon good fattening food, and under cover, during a period of five or six months, commencing at the age and stage of progress at which the 5 *store* or standard animals above mentioned were taken;

45 Sheep, from the same six Breeds as the last, but fed from the point at which they were slaughtered, for about six or seven months longer (though not under cover), until more than ordinarily fat, or in the condition of so-called "Christmas mutton;"

78 Sheep, all of one Breed, but divided into a number of lots, each with a different kind of diet, but fed to a medium degree of fatness;

21 Sheep, from several different Breeds, all fed and slaughtered as "Christmas mutton."

By the side of the columns showing the *mean results* for each of these 5 different Classes of Sheep, and for the whole 249 animals, respectively, are also given the results for the Fat Lamb, and for each of the 4 individual Sheep which were selected and killed in different conditions, for the purposes of analysis.

Table XIX. gives the *means* of the *actual weights*, and Table XXII. of the *percentages*, of the organs and parts of 59 Pigs; allotted into 7 Classes, distinguished one from another chiefly by the different character of the *food*, and the consequent and observed varying degree of growth and maturity. The separate results for each of the 2 animals selected and slaughtered for analysis are also given.

Lastly, in Table XXIII. are brought together, at one view, the general averages (both actual and percentage), for each of the *three descriptions of animal*; that is to say, the means, side by side, respectively of 16 Heifers and Bullocks, 249 Sheep, and 59 Pigs.

TABLE XVII.—Showing the Mean *Actual* Weights (lbs. and ozs.), of the different Organs and Parts, of CALVES, HEIFERS, and BULLOCKS.

	Description of parts.	Means of all slaughtered.				The animals selected for further analysis.		
		2 Fat Calves.	2 Fat Heifers.	14 Fat Bullocks.	Means of 16 Heifers and Bullocks.	Fat Calf.	Half-fat Ox.	Fat Ox.
		lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.
Separate parts of the "offal."	Stomachs	3 6·5	32 0	36 6·7	35 13·9	2 13.1	32 1	36 6
	Contents of stomachs	6 0·5	70 12	97 4·7	93 15·8	5 10·2	88 0	77 3·8
	Caul fat	2 9·2	23 4	23 2·7	23 2·9	2 7.6	16 9·5	33 12·5
	Small intestines and contents	5 5·5	15 4	17 1·3	16 13·6	6 3	26 6	14 10·5
	Large intestines and contents	3 4	12 2	13 4·6	13 2·3	2 14·5		6 4
	Intestinal fat	2 14	26 3	26 5·8	26 5·4	4 3·1	19 11·5	36 14
	Heart and aorta	1 7·7	4 0	5 14·4	5 10·6	1 7·5	5 13·5	7 6
	Heart fat	0 3·5	1 14	3 6·9	3 3·8	0 6·8	2 6·8	6 3
	Lungs and windpipe	3 4·8	6 6·5	9 10·1	9 3·6	3 5·7	7 12·8	8 14·5
	Blood	11 12·5	30 12	47 15·2	45 12·8	13 8·8	54 5	52 11·8
	Liver	4 2·8	12 15	15 1·6	14 13·3	4 3·6	15 11	17 10
	Gall-bladder and contents	0 2	0 10·8	1 0·5	0 15·7	with bladder	1 0	0 13
	Pancreas ("sweetbread")	1 11	0 13	1 1·6	1 1	1 12·9	1 0	0 15·5
	Thymus gland ("heartbread")		0 9·2	0 11	0 10·7		0 10·8	0 10·5
	Glands about the throat ("throatbread")		0 5·7	0 5·5	0 5·5		0 6·2	0 6·5
	Milt or spleen	0 13	1 4	1 15·3	1 13·9	0 12	2 2·2	2 4
	Bladder	0 6	0 8	0 9·2	0 9·1	0 7·2	0 5	1 10·7
	Penis		…………	0 7·5	…………	…………	0 6·5	
	Brains	13 9·5	21 7	0 12·1	0 12	11 7·3	0 14·2	0 14
	Tongue			32 0·5	30 10·7		7 7·5	3 7·5
	Head						31 8·5	39 15·5
	Hide	17 6·2	65 14	87 4·3	84 9·5	17 12·5	79 15·4	80 3·7
	Feet and hoofs	5 7·5	14 10	20 13	20 0·6	4 6	20 2	22 7·8
	Tail	0 5·2	0 12·8	1 1·3	1 1·9	0 5·3	1 12·4	1 7·5
	Diaphragm ("skirts")	1 1·5	4 9·5	5 3·2	5 2	1 1·6	5 11·5	7 9
	Miscellaneous trimmings	…………	4 6	3 14·3	3 15·3	…………	1 7	…………
	Total "offal" parts	85 4·9	351 6·5	452 13·3	439 13·9	85 4·7	423 10·3	460 13·3
	Carcass	156 10·8	474 10	710 3·1	680 12	160 9	797 11	939 6
	Loss by evaporation, error in weighing, &c.	8 12·3	27 13·5*	19 1·1	20 7·2	12 14·3	10 10·7	18 12·7
	Live-weight after fasting	250 12	853 14	1182 1·5	1141 1·1	258 12	1232 0	1419 0

* This amount includes the Wombs of the Heifers, one of which was with Calf

TABLE XVIII.—Showing the Mean *Actual* Weights (lbs. and ozs.), of the different Organs and Parts of SHEEP.

Description of parts.	Means of all slaughtered.						The animals selected for further analysis.				
	Gradationary Series.			Miscellaneous.							
	5 sheep of different breeds, killed in store condition, for a standard of comparison.	100 sheep of different breeds, moderately fattened. — about 1¼ year old.	45 sheep of different breeds, excessively fattened. — about 1¾ year old.	78 Hampshire down sheep, moderately fattened, on different foods. — 1¼ to 1½ year old.	21 sheep of various breeds, and modes of feeding; of more than average fatness. — about 1¾ year old.	Means of 249 sheep, of different breeds, conditions of fatness, age, &c.	Fat Lamb.	Store Sheep.	Half-fat Sheep.	Fat Sheep.	Very fat Sheep.
	lbs ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.
Original weight	....	102 7	124 11·4	115 1	96 11·4	109 13·5	......	. .	.. . ...	....	...
Final weight unfasted	99 0	152 5·8	202 7·5	144 13·1	179 15·2	160 4·9	.. ..	.. .. . .	.	... ...	260 8
Fasted live-weight	93 0·8	145 5·3	192 0·3	141 6·7	170 12·2	153 10·2	84 6·5	97 10	105 1	127 2·5	252 8
Separate parts of the "Offal."											
Stomachs	2 11·5	3 9·8	4 1·9	3 13·5	3 11·2	3 12·3	1 8·6	3 3·8	2 13·7	3 2·5	4 2·3
Contents of stomachs	6 2·5	6 8·5	6 14·8	9 9·9	6 2·7	7 10·4	5 2·1	4 13·8	7 9·4	4 15·5	10 8
Caul fat	2 11·9	6 1·3	9 10·5	6 9·7	9 1·1	7 1·8	3 4	3 2·7	3 1·5	6 9	17 0
Small intestines and contents	2 2·6	2 12·4	2 4·1	2 4·7	2 4·2	2 7·6	4 1·7	1 11·1	2 3·2	3 5·4	2 9
Large intestines and contents	2 11·5	2 12	3 0·9	3 2·3	2 15·9	2 15·2		1 12	1 15·5	3 1	3 3
Intestinal fat	1 2·6	2 7·8	4 3·6	3 2·4	4 5·6	3 2·2	1 10·8	1 9·4	2 5·5	3 9·5	7 6
Heart and aorta	0 7·1	0 9·3	0 11	0 11·4	0 11·1	0 10·4	0 5·4	0 8·3	0 7·4	0 8·7	0 13
Heart fat	0 4·9	0 4·8	0 10·7	0 9·6	0 11·3	0 7·3	0 4·6	0 3·9	0 8	0 8	0 9·5
Lungs and windpipe	1 1·5	1 8·2	1 9·5	1 3	1 9	1 3·3	1 0·8	1 6·9	1 1·4	1 0·5	1 14
Blood	4 7·3	6 0·2	7 2·5	5 9·4	6 8·8	6 1·6	2 14·3	5 1·2	4 1·2	4 8·8	10 4
Liver	1 8·1	2 8·2	2 5·8	2 0·6	2 5·3	2 5·4	1 2·8	1 10·5	1 11·3	1 14·8	2 10
Gall-bladder and contents	0 1	0 1·4	0 1·[illegible]	0 1·5	0 1·3	0 1·5	0 0·4	0 1	0 8·5	0 1	. . ....
Pancreas ("sweetbread")	0 2·1	0 2·5	0 3·1	0 3·4	0 3·3	0 3·3	0 2·5*	0 1·1	0 2·6*	0 3·4*	0 5·7*
Thymus gland ("heartbread")	....	.. ..	-		.	....		0 1·1			
Glands about the throat ("throatbread")	0 0·8	....	- -		..	.. ..		0 1			
Milt or spleen	0 2·6	0 4	0 4·4	0 3·6	0 4·7	0 4·0	0 2·6	0 2·5	0 2·9	0 3·8	0 6·2
Bladder	0 0·8	0 0·7	0 0·[illegible]		0 0·8	0 0·8	. .	0 1	0 1·1	0 0·9	... .
Head	3 5·9	4 5·2	4 12·3	4 9·7	4 12·2	4 8·1	2 10·1	3 9·3	4 2·8	3 14	6 0·5
Skin	13 1·8	18 9·8	20 0·2	16 4·5	15 7·9	18 0·4	4 15·2	5 8·6	7 0·4	7 0·6	25 8
Wool							3 1·7	7 3·3	6 10·6	8 1·9	
Feet, hoofs, &c.							0 12·7	1 0·7	with skin	with skin	with skin
Diaphragm ("skirts")	0 4·1	. ...	0 3·4		.	0 3·4	0 4·8	0 3·4	0 8·6	0 6·2	. ...
Miscellaneous trimmings	0 1·4	0 2·1	0 3·0		0 2·0	0 3·0	. .	.	. ...	....	.. ..
Total "offal" parts	42 12·0	58 13·2	[illegible] 12·1	[illegible]	74 6·4	61 11·5	33 9·1	43 4·6	47 3·6	53 3	93 3·2
Carcass	49 11·8	85 11·1	122 14·[illegible]	[illegible]	[illegible] 5·7	91 12·5	50 8	52 1	56 4·1	73 1	159 4
Loss by evaporation, error in weighing, &c.	0 9·0	0 13·0	0 [illegible]	[illegible]	0 [illegible]	0 2·2	0 5·4	2 4·4	1 9·3	0 14·5	0 0·8
Live-weight after fasting	93 0·8	145 5·3	[illegible] ·3	141 6·7	170 12·2	153 10·2	84 6·5	97 10	105 1	127 2·5	252 8

* In these cases it is doubtful whether the amounts include the Thymus Gland and Glands about the Throat, or whether they refer to the Pancreas only.

TABLE XIX.—Showing the Mean *Actual* Weights (lbs. and ozs.) of the different Organs and Parts of PIGS.

	Description of parts.	Moderately fattened on different descriptions of food. *Means of all slaughtered.*								The animals selected for further analysis.	
		9 Pigs.	12 Pigs.	15 Pigs.	12 Pigs.	6 Pigs.	2 Pigs.	3 Pigs.	Means of 59 fattened pigs.	Store pig.	Fat pig.
		Food; bran, with limited quantity of bean and lentil meal, or Indian meal, or both.	Food; bean and lentil meal, with limited quantity of Indian meal, or bran, or both.	Food; Indian meal, with limited quantity of bean and lentil meal, or bran, or all.	Food; sugar, or starch, or both, with limited quantity of bran and lentil meal.	Food; dried cod-fish, with Indian meal, or bran and Indian meal.	Put to feed in store condition and only half fattened.	Put to feed when half-fat on same food as last and moderately *fattened.*			
		lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.
	Original weight	140 12·5	142 9·4	143 7·5	95 5·8	163 13·3	130 8	135 10·7	134 5·1	. ... .	103 0
	Final-weight unfasted	191 7·1	239 5·4	245 10·7	185 4	287 13·3	180 0	181 5·3	222 9 8	100 0	191 0
	Fasted live-weight	182 12·4	227 6·9	234 12·7	177 6·6	278 0	170 8	172 10 8	212 12	93 15	185 0
Separate parts of the "Offal."	Stomachs	3 0·6	2 13	2 11·4	2 0·8	3 2·9	2 15·3	1 11·3	2 10·4	1 3 3	1 3·5
	Contents of stomachs									0 4·2	0 11·8
	Caul fat	0 15·2	1 2	1 5·2	1 0·9	1 6·8	0 12·8	0 14 3	1 2 8	0 5·6	0 9
	Small intestines and contents	5 10·3	4 14·8	3 14·9	3 13·2	4 8·1	6 10	3 15·1	4 8·4	3 9·8	3 12·6
	Large intestines and contents	9 1·4	9 5·4	7 10·8	8 14·8	7 9·6	7 6	5 10 3	8 5·7	5 14·3	6 14·6
	Intestinal fat, "mudgeon," &c.	1 10·8	3 2·4	3 3·6	1 1·7	2 14·3	1 2·6	1 8·2	2 5 6	1 5·8	4 1·3
	Heart and aorta	0 8·5	0 9·9	0 10·4	0 8·9	0 11·1	0 7·5	0 8·1	0 9 6	0 7·8	0 8·8
	Lungs and windpipe	1 9·7	1 10·2	1 9·5	1 6·4	1 9·4	1 6·5	1 12·3	1 9·1	1 5·7	1 11·7
	Blood	7 8·8	9 2	8 0·4	6 5·2	8 9·4	5 3·3	5 12·7	7 10·1	7 0·8	6 13·3
	Liver	2 13·5	3 14·3	3 5·6	3 0·1	3 7	3 2·1	2 10·1	3 4·5	2 8	3 0·8
	Gall-bladder and contents	0 2·1	0 2·1	0 1·7	0 2·4	0 2·8	0 1·6	0 2 4	0 2·1	0 1·2	0 2 1
	Pancreas ("sweetbread")	0 5·1	0 8·1	0 7·5	0 5	0 8·1	0 4·5	0 5·1	0 6·6	0 4·1	0 6·4
	Milt or spleen	0 4·7	0 5·4	0 4·7	0 3·7	0 5·6	0 4·6	0 4·5	0 4·7	0 2·7	0 4·9
	Bladder	0 1·9	0 3·1	0 2·7	0 2·2	0 2·7	0 1·8	0 2·6	0 2·5	0 2·3	0 2·5
	Penis	0 6·8	0 8·1	0 8	0 5·6	0 9·2	0 5·1	0 6 8	0 7·1	......	......
	Tongue	0 15·7	1 0·3	1 0·8	0 15·2	1 3·3	0 13 2	0 13·6	1 0 2	0 8·4	0 12 9
	Toes	0 2·9	0 3·3	0 2·9	0 2·3	0 3	.....	.....	0 2·9	.....	......
	Miscellaneous trimmings	0 5·1	0 11·8	0 11·7	0 3·5	0 9·1	0 9·3	0 10 4	0 8·8	0 6·6	.
	Total "Offal" parts	35 6·1	40 4·2	35 15·8	30 11·4	37 10·4	31 10 2	27 3·8	35 4 6	25 10 6*	31 4·2*
	Carcass (including head, with brains, feet and tail)	146 7·5	186 14·4	197 12·5	144 9·5	239 6	135 9·5	144 6·2	176 5 3	71 2·3*	153 2·8*
	Loss by evaporation, error in weighing, &c.	0 14·8	0 4·3	1 0·4	2 1·7	0 15·6	3 4·3	1 0·8	1 2 1	+ 2 13·9	0 9
	Live-weight after fasting	182 12·4	227 6·9	234 12·7	177 6·6	278 0	170 8	172 10·8	212 12	93 15	185 0

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* For comparison with the other results in this Table, the head (with brains), feet, and tail of the two *analysed* Pigs, are here excluded from the Offal, and included with the Carcass, according to the usual custom of the Butcher with Pigs.

TABLE XX.—Showing the Mean *Percentage* Proportion of the different Organs and Parts, in the Fasted Live-weight of CALVES, HEIFERS and BULLOCKS.

	Description of parts.	Means of all slaughtered.				The animals selected for further analysis.		
		2 Fat Calves.	2 Fat Heifers.	14 Fat Bullocks.	Means of 16 Heifers and Bullocks.	Fat Calf.	Half-fat Ox.	Fat Ox.
Separate parts of the "Offal."	Stomachs	1·37	3·75	3·09	3·17	1·09	2·60	2·56
	Contents of stomachs	2·39	8·40	8·44	8·44	2·18	7·14	5·44
	Caul fat	1·03	2·69	1·93	2·02	0·96	1·35	2·38
	Small intestines and contents	2·13	1·80	1·49	1·52	2·39	2·14	1·03
	Large intestines and contents	1·30	1·44	1·18	1·22	1·12		0·44
	Intestinal fat	1·13	3·02	2·12	2·24	1·62	1·60	2·60
	Heart and aorta	0·60	0·48	0·50	0·50	0·57	0·47	0·52
	Heart fat	0·08	0·22	0·32	0·31	0·16	0·20	0·44
	Lungs and windpipe	1·32	0·75	0·82	0·81	1·30	0·63	0·63
	Blood	4·68	3·60	4·07	4·01	5·24	4·41	3·72
	Liver	1·67	1·52	1·28	1·31	1·63	1·27	1·24
	Gall-bladder and contents	0·05	0·08	0·09	0·09	with bladder	0·08	0·06
	Pancreas ("sweetbread")	0·67	0·09	0·09	0·09	0·70	0·08	0·07
	Thymus gland ("heartbread")		0·07	0·06	0·06		0·06	0·05
	Glands about the throat ("throatbread")		0·05	0·03	0·03		0·03	0·03
	Milt or spleen	0·32	0·15	0·17	0·16	0·29	0·17	0·16
	Bladder	0·15	0·06	0·05	0·05	0·17	0·03	0·12
	Penis		......	0·04	......	......	0·03	
	Brains	5·46	2·51	0·07	0·06	4·43	0·07	0·06
	Tongue			2·71	2·69		0·61	0·24
	Head						2·56	2·82
	Hide	6·94	7·74	7·46	7·49	6·87	6·50	5·65
	Feet and hoofs	2·18	1·72	1·78	1·77	1·69	1·64	1·59
	Tail	0·13	0·09	0·09	0·10	0·13	0·14	0·10
	Diaphragm ("skirts")	0·44	0·53	0·39	0·41	0·43	0·46	0·53
	Miscellaneous trimmings	......	0·49	0·27	0·30	......	0·12	......
Total "Offal" parts		34·04	41·25	38·54	38·85	32·97	34·39	32·48
Carcass		62·53	55·58	59·84	59·31	62·05	64·75	66·20
Loss by evaporation, error in weighing, &c.		3·43	3·17*	1·62	1·84	4·98	0·86	1·32
		100·00	100·00	100·00	100·00	100·00	100·00	100·00

* This amount includes the Wombs of the Heifers, one of which was with Calf.

TABLE XXII.—Showing the Mean *Percentage* Proportion of the different Organs and Parts, in the Fasted Live-weight of PIGS.

	Description of parts.	Moderately fattened in different descriptions of food.					Means of all slaughtered.			The animals selected for further analysis.	
		9 Pigs. Food; bran, with limited quantity of bean and lentil meal, or Indian meal, or both.	12 Pigs. Food; bean and lentil meal, with limited quantity of Indian meal, or bran, or both.	15 Pigs. Food: Indian meal, with limited quantity of bean and lentil meal, or bran, or all.	12 Pigs. Food; sugar, or starch, or both, with limited quantity of bran and lentil meal.	6 Pigs. Food; dried codfish, with Indian meal, or bran and Indian meal.	2 Pigs. Put to feed in store condition, and only half-fattened.	3 Pigs. Put to feed when half-fat, on same food as last, and moderately fattened.	Means of 59 fattened pigs.	Store pig.	Fat pig.
Separate parts of the "Offal."	Stomachs	1·66	1·27	1·18	1·16	1·17	1·81	0·99	1·28	1·28	0·66
	Contents of stomachs									0·28	0·40
	Caul fat	0·52	0·49	0·57	0·59	0·51	0·47	0·52	0·54	0·37	0·30
	Small intestines and contents	3·05	2·19	1·69	2·15	1·66	3·98	2·36	2·20	3·85	2·05
	Large intestines and contents	4·91	4·16	3·28	5·05	2·76	4·84	3·38	4·04	6·27	3·74
	Intestinal fat, "mudgeon," &c.	0·91	1·35	1·37	0·63	1·03	0·67	0·87	1·06	1·45	2·21
	Heart and aorta	0·29	0·27	0·27	0·31	0·25	0·28	0·29	0·29	0·52	0·30
	Lungs and windpipe	0·88	0·73	0·68	0·79	0·57	0·85	1·06	0·76	1·44	0·94
	Blood	3·97	4·08	3·43	3·59	3·11	3·04	3·37	3·63	7·51	3·69
	Liver	1·55	1·71	1·43	1·70	1·26	1·87	1·56	1·57	2·66	1·65
	Gall-bladder and contents	0·07	0·05	0·05	0·08	0·06	0·05	0·09	0·06	0·08	0·07
	Pancreas ("sweetbread")	0·18	0·22	0·20	0·18	0·19	0·17	0·18	0·19	0·27	0·22
	Milt or spleen	0·16	0·15	0·13	0·14	0·12	0·17	0·15	0·14	0·18	0·17
	Bladder	0·07	0·09	0·07	0·08	0·06	0·06	0·10	0·08	0·15	0·08
	Penis	0·23	0·22	0·21	0·20	0·21	0·18	0·24	0·21		.
	Tongue	0·54	0·46	0·45	0·53	0·43	0·49	0·51	0·48	0·56	0·43
	Toes	0·09	0·09	0·08	0·08	0·07	....		0·08		...
	Miscellaneous trimmings	0·18	0·32	0·29	0·12	0·21	0·35	0·40	0·26	0·44	.....
Total "Offal" parts		19·26	17·85	15·38	17·38	13·67	18·78	16·07	16·87	27·31*	16·91*
Carcass (including head, with brains, feet, and tail)		80·22	82·07	84·18	81·44	85·98	79·26	83·39	82·57	75·74*	82·79*
Loss by evaporation, error in weighing, &c.		0·52	0·08	0·44	1·18	0·35	1·96	0·54	0·56	+3·05	0·30
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

* For comparison with the other results in this Table, the head (with brains), feet, and tail, of the two *analysed* Pigs, are here excluded from the Offal, and included with the Carcass, according to usual custom of the Butcher with Pigs.

TABLE XXI.—Showing the Mean *Percentage* Proportion of the different Organs and Parts, in the Fasted Live-weight of SHEEP.

	Description of parts.	Means of all slaughtered.						The animals selected for further analysis.				
		Gradationary Series.			Miscellaneous.							
		5 sheep of different breeds, killed in store condition, for a standard of comparison.	100 sheep of different breeds, moderately fattened. — About 1¼ year old.	45 sheep of different breeds, excessively fattened. — About 1¾ year old.	78 Hants down sheep, moderately fattened, on different foods. — 1¼ to 1½ year old.	21 sheep of various breeds and modes of feeding, of more than average fatness. — About 1¾ year old.	Means of 249 sheep of different breeds, conditions of fatness, age, &c.	Fat Lamb.	Store Sheep.	Half-fat Sheep.	Fat Sheep.	Very fat Sheep.
Separate parts of the "Offal."	Stomachs	2·94	2·49	2·14	2·72	2·17	2 45	1·822	3·316	2·719	2·482	1·641
	Contents of stomachs	6·16	4·49	3·62	6·83	3·62	4·98	6·079	4·981	7·222	3·908	4·158
	Caul fat	2·92	4·13	4·99	4·67	5 31	4·63	3 850	3 246	2·945	5·161	6·733
	Small intestines and contents	2·32	1·92	1·19	1·63	1·33	1·61	4·865	1·735	2·094	2·625	1·015
	Large intestines and contents	2·93	1·89	1·59	2·23	1·75	1·92		1·793	1·874	2·408	1·262
	Intestinal fat	1·28	1·70	2·10	2·23	2·55	2·04	1·984	1·626	2·231	2·826	2·921
	Heart and aorta	0·48	0·40	0·36	0·51	0·41	0·43	0·400	0·531	0·440	0·428	0·822
	Heart fat	0·32	0 20	0·35	0·42	0·42	0·32	0 341	0·250	0·476	0·398	0·235
	Lungs and windpipe	1·17	1·04	0·83	1·06	0·92	0·99	1·244	1 466	1·035	0·811	0·743
	Blood	4·81	4·14	3·73	3·95	3·84	3·97	3·428	5·199	3·879	3·578	4 059
	Liver	1·61	1·75	1·33	1·44	1·37	1·52	1·392	1 697	1·624	1·514	1·040
	Gall-bladder and contents	0·07	0·06	0·06	0·07	0·05	0·06	0·030	0 064	0 506	0·049	. ...
	Pancreas ("sweetbread")	0·13	0·15	0·10	0·15	0·12	0 14	0·185*	0·070	0·155*	0·167*	0·141*
	Thymus gland ("heartbread")	....	.. ..	. .....	....	....	. .. .		0·070			
	Glands about the throat ("throatbread")	0 06	..... .	....	....	....	.. ..		0·064			
	Milt or spleen	0·17	0·17	0·14	0·16	0·17	0·16	0·193	0·160	0·172	0·162	0·153
	Bladder	0·05	0 03	0·03	....	0·03	0 03	... ..	0·064	0·061	0·044	... .
	Head	3·64	3·00	2·53	3·27	2·74	2·93	3 117	3·668	3·974	3·047	2·389
	Skin	14·09	12·83	10·46	11·50	10·84	11·73	5·865	5 672	6·686	5·535	10·099
	Wool							3·680	7·382	6·341	6 385	
	Feet and hoofs	....	....	....	....	....	... . ..	0·940	1·069	. ..	.. .	.....
	Diaphragm ("skirts")	0·30	.. ....	0·12	....	....	0·14	0·355	0·218	0·511	0·305	.. ...
	Miscellaneous trimmings	0·10	0·18	0·11	....	0·07	0·12	....	.... . .	. ....	.. ...	......
	Total "Offal" parts	45·55	40·52	35·78	42·84	37·71	40·17	39·770	44 341	44 948	41·828	36·911
	Carcass	53·42	58·97	64·05	56·85	62·28	59·74	59·830	53·329	53·546	57·459	63·069
	Loss by evaporation, error in weighing, &c.	1·03	0·51	0·17	0·31	0·01	0·09	0·400	2·330	1·506	0·713	0·020
		100·00	100·00	100·00	100·00	100·00	100·00	100·000	100·000	100·000	100·000	100 000

* In these cases it is doubtful whether the amounts include the Thymus Gland, and Glands about the Throat, or whether they refer to the Pancreas only.

TABLE XXIII.—*Summary* of the Mean *Actual* Weights (lbs. and ozs.), and of the Mean *Percentage* Proportion, of the different Organs and Parts of different Descriptions of Animals:—BULLOCKS, SHEEP, and PIGS.

	Description of parts.	Mean Actual Weights (lbs. and ozs.).			Mean *Percentage* Proportions in the Fasted Live-weights.		
		Means of 16 heifers and bullocks.	Means of 249 sheep of different breeds, conditions of fatness, age, &c.	Means of 59 fattened pigs.	Means of 16 heifers and bullocks.	Means of 249 sheep of different breeds, conditions of fatness, age, &c.	Means of 59 fattened pigs.
Separate parts of the "Offal."	Stomachs	85 13·9	3 12·3	2 10·4	3·17	2·45	1·28
	Contents of stomachs (and vomit)	93 15·8	7 10·4		8·44	4·98	
	Caul fat	23 2·9	7 1·8	1 2·3	2·02	4·63	0·54
	Small intestines and contents	16 13·6	2 7·6	4 8·4	1·52	1·61	2·20
	Large intestines and contents	13 2·3	2 15·2	8 5·7	1·22	1·92	4·04
	Intestinal fat	26 5·4	3 2·2	2 5·6	2·24	2·04	1·06
	Heart and aorta	5 10·6	0 10·4	0 9·6	0·50	0·43	0·29
	Heart fat	3 3·8	0 7·8	....	0·31	0·32	......
	Lungs and windpipe	9 3·6	1 8·3	1 9·1	0·81	0·99	0·76
	Blood	45 12·8	6 1·6	7 10·1	4·01	3·97	3·63
	Liver	14 13·3	2 5·4	3 4·5	1·31	1·52	1·57
	Gall-bladder and contents	0 15·7	0 1·5	0 2·1	0·09	0·06	0·06
	Pancreas ("sweetbread")	1 1	0 3·3	0 6·6	0·09	0·14	0·19
	Thymus gland ("heartbread")	0 10·7	....	....	0·06	......	......
	Glands about the throat ("throatbread")	0 5·5	....	....	0·03	......	......
	Milt or spleen	1 13·9	0 4·0	0 4·7	0·16	0·16	0·14
	Bladder	0 9·1	0 0·8	0 2·5	0·05	0·03	0·08
	Penis	....	....	0 7·1	......	......	0·21
	Brains	0 12·0	....	*	0·06	......	*
	Tongue	30 10·7	4 8·1	1 0·2	2·69	2·93	0·48
	Head			*			*
	Hide, or Skin and Wool	84 9·5	18 0·4	....	7·49	11·73	......
	Feet and hoofs	20 0·6	....	0 2·9†	1·77	......	0·08†
	Tail	1 1·9	....	*	0·10	......	*
	Diaphragm ("skirts")	5 2	0 3·4	....	0·41	0·14	......
	Miscellaneous trimmings	3 15·3	0 3·0	0 8·8	0·30	0·12	0·26
Total "Offal" parts		439 13·9	61 11·5	35 4·6*	38·85	40·17	16·87*
Carcass		680 12	91 12·5	176 5·3*	59·31	59·74	82·57*
Loss by evaporation, error in weighing, &c.		20 7·2‡	0 2·2	1 2·1	1·84	0·09	0·56
Live-weight after fasting		1141 1·1	153 10·2	212 12	100·00	100·00	100·00

* In the case of the Pigs, the head (with brains), feet, and tail, are included with Carcass, and not with the Offal as with the other animals.

† These quantities relate to the toes only.

‡ Penis or Womb included here.

On the condensed, though still voluminous record of facts, relating to this branch of the inquiry, which these Tables (XVII.—XXIII. inclusive) provide, our space and more special objects will allow but a few short comments.

A few words may first be offered directing attention to the more prominent points of distinction between the different descriptions of animal—Oxen, Sheep, and Pigs—as regards the amount, and the proportion in the whole body, of their respective organs and parts.

An examination of Table XXIII. will show, that the stomachs and contents, constituted in the Oxen about 11½, in the Sheep about 7½, and in the Pig only about 1¼ per cent. of the entire weight of the body. The intestines and their contents, on the other hand, stand in an opposite relation. Thus, of the entire body of the Pig, these amounted to about 6¼ per cent., of that of the Sheep to about 3½ per cent., and of that of Oxen to only about 2¾ per cent. These facts are of considerable interest, when it is borne in mind, that in the food of the Ruminant there is so large a proportion of indigestible Woody-fibre, and in that of the well-fed Pig a comparatively large proportion of Starch —the primary transformations of which are supposed to take place chiefly after leaving the stomach, and more or less throughout the intestinal canal. Again, of the masses of *internal* "loose fat," with its connecting membrane, the Bullocks yielded about 4½ per cent., the Sheep about 7¾, and the Pig little more than 1½ per cent. The Pig, therefore, with its much less proportion of alimentary organs, has also a much less proportion to the whole body, of the fat which surrounds them. With regard to the much larger amount of this sort of fat indicated in the Sheep than in the Oxen, it may be remarked, that a considerable proportion of the Sheep which contribute to these recorded averages, were, compared with the Oxen, in more than a corresponding degree of maturity and fatness.

Taking together, stomachs, small intestines, large intestines, and their respective contents, the Oxen yielded rather more than 14 per cent., the Sheep a little less than 11 per cent., and the Pigs about 7¼ per cent. With these great variations in the proportion in the different animals, of these receptacles and first laboratories of the food, with their contents, the further elaborating organs (if we may so say) with their fluids, appear to be much more equal in their proportion in the three cases. This is approximately illustrated in the fact, that, taking together the recorded percentages of "heart and aorta," "lungs and windpipe," "liver," "gall-bladder and contents," "pancreas," "milt or spleen," and the "blood," the sum indicated is for the Bullocks about 7 per cent., for the Sheep about 7¼ per cent., and for the Pigs about 6⅔rds per cent. If from this list we were to exclude the blood, which was more than one-third of a per cent. lower in the Pig than in the other animals, the sums of the percentages of the other items enumerated would agree even much more closely for the three descriptions of animal.

A rapid survey may next be taken of the general indications as to the influence of *progression* in the maturity and fatness of the fattening animal, upon the relative development of its several organs or parts. An examination of the Tables shows, that the internal organs, and other offal parts, pretty generally *increase* in *actual weight* as the animal passes from the store or lean, to the fat, or to the very fat condition. Excluding the fat, however, their *percentage proportion to the whole live-weight*, as invariably *diminishes* as the animal matures and fattens. Of the internal offal parts, the loose fat alone increases, not only in *actual weight*, but in *percentage proportion*. The *carcasses*, on the other hand, invariably *increase* in both *actual* and *percentage* amount as the animals mature. These remarks apply generally to Oxen, Sheep, and Pigs; but the data relating to the Sheep comprise the most complete gradationary series for their illustration.

To go a little into detail: the average *actual* weights per head of the collective stomachs, and intestines, and their contents, *increased* from about 13¾ lbs. in the five *store* or *lean* sheep, to about 15¾ lbs. in the 100 *fat* Sheep, and to about 16¼ lbs. among the forty-five *very fat* ones. The *percentage* of these parts in the entire weight of the animal, *diminished* from 14·35 for the *store* sheep, to 10·79 for the *fat* ones, and to 8·54 for the *very fat* ones. Again, the "heart and aorta," the "lungs and windpipe," the "blood," the "liver," the "gall-bladder and contents," the "pancreas," and the "milt or spleen," taken together, give an average *actual* weight per head, for the five *store* Sheep of 7¾ lbs., for the 100 *fat* ones of 11¼ lbs., and for the forty-five *very fat* ones of 12½ lbs. The proportional *increase* in *actual* weight as the animals fatten, is rather greater therefore for these organs and parts than for the collective stomachs and intestines, and contents. Still they *decrease* (though not so much as the collective stomachs, &c.) in *percentage* to the whole body with the increase in weight and fatness of the animals. Thus the *percentage* of the heart and other parts here classed with it, is for the average of the five *store* Sheep 8·44, for that of the 100 *fat* ones 7·71, and for that of the forty-five *very fat* ones 6·55. As already said, of the internal parts the *loose fat alone* increases in both *actual weight* and *percentage relation* to the whole body with the progress of the animals. It averages in *actual weight*, for the store or lean Sheep about 4¼ lbs., for the fat ones about 8¾ lbs., and for the very fat ones about 14½ lbs.; and in *percentage proportion to the whole body*, 4·52 for the lean Sheep, 6·08 for the fat, and 7·44 for the very fat ones.

Turning from this more detailed view to notice the actual, and relative development of the collective or *total Offal parts*, and the *total Carcass parts*, respectively, the result is as follows:—The average *actual weights* per head, of the total Offal parts, increased from 42¾ lbs. in the *store* or *lean* condition, to 58¾ lbs. in the *fat*, and to 68¾ lbs. in the *very fat* condition. The increase in actual weight of the corresponding Carcasses was much greater; namely, from 49¾ lbs. in the *store*, to 85¾ lbs. in the *fat*, and to nearly 123 lbs. in the *very fat* condition. That is to say, although the collective Offal parts increase considerably as the animals fatten, the Carcass—or frame, with its muscles, membranes, vessels and fat—increases proportionally very much more. The result of this much greater proportional rate of increase, in the so-called Carcass parts, than in the collective internal organs and other Offal parts, is, of course, that there is a *diminishing percentage* in the entire body of the total Offal parts, and an *increasing percentage* of the total Carcass parts as the animals mature and fatten. Thus, the percentage of the collective Offal parts, is, in round numbers, for the average of the *lean* sheep 45·5, for that of the *fat* ones 40·5, and for that of the *very fat* ones 35·8. The percentages of Carcass parts were, on the other hand, 53·4 for the corresponding *lean* animals, 58·9 for the *fatter* ones, and 64·0 for the *very fat* ones*.

Without going into more of numerical illustration of the points above alluded to, it may be mentioned, that the same general indications as to the comparative development of the different parts during the fattening process, are traceable in the results of the comparable cases of the individual animals selected for Analysis as the types of the different conditions, as in those of the Gradationary Series, from which the illustrations given have been drawn.

From the few summary statements that have been adduced, it is sufficiently obvious—though the details are worthy the closer attention of the Physiologist—that in the feeding or fattening of animals, the apparatus which subserves for the reception, the elaboration, and the transmission, of the food, does not increase so rapidly as those parts which it is the object of the feeder to store up from that food. These parts constitute the saleable "*Carcass*"—or framework, with its covering of flesh and fat. The Tables of ultimate and proximate composition have shown, that of the *flesh* and *fat* of the Carcass, which thus constitute the greater portion of the increase, the former—the *flesh* or *nitrogenous* portion—increases but little during the fattening process; whilst the latter—the *fat*—increases in a very much greater proportion. Of the internal parts again, it is also the *fat* which increases the most rapidly.

The maturing process consists, then, in *diminishing* the proportional amount in the whole body, of the collective muscles, membranes, vessels, internal fleshy organs, and gelatigenous matters—or motive and functional, or, so to speak, working parts of the body—the constituents of which may increase the amount, or replace the transformed portions, of similar matters in the human body. It consists further, in *increasing* very considerably the deposition of *fat*—the most concentrated of the respiratory, and *non*-flesh-forming constituents of human food.

It is then, in our *meat-diet*, of recognized good quality, to which is generally attributed such a high relative "*flesh-forming*" capacity, that we carefully store up such a large proportion of *non*-flesh-forming, but concentrated respiratory material.

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* It will probably be noticed, that the sums of the percentages of the corresponding total offal, and total carcass-parts here quoted, do not quite make up the 100. The complementary amounts represent the "Loss by evaporation, error in weighing, &c."

SECTION X.—SUMMARY, AND CONCLUSION. RELATION OF THE NON-NITROGENOUS OR NON-FLESH-FORMING, TO THE NITROGENOUS CONSTITUENTS, IN ANIMAL FOOD, AND IN BREAD.

It has been established by analysis that the *entire bodies* of some of the most important animals fed and slaughtered for human food, even when in a reputed *lean* condition, may contain more *dry Fat* than *dry Nitrogenous substances*. This was the case with a half-fat Bullock, a store or lean young Sheep, a half-fat old Sheep, and a store or lean young Pig. Of these, the two last, indeed—namely, the half-fat old Sheep, and the lean Pig, contained in their Entire Bodies, nearly one and three quarter time as much *dry Fat* as *dry Nitrogenous matter*.

Of the animals "ripe" for the butcher, a Bullock contained rather more than twice as much *dry Fat* as *Nitrogenous substance*; a moderately fat Sheep nearly three times as much; and a very fat one more than four times as much. A moderately fat Pig contained in its entire body also about four times as much *dry Fat* as *dry Nitrogenous matter*. Even a fat Lamb yielded more than twice as much *Fat* as *Nitrogenous substance*. Of the professedly *fattened* animals, the fat Calf alone contained rather *less* Fat than nitrogenous matter.

Of the 10 animals analysed, the store Sheep, and the store Pig, respectively, were certainly in a much leaner condition than Sheep and Pigs are usually, if ever, slaughtered for food in this country. Sometimes, though seldom, Oxen and Sheep may be killed in as lean a state as the "half-fat Ox," and "half-fat old Sheep." The "fat Calf," the "fat Ox," the "fat Lamb," and the "fat Sheep," may perhaps be taken as fairly representing the average conditions, respectively, of such animals of reputed good quality, and admitted to be properly fattened. The "extra-fat Sheep" was undoubtedly considerably fatter than mutton as usually killed. The "fat Pig" was probably about as fat as the average of the animals consumed in large proportion as fresh pork; but certainly less so, than the average of those fed and slaughtered more exclusively for curing.

One of the most important applications which can be made of a knowledge of the composition of the animals which constitute the chief sources of our *animal* food, is to determine the main points of distinction between such food, and the staple *vegetable* substances which it substitutes or supplements, in an ordinary mixed diet. Of the latter, *Wheaten Bread* is, in this country at least, undoubtedly the most important. This substance therefore—*Wheaten Bread*—is the best that can be taken as the type of our current *vegetable* food-stuffs, for the purposes of any general view of the comparative characters of our chief *animal* and *vegetable* aliments. Obviously too, the first and main point is to attempt to gain some insight into the relative characters of these two prominently contrasted classes of human food-stuffs, in regard to the average proportions which they will probably respectively contain, of *non*-flesh-forming to "*flesh-forming*" constituents. And, for the purposes of the merely general view here contemplated, we shall, in accordance with the usual practice in such discussions of late years, assume the "*non*-flesh-forming" or more specially respiratory and fat-forming capacity of the foods, to be represented, approximately, by the *collective* or *total non-nitrogenous constituents*, and the "*flesh-forming*" capacity to be indicated, conditionally, by the *collective* or *total nitrogenous* constituents of the respective foods. Indeed—neither is our existing knowledge of the adaptation to the various exigencies of the animal economy of the different compounds which our foods supply, so far advanced—nor are our special facts in regard to the composition of the animal aliments we have analysed, of such a character—as to render it desirable to attempt, at present, any more exact mode of classification. With regard to the varying capacity for the purposes of the system of the different *non*-nitrogenous constituents, we shall, however, make some numerical estimates further on. But, so far as the several *nitrogenous* constituents are concerned, we are not in a position to reduce to the form of numerical illustration any distinctions that might be drawn between them. In fact, as already stated, we are not even prepared to give an estimate of how much of the nitrogen of the animal substances we have analysed was due to gelatin and chondrin-yielding matters, and how much to the so-called protein-compounds; yet, on the assumption that the former substances are not to be reckoned as "*flesh-forming*," our estimates of the amounts of such material in the animal substances in question, would be in excess by at least the quantity of gelatin and chondrin-yielding matters which the *total* or *collective* nitrogenous substance may contain.

It is proposed, then, on the present occasion, to confine attention to the contrast between the composition of the *estimated consumable portions* of the animals analysed, and that of *Bread*, in regard alone to the relation in each, of the *non-flesh-forming* to the assumed "*flesh-forming*" constituents—and in regard to this point, so far only, as this can be taken to be illustrated by the relation of the *non-nitrogenous* to the *collective nitrogenous* constituents, in the respective foods.

It is sufficiently obvious, that the composition of the *entire bodies* of our slaughtered animals cannot be taken as representing that of the *consumable portions* only. The composition of the *collective Carcass parts*, and of the *collective Offal parts*, respectively, of the animals analysed, was therefore also determined. As already alluded to, however, a considerable amount of the nitrogen of the *Carcass* will be found in its *bones*, and probably little of this will be consumed as food. On the other hand, a considerable proportion of the *internal organs* rich in nitrogen, will be so consumed. It has been estimated too, that of the *Fat* of the slaughtered animals, the amount contained in the Carcasses cannot always be taken as representing the proportion of the whole Fat of the body which will be consumed. It will be well, therefore, to state briefly here, the basis and mode of computation adopted, and the general result arrived at, in forming an estimate of the probable proportions of the total Nitrogenous compounds, and of the total Fat respectively, of the slaughtered animals, which will, on the average, be applied in some form as *human food*.

In the Calf and Bullocks analysed, about two-thirds of the entire Nitrogenous substances of the body were contained in the so-called *Carcasses*. Of this, say $66\frac{2}{3}$rds per cent. of the whole *nitrogen* of the body thus found in the Carcasses, somewhere about 12 parts were contained in the Carcass-*bones*. Supposing none of this were eaten, there would remain only about $54\frac{2}{3}$rds per cent. of the whole *nitrogen* of the body, in the *soft edible portions* of the Carcass. Of the $33\frac{1}{3}$rd per cent. of the *nitrogen* which is accumulated in the Offal parts, in the Calf perhaps 7 to 8 parts, and in the Oxen perhaps from 4 to 5 parts will be consumed as human food*. Adding this to the portion eaten in the soft parts of the Carcass, it would appear, that in Calves rather more, and in Oxen rather less, than 60 per cent. of the *total nitrogen* of the bodies will be *consumed*. Of the total *Fat* of the same animals, about 70 per cent. in the case of the Calf, and rather over 75 per cent. in the case of the Oxen, were found in the Carcasses. Of the 30 per cent. of the whole Fat of the Calf, which were found in its Offal, all but about 5 parts may be estimated as eaten—that is to say, 95 per cent. of the whole Fat of the animal. Of the 25 per cent. of the Fat in the Offal of the Oxen, about one-fifth is supposed to be consumed. This, with the whole of the Carcass-fat, will show 80 per cent. of the total Fat of Oxen to be applied as food. In estimating the proportion of the consumed *Fat*, to the consumed *Nitrogenous compounds* in *Calves* and *Oxen*, it is assumed then, that in the former 95 per cent. of the total Fat, and 60 per cent. of the total Nitrogenous compounds, will be applied as food; and that in the latter (oxen), 80 per cent. of the total Fat of the body, and 60 per cent. of the Nitrogenous matters will be so applied.

Of the Lamb and Sheep analysed, owing to the large amount of *nitrogen* in the *wool*, little more than half—from 52 to 53 per cent.—of the total amount of the nitrogenous compounds of the body was found in the *Carcass*. About 10 per cent. were contained in the *bones* of the Carcass, of which but little would be consumed as food. To make up for the nitrogen in the bones of the carcass that will not be consumed, from 6 to 7 parts of the 47 or 48 per cent. of the whole nitrogen included in the Offal may be considered as eaten. Thus, in round numbers, it may be estimated, that of the whole of the *Nitrogenous compounds* of the body of the Lamb and Sheep, 50 per cent. are applied as human food. Of the total *Fat* of the Lamb and Sheep, about 75 per cent. will be contained in the Carcass-parts; and this is the proportion of the whole, which, in the case of Sheep, should be taken as eaten. Of the Lamb, however, 95 per cent. of the whole *Fat* of the body is supposed to be consumed.

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* For the amounts, and distribution, of the *Bones* in the different animals, see Tables I. to X., inclusive, in the Appendix, pp. 580—589; also the discussion at pp. 523, 524. For the distribution of the *Nitrogen* in the different parts of the body, see Tables V and VI., pp. 514 and 515, and the discussion upon them. And for the estimates as to which portions of the Offal parts will usually be consumed as food, see p. 524 and [illegible].

In the fattened Pig, supposing its parts to be classified in the same manner as those of the other animals *, about three-fourths of the total *nitrogen* were found in the Carcass. Of these—say 74 to 75 parts—about 4 to 5 parts would be contained in the Carcass-*bones*. This leaves about 70 parts of the total *Nitrogenous compounds* of the fat Pig accumulated in the *soft edible parts* of the Carcass. But 8 parts at least of the whole nitrogen of its body will be consumable from the here reckoned Offal of the Pig. It may be estimated, therefore, that of the whole *Nitrogenous compounds* of the body of this animal, about 78 per cent. will be consumed as human food. Of the total *Fat* of the body of the Pig, about 90 per cent. were found in the parts classified as Carcass. Reckoning that a portion of this, from the "flare," may go for medicinal and perfumery purposes, but that the amount so lost to food is compensated by an equivalent portion of the Offal-fat consumed, it may be estimated that 90 per cent., or somewhere about the proportion of the whole which is contained in the Carcass, will be consumed as food.

The results of these estimates of the proportions of the total Nitrogenous Compounds, and total Fat respectively, of the several descriptions of animals which will be consumed as human food, may be arranged for convenience, at one view, as follows:—

	Per cent. consumed as human Food.	
	Of the Total Nitrogenous Compounds of the body.	Of the Total Fat of the body.
Calves	60	95
Oxen	60	80
Lambs	50	95
Sheep	50	75
Pigs	78	90

Adopting these estimates, it would result, that, in Calves and Lambs particularly, but also in Oxen and Sheep, the *proportion of Fat to nitrogenous compounds* in the *consumed portions*, will be somewhat higher than in the *entire Carcasses* including bone. In Pigs, on the other hand, the proportion of fat to nitrogenous matters will be slightly higher in the entire Carcasses as here classified (excluding head and feet), than in the estimated total consumed portions. The exact proportion to one another of the constituents in question, in the entire Carcasses, and in the total estimated consumed portions, respectively, of each animal, is shown in the first two columns of Table XXIV., which will shortly follow.

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* It has already been sufficiently explained, that, in the cases of the Pigs killed for *analysis*, there was adopted for the convenience of comparison, as nearly as possible the same classification of the parts into *Carcass* and *Offal*, respectively, as with the other descriptions of animal. The deviation from the usual practice consisted, in the cases in question, in classing the Head and Feet with the Offal, instead of with the Carcass. This will not, of course, in any way affect the final result arrived at in the estimates under consideration in the text.

Before any legitimate comparison can be drawn between the composition of our *Animal-food* and that of *Bread*, in regard to the proportion in each, of the *Non*-nitrogenous or *Non*-flesh-forming, to the *Nitrogenous* (or so-called "*Flesh-forming*") constituents, it is, of course, necessary to form an estimate of the probable relative values, for the purposes of the system, of a given amount of the *Fat* in the one, and of the *Starch* which predominates in the other. In calculating the amount of *Starch* which would be required to produce a given amount of the *mixed Fats* of the animal body, it was decided on grounds that were stated (p. 551), that 2·5 parts of *Starch* might be assumed to be requisite for the formation of 1 part of *Fat*. It was stated, that assuming the *mixed fats* of the fattening animal to contain, in round numbers, 77 per cent. of carbon, 12 per cent. of hydrogen, and 11 per cent. of oxygen, it would take pretty exactly 2·45 parts of pure Starch to supply the necessary carbon and hydrogen for 1 part of such a *mixed Fat*, and to yield, at the same time, oxidable material (carbon and hydrogen), to carry off the whole of the superfluous oxygen of the Starch, as carbonic acid and water. But, this being the *minimum* amount of Starch required, and owing also to other considerations stated, the convenient number 2 5 was taken as the amount of Starch required to produce 1 part of the mixed Fats, in the fattening animals. The *non*-nitrogenous substance (not fat) in *Bread* consists chiefly of *Starch*—part of it in a more or less altered condition. There is also a small but variable amount of indigestible matter, and of compounds having a somewhat lower percentage of carbon than Starch itself. We may safely take, then, for our present purpose, the number 2·5 as representing the amount of the *mixed Starch and the other non-nitrogenous substances in Bread* (excluding fat), which will be equivalent to 1 part of *Fat*, for the purposes of storing up Fat in the human body. It would, of course, be quite out of place in such a calculation, to take into account any slight difference between the composition of the mixed Fats in the human body, and of those of the slaughtered animals used as its food.

In the converse estimate to the above, namely, that of how much pure Starch would be equivalent to 1 part of the mixed Fats in oxygen-saturating capacity—that is, for the purposes of respiration—we get of course the same figure 2·45. But for the equivalent of the mixed Starch and other non-nitrogenous constituents, excluding fat, we take the number 2·5. In this purely chemical point of view, therefore, we take 1 part of *Fat in Animal-food*, as equal, in respiratory and fat-forming capacity, to 2½ parts of the *Starch, &c., in Bread.*

The amount of *fat* in the Carcasses—or in the estimated total consumed portions of the entire animals—has then only to be multiplied by 2·5 to bring it to its *starch-equivalent*; or rather, to its equivalent of the *mixed starch and the other non-nitrogenous matters in Bread.* This "starch-equivalent" of the Fat, divided by the amount of nitrogenous constituents, gives, of course, the relation of the more specially respiratory and Fat-forming constituents to 1 of Nitrogenous matters, in the *animal substances*, in such a form that these can be compared, in this respect, with *Bread.*

From all the information at command, we take the average composition of good *wheat-*

flour-bread (whole loaves, crust and crumb together), at 64 per cent. total dry substance[*]. Of this, 1·5 is reckoned as mineral matter (mineral constituents of the flour and salt); 8·2 per cent. nitrogenous compounds (=1·3 per cent. nitrogen); 1 per cent. fat; and the remainder, or 53·3 starch, and allied non-nitrogenous matters. If the 1 per cent. of fat be multiplied by 2·5, and the result added to the 53·3 of starch, &c., we get 55·8 for the per cent. of *starch* or "*starch-equivalent*," in Bread. If this be then divided by 8·2—the amount of the nitrogenous compounds—we get the result 6·8[†] as the *proportion of non-nitrogenous or respiratory and fat-forming matters* (reckoned into "starch-equivalent") to 1 of *nitrogenous* or so-called "*flesh-forming*" *compounds in Bread.* This, then, is the standard by which is compared the composition of the several Carcasses, and that of the estimated *total consumed portions*, of the different slaughtered animals, in order to arrive at an approximate judgment as to the comparative characters of our staple *Animal* and *Vegetable* food-stuffs.

In conformity with the explanations which have been given, the First Division of Table XXIV., which now follows, shows the proportion of actual dry *Fat* to 1 of dry *Nitrogenous compounds*, in—

1st. The *Carcasses* as analysed (including bone).
2nd. The estimated *total consumed portions* of the Entire Animals.

The Second Division of the Table shows the proportion of "*starch-equivalent*" to 1 of *Nitrogenous substances*, in—

1st. The *Carcasses* as analysed (including bone).
2nd. The estimated *total consumed portions* of the Entire Animals.
3rd. *Wheat-flour Bread* (whole loaves, crust and crumb together).

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[*] See Paper, by the authors, "On some Points in the Composition of Wheat-Grain, its Products in the Mill, and Bread," in the Quarterly Journal of the Chemical Society of London, vol. x. part 1.

[†] Since the above estimates were made, we have been favoured by Dr. J. FORBES WATSON with a view of some unpublished results obtained in his own Laboratory. These show, taking the average of 43 loaves of London bread purchased at nearly as many different shops, 63·63 per cent. of total dry matter; 1·44 per cent. of mineral matter; and 1·306 per cent. nitrogen=8·28 nitrogenous compounds. Adopting these data, and allowing 1 per cent. of fat in the bread, we get 6·74 parts of "starch-equivalent" to 1 of nitrogenous matter in bread. Taking again the mean of analyses of 25 different specimens of bread by Dr. ODLING (Journal of the Society of Arts, vol. vi. No. 281), we get, on the same plan of calculation, 6·15 parts of "starch-equivalent" to 1 of nitrogenous compounds in bread. It may be mentioned, however, that the probable average amount of *fatty-matter* in Wheaten Bread is perhaps nearer 0·5 than 1 per cent.; and taking it at 0·5 per cent., the relation of the "starch-equivalent" to 1 of nitrogenous compounds in Bread, would be, according to our own estimate of composition, 6·71; according to the results of Dr. J. F. WATSON, 6·65; and according to those of Dr. ODLING, 6·07.

TABLE XXIV.—Showing the Relation of the Non-nitrogenous or Non-flesh-forming to the Nitrogenous Constituents, in Animal Food, and in Bread.

	Proportion of Dry Fat to 1 of Dry Nitrogenous Compounds.		Proportion of Starch, or of "Starch-equivalent" of Fat, to 1 of Dry Nitrogenous Compounds.	
	In Carcasses including Bone.	In the Estimated Total Consumed Portions of the Animals.	In Carcasses including Bone.	In the Estimated Total Consumed Portions of the Animals.
Store or Lean, and Half-fat Animals.				
Store Sheep	1·64	.....	4·09	......
Store Pig	2·01	......	5·02	. ..
Half-fat Ox	1·27	1·53	3·17	3·83
Half-fat old Sheep	2·11	2·51	5·27	6·28
Fat and very Fat Animals.				
Fat Calf	1·00	1·54	2·49	3·85
Fat Ox	2·31	2·76	5·78	6·91
Fat Lamb	3·39	4·40	8·49	11·01
Fat Sheep	3·96	4·37	9·89	10·93
Very Fat Sheep	6·07	6·28	15·18	15·69
Fat Pig	4·71	4·48	11·77	11·20
Means.				
Of Store and Half-fat Animals	1·76	2·02	4·39	5·05
Of Fat and very Fat Animals	3·57	3·97	8·93	9·93
Of the 10 Animals Analysed	2·85	3·48	7·11	8·71
Wheat-flour Bread.				
Whole Loaves, Crust and Crumb together			6·8	

To the first Division of this Table, which shows only the relation of *fat itself* to the nitrogenous compounds in the staple of our animal food, and without any assumptions as to its probable equivalent food-value compared with other substances, little exception can be taken. The striking fact appears, that, whether we take the composition of the *entire carcasses* including bone, or that of the estimated *total consumed portions* of the animals, the average of those of them supposed to be sufficiently fattened, shows from 3½ to 4 parts of dry Fat, to 1 of dry nitrogenous substances. Taking the estimates of the *total consumed portions* to be pretty near the truth, the fat Calf shows 1½ time, the fat Ox 2¾ times, the fat Lamb, fat Sheep, and the fat Pig nearly 4½ times, and the very fat Sheep 6¼ times as much *dry fat*, as *dry nitrogenous* or so-called "*flesh-forming*" *constituents.* The average of the 10 entire Carcasses even, including bone, and of which several were certainly in a leaner condition than as usually sold, gives 2·85 parts of dry fat to 1 of dry nitrogenous compounds. And lastly, the average of 8 of the animals—that is, excluding only the store Sheep and store Pig—shows in the estimated consumed portions about 3½ parts (3·48) of *dry fat* for 1 of *dry nitrogenous compounds.*

It would perhaps be hardly anticipated, that in the staple of our *meat-diet*, to which such a high relative flesh-forming capacity is generally attributed, there should be found such a large proportion of fat or *non*-flesh-forming, to nitrogenous or assumed flesh-forming constituents, as the figures in the first Division of the Table would show. The result of such a comparison as present knowledge enables us to institute on the point in question, between our staple articles of *Animal*-food, and *Bread*, will certainly not be less surprising.

With regard to the second Division of the Table, in which the Fat of the animal matters is calculated to its supposed respiratory and fat-forming equivalent of the starch and other non-nitrogenous matters occurring in Bread, it is freely granted to the Physiologist, that it is only in a certain broad sense, that such an assumption of equivalency can be admitted. It is nevertheless maintained, that for our present purpose, it is both useful and legitimate to adopt it. Without it, the important comparison sought to be instituted cannot be made; and there is evidence enough both of a practical and scientific kind to show, that, at least to a certain degree, *Fat* and the *starch series of compounds* are really thus mutually replaceable in our foods.

The Table shows, then, that in *Bread*, there are 6·8 parts of *Starch*, or "*starch-equivalent*," to 1 part of *Nitrogenous compounds.* Taking the relation of the one class of constituents to the other in the estimated *total consumed portions* of those animals assumed to be in fit condition for the butcher, there is only one case—that of the fat Calf—in which the proportion of the so-measured respiratory or fat-forming constituents, to the so-measured flesh-forming ones, was, in this our *meat-diet*, lower than in *Bread.* In the estimated total consumed portions of the fat Ox, the proportion of the "starch-equivalent" of the *non*-flesh-forming material (fat), to 1 of nitrogenous compounds, is 6·9; or rather higher than in Bread. In the estimated consumed portions of the Fat Lamb, the Fat Sheep, and the Fat Pig, the proportion of the thus estimated respiratory and fat-forming material to the nitrogenous matters, was rather more than 1½ time as great as in Bread. In the Extra-fat Sheep, it was more than twice as great.

The average of the six cases in which the animals were supposed to be "ripe" for the butcher, shows, in the *estimated consumable portions*, nearly 10 parts of the "starch-equivalent" of specially respiratory or fat-forming material, to 1 of nitrogenous compounds; that is, nearly 1½ time as much as in *Bread.* In the Half-fat Ox, and the Half-fat old Sheep, neither of which, however, were in the condition of fatness of Oxen and Sheep as usually killed, the relation of the "starch-equivalent" to the nitrogenous compounds (in the consumable portions), was lower than in Bread; namely, as 3·83 to 1 in the Half-fat Ox, and as 6·28 to 1 in the Half-fat old Sheep.

Taking the *carcasses as analysed*, including bone, the relation of the *non*-nitrogenous (starch-equivalent) *to* the nitrogenous constituents, is, in them also, in most cases *higher* *than in Bread.* In the Fat Calf and Fat Ox, however, it is lower, namely, only 2·49 to 1 in the former, and 5·78 to 1 in the latter. On the other hand, in the Carcass of the Fat Lamb, the proportion of the "starch-equivalent" is 8·40, in that of the Fat Sheep 9·89, in that of the Fat Pig 11·77, and in that of the Extra-fat Sheep 15·18 to 1 of the nitrogenous matters. The *average* of the 6 *matured* Carcasses gives 8·93, and that of those of the 10 animals analysed 7·11 of *non*-nitrogenous matter, reckoned as "starch-equivalent," to 1 of nitrogenous substance—that is, *rather more than in Bread.*

Of the fact of the *increase* in the proportion (so far as its respiratory and fat-forming capacity is concerned) of the *non*-nitrogenous to the nitrogenous matter of our food, by the use of these *Animal* aliments, the evidence adduced can hardly leave a doubt. It will perhaps be maintained, that when animals are so far fattened as to attain the result here supposed, the feeder is simply inducing disease in the animals themselves, and frustrating that which it is considered should be the special advantage of a *meat-diet*—namely, the increase in the relative supply of the *nitrogenous* constituents in our food. It is not denied, that occasionally animals are over-fed, and that a condition bordering on disease is so induced. But such is certainly not the rule. There can, indeed, be little doubt, that in animals that would be admitted, by both producer and consumer, to be in only a proper condition of fatness, there would be a *higher relation* of respiratory and fat-forming capacity, so to speak, in their total consumed portions, than in the average of our *staple Vegetable* foods. It may be true, that with the modern system of bringing animals very early to the knife, by means of abundance of food and the avoidance of cold and exercise, the development of *Fat* will be *greater*, and that of the *muscles and other nitrogenous parts less*, than would otherwise be the case. But it is at the same time certain, that if *meat* is to be economically produced, so as to be within the reach of the masses of the population, it can only be so on the plan of "*early maturity.*" Nor can it be questioned, that the admixture of the meat *so produced*, with their otherwise vegetable diet, is, in practice, of great advantage to the health and vigour of those who consume it.

It is not to be assumed, that in every dish of fresh meat, the relation of the Fat to the Nitrogenous matter will be as high as in the estimated consumed portions of the animals to which our Table refers. The collective joints, as sold, will frequently have a less proportion of fat, than the whole carcasses from which they are taken. A further amount will be removed in the process of cooking; though this portion will generally be consumed in some form or other. But the consumers of fresh meat generally consume also suet, lard, and butter; which either add to the fatness of the cooked meats, or are used further to reduce the relation of the nitrogenous constituents in the collaterally consumed vegetable foods. But, even were it granted, that the proportions of Fat to Nitrogenous substance which our Table shows were too high, it must on the other hand be remembered, that a considerable portion of the Nitrogenous matter of the animals will be gelatin or chondrin-yielding substance, the applicability of which, at any rate for strictly speaking flesh-forming, is, to say the least, doubted.

It would appear to be unquestionable, therefore, that the influence of the introduction of our staple *animal foods*, to supplement our otherwise mainly farinaceous diet, is, on the large scale, to *reduce*, and *not to increase*, the relation of the *assumed* flesh-forming material, to the more peculiarly respiratory and fat-forming capacity, so to speak, of the food consumed.

That, nevertheless, a dietary containing a due proportion of *animal food*, is, for some reason or other, better adapted to meet the collective requirements of the human organism, at least under many conditions, than an *exclusively Bread*, or *other vegetable one*, the testimony of common experience may be accepted as sufficient evidence. Independently of any difference in the physical, and perhaps even chemical relations of the supposed flesh-forming nitrogenous compounds in *animal food*, which may render them, at least in limited quantity, more easily available to the purposes of the system than the assumed analogous *vegetable products*, it is at any rate clear, that the main and characteristic distinction between a *Bread*—and a *mixed Bread and Meat diet*—consists, not only in the quantitatively higher relation of the respiratory and fat-forming capacity to a given amount of assumed flesh-forming material in the latter, but in the fact, that the *non*-flesh-forming constituents in the *animal* portions of the food, are in the form of *fat itself*—and not as in *Bread*, of mainly *starch*.

In *fat*, we have the most concentrated respiratory—and of course *fat*-storing material also—which our food-stuffs supply. But independently of the far greater capacity, so far as the supply of constituents is concerned, of a given weight and bulk of *Fat* compared with *Starch* and the other substances of its class—would it not seem probable, that the tax upon the system would be less, at least for *Fat-storing*, if not in a degree for respiration also, in the case of the *ready-formed Fat*, than in that of the *Starch from which it may be formed*?

Again, it has been shown that *Fat* subserves important purposes in aiding the digestion, and preparation for assimilation, of the matters ingested with it. And certainly the natural distribution and blending of the Fat with the nitrogenous compounds in *meat*, is such as is not met with in our staple vegetable foods. May it not too be supposed, that its liberal distribution with the transforming nitrogenous matters throughout the body, will modify the character of the changes constantly going forward, from that which would obtain, were the needed oxidable material kept up in larger proportion through the means, more or less directly, of the current supplies of Starch, and other matters not Fat, in the food?

But whatever may prove to be the exact explanations of the benefits arising from a mixed animal and vegetable diet, it is at any rate clear, that they are essentially connected with the *amount*, the *condition*, and the *distribution*, of the *Fat* in the *animal* portions of the food. It is true, that the very basis of some of our illustrations has been the assumption, that *Starch* and its analogues on the one hand, and *Fat* on the other, are, in a certain sense, and within certain limits indifferently, mutually replaceable;—and further, that they are so, in approximately measurable proportions. It is, however, certain, that independently of the mere *supply* of constituents, the conditions of *concentration*, and *digestibility*, and consequently of *assimilability* of our different foods, must have their share in determining the relative values for the varying exigencies of the system, of substances which, in a more general, or more purely chemical sense, may still justly be looked upon as mutually replaceable. It would, indeed, hardly be supposed, that substances so distinct, both morphologically and chemically, as *Fat* and *Starch*, will, under all circumstances, be equally adapted to the conditions supplied within the living organism, and be so at an equal cost to its energies;—even though they may each eventually subserve the Fat-storing and respiratory requirements of the body.

It is, then, fully granted, that in the study of this branch of Physiological Chemistry, as in that of others, the facts peculiar to *Physiology herself* must be allowed an important place; though, it is at the same time claimed, that those of *Chemistry* be not excluded. By the aid of *Chemistry* it may be established—that, in the admixture of *Animal* food with *Bread*, the relation, in estimated respiratory and fat-forming capacity, of the *non*-nitrogenous to the Nitrogenous substance, will be increased; and further—that, in such a mixed diet, the proportion of the non-nitrogenous constituents which will be in the concentrated condition, so to speak, of *fat itself*, will be considerably greater than in Bread alone. Common experience testifies, moreover, that certain advantages are so derived. It is for Physiology to lend her aid to the full explanation of the facts and conclusions which Chemistry and common usage may in their turn determine.

APPENDIX.

APPENDIX.

TABLE I.—Showing the Fresh Weights of the various Organs and Parts of A FAT CALF, killed for Analysis, September 12, 1849; also the results of the Determinations of *Dry Matter* and *Ash* in the same.

Designation of Parts.	Original fresh weights.	Dry matter at 212°.			Mineral matter (ash).			
		Exclusive of melted fat.	Melted fat.	Total dry matter.	Proportion of whole, and calculated weights, of dry matters taken for burning.		Actual ashes obtained.	Ashes calculated for the whole dry matters.
CARCASS.								
	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.		oz.	oz.	lbs. oz.
Half-carcass... Flesh and kidney	65 13·46	13 9·70	8 1·50	21 11·20	1/10	21·770	0 994	0 9·94
Half-carcass... Kidney fat	2 12·94	0 1·58	2 1·11	2 2·69	½	0 792	0·024	0 0·05
Half-carcass... Bones	11 14·66	6 8·58	......	6 8 58	1/10	10 458	4·780	2 15·80
Totals of half-carcass operated on	80 9·06	20 3·86	10 2 61	30 6·47	...	......	......	3 9·79
Other half-carcass (dry, &c., calculated)	79 15·94	20 1·57	10 1·46	30 3 03	...	......	......	3 9·38
Whole carcass	160 9·00	40 5·43	20 4·07	60 9·50	...	......	......	7 3·17
ORGANS OR PARTS CONSTITUTING "OFFAL."								
Stomachs (washed)	2 13 10	0 4·66	... .	... ..	½	2·329	0·111	0 0·22
Caul-fat and membrane	2 7·56	0 5·01*	......	......	¼	2·015	0 070	0 0·28
Small intestines (washed) Large intestines (washed)	4 8·30 2 0 50	0 15·44	......	......	½	7·717	0·380	0 0·76
Intestinal fat and membrane	4 3·11	0 3·05	......	......		‡	‡	‡
Heart and aorta	1 7·56	0 5·05	......	......	½	2·527	0·215	0 0·43
Heart-fat (with membrane)	0 6·80	†	......	......	..	†	†	†
Lungs and windpipe	3 5·70	0 13·50	......	......	½	6·777	0·257	0 0·51
Blood	13 8·79	2 8·13	......	......	½	20·061	1·192	0 2·39
Liver	4 3·57	1 0·34	......	......	½	8·172	0·432	0 0·86
Pancreas ("sweetbread") Thymus gland ("heartbread") Glands about the throat ("throatbread")	1 12 90	0 5·31	8 4·04	27 0·64	½	2·652	0·112	0 0·22
Milt or spleen	0 12 01	0 2·72	... ..	......	½	1·362	0·082	0 0·16
Bladder, &c.	0 7·20	0 1·58	......	......	½	0 787	0·034	0 0·07
Head flesh	7 13·84	1 7 86	......	.... .	¼	5·965	0 229	0 0·92
Head bones	3 9 46	1 14·98	... .	......	½	15 490	8 560	1 1·12
Pelt	16 13 71	4 5·80	......	......	⅕	13·972	0 419	0 2·10
Hair	0 14 78	0 10·58	......	.. ...	½	5·292	0·090	0 0·18
Feet, hoofs, &c.	4 5 99	2 14·11	......	......	⅕	9·222	3·082	0 15·41
Tail flesh	0 3·48	0 0·97	......	......	½	0 487	0·016	0 0·03
Tail bones	0 1·86	0 0·94	......	......	½	0·471	0·159	0 0·32
Diaphragm ("skirts")	1 1·60	0 4·45	......	......	¼	1·112	0 040	0 0·16
Total offal	77 1·82	18 12·60	8 4·04	27 0·64	...	......	......	2 10·14
SUMMARY:—ENTIRE ANIMAL.								
Total carcass	160 9·00	40 5·43	20 4·07	60 9·50	...	......	......	7 3·17
Total "offal" parts	77 1·82	18 12·60	8 4·04	27 0·64	...	......	......	2 10·14
Contents of stomachs	5 10 20	......	... ..	......	...	......	......	. ..
Contents of intestines	2 8·70	.. ...	... ..	......	...	......	......	......
Loss by evaporation, error in weighing, &c.	12 14 28	. ....	.. ..	.... .	...	......	. ....	.... .
Entire animal (live-weight after fasting)	258 12·00	59 2·03	28 8·11	87 10·14	...	.....	......	9 13·31

Amount and distribution of the fat obtained by melting and expression (lbs., ozs., and tenths).

In carcass (including kidney fat)				20 4·07
In offal...	From caul From intestines From region of heart	5 7·78	8 4·04	
	From remaining parts	2 12·26		
			Total	28 8·11

Amount and distribution of bones (including hoofs), lbs., ozs., and tenths.

In carcass	First half (by experiment)	11 14·66	23 11·97
	Second half (by calculation)	11 13 31	
In offal...	Head	3 9·46	8 1·31
	Feet and hoofs	4 5·99	
	Tail	0 1·86	
	Total		31 13·28

* Including dry matter of heart-fat with membrane. † With caul-fat and membrane. ‡ With caul-fat-membrane.

APPENDIX.—TABLE II. Showing the Fresh Weights of the various Organs and Parts of A HALF-FAT OX, killed for Analysis, November 14, 1849; also the results of the Determinations of *Dry Matter* and *Ash* in the same.

Designation of Parts.	Original fresh weights.	Dry matter at 212°. Exclusive of melted fat.	Dry matter at 212°. Melted fat.	Dry matter at 212°. Total dry matter.	Mineral matter (ash). Proportion of whole, and calculated weights, of dry matters taken for burning.		Mineral matter (ash). Actual ashes obtained.	Mineral matter (ash). Ashes calculated for the whole dry matters.
CARCASS.								
	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.		oz.	oz.	lbs. oz.
Half-carcass. Flesh and fat	332 0·98	82 6·79	47 11·65	130 5·44	1/10	131·300	4·285 0·051	2 11·39
Half-carcass. Kidney fat and membrane	13 9·51	0 8·96	12 4·00	12 12·96	1/4	4·480	0·183	0 0·37
Half-carcass. Kidney	1 2·71	0 4·84	......	0 4·81	[illegible]	1·210	0·019	0 0·20
Half-carcass. Flesh juice	......	2 1·00	....	2 4·00	[illegible]	9·000	1·816	0 7·38
Half-carcass. Bones	52 5·77	37 12·23	.....	37 12·23	1/10	60·423	30·102	19 0·02
Totals of half-carcass operated on	399 3·00	123 4·82	60 2·65	183 7·47	...	......	...	22 3·36
Other half-carcass (dry, &c., calculated).	398 8·00	123 1·42	60 0·99	183 2·41	...	.....	...	22 2·75
Whole carcass	797 11·00	246 6·24	120 3·61	366 9·88	...	...	...	44 6·11
ORGANS OF PARTS CONSTITUTING "OFFAL."								
Stomachs (washed)	32 1·00	6 1·17	.....	...	1/10	9·717	0·103	0 4·93
Caul-fat and membrane	16 9·50	0 4·99	.....	...	1/4	2·196	0·098	0 0·19
Small intestines (washed)	8 5·00	2 10·48	.	.	1/15	4·218	0·138	0 1·38
Large intestines (washed)	6 0·00							
Intestinal fat and membrane	19 11·50	1 11·65	.	...	1/4	0·912	0·180	0 0·72
Heart and aorta	5 13·50	1 9·16	.	. .	1/4	0·365	0·220	0 0·88
Heart-fat with membrane	2 6·75	0 1·61	...	. .	1/4	0·801	0·033	0 0·07
Lungs and windpipe	7 12·75	1 13·53	.	.	1/4	7·382	0·308	0 1·59
Blood	51 5·00	11 11·41	...		1/10	18·741	0·812	0 8·12
Liver	15 11·00	4 7·34	...	...	1/10	7·131	0·380	0 3·80
Pancreas ("sweetbread")	1 0·00	0 3·81	...		1/4	1·020	0·096	0 0·19
Thymus gland ("heartbread")	0 10·75	0 0·99	...	.	1/4	0·198	0·034	0 0·07
Glands about the throat ("throatbread")	0 6·25	0 1·30	....	.....	1/4	0·050	0·010	0 0·08
Milt or spleen	2 2·25	0 8·40	. .	...	1/4	4·232	0·263	0 0·52
Bladder	0 5·00	0 4·22						
Gall-bladder	0 3·00		38 13·88	130 5·81	1/4	2·100	0·071	0 0·14
Penis	0 6·50							
Brains	0 14·25	0 3·17	....	.....	1/4	1·583	0·109	0 0·22
Tongue	7 7·50	1 12·75	......	.....	1/4	7·187	0·310	0 1·40
Head flesh	17 6·50	3 5·12	.....	......	1/10	5·312	0·275	0 2·75
Head bones	14 2·00	10 14·00	.....	......	1/10	17·400	10·141	6 5·41
Hide (and horns, &c.)	72 7·98	12 12·25 12 12·25	......	.....	1/20	20·425	0·492	0 9·84
Hair	7 7·38	4 10·16	......	......	1/4	18·540	0·459	0 1·84
Leg bones	13 0·00	7 3·23	......	......	1/10	11·523	5·250	3 4·50
Hoofs and heels	7 2·00	3 12·34	......	......	1/4	15·085	3·397	0 13·31
Tail flesh	1 5·21	0 6·71	......	......	1/4	1·077	0·053	0 0·21
Tail bones	0 7·15	0 5·80	......	......	1/4	2·808	1·110	0 2·22
Diaphragm ("skirts")	5 11·50	1 8·82	......	.....	1/4	0·205	0·187	0 0·75
Miscellaneous trimmings	1 7·00	0 2·88	......	......	1/4	1·438	0·016	0 0·09
Total offal	322 12·25	91 7·03	38 13·88	130 5·81	...	......	...	13 1·22
SUMMARY:—ENTIRE ANIMAL.								
Total carcass	797 11·00	246 6·24	120 3·64	366 9·88	...	......	......	44 6·11
Total offal parts	322 12·25	91 7·03	38 13·88	130 5·81	...	......	......	13 1·22
Contents of stomachs, and vomit	88 0·00	.....	......	......	...	......	...	...
Contents of intestines, and bile	12 14·00	......	......	......	...	......	...	..
Loss by evaporation, error in weighing, &c.	10 10·75	......	......	.....	..	......	......	.....
Entire animal (live-weight after fasting)	1232 0·00	337 14·17	159 1·52	496 15·69	...	.....	...	57 7·33

Amount and distribution of the fat obtained by melting and expression (lbs., ozs., and tenths).

In carcass (including kidney fat)			120 3·64
In offal...	From caul	14 14·97	38 13·88
	From intestines	15 0·50	
	From region of heart	2 13·36	
	From remaining parts	6 1·05	
	Total		159 1·52

Amount and distribution of bones (including hoofs), lbs., ozs., and tenths.

In carcass	First half (by experiment)	52 5·77	104 10·10
	Second half (by calculation)	52 4·33	
In offal...	Head	14 2·00	34 11·15
	Legs	13 0·00	
	Feet, hoofs, and heels	7 2·00	
	Tail	0 7·15	
	Total		139 5·25

APPENDIX.—TABLE III. Showing the Fresh Weights of the various Organs and Parts of A FAT OX, killed for Analysis, October 30, 1849; also the results of the Determinations of *Dry Matter* and *Ash* in the same.

Designation of Parts.	Original fresh weights.	Dry matter at 212°.			Mineral matter (ash).		
		Exclusive of melted fat.	Melted fat.	Total dry matter.	Proportion of whole, and calculated weights, of dry matters taken for burning.		Actual ashes obtained.	Ashes calculated for the whole dry matters.
CARCASS.								
	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.		oz.	oz.	lbs. oz.
Half-carcass... Flesh and fat........	386 10·32	72 5 70	115 5·16	187 10·86	1/10	115·770	4·275	2 10·75
Half-carcass... Kidney	1 8·77	0 6·07	..	..	1/2	3 034	0·134	0 0·27
Half-carcass... Kidney fat and membrane	26 11·40	0 14·56	25 9·50	27 10·72	1/2	7·280	0·215	0 0·49
Half-carcass... Flesh juice....	....	0 12·59	.. ...	...	1/2	6·295	1·110	0 2·22
Half-carcass... Bones..	55 8·01	40 11·20	.	40 11·20	1/10	40·700	18·594	18 9·50
Totals of half-carcass operated on	470 6 50	115 2 12	140 14·66	256 0·78	...	......	......	21 7·23
Other half-carcass (dry, &c., calculated).	468 15·50	114 12·49	140 7·77	255 4·26	..	.. .	......	21 6·18
Whole carcass.	939 6·00	229 14·61	281 6·43	511 5·04	...	.. ..	.. ...	42 13·41
ORGANS OF PARTS CONSTITUTING "OFFAL."								
Stomachs (washed)	36 6 00	6 9·05	. ..	... ..	1/4	26·202	1·130	0 4·52
Caul-fat and membrane	29 13·00	0 14 52*	......	......	1/4	3·630	0·089	0 0·28
Heart, trimmings, &c.	3 15·50							
Small and large intestines (washed)	13 14·25	1 12·59	......	......	1/4	7·147	0·285	0 1·14
Intestinal fat and membrane	36 14·00	1 5·88	......	......	1/2	10·940	0·480	0 0·96
Heart and aorta.............................	7 6·00	1 7·12	......	......	1/4	5·780	0·245	0 0·98
Heart-fat (with membrane)...................	6 3·00	0 3·01	......	......	1/2	1·955	0·061	0 0·13
Lungs and windpipe..........................	8 14·50	2 0·29	......	......	1/4	8·072	0·401	0 1·62
Blood.......................................	52 11·75	10 12·46	......	......	1/10	17·246	0·723	0 7·23
Liver	17 10·00	5 0·70	.. ...	......	1/10	8·070	0·385	0 3·85
Pancreas ("sweatbread")..	0 15·50	0 2·39	......	......	1/4	0·598	0·040	0 0·16
Thymus gland ("heartbread")	0 10·50	0 1·15	......	......	1/2	0·575	0·047	0 0·09
Glands about the throat ("throatbread") ...	0 6 50	0 1·03	......	.....	22/25	0·744	0 052	0 0·07
Milt or spleen	2 4·00	0 8·77	......	......	1/4	2·192	0·116	0 0·46
Gall-bladder	1 15·08	0 7·32	83 13·05	177 4·34	1/4	1·830	0·061	0 0·24
Bladder								
Penis								
Brains......................................	0 14·00	0 3·14	.. ..	.. ..	1/2	1·500	0·110	0 0·22
Tongue	3 7·50	0 15·40	.... .	.. ...	1/4	3·872	0·148	0 0·59
Head flesh	25 9·50	5 6·56	......	......	1/10	8·650	0·300	0 3·00
Head bones	14 6·00	10 4·40	.. ...	......	1/10	16·440	9 350	5 13·50
Hide (and horns, &c.)	73 15·49	25 13 44	......	......	1/20	20·672	0·525	0 10·50
Hair	6 4·21	4 14·60	......	......	1/4	19·650	0·315	0 1·26
Leg bones	13 13 00	8 2·30	......	......	1/10	13·030	5·711	3 9·11
Hoofs and heels.............................	8 10·80	4 1·98	......	......	1/10	6·508	1·370	0 13 70
Tail flesh	1 0 70	0 4·45	... .	......	1/2	2·227	0·055	0 0·11
Tail bones	0 6 80	0 5·40	......	......	1/2	2·700	1·054	0 2·11
Diaphragm ("skirts")	7 9·00	1 8 35	, ...	.. ...	1/4	6·087	0·193	0 0 77
Miscellaneous trimmings†.	......	......	......	......	...	......	......	......
Total offal	376 0·58	93 7·29	83 13·05	177 4·34	...	......	......	12 12·60
SUMMARY:—ENTIRE ANIMAL.								
Total carcass	939 6·00	229 14·61	281 6·43	511 5·04	...	......	.....	42 13·41
Total offal parts...	376 0·58	93 7·29	83 13·05	177 4·34	...	......	.....	12 12·60
Contents of stomachs, and vomit............	84 12·67	......	......	......	...	......	......	.. ...
Contents of intestines, and bile		......	......	......	...	......	......	......
Loss by evaporation, error in weighing, &c.	18 12·75	......	......	......	...	......	......	......
Entire animal (live-weight after fasting) ...	1419 0 00	323 5·90	365 3·48	688 9·38	...	......	......	55 10·01

Amount and distribution of the fat obtained by melting and expression (lbs., ozs., and tenths).

In carcass (including kidney fat)			281 6·43
In offal...	From caul........................	29 13·00	83 13·05
	From intestines..................	33 1·00	
	From region of heart.............	6 6·00	
	From remaining parts.............	14 9·05	
Total			365 3·48

Amount and distribution of bones (including hoofs), lbs., ozs., and tenths.

In carcass	First half (by experiment)	55 8·10	110 13·30
	Second half (by calculation)	55 5·29	
In offal...	Head	14 6·00	37 4·60
	Legs	13 13·00	
	Feet, hoofs, and heels	8 10·80	
	Tail	0 6·80	
	Total		48 1·90

* These parts accidentally mixed.

† Accidentally mixed with caul-fat.

APPENDIX.—TABLE IV. Showing the Fresh Weights of the various Organs and Parts of A FAT LAMB, killed for Analysis, August 17, 1840; also the results of the Determinations of *Dry Matter* and *Ash* in the same.

Designation of Parts.	Original fresh weights.	Dry matter at 212°.			Mineral matter (ash).			
		Exclusive of melted fat.	Melted fat.	Total dry matter.	Proportion of whole, and calculated weights, of dry matters taken for burning.		Actual ashes obtained.	Ashes calculated for the whole dry matters,
CARCASS.								
	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.		oz.	oz.	lbs. oz.
Half-carcass... Flesh and fat..........	21 1·34	3 3·66	6 8·10	9 11·76	½	25·830	0·960	0 1·92
Half-carcass... Kidney fat..	1 11·20	0 0·46	1 9·20	1 9·66	½	0·230	0·008	0 0·02
Half-carcass... Bones....	2 9·41	1 11·17	......	1 11·17	½	13·585	6·405	0 12·81
Totals of half-carcass operated on	25 6·04	4 15·29	8 1·30	13 0·59	...	......	......	0 14·75
Other half-carcass (dry, &c., calculated)......	25 1·96	4 14·49	8 0·00	12 14·49	...	.....	......	0 14·60
Whole carcass.............	50 8·00	9 13·78	16 1·30	25 15·08	...	......	......	1 13·35
ORGANS OR PARTS CONSTITUTING "OFFAL"								
Stomachs (washed)	1 8·64	0 5·01	.. ...	......	½	2·505	0·088	0 0·18
Caul-fat and membrane	3 4·02	0 1·18*	......	......	½	0·590*	0·038*	0 0·08*
Small intestines................................. Large intestines................................	2 0·35	0 6·49	......	......	½	3·245	0·157	0 0·31
Intestinal fat and membrane	1 10·83	†	......	......	...	‡	‡	‡
Heart and aorta	0 5·37	0 1·17	.....	.....	½	0·585	0·022	0 0·04
Heart-fat and membrane..........................	0 4·64	†	......	.....	...	‡	‡	‡
Lungs and windpipe	1 0·80	0 5·10	......	... .	½	2·550	0·083	0 0·17
Blood..	2 14·27	0 10·12	......	......	½	5·062	0·245	0 0·49
Liver	1 2·85	0 5·68	3 14·80	10 14·77	½	2·840	0·137	0 0·27
Pancreas ("sweetbread").......... .... Thymus gland ("heartbread") .... Glands about the throat ("throatbread") ...	0 2·47§	0 0·57§	......	......	½	0·285§	0·017§	0 0·03§
Milt or spleen	0 2·58	0 0·66	......	......	½	0·330	0·019	0 0·04
Head flesh	1 11·60	0 5·41	......	......	½	2·705	0·150	0 0·30
Head bones	0 14·49	0 9·00	......	......	½	4·498	2·534	0 5·07
Pelt	4 15·19	1 7·36	......	......	½	11·678	0·580	0 1·16
Wool	3 1·71	1 15·10	......	......	¼	7·775	0·256‖	0 1·09‖
Feet, hoofs, &c.	0 12·70	0 5·74	......	......	¼	1·435	0·278	0 1·11
Diaphragm ("skirts").............................	0 4·78	0 1·38	......	......	½	0·692	0·018	0 0·04
Total offal	26 5·29	6 15·97	3 14·80	10 14·77		......	......	0 10·31
SUMMARY:—ENTIRE ANIMAL.								
Total carcass	50 8·00	9 13·78	16 1·30	25 15·08	...	......	......	1 13·35
Total offal parts	26 5·29	6 15·97	3 14·80	10 14·77	...	......	......	0 10·31
Contents of stomachs	5 2·06	......	......	......	...	......	......	......
Contents of intestines	2 1·35	......	......	......	...	......	......	......
Loss by evaporation, error in weighing, &c.	0 5·80	......	......	......	...	......	......	......
	84 6·50	16 13·75	20 0·10	36 13·85	...	......	......	2 7·66

Amount and distribution of the fat obtained by melting and expression (lbs., ozs., and tenths).

In carcass (including kidney fat)			16 1·30
In offal... From caul .... From intestines............ From region of heart ...	3 10·40	3 14·80	
In offal... From remaining parts	0 4·40		
Total			20 0·10

Amount and distribution of bones (including hoofs), lbs., ozs., and tenths.

In carcass First half (by experiment) ...	2 9·41	5 2·40
In carcass Second half (by calculation)...	2 8·99	
In offal... Head	0 14·49	1 11·19
In offal... Feet and hoofs	0 12·70	
Total		6 13·59

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* Including dry matters of intestinal and heart-fat and their membranes. † With caul-fat. ‡ With caul-fat-membrane.

§ It is doubtful whether these amounts refer to the Pancreas, Thymus Gland, and Glands about the Throat collectively, or to the Pancreas alone.

‖ The amounts of Mineral Matter in the Wool are stated too high, owing to adherent dirt.

APPENDIX.—TABLE V. Showing the Fresh Weights of the various Organs and Parts of A STORE SHEEP, killed for Analysis, February 28, 1850; also the results of the Determinations of *Dry Matter* and *Ash* in the same.

Designation of Parts.	Original fresh weights.	Dry matter at 212°.			Mineral matter (ash).			
		Exclusive of melted fat.	Melted fat.	Total dry matter.	Proportion of whole, and calculated weights, of dry matters taken for burning.		Actual ashes obtained.	Ashes calculated for the whole dry matters.
CARCASS								
	lbs. ozs.	lbs ozs.	lbs. ozs.	lbs. ozs.		ozs.	ozs.	lbs. ozs.
Half-carcass: Flesh and fat	21 9·91	3 10·96	4 8·50	8 3·46		14·740	1·286	0 5·14
Half-carcass: Kidney	0 2·17	0 0·48	. ..	0 0·48		0·120	0·006	0 0·02
Half-carcass: Kidney fat and membrane	0 8·85	0 0·45	0 7·00	0 7·15		0·114	0·008	0 0·01
Half-carcass: Bones	3 8·01	2 4·85	. ..	2 4·85		9·212	3·207	0 12·83
Totals of half-carcass operated on	25 12·94	6 0·74	4 15·50	11 0·24	...	.	. .	1 2·00
Other half-carcass (dry, &c., calculated)	26 4·06	6 2·41	5 0·87	11 3·28	.	. . .	.. ...	1 2·31
Whole carcass	52 1·00	12 3·15	10 0·37	22 3·52	...	......	......	2 4·31
ORGANS OR PARTS CONSTITUTING "OFFAL."								
Stomachs (washed)	3 3·83	0 5·98	.. ...	.. ...	[illegible]	2·243	0·105	0 0·28
Caul-fat and membrane	3 2·70	0 1·08	... ..	......	[illegible]	0·406	0·016	0 0·01
Small intestines (washed)	1 1·04	0 4·71	.....	......	[illegible]	1·767	0·078	0 0·21
Large intestines (washed)	1 7·09							
Intestinal fat and membrane	1 9·40	0 1·24	......	.. ...	[illegible]	0·465	0·020	0 0·05
Heart and aorta	0 8·30	0 1·78	......	.. ...	[illegible]	0·667	0·026	0 0·07
Heart-fat (with membrane)	0 3·90	0 0·12	......	......	[illegible]	0·044	0·004	0 0·01
Lungs and windpipe	1 6·95	0 6·44	......	......	[illegible]	2·413	0·091	0 0·24
Blood	5 1·25	0 13·76	......	......	[illegible]	5·158	0·208	0 0·56
Liver	1 10·50	0 8·46	......	.....	[illegible]	3·173	0·138	0 0·37
Pancreas ("sweetbread")	0 1·13	0 0·62			[illegible]	0·235	0·021	0 0·06
Thymus gland ("heartbread")	0 1·10		4 5·05	13 9·25				
Glands about the throat ("throatbread")	0 1·00							
Milt or spleen	0 2·50	0 0·50	.. ...	.....	[illegible]	0·187	0·011	0 0·03
Gall-bladder	0 0·11	0 0·22	. ...	.....	[illegible]	0·083	0·008	0 0·02
Bladder	0 1·00							
Head flesh	2 6·00	0 6·08		.. .	[illegible]	2·280	0·131	0 0·35
Head bones	1 3·25	0 12·68	.	.....	[illegible]	4·755	2·059	0 5·49
Pelt	5 8·57	1 10·00	.. ..	.. ..	[illegible]	9·750	0·543	0 1·45
Wool	7 8·25	3 1·80	. ....	. .	[illegible]	21·900	1·202*	0 2·40*
Feet, hoofs, &c.	1 0·68	0 7·79	.....	......	[illegible]	2·920	0·539	0 1·44
Diaphragm ("skirts")	0 3·87	0 0·04	......	.....	[illegible]	0·354	0·013	0 0·03
Total offal	37 6·92	9 4·20	4 5·05	13 9·25	...	......	......	0 13·10
SUMMARY:—ENTIRE ANIMAL.								
Total carcass	52 1·00	12 3·15	10 0·37	22 3·52	..	... ..	......	2 4·31
Total offal parts	37 6·92	9 4·20	4 5·05	13 9·25	...	......	......	0 13·10
Contents of stomachs	4 13·80	......	......	......	..	.....	......	......
Contents of intestines, and bile	0 15·90	......	......	.. ...	...	.. ...	......	......
Loss by evaporation, error in weighing, &c.	2 4·38	.....	. ...	... ..	...	. . ..	......	......
Entire animal (live-weight after fasting)	97 10·00	21 7·35	14 5·42	35 12·77	...	......	......	3 1·41

Amount and distribution of the fat obtained by melting and expression (lbs., ozs., and tenths).

In carcass (including kidney fat)			10 0·37
In offal	From caul	1 14·80	4 5·05
	From intestines	1 2·48	
	From region of heart	0 2·60	
	From remaining parts	1 1·22	
	Total		14 5·42

Amount and distribution of bones (including hoofs), lbs., ozs., and tenths.

In carcass	First half (by experiment)	3 8·01	7 0·98
	Second half (by calculation)	3 8·97	
In offal	Head	1 3·25	2 3·93
	Feet and hoofs	1 0·68	
	Total		9 4·91

* The amounts of Mineral Matter in the Wool are stated too high, owing to adherent dirt.

APPENDIX.—TABLE VI. Showing the Fresh Weights of the various Organs and Parts of A HALF-FAT OLD SHEEP, killed for Analysis, May 3, 1849; also the results of the Determinations of *Dry Matter* and *Ash* in the same.

Designation of Parts.	Original fresh weights.	Dry matter at 212°.			Mineral matter (ash).		
		Exclusive of melted fat.	Melted fat.	Total dry matter.	Proportion of whole, and calculated weights, of dry matters taken for burning.		Actual ashes obtained.	Ashes calculated for the whole dry matters.
CARCASS.								
	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.		oz.	oz.	lbs. oz.
Half-carcass .. Flesh and fat	25 7·87	5 3·01	5 11·59	10 14·60	1/5	16·602	0·568	0 2·84
Half-carcass .. Kidney fat	…	0·47	1 0·20	3 3·94	4/10	0·188	0·001	0 0·01
Half-carcass .. Bones	2 10·20	2 3·27			1/5	7·051	3·151	0 15·75
Totals of half-carcass operated on	28 2·07	7 6·75	6 11·79	14 2·51	.	.	.	1 2·60
Other half-carcass (dry, &c., calculated)......	28 2·07	7 6·75	6 11·79	14 2·51		. . .	. .	1 2·61
Whole carcass	56 4·14	14 13·50	13 7·58	28 5·08	..	.. ..	..	2 5·21
ORGANS OR PARTS CONSTITUTING "OFFAL."								
Stomachs (washed)	2 13·75	0 7·78	5 8·50	14 0·91	4/10	3·112	0·129	0 0·32
Caul-fat and membrane	3 1·50	0 0·63	……	.. ...	4/10	0·250	0·044†	0 0·11†
Small intestines....	1 0·80	0 6·33	……	……	4/10	2·532	0·092	0 0·23
Large intestines..........	1 3·00							
Intestinal fat and membrane	2 5·50	0 1·36	. ...	.. ...	4/10	0·542	§	§
Heart and aorta...	0 7·35	0 1·51	....	.. .	4/10	0·605	0·025	0 0·06
Heart-fat and membrane	0 8·00	0 0·93	. .	. .	4/10	0·374	§	§
Lungs and windpipe....	1 1·40	0 4·43	. ..	.	4/10	1·772	0·082	0 0·20
Blood	4 1·23	0 11·21	.. ..	. .	4/10	4·484	0·184	0 0·61
Liver	1 11·30	0 6·33			4/10	2·532	0·124	0 0·31
Pancreas ("sweetbread")....	0 2·58‖	0 0·51‖	....	.... .	4/10	0·204‖	0·014‖	0 0·03‖
Thymus gland ("heartbread")								
Glands about the throat ("throatbread") ...								
Milt or spleen	0 2·95	0 0·80	. .	.. .	4/10	0·318	0·019	0 0·05
Head flesh	2 10·44	0 7·54	.. .	. ...	4/10	3·016	0·122	0 0·31
Head bones	1 8·36	1 1·30	.. ..	……	1/10	1·730	0·868	0 8·68
Pelt	7 0·44*	0 15·37	……	.. ...	4/10	6·148	0·370	0 0·93
Wool	6 10·50	3 4·50	……	……	1/10	5·250	0·310¶	0 3·10¶
Feet, hoofs, &c.	‡	0 5·60	.... .	……	3/20	0·840	0·165	0 1·10
Diaphragm ("skirts")	0 8·60	0 2·28	……	……	4/10	0·911	0·034	0 0·09
Total offal	37 1·76	8 14·41	5 8·50	14 0·91	...	.. ..	. . . .	1 0·13
SUMMARY:—ENTIRE ANIMAL.								
Total carcass	56 4·14	14 13·50	13 7·58	28 5·08	...	……	……	2 5·21
Total "offal" parts	37 1·76	8 14·41	5 8·50	14 0·91	...	……	……	1 0·13
Contents of stomachs	7 9·35	……	……	……	...	……	……	……
Contents of intestines	1 14·90	.....	.....	.... .	...	.. ...	.. . .	.... .
Loss by evaporation, error in weighing, &c.	2 2·85	……	……	……	...	……	... ..	... ..
Entire animal (live-weight after fasting) ...	105 1·00	23 11·01	19 0·08	42 11·99	..	. . . .	. . .	3 5·34

Amount and distribution of the fat obtained by melting and expression (lbs., ozs., and tenths).

In carcass (including kidney-fat)...............			13	7·58
In offal ... From caul...................	2	6·66	5	8·50
In offal ... From intestines	2	1·80		
In offal ... From region of heart ...	0	6·58		
In offal ... From remaining parts...	0	9·46		
Total			19	0·08

Amount and distribution of bones (lbs., ozs., and tenths).

In carcass First half (by experiment)...	2	10·20	5	8·40
In carcass Second half (by calculation)	2	10·20		
In offal ... Head			1	8·36
In offal ... Feet and hoofs (with pelt, not weighed separately)			.....	
Total			7	0·76

* Including feet and hoofs.

† Including intestinal and heart-fat, and their membranes.

‡ With pelt.

§ With caul-fat-membrane.

‖ It is doubtful whether these amounts refer to the Pancreas, Thymus Gland, and Glands about the Throat collectively, or to the Pancreas alone.

¶ The amounts of Mineral Matter in the Wool are stated too high, owing to adherent dirt.

APPENDIX.—TABLE VII. Showing the Fresh Weights of the various Organs and Parts of A FAT SHEEP, killed for Analysis, May 7, 1849; also the results of the Determinations of *Dry Matter* and *Ash* in the same.

Designation of Parts.	Original fresh weights.	Dry matter at 212°.			Mineral matter (ash).			
		Exclusive of melted fat.	Melted fat.	Total dry matter.	Proportion of whole, and calculated weights, of dry matters taken for burning.		Actual ashes obtained.	Ashes calculated for the whole dry matters.
CARCASS.								
	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.		oz.	oz.	lbs. oz.
Half-carcass ... Flesh and fat	.. ..	4 1·49	12 4·60	16 6·09	2/10	13·097	0·538	0 2·61
Half-carcass ... Kidney	33 1·00	0 0·51	.. ..	0 0·51	4/10	0·201	0 028	0 0·07
Half-carcass ... Kidney fat	....	0 1·17	3 0·66	3 1·83	4/10	0·469		
Half-carcass ... Bones	3 4·00	2 6·20	. ...	2 6·20	2/10	7·650	3·456	1 1·28
Totals of half-carcass operated on	36 5·00	6 9·37	15 5·26	21 14·63	...	......	......	1 4·04
Other half-carcass (dry, &c., calculated)	36 12 00	6 10·64	15 8·21	22 2·85	...	......	......	1 4·28
Whole carcass	73 1·00	13 4·01	30 13·47	44 1·48	...	......	......	2 8·32
ORGANS OR PARTS CONSTITUTING "OFFAL."								
Stomachs (washed)	3 2·50	0 6·92	......	......	4/10	2·769	0·204	0 0·51
Caul-fat and membrane	6 9·00	0 1·84	.....	......	4/10	0·737	0·058 †	0 0·14†
Small intestines	1 5·80	0 5·90	......	......	4/10	2·360	0·120	0 0·30
Large intestines	2 5·50							
Intestinal fat and membrane	3 9·50	0 1·57	......	.....	4/10	0·626	§	§
Heart and aorta	0 8·70	0 1·86	......	......	6/10	1·115	0·040	0 0·07
Heart-fat and membrane	0 8·00	0 0·48	......	......	4/10	0·190	§	§
Lungs and windpipe	1 0·50	0 4·38	......	......	6/10	2·628	0·107	0 0·18
Blood	4 8·80	0 13·02	......	......	4/10	5·208	0·220	0 0·55
Liver	1 14 80	0 9·36	10 2·00	20 5·50	4/10	3·746	0·180	0 0·45
Pancreas ("sweetbread")	0 3·40‖	0 0·51‖	......	......	4/10	0·205‖	0·014‖	0 0·03‖
Thymus gland ("heartbread")								
Glands about the throat ("throatbread")								
Milt or spleen	0 3·33	0 0·78	......	......	6/10	0·469	0·034	0 0·06
Head flesh	2 10·20	0 6·12	......	......	4/10	2·446	0·149	0 0·37
Head bones	1 3·80	0 13·42	.. ...	......	1/10	1·341	0·709	0 7·09
Pelt	7 0·60*	1 11·87	......	......	4/10	11·148	0·613	0 1·53
Wool	8 1·90	3 13·10	.. ...	......	1/10	6·110	0·404¶	0 4·04¶
Feet, hoofs, &c.	‡	0 6·66	.. ..	......	1/10	0·666	0·150	0 1·50
Diaphragm ("skirts")	0 6·20	0 1·71	......	......	6/10	1·027	0·032	0 0·05
Total offal	45 6·53	10 3·50	10 2·00	20 5·50	...	......	......	1 0·87
SUMMARY:—ENTIRE ANIMAL.								
Total carcass	73 1·00	13 4·01	30 13·47	44 1·48	...	......	......	2 8·32
Total "offal" parts	45 6·53	10 3·50	10 2·00	20 5·50	...	......	......	1 0·87
Contents of stomachs	4 15·50	......	......	......	...	......	......	......
Contents of intestines	2 11·10	......	......	......	...	......	......	......
Loss by evaporation, error in weighing, &c.	1 0·37	......	......	......	...	......	......	......
Entire animal (live-weight after fasting)	127 2·50	23 7·51	40 15·47	64 6·98	...	......	......	3 9·19

Amount and distribution of the fat obtained by melting and expression (lbs., ozs., and tenths).			
In carcass (including kidney fat)			30 13·47
In offal ..	From caul	5 15·80	10 2·00
	From intestines	2 15·70	
	From region of heart	0 0·00	
	From remaining parts	1 2·50	
	Total		40 15·47

Amount and distribution of bones (lbs., ozs., and tenths).			
In carcass	First half (by experiment)	3 4·00	6 8·62
	Second half (by calculation)	3 4·62	
In offal ...	Head		1 3·80
	Feet and hoofs (with pelt, not weighed separately)		......
	Total		7 12·42

* Including feet and hoofs.

† Including ash of intestinal and heart-fat, and their membranes.

‡ With pelt.

§ With caul-fat.

‖ It is doubtful whether these amounts refer to the Pancreas, Thymus Gland, and Glands about the Throat collectively, or to the Pancreas alone.

¶ The amounts of Mineral Matter in the Wool are stated too high, owing to adherent dirt.

APPENDIX.—TABLE VIII. Showing the Fresh Weights of the various Organs and Parts of AN EXTRA-FAT SHEEP *, killed for Analysis, December 13, 1848; also the results of the Determinations of *Dry Matter* and *Ash* in the same.

Designation of Parts.	Original fresh weights.	Dry matter at 212°.			Mineral matter (ash).			
		Exclusive of melted fat.	Melted fat.	Total dry matter.	Proportion of whole, and calculated weights, of dry matters taken for burning.		Actual ashes obtained.	Ashes calculated for the whole dry matters.
CARCASS.								
	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.		oz	oz	lbs. oz.
Half-carcass ... Flesh and fat	79 10 00	7 7·36	..... 39 9·10	53 4·49	3/8	44·700	1·220	0 3·25
Half-carcass ... Fat-membrane		2 0·94			3/8	12·350	0·332	0 0·89
Half-carcass ... Bones		4 3·09		......	3/8	25·158	11·090	1 15·17
Totals of half-carcass operated on	79 10·00	13 11·39	39 9·10	53 4·49	...	.. ..	.. ..	2 3·31
Other half-carcass (dry, &c., calculated)......	79 10·00	13 11·39	39 9·10	53 4·49	...	.. ...	....	2 3·31
Whole carcass.....	159 4·00	27 6·78	79 2·20	106 8·98	...	.....	.....	4 6·62
ORGANS OR PARTS CONSTITUTING "OFFAL."								
Stomachs (washed)	4 2·30	6 0·64 (2 4·84)	24 1·00	30 1·64	3/16 (3/16)	18·120 (0·907)	0·608 (0·278)	0 3·72 (0 1·48)
Caul-fat and membrane	24 6·00†							
Small intestines (washed)........................	1 14·70							
Large intestines (washed).......	1 3·90							
Intestinal fat and membrane	‡							
Heart and aorta..	0 13·00							
Heart-fat and membrane	0 9·50							
Lungs and windpipe...........	1 14·00							
Blood	10 4·00							
Liver, with milt.................	3 0·20							
Pancreas ("sweetbread")...................... ..	.. .. 0 5·70§							
Thymus gland ("heartbread")								
Glands about the throat ("throatbread") ...								
Head flesh	3 10·00							
Head bones	2 6·50	1 6·00	......	1 6·00	3/16	4·125	2·310	0 12·32
Pelt...	25 8·00	3 13·46	......	3 13·46	3/16	11·523	0·580	0 2·49
Wool		7 15·76	......	7 15·76	3/16	23·955	4·667‖	‖1 8·89
Feet and hoofs, &c.		0 11·40	......	0 11·40	3/16	2·138	0·515	0 2·75
Total offal	80 1·80	19 15·26	24 1·00	44 0·26	...	......	......	2 14·07
SUMMARY:—ENTIRE ANIMAL.								
Total carcass	159 4·00	27 6·78	79 2·20	106 8·98	...	......	......	4 6·62
Total "offal" parts	80 1·80	19 15·26	24 1·00	44 0·26	...	......	......	2 14·07
Contents of stomachs	10 8·00	......	......	......	...	......	......	......
Contents of intestines	2 9·40	......	......	......	...	......	......	......
Loss by evaporation, error in weighing, &c.	0 0·80	......	......	......	...	......	......	......
Entire animal (live-weight after fasting) ...	252 8·00	47 6·04	103 3·20	150 9·24	...	......	......	7 4·69

Amount and distribution of the fat obtained by melting and expression (lbs., ozs., and tenths).

In carcass (including kidney-fat)			79 2·20
In offal ...	From caul	15 10·00	24 1·00
	From intestines	5 15·00	
	From remaining parts............	2 8·00	
Total			103 3·20

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* This was the first of the ten animals analysed; and, as the Table on comparison will show, the plan of operation and separation of the parts was less complete and systematic, than in the other cases.

† Including intestinal fat and membrane.

‡ With caul-fat and membrane.

§ It is doubtful whether this amount refers to the Pancreas, Thymus Gland, and Glands about the Throat collectively, or to the Pancreas only.

‖ The amounts of Mineral Matter in the Wool are stated too high, owing to adherent dirt.

APPENDIX.—TABLE IX. Showing the Fresh Weights of the various Organs and Parts of A STORE PIG, killed for Analysis, May 12, 1850; also the results of the Determinations of *Dry Matter* and *Ash* in the same.

Designation of Parts.		Original fresh weights.	Dry matter at 212°.			Mineral matter (ash).			
			Exclusive of melted fat.	Melted fat.	Total dry matter.	Proportion of whole, and calculated weights, of dry matters taken for burning.		Actual ashes obtained.	Ashes calculated for the whole dry matters.
CARCASS.									
		lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.		ozs.	ozs.	lbs. ozs.
Half-carcass	Flesh and fat	26 0·96	4 1·18	7 0 00	11 1·18	[illegible]	16·295	0·651	0 2·61
	Kidney	0 2 82	0 0 64	.....	0 0 64	[illegible]	0·160	0 009	0 0 04
	Kidney fat and membrane	0 12·97	0 1·02	0 8 30	0 9·32	[illegible]	0·255	0·004	0 0 02
	Flesh juice	.....	0 1 91	..	0 1 91	[illegible]	0 955	0·210	0 0 42
	Skin and diaphragm	1 9 01	0 10·47	....	0 10 47	[illegible]	2 617	0·031	0 0·12
	Bones	2 7 50	1 6 66	.. ..	1 6 66	[illegible]	5·665	2·394	0 9·58
Totals of half-carcass operated on		31 1·26	6 5 88	7 8 30	13 14·18	...	......	......	0 12·79
Other half-carcass (dry, &c., calculated)		31 5·18	6 6·68	7 9·25	13 15·93	...	.....	.....	0 12·89
Whole carcass		62 6·44	12 12·56	15 1·55	27 14·11	...	......	. ...	1 9 68
ORGANS OR PARTS CONSTITUTING "OFFAL."									
Stomach (washed)		1 3·30	0 3·80	......	......	[illegible]	1·425	0 065	0 0·17
Caul-fat and membrane		0 5·55	0 0 34	......	......	[illegible]	0·127	0 003	0 0·01
Small intestines (washed)		2 9·00	0 9·47	......	......	[illegible]	3·551	0·152	0 0·40
Large intestines (washed)		2 5·74							
Intestinal fat, "mudgeon," &c.		1 5·77	0 1·62	......	.....	[illegible]	0·607	0·024	0 0·06
Heart and aorta		0 7·85	0 1·88	......	......	[illegible]	0·700	0·032	0 0 08
Lungs and windpipe		1 5·70	0 3·82	......	......	[illegible]	1·432	0·086	0 0 23
Blood		7 0·85	0 13·21	......	......	[illegible]	4·956	0·270	0 0·72
Liver		2 8·00	0 11·47	......	......	[illegible]	4·302	0·231	0 0 62
Pancreas ("sweetbread")		*0 4·05	*0 1·02	......	......	[illegible]	*0·127	*0·009	*0 0·07
Milt or spleen		0 2·75	0 0·65	......	......	[illegible]	0·246	0·013	0 0·03
Gall-bladder		0 0·20	0 0·53	3 7·10	0 6·83	[illegible]	0·198	0·010	0 0·03
Bladder		0 2·30							
Brains		0 3·56	0 0·68	......	.....	[illegible]	0·254	0·017	0 0·05
Tongue		0 8·40	0 1·85	... ..	.. ..	[illegible]	0·231	0·020	0 0 16
Head flesh		4 12·54	0 10·56	.....	.....	[illegible]	3·958	0·175	0 0·47
Head bones		1 11·25	0 15·55	....	......	[illegible]	5·831	3·112	0 8 30
Head-skin and ears		0 10·62	0 3·47	......	......	[illegible]	1·301	0·012	0 0·03
Hair		.. ...	0 1·76	......	......	[illegible]	0·878	0·011	0 0·02
Scurf		......	0 1·92	.... .	... ..	[illegible]	0·240	0·010	0 0·08
Feet and toes		1 3·57 0 1·75	0 10·88	.. ...	......	[illegible]	2·720	0·705	0 2·82
Tail and bones		0 0·57	0 0·31	.....	......	[illegible]	0·114	0·013	0 0·04
Œsophagus and trimmings		0 6·55	0 0·94	.. ..	......	[illegible]	0·354	0·017	0 0·05
Total offal		29 7·87	5 15·73	3 7·10	9 6·83	...	......	......	0 14·44
SUMMARY:—ENTIRE ANIMAL									
Total carcass		62 6·44	12 12·56	15 1·55	27 14·11	...	......	......	1 9·68
Total offal parts		29 7 87	5 15·73	3 7·10	9 6·83	...	......	......	0 14·44
Contents of stomachs		0 4·25	......	......	......	...	......	......	......
Contents of intestines and bile		4 10·36	......	......	......	...	......	......	......
Loss by evaporation, error in weighing, &c.		— 2 13·92	......	......	......	...	......	......	......
Entire animal (live-weight after fasting)		93 15·00	18 12·29	18 8·65	37 4·94	...	......	.. ...	2 8·12

Amount and distribution of the fat obtained by melting and expression (lbs., ozs., and tenths).

In carcass (including kidney fat)			15 1·55
In offal	From caul	0 3·35	3 7·10
	From intestines, "mudgeon," &c.	0 6·65	
	From remaining parts (including head)	2 13·10	
Total			18 8·65

Amount and distribution of the bones (including toes), lbs., ozs., and tenths.

In carcass	First half (by experiment)	2 7·50	4 15·31
	Second half (by calculation)	2 7·81	
In offal	Head	1 11·25	3 0·57
	Feet and toes	1 5·32	
Total			7 15·88

* It is doubtful whether these amounts refer to the Pancreas, Thymus Gland, and Glands about the Throat collectively, or to the Pancreas alone.

APPENDIX.—TABLE X. Showing the Fresh Weights of the various Organs and Parts of A FAT PIG, killed for Analysis, July 18, 1850; also the results of the Determinations of *Dry Matter* and *Ash* in the same.

Designation of Parts.	Original fresh weights.	Dry matter at 212°.			Mineral matter (ash).		
		Exclusive of melted fat.	Melted fat.	Total dry matter.	Proportion of whole, and calculated weights, of dry matters taken for burning.		Actual ashes obtained.	Ashes calculated for the whole dry matters.
CARCASS.								
	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.		oz.	oz.	lbs. oz.
Half-carcass.. Flesh and fat	60 1·22	7 0·46	29 15·00	36 15·46	[illegible]	28·111	0·611	0 2·56
Half-carcass.. Kidney	0 4·50	0 1·02	......	0 1·02	[illegible]	0·251	0·013	0 0·05
Half-carcass.. Kidney fat and membrane	3 11·52	0 1·70	3 0·75	3 2·45	[illegible]	0·425	0·012	0 0·05
Half-carcass.. Flesh juice	.....	0 0·54	.....	0 0·54	[illegible]	0·136	0·027	0 0·11
Half-carcass.. Skin and diaphragm	3 2·88	1 1·99	.. ..	1 1·99	[illegible]	4·198	0·091	0 0·36
Half-carcass.. Bones	3 4·21	2 0·20	.....	2 0·20	[illegible]	8·050	3·160	0 12·61
Totals of half-carcass operated on	70 8·33	10 5·91	32 15·75	43 5·66	..	.. ..	. .	0 15·77
Other half-carcass (dry, &c., calculated)	70 0·40	10 4·74	32 12·04	43 0·78	.	.	..	0 15·66
Whole carcass	140 8·73	20 10·65	65 11·79	86 6·44	..	.. ..	.	1 15·43
ORGANS OR PARTS CONSTITUTING "OFFAL."								
Stomach (washed)	1 3·48	0 3·90	......	......	[illegible]	1·462	0·019	0 0·05
Caul-fat and membrane	0 8·95	0 0·30	......	.. ...	[illegible]	0·112	0·005	0 0·01
Small intestines (washed)	2 8·54	0 8·62	. ...	. ..	[illegible]	3·232	0·135	0 0·38
Large intestines (washed)	1 10·74							
Intestinal fat, "mudgeon," &c.	4 1·31	0 3·05	.. .	.	[illegible]	1·145	0·057	0 0·15
Heart and aorta	0 8·80	0 1·87	...		[illegible]	0·703	0·033	0 0·09
Lungs and windpipe	1 11·73	0 5·64	.. ..	...	[illegible]	1·880	0·087	0 0·29
Blood	6 13·30	1 7·26	.. .	.. .	[illegible]	8·724	0·455	0 1·21
Liver	3 0·77	0 14·34	.....	..	[illegible]	5·377	0·350	0 0·93
Pancreas ("sweetbread")	*0 6·44	*0 1·00	.. .	.. ...	[illegible]	*0·375	*0·017	*0 0·05
Milt or spleen	0 4·83	0 0·85	7 0·45	14 13·11	[illegible]	0·318	0·018	0 0·05
Gall-bladder	0 0·21	0 0·59	.. ...	......	[illegible]	0·222	0·009	0 0·02
Bladder	0 2·45							
Brains	0 3·31	0 0·68	......	.. ..	[illegible]	0·171	0·013	0 0·05
Tongue	0 12·90	0 2·66	......	......	[illegible]	0·098	0·040	0 0·11
Head flesh	7 1·08	0 12·86	......	......	[illegible]	4·822	0·182	0 0·40
Head bones	1 13·50	1 2·01	......	....	[illegible]	0·754	3·498	0 9·31
Head-skin and ears	1 5·00	0 5·12	......	.. ..	[illegible]	1·020	0·053	0 0·14
Hair	......	0 3·54	......	......	[illegible]	1·329	0·054	0 0·14
Scurf	......							
Feet and toes	1 14·47 0 2·84	1 1·96	......	......	[illegible]	0·733	1·463	0 3·90
Tail and bones	0 1·01	0 0·41	......	......	[illegible]	0·151	0·015	0 0·01
Total offal	36 8·66	7 12·66	7 0·45	14 13·11	...	......	.. ...	1 1·30
SUMMARY:—ENTIRE ANIMAL.								
Total carcass	140 8·73	20 10·65	65 11·70	86 6·44	...	.....	....	1 15·43
Total offal parts	36 8·66	7 12·66	7 0·45	14 13·11	...	......	. .	1 1·30
Contents of stomachs	0 11·82	......	......	......	...	......	....	.....
Contents of intestines and bile	6 9·80	......	......	......	...	......	....	......
Loss by evaporation, error in weighing, &c.	0 8·99	......	......	......	...	......	... .	......
Entire animal (live-weight after fasting)	185 0·00	28 7·31	72 12·24	101 3·55	...	......	.....	3 0·79

Amount and distribution of the fat obtained by melting and expression (lbs., ozs., and tenths).

In carcass (including kidney fat)			65 11·70
In offal...	From caul	0 8·05	7 0·45
	From intestines, "mudgeon," &c.	2 8·75	
	From remaining parts (including head)	3 15·65	
	Total		72 12·24

Amount and distribution of bones (including toes), lbs., ozs., and tenths.

In carcass	First half (by experiment)	3 4·21	6 8·05
	Second half (by calculation)	3 3·84	
In offal...	Head	1 13·50	3 14·81
	Feet and toes	2 1·31	
	Total		10 6·86

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* It is doubtful whether these amounts refer to the Pancreas, Thymus Gland, and Glands about the Throat collectively, or to the Pancreas alone.

APPENDIX.—TABLE XI. Showing the Percentages of *Crude Dry Matter** in the Individual Organs, and other separated Parts, of 10 Animals of different Description, Age, or condition of Fatness.

	Designation of Parts.	Calf and Oxen.			Lamb and Sheep.					Pigs.	
		Fat calf	Half-fat ox.	Fat ox.	Fat lamb.	Store sheep.	Half-fat old sheep.	Fat sheep.	Extra fat sheep.	Store pig.	Fat pig.
Carcass	Flesh and fat	20·665	24·822	18·714	15·316	17·044	20·469	12·697	17·220	15·632	11·699
	Kidney		25·854	24·477		22 059				22·727	22·750
	Kidney fat and membrane	3·514	4·117	3·407	1·691	5·085				7·870	2 856
	Flesh juice	.. ...	0·677	0·203	......	......	. ...	. ...		0·460	0·056
	Skin and diaphragm	. ....	......	......	. ....	..... .	.	......		41·947	35 358
	Bones	54·834	72·124	73 838	65·311	65·808	83·499	73·461		57·338	61·733
	Total carcass	25·124	30·888	24·475	19·527	23·428	26·384	18 136	17 220	20·489	14·704
Separated parts of the "offal."	Stomachs (washed)	10·330	18·941	18·050	20·333	11·540	17·005	13·703		19 689	20·020
	Caul-fat and membrane	10 807	1·880	2·686	1·413	2·188	1·273	1·752		6·126	3 330
	Small and large intestines (washed)	14·728	18·550	12·864	20·061	11·740	17·681	9·949		12·027	12·812
	Intestinal fat and membrane	4·544	8·764	3·708	†	4 882	3·627	2·730		7·441	4·670
	Heart and aorta	21·451	27·230	19·593	21·787	21·446	20·544	21·379		23·974	21·295
	Heart-fat with membrane	†	4·150	3 949	†	3·026	11·625	6·000		..	...
	Lungs and windpipe	25·242	23·671	22.658	30·357	23·896	25·460	26·545		17·603	20 339
	Blood	18·510	21·566	20·440	21·872	16 929	17·185	17·884	22·463	11·709	21·283
	Liver	24·188	28·422	28·617	30·132	31·932	23·186	30·390		28·675	29·403
	Pancreas ("sweetbread")	18·353	24·000	15·445	23·076**	19·195	19·767**	15·000**		25·185**	15·528**
	Thymus gland ("heartbread")		9·256	10·952							
	Glands about the throat ("throatbread")		20·800	15·784							
	Milt or spleen	22·681	24·712	24·355	25·581	20·000	27·118	23·423		23·636	17·241
	Gall-bladder and bladder	21·875	29·090	23·552	‡	19·923	‡	‡	8 908††	21 120	22·256
	Bile	‡	9 200	‡	‡	‡	‡	‡		‡	‡
	Brains	18·960	22·217	22·414	19·601	16·000	17·766	14·502		19 101	20 531
	Tongue		24·058	27·910						22·024	20·589
	Head flesh		19·074	21·188						13·807	11 282
	Head bones	53·915	76 991	71·480	62·112	65·870	71·018	67·778	57·142	57·169	61·174
	Head-skin and ears	25·902	33·493	34·934	29·499	29 355	18·650	30 666	49 171	32·661	24·427
	Pelt									§	§
	Hair or wool	71·583	62 119	73·485	62·563	43 210	49 268	47·086		. .	
	Leg bones	65·880	55·399	58 959	¶	¶	§	¶	¶	§	§
	Feet bones and hoofs		52 930	47·536	45 196	46 684	‖	‖		51·128	53 917
	Tail flesh	27·988	31·590	26·670	¶	¶	¶	¶	¶	53·805	40·594
	Tail bones	50 645	81·074	79·412							
	Diaphragm ("skirts")	25·272	27·126	20·124	28·670	28·012	26·512	27·581	-	§	§
	Miscellaneous trimmings	. ...	12·509	+	....	...	-	.		14 427	. ..
	Total offal	24·363	28 347	24·853	26·578	24·745	23·984	22·504	24·907	20·287	21·822
	Entire animal	22·551	27·423	22·783	19·274	21 981	22·600	18·457	18·763	19 979	15·382

* The so-called "*Crude Dry Matter*" is exclusive of the fat removeable by melting and expression: but still retains, in most cases, 20 to 30 per cent. of fat extractable by ether.

† With caul-fat and membrane. ‡ Thrown away. § With carcass. ‖ With pelt. ¶ With carcass in case of all the sheep.

** It is doubtful whether these amounts refer to the Pancreas, Thymus Gland, and Glands about the Throat collectively, or to the Pancreas alone.

†† This is the per cent. of *Crude Dry Matter* (that is the Dry Matter exclusive of melted and expressed Fat) in all those parts taken collectively, to which the other figures in the column do not apply.

APPENDIX.—TABLE XII. Showing the Percentages of *Mineral Matter* (Ash) in the Individual Organs, and other separated Parts (fresh), of 10 Animals of different Description, Age, or condition of Fatness.

	Designation of Parts.	Calf and Oxen.			Lamb and Sheep					Pigs.	
		Fat calf.	Half-fat ox.	Fat ox.	Fat lamb.	Store sheep.	Half-fat old sheep.	Fat sheep	Extra-fat sheep.	Store pig.	Fat pig.
Carcass...	Flesh and fat	0·943	0·816	0·691	0 569	1·486	0·699	0·522	2·772	0 626	0·266
	Kidney		1·068	1·088		0·919				1·420	1 116
	Kidney fat and membrane	0·111	0 170	0·115	0·073	0·113				0·154	0·084
	Flesh juice	......	0·139	0 035	.. . .	. ...	. ...	. .		0·101	0·011
	Skin and diaphragm	.....	......	... ..	. ..	.. ..		.. .		0·481	0·707
	Bones	25·062	36·289	33 521	30 793	22·910	37 287	33 231		24·241	24 233
	Total carcass	4·483	5·563	4·560	3 632	4 359	4·184	3 449	2 772	2·572	1·398
Separate parts of the "offal."	Stomachs (washed)	0·492	0·961	0·777	0·730	0 540	0 699	1·009	0·378††	0·896	0·262
	Caul-fat and membrane	0·603	0·074	0·051	0·096	0·087	0·116	0·082		0·180	0 145
	Small and large intestines (washed)	0·725	0·603	0·513	0 958	0·518	0·642	0·506		0 514	0 535
	Intestinal fat and membrane	†	0·228	0·163	†	0·209	†	†		0·276	0 230
	Heart and aorta	1·825	0·941	0·830	0·745	0·831	0·816	0·804		1·083	1·000
	Heart-fat with membrane	†	0·170	0·127	†	0·256	†	†		. .	.
	Lungs and windpipe	0·857	2·717	2·427	1 012	2 640	1·145	1·091		1·055	0·938
	Blood	1·099	0·934	0·857	1 059	0·683	0·935	0·755	0·902	0·688	1·110
	Liver	1·278	1·514	1·365	1·432	1·392	1·143	1·461		1·540	1·913
	Pancreas ("sweetbread")	0·775	1·200	1·032	1·214**	2·844	1·163**	0·882**		1·778**	0·776**
	Thymus gland ("heartbread")		0·632	0·895		0·454					
	Glands about the throat ("throatbread")		1·280	1·103		1·900					
	Milt or spleen	1·365	1·536	1·290	1·550	1·160	1·695	1·802		1·273	0·974
	Gall-bladder and bladder	0·344	0·979	0·785	‡	1·885	‡	‡		1·080	0 902
	Bile	‡	1·234	‡	‡	‡	‡	‡		‡	‡
	Brains	0·728	1·540	1·571	1·036					1 404	1·510
	Tongue		1·168	1·067		0·918	0·730	0·877		1 905	0 829
	Head-flesh		0·987	0·733						0 614	0 430
	Head bones	29·794	44·372	40·652	34·939	28 525	35·632	35·808	32·000	30 515	31 623
	Head-skin and ears	0 778	0·807	0·887	1·465	1·635	1·805	2·691	7·507	0·282	0·667
	Pelt									§	§
	Hair or wool*	1·218	1·538	1·257	2·052	2·086	2·909	3·110		.. .	. .
	Leg bones	22·017	25·240	25·841	☛	☛	☛	☛	☛	§	§
	Feet bones and hoofs		11·674	9·870	8·740	8·616	‖	‖		13·248	11·708
	Tail flesh	0·919	0·998	0·659	☛	☛	☛	☛	☛	6·944	3 960
	Tail bones	17·097	31·053	31·000						§	§
	Diaphragm ("skirts")	0·909	0·817	0·633	0·837	1·039	1·046	0·806	.	0·687	. .
	Miscellaneous trimmings	......	1·264	‡	.. .	... ..	......	. ..	. ..		
	Total offal	3·415	4·051	3·401	2·447	2·187	2·717	2·322	3 641	3 060	2 969
	Entire animal	3·800	4·664	3·820	2 937	3·163	3·173	2·811	2·903	2·669	1·648

* The amounts of mineral matter in wool are stated too high, owing to adherent dirt.

† With caul-fat and membrane.

‡ Thrown away

§ With carcass.

‖ With pelt.

☛ With carcass in case of all the sheep.

** It is doubtful whether these amounts refer to the Pancreas, Thymus Gland, and Glands about the Throat collectively, or to the Pancreas alone

†† This is the per cent. of Ash in all those parts taken collectively, to which the other figures in the column do not refer

APPENDIX.—TABLE XIII. Showing the Results of the individual Determinations of *Fat* (by extraction with Ether), in the "Crude Dry Substance*" of certain collective portions, and of the entire bodies, of 10 Animals of different Description, Age, or condition of Fatness.

	Percentages of Fat remaining in the "Crude Dry Substance."				
	Experiment 1.	Experiment 2.	Experiment 3.	Experiment 4.	Mean.
1. COLLECTIVE CARCASS PARTS (INCLUDING BONE).					
Fat calf	15·98	15·84	.. ..	. ..	15·91
Half-fat ox	24·67	23·94	.. ..	. . ..	24·30
Fat ox	19·99	19·85	. . .	.	19·92
Fat lamb	25·61	25·93	.....	.	25·77
Store sheep	19·34	19·43	... .	. ...	19·39
Half-fat old sheep	27·43	27·74	....	.. . .	27·59
Fat sheep	17·64	17·67	... .	. .	17·66
Extra-fat sheep	31·34	31·16	....	. ...	31·25
Store pig	18·98	19·10	. .	.	19·04
Fat pig	18·70	19·16		.	18·93
Store pig (head and feet without tongue and brains)	18·02	18·17	.	. ...	18·09
Fat pig (head and feet without tongue and brains)	21·72	21·65	..	... .	21·68
2. COLLECTIVE OFFAL PARTS (INCLUDING BONE).					
Fat calf	16·52	16·31	......	. ..	16·42
Half-fat ox	13·54	13·51	14·22	13·33	13·66
Half-fat ox (without bone)	15·78	15·31	.. ...	.. ...	15·55
Half-fat ox (offal bones only)	9·16	8·52	.. ...	......	8·84
Fat ox	17·24	16·19	17·27	......	16·90
Fat lamb	23·89	23·80	......	......	23·85
Store sheep	22·38	22·15	.. ...	......	22·27
Half-fat old sheep	18·40	18·27	......	......	18·34
Fat sheep	20·93	21·05	20·76	.. ...	20·91
Extra-fat sheep	22·90	22·62	......	......	22·76
Store pig	15·47	15·35	......	. ....	15·41
Fat pig	13·41	13·49	. ....	. ....	13·45
3. WOOL†.					
Fat lamb	8·69	9·28	......	.....	8·99
Store sheep	11·18	8·98	11·38	12·75	11·07
Half-fat old sheep	8·95	8·85	......	......	8·90
Fat sheep	14·40	12·69	14·81	.....	13·97
Extra-fat sheep	11·85	9·70	10·38	9·84	10·44
4. ENTIRE ANIMAL (EXCLUDING WOOL).					
Fat calf	17·10	16·52	.....	......	16·81
Half-fat ox	22·74	22·79	....	......	22·77
Fat ox	19·10	19·10	......	.....	19·10
Fat lamb	25·70	25·83	......	......	25·77
Store sheep	19·35	19·38	......	......	19·37
Half-fat old sheep	26·17	26·48	.....	......	26·33
Fat sheep	19·42	19·35	......	......	19·39
Extra-fat sheep	29·42	29·46	......	.... .	29·44
Store pig	18·18	18·08	......	.....	18·13
Fat pig	19·08	18·49	......	......	18·78

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* After the removal of as much fat as possible by melting and expression.

† The agreement of the separate determinations is not so good in the case of the wool, as in that of the other parts, owing to the adherent dirt, which rendered it difficult to secure even samples for analysis.

APPENDIX.—TABLE XIV. Showing the Results of the individual Determinations of *Nitrogen* in the "Crude Dry Substance*" of certain collective portions, and of the entire bodies, of 10 Animals of different Description, Age, and condition of Fatness.

	Percentages of Nitrogen in the "Crude Dry Substance."				
	Experiment 1.	Experiment 2	Experiment 3.	Experiment 4.	Mean.
1. COLLECTIVE CARCASS PARTS (EXCLUDING BONE).					
Fat calf	12·49	12·44	......	......	12·47
Half-fat ox	10·90	10·86	......	......	10·88
Fat ox	12·20	12·05	......	......	12·13
Fat ox (with bones)	9·62	9·59	......	......	9·60
Fat lamb	10·63	10·72	......	......	10·68
Store sheep	13·04	13·09	......	......	13·07
Half-fat old sheep	10·33	10·43	......	......	10·38
Fat sheep	12·65	12·73	......	......	12·69
Extra-fat sheep	9·55	9·45	......	......	9·50
Store pig (with bones)	11·38	11·26	......	......	11·32
Fat pig (with bones)	11·81	11·55	11·58	......	11·64
Store pig (head and feet without tongue and brains)	8·70	8·88	......	......	8·79
Fat pig (head and feet without tongue and brains)	8·68	8·66	......	......	8·67
2. CARCASS BONES.					
Fat calf	5·94	6·06	......	......	6·00
Half-fat ox	4·88	4·90	......	......	4·89
Fat ox	4·95	5·03	......	......	4·99
Fat lamb	5·17	5·02	......	......	5·10
Store sheep	5·08	5·09	......	......	5·09
Half-fat old sheep	4·62	4·65	......	......	4·64
Fat sheep	4·92	4·94	......	......	4·93
Extra-fat sheep	4·97	4·93	......	......	4·95
3. COLLECTIVE OFFAL PARTS (INCLUDING BONE).					
Fat calf	11·35	11·30	11·43	......	11·36
Half-fat ox	11·95	11·71	11·95	......	11·87
Fat ox	11·25	11·29	11·29	......	11·28
Fat lamb	10·21	9·89	9·97	......	10·02
Store sheep	10·51	10·60	10·43	......	10·51
Half-fat old sheep	10·52	10·35	10·51	......	10·46
Fat sheep	10·61	10·60	10·28	......	10·50
Extra-fat sheep	10·58	10·33	10·29	......	10·40
Store pig (no bones)	12·96	12·82	......	......	12·89
Fat pig (no bones)	13·10	13·23	......	......	13·17
4. HAIR OR WOOL†.					
Fat calf	16·46	16·60	......	......	16·53
Half-fat ox	16·74	16·90	......	......	16·82
Fat ox	16·16	16·82	16·57	......	16·52
Fat lamb	14·80	14·97	......	......	14·89
Store sheep	14·30	13·86	15·20	14·03	14·35
Half-fat old sheep	16·52	14·38	14·70	......	15·20
Fat sheep	14·49	13·56	12·60	13·08	13·43
Extra-fat sheep	12·49	11·83	12·08	12·54	12·24
5. ENTIRE ANIMAL (EXCLUDING HAIR OR WOOL).					
Fat calf	10·79	10·70	......	......	10·75
Half-fat ox	10·04	10·06	......	......	10·05
Fat ox	10·15	10·13	......	......	10·14
Fat lamb	9·24	9·22	......	......	9·23
Store sheep	10·33	10·02	10·34	......	10·23
Half-fat old sheep ‡	9·33	9·23	......	......	9·28
	8·85	9·26	9·50	......	9·20
Fat sheep ‡	9·92	10·05	......	......	9·99
	10·69	10·24	10·76	......	10·56
Extra-fat sheep ‡	8·71	8·72	......	......	8·72
	9·10	9·13	9·21	......	9·15
Store pig	11·19	11·16	......	......	11·18
Fat pig	11·52	11·77	11·57	......	11·62

* After the removal of as much fat as possible by melting and expression.

† The agreement of the separate determinations is not so good, in the case of the wool, as in that of the other parts, owing to the adherent dirt, which rendered it difficult to secure even samples for analysis.

‡ In these cases a second set of determinations was made as a check, some months after the first, and with different reagents.

APPENDIX.—TABLE XV. Showing the Actual Weights (lbs. and ozs.) of the

	Designation of Parts	Calves.		Heifers.		Bullocks.				
		No. 1.	No. 2	No. 1.	No. 2	No. 1.	No. 2.	No. 3.	No. 4.	No. 5
		Short horned, milk-fed, killed Aug 11, 1849.	Durham breed, (analysed as fat) taken from dam feeding on grass; killed Sept. 12, 1849.	Welsh, three to four years old, killed Aug 23, 1849.	Welsh; three years old; lately oil-cake fed; killed Aug. 30, 1849	Mixed breed; killed Aug. 2, 1849.	Mixed breed, killed Aug. 9, 1849.	Welsh, horned; four years old; killed Aug. 16, 1849.	Scotch, long-horned; four years old; grass-fed; killed Aug. 30, 1849.	Dutch; lately grass-fed in Norfolk; killed Aug. 23, 1849.
		lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.
Separate parts of the "offal."	Stomachs	4 0	2 13	33 0	31 0	40 8	37 2	28 8	36 0	35 8
	Contents of stomachs and vomit	3 14	8 3	51 0	99 8	157 14	83 11	81 2	99 0	79 8
	Caul-fat	2 11	2 7·5	31 11	14 13	18 7	15 8	11 9	21 9	19 0
	Small intestines and contents	4 8	6 3	13 0	17 8	24 10	19 0	13 2	20 10	19 8
	Large intestines and contents	3 9·5	2 14·5	9 4	15 0	18 0	21 12	12 10	16 6	13 12
	Intestinal fat	1 9	4 8	36 0	16 6	17 4	15 14	12 12	17 0	21 1
	Heart and aorta	1 8	1 7·5	4 3	3 13	6 4	5 2	4 14·5	6 1	6 8
	Heart-fat	......	0 6·8	2 0	1 12	1 14	3 0	1 14·5	1 15	1 14
	Lungs and windpipe	3 4	3 5·5	5 7	7 6	12 8	9 0	7 2·5	11 4	11 12
	Blood	10 0	13 8·8	32 4	20 4	48 9	46 8	25 13·5	40 0	41 8
	Liver	4 2	4 3·5	13 12	12 2	15 3	11 4	10 5	16 11	13 10
	Gall-bladder and contents	0 2	*	0 10	0 11·5	0 6	0 3	0 15	1 1·5	1 14
	Pancreas ("sweetbread")	0 4·5	.. ..	0 12	0 14	1 1	1 2·5	0 12	0 12·5	0 14
	Thymus gland ("heartbread")	0 11·5	1 13	0 9	0 9·5	1 0	0 8	0 7	0 10·5	1 0
	Glands about the throat ("throatbread")	0 9	. ..	0 5	0 6·5	0 5	.. ...	0 4	0 3·5	0 11
	Milt or spleen	0 14	0 12	1 8	1 0	2 0	2 0	0 15	1 11	2 0
	Bladder, with penis, or womb	0 4·5	0 7·2	9 12†	1 6	0 9	0 8	0 4·5	0 7·5	0 8
	Brains	15 12	11 7·2	20 4	0 10	1 2·5	0 8	0 11·8	0 12	0 10
	Head and tongue				22 0	31 0	26 12	24 7·5	31 12	35 2
	Hide and horns	17 0	17 12·5	64 12	67 0	91 0	68 8	73 7	98 12	81 0
	Feet, hoofs, &c.	6 9	4 6	13 12	15 8	22 8	18 7	17 10·5	18 2	20 12
	Tail	......	0 5·2	0 8	1 1·5	1 8	0 14	0 13	0 13	1 4
	Diaphragm ("skirts")	.. ...	1 1·5	4 12	4 7	5 12	2 4·5	1 10	2 1	1 14·5
	Miscellaneous trimmings	... ..	.....	7 4	1 8	......	1 2	2 5·5	2 1	3 12
	Total "offal" parts	81 4	87 12·7	356 5	356 10	519 4·5	393 13	334 7·8	455 7·5	418 14·5
	Carcass	155 14	157 7·5	498 0	451 4	665 8	550 4	527 4	629 8	713 13
	Loss by evaporation, error in weighing, &c.	5 10	13 7·8	37 7	8 2	27 11·5	20 0	20 0	23 0·5	36 4·5
	Live-weight after fasting	242 12	258 12	891 12	816 0	1212 8	964 1	881 11·8	1108 0	1169 0

* With bladder.

† Including calf.

individual Organs, and other separated Parts, of CALVES, HEIFERS, and BULLOCKS.

Bullocks.									Means of			
No. 6.	No. 7.	No. 8.	No. 9.	No. 10.	No. 11.	No. 12.	No. 13.	No. 14.				
Scotch, long-horned, four years old, grass-fed; killed Sept. 6, 1849.	Welsh, five years old, killed Sept. 13, 1849.	Mixed breed; killed Sept. 20, 1849.	Durham, short-horned; three to four years old; grass-fed; killed Sept. 27, 1849.	Irish, lately grass-fed in Leicestershire; killed Sept. 27, 1849.	Mixed breed; three years old; killed Oct. 4, 1849.	Scotch; (analysed as fat) four years old; killed Oct. 30, 1849.	Scotch; (analysed as half-fat) three to four years old; killed Nov. 14, 1849.	Devon: working ox; afterwards fed on oilcake, &c.; seven years old; killed April 6, 1853.	The two calves.	The two heifers.	The fourteen bullocks.	The sixteen heifers and bullocks
lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz	lbs oz.	lbs. oz
34 0	31 11	36 0	41 12	30 10	33 4	36 6	32 1	58 8	3 6·5	32 0	36 6·7	35 13·9
117 12	104 7	93 0	92 0	108 6	100 9·5	84 4	100 1	60 8	6 0·5	70 12	97 4·7	93 15·8
13 10	22 9·5	24 2	24 8·5	19 0·5	35 0	29 13	16 9·5	53 0	2 9·2	23 4	23 2·7	23 2·9
19 0	20 0	17 0	20 4	17 0·5	16 15·5	8 11·7	8 5	14 15	5 5·5	16 4	17 1·3	16 13·6
22 4	16 2	11 7	8 5	11 15·5	14 10	5 2·5	6 0	7 11	3 4	12 2	13 4·0	13 2·3
10 12	21 14	27 0	27 14	10 10	39 0	36 14	19 11·5	79 0	2 14	26 3	26 5·8	26 5·1
5 13	4 13	5 10	5 14·5	5 3	5 1·5	7 6	5 13·5	8 2	1 7·7	4 0	5 14·4	5 10·6
1 9	3 5	3 14	2 14·5	3 14	4 7	6 8	2 6·8	8 14	0 3·5	1 14	3 0·9	3 3·8
9 12	8 1	8 3	9 6	10 0	8 11·5	8 14·5	7 12·8	12 6	3 4·8	6 6·5	9 10·1	9 3·0
44 10	49 8	56 6	57 6	42 3·5	45 5	52 11·8	54 5	51 7	11 12·5	30 12	47 15·2	45 12·8
13 12	13 2	15 0	17 11	13 12	14 6·5	17 10	15 11	20 2	4 2·8	12 15	15 1·6	14 13·3
0 13·5	1 7	1 5	1 1·8	0 15	0 14·5	0 13	1 0	1 6	0 2	0 10·8	1 0·5	0 15·7
0 13·5	0 12·5	0 15	1 3	1 1	1 5	0 15·5	1 0	2 10	1 11	0 13	1 1·6	1 1
0 13	0 8	0 9	0 8	0 9·5	……	0 10·5	0 10·8	1 11		0 9·2	0 11	0 10·7
0 7·5	0 2	0 5	0 7·2	0 4·8	0 5·5	0 6·5	0 6·2	0 8		0 5·7	0 5·5	0 5·5
1 13	1 14	2 9	1 13·8	2 1·5	1 6·5	2 4	2 2·2	2 12	0 13	1 4	1 15·3	1 13·9
0 7	0 8	0 9	0 15·5	0 15	1 2·5	1 10·7	0 11·5	1 5	0 6	5 9 §	0 10·7	0 9·1
0 10·5	0 12·5	0 11	0 15·2	0 12·2	0 11·8	0 14	0 14·2	0 8	13 9·5	0 5	0 12·1	0 12·0
32 0	27 0	26 6	30 12	30 11	29 10	43 7	39 0	40 8		21 2	32 0·3	30 10·7
105 0	84 8	91 8	90 8	80 4	83 14	80 3·8	79 15·4	113 4	17 6·2	65 14	87 4·3	84 9·5
20 14	17 14	21 13	23 11	20 9	21 7	22 7·7	20 2	25 0	5 7·5	14 10	20 13	20 0·6
1 0	1 1	1 3	1 6·5	1 0	1 0	1 7·5	1 12·4	‡	0 5·2	0 12·8	1 1·3	1 1·9
4 11	6 12	5 8	7 1	4 7	4 12	7 9	5 11·5	18 0	1 1·5	4 9·5	5 5·2	5 2
3 11	1 3	2 3	2 5·3	3 2	11 9·5	9 0·5	3 6·2	8 11	……	4 0	8 14·3	8 15·3
466 0	439 14·5	452 13	470 11·8	428 7	475 8·8	465 14·2	425 9·5	588 12	85 4·9	356 7·5	452 13·3	430 13·9
639 4	630 0	693 12	702 12	666 12	701 1·2	939 6	797 11	1025 12	156 10·8	474 10	710 3·1	680 12
6 12	15 1·5	15 7	32 8·2	16 13	—2 10	13 11·8	8 11·5	37 8	8 12·3	23 12·5	19 1·1	20 7·2
1112 0	1085 0	1162 0	1260 0	1112 0	1174 0	1419 0	1232 0	1652 0	250 12	853 14	1182 1·5	1141 1·1

‡ With hide.

§ In one case Womb with Calf.

APPENDIX.—TABLE XVI. Showing the *Actual Weights* (lbs. and ozs.) of the individual Organs, and other separated Parts, of SHEEP.

CLASS I.—5 Sheep of different Breeds, killed in Store condition (at Rothamsted), for standards of comparison.

Designation of Parts	Cotswold Wethers, killed Nov 25, 1851.		Leicester Wether, killed Nov 22, 1852	Cross bred Wether (Leicester and South Down), killed Nov 22, 1852	Cross bred Ewe (Leicester and South Down), killed Nov. 22, 1852	Means of the 5 Store Sheep
	No 1.	No. 2				
	lbs oz	lbs oz	lbs oz	lbs oz	lbs oz	lbs oz
Original weight.	115 0	109 0	94 0	87 0	84 0	97 12·8
Final weight, unfasted	112 0	112 0	95 0	89 0	87 0	99 0
Final weight after fasting	105 12	104 8	89 0	84 0	82 0	93 0·8
Separate parts of the "offal."						
Stomachs	2 12·6	3 2·5	2 9·7	2 11·5	2 5·5	2 11·5
Contents of stomachs	5 14·1	7 3	5 6·3	‡	‡	6 2·5
Caul-fat	3 15·5	3 0	1 8·6	2 15·5	2 3·8	2 11·9
Small intestines and contents	2 8·2	2 9·8	1 15	1 14·4	1 13·5	2 2·6
Large intestines and contents	3 4	2 6·5	3 2	2 8	2 5	2 11·5
Intestinal fat	1 0·6	1 0·3	1 3·6	1 9·1	0 15·5	1 2·6
Heart and aorta	0 7·8	0 8·3	0 6·8	0 6·7	0 5·9	0 7·1
Heart-fat	0 6·8	0 6·5	0 3·7	0 4	0 3·5	0 4·9
Lungs and windpipe	1 4	1 5	0 14·9	0 13·2	1 2·3	1 1·5
Blood	5 3·5	4 11·2	4 1	3 15·5	4 5·5	4 7·3
Liver	1 13	1 13·3	1 3·5	1 6	1 4·8	1 8·1
Gall-bladder and contents	0 0·7	0 2	0 0·4	0 0·5	0 1·5	0 1
Pancreas ("sweetbread")	0 2·1	0 3	0 2·3	0 1	0 1·9	0 2·1
Glands about the throat ("throatbread")	………	………	………	0 0·8	………	0 0·8
Milt or spleen	0 3·9	0 2·2	0 2·2	0 2	0 2·7	0 2·6
Bladder	0 0·8	0 0·8	0 0·6	0 1·3	0 0·5	0 0·8
Womb	………	………	………	.. ....	0 1·5	………
Head	3 10	3 9	3 6	3 2·7	3 1·7	3 5·9
Skin (with feet, &c.)	14 11	14 4	15 2	10 12	10 12	13 1·8
Wool						
Diaphragm ("skirts")	.. .. .	………	... . ..	0 4·1	... ..	0 4·1
Miscellaneous trimmings	………	………	0 1·6	....	0 1·3	0 1·4
Total "offal" parts	47 6·6	46 7·4	41 10·2	33 0·3	31 8·4	42 12·0
Carcass	56 15	56 11·6	45 10·5	45 7·5	43 14·5	49 11·8
Loss by evaporation, error in weighing, &c.	1 6·4*	1 5*	1 11·3	5 8·2†	6 9·1†	0 9·0
Live-weight after fasting	105 12	104 8	89 0	84 0	82 0	93 0·8

* Inclusive of contents of bladder.

† Including contents of stomachs

‡ Included with loss, &c

Appendix.—Table XVII. Showing the *Actual Weights*

Class II.—20 Wether Sheep of *Cotswold Breed*, about 1¾ year old, in *moderately* Fat condition.

	Designation of Parts	The 5 giving the Largest amount of Increase during Fattening.										The 5 giving the Smallest amount of Increase during Fattening.									
		No. 1.		No. 2.		No. 3.		No. 4.		No 5		No. 6		No 7		No. 8.		No. 9.		No. 10.	
		lbs	ozs.	lbs.	ozs.	lbs.	ozs	lbs.	ozs	lbs.	ozs	lbs.	ozs	lbs	ozs	lbs.	ozs	lbs.	ozs.	lbs.	ozs.
	Original weight	112	0	127	0	108	0	140	0	119	0	128	0	109	0	133	0	120	0	129	0
	Final weight, unfasted (including shorn wool)	201	0	207	1	185	0	222	12	101	2	177	2	155	12	176	9	160	8	166	8
	Final weight, fasted (including shorn wool)	188	0	196	1	178	0	208	12	182	2	166	2	148	12	168	9	151	8	162	8
Separate parts of the "offal."	Stomachs	5	1	4	12·5	4	9	5	14	4	5	4	2	5	3	1	3	1	0	1	0
	Contents of stomachs	8	11·5	10	15·5	7	15	9	1	9	8	7	0	6	0	7	7	7	5	8	0
	Caul-fat	7	8	5	6	4	9	8	3	6	0·5	8	1	9	0	5	9	4	9	7	11
	Small intestines and contents	3	7	3	7	3	10	3	10·5	3	11	3	0	4	1	3	3	2	13	2	12
	Large intestines and contents	3	5·5	3	7	2	2·5	3	6·5	4	2·5	2	3	1	3	3	2	2	9	3	11·5
	Intestinal fat	1	14	2	7	2	13	2	2	2	6	2	1	1	4·5	1	12·5	1	10	1	11
	Heart and aorta	0	11·3	0	12·3	0	11	0	12	0	11·5	0	9	0	10·8	0	11	0	8·5	0	9
	Heart-fat	0	6	0	7·3	0	4·5	0	11·5	0	8	0	8	0	1·6	0	1	0	1·7	0	2
	Lungs and windpipe	1	8·5	2	2·5	2	0	2	3	1	12·5	1	0·5	1	12	1	9	1	7	1	8
	Blood	7	5·5	7	12·5	6	10·5	8	10	7	4·5	6	9	8	1·5	7	11·5	6	0·5	6	13
	Liver	3	5·5	3	4	2	13	3	15	3	1	2	10·3	2	15	3	2	2	9	2	9
	Gall-bladder and contents	0	1·9	0	2·2	0	1·1	0	2·2	0	3	0	1·6	0	0·2	0	2·3	0	1·2	0	1·4
	Pancreas ("sweetbread")	0	2·5	0	4·3	0	3	0	3·5	0	4	0	3·5	0	3·5	0	3·5	0	3·5	0	3·5
	Milt or spleen	0	4·5	0	5	0	3·8	0	5·8	0	5·5	0	4	0	5	0	6	0	3·7	0	4
	Bladder	0	1	0	0·8	0	0·7	0	0·8	0	1	0	0·6	0	1·1	0	0·6	0	0·8	0	0·8
	Head	4	9	5	4	4	13	5	5	5	0	4	13	4	13·5	5	0	4	6	4	7
	Skin (with feet, &c.)	14	0	15	8	14	0	11	6	13	4	12	8	14	0	13	8	12	4	12	0
	Wool previously shorn	10	0	11	1	8	0	8	12	9	2	9	2	8	12	11	9	8	8	8	8
	Total "offal" parts	72	6·7	77	6·9	65	7·1	77	14·8	72	0	65	5·6	69	6·7	69	7·4	59	6·0	66	5·2
	Carcass	115	1·5	117	14	110	6	131	6·5	109	14·5	101	0·5	78	6	98	4·5	91	14·8	95	5·2
	Loss by evaporation, error in weighing, &c.	0	7·8	0	12·1	2	2·9	−0	9·3	0	3·5	−0	4·1	0	15·3	0	13·1	0	2·3	1	13·6
	Live-weight after fasting	188	0	196	1	178	0	208	12	182	2	166	2	148	12	168	9	151	8	162	8

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* For particulars of the feeding experiment, see article on the "Comparative Fattening Qualities o

(lbs. and ozs.) of the individual Organs, and other separated Parts, of SHEEP.

Fattening food—Oilcake, Clover Chaff, and Swedish Turnips*. Killed at Rothamsted, April 19, 1852.

The 10 giving the Medium amount of Increase during Fattening.										Means of			
No. 11.	No. 12.	No. 13.	No. 14.	No. 15.	No. 16.	No. 17.	No. 18.	No. 19.	No. 20.	The 5 of Largest Increase.	The 5 of Smallest Increase.	The 10 of Medium Increase.	The 20 Cotswolds
lbs oz	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz	lbs. oz.	lbs. oz.	lbs oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs oz.
108 0	125 0	119 0	116 0	123 0	128 0	121 0	105 0	108 0	112 0	122 6·4	123 12·8	116 8	119 12·8
176 3	191 8	185 1	181 13	188 11	193 1	184 8	168 2	168 13	172 5	201 15·8	167 4·6	180 0·1	182 13·1
164 3	180 8	176 1	173 13	180 11	180 1	173 8	161 2	162 13	161 5	190 9·4	159 7·8	171 6·5	173 3·5
4 5	4 6	4 13	4 5	4 13	4 14	4 12	4 4	4 9·5	4 6	4 14·7	4 4·8	4 8·7	4 9·3
7 3	8 7	8 13	8 14	7 7	8 11	8 4	5 10	8 4·5	6 2	9 4·4	7 4·2	7 12·3	8 0·3
7 4	8 11	7 12·5	9 3	8 8	8 10	7 5	7 0	5 11·5	5 14	6 6·5	6 6·6	7 9·6	7 0
2 14·5	3 7	4 0	3 3	3 7·5	3 6	3 4	2 9·5	3 3	3 5	3 9·1	3 3·2	3 4·3	3 5·2
4 0	3 10·5	3 13	3 1	3 15	3 2·5	3 1	2 10	3 12	2 15	3 4·8	3 2·5	3 6·4	3 5·2
2 5	2 10·2	1 8	2 3	1 10	2 9	1 7	1 12·5	1 10	1 11	2 5·2	1 11·6	1 15	1 15·7
0 10	0 10	0 10·5	0 11	0 10·9	0 9·5	0 10·5	0 10	0 11	0 10	0 11·6	0 9·7	0 10·3	0 10·5
0 4	0 5·3	0 6	0 9·5	0 4	0 6·5	0 4·5	0 8	0 4	0 5·7	0 7·4	0 4·6	0 5·8	0 5·9
1 15·5	2 1·5	2 6	1 13	2 2	2 5	1 15·5	1 15	1 13	2 0	1 14·9	1 9·1	2 0·7	1 14·9
6 8·5	7 9	7 0·5	7 4	7 4·5	7 14	7 1	6 2	8 0·5	7 4	7 8·6	7 0·7	7 3·2	7 3·9
3 0	3 6	3 2·5	3 2	3 0	3 6	3 3	2 10	3 5	3 3	3 1·5	2 12·3	3 2·1	3 1·3
0 1·6	0 1	0 1·3	0 1·5	0 2	0 1·7	0 1·6	0 1·8	0 0·8	0 1·5	0 2·1	0 1·4	0 1·5	0 1·6
0 3·5	0 3·2	0 4	0 3	0 3·5	0 3·3	0 3·2	0 3·6	0 3	0 4	0 3·6	0 3·5	0 3·4	0 3·4
0 4·5	0 4·5	0 4·7	0 4·5	0 4·3	0 3·8	0 4·2	0 4·6	0 6·5	0 5	0 4·9	0 4·5	0 4·7	0 4·7
0 0·6	0 0·8	0 0·5	0 0·7	0 0·5	0 0·7	0 0·7	0 0·7	0 0·8	0 0·8	0 0·9	0 0·8	0 0·7	0 0·7
4 10	4 8·5	5 2	4 8	5 0	5 2	4 14	4 12·5	5 1·5	5 0	4 15·8	4 11·1	4 13·9	4 13·6
12 8	13 8	13 0	11 8	13 0	13 8	13 0	11 8	15 0	13 8	14 3·6	12 13·6	13 0	13 4·3
7 3	9 8	10 1	8 13	8 11	10 1	9 8	8 2	8 13	9 5	9 6·2	9 4·6	9 0·1	9 2·7
65 4·6	73 5·5	73 2·5	69 11·2	70 7·2	75 2	69 3·1	60 12·1	70 13·5	66 4	73 0·7	65 12·8	69 6·6	69 6·6
93 7·5	106 9	104 4	105 5·5	109 0	104 11·5	102 6·5	98 14·5	91 14·7	94 6	110 14·9	92 15·8	101 9·5	103 4·4
0 6·9	0 9·5	−1 5·5	−1 3·7	1 3·8	0 3·5	1 14·4	1 7·4	0 0·8	0 11	0 9·8	0 11·2	0 6·4	0 8·6
164 3	180 8	176 1	173 13	180 11	180 1	173 8	161 2	162 13	161 5	190 9·4	159 7·8	171 6·5	173 3·5

different Breeds of Sheep," *Journal of the Royal Agricultural Society of England*, vol. xiii. part 1.

APPENDIX.—TABLE XVIII. Showing the *Actual Weights* (lbs. and ozs.)

CLASS III.—16 Wether Sheep of *LeicesterBreed,* about 1¼ year old, in *moderately*
Killed at Rothamsted,

Designation of Parts.		The 4 giving the Largest amount of Increase during Fattening.				The 4 giving the Smallest amount of Increase during Fattening.			
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.
		lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
Original weight		119 0	121 0	117 0	108 0	96 0	92 0	93 0	84 0
Final weight, unfasted (including shorn wool)		185 4	183 12	179 0	161 0	127 0	123 9	118 12	107 11
Final weight, after fasting (including shorn wool).		170 4	168 12	165 0	150 0	116 0	112 9	109 12	98 11
Separate parts of the "offal."	Stomachs	4 12	3 14	3 12	3 15	3 6	3 7	3 1	2 11
	Contents of stomachs	7 0·5	5 14	6 8	7 1	4 15	6 0	5 1	5 1
	Caul-fat	6 3	6 13	7 4	5 5	4 8	2 0	2 8·5	1 11
	Small intestines and contents	3 10	3 0	3 9	2 13	2 4	4 1	2 11	2 12·5
	Large intestines and contents	3 3·5	2 11	3 1	2 9	1 14	2 11	2 2	2 6
	Intestinal fat	2 0	1 15	1 15	1 6	1 6	1 10	0 12·5	1 5·5
	Heart and aorta	0 10	0 12	0 10·5	0 10	0 7·5	0 9·5	0 7	0 9
	Heart-fat	0 5·7	0 3	0 8·5	0 3	0 2	0 2·5	0 2	0 2·4
	Lungs and windpipe	1 6	1 11	1 3	1 5	1 4	2 0	1 1	1 14
	Blood	6 13	6 10	7 2	5 11	4 8	5 7	4 15	4 13
	Liver	3 0	2 10	3 1	2 11	2 3·5	2 4	1 15·5	2 0·4
	Gall-bladder and contents	0 1·3	0 1·5	0 0·5	0 1	0 0·8	0 1·2	0 0·8	0 1
	Pancreas ("sweetbread")	0 4	0 4·5	0 4	0 3	0 3·2	0 3·3	0 3	0 2·5
	Milt or spleen	0 3·5	0 3·5	0 3·3	0 3	0 2·5	0 3·6	0 3	0 3
	Bladder	0 1	0 0·9	0 0·9	0 1	0 0·7	0 0·7	0 0·9	0 1
	Head	4 3	4 11	4 1	4 4	3 9	3 15	3 9·5	3 7·5
	Skin (with feet, &c.)	12 15	12 1	11 10	10 14	9 1	10 0	9 8	9 0
	Wool previously shorn	10 4	8 12	10 0	10 0	6 0	8 2	9 12	6 11
	Miscellaneous trimmings	......	......	......	......	......	......	......	......
Total "offal" parts		66 15·5	62 3·4	64 13·7	59 6	45 15·2	53 4·8	48 1·7	44 15·8
Carcass		99 9·5	101 13	96 5	88 6	66 13·5	59 2·5	59 14·5	51 15
Loss by evaporation, error in weighing, &c.		3 11	4 11·6	3 13·8	2 4	3 3·8	1 7	1 11·8	1 12·2
Live-weight after fasting		170 4	168 12	165 0	150 0	116 0	112 9	109 12	98 11

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* For particulars of the feeding experiment, see article on the "Comparative Fattening Qualities of

of the individual Organs, and other separated Parts, of SHEEP.

Fat condition. Fattening Food—Oilcake, Clover Chaff, and Swedish Turnips*.
April 23, 1853.

The 8 giving the Medium amount of Increase during Fattening								Means of			
No. 9	No. 10.	No. 11	No. 12.	No. 13.	No. 14.	No 15.	No. 16.	The 4 of Largest Increase.	The 4 of Smallest Increase	The 8 of Medium Increase	The 16 Leicesters
lbs. oz	lbs. oz.	lbs. oz	lbs. oz	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs oz.	lbs. oz	lbs. oz
114 0	100 0	89 0	105 0	105 0	95 0	105 0	91 0	110 4	91 4	100 8	102 2
160 3	145 14	134 7	150 3	149 2	138 4	146 10	132 8	178 0	119 4	144 10·4	146 10·2
148 3	132 14	121 7	138 3	139 2	122 4	134 10	123 8	163 8	109 4	132 8·4	134 7·2
3 9	3 8	3 8	3 2·5	3 11	3 5	3 14	3 5	4 1·2	3 2·2	3 7·8	3 8·8
5 7·5	4 12	7 1	4 9·5	5 11	4 11	4 6	7 1	6 9·9	5 4·2	5 7·4	5 11·2
5 6·5	4 13	3 4	7 1	5 2	4 4·5	3 11	5 2	6 6·2	2 10·9	4 13·5	4 11
2 15	2 14	3 6	2 9·5	2 13	3 1	2 10·5	3 0	3 4	2 15·1	2 14·6	3 0·1
2 4·5	2 5	2 2	2 8·5	3 6·5	2 5·5	2 8	2 4·5	2 14·1	2 4·3	2 7·6	2 8·4
1 7·5	1 7·5	1 7	2 4	1 9·7	1 14	1 5·5	1 13	1 13	1 4·5	1 10·5	1 9·6
0 9·5	0 9	0 9	0 7·5	0 8	0 8	0 9·5	0 8·5	0 10·6	0 8·3	0 8·6	0 9
0 3·8	0 4	0 2·3	0 4·5	0 4	0 2	0 3·5	0 1·5	0 5·1	0 2·2	0 3·2	0 3·4
1 5	1 6	2 0	1 4·5	1 5·5	1 1	1 8	1 3	1 0·2	1 8·8	1 0·1	1 6·8
5 8	5 0	5 12	5 2·5	5 15·5	4 11	5 6	5 4	6 9	4 14·8	5 5·4	5 8·6
2 8·5	2 12	2 10	2 5·5	2 10	2 4	2 13	2 10·5	2 14·3	2 1·9	2 0·2	2 8·6
0 1·3	0 2	0 2·2	0 1	0 1	0 1·5	0 1	0 0·8	0 1·1	0 0·9	0 1·4	0 1·2
0 3·5	0 3·7	0 3	0 3·5	0 4	0 4	0 3·5	0 3	0 3·9	0 3	0 3·5	0 3·5
0 3·2	0 3·5	0 3·5	0 3	0 3	0 2·5	0 3	0 3	0 3·3	0 3	0 3·1	0 3·1
0 0·5	0 0·8	0 0·8	0 0·7	0 0·8	0 1·7	0 0·7	0 0·5	0 1	0 0·8	0 0·8	0 0·9
4 1·5	4 0·5	4 1·5	3 12	4 1	3 11	4 3	3 10	4 4·7	3 10·3	3 15·1	3 15·3
11 8	10 1	10 7	10 0	11 4	9 14	11 11	9 8	11 14	9 6·2	10 8·6	10 9·4
8 3	6 14	8 7	8 3	9 2	7 4	6 10	6 8	9 12	7 12	7 10·4	8 3·2
0 3	0 2·5	0 1·2	0 2	0 1·8	0 1	0 4	0 3	...	.. ...	0 2·3	0 2·3
55 12·8	51 6·5	55 7·5	54 4·7	58 1·8	49 10·7	52 3·2	52 9·3	68 5·6	48 1·4	53 11·1	54 12·4
90 7	79 5·5	65 1	81 5·5	79 11·5	72 12	80 14	70 7·5	96 8·4	59 7·4	77 8	77 11·9
1 15·2	2 2	0 14·5	2 8·8	1 4·7	−0 2·7	1 8·8	0 7·2	3 10	1 11·2	1 5·8	1 14·9
148 3	132 14	121 7	138 3	139 2	122 4	134 10	123 8	163 8	109 4	132 8·4	134 7·2

different Breeds of Sheep," Journal of the Royal Agricultural Society of England, vol. xvi. part 1.

APPENDIX.—TABLE XIX. Showing the *Actual Weights* (lbs. and oz.)

CLASS IV.—16 Wether Sheep of *Cross-breed* (Leicester and South Down), about 1¼ year old, Killed at Rothamsted,

	Designation of Parts.	The 4 giving the Largest amount of Increase during Fattening.								The 4 giving the Smallest amount of Increase during Fattening.							
		No 1.		No. 2.		No. 3.		No. 4		No. 5.		No. 6.		No. 7.		No. 8	
		lbs	oz.	lbs	oz.	lbs	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.
	Original weight	88	0	93	0	99	0	102	0	91	0	88	0	100	0	98	0
	Final weight, unfasted (including shorn wool)	114	12	140	0	154	10	155	12	127	13	124	6	134	8	131	7
	Final weight, after fasting (including shorn wool)	134	12	138	0	142	10	141	12	115	13	111	6	122	8	122	7
Separate parts of the "offal."	Stomachs	3	11	3	12	3	15	3	11	3	1	3	2	3	8	3	2
	Contents of stomachs	7	6	5	15	9	5	5	12	4	15	5	12	5	12	7	3
	Caul-fat	4	12	6	12·5	4	5·5	5	12	4	0	4	4	5	6	6	1
	Small intestines and contents	1	14	2	7·5	2	6	2	7	1	15	2	3	2	9	2	7·5
	Large intestines and contents	2	4	2	12	3	4	3	6	2	10·5	2	7	2	15	2	11
	Intestinal fat	1	7	2	3	1	9	1	11	1	0	1	0·5	1	9·5	1	7
	Heart and aorta	0	9·3	0	8·5	0	10	0	9	0	7	0	9·5	0	8	0	7·5
	Heart-fat	0	4	0	3·5	0	4·5	0	3·5	0	4	0	3	0	2·2	0	4
	Lungs and windpipe	1	8	1	4·5	1	14	1	8	1	9	1	2	1	7	1	8
	Blood	5	5	5	8	5	9	6	9	4	12	4	14	5	12·5	5	5·3
	Liver	2	7	2	7·5	2	15	2	3	2	1	2	4	2	4	2	2·5
	Gall-bladder and contents	0	2·5	0	1·5	0	3	0	1·8	0	2	0	1·5	0	1·1	0	1·2
	Pancreas ("sweetbread")	0	3·8	0	4	0	3·5	0	4	0	3·5	0	3·5	0	3·5	0	3·5
	Milt or spleen	0	3·5	0	8	0	3	0	4	0	3	0	3·5	0	4·4	0	3·3
	Bladder	0	1	0	0·8	0	1·3	0	0·8	0	0·7	0	0·8	0	0·7	0	0·9
	Head	4	4	4	1	4	0·5	4	6	3	14·5	3	12·5	4	2·5	4	0
	Skin (with feet, &c.)	11	0	8	12·	13	2	12	0	10	8	10	0	9	11	10	4
	Wool previously shorn	8	12	7	0	7	10	6	12	5	13	5	6	7	8	6	7
	Miscellaneous trimmings	0	5·5	0	1·8	……		0	5·5	0	2	0	3·5	0	3·5	0	2·5
	Total "offal" parts	56	7·1	54	6·1	61	14·3	58	0·6	47	9·2	47	12·3	53	15·0	54	1·2
	Carcass	77	13·5	85	5	83	0	83	12	67	15·5	64	6·5	65	12	68	14·5
	Loss by evaporation, error in weighing, &c.	0	7·4	−1	11·1	−2	4·3	−0	0·6	0	4·3	−0	12·8	2	12·1	−0	8·7
	Live-weight after fasting	134	12	138	0	142	10	141	12	115	13	111	6	122	8	122	7

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* For particulars of the feeding experiment, see article on the "Comparative Fattening Qualities of

of the individual Organs, and other separated Parts, of SHEEP.

in *moderately* Fat condition. Fattening food—Oilcake, Clover Chaff, and Swedish Turnips*.
April 23, 1853.

The 8 giving the Medium amount of Increase during Fattening.								Means of			
No. 9.	No. 10.	No. 11.	No. 12	No. 13	No. 14.	No. 15.	No. 16.	The 4 of Largest Increase.	The 4 of Smallest Increase.	The 8 of Medium Increase.	The 16 Cross-bred Wethers.
lbs. oz	lbs. oz	lbs. oz.	lbs. oz.	lbs oz.	lbs. oz.	lbs oz	lbs. oz.	lbs. oz.	lbs. oz	lbs. oz	lbs oz
95 0	102 0	88 0	98 0	101 0	89 0	96 0	100 0	95 8	94 4	96 2	95 8
140 12	147 4	132 12	142 9	145 8	133 5	139 13	142 8	151 0·5	120 8·5	140 8·9	140 6·7
126 12	139 4	120 12	133 9	131 8	123 5	132 13	126 8	139 4·5	118 0·5	129 4·9	128 15·7
3 6	3 12	2 14	3 1·5	3 7	3 2	3 14	3 5	3 13	3 3·3	3 5·7	3 6·9
4 15	6 4	5 9	6 8·5	3 13	5 6	7 10·5	3 9	7 1·5	5 14·5	5 7·4	5 15·7
5 11	7 5·5	5 1	5 8	4 9·5	3 12·5	4 10	5 9·5	5 6·5	4 14·7	5 4·4	5 3·5
1 11	3 1	1 13	2 3·5	2 8·5	1 11	2 9	2 3	2 4·6	2 4·6	2 3·5	2 4·1
2 2	2 9	2 0	2 8	2 9·5	2 13	3 9	2 5·5	2 14·5	2 10·9	2 9	2 10·9
2 3·5	2 12	1 12	2 8·5	1 10·8	1 0	1 7·5	2 2	1 11·5	1 4·2	1 15	1 11·5
0 9	0 8·7	0 7·7	0 8·5	0 8·5	0 8·2	0 8·8	0 9·5	0 9·2	0 8	0 8·6	0 8·6
0 4·4	0 5·5	0 5	0 3	0 7·5	0 3·1	0 6·7	0 4·5	0 3·9	0 3·3	0 5	0 4·3
1 4	1 6·2	1 2·5	1 4·5	1 10	1 5·5	1 8	1 5	1 8·6	1 6·5	1 5·7	1 6·6
5 9·5	5 11·3	4 7	5 3	4 12	4 15	5 13·5	5 5	5 11·7	5 2·9	5 3·6	5 5·4
2 5	2 9	2 1	2 6	2 5·5	2 1	2 5	2 7	2 8·1	2 2·9	2 4·9	2 5·2
0 1·8	0 1·3	0 2·2	0 2	0 0·8	0 0·7	0 2·2	0 1·2	0 2·2	0 1·6	0 1·5	0 1·7
0 2·7	0 4·7	0 3·5	0 3·2	0 3·7	0 3	0 4·2	0 3·7	0 3·7	0 3·5	0 3·6	0 3·6
0 3·7	0 3·8	0 3	0 3	0 3·5	0 2·7	0 3	0 3	0 3·4	0 3·6	0 3·2	0 3·3
0 0·6	0 0·9	0 0·7	0 0·7	0 0·9	0 0·6	0 0·9	0 0·8	0 1	0 0·8	0 0·8	0 0·8
4 0	4 4·7	3 10	3 14	4 1·8	4 2·5	4 1·5	4 2	4 4·4	3 15·4	4 0·5	4 1·2
9 6	10 12	9 6	10 14	11 3	11 12	11 11	9 10	11 3·5	10 1·7	10 9·2	10 9·9
4 12	5 4	6 12	6 9	5 8	8 5	6 13	6 8	7 8·5	6 4·5	6 4·9	6 9·7
0 2·5	0 2	0 2·5	0 1	0 3	0 2·8	0 3·5	0 1·8	0 4·3	0 2·9	0 2·4	0 2·9
48 13·7	57 5·6	48 0·1	53 13·9	49 14·5	51 10·6	57 13·3	49 15·5	57 12·1	50 13·7	52 2·9	53 3·8
78 14	80 7·5	72 0	78 12	80 13·2	68 4·5	72 10	76 9·5	82 7·6	66 12·1	76 0·8	75 5·4
−0 15·7	1 6·9	0 11·9	0 15·1	0 12·3	3 5·9	2 5·7	−0 1	−0 15·2	0 6·7	1 1·2	0 6·5
126 12	139 4	120 12	133 9	131 8	123 5	128 5	126 8	139 4·5	118 0·5	129 4·9	128 15·7

different Breeds of Sheep," Journal of the Royal Agricultural Society of England, vol. xvi. part 1.

APPENDIX.—TABLE XX. Showing the *Actual Weights* (lbs. and ozs.)

Class V.—16 Ewe Sheep of *Cross-breed* (Leicester and South Down), about 1[illegible] year old, in
Killed at Rothamsted,

	Designation of Parts.	The 4 giving the Largest amount of Increase during Fattening.				The 4 giving the Smallest amount of Increase during Fattening.			
		No 1.	No. 2.	No. 3.	No 4.	No. 5.	No 6.	No. 7.	No. 8.
		lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
	Original weight	84 0	91 0	95 0	91 0	90 0	91 0	92 0	89 0
	Final weight, unfasted (including shorn wool)	143 14	142 6	145 14	140 4	124 12	124 8	125 6	118 7
	Final weight, after fasting (including shorn wool).	129 14	129 6	131 11	125 4	112 12	112 8	114 6	108 7
Separate parts of the "offal."	Stomachs	3 2·5	3 0	3 7	2 15	3 4	2 15	2 13	2 10·5
	Contents of stomachs	5 11·5	3 6	5 6	6 2	4 2	3 0	5 4	4 6·5
	Caul-fat	6 4	5 3·5	6 8	4 7	5 3·5	6 0	3 15·5	4 4·5
	Small intestines and contents	2 0	2 4·5	2 4	2 5	1 12	1 11	2 0	2 1·5
	Large intestines and contents	1 12	2 7·5	2 4	1 15	2 4	1 5	2 6	2 4·5
	Intestinal fat	1 8·3	1 5	2 10·5	1 14·5	1 12	2 1·5	1 7	1 0
	Heart and aorta	0 8·5	0 9	0 9	0 9	0 8	0 8	0 6·5	0 7
	Heart-fat	0 4·5	0 2	0 3·7	0 2·5	0 4·5	0 3	0 2·5	0 2
	Lungs and windpipe	1 6	1 2·5	1 3·7	1 2	1 2·8	1 1	1 6	1 8·5
	Blood	5 12	5 0·5	5 5	4 8	5 7·5	4 12	4 11	4 1
	Liver	2 8·5	2 5·5	2 9	2 2·5	1 12·5	1 14	1 15·5	1 13
	Gall-bladder and contents	0 2	0 1·2	0 0·5	0 2	0 1·4	0 1·8	0 0·7	0 0·9
	Pancreas ("sweetbread")	0 3·5	0 3·3	0 3·5	0 3	0 2·5	0 3	0 3	0 3
	Milt or spleen	0 3·2	0 3·2	0 4·5	0 3	0 3	0 2·5	0 2·5	0 2·5
	Bladder	0 0·5	0 0·8	0 0·7	0 0·5	0 0·6	0 0·9	0 0·8	0 0·9
	Womb	0 0·7	0 1·2	0 1	0 1	0 1·2	0 0·7	0 0·7	0 0·7
	Head	3 13·5	4 0	4 0	3 11·5	3 0·5	3 7·5	3 7·5	3 3
	Skin (with feet, &c.)	11 3	10 7	9 8	10 0	8 12	8 4	9 0	9 8
	Wool previously shorn	7 14	7 6	6 14	7 4	6 12	5 8	6 6	6 7
	Miscellaneous trimmings	0 8	......	0 2·5	0 4·5	0 2	......	......	0 4
	Total "offal" parts	54 9·2	49 10·7	53 8·6	50 0	47 2	43 2·9	45 12·2	44 9
	Carcass	76 11·0	77 5·5	77 13	71 12	65 5	67 6·0	68 1	62 11
	Loss by evaporation, error in weighing, &c.	−1 6·2	2 5·8	0 8·4	3 8	0 5	1 15·1	0 8·8	1 3
	Live-weight after fasting	120 14	129 6	131 14	125 4	112 12	112 8	114 6	108 7

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* For particulars of the feeding experiment, see article on the "Comparative Fattening Qualities of

of the individual Organs, and other separated Parts, of SHEEP.

moderately Fat condition. Fattening food—Oilcake, Clover Chaff, and Swedish Turnips*.
April 28, 1858.

The 8 giving the Medium amount of Increase during Fattening.								Means of—			
No. 9.	No. 10.	No. 11.	No. 12.	No. 13	No. 14.	No. 15.	No. 16.	The 4 of Largest Increase.	The 4 of Smallest Increase.	The 8 of Medium Increase	The 16 Cross bred Ewes.
lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
86 0	98 0	86 0	98 0	87 0	96 0	91 0	92 0	90 4	90 8	91 12	91 1
129 10	141 8	129 8	140 10	129 3	137 12	132 6	133 4	143 1·5	123 4·3	131 3·6	133 11·3
119 10	129 8	118 8	129 10	117 3	125 12	122 6	120 4	129 1·5	112 0·3	122 13·6	121 11·2
3 2	3 6	2 15·5	3 0	2 13·5	3 3·5	3 0	3 2·5	3 2·1	2 14·6	3 1·4	3 0·9
6 9	6 4	5 14·5	4 11·5	3 10·5	4 13·5	5 2	2 7·5	5 2·3	4 3·1	4 15	4 12·9
5 12	5 10	5 10·5	5 12	5 9·5	5 10	4 10	5 1	5 9·6	4 13·9	5 7·4	5 5·6
2 2·5	2 3	2 4	2 3	1 8·5	2 2·5	2 1	2 2·5	2 3·3	1 14·1	2 1·4	2 1·1
2 6	2 9	2 4·5	2 5	2 1	2 11	2 2	2 3·5	2 1·6	2 0·9	2 5·3	2 3·2
1 5·5	1 7·5	1 5	2 4	1 2	2 5·5	2 3·5	1 10	1 13·6	1 9·1	1 11·4	1 11·4
0 8	0 7	0 7	0 8·3	0 8·5	0 7·7	0 8·3	0 8	0 8·8	0 7·4	0 7·8	0 8
0 2	0 3	0 4	0 4·4	0 5·5	0 4·5	0 4·3	0 3	0 3·1	0 3	0 3·8	0 3·5
1 3·5	1 5	1 3	1 2·5	1 5	1 3·8	1 3·3	1 1·7	1 3·5	1 4·6	1 3·5	1 3·8
5 0	4 8	4 14·5	4 6·5	5 3	5 2·5	4 14	4 14·5	5 3·8	4 11·9	4 13·9	4 14·9
1 14·5	2 1·5	2 2·5	2 2	2 3·5	2 1·5	2 3	2 1	2 6·3	1 13·8	2 1·7	2 1·9
0 1·5	0 1·3	0 2·1	0 1·3	0 2	0 1·2	0 1·2	0 1·6	0 1·4	0 1·2	0 1·5	0 1·4
0 3·5	0 3	0 2·7	0 4	0 3	0 3	0 3·2	0 3·3	0 3·3	0 2·9	0 3·2	0 3·1
0 4	0 3	0 3	0 3·5	0 3·3	0 2·8	0 3	0 3·5	0 3·4	0 2·6	0 3·3	0 3·1
0 0·6	0 0·5	0 0·5	0 0·8	0 0·7	0 0·7	0 0·6	0 0·7	0 0·6	0 0·8	0 0·6	0 0·7
0 1·5	0 1	0 0·8	0 1	0 1·2	0 1·4	0 0·9	0 1	0 0·9	0 0·8	0 1·1	0 1
4 0	4 0	3 8·5	3 10	3 10·5	3 11·5	3 14	3 8	3 14·2	3 6·1	3 11·8	3 11
10 2	10 8	9 10	9 10	10 4	11 0	9 7	9 14	10 4·5	8 14	10 0·9	9 13
7 10	6 8	7 8	5 10	7 3	7 12	6 6	8 4	7 5·5	6 4·2	7 1·6	6 15·2
0 4·4	0 1·7	0 3	0 2·4	0 2	0 1·8	0 2·8	0 1·5	0 3·3	0 3	0 2·4	0 2·7
52 12·5	51 10·5	50 11·6	48 6·2	48 4·2	53 4·4	48 10·1	47 12·8	51 15·1	45 4	50 3	49 6·4
65 10	75 11·5	67 8·5	79 8·2	68 12	71 2	73 1	71 9	75 14·4	65 13·8	71 0·8	71 3·9
1 3·5	2 2	0 3·9	1 11·6	0 2·8	1 5·6	0 10·9	0 14·2	1 4	0 14·5	1 0·8	1 0·9
119 10	129 8	118 8	129 10	117 3	125 12	122 6	120 4	129 1·5	112 0·3	122 13·6	121 11·2

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different Breeds of Sheep," Journal of the Royal Agricultural Society of England, vol. xvi. part 1.

APPENDIX.—TABLE XXI. Showing the *Actual Weights* (lbs. and ozs.)

CLASS VI.—16 Wether Sheep of *Hampshire Down Breed*, about 1¼ year old, in *mode-*
Killed at Rothamsted,

Designation of Parts.		The 4 giving the Largest amount of Increase during Fattening.				The 4 giving the Smallest amount of Increase during Fattening.			
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.
		lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
Original weight		119 0	124 0	112 0	116 0	105 0	96 0	119 0	105 0
Final weight, unfasted (including shorn wool)		218 8	220 8	207 8	201 0	159 0	150 12	174 4	160 8
Final weight, after fasting (including shorn wool).		202 8	205 8	192 8	191 0	151 0	140 12	164 4	151 8
Separate parts of the "offal."	Stomachs	4 15	4 11	4 2	4 6	3 7	3 10	3 10	3 6
	Contents of stomachs	8 15	10 7	10 0	10 6	8 5	6 6	7 14	6 12
	Caul-fat	8 3	8 15·8	7 2·4	7 8	8 11	5 11	8 12·1	6 2·3
	Small intestines and contents	4 0	3 15	3 4	3 0	2 12	2 10	2 10	2 1
	Large intestines and contents	4 7	3 0	2 13	3 1	2 8	2 9	3 0	2 10
	Intestinal fat	5 0·4	5 2	5 4	4 10	3 14·1	2 4·3	5 6·5	4 7·2
	Heart and aorta	0 13	0 11·5	0 10·8	0 10·5	0 9·1	0 8·5	0 9·7	0 8·6
	Heart-fat	0 10·6	0 7·4	0 8·6	0 5·9	0 8·5	0 4·5	0 6	0 4·2
	Lungs and windpipe	1 14·3	1 13	1 6·1	1 0·3	1 9	1 13	1 9·5	1 5·6
	Blood	9 1	9 2·5	7 12·5	7 10·8	6 7	6 10·3	5 8·5	6 4·5
	Liver	3 7·8	3 9·6	2 9·9	3 5·8	2 6	2 9	2 5·6	2 0·8
	Gall-bladder and contents	0 1·1	0 0·5	0 0·8	0 0·9	0 1·4	0 0·6	0 1·2	0 0·4
	Pancreas ("sweetbread")	0 4	0 4	0 3·1	0 4·1	0 4·7	0 4	0 4·3	0 3
	Milt or spleen	0 4·4	0 5·1	0 4·2	0 4·1	0 3·4	0 4·2	0 3·7	0 3·1
	Bladder	......	0 3·2	0 2·4	0 0·8	0 0·5	0 0·5	0 1	0 0·6
	Head	6 4	6 0·5	5 10·5	5 1	4 8	4 14	5 0·1	4 4·8
	Skin (with feet, &c.)	17 11	17 8	15 14	16 2	11 0	11 8	11 5	12 2
	Wool previously shorn	5 8	5 8	6 8	8 0	6 0	5 12	4 4	7 8
	Miscellaneous trimmings	0 4·9	0 1·9	0 2·1	0 4	0 4·5	0 5	0 4·3	0 3·4
Total "offal" parts*		81 12·4	81 14	74 6·4	76 10·2	63 2·2	57 15·9	63 3·5	60 10·5
Carcass		120 10·5	123 13	118 13·5	114 5·5	86 14	80 6	99 8·5	89 15
Loss by evaporation, error in weighing, &c.		0 1·1	−0 3	−0 11·9	0 0·8	0 15·8	2 6·1	1 8	0 14·5
Live-weight after fasting		202 8	205 8	192 8	191 0	151 0	140 12	164 4	151 8

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* For particulars of the *feeding* experiment, see Article on the "Comparative Fattening

of the individual Organs, and other separated Parts, of SHEEP.

rately Fat condition. Fattening food—Oilcake, Clover Chaff, and Swedish Turnips*.
May 8, 1851.

The 8 giving the Medium amount of Increase during Fattening.								Means of—			
No. 9.	No. 10.	No. 11.	No. 12.	No. 13.	No. 14.	No. 15.	No. 16.	The 4 of Largest Increase.	The 4 of Smallest Increase.	The 8 of Medium Increase.	The 16 Hampshire Downs.
lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz	lbs. oz	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
108 0	125 0	108 0	113 0	131 0	100 0	120 0	112 0	117 12	106 4	114 10	113 5
177 8	194 4	177 0	181 8	199 8	168 0	187 8	178 8	212 10	161 2	182 15·5	184 14·8
168 8	184 4	164 0	173 8	188 8	156 0	176 8	163 8	197 14	151 14	171 13·5	173 5·8
3 14	3 10	3 9	3 8	4 2	3 8	3 12	3 10	4 8·5	3 8·3	3 11·1	3 13·8
8 10	10 10	7 11	8 8	6 14	7 0	8 10	7 6	9 15	7 5·3	8 2·6	8 6·4
8 3·5	6 15·7	7 1	9 10	9 5·5	6 3·4	6 8	7 8·5	7 15·3	7 5·1	7 11	7 10·6
2 14	3 8	4 8	3 0	3 4	3 2	2 12	3 2	3 8·8	2 9	3 3·6	3 2·3
2 14	4 2	3 0	3 4	4 8	2 8	3 12	3 2	3 5·3	2 10·7	3 6·2	3 3·1
5 0·5	4 10	5 2	5 2·5	4 14	3 8	4 3·5	5 9	5 0·1	4 0	4 12·2	4 10·1
0 12·8	0 12·3	0 9·5	0 9·8	0 13·8	0 9	0 12·1	0 10·2	0 11·5	0 9	0 11·2	0 10·7
0 3·3	0 7·6	0 5·5	0 4·8	0 10·5	0 6	0 5·7	0 4·2	0 8·1	0 4·5	0 6	0 6·1
1 12·5	1 7·5	1 10·5	1 0·8	1 11·5	1 9·5	1 0·2	1 0·2	1 10·7	1 0·3	1 0·6	1 0·8
6 14	7 7·5	7 1·5	6 5·5	8 10·5	5 14·5	7 4·5	6 15	8 6·7	6 3·5	7 1·1	7 3·1
2 12·5	1 15·7	2 8	2 9·5	2 11·8	2 7·5	3 0·5	2 12	3 4·3	2 5·3	2 0·7	2 11·2
0 0·7	0 2·6	0 1·4	0 2·9	0 1·4	0 1·1	0 1·3	0 0·9	0 0·8	0 0·9	0 1·5	0 1·2
0 4·3	0 3·5	0 3·5	0 3·3	0 3·6	0 4·5	0 4	0 3·1	0 3·8	0 4	0 3·7	0 3·6
0 4·8	0 4·1	0 5·1	0 4·5	0 5·3	0 3·7	0 4·7	0 3·5	0 4·4	0 3·6	0 4·5	0 4·2
0 1	0 1	0 1·1	0 0·7	0 1	0 0·6	0 0·9	0 1	0 2·1	0 0·7	0 0·9	0 1·1
5 2·2	5 2	4 13	5 4	5 12	4 15·5	5 11·5	5 4	5 12	4 10·7	5 4	5 3·7
12 3	15 12	12 12	11 12	14 4	12 14	16 0	12 0	16 12·7	11 7·8	13 7·1	13 12·7
5 8	6 4	7 0	5 8	6 8	6 0	5 8	4 8	6 6	5 14	5 13·5	5 15·8
0 2	0 3·8	0 3	0 2·8	0 4·5	0 4	0 3	0 7	0 3·2	0 4·3	0 3·8	0 3·8
67 9·1	73 6·3	68 9·1	67 11·1	75 1·4	61 7·3	70 10·9	65 3·6	78 11·3	61 4	68 11·3	69 5·5
100 0	108 12·5	94 12	105 3·5	111 10	94 8	106 6·5	98 13	119 6·6	89 2·9	102 7·6	103 6·2
0 14·9	2 1·2	0 10·9	0 9·4	1 12·6	0 5·7	–0 9·4	–0 8·6	–0 3·9	1 7·1	0 10·6	0 10·1
168 8	184 4	164 0	173 8	188 8	156 0	176 8	163 8	197 14	151 14	171 13·5	173 5·8

Qualities of Sheep," Journal of the Royal Agricultural Society of England, vol. xii. part 2.

APPENDIX.—Table XXII. Showing the *Actual Weights* (lbs. and ozs.)

CLASS VII.—16 Wether Sheep of *Sussex Down Breed*, about 1¼ year old, in *moderatel*
Killed at Rothamsted

	Designation of Parts.	The 4 giving the Largest amount of Increase during Fattening.								The 4 giving the Smallest amount of Increase during Fattening.							
		No. 1.		No. 2.		No. 3.		No. 4.		No. 5.		No. 6.		No. 7.		No. 8.	
		lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.
	Original weight	99	0	97	0	84	0	80	0	86	0	88	0	86	0	78	0
	Final weight, unfasted (including shorn wool)	167	4	163	8	147	8	142	12	121	4	123	8	126	4	121	8
	Final weight, after fasting (including shorn wool)	154	4	151	8	137	8	132	12	117	4	116	8	119	4	116	8
Separate parts of the "offal."	Stomachs	3	13	3	1	3	2	3	0	2	10	2	14	2	13	2	9
	Contents of stomachs	5	13	4	12	7	5	7	0	3	14	5	0	6	3	6	14
	Caul-fat	7	0·4	6	3	6	9	6	5	6	2·3	4	7·5	6	8·3	5	2
	Small intestines and contents	3	10	2	12	2	14	2	13	1	12	2	8	3	4	2	8
	Large intestines and contents	2	5	2	7	2	6	2	4	1	14	2	7	2	6	2	3
	Intestinal fat	4	9	4	4	3	4·6	2	14·3	3	9·5	2	8	3	1	2	9·8
	Heart and aorta	0	9·5	0	9·3	0	9·5	0	8·0	0	8	0	7·5	0	7·5	0	7·8
	Heart-fat	0	10·1	0	10·1	0	8·4	0	4·5	0	5·6	0	8	0	4·5	0	3·8
	Lungs and windpipe	1	5·9	1	3·5	1	3·8	1	5	1	2·5	1	4·5	1	5	1	2·8
	Blood	8	13·8	6	1·6	5	15·8	5	8·0	4	8·3	4	11·8	4	12	4	9·5
	Liver	2	3·8	2	5·8	2	2	2	2	1	14·5	2	1·4	2	0	2	1·3
	Gall-bladder and contents	0	0·3	0	0·5	0	0·8	0	0·8	0	1	0	0·7	0	0·4	0	0·8
	Pancreas ("sweetbread")	0	2·7	0	4	0	4	0	3·0	0	2·4	0	2·5	0	3·1	0	3·9
	Milt or spleen	0	4·6	0	5·9	0	5·9	0	4·7	0	5·5	0	5·5	0	4·0	0	6
	Bladder	0	0·6	0	0·9	0	0·5	0	0·5	0	0·6	0	0·5	0	1	0	0·8
	Head	4	3·3	4	8	4	8·5	4	2·5	3	10·3	3	12·8	3	11·5	3	11·5
	Skin (with feet, &c.)	12	4	10	4	10	6	9	2	9	8	9	0	8	7	9	5
	Wool previously shorn	6	4	6	8	6	8	4	12	5	4	4	8	6	4	4	8
	Miscellaneous trimmings	0	2·1	0	3·0	0	0·8	0	3·4	0	4·5	0	5	0	2·3	0	3·3
	Total "offal" parts	64	3·2	56	8·5	58	2·6	52	14·8	47	9	46	11·7	52	5·5	48	14·3
	Carcass	92	12	94	6	79	10	80	11	68	6	68	6·5	67	0	67	6
	Loss by evaporation, error in weighing, &c	–2	11·2	0	9·4	–0	4·6	–0	13·8	1	4·9	1	5·7	–0	1·5	0	3·7
	Live-weight after fasting	154	4	151	8	137	8	132	12	117	4	116	8	119	4	116	8

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* For particulars of the *feeding* experiment, see Article on the "Comparative Fattening

of the individual Organs, and other separated Parts, of SHEEP.

Fat condition. Fattening food—Oilcake, Clover Chaff, and Swedish Turnips*.
May 8, 1851.

The 8 giving the Medium amount of Increase during Fattening								Means of—			
No. 9.	No. 10.	No. 11.	No 12.	No 13.	No. 14.	No. 15.	No. 16.	The 4 of Largest Increase.	The 4 of Smallest Increase.	The 8 of Medium Increase.	The 16 Sussex Downs.
lbs. oz.	lbs. oz.	lbs oz.	lbs. oz.	lbs oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs oz.
89 0	96 0	86 0	87 0	85 0	89 0	93 0	93 0	90 0	84 8	89 10	88 8
143 8	149 0	139 12	140 8	138 8	142 4	145 8	145 4	155 4	123 2	143 0·5	141 1·8
139 8	142 0	132 12	134 8	129 8	136 4	137 8	135 4	144 0	117 6	135 14·5	133 4·7
2 14	3 0	2 12	2 14	3 2	3 0	3 2	3 0	3 4	2 11·5	2 15·5	2 15·6
6 6	4 10	5 8	6 10	5 4	8 0	6 2	5 4	6 3·5	5 7·8	5 15·5	5 14·6
6 13	7 5	7 6	6 15	4 14	5 13·5	7 6	6 5·5	6 8·3	5 9	6 9·8	6 5·2
3 4	2 8	2 10	3 0	2 12	2 10	2 2	2 4	3 0·3	2 8	2 10·3	2 11·2
2 6	2 2	2 5	2 8	3 0	3 6	2 8	2 4	2 5·5	2 3·5	2 8·9	2 6·7
3 2	5 6·5	4 2·5	2 14·5	2 13	2 13	3 12·8	3 5·2	3 11·9	2 13·8	3 8·7	3 6·8
0 9·3	0 9·5	0 7·2	0 8·1	0 8·5	0 9	0 7·8	0 8	0 9·2	0 7·7	0 8·4	0 8·4
0 3·3	0 1·9	0 4	0 4·5	0 7	0 4	0 5	0 6	0 8·3	0 5·5	0 4·5	0 5·7
1 11·4	1 10	1 3·8	1 2·5	1 6·5	1 6·3	1 3·5	1 7·7	1 4·6	1 3·7	1 8·6	1 0·3
6 0·8	5 6·6	5 2·5	5 9·5	5 8·9	5 15	5 8·5	4 13·8	6 10	4 10·4	5 6·2	5 8·2
2 8·5	2 6·4	2 3·5	2 0	2 6	1 15·5	1 15·3	2 9	2 3·4	2 0·3	2 4	2 2·0
0 1·1	0 0·4	0 1·8	0 1·3	0 1·9	0 2·6	0 2·7	0 3·1	0 0·6	0 0·7	0 1·0	0 1·3
0 3·5	0 2·8	0 3·5	0 3	0 2·8	0 4·1	0 3	0 3·3	0 3·6	0 3	0 3·2	0 3·2
0 6	0 4·7	0 6	0 3·7	0 6·2	0 4·5	0 4	0 4·5	0 5·3	0 5·5	0 4·9	0 5·2
0 0·5	0 0·6	0 0·5	0 0·7	0 0·5	0 0·7	0 0·6	0 0·8	0 0·6	0 0·7	0 0·6	0 0·6
4 0·5	3 12·5	3 14	3 11	3 15·7	4 2·9	4 0·5	4 6·5	4 5·6	3 12·3	3 15·0	4 0·4
12 4	10 8	10 0	10 2	10 10	9 10	10 0	10 6	10 8	9 1	10 7	10 1·8
5 8	5 0	5 12	6 8	6 8	5 4	5 8	5 4	6 0	5 2	5 10·5	5 9·8
0 2	0 3	0 4	0 3·5	0 3	0 2·9	0 4·5	0 4·1	0 2·6	0 3·8	0 3·3	0 3·3
58 7·9	55 1·9	54 10·3	55 7·3	54 1·5	55 12	55 0·1	53 3·5	57 15·3	48 14·2	55 3·6	54 5·2
80 2	85 14·5	77 13	78 7·5	76 0·5	79 12·5	81 13·5	82 1·5	86 13·7	67 12·6	80 3·9	78 12·6
0 14	0 15·6	0 4·6	0 9·2	−0 10	0 11·5	0 10·4	−0 1	−0 13	0 11·2	0 7	0 2·9
139 8	142 0	132 12	134 8	129 8	136 4	137 8	135 4	144 0	117 6	135 14·5	133 4·7

Qualities of Sheep," Journal of the Royal Agricultural Society of England, vol. xii. part 2.

APPENDIX.—TABLE XXIII. Showing the *Actual Weights* (lbs and ozs.) of the individual Organs, and other separated Parts, of SHEEP

CLASS VIII.—6 Wether Sheep of *Cotswold Breed*, about 1¾ year old, in *excessively* Fattened condition. Fattening food—Oilcake, Clover Hay, and Swedish Turnips, under cover, until "moderately" fattened; afterwards, Oilcake (or Lentils), with Grass or Green Clover, or with Meadow-hay and Roots, in the field. Killed at Rothamsted, December 18, 1852.

	Designation of Parts.	6 Very Fat Cotswold Sheep						Means of the 6 Cotswolds
		No 1	No 2	No 3	No 4	No. 5.	No 6	
		lbs oz	lbs oz	lbs oz	lbs oz	lbs oz	lbs oz	lbs oz
	Original weight	121 0	132 0	112 0	121 0	121 0	121 0	121 5·3
	Final weight, unfasted	262 0	254 0	252 0	242 0	234 0	228 0	245 5 3
	Final weight, after fasting	252 0	245 0	242 0	233 0	224 0	220 0	236 0
Separate parts of the "offal."	Stomachs	5 7	4 11	4 9	4 5	4 6	4 0	4 9
	Contents of stomachs	11 3	8 4	10 13	8 9	7 10	....	9 4·6
	Caul-fat	8 2	9 12	6 13	12 15	8 11·7	7 11	9 0·1
	Small intestines and contents	3 7·5	3 3	2 12	2 4	2 14	3 2	2 15·1
	Large intestines and contents	5 0	4 6	3 9	3 6	3 5	3 8	3 13·7
	Intestinal fat	4 14	3 14·5	3 5·5	3 2·5	2 2	3 3	3 6·9
	Heart and aorta	0 12·9	0 15	0 13	0 11·5	0 13	0 13	0 13·1
	Heart-fat	1 0	0 14·5	0 15	0 10·5	0 7	0 10·5	0 12·3
	Lungs and windpipe	2 1·2	2 3	1 14	1 10·5	2 1·5	1 12·5	1 15·1
	Blood	10 3	10 5	8 10	7 13	8 1	8 8·5	8 14·7
	Liver	3 15	3 7	3 2	2 9	2 9·5	2 9	3 0·6
	Gall-bladder and contents	0 1·8	0 3·7	0 1·4	0 1·1	0 1·7	0 1·3	0 1·8
	Pancreas ("sweetbread")	0 3·5	0 2·7	0 4·5	0 2·5	0 3·3	0 3	0 3·3
	Milt or spleen	0 6·4	0 5·5	0 5	0 6·5	0 4	0 5·5	0 5·5
	Bladder	0 0·8	0 0·7	0 1	0 0·9	0 0·9	0 1·2	0 0·9
	Head	5 7·5	5 3·5	5 4	5 3·5	5 11	4 15·3	5 4·8
	Skin and wool (with feet, &c.)	29 10	24 4	27 12	26 0	26 4	24 14	26 7·3
	Wool shorn in Spring*	(10 7)	(10 0)	(16 8)	(11 8)	(10 0)	(9 12)	(11 5·8)
	Diaphragm ("skirts")	0 3·2	0 4·5	0 4·5	0 3	0 3·5	0 2·5	0 3·5
	Miscellaneous trimmings	....	....	... . ..	....	0 5·5	0 2·9	0 4·2
	Total "offal" parts	92 2·8	82 7·6	81 3·9	80 1·5	76 2·6	66 11·2	81 9·4
	Carcass	158 10·5	156 12·5	156 2·5	150 11·7	145 2·5	139 11·2	151 3·2
	Loss by evaporation, error in weighing, &c.	1 2·7	5 11·9	4 9·6	2 2·8	2 10·9	13 9·6	3 3·4
	Live-weight after fasting	252 0	245 0	242 0	233 0	224 0	220 0	236 0

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* It should be mentioned, that in the case of these animals killed in December, the weight of wool shorn in the previous Spring, is not included either in the recorded "Final Weight," or in the sum of items at the time of slaughtering. Nor is it taken into account in calculating the *Percentage proportions* of the different parts, which are recorded in Appendix-Table XLVIII. It is, however, given above, parenthetically, as Memorandum.

APPENDIX.—TABLE XXIV. Showing the *Actual Weights* (lbs. and ozs.) of the individual Organs, and other separated Parts, of SHEEP.

CLASS IX.—7 Wether Sheep of *Leicester Breed*, about 1¼ year old, in *excessively* Fattened condition. Fattening food—Oilcake, Clover Hay, and Swedish Turnips, under cover, until "moderately" fattened; afterwards, Oilcake, with Grass or Green Clover, or with Meadow-hay and Roots, in the field. Killed at Rothamsted, December 20, 1853.

Designation of Parts.		7 Very Fat Leicester Sheep.							Means of the 7 Leicesters.
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6	No. 7.	
		lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz	lbs. oz.	lbs. oz.
	Original weight	94 0	88 0	112 0	91 0	100 0	94 0	112 0	98 11·4
	Final weight, unfasted	188 0	184 0	224 0	182 0	189 0	199 0	201 0	195 4·6
	Final weight, after fasting	176 0	171 0	212 0	170 0	176 0	186 0	187 0	182 9·1
Separate parts of the "offal."	Stomachs	3 10	4 0	5 0	4 0	4 6	4 2	4 1	4 2·7
	Contents of stomachs	4 6	6 6	10 4	6 4	7 14	6 14	5 15	6 13·6
	Caul-fat	6 10·5	5 8	8 6	5 5	12 0	7 4	10 6	7 14·8
	Small intestines and contents	2 8	2 8	3 3	1 8	2 14	2 3	2 6	2 7·1
	Large intestines and contents	3 2	2 14	3 12	3 0	3 8	3 0	2 11	3 2·2
	Intestinal fat	3 0·5	2 12·4	3 10	1 5·5	3 0	3 0	2 9	2 12·2
	Heart and aorta	0 10	0 11·5	0 13	0 9	0 12·5	0 10	0 9·3	0 10·7
	Heart-fat	0 10·5	0 7·3	0 1·3	0 8	0 9	0 11·5	0 14	0 11·3
	Lungs and windpipe	1 6·3	1 5·5	1 15	1 7·5	1 9·5	1 7·5	1 8·5	1 8·6
	Blood	6 7	7 1·5	7 6·5	6 11	6 13	6 10	6 8·5	6 12·8
	Liver	2 7·5	2 11·5	3 0·5	2 4	2 13	2 7	2 5·5	2 9·3
	Gall-bladder and contents	0 1·8	0 1·7	0 2	0 1·2	0 1·8	0 1·3	0 2	0 1·7
	Pancreas ("sweetbread")	0 2·5	0 2·5	0 2	0 2·2	0 1·7	0 1·7	0 2·3	0 2·1
	Milt or spleen	0 5·5	0 3·5	0 4	0 3·5	0 3·5	0 4	0 4·2	0 4
	Bladder	0 1	0 1	0 1·5	0 1	0 1	0 0·8	0 0·9	0 1
	Head	4 8	4 3	5 0	4 4	4 4·5	4 8·5	4 4·5	4 6·9
	Skin and wool (with feet, &c.)	20 4	16 4	25 0	18 8	19 12	19 4	19 12	19 13·2
	Wool shorn in Spring*	(9 13)	(6 14)	(9 9)	(7 12)	(8 4)	(7 14)	(9 4)	(8 7·7)
	Diaphragm ("skirts")	....	....	....	....	....	0 2·3	....	0 2·3
	Miscellaneous trimmings	0 1·5	....	....	....	....	....	....	0 1·5
	Total "offal" parts	60 6·6	57 5·4	79 2·5	56 1·9	70 11·5	62 11·6	64 7·7	64 10
	Carcass	115 8·5	112 2·5	133 5·5	112 15	106 3	124 1·7	120 9	117 13·3
	Loss by evaporation, error in weighing, &c.	0 0·9	1 8·1	— 8	0 15·1	— 14·5	— 13·3	1 15·3	0 1·8
	Live-weight after fasting	176 0	171 0	212 0	170 0	176 0	186 0	187 0	182 9·1

* It should be mentioned, that in the case of these animals killed in December, the weight of wool shorn in the previous Spring, is not included either in the recorded "Final Weight," or in the sum of items at the time of slaughtering. Nor, is it taken into account in calculating the *Percentage proportions* of the different parts, which are recorded in Appendix-Table XLIX. It is, however, given above, parenthetically, as Memorandum.

APPENDIX.—TABLE XXV. Showing the *Actual Weights* (lbs. and ozs.) of the individual Organs, and other separated Parts, of SHEEP.

CLASS X.—8 Wether Sheep of *Cross-bred* (Leicester and South Down), about 1¾ year old, in *excessively* Fattened condition. Fattening food—Oilcake, Clover Hay, and Swedish Turnips, under cover, until "moderately" fattened; afterwards, Oilcake, with Grass or Green Clover, or with Meadow-hay and Roots, in the field. Killed at Rothamsted, December 20, 1858

	Designation of Parts.	8 Very Fat Cross-bred Wether Sheep (Leicester and South Down)								Means of the 8 Cross bred Wethers
		No 1	No 2	No. 3.	No 4	No 5	No 6	No 7.	No 8	
		lbs oz	lbs oz	lbs oz	lbs oz	lbs oz	lbs oz	lbs oz	lbs oz	lbs oz
	Original weight	87 0	93 0	96 0	98 0	91 0	89 0	86 0	98 0	92 4
	Final weight, unfasted	172 0	181 0	190 0	203 0	179 0	186 0	183 0	196 0	186 4
	Final weight, after fasting	162 0	170 0	182 0	191 0	168 0	176 0	171 0	184 0	175 8
Separate parts of the "offal."	Stomachs	3 9	3 12	4 1	3 15	4 6	3 12	3 14	4 1	3 14·7
	Contents of stomachs	6 11·5	7 6	9 8	7 5	7 1	6 12	3 14	6 1	6 13·3
	Caul-fat	7 0	9 4	8 7	11 12·5	7 7·5	7 9·5	9 1·5	6 13·5	8 6·9
	Small intestines and contents	1 14	1 13	2 0	1 12·5	1 13	1 14	2 1	2 0	1 14·4
	Large intestines and contents	2 13	2 4	3 10	2 15	3 0	2 10	2 13	2 11	2 13·5
	Intestinal fat	2 12	3 9·5	3 11·5	4 11	2 4	1 15	3 12	2 9	3 2·5
	Heart and aorta	0 8	0 9·8	0 9	0 9·5	0 9	0 9·5	0 9	0 10·5	0 9·3
	Heart-fat	0 15	0 9·5	0 13·5	0 14	0 10·5	0 14	0 8	0 15·5	0 12·5
	Lungs and windpipe	1 4·5	1 7·5	1 8	1 7	1 7·5	1 6·7	1 8	1 6·5	1 7
	Blood	6 4	6 7	5 13·5	6 13	6 15	6 7	6 8	6 6·5	6 7·2
	Liver	2 1	2 2·5	2 3	2 7	2 3·5	2 5·5	2 4	2 6	2 4·1
	Gall-bladder and contents	0 1·7	0 1·5	0 1·3	0 1·5	0 2·5	0 2	0 1·8	0 1·2	0 1·7
	Pancreas ("sweetbread")	0 2·5	0 2·4	0 2·3	0 2·7	0 3·5	0 2·7	0 2·2	0 1·5	0 2·5
	Milt or spleen	0 3·3	0 3·5	0 4	0 4	0 4	0 4	0 4·7	0 4·5	0 4
	Bladder	0 1	0 1	0 1	0 1·1	0 1	0 1·1	0 0·7	0 1·7	0 1·1
	Head	3 13·5	4 2	4 9	4 2·5	4 8·5	4 6·5	4 6	4 4	4 4·5
	Skin and wool (with feet, &c.)	14 4	17 8	18 0	15 0	18 10	18 4	18 8	17 8	17 3·3
	Wool shorn in Spring*	(6 4)	(5 10)	(5 9)	(4 4)	(5 8)	(5 7)	(6 14)	(5 12)	(5 10·5)
	Miscellaneous trimmings	0 4	0 4·2	....	0 2	0 1	....	....	0 2	0 2·6
	Total "offal" parts	54 10	61 11·4	65 6·6	64 7·3	61 11·5	59 7·5	60 3·9	58 7·4	60 13·1
	Carcass	107 13	107 10	113 12	126 3·5	105 5	115 12·5	109 14	125 2·5	113 13·1
	Loss by evaporation, error in weighing, &c.	—0 7·0	0 10·6	3 13·4	0 5·2	0 15·5	0 12	0 14·1	0 6·1	0 13·8
	Live-weight after fasting	162 0	170 0	182 0	191 0	168 0	176 0	171 0	184 0	175 8·0

* It should be mentioned, that in the case of these animals killed in December, the weight of wool shorn in the previous Spring, is not included either in the recorded "Final Weight," or in the sum of items at the time of slaughtering. Nor, is it taken into account in calculating the *Percentage proportions* of the different parts, which are recorded in Appendix-Table L. It is, however, given above, parenthetically, as Memorandum

APPENDIX.—TABLE XXVI. Showing the *Actual Weights* (lbs. and ozs.) of the individual Organs, and other separated Parts, of SHEEP.

CLASS XI.—8 Ewe Sheep of *Cross-breed* (Leicester and South Down), about 1¾ year old, in *excessively* Fattened condition. Fattening food—Oilcake, Clover Hay, and Turnips, under cover, until "moderately" fattened; afterwards, Oilcake, with Grass or Green Clover, or with Meadow-hay and Roots, in the field. Killed at Rothamsted, December 20, 1853.

	Designation of Parts.	8 Very Fat Cross-bred Ewe Sheep (Leicester and South Down).								Means of the 8 Cross-bred Ewes.
		No. 1.	No. 2.	No. 3.	No. 4.	No 5	No. 6	No 7	No. 8.	
		lbs. oz.	lbs. oz	lbs. oz.	lbs oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
	Original weight	94 0	89 0	86 0	88 0	98 0	92 0	78 0	98 0	90 6
	Final weight, unfasted	186 0	178 0	175 0	171 0	209 0	188 0	164 0	186 0	182 2
	Final weight, after fasting	177 0	167 0	168 0	163 0	197 0	176 0	154 0	176 0	172 4
Separate parts of the "offal."	Stomachs	4 2	4 6	3 6	3 5	5 0	3 12	3 5	4 6	3 15·2
	Contents of stomachs	5 12	5 6	5 6	5 4	8 14	4 3	5 8	6 2	5 12·9
	Caul-fat	9 5	8 14	8 12·5	6 9·5	9 8	7 7	7 12	9 4·5	8 7·1
	Small intestines and contents	2 5	1 12	2 1	2 3	2 10	3 0	2 3	1 10	2 3·5
	Large intestines and contents	3 2	2 10	2 14	2 13	3 12	2 13·5	2 5	3 10	2 15·9
	Intestinal fat	4 0·5	5 0	3 10·5	2 8·5	4 0	4 0·5	2 14	2 10	3 9·5
	Heart and aorta	0 9	0 11	0 9	0 9·5	0 10·8	0 10·5	0 9·3	0 10·5	0 9·9
	Heart-fat	1 1·5	0 15	0 11	0 8	0 12·5	0 15	0 9·3	0 1·2	0 13·3
	Lungs and windpipe	1 5·5	1 8·5	1 3·5	1 10	1 7	1 6·5	1 3·5	1 9	1 6·7
	Blood	6 1	6 1	6 4·5	6 2·5	7 6·5	8 9·4	5 11	7 3	6 10·8
	Liver	2 4	2 7·5	2 2	2 5·5	2 12	2 9·8	2 3·5	2 5	2 6·2
	Gall-bladder and contents	0 1·5	0 2·3	0 1·1	0 2·6	0 3·7	0 1	0 1·5	0 2·5	0 2
	Pancreas ("sweetbread")	0 2	0 2	0 1·5	0 2·2	0 3	0 2·5	0 1·8	0 2·3	0 2·2
	Milt or spleen	0 4	0 4	0 3	0 4	0 4	0 4	0 3·3	0 3·5	0 3·7
	Bladder	0 1	0 1	0 0·9	0 1	0 0·8	0 1·5	0 0·7	0 1	0 1
	Head	4 3	4 11·5	4 1·5	4 6	5 0·5	4 8·5	3 14	4 7	4 6·5
	Skin and wool (with feet, &c.)	17 14	18 4	17 4	19 10	24 0	19 12	16 8	19 0	19 0·5
	Wool shorn in Spring*	(6 4)	(7 0)	(6 1)	(6 14)	(9 12)	(7 4)	(7 13)	(6 14)	(7 3·8)
	Womb	0 2·5	0 2·5	0 2·5	0 2·5	0 2·5	0 2	0 1·7	0 3	0 2·4
	Miscellaneous trimmings	....	....	....	....	....	....	0 3·5	....	0 3·5
	Total "offal" parts	62 11·5	63 6·3	58 14·5	58 10·8	76 11·3	64 6·7	55 6·1	64 11·3	63 4·8
	Carcass	114 12	104 14	109 10·5	106 2	119 5	116 5·7	100 8·5	113 4·5	110 9·8
	Loss by evaporation, error in weighing, &c.	—0 7·5	—1 4·3	—0 9	—1 12·8	0 15·7	—4 12·4	—1 14·6	—1 15·8	—1 10·6
	Live-weight after fasting	177 0	167 0	168 0	163 0	197 0	176 0	154 0	176 0	172 4

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* It should be mentioned, that in the case of these animals killed in December, the weight of wool shorn in the previous Spring, is not included either in the recorded "Final Weight," or in the sum of items at the time of slaughtering. Nor, is it taken into account in calculating the *Percentage proportions* of the different parts, which are recorded in Appendix-Table LI. It is, however, given above, parenthetically, as Memorandum.

APPENDIX.—TABLE XXVII. Showing the *Actual Weights* (lbs. and ozs.) of the individual Organs, and other separated Parts, of SHEEP

CLASS XII.—8 Wether Sheep of *Hampshire Down Breed*, about 1¼ year old, in *excessively* Fattened condition. Fattening food—Oilcake, Clover Hay, and Swedish Turnips, under cover, until "moderately" fattened; afterwards, Oilcake, with Grass or Green Clover, or with Meadow-hay and Roots, in the field. Killed at Rothamsted, December 17, 1851

	Designation of Parts.	8 Very Fat Hampshire Down Sheep								Means of the 8 Hampshire Downs
		No 1	No 2	No 3	No 4	No 5	No 6	No 7	No 8	
		lbs oz	lbs oz	lbs oz	lbs oz	lbs oz	lbs oz	lbs oz	lbs oz	lbs oz
	Original weight	114 0	112 0	112 0	117 0	109 0	120 0	113 0	117 0	114 4
	Final weight, unfasted	228 0	226 0	234 0	235 0	238 0	262 0	236 0	238 0	237 2
	Final weight, after fasting	218 0	214 0	224 0	226 0	227 0	250 0	225 0	227 0	226 6
Separate parts of the "offal."	Stomachs	4 8	4 7	5 8	4 10	4 14	5 4·5	5 0	4 0	4 12·5
	Contents of stomachs	9 0	7 3	7 2	8 15	9 14	8 5·5	8 4	6 4	8 1·9
	Caul-fat	18 12	10 13	11 14	13 2	12 10·5	12 7	17 8	11 6	13 9·1
	Small intestines and contents	2 8·5	1 13	2 4	2 0	2 12	2 9	1 6	2 8	2 3·6
	Large intestines and contents	2 12	1 14	3 6	3 11	4 3	3 13·5	3 8	3 3·5	3 4·9
	Intestinal fat	5 12	6 4	7 10	7 8	6 10	7 13	6 9	7 2	6 14·5
	Heart and aorta	0 12	0 12·5	0 13	1 0	0 12	0 12·5	0 13	0 13·2	0 13
	Heart-fat	0 9	0 3·2	0 3·5	0 7·3	0 14·5	0 9	0 12	0 13	0 8·9
	Lungs and windpipe	1 9·5	1 11·5	2 2	1 10·5	1 13·5	1 14·5	1 9·5	1 15	1 12·8
	Blood	7 1	7 7	7 10	8 4	8 14·5	9 9·5	8 11	8 7	8 3·8
	Liver	2 6	2 11	2 10	2 13	2 5·5	3 3·5	2 7	3 8	2 12
	Gall-bladder and contents	0 2	0 1·7	0 1·9	0 2·3	0 0·8	0 3·2	0 3	0 1·9	0 2·1
	Pancreas ("sweetbread")	0 5·7	0 4·7	0 3·5	0 4·8	0 4·8	0 3·7	0 5·5	0 4·5	0 4·7
	Milt or spleen	0 4·5	0 4·3	0 4·5	0 4·5	0 5·6	0 5·6	0 4·5	0 5	0 4·8
	Bladder	0 0·8	0 0·9	0 0·5	0 1	0 0·7	0 0·8	0 1	0 0·7	0 0·8
	Head	5 7	5 10	5 10	5 8	5 10	5 13	5 15	5 13·5	5 10·8
	Skin and wool (with feet, &c.)	20 12	23 14	21 10	23 0	22 14	24 4	23 12	21 0	22 10·3
	Wool shorn in Spring*	(6 0)	(7 0)	(6 4)	(7 8)	(6 8)	(5 8)	(5 12)	(6 8)	(6 6)
	Miscellaneous trimmings	....	....	... .	....	....	. .	... ...	0 3	0 3
	Total "offal" parts	82 10	75 6·8	79 0·9	83 5·4	84 15·4	87 3·6	87 0·5	77 12·3	82 5·5
	Carcass	134 7·1	135 14·5	143 10	142 10	142 6·5	161 6·5	137 5·5	148 5	143 4·1
	Loss by evaporation, error in weighing, &c.	0 14·9	2 10·7	1 5·1	0 0·6	—0 5·9	1 5·7	0 10	0 14·7	0 12·4
	Live-weight after fasting	218 0	214 0	224 0	226 0	227 0	250 0	225 0	227 0	226 6

* It should be mentioned, that in the case of these animals killed in December, the weight of wool shorn in the previous Spring is not included either in the recorded "Final Weight," or in the sum of items at the time of slaughtering. Nor, is it taken into account in calculating the *Percentage proportions* of the different parts, which are recorded in Appendix-Table LII. It is, however, given above, parenthetically as Memorandum

APPENDIX.—TABLE XXVIII. Showing the *Actual Weights* (lbs. and ozs.) of the individual Organs, and other separated Parts, of SHEEP.

CLASS XIII.—8 Wether Sheep of *Sussex Down Breed*, about 1¾ year old, in *excessively* Fattened condition. Fattening food—Oilcake, Clover Hay, and Swedish Turnips, under cover, until "moderately" fattened; afterwards, Oilcake, with Grass or Green Clover, or with Meadow-hay and Roots, in the field. Killed at Rothamsted, December 17, 1851.

Designation of Parts.	8 Very Fat Sussex Down Sheep.																Means of the 8 Sussex Downs.	
	No. 1.		No. 2.		No. 3.		No. 4.		No. 5.		No. 6.		No. 7.		No. 8.			
	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs	oz.	lbs	oz.	lbs.	oz
Original weight	82	0	82	0	79	0	84	0	87	0	80	0	81	0	91	0	83	4
Final weight, unfasted	160	0	179	0	167	0	178	0	189	0	184	0	182	0	189	0	178	8
Final weight, after fasting	152	0	170	0	158	0	168	0	178	0	175	0	172	0	181	0	169	4
Separate parts of the "offal."																		
Stomachs	2	9	3	5	3	3	3	14	4	0	3	6·5	3	7	3	12	3	7
Contents of stomachs	5	7	6	11	3	13	6	6	6	4	5	3·5	3	9	6	14·5	5	8·5
Caul-fat	9	14	11	5	10	8	10	10	10	4	9	5	11	12	7	14	10	3
Small intestines and contents	1	12	2	1	1	15·5	2	1	2	8	2	0	1	15	1	13	2	0·3
Large intestines and contents	2	0·5	2	9	1	15	1	14	2	14	2	7	2	4	3	6	2	6·8
Intestinal fat	4	2	4	6	5	0	3	9	5	1	5	5·5	6	2·5	7	4·5	5	1·8
Heart and aorta	0	8·5	0	11	0	10·2	0	9·7	0	9·5	0	10·8	0	11·2	0	13	0	10·5
Heart-fat	0	4	0	6	0	4·2	0	4·5	0	6·8	0	5·9	0	11·2	0	8	0	6·3
Lungs and windpipe	1	5	1	9	1	6·4	1	9	1	9·5	1	9	1	11·5	1	6·5	1	8·2
Blood	5	12	6	0	5	5	6	5	6	7	6	12·5	6	5·9	6	12·5	6	3·5
Liver	2	1	2	8	2	2	2	6	2	10	2	8	2	10·5	2	8	2	6·7
Gall-bladder and contents	0	1·8	0	1·1	0	0·7	0	2	0	1·5	0	1·1	0	2	0	2·2	0	1·5
Pancreas ("sweetbread")	0	3·2	0	3·5	0	3·3	0	4	0	3·5	0	3·8	0	4·4	0	4·1	0	3·7
Milt or spleen	0	4	0	5	0	3·7	0	4	0	4·5	0	5·5	0	4·7	0	5·5	0	4·6
Bladder	0	0·7	0	0·5	0	0·5	0	0·5	0	0·7	0	0·6	0	0·6	0	0·6	0	0·6
Head	4	10	4	3	4	9	4	5	4	11	5	0·5	4	8	5	0	4	9·8
Skin and wool (with feet, &c.)	15	0	17	0	16	8	16	2	16	0	17	0	16	2	20	4	16	12
Wool shorn in Spring*	(5	12)	(6	8)	(5	12)	(5	8)	(6	0)	(5	0)	(4	8)	(6	4)	(5	10·5)
Total "offal" parts	55	14·7	63	4·1	57	11·5	60	9·7	63	15	62	5·2	62	9·5	69	0·4	61	14·8
Carcass	96	7·5	107	2·5	100	2	105	12	114	6	112	12·5	109	14·5	111	12	107	4·6
Loss by evaporation, error in weighing, &c.	—0	6·2	—0	6·6	0	2·5	1	10·3	— 0	5	— 0	1·7	— 0	8	0	3·6	0	0·6
Live-weight after fasting	152	0	170	0	158	0	168	0	178	0	175	0	172	0	181	0	169	4

* It should be mentioned, that in the case of these animals killed in December, the weight of wool shorn in the previous Spring, is not included either in the recorded "Final Weight," or in the sum of items at the time of slaughtering. Nor, is it taken into account in calculating the *Percentage proportions* of the different parts, which are recorded in Appendix-Table LIII. It is, however, given above, parenthetically, as Memorandum.

APPENDIX.—TABLE XXIX. Showing the *Actual Weights* (lbs. and ozs.)

CLASS XIV.—19 Sheep of *Hampshire Down Breed*, divided into 4 Lots, each with different Food*.
Killed April

Designation of Parts.	Lot 1.—Food: Oilcake—in fixed quantity; Swedish Turnips *ad libitum*										Lot 2.—Food: Oats—in fixed quantity; Swedish Turnips *ad libitum*									
	No. 1.		No. 2.		No. 3.		No. 4.		No. 5.		No. 6.		No. 7.		No. 8.		No. 9.		No. 10.	
	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.
Original weight	118	8	112	8	111	0	110	0	108	0	117	8	112	8	112	0	110	0	96	0
Final weight, unfasted	155	0	147	0	143	0	123	0	120	0	153	0	148	8	144	0	121	0	123	0
Final weight, after fasting	151	0	144	0	139	0	121	0	116	8	146	0	145	0	139	0	119	0	118	0
Separate parts of the "offal": Stomachs	3	7	3	14	3	10	3	6·5	3	1·5	4	2·5	4	2	3	12·5	3	6	3	7
Contents of stomachs	8	6	9	12	9	5	11	1	7	14	12	4	13	8	10	8	9	12	9	12
Caul, intestinal, and heart-fat	13	2	13	11·1	12	14	9	7	7	3·5	12	4	11	11	11	11	8	15	7	10
Small intestines and contents	1	14	2	1·4	1	9	2	1	2	12·5	1	10·5	1	8·5	2	1·5	1	7·5	2	8
Large intestines and contents	2	14·5	2	13	2	2·5	2	8·5	2	1	2	7·5	3	2	3	5·5	2	15	2	11
Intestinal fat (included with caul-fat).																				
Heart and aorta	1	1·4	1	4·5	1	1	0	13	0	13	1	4·5	1	5	1	1·5	1	1	0	14
Heart-fat (included with caul-fat).																				
Lungs and windpipe	1	14	1	12	1	8·5	1	6·7	1	11	1	15	2	0	1	11·5	1	9	1	2
Blood	6	7	5	0	5	15	4	4·5	4	9·5	5	11	5	0·5	5	11	4	8·5	4	11·5
Liver	2	5	2	8·5	2	6	1	3·2	1	12·7	2	0·5	1	15·5	2	11	1	11	1	14
Gall-bladder and contents	0	1·1	0	1·4	0	2	0	2·5	0	0·5	0	1	0	2	0	4	0	2	0	1·5
Head	4	15·5	4	11·1	4	7	4	4	4	6·5	4	8·5	4	15·5	4	12	4	10·5	4	8
Skin and wool (with feet, &c.)	17	7	17	0	14	12·5	12	13	15	10	14	14	17	4	15	4	13	13	17	0
Total "offal" parts	63	14·5	64	4	59	12·5	53	6·9	52	2·7	63	3	67	0	63	0·5	53	1	56	3
Carcass	89	0	81	8	77	0	68	8	64	8	86	4	80	0	78	8	68	8	64	0
Loss by evaporation, error in weighing, &c.	−1	14·5	−1	12	2	3·5	−	14·9	−	2·7	−3	7	−2	0	−2	8·5	−2	9	−2	3
Live-weight after fasting	151	0	144	0	139	0	121	0	116	8	146	0	145	0	139	0	119	0	118	0

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* For particulars of the *feeding* experiment, refer, under the head of "Experiments with Sheep - Series I,

of the individual Organs, and other separated Parts, of SHEEP.

Moderately Fattened. Bred, Fed, and killed at Rothamsted—Period of Feeding Experiment 97 days.
11, 1848.

Lot. 3—Food—Clover Chaff—in fixed quantity. Swedish Turnips—*ad libitum*.					Lot 4—Food—Oat-straw Chaff, Swedish Turnips } *ad libitum*.				Means of—				
No 11.	No. 12.	No 13.	No. 14.	No. 15.	No. 16.	No. 17.	No 18.	No. 19.	Lot 1.	Lot 2.	Lot 3.	Lot 4	The 19 Sheep
lbs. oz.	lbs oz.	lbs. oz.	lbs. oz.	lbs oz	lbs. oz.	lbs. oz	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz
117 0	114 0	110 8	110 0	107 0	115 0	114 8	112 0	110 4	111 9·6	109 9·6	111 11·2	112 15	111 6·1
142 0	144 0	138 0	141 0	149 0	136 0	138 0	124 0	120 0	137 9·6	137 14·4	142 12·8	131 12	137 13
142 0	143 0	135 8	130 0	145 0	129 0	134 0	121 0	127 0	134 4·8	133 6·4	140 14·4	127 12	134 6·7
4 8	4 3	4 2	4 6·5	4 6	3 12	4 2	3 8	3 10·5	3 8·4	3 12·4	4 5·1	3 12·1	3 13·6
19 0	13 6	14 2·5	14 11	15 6·5	10 13	11 0·5	8 5·5	12 15·5	9 4·4	11 2·4	15 5·2	10 14·9	11 11·3
8 11	10 4·5	10 9·5	11 1	10 10	11 9	11 2·5	11 4·5	10 9	11 4·3	10 4·9	10 4	11 2·2	10 11·5
4 1·5	2 2·5	1 14·5	2 2	1 15·5	2 7	2 9·5	1 8·5	2 10·5	2 1·2	1 13·6	2 7·2	2 4·9	2 2·6
4 14·5	3 13·5	3 10·5	3 13	2 10	2 9·5	3 4	2 4	2 12·5	2 7·9	2 11·6	3 12·3	2 11·5	2 15·8
1 2	1 3	1 8	1 2	1 4	1 1·5	1 3	0 14	0 13	1 0·2	1 2	1 3·8	0 15·9	1 1·6
1 12	1 12	1 15	1 15	1 14	1 9	1 14	1 10	1 10	1 10·5	1 10·7	1 13·6	1 10·8	1 11·4
5 11	5 6	4 7	5 5	5 8·5	3 13	5 1·5	4 8·5	4 15·5	5 4	5 3·3	5 4·2	4 9·6	5 1·7
2 8	2 2·5	2 7·5	2 8	2 7	1 15	2 5	1 7·5	2 1·5	1 15·6	2 0·8	2 6·6	1 15·2	2 1·7
0 0·3	0 2·5	0 0·9	0 0·7	0 1·9	0 1·2	0 2	0 0·5	0 0·6	0 1·5	0 2·1	0 1·2	0 1·1	0 1·5
4 3·5	4 8	4 11	4 10·5	5 0	4 6	4 6·5	4 8	4 4	4 8·8	4 10·9	4 9·8	4 6·1	4 9·1
16 0	15 2	15 0	14 12	16 0	14 0	13 4	15 0	14 10	15 8·5	15 10·2	15 7·8	14 3·5	15 4·3
72 7·8	64 1·5	64 8·4	66 6·7	67 12·4	58 0·2	60 15·5	54 15	61 0·6	58 11·8	60 7·9	67 0·9	58 11·6	61 8·1
71 0	81 8	72 8	73 12	76 8	72 0	74 8	67 8	66 8	76 1·6	75 7·2	75 0·8	70 2	74 6·3
−1 7·8	−2 9·5	−1 8·4	−1 2·7	0 11·6	−1 0·2	−1 7·5	−1 7	−0 8·6	−0 8·1	−2 8·7	−1 3·3	−1 1·8	−1 5·7
142 0	143 0	135 8	130 0	145 0	129 0	134 0	121 0	127 0	134 4·8	133 6·4	140 14·4	127 12	134 6·7

to Article—"Sheep Feeding and Manure," Journal of the Royal Agricultural Society of England, vol. x, part 1.

Appendix.—Table XXX. Showing the *Actual Weights* (lbs. and oz.)

Class XV.—20 Sheep of *Hampshire Down Breed*, divided into 4 Lots, each with different

Experiment 184 days.

	Designation of Parts.	Lot 1.—Food:— Oilcake—in fixed quantity Clover Chaff—*ad libitum.*										Lot 2.—Food:— Linseed—in fixed quantity Clover Chaff—*ad libitum.*									
		No. 1.		No. 2		No. 3.		No. 4.		No. 5.		No. 6.		No. 7.		No. 8		No. 9.		No. 10.	
		lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.
	Original weight	120	0	124	0	125	0	121	0	117	0	125	0	123	0	117	0	123	0	119	0
	Final weight, unfasted	147	0	157	0	157	0	151	0	147	8	141	0	146	0	161	0	151	0	151	0
	Final weight, after fasting	142	2·6	148	8·8	149	15·9	141	13·7	143	10·2	132	2·7	136	13·3	151	8·9	142	0·6	138	15
Separate parts of the "offal."	Stomachs	3	11·5	4	8	4	5	3	10	3	15	3	6	3	6	3	15	3	5	3	14
	Contents of stomachs	13	2·5	18	3	14	10	12	2	14	3	7	14	9	0	11	13	10	12	10	2
	Caul-fat	4	2·5	7	1	6	8	7	11	7	0	9	2·5	6	13	7	14	6	13·5	8	8
	Small intestines and contents	2	9·5	2	12	2	11·5	2	5·5	2	3	2	14	2	0	2	11	2	1	2	10
	Large intestines and contents	4	14	4	1·5	5	0	3	11	4	0	2	13	2	2	4	15	3	6	3	3
	Intestinal fat	2	7·5	3	10	2	10·5	2	5	2	4	2	15	2	7	3	10	3	2	2	2
	Heart and aorta	0	9	0	7·5	0	8	0	9	0	9	0	9	0	7	0	9	0	9·5	0	9
	Heart-fat	0	6·3	0	8·5	0	6·5	0	10·5	0	7	0	8	0	8·5	0	9	0	8·5	0	10
	Lungs and windpipe	1	4	1	3	1	3·5	1	3·5	2	0	1	3	1	3	1	4	1	2	1	6
	Blood	6	4·5	5	15	5	10	5	9	5	12	5	4	5	7	6	2	5	6	5	0
	Liver	1	13	2	1	1	15·5	1	14	2	4	1	11	1	8	2	2	1	15	1	13
	Gall-bladder and contents	0	1	0	1·5	0	0·7	0	1	0	1·1	0	1·7	0	2·3	0	3·3	0	2·1	0	2·1
	Pancreas ("sweetbread")	0	2·3	0	2·8	0	3·2	0	2·7	0	1·2	0	2·7	0	2·1	0	3·6	0	2·8	0	3·4
	Milt or spleen	....		0	3·2	0	2·5	0	3	0	2·9	0	2·8	0	2·9	0	4	0	3·2	0	2·5
	Head	4	9	4	8	4	12	4	10·5	4	8	4	6	4	6·5	5	0	4	7	4	12
	Skin and wool (with feet, &c.)	13	2	15	0	17	5	13	10	12	14	10	10	13	0	13	6	13	5	13	2
	Total "offal" parts	*59*	*2·6*	*65*	*1*	*67*	*15·9*	*60*	*5·7*	*62*	*10·2*	*53*	*10·7*	*52*	*11·3*	*64*	*8·9*	*57*	*4·6*	*58*	*3*
	Carcass	83	0	83	8	82	0	81	8	81	0	78	8	84	2	87	0	84	12	80	12
	Loss by evaporation, error in weighing, &c.	0	0	−0	0·2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Live-weight after fasting	142	2·6	148	8·8	149	15·9	141	13·7	143	10·2	132	2·7	136	13·3	151	8·9	142	0·6	138	15

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* For particulars of the *feeding* experiment, refer, under the head of "Experiments with Sheep—Series 2,"

of the individual Organs, and other separated Parts, of SHEEP.

Food*. *Moderately* Fattened. Bred, fed, and killed at Rothamsted.—Period of Feeding
Killed October 17, 1848.

Lot 3.—Food:— Barley—in fixed quantity. Clover Chaff—*ad libitum.*					Lot 4.—Food:— Malt—in fixed quantity. Clover Chaff—*ad libitum.*					Means of—				
No. 11.	No. 12.	No 13.	No. 14	No. 15.	No. 16.	No. 17.	No. 18.	No. 19.	No. 20.	Lot 1.	Lot 2.	Lot 3.	Lot 4.	The 20 Sheep.
lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
122 0	117 0	120 0	118 0	125 0	117 0	122 0	120 0	119 0	124 0	121 6·4	121 6·4	120 6·4	120 6·4	120 14·4
150 0	134 0	149 0	154 0	154 0	140 0	144 0	141 0	143 0	155 0	151 14·4	150 0	148 3·2	144 9·6	148 10·8
142 7·1	129 7·7	140 15·7	140 1·1	142 9·2	133 6	136 10	130 2	133 7·7	144 9·2	145 0·6	140 4·9	139 1·7	135 10·2	140 0·3
4 3	3 13	4 5	4 3·5	4 14	3 2	3 8	3 3	3 7	4 3	4 0·8	3 9·2	4 4·5	3 7·8	3 13·4
13 5	9 9	15 6	14 0·5	14 14	11 14	11 4	7 5	10 3·5	12 14	13 7·3	9 14·6	13 6·9	10 11·3	11 14
6 1	4 11	6 15	5 5	6 2	4 7	4 9	5 2·5	5 1	8 1·5	6 7·7	7 13·4	5 13·2	5 7·4	6 6·4
2 2	1 8	1 13	1 14	1 13	2 2	2 1	1 12	1 15	1 15	2 8·3	2 7·2	1 13·2	1 15·4	2 3
3 11	3 8	4 5	2 12	3 14·5	3 2	3 6	2 14	3 5·5	3 14	4 5·3	3 4·6	3 10·1	3 5·1	3 10·3
2 5	2 3	2 5·5	2 6	1 12	2 11	2 11	2 11	2 7	3 0	2 10·6	2 13·6	2 3·1	2 11·2	2 9·6
0 7	0 9	0 8·5	0 8·5	0 9·5	0 7·5	0 9	0 8	0 8	0 9	0 8·5	0 8·7	0 8·5	0 8·3	0 8·5
0 4·3	0 9	0 7	0 9·5	0 7	0 7·5	0 8·5	0 9·5	0 9	0 9	0 7·7	0 8·8	0 7·3	0 8·7	0 8·1
1 9·5	0 13	1 4	1 1·5	1 4	1 2	1 5	1 1·5	1 4	1 5	1 7·1	1 3·6	1 3·2	1 3·5	1 4·4
6 0	6 2	5 9	5 4	5 15	5 6	5 10	5 8	5 15	6 3	5 13·3	5 7	5 12·4	5 11·6	5 11·1
1 13	1 11	1 10	1 12·5	1 14	1 10	1 12	1 12	1 10	2 4	1 15·8	1 13	1 12·1	1 12·8	1 13·4
0 0·9	0 1·7	0 1·4	0 2·8	0 1·7	0 1·2	0 1	0 1·5	0 1·2	0 2	0 1	0 2·3	0 1·7	0 1·4	0 1·6
0 2·7	0 2·4	0 2·9	0 2·8	0 2·2	0 1·7	0 2·5	0 2·7	0 2·7	0 3·3	0 2·4	0 2·9	0 2·6	0 2·6	0 2·6
0 3·7	0 3·1	0 3·4	0 2·5	0 3·3	0 3·1	0 3	0 3·3	0 2·8	0 3·4	0 2·9	0 3·1	0 3·2	0 3·1	0 3·1
4 9	4 1·5	4 10	4 11	5 4	4 3	4 13	4 8	4 7	4 12	4 8·5	4 9·5	4 10·3	4 8·6	4 9·2
13 2	11 15	13 6	14 13	13 7	11 10	15 3	15 4	14 9	11 11	14 6·1	12 11	13 5·4	13 10·6	13 8·8
59 15·1	51 7·7	62 15·7	59 13·1	62 0·2	52 10	57 10	52 10	55 11·7	61 13·2	63 0·8	57 4·5	59 5·7	56 1·4	58 15
82 8	78 0	79 0	80 4	80 0	80 12	79 0	77 8	77 12	81 12	82 0	83 0·4	79 15·2	79 5·6	81 1·8
0 0	0 0	—1 0	0 0	0 0	0 0	0 0	0 0	0 0	1 0	—0 0·2	0 0	—0 3·2	0 3·2	0 0
142 7·1	129 7·7	140 15·7	140 1·1	142 9·2	133 6	136 10	130 2	133 7·7	144 9·2	145 0·6	140 4·9	139 1·7	135 10·2	140 0·3

to Article—"Sheep Feeding and Manure," Journal of the Royal Agricultural Society of England, vol. x. part 1.

APPENDIX.—TABLE XXXI. Showing the *Actual Weights* (lbs. and ozs.)

CLASS XVI.—25 Sheep of *Hampshire Down Breed*, divided into 6 Lots, each with different Food*. Killed

Designation of Parts.		Lot 1.—Food:— Barley (ground)—in fixed quantity. Mangolds—*ad libitum.*				Lot 2.—Food:— Malt (ground), and Malt-dust—in fixed quantity. Mangolds—*ad libitum.*				Lot 3.—Food:— Barley (ground and *steeped*)—in fixed quantity. Mangolds—*ad libitum.*			
		No. 1.	No 2	No 3.	No 4.	No 5.	No. 6.	No. 7.	No. 8.	No. 9.	No. 10.	No. 11.	No. 12.
		lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
Original weight		136 0	136 0	136 0	111 0	121 0	133 0	130 0	123 0	139 0	133 0	127 0	137 0
Final weight, unfasted (including shorn wool)		147 12	152 8·5	150 12·5	147 8	150 7·5	143 15·5	153 8·5	148 8·5	177 1	154 8·5	147 9·5	153 12
Final weight, after fasting (including shorn wool)		140 12	143 0·5	142 12·5	139 12	140 7·5	138 15·5	144 8·5	138 12·5	166 1	144 8·5	139 13·5	147 0
Separate parts of the "offal."	Stomachs	3 8	4 0	4 2	3 11	3 15·5	3 13·5	3 6	3 10	4 6·5	3 11	3 9	3 12
	Contents of stomachs	4 11·5	4 12	6 4	5 6	5 14·5	7 1·5	6 14·5	4 11·5	9 6·5	6 7	6 4	4 11·5
	Caul-fat	7 6	5 10·5	6 5·2	6 13·5	6 2	7 5·8	7 0	7 9·5	5 3	5 10	7 11·5	8 2
	Small intestines and contents	2 7	1 15·5	2 7	2 10	2 0·7	2 10	1 8	1 15	3 0	1 13·5	1 15	2 10·5
	Large intestines and contents	2 5	3 7·5	3 10	2 15·5	2 12·7	3 5·5	2 8·8	2 11	3 7	2 3·5	2 9	2 10·2
	Intestinal fat	3 3·5	4 12	4 8	3 13	2 11·7	2 13	4 2·8	3 5·5	3 12	2 8·3	2 9	3 9·5
	Heart and aorta	0 9	0 11	0 12·5	0 8·5	0 11·4	0 9·5	0 9·5	0 8·5	0 11·5	0 10·3	0 9·1	0 10·2
	Heart-fat	0 7·4	0 8	0 10·4	0 8·7	0 13·7	0 8·1	0 13·8	0 13·4	0 11·3	1 3·2	0 9·7	0 14·2
	Lungs and windpipe	1 8·5	1 7	1 15	1 6	1 6	1 8·8	1 7·7	1 10	1 12·7	1 0·7	1 5·6	1 12
	Blood	5 8	5 7	5 14	6 1	5 14	5 8	5 10	4 15	7 11	6 1	5 3	6 6
	Liver	2 0·8	2 2·4	1 15	2 2·2	2 2·2	1 14	2 0	1 11·5	2 11	1 15	1 13	2 1·7
	Gall-bladder and contents	0 1·2	0 0·8	0 0·6	0 0·7	0 1·4	0 1·2	0 0·5	0 1·8	0 1·8	0 1·7	0 0·8	0 1·1
	Pancreas ("sweetbread")	0 3·7	0 4·5	0 4·3	0 4	0 4	0 4·4	0 3·2	0 4·2	0 4·2	0 3·2	0 4·7	0 3·7
	Milt or spleen	0 3·6	0 3	0 4·5	0 4	0 4	0 3·7	0 3·2	0 2·4	0 6·4	0 4·2	0 4·4	0 4·2
	Head	4 12	4 9	4 11	4 5·5	4 12·5	4 12·3	4 3	4 2	5 3·5	4 8·6	4 6·7	4 10·5
	Skin (with feet, &c.)	9 14	10 14·5	11 9	10 3	8 15	10 10	10 2	9 8	12 1·5	10 11	9 10·5	10 15
	Wool previously shorn	6 12	7 8·5	5 12·5	7 8	5 7·5	6 15·5	10 8·5	9 8·5	9 1	6 8·5	7 9·5	7 12
Total "offal" parts		55 9·2	58 5·2	60 12	58 8·6	54 4·8	60 0·8	61 5·5	57 3·8	69 14·9	55 14·7	56 6·5	61 2·3
Carcass		84 0	85 0	81 0	80 0	85 0	76 0	80 0	80 0	95 0	86 0	80 0	85 0
Loss by evaporation, error in weighing, &c.		1 2·8	—0 4·7	1 0·5	1 3·4	1 2·7	2 14·7	3 3	1 8·7	1 2·1	2 9·8	3 7	0 13·7
Live-weight after fasting		140 12	143 0·5	142 12·5	139 12	140 7·5	138 15·5	144 8·5	138 12·5	166 1	144 8·5	139 13·5	147 0

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* For particulars of the *feeding* experiment, refer, under the head of "Experiments with Sheep—Series 4," to Article—"Sheep [illegible] *feeding* experiment concluded on May 29th. Between that date and the *date of* killing (June 13), the animals either gained [illegible] the conclusion of the feeding experiment. The facts here stated, will of course account for the differences that occur [illegible] in the Paper referred to.

of the individual Organs, and other separated Parts, of SHEEP.

Moderately Fattened. Bred, fed, and killed at Rothamsted. Period of Feeding Experiment 70 days. June 18, 1849.

Lot 4.—Food:— Malt (ground and *steeped*), and Malt-dust—in fixed quantity. Mangolds—*ad libitum.*				Lot 5—Food.— Malt (ground), and Malt-dust—in fixed quantity. Mangolds—*ad libitum.*					Lot 6.—Food:— Oilcake—in fixed quantity. Mangolds—*ad libitum.*				Means of—						
No. 13	No 14.	No. 15.	No. 16	No. 17	No. 18	No. 19.	No 20.	No. 21	No. 22	No. 23.	No. 24	No. 25.	Lot 1.	Lot 2	Lot 3.	Lot 4	Lot 5.	Lot 6.	The 25 Sheep
lbs. oz	lbs. oz	lbs. oz.	lbs. oz	lbs. oz.	lbs. oz	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs oz.	lbs. oz.	lbs. oz	lbs oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz
144 0	144 0	135 0	137 0	134 0	145 0	134 0	130 0	117 0	150 0	135 0	131 0	120 0	129 12	126 12	131 0	140 0	133 3·2	134 0	132 15·4
164 10	153 10	163 12	153 15	152 4	158 12	150 7	154 3·5	155 10	156 8·5	155 11	148 9	144 10	149 10·2	149 2	158 3·7	158 15·7	154 4·1	151 5·6	153 10
153 10	145 10	149 12	143 3	148 4	152 0	143 11	149 11·5	152 6	151 8·5	149 11	144 1	137 10	141 9·3	140 11	149 5·7	148 0·7	149 3·3	145 11·6	145 14·5
4 6	3 4	4 0	3 7	3 13	3 13	4 13	3 12·5	4 7·5	3 10·4	4 6	3 10	3 14	3 13·3	3 11·3	3 13·8	3 12·2	4 2·2	3 14·3	3 13·8
8 13	4 12	7 3	6 15	7 5	9 15	7 14	12 4	11 4·5	6 8·6	6 10	6 13·5	6 11·5	5 4·4	6 2·5	6 11·3	6 14·7	9 11·7	6 11	7 0·2
6 6·5	7 0	9 1	6 5·5	5 12	5 9·5	6 11·5	7 14	4 13·5	7 10·5	7 8·3	6 2	6 10·2	6 8·8	7 0·3	6 10·7	7 3·3	6 2·5	7 0·1	6 11·8
2 6·3	1 6	1 14·9	2 7	1 14·5	2 8·7	2 6·8	4 3	2 8·7	1 8	2 1	1 8·5	2 1·5	2 5·9	2 0·4	2 5·9	2 0·6	2 11·5	1 10·4	2 3·7
2 15·7	2 1·2	2 7·7	2 15	3 0·5	3 10	2 1	3 4·5	3 4	3 6·6	3 1	2 8	3 2	3 1·5	2 13·5	2 11·5	2 9·9	3 0·8	3 0·5	2 14·3
3 4·8	2 7	4 11·5	2 14	4 8·5	4 7	2 4	2 10·3	3 2	3 5·6	4 8	2 8	2 10·5	3 15·9	3 4·3	3 1·8	3 0·3	3 0·4	3 4·2	3 8·1
0 9·4	0 9	0 10	0 10	0 10·6	0 10·3	0 10·3	0 10	0 9·5	0 9·9	0 10·5	0 10·3	0 11	0 10·2	0 9·7	0 10·3	0 9·6	0 10·1	0 10·5	0 10
0 13·5	0 10	0 13·5	0 11	0 11·6	0 11·4	0 10·4	0 10·5	0 11	0 12·5	0 11·4	0 7·7	0 7	0 8·6	0 12·3	0 12·1	0 12	0 11	0 9·7	0 11
1 9·9	1 7·5	1 7·3	1 6·7	1 13	1 6·5	1 7·3	1 4·5	1 10	1 6	1 5·5	1 8·5	1 6·5	1 9·1	1 8·1	1 9·3	1 7·9	1 8·2	1 0·7	1 8·2
5 15·5	5 6	5 14	5 15·5	5 3·5	5 7	6 3	6 0	6 6	5 12	5 6	5 11	5 15	5 11·5	5 7·8	6 5·4	5 12·7	5 13·5	5 11·2	5 12·9
2 0·5	1 10	2 6	1 13·7	2 11·8	1 13	2 0·6	1 13·8	2 3·7	2 3	2 9	2 2	2 6·3	2 1·1	1 14·9	2 2·2	1 15·5	2 2·2	2 5·1	2 1·5
0 1·6	0 1·8	0 2·7	0 0·6	0 1·7	0 2·3	0 2·2	0 2·4	0 2·2	0 1·9	0 2·4	0 1·6	……	0 0·8	0 1·2	0 1·4	0 1·7	0 2·2	0 2	0 1·4
0 4·5	0 3·4	0 4	0 4	0 3·8	0 4	0 4	0 2·6	0 4·7	0 2·7	0 4·4	0 3·5	0 3·3	0 4·1	0 3·9	0 4·1	0 4	0 3·8	0 3·5	0 3·8
0 3·6	0 3·4	0 4·7	0 3	0 3·4	0 3	0 4·5	0 4·4	0 4·3	0 3·8	0 4	0 4	0 3·4	0 3·8	0 3·3	0 4·9	0 3·7	0 3·9	0 3·9	0 3·8
4 15	4 9·5	4 8·5	4 7	4 10	4 15	4 9	4 5·5	5 3·5	4 11	4 5·5	4 9	4 11	4 9·4	4 7·5	4 11·5	4 10	4 11·8	4 0·3	4 9·9
11 13·5	9 11	9 11	10 5	10 11	9 10·4	11 14	11 4	12 0	11 4·5	10 8	11 0	11 12·5	10 10·1	9 12·7	10 13·6	10 6·1	11 1·5	11 2·4	10 10·6
8 10	8 10	5 12	6 15	11 4	8 12	8 7	7 3·5	5 10	6 8·5	8 11	6 9	6 10	6 14·3	8 2	7 11·7	7 7·7	8 4·1	7 1·6	7 10
65 5·3	54 15·8	61 3·8	57 11	64 9·9	63 14·1	62 10·6	67 13·5	64 9·1	59 13·5	63 0	56 4·6	50 7·7	58 4·8	58 3·7	60 13·5	59 12·9	64 11·4	59 10·4	60 7
86 0	90 0	87 0	84 0	81 0	89 0	80 0	82 0	85 0	89 0	85 0	86 0	76 0	82 8	80 4	86 8	86 12	83 0·4	84 0	83 14·1
2 4·7	0 10·2	1 8·2	1 8	2 10·1	−0 14·1	1 0·4	−0 2	2 12·9	2 11	1 11	1 12·4	2 2·3	0 12·5	2 3·3	2 0·2	1 7·8	1 1·5	2 1·2	1 9·4
153 10	145 10	149 12	143 3	148 4	152 0	143 11	149 11·5	152 6	151 8·5	149 11	144 1	137 10	141 9·3	140 11	149 5·7	148 0·7	149 3·3	145 11·6	145 14·5

Feeding and Manure," Journal of the Royal Agricultural Society of England, vol. x. part 1. By reference it will be seen that the but little, or in some cases even lost. This was owing to the influence of losing their wool, which was shorn on May 20th, imme- between the final weights (unfasted) here given, in the Table, and those given for the respective animals at the conclusion of the

APPENDIX.—TABLE XXXII. Showing the *Actual Weights* (lbs. and ozs.)

CLASS XVII.—14 Sheep of *Hampshire Down Breed*, divided into 3 Lots—Fed respectively, for 63 days, and Mangolds, and Lot 3 with Oilcake and Mangolds. Total period of Feeding Experiment 96 days:

	Designation of Parts.	Lot 1 Food Clover Chaff in fixed quantity. Swedish Turnips, highly manured, or Mangolds *ad libitum.*				
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.
		lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
	Original weight	140 0	131 0	135 0	129 0	128 0
	Final weight, unfasted (including shorn wool)*	158 6	153 0	161 13	150 0	150 15
	Final weight, after fasting (including shorn wool)*	146 6	145 0	151 13	142 0	142 15
Separate parts of the "offal."	Stomachs	4 8	3 4·5	3 6	3 12	4 2
	Contents of stomachs	9 12	8 5·5	6 13	7 4	9 9
	Caul-fat	6 14	5 11	7 4	6 7	6 0
	Small intestines and contents	3 7	2 2	3 0	2 9	2 10·6
	Large intestines and contents	3 12·4	3 1·5	3 3	2 13	4 0
	Intestinal fat	2 13	2 13	3 12	2 13·5	3 2
	Heart and aorta	0 11	0 9	0 9	0 8·7	0 9·1
	Heart-fat	0 9·5	0 10·7	0 12	0 10·1	0 11
	Lungs and windpipe	1 13	1 4	1 6	1 7·3	1 6·6
	Blood	3 13·5	5 10·5	6 3·5	5 8	6 1
	Liver	2 2	1 15	2 7	1 15	2 1·3
	Gall-bladder and contents	0 0·8	......	......	0 0·9	0 0·7
	Pancreas ("sweetbread")	0 3·5	0 3·6	0 3·9	0 4	0 4
	Milt or spleen	0 4·5	0 3·8	0 4	0 3·4	0 3·6
	Head	4 15	4 14	4 9	4 10·7	4 10·5
	Skin (with feet, &c.)	10 14·5	10 7	10 13	9 11	10 1
	Wool previously shorn*	7 6	5 0	7 13	8 0	7 15
	Total "offal" parts	63 15·7	56 3·1	62 6·4	58 9·6	63 7·4
	Carcass	79 8	86 0	86 0	81 0	76 0
	Loss by evaporation, error in weighing, &c.	2 14·3	2 12·9	3 0·6	2 6·4	3 7·6
	Live-weight after fasting	146 6	145 0	151 13	142 0	142 15

* Wool shorn May 20.

of the individual Organs, and other separated Parts, of SHEEP.

with Clover-chaff, and Turnips differently manured; then, for 33 days, Lots 1 and 2 with Clover-chaff namely, from March 7 to June 11. Bred, fed, and killed at Rothamsted. Killed June 12, 1849.

Lot 2.—Food.— Clover Chaff—in fixed quantity. Swedish Turnips, manured with Superphosphate of Lime alone, or Mangolds—*ad libitum.*				Lot 3.—Food.— Clover Chaff—in fixed quantity. Swedish Turnips, unmanured—*ad libitum*, or Oilcake—in fixed quantity. Mangolds—*ad libitum.*					Means of—			
No. 6.	No. 7.	No. 8	No. 9.	No. 10.	No. 11.	No. 12.	No. 13.	No. 14.	Lot 1.	Lot 2.	Lot 3.	The 14 Sheep.
lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
139 0	130 0	129 0	127 0	140 0	130 0	134 0	124 0	130 0	133 3·2	131 4	131 9·6	132 1·1
158 12	154 12	157 2	146 13	167 0·5	152 10·5	157 1	143 9	143 12	154 13·2	154 5·8	152 13	153 15·6
149 4	144 0	145 14	138 5	159 8·5	146 6·5	145 5	135 13	136 0	145 10	144 5·7	144 9·8	144 14·4
4 4	3 10	4 11	3 13	3 12	3 5	3 13	3 7	3 11	3 12·9	4 1·5	3 9·6	3 13
9 7·5	8 2·5	8 1	7 12	8 7	8 3	7 3	5 8	10 3	8 5·5	8 5·7	7 14·4	8 3
5 3·4	9 9	5 12	4 8·5	8 7·5	6 5	6 13·5	10 5	4 4	6 7·2	6 4·2	7 3·8	6 10·8
2 8·6	2 0	2 13·5	2 14	2 4	3 1·5	2 14	2 4·5	3 3·8	2 12·1	2 11·3	2 12	2 11·8
4 0	3 3	2 12·3	3 1·5	2 0·4	3 2	3 3	2 7	2 12·5	3 6	3 4·2	2 11·4	3 1·6
2 12	2 10	2 13·3	4 0·8	4 13·5	2 14·2	2 10	5 0·5	3 3·5	3 1·1	3 1	3 11·5	3 4·8
0 12·2	0 7·9	0 9·2	0 10·4	0 9·4	0 10·5	0 10·2	0 9	0 10	0 9·3	0 9·9	0 9·8	0 9·6
0 8·7	0 8·7	0 6·3	0 8·3	0 12·4	0 10	0 11·5	0 5·5	0 6·5	0 10·6	0 8	0 9·2	0 9·4
1 13	1 5·7	1 7·8	1 9·7	1 4·7	1 12	1 8·3	1 8	1 8·5	1 7·4	1 9·1	1 8·3	1 8·2
5 8·5	5 11	6 13	5 13	5 8·5	6 8·5	5 10	5 7	5 11·6	5 7·3	5 15·4	5 12·3	5 11·4
2 3	1 14	2 0	1 15·4	2 5·2	2 5	2 4·5	2 2	2 2	2 1·7	2 0·1	2 3·7	2 1·9
0 2·1	0 1·8	0 2	……	0 1	0 2·3	0 2	……	0 1	0 0·8	0 1·8	0 1·6	0 1·4
0 3	0 4·4	0 3·6	0 3·4	0 4·1	0 3·5	0 3·3	0 2·3	0 5	0 3·8	0 3·6	0 3·6	0 3·7
0 4·3	0 4·5	0 4·3	0 3·7	0 4·5	0 3	0 3·4	0 3·8	0 3·8	0 3·9	0 4·2	0 3·7	0 3·9
5 0	4 8	4 5	4 12·5	4 2·5	4 9	4 14·5	4 9·5	5 0	4 11·8	4 10·4	4 10·3	4 10·8
10 13	10 1	10 13	10 7·5	11 14·5	14 2	11 11	8 14	9 0	10 6·1	10 8·6	11 3·7	10 11·7
8 12	6 12	7 2	7 13	6 0·5	5 10·5	10 1	6 9	6 12	7 3·6	7 9·7	7 0·2	7 4·5
64 3·3	61 10	61 1·3	60 2·7	62 15·7	63 11	64 8·2	59 0·1	59 11·1	60 15·1	61 12·7	62 1·1	61 0·5
82 0	81 0	85 0	75 0	95 0	84 0	79 0	75 0	74 0	81 11·2	80 12	81 6·4	81 5·2
3 0·7	1 6	−0 3·3	3 2·3	1 8·8	−1 4·5	1 12·8	1 0·9	2 4·9	2 15·7	1 13	1 2·3	1 15·7
149 4	144 0	145 14	138 5	159 8·5	140 6·5	145 5	135 13	136 0	145 10	144 5·7	144 9·8	144 14·4

APPENDIX.—TABLE XXXIII. Showing the *Actual Weights* (lbs. and ozs.)

CLASS XVIII.—21 Sheep of various Breeds and Modes of

Designation of Parts.	Hampshire Down Breed. *Killed December 18, 1851.* No. 1.	No 2.	No. 3.	No. 4.	Sussex Down Breed. No 5	No. 6.	No. 7.	No. 8	No. 9	No 10
	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
Original weights	......	......	......	......	76 0	85 0	91 0	81 0	79 0	81 0
Final weight, unfasted	224 0	224 0	243 0	218 0	158 0	166 0	184 0	163 0	150 0	184 0
Final weight, after fasting	211 0	214 0	231 0	209 0	150 0	158 0	176 0	155 0	142 0	174 0
Separate parts of the "offal." Stomachs	4 1	4 11	2 8	4 2	3 4	3 11	1 0	3 1·5	2 8	3 13
Contents of stomachs	2 13	10 15	9 12	5 8	5 0	4 13	4 14	4 0·5	4 9	5 8
Caul-fat	14 14	11 13	13 8	13 12	8 0	7 13	13 10	11 11	7 7·5	9 14
Small intestines and contents	1 13	2 3	2 11	2 4	2 2	2 9	2 0	1 15	1 6	2 1
Large intestines and contents	2 8	3 10	3 0	2 10	2 6·5	2 6	2 11	2 2·5	2 11	2 8·5
Intestinal fat	6 9	4 4	6 13	6 10	4 1	5 13	5 5	6 0	3 7	5 3
Heart and aorta	0 12	0 12·5	0 14·5	0 12	0 9	0 14	0 10	0 10·5	0 12	0 11
Heart-fat	0 7	0 8	0 13·5	1 4	0 6	0 9	0 10·5	0 8	0 4	0 11·5
Lungs and windpipe	2 1	2 0	1 15	1 7·5	1 3	1 10·5	1 5	1 5	1 10	1 3·5
Blood	7 3·5	8 0	8 8	7 2·5	5 9	6 8·5	5 15	6 0	5 11	7 11
Liver	3 1·5	2 9·5	3 3·5	2 2	2 2	2 5·5	2 3	2 2·5	1 8·5	2 6
Gall-bladder and contents	0 2·3	0 2·6	0 0·3	0 1·6	0 0·9	0 0·7	0 2·2	0 1	0 0·9	0 1·5
Pancreas ("sweetbread")	0 4·5	0 3·5	0 4·5	0 4	0 2·6	0 1·3	0 3·5	0 2	0 4·8	0 3·8
Milt or spleen	0 4·5	0 5·5	0 6·5	0 5·5	0 5	0 5	0 5	0 4	0 6·5	0 6
Bladder	0 1	0 0·7	0 1·3	0 0·9	0 0·5	0 0·5	0 1	0 0·7	0 0·5	0 0·8
Head	5 7·5	5 14	6 4	5 9	4 6	4 4·5	4 5	4 5	4 13	4 15
Skin and wool (with feet, &c.)	20 0	24 0	20 8	15 12	10 8	18 8	15 8	15 8	18 0	18 4
Wool shorn in Spring*	(6 0)	(5 0)	(5 4)	(7 8)	(6 0)	(6 0)	(7 8)	(9 0)	(8 0)	(7 8)
Miscellaneous trimmings	......	......	......	......	......	......	......	......	......	....
Total "offal" parts	72 6·8	82 0·3	81 6·1	69 11	56 1·5	62 7·5	63 12·2	59 13·2	55 10·7	65 8·6
Carcass	137 8	130 4·5	149 10·5	139 14·5	95 5	96 4	113 6·5	96 6·5	86 9	109 1·5
Loss by evaporation, error in weighing, &c.	1 1·2	1 11·2	—0 0·6	—0 9·5	—1 6·5	—0 11·5	—1 2·7	—1 3·7	—0 3·7	—0 10·1
Live-weight after fasting	211 0	214 0	231 0	209 0	150 0	158 0	176 0	155 0	142 0	174 0

* It should be mentioned, that in the case of these animals killed in December, the weight of wool shorn in the previous Spring, is not included

proportions of the different parts, which are recorded in Appendix-

of the individual Organs, and other separated Parts, of SHEEP.

Feeding, about 1¾ year old; of more than Average Fatness.

Killed December 18, 1851.				Leicesters and Cross-breds Killed December 21, 1853.							Means of—			
				Cross-bred Ewe	Cross-bred Wether	Leicester Wethers.					The 4 Hants Downs	The 10 Sussex Downs.	The 7 Leicesters and Cross-breds	The 21 Sheep
No. 11.	No. 12.	No. 13.	No. 14.	No. 15.	No. 16	No. 17.	No. 18.	No. 19	No. 20.	No. 21				
lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz	lbs. oz.	lbs. oz.	lbs oz.	lbs. oz.	lbs. oz.	lbs. oz	lbs. oz.	lbs. oz.	lbs oz	lbs. oz
89 0	79 0	83 0	81 0	95 0	82 0	81 0	88 0	88 0	77 0	86 0	...	82 8	85 11·4	96 11·4
189 0	165 0	192 0	182 0	158 0	171 0	195 0	152 0	173 0	140 0	148 0	227 4	173 4·8	162 6·8	179 15·3
182 0	157 0	184 0	173 0	150 0	159 0	184 0	143 0	163 0	132 0	139 0	216 4	165 1·6	152 13·7	170 12·2
3 10	3 6	3 12	3 11	3 9	3 14	4 8	3 11	4 4	3 12	4 0	3 13·5	3 7·7	3 15·2	3 11·3
5 14	......	7 6	4 10	7 12	7 10	9 1	6 9	6 2	8 12	8 2	7 4	4 10·6	7 11·4	6 2·8
12 9·5	9 10	*10 1	10 11	5 8·5	6 3·5	6 11	3 15·5	5 3	3 10	3 14	13 7·7	10 2·3	5 0·2	9 1·1
1 13·5	1 15	1 15	2 0	2 2	2 6	3 6	2 6	2 10	3 2	2 11	2 4·5	1 15·6	2 10·7	2 1·3
3 2	3 6	3 3	2 15	2 14	3 14·5	3 8·5	3 2	3 4	3 8	3 8	2 15	2 11·9	3 6·2	2 15·9
6 4	3 12	5 3	5 4	3 5·5	1 13	3 0	2 4·2	2 13	1 9·7	2 2	6 1	5 0·3	2 6·8	4 5·7
0 10·5	0 10	0 13	0 13	0 8·7	0 10·8	0 10·6	0 11	0 11·5	0 9	0 9	0 12·7	0 11·3	0 10·1	0 11·2
0 12	0 9·5	1 2	0 9	0 12	0 12·7	1 1·9	0 13·5	0 12·5	0 13·5	0 11	0 12·1	0 9·7	0 13·3	0 11·4
1 5·5	1 4	1 7·5	1 6	1 3·7	1 4	2 10	1 8·5	1 11	1 9·5	1 12·5	1 13·9	1 6	1 10·8	1 9·1
6 7·5	6 9	7 0·5	7 2·5	6 3	6 9	6 14	5 12·5	5 15·5	5 8	6 0	7 11·4	6 7·8	5 15·8	6 8·9
2 6	2 3	2 7	2 4·5	2 0	2 0	2 12·5	2 2	2 7·5	2 7	2 3	2 12·1	2 3·2	2 4·6	2 5·4
0 0·8	0 1·1	0 2·5	0 1·5	0 0·5	0 0·4	0 0·7	0 1·5	0 1·0	0 1·8	0 1·5	0 2·1	0 1·3	0 1·1	0 1·4
0 3·3	0 3·5	0 3·5	0 4·3	0 2·3	0 2·3	0 2·8	0 2·3	0 2·7	0 2·8	0 2	0 4·1	0 3·6	0 2·4	0 3·3
0 5	0 5	0 6	0 6·3	0 2·0	0 3·5	0 4	0 3·5	0 3·7	0 3	0 3·5	0 5·5	0 5·4	0 3·5	0 4·6
0 0·5	0 0·8	0 0·7	0 0·6	0 1	0 0·8	0 1·4	0 0·7	0 1	0 1	0 1	0 0·9	0 0·7	0 1	0 0·9
4 9	4 7	4 13	4 14	4 2·5	4 8·5	4 5·5	4 3·5	4 7·5	4 2·5	4 4	5 12·6	4 9·1	4 4·8	4 10·9
18 4	16 8	17 0	18 4	17 12	19 4	21 5	20 4	20 8	17 4	19 4	20 1	17 4·2	19 5·9	18 8·1
(7 0)	(9 0)	(6 8)	(7 4)	(7 0)	(6 8)	(8 0)	(8 0)	(8 0)	(8 0)	(8 0)	(5 15)	(7 6)	(7 10·3)	(7 3)
......	......	......	......	......	0 1	......	......	......	0 3	......	......	......	0 2	0 2
68 5·1	54 13·9	67 5·7	65 4·7	57 5·6	61 6	70 0·9	57 14·7	61 0·8	57 6·3	59 8·5	76 0·1	61 14·7	60 13·8	64 6·5
113 8·5	96 7	116 2·5	108 4	93 7	98 14·7	112 12	85 7·5	99 3	74 6	80 9	139 5·4	103 2·2	92 1·6	106 5·7
0 2·4	5 11·1	0 7·8	−0 8·7	−0 12·6	−1 4·7	0 13·1	−0 6·2	2 6·2	0 3·7	−1 1·5	0 8·5	0 0·7	−0 1·7	....
182 0	157 0	184 0	173 0	150 0	159 0	184 0	143 0	163 0	132 0	139 0	216 4	165 1·6	152 13·7	170 12·2

either in the recorded "Final Weight," or in the sum of items at the time of slaughtering. Nor, is it taken into account in calculating the *Percentage* Table LVIII. It is, however, given above, parenthetically, as Memorandum.

APPENDIX.—TABLE XXXIV. Showing the *Actual Weights* (lbs. and oz.)

CLASS I.—9 Pigs, divided into 3 Lots, each with rather different Food *.

	Designation of Parts.	Lot 1.—Food:— Bean and Lentil meal—in fixed quantity. Bran—*ad libitum.*						Lot 2—Food:— Indian meal—in fixed quantity. Bran—*ad libitum.*					
		No. 1.		No. 2.		No 3.		No 4.		No. 5.		No. 6.	
		lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.
	Original weight	156	0	142	0	130	0	149	0	138	0	129	0
	Final weight, unfasted	210	0	151	0	188	0	186	0	190	0	183	0
	Final weight, after fasting	200	0	144	0	175	0	178	0	178	0	175	0
Separate parts of the "offal."	Stomach and contents	4	6	2	2	2	4·5	2	7	3	6	3	4
	Caul-fat	0	11	0	10	0	11	1	5	1	1·5	0	13·5
	Small intestines and contents	9	2	2	12	5	14	3	12	6	10	5	4
	Large intestines and contents	15	1	5	3	8	12	5	6·5	10	0	8	4
	Intestinal fat, "mudgeon," &c.	1	13·5	1	1·5	1	2·5	2	7·5	1	11·5	1	6·5
	Heart and aorta	0	10·5	0	8	0	8·5	0	9	0	5	0	9
	Lungs and windpipe	1	9	1	8	1	11·5	1	8	1	4·5	1	8
	Blood	8	13	6	11	5	9	7	11	7	2	6	0
	Liver	3	8	2	0	3	0	2	5	2	14	2	10
	Gall-bladder and contents	0	4·5	0	2·8	0	1·8	0	2·2	0	1·5	0	1·8
	Pancreas ("sweetbread")	0	4·5	0	5·5	0	4·5	0	6·5	0	5	0	4·5
	Milt or spleen	0	5	0	4·5	0	7	0	4·5	0	3·5	0	4
	Bladder	0	1·8	0	1·2	0	1·5	0	2·8	0	1·5	0	1·5
	Penis (or uterus)	0	8	0	6·8	0	5	......		......		0	6
	Tongue	1	2·5	1	0	0	13·5	1	2	0	14	0	13·5
	Toes	0	4	0	2·5	0	3	0	3	0	2	0	2·2
	Miscellaneous trimmings	0	13	0	5	0	8	0	1·8	0	0	0	8
	Total "offal" parts	49	5·3	25	3·8	32	0·8	29	13·8	36	11	31	15·5
	Carcass (including head and feet)	147	14	117	14	139	4·5	148	12·5	138	14·5	142	7·5
	Loss by evaporation, error in weighing, &c.	2	12·7	0	14·2	3	11·2	−0	10·8	2	6·5	0	9
	Live-weight after fasting	200	0	144	0	175	0	178	0	178	0	175	0

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* For particulars of the feeding experiment, refer to Pens 9, 10, and 11, under the head of "Experiments

of the individual Organs, and other separated Parts, of PIGS.

Somewhat *under* Fattened—the Food containing a considerable portion of Bran.

Lot 3.—Food:— Bean and Lentil meal, and Indian meal —in fixed quantity. Bran—*ad libitum*.			Means of—			
No. 7.	No. 8.	No. 9.	Lot 1. 3 Pigs.	Lot 2. 3 Pigs.	Lot 3. 3 Pigs.	The 9 Pigs
lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
137 0	150 0	136 0	142 10·7	138 10·7	141 0	140 12·5
201 0	204 0	210 0	183 0	186 5·3	205 0	191 7·1
196 0	198 0	201 0	173 0	177 0	198 5·3	182 12·4
3 12	3 1	2 11	2 14·8	3 0·3	3 2·7	3 0·6
1 5	1 1	0 15	0 10·7	1 1·3	1 1·7	0 15·2
5 7	6 4	5 12	5 14·7	5 3·3	5 13	5 10·3
9 0	10 1	10 1	9 10·7	7 14·2	9 11·4	9 1·4
2 0·5	1 15	1 6	1 5·8	1 13·9	1 12·6	1 10·8
0 9	0 8·5	0 9	0 9	0 7·7	0 8·9	0 8·5
1 15	1 10·3	1 13	1 9·5	1 6·8	1 12·8	1 9·7
7 8	7 5	8 7	7 0·4	6 15	7 12	7 3·8
3 0	2 12	3 8	2 13·3	2 9·7	3 1·4	2 13·5
0 2·8	0 1·2	0 2	0 3	0 1·8	0 1·4	0 2·1
0 6·5	0 4·5	0 4·5	0 4·8	0 5·3	0 5·2	0 5·1
0 5	0 4	0 5	0 5·5	0 4	0 4·7	0 4·7
0 3	0 2	0 2	0 1·5	0 1·9	0 2·4	0 1·9
……	……	0 8	0 6·6	0 6	0 8	0 6·8
0 15	1 3	0 14	1 0	0 15·2	1 0	0 15·7
0 2·5	0 3	0 3·5	0 3·2	0 2·4	0 3	0 2·9
0 1·5	0 3·3	0 6	0 7	0 4·6	0 3·6	0 5·1
36 12·8	30 14·8	37 15	35 8·5	33 1·4	37 8·8	35 6·1
157 5	162 6	163 5·5	135 0·1	143 6·2	161 0·2	146 7·5
1 14·2	−1 4·8	−0 4·5	2 7·4	0 8·4	−0 3·7	0 14·8
196 0	198 0	201 0	173 0	177 0	198 5·3	182 12·4

with Pigs—Series I," Article—"Pig Feeding," Journal of the Royal Agricultural Society of England, vol. xiv. part 2.

APPENDIX.—TABLE XXXV. Showing the *Actual Weights* (lbs. and ozs.)

CLASS II.—12 Pigs, divided into 4 Lots, each with rather different Food*. *Moderately*

	Designation of Parts.	Lot 1.—Food.—Bean and Lentil meal—*ad libitum.*			Lot 2 Food: Indian meal—in fixed quantity. Bean and Lentil meal *ad libitum*		
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.
		lbs. oz.	lbs. oz	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
	Original weight	176 0	135 0	120 0	157 0	142 0	123 0
	Final weight, unfasted	283 0	231 0	239 0	304 0	246 0	238 0
	Final weight, after fasting	266 2·5	224 0	230 0	202 0	237 0	224 0
Separate parts of the "offal."	Stomach and contents	3 15·5	2 4	3 4·5	2 3·2	2 10	1 9
	Caul-fat	1 3·5	1 9	0 13·5	1 11·5	0 15·2	1 1·5
	Small intestines and contents	4 11·5	4 11	5 0	5 1	5 0	4 8
	Large intestines and contents	9 10	8 6·5	11 14	10 2	9 8	6 3
	Intestinal fat, "mudgeon," &c.	4 9·8	3 12·8	3 3·3	6 5	3 0	3 2
	Heart and aorta	0 10·8	0 9	0 11	0 11·3	0 9·8	0 11·5
	Lungs and windpipe	1 13·2	1 8	1 10·5	1 15·5	1 7	1 7·1
	Blood	9 13·8	8 9·5	9 13·2	10 11	9 8·5	10 0
	Liver	4 15	4 6	3 15	4 11	8 6	4 0
	Gall-bladder and contents	0 3·2	0 2·2	0 2	0 2·5	0 1·7	0 3
	Pancreas ("sweetbread")	0 4·2	0 7·5	0 9·2	0 14·2	0 8·2	0 9·2
	Milt or spleen	0 6 5	0 5	0 5·8	0 5·8	0 6·5	0 5
	Bladder	0 2·5	0 2·8	0 4	0 4	0 2·8	0 1·8
	Penis (or uterus)	0 12·8	......	......	0 8·2	......	0 5·8
	Tongue	1 3	1 0·2	1 3·5	1 2·5	1 0·8	0 14·5
	Toes	0 3·5	0 3·5	0 3	0 3	0 3·5	0 3
	Miscellaneous trimmings	0 13·2	0 11·5	0 11·5	0 9	0 11·5	0 6·8
	Total "offal" parts	45 8	38 12·5	43 12	47 8·7	39 8·5	35 11·2
	Carcass (including head and feet)	223 2	184 12	182 10	240 15	196 8·5	188 4
	Loss by evaporation, error in weighing, &c.	−2 7·5	0 7·5	3 10	3 8·3	2 4	0 0·8
	Live-weight after fasting	266 2·5	224 0	230 0	292 0	237 0	224 0

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* For particulars of the *fasting* experiment, refer to Pens 1, 2, 3, and 4, under the head of "Experiments

of the individual Organs, and other separated Parts, of PIGS.

Fattened—the Food containing a considerable proportion of Bean and Lentil Meal.

Lot 3.—Food:— Bran—in fixed quantity. Bean and Lentil meal—*ad libitum.*			Lot 4.—Food:— Indian meal and Bran—in fixed quantity Bean and Lentil meal—*ad libitum*			Means of—				
No. 7.	No. 8.	No. 9	No. 10.	No. 11.	No. 12	Lot 1 3 Pigs	Lot 2 3 Pigs.	Lot 3. 3 Pigs	Lot 4 3 Pigs	The 12 Pigs
lbs. oz.	lbs. oz	lbs oz.	lbs. oz	lbs. oz.	lbs. oz.	lbs oz	lbs oz	lbs. oz	lbs oz	lbs oz
163 0	131 0	128 0	173 0	131 0	123 0	116 10·7	110 10·7	110 10·7	112 5·3	112 9·4
248 0	183 0	202 0	302 0	207 0	180 0	251 0	262 10·7	211 0	232 10·7	239 5·4
238 0	173 0	194 0	281 8	191 8	178 0	240 0·8	251 0	201 10·7	217 0	227 6·9
2 14	3 0·5	2 13	3 4	2 11	3 0	3 2·7	2 2·1	2 14·5	3 0·7	2 13
1 7	0 15·3	0 11	1 4·5	1 3	0 9·5	1 3·3	1 4·1	1 0·4	1 0·3	1 2
6 12	4 7	4 7·5	6 9	4 0	3 15	4 12·8	4 13·7	5 3·5	4 13·3	4 14·8
10 4	8 0	8 15·5	12 4	8 10	8 3	9 15·5	8 9·7	9 1·2	9 11	9 5·4
2 7	1 0	2 7	3 4·5	2 15	1 10	3 14	4 2·3	1 15·3	2 9·8	3 2·4
0 6·2	0 9	0 7·7	0 14·3	0 8·3	0 9·8	0 10·3	0 10·9	0 7·6	0 10·8	0 9·9
1 11·5	1 8	1 3·5	2 0	1 11·5	1 8	1 10·6	1 9·9	1 7·7	1 12·8	1 10·2
8 13	7 4	7 13	8 6·5	9 4	9 7	9 6·8	10 1·2	7 15·4	9 0·5	9 2
4 4	2 10	2 15·5	5 2	3 12	2 11·5	4 0·7	4 0·3	3 4·5	3 13·8	3 14·3
0 3	0 1·5	0 1	0 1·8	0 2·5	0 1	0 2·5	0 2·4	0 1·8	0 1·8	0 2·1
0 9	0 6	0 6·8	0 9·2	0 7·3	0 6	0 7	0 10·5	0 7·3	0 7·5	0 8·1
0 5	0 4·5	0 4	0 6·5	0 5·2	0 5·2	0 5·8	0 5·8	0 4·5	0 5·6	0 5·4
0 2·8	0 3·2	0 3	0 3·5	0 3·2	0 3·5	0 3·1	0 2·9	0 3	0 3·4	0 3·1
……	……	……	0 5·8	……	……	0 12·8	0 7	……	0 5·8	0 8·1
1 4	0 15	0 12·8	1 0	0 14	0 13·5	1 2·2	1 0·6	0 15·9	0 14·5	1 0·3
0 3·5	0 3	0 3	0 4	0 3·5	0 3	0 3·3	0 3·2	0 3·2	0 3·5	0 3·3
1 8	0 13	0 11·5	0 9·2	……	0 9	0 12·1	0 9·1	1 0·2	0 9·1	0 11·6
43 2	32 4	34 7·8	46 8·8	37 4·5	34 3	43 3·5	40 15·7	36 10	39 12·2	40 4·2
192 0·5	141 3	157 6	235 7	158 6·3	142 12	196 13·3	208 3·8	163 10·5	178 13·8	186 14·4
2 7·5	–0 7	2 2·2	–0 7·8	–4 2·8	1 1	……	1 12·5	1 6·2	–1 10	0 4·3
238 0	173 0	194 0	281 8	191 8	178 0	240 0·8	251 0	201 10·7	217 0	227 6·9

with Pigs—Series 1," Article—"Pig Feeding," Journal of the Royal Agricultural Society of England, vol. xiv. part 2.

APPENDIX.—TABLE XXXVI. Showing the *Actual Weights* (lbs. and ozs.)

CLASS III.—15 Pigs, divided into 5 Lots, each with rather different Food.*

	Designation of Parts	Lot 1.—Food.—Indian meal—*ad libitum.*						Lot 2.—Food. Bran and Lentil meal in fixed quantity; Indian meal *ad libitum.*						Lot 3.—Food. Bran in fixed quantity; Indian meal *ad libitum.*					
		No. 1		No. 2		No. 3		No. 4		No. 5		No. 6		No. 7		No. 8		No. 9	
		lbs.	ozs.	lbs.	ozs.	lbs.	ozs.	lbs.	ozs.	lbs.	ozs.	lbs.	ozs.	lbs.	ozs.	lbs.	ozs.	lbs.	ozs.
	Original weight	168	0	128	0	135	0	157	0	111	0	111	0	118	0	126	0	111	0
	Final weight, unfasted	263	0	187	0	210	0	266	0	235	0	255	0	251	0	196	0	282	0
	Final weight, after fasting	253	8	181	8	208	0	262	0	223	0	248	0	244	0	190	0	279	0
Separate parts of the "offal."	Stomach and contents	2	15	3	0·5	2	2	2	0	2	3	2	2	2	9·5	1	15	2	12·5
	Caul fat	1	10	0	14·3	1	1·2	1	7	0	11·5	1	13	1	1·5	1	3·5	1	4
	Small intestines and contents	1	10	3	0·5	4	1·5	3	8	3	13	3	14	3	2	3	2	4	11
	Large intestines and contents	7	0·5	5	11·5	5	6	9	1	6	8	7	5	11	0	7	8	7	11
	Intestinal fat, "mudgeon," &c.	4	10·5	2	8·5	1	11·8	4	10·8	2	8·5	5	9	2	10	3	3	4	0
	Heart and aorta	0	11	0	9·2	0	10·5	0	11	0	10·5	0	10	0	10·5	0	9·5	0	13
	Lungs and windpipe	1	13	1	5	1	11	1	10	1	7·3	1	14	1	5	1	5	2	4
	Blood	9	1	6	9	7	13	9	10·5	6	8	8	14	7	6·5	6	14	8	3
	Liver	3	6·5	2	10	2	15·2	3	14	3	0·5	3	4	4	5	3	0	3	15
	Gall-bladder and contents	0	1·5	0	0·5	0	2·2	0	2·5	0	1·8	0	2·5	0	1·5	0	0·8	0	2·2
	Pancreas ("sweetbread")	0	8·8	0	5·3	0	9	0	7·5	0	4·5	0	10	0	6·2	0	5·2	0	11·5
	Milt or spleen	0	4·2	0	4·3	0	4·5	0	4·5	0	5·2	0	5·5	0	4	0	5	0	3·5
	Bladder	0	1·8	0	2	0	1·5	0	3·2	0	3·2	0	2·2	0	2·5	0	2·8	0	4·5
	Penis (or uterus)	0	8·5			0	8·5	0	8·3					0	0·5				
	Tongue	1	1·5	0	13	0	15·5	1	0·5	0	14·5	1	2·5	1	2	1	1·5	1	3
	Toes	0	3·2	0	3·2	0	2·8	0	3	0	3	0	3·5	0	2·5	0	2·5	0	3·5
	Miscellaneous trimmings	0	11·5	0	0	0	1	1	3	1	5·8	0	12	1	6	0	4	0	10·5
	Total "offal" parts	39	6·5	28	9·8	30	5·2	40	8·8	30	14·8	38	11·2	38	7·2	31	1·8	39	5·2
	Carcass (including head and feet)	217	6	156	9·5	175	11·5	219	7·5	190	14·5	208	1·5	204	11	157	8·5	235	1·5
	Loss by evaporation, error in weighing, &c.	−3	4·5	−3	4·8	1	15·8	1	15·7	1	2·7	1	3·3	0	13·8	1	5·7	4	0·3
	Live-weight after fasting	253	8	181	8	208	0	262	0	223	0	248	0	244	0	190	0	279	0

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* For particulars of the *feeding* experiment, refer to Pens 5, 6, 7, 8, and 12, under the head of "Experiments with

of the individual Organs, and other separated Parts, of Pigs.

Well Fattened—the Food containing a considerable proportion of Indian-corn Meal.

Lot 4.—Food:— Bean and Lentil meal, and Bran—in fixed quantity. Indian meal—*ad libitum.*			Lot 5.—Food:— Bean and Lentil meal, Indian meal, and Bran—each *ad libitum.*			Means of -					
No. 10.	No. 11.	No. 12	No. 13.	No. 14.	No. 15.	Lot 1. 3 Pigs	Lot 2 3 Pigs.	Lot 3. 3 Pigs.	Lot 4 3 Pigs.	Lot 5. 3 Pigs.	The 15 Pigs
lbs. oz.	lbs. oz.	lbs oz.	lbs. oz.	lbs. oz	lbs. oz.	lbs. oz.	lbs oz.	lbs. oz.	lbs. oz	lbs. oz.	lbs. oz.
145 0	144 0	113 0	149 0	130 0	150 0	143 10·7	144 5·3	138 5·3	144 0	143 0	143 7·5
294 0	250 0	255 0	271 0	163 0	244 0	220 0	252 0	247 5·3	266 5·3	226 0	235 10·7
284 0	242 0	239 0	267 0	165 0	236 0	214 5·3	244 5·3	237 10·7	255 0	222 10·7	234 12·7
2 11	3 2	2 14	4 8	3 0	2 12	2 11·1	2 1·6	2 7	2 14·4	3 6·7	2 11·4
1 11·5	1 4·5	1 8·5	1 5	1 9·5	0 14·5	1 3·2	1 6·2	1 4	1 8·2	1 4·3	1 5·2
5 2	4 2	5 5	3 14	3 6	3 2	3 14·7	3 11·7	3 11·3	4 13·7	3 7·3	3 14·0
9 10	10 8	7 10	8 0	5 10	6 0	6 0·7	7 10	8 11·7	9 4	6 11·7	7 10·8
5 2	2 8·5	1 12	2 9·3	2 14·5	2 1	2 15·5	4 4·1	3 4·3	3 2·1	2 8·3	3 3·6
0 13	0 10·5	0 9·5	0 12	0 7·5	0 9	0 10·2	0 10·5	0 11	0 11	0 9·5	0 10·4
1 10	1 12	1 10	1 13	1 4·5	1 3	1 9·7	1 10·4	1 10	1 10·7	1 6·8	1 9·5
8 14	9 4	8 11	8 10	5 14	8 1	7 13	8 5·5	7 7·3	8 15·1	7 8·3	8 0·4
4 2	3 4	3 5	3 6	2 8	3 4	2 15·0	3 0·2	3 12	3 0	3 0·7	3 5·0
0 2	0 1·5	0 3·7	0 1·5	0 0·5	0 1·5	0 1·4	0 2·3	0 1·5	0 2·2	0 1·2	0 1·7
0 12·5	0 7·5	0 6·8	0 5	0 5	0 7	0 7·7	0 7·3	0 7·6	0 8·0	0 5·7	0 7·5
0 5	0 5·5	0 3	0 5	0 3	0 4·5	0 4·8	0 5·1	0 4·8	0 5·2	0 4·2	0 4·7
0 3·5	0 2·5	0 1·8	0 2·8	0 1·3	0 4	0 1·8	0 2·0	0 3·3	0 2·0	0 2·7	0 2·7
......	0 8·5	0 9	0 6	0 5·5	......	0 8·5	0 8·3	0 0·5	0 8·7	0 5·7	0 8
1 1·5	1 0	1 1	1 8	0 12·5	0 15	0 15·3	1 0·5	1 2·2	1 0·0	1 1·2	1 0·8
0 3	0 3	0 2·5	0 3·2	0 3	0 3	0 5·1	0 3·2	0 2·8	0 2·8	0 3	0 2·9
1 3	0 5·5	0 9·7	0 12	0 9	0 9·5	0 7·2	1 1·4	0 12·2	0 11·4	0 10·2	0 11·7
43 10	39 0·5	36 12·5	38 8·8	29 1·8	31 4	32 15·3	37 1·2	36 11·1	40 2·0	33 1·5	35 15·8
238 3	200 6	200 6	220 12·5	133 4·5	202 11	183 1·3	206 2·5	199 1·7	212 15·7	187 9·8	197 12·5
2 3	2 0·5	1 13·5	1 10·7	2 9·7	2 1	−1 11·3	1 1·6	1 13·0	1 13·4	1 15·9	1 0·4
284 0	242 0	239 0	267 0	165 0	236 0	214 5·3	244 5·3	237 10·7	255 0	222 10·7	234 12·7

Pigs—Series 1," Article—"Pig Feeding," Journal of the Royal Agricultural Society of England, vol. xiv. part 2.

APPENDIX.—TABLE XXXVII. Showing the *Actual Weights* (lbs. and ozs.)

CLASS IV.—12 Pigs, divided into 4 Lots, according to the Food*. *Moderately* Fattened.

	Designation of Parts.	Lot 1.—Food. Lentils, and Bran—in fixed quantity. Sugar—*ad libitum.* No. 1.	No. 2.	No. 3.	Lot 2 Food: Lentils, and Bran in fixed quantity. Starch *ad libitum.* No. 4.	No. 5.	No. 6.
		lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
	Original weight	100 0	82 0	95 0	89 0	105 0	91 0
	Final weight, unfasted	181 0	168 0	184 0	167 0	201 0	162 0
	Final weight, after fasting	173 0	163 0	175 0	160 0	198 0	153 0
Separate parts of the "offal."	Stomach and contents	1 8	1 12	2 3·5	2 5	1 13·5	1 6·5
	Caul-fat	1 1	1 2·5	0 14·3	1 0·5	1 1·3	0 12
	Small intestines and contents	3 2	3 9	3 3	3 10	3 14	3 2
	Large intestines and contents	8 2	7 13	7 4	9 6	10 1	10 6
	Intestinal fat, "mudgeon," &c.	0 15·7	1 1	1 0	1 0	0 14	0 14·5
	Heart and aorta	0 8·8	0 9	0 8·7	0 7·3	0 9·5	0 9
	Lungs and windpipe	1 3·2	1 6	1 9·5	1 4	1 11·5	1 10
	Blood	5 0·5	5 12	6 5	6 2	7 6	6 14·5
	Liver	2 8·5	3 4·5	2 13	2 12·5	3 0	2 11·5
	Gall-bladder and contents	0 1·8	0 2·3	0 0·7	0 3	0 3·4	0 3
	Pancreas ("sweetbread")	0 4	0 6	0 4	0 4·8	0 5·8	0 4·5
	Milt or spleen	0 3·5	0 4	0 4·5	0 3	0 4	0 3·8
	Bladder	0 3·3	0 1·5	0 1·5	0 3	0 1·5	0 2
	Penis (or uterus)	......	0 4·5	......	0 4·8	0 5	0 4·5
	Tongue	1 2·5	1 0·5	1 2·5	1 0	0 14·5	0 13
	Toes	......	......	0 2·5	0 2	0 2·5	0 2
	Miscellaneous trimmings	......	......	......	0 2·5	0 2·5	0 2
	Total "offal" parts	26 6·8	28 7·8	27 12·7	30 0·4	32 14	30 8·8
	Carcass (including head and feet)	144 15	132 5	147 6·7	125 5·7	163 5·5	121 11·5
	Loss by evaporation, error in weighing, &c.	1 10·2	2 3·2	–0 3·4	4 3·2	1 12·5	0 11·7
	Live-weight after fasting	173 0	163 0	175 0	160 0	198 0	153 0

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* For particulars of the *feeding* experiment, see Article—"On the Equivalency of Starch and

of the individual Organs, and other separated Parts, of PIGS.

The Food consisted, in considerable proportion, of either Starch or Sugar.

Lot 3.—Food — Lentils, and Bran—in fixed quantity. Sugar, and Starch—*ad libitum.*			Lot 4—Food — Lentils, Bran, Sugar, and Starch —each *ad libitum.*			Means of				
No. 7.	No. 8.	No. 9.	No. 10.	No. 11.	No. 12.	Lot 1 3 Pigs.	Lot 2. 3 Pigs	Lot 3 3 Pigs	Lot 4 3 Pigs	The 12 Pigs
lbs. oz	lbs. oz.	lbs. ozs.	lbs. oz.	lbs. oz.	lbs. oz	lbs. oz	lbs. oz	lbs. oz.	lbs. oz	lbs. oz
86 0	87 0	108 0	98 0	100 0	94 0	95 5·3	95 0	93 10·6	97 5·3	95 5·3
177 0	172 0	204 0	211 0	208 0	185 0	177 10·6	177 10·6	184 5·3	201 5·3	185 4
168 0	163 0	196 0	205 0	193 0	182 0	170 5·3	170 5·3	175 10·6	193 5·3	177 6·6
2 2	1 15	2 4·7	1 11	2 11	2 13·5	1 13·2	1 13·7	2 1·9	2 6·5	2 0·8
1 0	1 1	1 0·5	1 5·5	1 0·5	1 3·5	1 0·6	0 15·3	1 0·5	1 3·2	1 0·9
3 1	3 6	4 13	4 3	5 2	4 13	3 4·7	3 8·7	3 12	4 11·3	3 13·2
7 9	8 14	8 9	9 8	8 8	10 12	7 11·7	9 15	8 5·3	9 9·3	8 11·3
1 6	0 14	1 7	1 0·5	1 7	0 15	1 0·2	0 11·8	1 3·7	1 4·2	1 1·7
0 8·5	0 8·5	0 10	0 9·5	0 9	0 9	0 8·8	0 8·6	0 9	0 9·2	0 8·9
1 4	1 2·7	1 7	1 6·8	1 8	1 4	1 6·2	1 8·5	1 4·6	1 6·3	1 6·4
5 10	6 1	6 0	7 6·5	6 2	6 13	5 13·1	6 12·8	5 11·3	6 12·5	6 5·2
2 11·5	2 7	2 15·5	3 6	4 0·5	3 7	2 11	2 13·3	2 11·3	3 0·8	3 0·1
0 1·8	0 2·3	0 2	0 3	0 3·2	0 2	0 1·6	0 3·1	0 2	0 2·7	0 2·4
0 5·2	0 4·3	0 4·5	0 5·8	0 5	0 6	0 4·7	0 5	0 4·7	0 5·6	0 5
0 4	0 3·5	0 3·5	0 4	0 3·5	0 3·5	0 4	0 3·6	0 3·7	0 3·7	0 3·7
0 2	0 2·5	0 2·3	0 2·5	0 2·5	0 2·5	0 2·1	0 2·2	0 2·3	0 2·5	0 2·2
......	......	......	0 6	0 8·5	...	0 4·5	0 4·8	...	0 7·2	0 5·8
0 14·5	0 14	0 15	0 15	0 13·5	0 12·8	1 1·8	0 14·5	0 14·5	0 13·8	0 15·2
0 2	0 2	0 3	0 0	0 2·5	0 2·5	0 2·5	0 2·2	0 2·3	0 2·5	0 2·3
0 3·7	0 3·5	0 5	0 5	0 6	0 1·8	......	0 2·3	0 4·1	0 4·1	0 3·6
27 5·2	28 5·3	31 6	33 8·1	33 12·7	34 8·6	27 13·7	31 4·4	29 0·2	34 2·1	30 11·4
137 6	132 0·7	160 14	168 9·3	155 14·5	144 11·5	141 9	136 12·9	143 0·8	156 6·4	144 9·5
3 4·8	2 1	3 12	2 14·6	3 4·8	2 11·0	0 14·6	2 4	3 0·6	2 12·5	2 1·7
168 0	163 0	196 0	205 0	193 0	182 0	170 5·3	170 5·3	175 10·6	193 5·3	177 6·6

Sugar in Food"—Report of the British Association for the Advancement of Science for 1854.

APPENDIX.—TABLE XXXVIII. Showing the *Actual Weights* (lbs. and ozs.) of the individual Organs, and other separated Parts, of PIGS

CLASS V.—6 Pigs, divided into 2 Lots, each with rather different Food*. Well Fattened.—Food comprised a portion of dried Cod-fish.

	Designation of Parts	Lot 1.—Food.—Dried Cod-fish—in fixed quantity. Bran and Indian meal, equal parts—*ad libitum*			Lot 2.—Food.—Dried Cod-fish—in fixed quantity. Indian meal—*ad libitum*			Means of—		
		No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	Lot 1. 3 Pigs.	Lot 2. 3 Pigs.	The 6 Pigs.
		lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
	Original weight	171 0	158 0	164 0	192 0	158 0	140 0	164 5·3	163 5·3	163 13·3
	Final weight, unfasted	294 0	238 0	286 0	335 0	275 0	299 0	272 10·7	303 0	287 13·3
	Final weight, after fasting	279 0	226 0	271 0	329 0	270 0	293 0	258 10·7	297 5·3	278 0
Separate parts of the "offal."	Stomach and contents	2 12	3 15	2 10	3 6	3 2	3 4	3 1·7	3 4	3 2·9
	Caul-fat	1 8·5	1 0	1 0	1 15·5	1 11	1 6	1 2·8	1 10·8	1 6·8
	Small intestines and contents	4 8	5 6	5 1	4 11·5	3 14	3 8	4 15·7	4 0·5	4 8·1
	Large intestines and contents	11 2	7 2	7 0·5	7 13	6 0	6 8	8 6·8	6 12·4	7 9·6
	Intestinal fat, "mudgeon," &c.	3 5	1 14	2 14	3 7	3 1	2 12·5	2 11	3 1·5	2 14·3
	Heart and aorta	0 11	0 15	0 11·5	0 10	0 7·5	0 12	0 12·5	0 9·8	0 11·1
	Lungs and windpipe	1 3·5	1 4	1 4	1 10·5	2 10·5	1 7·7	1 3·8	1 14·9	1 9·4
	Blood	9 13·5	7 13·5	8 7·5	9 10	7 15·5	7 12	8 11·5	8 7·2	8 9·4
	Liver	3 12	3 5	4 1	3 4	3 4	3 0	3 11·3	3 2·7	3 7
	Gall-bladder and contents	0 3·5	0 2	0 1·5	0 3	0 2·5	0 4	0 2·4	0 3·2	0 2·8
	Pancreas ("sweetbread")	0 8	0 8	0 7·5	0 10	0 7·5	0 8	0 7·8	0 8·5	0 8·1
	Milt or spleen	0 5·5	0 5·5	0 5	0 6·5	0 5	0 6	0 5·4	0 5·8	0 5·6
	Bladder	0 3·2	0 2·3	0 2·5	0 2·5	0 3	0 3	0 2·7	0 2·8	0 2·7
	Penis (or uterus)	0 10	0 7·5	......	0 10	. .	.	0 8·7	0 10	0 9·2
	Tongue	1 5	1 2·2	1 4	1 9·5	1 0	0 15	1 3·7	1 2·8	1 3·3
	Toes	0 2·5	0 2·5	0 3	0 4	0 3	0 3	0 2·7	0 3·3	0 3
	Miscellaneous trimmings	0 11	0 5	1 1	0 2·5	0 10	0 9	0 11	0 7·2	0 9·1
	Total "offal" parts	42 12·2	35 13·5	36 10	40 7·5	35 0·5	33 6·2	38 9·5	36 11·4	37 10·4
	Carcass (including head and feet)	235 14	189 2·5	232 1	288 11·5	234 0	256 7	219 0·5	259 11·5	239 6
	Loss by evaporation, error in weighing, &c.	0 5·8	1 0	2 5	—0 3	0 15·5	3 2·8	1 0·7	0 14·4	0 15·6
	Live-weight after fasting	279 0	226 0	271 0	329 0	270 0	293 0	258 10·7	297 5·3	278 0

* For particulars of the *feeding* experiment, refer to Papers 1 and 2, [illegible] 3 [illegible] Experiments with Pigs—Series 3. Article—"Pig Feeding," Journal of the Royal Agricultural Society of England, vol. [illegible], part 2.

APPENDIX.—TABLE XXXIX. Showing the *Actual Weights* (lbs. and ozs.) of the individual Organs, and other separated Parts, of PIGS.

5 Pigs, divided into 2 Lots, according to condition of Maturity when put to Fatten.

CLASS VI.—Put to Fatten when in Store condition, and fed till only Half-fattened.

CLASS VII.—Put to Fatten when Half-fat, and fed till Moderately Fattened.

	Designation of Parts.	Class 6. Put to Fatten when in store condition, and fed till only Half-fattened.		Class 7. Put to Fatten when Half-fat, and fed till Moderately fattened.			Means of—		
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	Class 6	Class 7.	The 5 Pigs.
		lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.	lbs. oz.
	Original weight	121 0	140 0	147 0	112 0	148 0	130 8	135 10·7	133 9·6
	Final weight, unfasted	165 0	195 0	185 0	137 0	222 0	180 0	181 5·3	180 12·8
	Final weight, after fasting	155 0	186 0	175 0	130 0	213 0	170 8	172 10·8	171 12·8
Separate parts of the body	Stomach and contents	4 3	1 11·6	1 7·8	1 5·5	2 4·5	2 15·3	1 11·3	2 3·3
	Caul-fat	0 12·9	0 12·7	0 14·2	0 11·2	1 1·6	0 12·8	0 14·3	0 13·7
	Small intestines and contents	7 12	5 8	3 12·7	3 13·8	4 2·9	6 10	3 15·1	5 0·3
	Large intestines and contents	6 14	7 14	4 11·5	5 7·5	6 12	7 6	5 10·3	6 5·4
	Intestinal fat, "mudgeon," &c.	0 15·7	1 5·5	2 4·1	0 13·5	1 7·2	1 2·6	1 8·2	1 6
	Heart and aorta	0 7·4	0 7·5	0 7·2	0 6·2	0 10·8	0 7·5	0 8·1	0 7·8
	Lungs and windpipe	1 9·7	1 3·3	1 7·8	1 14	1 15·2	1 6·5	1 12·3	1 10
	Blood	4 7	5 15·5	5 9·9	4 10	7 2	5 3·3	5 12·7	5 8·9
	Liver	3 7·1	2 13·2	2 10	2 6·3	2 14	3 2·1	2 10·1	2 13·3
	Gall-bladder and contents	0 1	0 2·2	0 1·7	0 2·3	0 3·1	0 1·6	0 2·4	0 2·1
	Pancreas ("sweetbread")	0 3·4	0 5·6	0 5·6	0 3·8	0 6	0 4·5	0 5·1	0 4·9
	Milt or spleen	0 5·1	0 4·2	0 3·2	0 2·7	0 7·5	0 4·6	0 4·5	0 4·5
	Bladder	0 1·8	0 1·7	0 2	0 2·5	0 3·3	0 1·8	0 2·6	0 2·3
	Penis (or uterus)	0 4·2	0 6	0 7·4	0 6	0 9	0 5·1	0 6·8	0 6·1
	Tongue	0 13·5	0 12·9	0 14·3	0 12·2	0 14·2	0 13·2	0 13·6	0 13·4
	Miscellaneous trimmings	0 8·9	0 9·7	1 0·7	0 9·3	0 5·1	0 9·3	0 10·4	0 9·9
	Total "offal" parts	32 14·7	30 5·6	26 6·1	23 14·8	31 6·4	31 10·2	27 3·8	28 15·9
	Carcass (including head and feet)	118 3	153 0	147 0	106 2·5	180 0	135 9·5	144 6·2	140 13·9
	Loss by evaporation, error in weighing, &c.	3 14·3	2 10·4	1 9·9	—0 1·3	1 9·6	3 4·3	1 0·8	1 15
	Live-weight after fasting	155 0	186 0	175 0	130 0	213 0	170 8	172 10·8	171 12·8

APPENDIX.—TABLE XL. Showing the *Percentage Proportion* of the individual Organs, and other

Designation of Parts	Calves No. 1	Calves No. 2	Heifers No. 1	Heifers No. 2	Bullocks No. 1	Bullocks No. 2	Bullocks No. 3	Bullocks No. 4	Bullocks No. 5
	Short-horned, milk-fed, killed Aug. 11, 1849	Durham breed (analysed as fat), taken from dam feeding on grass, killed Sept. 12, 1849	Welsh, 2 to 3 years old, killed Aug. 23, 1849	Welsh, 3 years old, lately calved, fed, killed Aug. [illegible], 1849	Mixed breed, killed Aug. [illegible], 1849	Mixed breed, killed Aug. 9, 1849	Welsh horned, 4 years old, killed Aug. 16, 1849	Scotch long-horned, 4 years old, grass-fed, killed Aug. [illegible], 1849	Dutch, lately grass-fed in Norfolk, killed Aug. 23, 1849
Separate parts of the offal:									
Stomachs	1.61	1.09	3.70	3.79	3.31	3.85	3.23	3.21	2.87
Contents of stomachs, and vomit	1.60	3.17	5.72	11.09	13.02	8.70	9.20	8.93	6.80
Caul-fat	1.11	0.96	3.55	1.82	1.82	1.61	1.31	1.94	1.63
Small intestines and contents	1.85	2.39	1.16	2.11	2.03	1.97	1.10	1.86	1.67
Large intestines and contents	1.18	1.12	1.01	1.84	1.18	2.25	1.13	1.17	1.18
Intestinal fat	0.64	1.62	4.01	2.01	1.12	1.65	1.15	1.56	2.06
Heart and aorta	0.62	0.57	0.17	0.50	0.52	0.53	0.56	0.55	0.56
Heart-fat		0.16	0.22	0.21	0.15	0.31	0.22	0.17	0.16
Lungs and windpipe	1.31	1.29	0.61	0.90	1.03	0.93	0.81	1.01	1.01
Blood	4.13	5.21	3.62	3.58	4.01	4.82	2.93	[illegible]	3.81
Liver	1.70	1.63	1.51	1.49	1.25	1.48	1.17	1.52	1.16
Gall-bladder and contents	0.05		0.07	0.09	0.03	0.02	0.11	0.11	0.16
Pancreas ("sweetbread")	0.12	0.70	0.08	0.11	0.09	0.13	0.08	0.07	0.07
Thymus gland ("heartbread")	0.30		0.06	0.07	0.08	0.05	0.05	0.06	0.08
Glands about the throat ("throatbread")	0.23		0.04	0.05	0.03		0.03	0.02	0.06
Milt or spleen	0.36	0.20	0.17	0.12	0.16	0.21	0.11	0.15	0.17
Bladder, and penis, or womb	0.12	0.18	1.09*	0.17	0.05	0.05	0.03	0.11	0.04
Brains	6.49	4.13	2.27	0.08	0.09	0.05	0.08	0.07	0.05
Head and tongue				2.60	2.56	2.78	2.77	2.86	3.00
Hide and horns	7.00	6.87	7.26	8.21	7.50	7.10	8.33	8.91	6.93
Feet, hoofs, &c.	2.70	1.66	1.54	1.89	1.86	1.91	2.00	1.03	1.78
Tail		0.13	0.06	0.13	0.12	0.09	0.09	0.07	0.11
Diaphragm ("skirts")		0.43	0.53	0.54	0.18	0.24	0.18	0.10	0.16
Miscellaneous trimmings			0.81	0.18		0.12	0.27	0.19	0.32
Total "offal" parts	33.47	33.93	39.95	43.70	42.82	40.85	37.93	41.11	35.81
Carcass	64.21	60.86	55.85	55.30	54.89	57.07	59.80	56.81	61.06
Loss by evaporation, error in weighing, &c.	2.32	5.21	4.20	1.00	2.29	2.08	2.27	2.08	3.10
Live-weight [illegible]	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

* Including calf.

separated Parts, in the Fasted Live-weight—of CALVES, HEIFERS, and BULLOCKS.

Bullocks.									Means of—			
No 6.	No. 7.	No 8.	No. 9	No. 10.	No 11.	No. 12.	No 13.	No. 14.				
Scotch; long-horned; 4 years old; grass-fed; killed Sept. 6, 1849.	Welsh; 5 years old; killed Sept. 13, 1849.	Mixed breed; killed Sept. 20, 1849	Durham, short-horned; 3 to 4 years old; grass-fed; killed Sept. 27, 1849.	Irish; lately grass-fed in Leicestershire, killed Sept. 27, 1849.	Mixed breed; 3 years old; killed Oct. 4, 1849.	Scotch (analysed as fat); 4 years old; killed Oct. 30, 1849.	Scotch (analysed as half-fat); 3 to 4 years old; killed Nov. 14, 1849.	Devon, working ox; afterwards fed on oilcake, &c; 7 years old, killed April 6, 1853.	The 2 Calves.	The 2 Heifers	The 14 Bullocks	The 16 Heifers and Bullocks.
3·06	2·92	3·10	3·30	2·71	2·83	2·56	2·60	3·51	1·37	3·75	3·09	3·17
10·59	9·63	8·00	7·27	9·75	8·56	5·92	8·12	3·66	2·30	8·40	8·44	8·44
1·23	2·08	2·08	1·94	1·71	2·08	2·10	1·35	3·21	1·03	2·68	1·93	2·02
1·71	1·84	1·46	1·00	1·53	1·45	0·62	0·67	0·90	2·12	1·80	1·49	1·52
2·00	1·49	0·98	0·66	1·08	1·25	0·36	0·49	0·46	1·30	1·44	1·18	1·22
0·97	2·02	2·32	2·20	1·76	3·32	2·60	1·61	4·78	1·13	3·02	2·12	2·24
0·52	0·44	0·48	0·47	0·47	0·43	0·52	0·47	0·49	0·60	0·48	0·50	0·50
0·34	0·31	0·33	0·23	0·35	0·38	0·72	0·31	0·51	0·08	0·22	0·32	0·31
0·88	0·74	0·71	0·71	0·90	0·74	0·63	0·63	0·75	1·32	0·75	0·82	0·81
4·01	4·56	4·85	4·53	3·80	3·86	3·72	4·41	3·30	4·68	3·60	4·07	4·01
1·24	1·21	1·29	1·39	1·21	1·23	1·24	1·28	1·22	1·07	1·52	1·28	1·31
0·08	0·13	0·11	0·09	0·08	0·08	0·06	0·08	0·08	0·05	0·08	0·09	0·09
0·08	0·07	0·08	0·09	0·10	0·11	0·07	0·08	0·16	0·67	0·09	0·09	0·09
0·07	0·05	0·05	0·04	0·05	……	0·05	0·05	0·10		0·07	0·06	0·06
0·04	0·01	0·03	0·03	0·03	0·03	0·03	0·03	0·03		0·05	0·03	0·03
0·16	0·17	0·22	0·15	0·19	0·12	0·16	0·17	0·17	0·32	0·15	0·17	0·16
0·04	0·05	0·05	0·07	0·08	0·10	0·12†	0·06†	0·08†	0·15	0·03§	0·09‖	0·05¶
0·06	0·07	0·06	0·08	0·07	0·06	0·06	0·08	0·03	5·40	0·04	0·07	0·06
2·87	2·49	2·27	2·43	2·70	2·52	3·06	3·17	2·45		2·48	2·71	2·69
9·44	7·79	7·88	7·15	7·22	7·15	5·67	6·49	6·86	6·94	7·74	7·46	7·40
1·87	1·65	1·88	1·87	1·85	1·83	1·57	1·63	1·51	2·18	1·72	1·78	1·77
0·09	0·09	0·10	0·11	0·09	0·09	0·10	0·14	‡	0·13	0·09	0·09	0·10
0·42	0·62	0·45	0·56	0·40	0·40	0·53	0·46	0·79	0·43	0·53	0·39	0·41
0·13	0·12	0·19	0·13	0·28	0·90	0·36	0·16	0·53	……	0·49	0·27	0·30
41·90	40·55	38·97	37·18	38·53	40·51	32·83	34·54	35·64	34·02	41·82	38·54	38·86
57·49	58·06	59·70	60·25	59·96	59·72	66·20	64·75	62·09	63·53	55·58	59·84	59·31
0·61	1·39	1·33	2·57	1·51	—0·23	0·97	0·71	2·27	2·45	2·60	1·62	1·83
100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

† In these cases bladder and penis together, and in the others bladder only.

‡ With hide.

§ In one case womb with calf.

‖ Sum of the mean of the bladder, and that of the penis, taken separately.

¶ Mean of bladders only, the penis or womb included with the "Loss, &c."

APPENDIX.—TABLE XLI. Showing the *Percentage Proportion* of the individual Organs, and other separated Parts, in the Fasted Live-weight of SHEEP

CLASS I.—5 Sheep, of different Breeds, killed (at Rothamsted) in *store* or *lean* condition, for standards of comparison

	Designation of Parts.	Cotswold Wethers, killed November 25, 1851. No 1	Cotswold Wethers, killed November 25, 1851. No. 2	Leicester Wether, killed November 22, 1852	Cross-bred Wether (Leicester and South Down), killed November 22 1852	Cross-bred Ewe (Leicester and South Down), killed November 22 1852	Means of the 5 Store Sheep
Separate parts of the "offal."	Stomachs	2·64	3·02	2·93	3·24	2·86	2·94
	Contents of stomachs	5·56	6·87	6·06	*	*	6·16
	Caul-fat	3·75	2·87	1·73	3·53	2·73	2·92
	Small intestines and contents	2·38	2·50	2·18	2·26	2·25	2·32
	Large intestines and contents	3·07	2·30	3·51	2·97	2·82	2·93
	Intestinal fat	0·98	0·97	1·38	1·87	1·18	1·28
	Heart and aorta	0·46	0·50	0·48	0·50	0·45	0·48
	Heart-fat	0·40	0·39	0·26	0·30	0·26	0·32
	Lungs and windpipe	1·18	1·26	1·05	0·98	1·40	1·17
	Blood	4·94	4·50	4·56	4·73	5·30	4·81
	Liver	1·71	1·75	1·37	1·64	1·58	1·61
	Gall-bladder and contents	0·04	0·12	0·03	0·04	0·11	0·07
	Pancreas ("sweetbread")	0·12	0·18	0·16	0·07	0·14	0·13
	Glands about the throat ("throatbread")	....	....	....	0·06	....	0·06
	Milt or spleen	0·23	0·13	0·15	0·15	0·21	0·17
	Bladder	0·04	0·05	0·04	0·10	0·04	0·05
	Womb	......	......	....	....	0·11	....
	Head	3·43	3·41	3·79	3·77	3·79	3·64
	Skin and wool (with feet, &c.)	13·90	13·64	16·99	12·80	13·11	14·09
	Diaphragm ("skirts")	....	....	....	0·30	......	0·30
	Miscellaneous trimmings	....	....	0·11	....	0·10	0·10
	Total "offal" parts	44·83	44·46	46·78	39·31	38·44	45·55
	Carcass	53·84	54·28	51·30	54·13	53·54	53·42
	Loss by evaporation, error in weighing, &c.	1·33†	1·26†	1·92	6·56‡	8·02‡	1·03
		100·00	100·00	100·00	100·00	100·00	100·00

* Included with "Loss, &c."

† Including contents of bladder.

‡ Including contents of stomachs.

* APPENDIX.—TABLE XLII. Showing the *Percentage Proportion* of the individual

CLASS II.—20 Wether Sheep of *Cotswold Breed*, about 1¾ year old, in *moderately*
Killed at Rothamsted.

	Designation of Parts	The 5 giving the Largest amount of Increase during Fattening					The 5 giving the Smallest amount of Increase during Fattening				
		No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10
Separate parts of the "offal."	Stomachs	2·69	2·44	2·56	2·51	2·37	2·16	3·10	2·18	2·61	2·16
	Contents of stomachs	4·61	5·60	4·16	4·12	5·22	4·20	4·11	4·41	4·81	4·92
	Caul fat	3·99	2·74	2·56	3·91	3·52	1·95	1·03	3·30	3·01	1·85
	Small intestines and contents	1·83	1·75	2·01	1·75	2·02	1·80	2·56	1·90	1·46	1·69
	Large intestines and contents	1·78	1·75	1·21	1·63	2·24	1·32	2·42	1·85	1·69	2·29
	Intestinal fat	1·00	1·21	1·58	1·03	1·30	1·21	0·86	1·06	1·07	1·15
	Heart and aorta	0·37	0·30	0·39	0·36	0·39	0·31	0·45	0·41	0·35	0·35
	Heart-fat	0·20	0·23	0·16	0·31	0·27	0·30	0·19	0·15	0·19	0·04
	Lungs and windpipe	0·81	1·10	1·12	1·04	0·98	0·96	1·18	0·93	0·95	0·92
	Blood	3·91	3·97	3·74	4·11	4·00	3·91	5·14	4·58	3·98	4·10
	Liver	1·78	1·66	1·58	1·88	1·68	1·60	1·97	1·85	1·69	1·68
	Gall bladder and contents	0·06	0·07	0·04	0·07	0·11	0·06	0·01	0·09	0·05	0·08
	Pancreas ("sweetbread")	0·08	0·14	0·10	0·10	0·14	0·13	0·15	0·13	0·15	0·14
	Milt or spleen	0·15	0·16	0·13	0·17	0·19	0·16	0·21	0·22	0·15	0·15
	Bladder	0·03	0·03	0·02	0·02	0·03	0·02	0·05	0·02	0·03	0·05
	Head	2·43	2·68	2·70	2·54	2·76	2·80	3·26	2·96	2·89	2·72
	Skin (with feet, &c.)	7·45	7·90	7·87	6·87	7·27	7·51	9·41	8·01	8·09	7·31
	Wool previously shorn	5·32	5·64	4·49	4·18	5·01	5·49	5·88	6·86	5·61	5·24
Total "offal" parts		38·52	39·49	36·75	37·23	39·53	39·28	46·07	41·21	39·23	40·2
Carcass		61·22	60·12	62·01	62·77	60·35	60·72	53·89	58·30	60·68	58·6
Loss by evaporation, error in weighing, &c.		0·26	0·39	1·24	0·001	0·12	0·00†	0·04	0·49	0·09	1·1
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·0

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* For particulars of the *feeding* experiment, see Article on the "Comparative Fattening Qualities of

† In these cases the sum of the weights taken for the separated parts exceeded the Fasted Live-

Organs, and other separated Parts, in the Fasted Live-weight of SHEEP.

Fat condition. Fattening Food—Oilcake, Clover Chaff, and Swedish Turnips*.
April 19, 1852.

The 10 giving the Medium amount of Increase during Fattening										Means of			
No. 11.	No. 12.	No. 13.	No. 14.	No. 15.	No. 16.	No. 17.	No. 18.	No. 19.	No. 20.	The 5 of Largest Increase	The 5 of Smallest Increase.	The 10 of Medium Increase	The 20 Cotswolds
2·63	2·42	2·71	2·46	2·66	2·71	2·74	2·64	2·82	2·71	2·57	2·71	2·65	2·65
4·38	4·67	4·97	5·07	4·12	4·83	4·75	3·40	5·09	3·80	4·87	4·55	4·52	4·05
4·42	4·81	4·39	5·25	4·70	4·79	4·21	4·34	3·51	3·64	3·84	4·01	4·41	3·92
1·77	1·90	2·26	1·82	1·92	1·87	1·87	1·61	1·96	2·05	1·88	2·02	1·90	1·93
2·44	2·03	2·15	1·75	2·18	1·75	1·77	1·63	2·30	1·82	1·73	2·00	1·99	1·91
1·41	1·46	0·84	1·25	0·90	1·42	0·83	1·10	1·00	1·05	1·23	1·08	1·12	1·14
0·38	0·35	0·37	0·39	0·38	0·33	0·38	0·39	0·42	0·39	0·38	0·38	0·38	0·38
0·15	0·18	0·21	0·34	0·14	0·23	0·16	0·31	0·15	0·22	0·24	0·18	0·21	0·21
1·20	1·16	1·34	1·04	1·18	1·28	1·13	1·20	1·11	1·24	1·01	0·99	1·19	1·06
3·97	4·19	3·96	4·15	4·02	4·37	4·07	3·80	4·94	4·50	3·95	4·42	4·20	4·10
1·83	1·87	1·78	1·78	1·66	1·87	1·84	1·63	2·03	1·98	1·72	1·74	1·83	1·77
0·06	0·04	0·05	0·06	0·07	0·06	0·06	0·07	0·03	0·05	0·07	0·05	0·05	0·06
0·13	0·11	0·14	0·10	0·12	0·12	0·12	0·14	0·12	0·16	0·11	0·14	0·13	0·13
0·17	0·16	0·16	0·16	0·15	0·13	0·15	0·18	0·25	0·10	0·16	0·18	0·17	0·14
0·02	0·03	0·01	0·02	0·02	0·02	0·02	0·02	0·03	0·03	0·03	0·03	0·02	0·03
2·82	2·51	2·80	2·58	2·77	2·85	2·81	2·97	3·13	3·10	2·92	2·95	2·84	2·80
7·61	7·48	7·33	6·57	7·10	7·50	7·49	7·14	9·21	8·37	7·47	8·08	7·59	7·72
4·38	5·26	5·07	5·03	4·81	5·59	5·48	5·04	5·41	5·77	4·98	5·81	5·28	5·33
39·77	40·63	41·23	39·82	38·99	41·72	39·88	37·70	43·51	41·07	38·31	41·32	40·43	40·02
59·97	59·04	58·77	60·18	60·33	58·16	59·02	61·30	56·46	58·50	61·29	58·21	59·18	59·56
0·26	0·33	0·00†	0·00†	0·68	0·12	1·10	0·91	0·03	0·43	0·40	0·47	0·39	0·42
100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

different Breeds of Sheep," Journal of the Royal Agricultural Society of England, vol. xiii, part 1.
weight, and this sum has therefore been taken as the standard, in calculating the Percentages.

APPENDIX.—Table XLIII. Showing the *Percentage Proportion* of the individual

CLASS III.—16 Wether Sheep of *Leicester Breed*, about 1¼ year old, in *moderately*
Killed at Rothamsted,

	Designation of Parts.	The 4 giving the Largest amount of Increase during Fattening.				The 4 giving the Smallest amount of Increase during Fattening.			
		No. 1.	No. 2.	No 3.	No. 4.	No. 5.	No 6	No 7	No 8
Separate parts of the "offal."	Stomachs	2·70	2·30	2·27	2·63	2·01	3·05	2·79	2·72
	Contents of stomachs	4·13	3·48	3·04	4·71	4·26	5·33	4·61	5·13
	Caul-fat	3·63	4·04	4·39	3·54	3·88	1·77	2·31	1·71
	Small intestines and contents	2·13	1·78	2·16	1·88	1·04	3·61	2·15	2·82
	Large intestines and contents	1·80	1·59	1·86	1·71	1·02	2·39	1·94	2·41
	Intestinal fat	1·17	1·15	1·17	0·92	1·10	1·44	0·72	1·36
	Heart and aorta	0·37	0·44	0·40	0·41	0·40	0·53	0·40	0·57
	Heart-fat	0·21	0·11	0·32	0·12	0·11	0·14	0·11	0·15
	Lungs and windpipe	0·81	1·00	0·72	0·87	1·08	1·78	0·97	1·00
	Blood	4·00	3 93	4·32	3·70	3·88	4·83	4·50	4·88
	Liver	1·70	1·55	1·80	1·92	1·91	2·00	1·79	2·05
	Gall-bladder and contents	0·05	0·06	0·09	0·04	0·04	0·07	0·04	0·06
	Pancreas ("sweetbread")	0·14	0·17	0·15	0·12	0·17	0·18	0·17	0·16
	Milt or spleen	0·13	0·13	0·13	0·12	0·13	0·20	0·17	0·19
	Bladder	0·01	0·03	0 03	0·05	0 04	0·04	0·05	0·06
	Head	2·46	2·78	2·40	2·83	3·07	3·50	3·27	3·51
	Skin (with feet, &c.)	7·60	7·15	7·05	7·25	7·81	8 88	8·06	9 12
	Wool previously shorn	6·02	5·18	6·06	6·07	5·17	7·61	8·88	6·78
	Miscellaneous trimmings	......	......	......	......	......	......	......	.....
Total "*offal*" parts		39·33	36·87	39·31	39·58	39·61	47·35	43 83	45·58
Carcass		58·50	60·33	58·37	58·92	57·62	52·56	54·59	52·63
Loss by evaporation, error in weighing, &c.		2·17	2·80	2·32	1·50	2·77	0·09	1·58	1·70
		100·00	100 00	100·00	100·00	100·00	100·00	100·00	100 00

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* For particulars of the *feeding* experiment, see Article on the "Comparative Fattening Qualities of

† This sheep was an Ewe, and this amount includes the womb.

‡ In this case the sum of the weights taken for the separated parts exceeded the Fasted Live-weight,

Organs, and other separated Parts, in the Fasted Live-weight of SHEEP.

Fat condition. Fattening food—Oilcake, Clover Chaff, and Swedish Turnips*.
April 23, 1853.

The 8 giving the Medium amount of Increase during Fattening.								Means of—			
No. 9.	No. 10.	No. 11.	No. 12.	No. 13	No. 14.	No. 15	No. 16.	The 4 of Largest Increase.	The 4 of Smallest Increase.	The 8 of Medium Increase.	The 16 Leicesters.
2·40	2·63	2·88	2·28	2·05	2·71	2·88	2·68	2·50	2·87	2·64	2·67
3·60	3·58	5·82	3·32	4·09	3·83	3·25	5·72	4·07	4·83	4·16	4·35
3·65	3·62	2·68	5·11	3·68	3·50	2·74	4·15	3·90	2·42	3·64	3·32
1·98	2·16	2·78	1·88	2·02	2·50	1·97	2·43	1·99	2·70	2·22	2·30
1·54	1·74	1·75	1·83	2·45	1·91	1·86	1·85	1·76	2·09	1·87	1·91
0·99	1·11	1·18	1·63	1·15	1·53	1·00	1·47	1·10	1·18	1·26	1·18
0·40	0·42	0·46	0·34	0·36	0·41	0·44	0·43	0·41	0·47	0·41	0·43
0·16	0·19	0·12	0·20	0·18	0·10	0·16	0·08	0·19	0·13	0·15	0·16
0·89	1·04	1·05	0·93	0·97	0·87	1·11	0·96	0·85	1·43	1·05	1·11
3·71	3·77	4·74	3·73	4·29	3·83	3·99	4·26	4·01	4·52	4·05	4·19
1·71	2·07	2·16	1·70	1·89	1·84	2·09	2·15	1·77	1·91	1·95	1·89
0·06	0·09	0·12	0·05	0·04	0·08	0·05	0·04	0·04	0·05	0·06	0·05
0·15	0·17	0·15	0·16	0·18	0·20	0·17	0·15	0·14	0·17	0·17	0·16
0·13	0·17	0·18	0·14	0·13	0·13	0·14	0·15	0·13	0·17	0·15	0·15
0·02	0·04	0·04	0·03	0·04	0·08†	0·03	0·02	0·04	0·05	0·04	0·04
2·76	3·03	3·37	2·71	2·92	3·01	3·11	2·94	2·63	3·24	2·98	2·98
7·76	7·57	8·50	7·24	8·09	8·07	8·08	7·60	7·26	8·02	7·96	7·95
5·52	5·17	6·05	5·92	6·56	5·92	4·92	5·26	5·98	7·11	5·78	6·20
0·13	0·12	0·06	0·00	0·08	0·05	0·19	0·15	......	......	0·11	0·11
37·65	38·69	45·68	39·29	41·77	40·57	38·78	42·58	38·77	44·00	40·63	41·24
61·03	59·71	53·58	58·86	57·30	59·43	60·07	57·06	59·03	54·35	58·38	57·25
1·32	1·60	0·74	1·85	0·93	0·00‡	1·15	0·36	2·20	1·65	0·99	1·51
100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

different Breeds of Sheep," Journal of the Royal Agricultural Society of England, vol. xvi. part 1.

and this *sum* has therefore been taken as the standard, in calculating the Percentages.

APPENDIX.—TABLE XLIV. Showing the *Percentage Proportion* of the individual

CLASS IV.—16 Wether Sheep of *Cross-breed* (Leicester and South Down), about 1¼ year old. Killed at Rothamsted.

	Designation of Parts.	The 4 giving the Largest amount of Increase during Fattening.				The 4 giving the Smallest amount of Increase during Fattening.			
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.
Separate parts of the "offal."	Stomachs	2·74	2·68	2·72	2·73	2·65	2·79	2·86	2·54
	Contents of stomachs	5·47	4·25	6·43	4·06	1·26	5·13	1·60	5·84
	Caul-fat	3·53	4·85	3·00	4·06	3·15	3·79	1·39	1·93
	Small intestines and contents	1·39	1·77	1·64	1·72	1·67	1·95	2·00	2·01
	Large intestines and contents	1·67	1·97	2·21	2·38	2·20	2·17	2·10	2·19
	Intestinal fat	1·07	1·57	1·08	1·19	0·86	0·92	1·30	1·17
	Heart and aorta	0·13	0·38	0·43	0·40	0·38	0·63	0·41	0·38
	Heart-fat	0·19	0·16	0·19	0·15	0·22	0·17	0·11	0·20
	Lungs and windpipe	1·11	0·92	1·20	1·06	1·35	1·00	1·17	1·22
	Blood	3·94	3·94	3·81	4·02	4·10	4·31	4·72	4·34
	Liver	1·81	1·77	2·03	1·54	1·78	2·01	1·84	1·75
	Gall-bladder and contents	0·11	0·07	0·13	0·08	0·11	0·09	0·05	0·06
	Pancreas ("sweetbread")	0·15	0·18	0·15	0·18	0·19	0·19	0·18	0·18
	Milt or spleen	0·16	0·13	0·13	0·18	0·16	0·19	0·23	0·17
	Bladder	0·05	0·03	0·06	0·03	0·04	0·04	0·04	0·05
	Head	3·16	2·01	3·04	3·09	3·37	3·37	3·39	3·25
	Skin (with feet, &c.)	8·16	6·26	9·06	8·46	9·07	8·92	7·91	8·33
	Wool previously shorn	6·49	5·01	5·26	4·76	5·02	4·79	6·12	5·23
	Miscellaneous trimmings	0·26	0·08	0·00	0·24	0·11	0·19	0·18	0·13
Total "offal" parts		41·80	38·93	42·72	40·93	41·08	42·58	44·08	43·97
Carcass		57·77	61·07	57·28	59·07	58·69	57·42	53·67	56·03
Loss by evaporation, error in weighing, &c.		0·34	0·00†	0·00†	0·00†	0·23	0·00†	2·25	0·00†
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

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* For particulars of the *feeding* experiment, see Article on the "Comparative Fattening Qualities o

† In those cases the sum of the weights taken for the separated parts exceeded the Fasted Live

Organs, and other separated Parts, in the Fasted Live-weight, of SHEEP.

in *moderately* Fat condition. Fattening food—Oilcake, Clover Chaff, and Swedish Turnips*.
April 23, 1853.

The 8 giving the Medium amount of Increase during Fattening.								Means of—			
No. 9.	No. 10.	No. 11.	No. 12.	No. 13.	No. 14.	No. 15.	No. 16.	The 4 of Largest Increase.	The 4 of Smallest Increase.	The 8 of Medium Increase.	The 16 Cross-bred Wethers.
2·64	2·70	2·38	2 32	2·61	2·53	2·02	2·62	2·72	2·71	2·59	2·67
3·87	4·49	4·61	4·89	2·90	4·36	5·77	2·81	5·05	4·98	4·21	4·75
4·45	5·28	4·19	4·12	3·49	3·07	3·48	4·42	3·86	4·14	4·06	4·02
1·32	2·20	1·50	1·66	1·03	1·37	1 93	1·73	1·63	1·93	1·71	1·76
1·66	1·84	1·66	1·87	1·97	2·28	2·08	1·85	2·06	2·26	1·98	2·10
1·74	1·98	1·45	1·89	1·27	0·81	1·10	1·68	1·23	1·06	1·49	1·26
0·44	0·30	0·40	0·40	0·40	0·42	0·41	0·47	0·41	0·43	0·42	0·42
0 22	0·25	0·26	0·14	0·36	0·16	0·32	0·22	0·17	0·17	0·24	0·19
0·98	1·00	0·96	0·96	1·24	1·09	1·13	1·04	1·09	1·19	1·05	1·11
4·38	4·10	3·67	3·80	3·61	4·00	4·41	4·20	4·08	4·38	4·03	4·16
1·81	1·84	1·71	1·78	1·78	1·67	1·74	1·93	1·79	1·85	1·78	1·81
0·09	0·06	0·11	0·09	0·04	0·04	0·10	0·06	0·10	0·07	0·07	0·08
0·13	0·21	0·18	0·15	0·18	0·15	0·20	0·16	0·16	0·18	0·17	0·17
0·18	0·19	0·16	0 14	0·17	0·14	0·14	0·15	0·15	0·19	0·15	0·16
0·03	0·04	0·04	0·03	0·04	0·03	0·04	0·04	0·04	0·04	0·04	0·04
3·13	3·09	3·00	2·90	3·13	3·37	3·08	3·26	3·05	3·35	3·12	3·17
7·34	7·72	7·76	8·14	8·51	9·53	8·80	7·60	7·99	8·56	8·18	8·25
3·72	3·77	5·59	4·91	4·18	6·74	5·13	5·13	5·39	5·29	4·00	5·19
0·12	0·09	0·13	0·05	0·14	0·14	0·16	0·09	0·19	0·15	0·11	0·14
38·25	41·18	39·76	40·33	37·95	41·00	43·54	39·48	41·16	42·08	40·30	41·45
61·75	57·79	59·63	58·96	61·46	55·37	54·68	60·52	58·80	56·45	58·77	58·01
0·00†	1·03	0·61	0·71	0·59	2·73	1·78	0·00†	0·04	0·62	0·93	0·54
100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100 00	100·00	100·00

different Breeds of Sheep," Journal of the Royal Agricultural Society of England, vol. xvi. part 1.
weight, and this *sum* has been taken as the standard, in calculating the Percentages.

APPENDIX.—TABLE XLV. Showing the *Percentage Proportion* of the individual

CLASS V.—16 Ewe Sheep of *Cross-breed* (Leicester and South Down), about 1½ year old, in
Killed at Rothamsted.

	Designation of Parts.	The 4 giving the Largest amount of Increase during Fattening.				The 4 giving the Smallest amount of Increase during Fattening.			
		No 1.	No 2.	No. 3	No. 4.	No. 5	No. 6	No. 7	No. 8
Separate parts of the "offal."	Stomachs	2·41	2·32	2·61	2·35	2·88	2·61	2·16	2·45
	Contents of stomachs	4·36	2·61	4·08	4·89	3·66	2·67	1·59	1·06
	Caul-fat	4·70	4·04	1·93	3·51	1·63	5·33	3·17	3·95
	Small intestines and contents	1·52	1·70	1·71	1·85	1·55	1·50	1·75	1·93
	Large intestines and contents	1·33	1·91	1·71	1·55	1·99	1·17	2·08	2·11
	Intestinal fat	1·16	1·01	2·01	1·52	1·55	1·80	1·26	0·92
	Heart and aorta	0·11	0·43	0·13	0·45	0·11	0·14	0·30	0·40
	Heart-fat	0·22	0·10	0·18	0·12	0·25	0·17	0·14	0·12
	Lungs and windpipe	1·05	0·80	0·93	0·90	1·04	0·94	1·20	1·41
	Blood	4·38	4·18	4·03	3·50	4·85	4·22	4·10	3·75
	Liver	1·93	1·81	1·94	1·72	1·58	1·67	1·72	1·67
	Gall-bladder and contents	0·09	0·06	0·02	0·10	0·08	0·10	0·04	0·05
	Pancreas ("sweetbread")	0·17	0·16	0·17	0·15	0·14	0·17	0·17	0·17
	Milt or spleen	0·15	0·15	0·21	0·15	0·17	0·11	0·11	0·15
	Bladder	0·02	0·04	0·03	0·03	0·03	0·05	0·04	0·05
	Womb	0·03	0·06	0·05	0·05	0·07	0·01	0·01	0·04
	Head	2·93	3·09	3·03	2·97	3·02	3·08	3·01	2·91
	Skin (with feet, &c.)	8·52	8·07	7·20	7·98	7·70	7·33	7·87	8·76
	Wool previously shorn	6·00	5·70	5·21	5·70	5·99	4·80	5·57	5·04
	Miscellaneous trimmings	0·14	......	0·12	0·22	0·11	......	......	0·28
Total "offal" parts		41·58	38·39	40·60	39·92	41·70	38·38	40·01	41·10
Carcass		58·42	59·78	59·00	57·29	57·98	59·89	59·51	57·81
Loss by evaporation, error in weighing, &c.		0·00†	1·83	0·40	2·79	0·28	1·73	0·48	1·09
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

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* For particulars of the *feeding* experiment, see Article on the "Comparative Fattening Qualities of

† In this case the sum of the weights taken for the separated parts exceeded the Fasted Live-weight,

Organs, and other separated Parts, in the Fasted Live-weight, of SHEEP.

moderately Fat condition. Fattening food—Oilcake, Clover Chaff, and Swedish Turnips*.
April 23, 1853.

The 8 giving the Medium amount of Increase during Fattening.								Means of—			
No. 9	No. 10.	No. 11	No. 12.	No. 13.	No. 14.	No. 15.	No. 16.	The 4 of Largest Increase.	The 4 of Smallest Increase.	The 8 of Medium Increase.	The 16 Cross-bred Ewes.
2·61	2·61	2·51	2·31	2·43	2·56	2·45	2·02	2 42	2·60	2·51	2·51
5·49	4·83	4·08	3·64	3·12	3·85	4·19	2·05	3·99	3·75	4·02	3·92
4·81	4·35	4·77	4·44	4·77	4·47	3·78	4·21	4·32	4·34	4·45	4·37
1·80	1·69	1·90	1·69	1·31	1·72	1·69	1·79	1·71	1·68	1·70	1·70
1·99	1·98	1·93	1·78	1·76	2·14	1·74	1·85	1·63	1·84	1·90	1·79
1·12	1·13	1·11	1·74	0·96	1·87	1·81	1·35	1·42	1·40	1·39	1·40
0·42	0·34	0·37	0·40	0·45	0·38	0·42	0·42	0·43	0·41	0·40	0·41
0·11	0·14	0·21	0·21	0·20	0·22	0·22	0·16	0·16	0·17	0·19	0·17
1·02	1·01	1·00	0·89	1·12	0·99	0·99	0·92	0 94	1·15	0·99	1·03
4·18	3·48	4·14	3·40	4·43	4·10	3·98	4·08	4·05	4·23	3·97	4·09
1·59	1·62	1·82	1·64	1·80	1 67	1·79	1·72	1·85	1·66	1·72	1·75
0·08	0·06	0·11	0·06	0·10	0·05	0·06	0·08	0·07	0·07	0·07	0·07
0·18	0·14	0·14	0·19	0·16	0·15	0·16	0·17	0·16	0·16	0·16	0·16
0·21	0·14	0·16	0·17	0·18	0·14	0·15	0·18	0·16	0·15	0·17	0·16
0·03	0·02	0·03	0·04	0·04	0·03	0·03	0·04	0 03	0·04	0·03	0·03
0·08	0·05	0·04	0·05	0·06	0·07	0·05	0·05	0·05	0·05	0·06	0·06
3·34	3·00	2·98	2·80	3·12	2·96	3·17	2·91	3·00	3·01	3·05	3·02
8·46	8·11	8·12	7·48	8·75	8·75	7·71	8·21	7·94	7·93	8·19	8·02
6·37	5·02	6·38	4·84	6·13	6·10	5·21	6·86	5·67	5·60	5·80	5·69
0·23	0·08	0·16	0·11	0·11	0·09	0·14	0·08	0·16	0·17	0·13	0·14
44·12	39·89	42·81	37·33	41·18	42 37	39·74	39·75	40·16	40·41	40 90	40·48
54·86	58·47	56·99	61·34	58·67	56·56	59·70	59·51	58·62	58·79	58·26	58·56
1·02	1 64	0·20	1·33	0·15	1·07	0·56	0·74	1·22	0 80	0·84	0·96
100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

different Breeds of Sheep," Journal of the Royal Agricultural Society of England, vol. xvi. part 1.
and this *sum* has been taken as the standard, in calculating the Percentages.

APPENDIX.—TABLE XLVI. Showing the *Percentage Proportion* of the individual

CLASS VI.—10 Wether Sheep of *Hampshire Down Breed*, about 1¼ year old, in

Killed at Rothamsted,

	Designation of Parts	The 1 giving the Largest amount of Increase during Fattening.				The 1 giving the Smallest amount of Increase during Fattening			
		No. 1	No. 2	No. 3.	No. 4.	No. 5.	No. 6	No. 7	No. 8
Separate parts of the "offal."	Stomachs	2·44	2·28	2·13	2·20	2·28	2·58	3·31	2·23
	Contents of stomachs	4·11	5·07	5·18	5·43	5·50	1·53	1·70	4·40
	Caul-fat	4·01	4·37	3·70	3·93	5·75	4·01	5·33	4·06
	Small intestines and contents	1·07	1·92	1·68	1·57	1·82	1·80	1·60	1·49
	Large intestines and contents	2·10	1·46	1·46	1·60	1·66	1·82	1·83	1·73
	Intestinal fat	2·48	2·49	2·72	2·12	2·57	1·61	3·39	2·94
	Heart and aorta	0·40	0·35	0·35	0·31	0·38	0·38	0·37	0·35
	Heart-fat	0·33	0·23	0·28	0·19	0·15	0·20	0·23	0·17
	Lungs and windpipe	0·93	0·88	0·71	0·83	1·01	1·29	0·97	0·89
	Blood	4·17	4·45	4·02	4·01	4·26	4·72	3·37	4·14
	Liver	1·72	1·75	1·35	1·70	1·57	1·82	1·43	1·35
	Gall-bladder and contents	0·04	0·02	0·03	0·03	0·06	0·03	0·01	0·01
	Pancreas ("sweetbread")	0·12	0·12	0·10	0·14	0·19	0·18	0·16	0·12
	Milt or spleen	0·14	0·16	0·14	0·11	0·11	0·19	0·14	0·13
	Bladder	......	0·09†	0·08†	0·03	0·02	0·02	0·04	0·03
	Head	3·09	2·93	2·93	2·65	2·98	3·46	3·05	2·84
	Skin (with feet, &c.)	8·74	8·51	8·21	8·44	7·29	8·17	8·90	8·00
	Wool previously shorn	2·72	2·67	3·36	4·19	3·07	4·00	2·50	4·05
	Miscellaneous trimmings	0·15	0·06	0·07	0·13	0·18	0·22	0·16	0·14
Total "offal" parts		40·38	39·81	38·50	40·12	41·81	41·21	38·49	40·03
Carcass		59·59	60·19	61·50	59·87	57·53	57·10	60·60	59·37
Loss by evaporation, error in weighing, &c.		0·03	0·00‡	0·00‡	0·01	0·66	1·69	0·91	0·60
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

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* For particulars of the *feeding* experiment, see Article on the "Comparative Fattening

† In each of these cases the weight of *one* Testicle is included with that of the Bladder.

‡ In these cases the sum of the weights taken for the separated parts exceeded the Fasted

Organs, and other separated Parts, in the Fasted Live-weight, of SHEEP.

moderately Fat condition. Fattening food—Oilcake, Clover Chaff, and Swedish Turnips*.

May 8, 1851.

The 8 giving the Medium amount of Increase during Fattening								Means of—			
No. 9.	No. 10	No. 11	No. 12	No. 13.	No. 14.	No. 15.	No. 16.	The 4 of Largest Increase.	The 4 of Smallest Increase.	The 8 of Medium Increase.	The 16 Hampshire Downs
2·30	1·97	2·17	2·02	2·10	2·24	2·12	2·21	2·28	2·33	2·15	2·23
5·12	5·77	4·69	4·90	3·65	4·49	4·87	4·50	5·02	4·82	4·75	4·83
4·88	3·79	4·31	5·55	4·96	3·98	3·67	4·59	4·01	4·80	4·47	4·43
1·71	1·73	2·74	1·73	1·72	2·00	1·55	1·90	1·78	1·69	1·89	1·81
1·71	2·24	1·83	1·87	2·30	1·60	2·12	1·90	1·68	1·76	1·96	1·84
2·99	2·51	3·13	2·97	2·58	2·24	2·38	3·39	2·53	2·60	2·77	2·67
0·47	0·42	0·36	0·35	0·46	0·36	0·43	0·39	0·36	0·37	0·41	0·38
0·12	0·26	0·21	0·17	0·35	0·21	0·20	0·16	0·26	0·19	0·21	0·22
1·06	0·80	1·01	0·82	0·91	1·09	0·89	0·96	0·84	1·05	0·93	0·91
4·08	4·05	4·33	3·60	4·59	3·79	4·11	4·23	4·23	4·12	4·10	4·08
1·65	1·07	1·52	1·49	1·45	1·58	1·71	1·68	1·64	1·61	1·52	1·62
0·02	0·09	0·08	0·11	0·05	0·05	0·05	0·03	0·03	0·04	0·06	0·05
0·16	0·12	0·13	0·12	0·12	0·18	0·14	0·12	0·12	0·17	0·14	0·14
0·18	0·14	0·10	0·16	0·17	0·15	0·17	0·13	0·14	0·15	0·16	0·15
0·04	0·03	0·04	0·02	0·08	0·09	0·03	0·04	0·07	0·03	0·03	0·04
3·05	2·78	2·93	3·08	3·05	3·19	3·23	3·20	2·90	3·08	3·06	3·02
7·23	8·55	7·77	6·77	7·56	8·25	9·03	7·32	8·47	7·59	7·81	7·02
3·26	3·39	4·27	3·17	3·45	3·85	3·10	2·74	3·23	3·00	3·40	3·49
0·07	0·13	0·11	0·10	0·15	0·16	0·11	0·27	0·10	0·15	0·14	0·14
40·10	39·84	41·80	39·01	39·80	39·39	39·91	39·76	39·69	40·38	39·96	40·00
59·35	59·04	57·78	60·65	59·22	60·38	60·09	60·24	60·29	59·55	59·59	59·53
0·55	1·12	0·42	0·34	0·95	0·23	0·00‡	0·00‡	0·02	0·07	0·45	0·47
100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

Qualities of Sheep," Journal of the Royal Agricultural Society of England, vol. xii. part 2.

Live-weight, and this sum has been taken as the standard, in calculating the Percentages.

Appendix.—Table XLVII. Showing the *Percentage Proportion* of the

Class VII.—16 Wether Sheep of *Sussex Down Breed*, about 1½ year old, in

Killed at Rothamsted.

Designation of Parts		The 4 giving the Largest amount of Increase during Fattening				The 4 giving the Smallest amount of Increase during Fattening			
		No. 1.	No. 2	No. 3.	No. 4	No. 5	No. 6.	No. 7	No. 8
Separate parts of the "offal."	Stomachs	2·43	2·02	2·27	2·25	2·21	2·17	2·38	2·20
	Contents of stomachs	3·70	3·11	5·31	5·21	3·30	1·29	5·18	5·90
	Caul-fat	4·48	4·08	4·76	4·72	5·21	3·84	5·46	1·40
	Small intestines and contents	2·31	1·82	2·09	2·11	1·19	2·15	2·72	2·15
	Large intestines and contents	1·47	1·61	1·72	1·68	1·60	2·09	1·90	1·88
	Intestinal fat	2·01	2·80	2·30	2·17	3·08	1·88	2·57	2·21
	Heart and aorta	0·38	0·38	0·43	0·40	0·43	0·40	0·39	0·12
	Heart-fat	0·40	0·42	0·38	0·21	0·30	0·13	0·24	0·20
	Lungs and windpipe	0·87	0·80	0·90	0·98	0·80	1·10	1·10	1·01
	Blood	5·64	4·02	4·34	4·15	3·85	4·06	3·98	3·95
	Liver	1·43	1·50	1·54	1·59	1·63	1·70	1·68	1·70
	Gall-bladder and contents	0·02	0·03	0·03	0·01	0·05	0·01	0·02	0·01
	Pancreas ("sweetbread")	0·11	0·17	0·18	0·17	0·13	0·13	0·16	0·21
	Milt or spleen	0·18	0·24	0·27	0·22	0·20	0·20	0·26	0·32
	Bladder	0·03	0·04	0·02	0·02	0·03	0·03	0·05	0·01
	Head	2·68	2·97	3·20	3·11	3·11	3·26	3·27	3·19
	Skin (with feet, &c.)	7·80	0·77	7·53	6·83	8·10	7·73	7·07	7·90
	Wool previously shorn	3·98	4·29	4·72	3·50	4·48	3·86	5·24	3·86
	Miscellaneous trimmings	0·09	0·10	0·04	0·16	0·24	0·27	0·12	0·18
Total "offal" parts		40·91	37·32	42·21	39·61	40·56	40·11	49·86	41·97
Carcass		59·09	62·29	57·79	60·39	58·32	58·72	50·14	57·83
Loss by evaporation, error in weighing, &c.		0·00†	0·39	0·00†	0·00†	1·12	1·17	0·00†	0·20
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

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* For particulars of the *feeding* experiment, see Article on the "Comparative Fattening

† In these cases the sum of the weights taken for the separated parts exceeded the Fasted

individual Organs, and other separated Parts, in the Fasted Live-weight, of SHEEP.

moderately Fat condition. Fattening Food—Oilcake, Clover Chaff, and Swedish Turnips*.

May 8, 1851.

The 8 giving the Medium amount of Increase during Fattening								Means of —			
No. 9.	No. 10.	No. 11.	No. 12.	No. 13.	No. 14.	No. 15.	No. 16.	The 4 of Largest Increase.	The 4 of Smallest Increase.	The 8 of Medium Increase.	The 16 Sussex Downs
2·06	2·11	2·07	2·14	2·40	2·20	2·27	2·22	2·24	2·32	2·18	2·23
4·57	3·26	4·14	4·03	4·03	5·87	4·46	3·88	4·35	4·07	4·30	4·46
4·88	5·15	5·56	5·16	3·75	4·29	5·36	4·69	4·51	4·74	4·85	4·74
2·33	1·76	1·08	2·23	2·11	1·03	1·55	1·66	2·08	2·13	1·91	2·02
1·70	1·50	1·74	1·86	2·31	2 48	1·82	1·66	1·62	1·80	1·88	1·82
2·24	3·81	3·13	2·16	2·16	2·06	2·77	2·16	2 57	2 41	2·60	2·55
0·42	0·42	0·34	0·38	0·41	0·41	0 35	0·37	0·40	0·41	0·39	0·38
0·15	0·08	0·19	0·21	0·34	0 18	0·23	0·28	0·35	0 29	0·21	0·27
1·23	1·14	0·93	0·86	1·08	1·02	0·89	1·09	0 89	1·05	1·03	1·09
4·34	3·81	3·89	4·15	4·27	4·30	4·02	3·59	4·53	3 96	4·05	4·18
1·81	1·69	1·67	1·49	1·83	1·45	1·42	1·89	1·53	1·72	1·66	1·64
0·05	0·02	0·08	0·06	0·09	0·12	0·12	0·14	0·03	0·03	0·09	0·06
0·16	0·12	0·16	0·14	0 11	0·19	0·14	0·15	0·16	0·16	0·15	0·15
0·27	0·21	0·28	0·17	0·30	0·21	0·18	0·21	0·23	0·29	0·24	0·24
0·02	0·03	0·03	0·03	0 02	0·03	0·02	0·04	0·03	0·04	0·03	0·03
2·89	2·66	2·92	2·74	3·06	3·07	2·93	3·26	3·01	3·21	2·94	3·03
8·78	7·30	7·53	7·53	8·17	7·06	7·27	7·07	7·23	7·72	7·67	7·58
3·94	3 52	4·33	4·63	4·99	3·85	4·00	3·88	4·14	4·30	4·17	4·21
0 09	0·13	0·19	0·16	0·14	0·13	0·21	0·19	0·11	0·20	0 15	0·16
41·93	38·81	41·16	41·23	41·57	40·91	40·01	39 33	40·01	41·63	40 62	40·71
57·44	60·50	58·62	58·34	58·43	58·56	59·52	60·67	59·89	57·75	59·01	58·92
0·63	0·69	0·22	0·43	0·00†	0·53	0·47	0·00†	0·10	0·62	0·37	0·37
100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

Qualities of Sheep," Journal of the Royal Agricultural Society of England, vol. xii. part 2.
Live-weight, and this sum has been taken as the standard, in calculating the Percentages.

APPENDIX.—TABLE XLVIII. Showing the *Percentage Proportion* of the individual Organs, and other separated Parts, in the Fasted Live-weight, of SHEEP.

CLASS VIII.—6 Wether Sheep of *Cotswold Breed*, about 1¾ year old, in *excessively* Fattened condition. Fattening food—Oilcake, Clover Hay, and Swedish Turnips, under cover, until "moderately" fattened; afterwards, Oilcake (or Lentils), with Grass or Green Clover, or with Meadow-hay and Roots, in the field. Killed at Rothamsted, December 18, 1852.

	Designation of Parts.	6 Very Fat Cotswold Sheep						Means of the 6 Cotswolds.
		No 1	No 2	No 3	No 4	No 5	No 6	
Separate parts of the "offal."	Stomachs	2·16	1·91	1·88	1·85	1·95	1·82	1·93
	Contents of stomachs	4·44	3·37	4·47	3·68	3·40	....	3·87
	Caul-fat	3·22	3·93	2·81	5·55	3·90	3·49	3·83
	Small intestines and contents	1·38	1·30	1·14	0·97	1·28	1·42	1·25
	Large intestines and contents	1·98	1·79	1·47	1·45	1·48	1·59	1·63
	Intestinal fat	1·94	1·59	1·35	1·36	0·95	1·45	1·44
	Heart and aorta	0·32	0·38	0·34	0·31	0·36	0·37	0·35
	Heart-fat	0·40	0·37	0·39	0·28	0·20	0·30	0·32
	Lungs and windpipe	0·82	0·89	0·77	0·71	0·94	0·81	0·82
	Blood	4·04	4·21	3·56	3·35	3·60	3·88	3·77
	Liver	1·56	1·40	1·29	1·10	1·16	1·17	1·28
	Gall-bladder and contents	0·045	0·097	0·035	0·024	0·045	0·036	0·045
	Pancreas ("sweetbread")	0·09	0·07	0·12	0·07	0·09	0·08	0·09
	Milt or spleen	0·16	0·14	0·13	0·17	0·11	0·16	0·14
	Bladder	0·02	0·02	0·03	0·02	0·02	0·03	0·02
	Head	2·17	2·13	2·17	2·24	2·54	2·25	2·25
	Skin and wool * (with feet, &c.)	11·76	9·98	11·47	11·16	11·72	11·31	11·22
	Diaphragm ("skirts")	0·03	0·11	0·12	0·05	0·10	0·07	0·09
	Miscellaneous trimmings	......	....	....		0·15	0·08	0·11
Total "offal" parts		36·58	33·65	33·57	34·37	34·00	36·32	34·45
Carcass		62·96	63·99	64·53	64·69	64·80	63·50	64·05
Loss by evaporation, error in weighing, &c.		0·46	2·35	1·90	0·94	1·20	0·18	1·47
		100·00	100·00	100·00	100·00	100·00	100·00	100·00

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* In the case of these animals killed in December, the wool [illegible] of the shorn wool, see Appendix-Table XXIII.

APPENDIX.—TABLE XLIX. Showing the *Percentage Proportion* of the individual Organs, and other separated Parts, in the Fasted Live-weight, of SHEEP.

CLASS IX.—7 Wether Sheep of *Leicester Breed*, about 1¾ year old, in *excessively* Fattened condition. Fattening food—Oilcake, Clover Hay, and Swedish Turnips, under cover, until "moderately" fattened; afterwards, Oilcake, with Grass or Green Clover, or with Meadow-hay and Roots, in the field. Killed at Rothamsted, December 20, 1853.

	Designation of Parts.	7 Very Fat Leicester Sheep.							Means of the 7 Leicesters.
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	
Separate parts of the "offal."	Stomachs	2·06	2·34	2·36	2·35	2·49	2·22	2·17	2·28
	Contents of stomachs	2·49	3·73	4·84	3·68	4·48	3·70	3·17	3·73
	Caul-fat	3·78	3·22	3·95	3·13	6·82	3·90	5·55	4·34
	Small intestines and contents	1·42	1·46	1·50	0·88	1·63	1·17	1·27	1·33
	Large intestines and contents	1·78	1·68	1·77	1·76	1·99	1·61	1·44	1·72
	Intestinal fat	1·72	1·62	1·71	0·79	1·70	1·61	1·37	1·50
	Heart and aorta	0·35	0·42	0·38	0·33	0·44	0·33	0·31	0·37
	Heart-fat	0·37	0·27	0·56	0·29	0·32	0·39	0·47	0·38
	Lungs and windpipe	0·79	0·78	0·91	0·86	0·91	0·79	0·82	0·84
	Blood	3·66	4·14	3·50	3·94	3·87	3·56	3·49	3·74
	Liver	1·40	1·59	1·43	1·32	1·60	1·31	1·25	1·41
	Gall-bladder and contents	0·07	0·07	0·06	0·05	0·06	0·05	0·07	0·06
	Pancreas ("sweetbread")	0·09	0·09	0·06	0·08	0·06	0·06	0·08	0·07
	Milt or spleen	0·19	0·13	0·12	0·13	0·12	0·13	0·14	0·14
	Bladder	0·04	0·04	0·04	0·04	0·04	0·03	0·03	0·04
	Head	2·56	2·45	2·36	2·50	2·43	2·43	2·29	2·43
	Skin and wool * (with feet, &c.)	11·51	9·50	11·79	10·88	11·22	10·35	10·56	10·83
	Diaphragm ("skirts")	......	......	......	......	......	0·08	......	0·08
	Miscellaneous trimmings	0·05	......	......	......	......	......	......	0·05
Total "offal" parts		34·33	33·53	37·34	33·01	40·18	33·72	34·48	35·34
Carcass		65·64	65·59	62·90	66·43	60·33	66·72	64·47	64·58
Loss by evaporation, error in weighing, &c.		0·03	0·88	—0·24	0·56	—0·51	—0·44	1·05	0·08
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

* In the case of these animals killed in December, the wool shorn in the previous Spring is not included in the calculations. For the *actual weights* of the shorn wool, see Appendix-Table XXIV.

APPENDIX.—TABLE L. Showing the *Percentage Proportion* of the individual Organs, and other separated Parts, in the Fasted Live-weight, of SHEEP

CLASS X.—8 Wether Sheep of *Cross-breed* (Leicester and South Down), about 1¾ year old, in *excessively* Fattened condition. Fattening food—Oilcake, Clover Hay, and Swedish Turnips, under cover, until "moderately" fattened; afterwards, Oilcake, with Grass or Green Clover, or with Meadow-hay and Roots, in the field. Killed at Rothamsted, December 20, 1858.

	Designation of Parts.	8 Very Fat Cross-bred Wether Sheep (Leicester and South Down)								Means of the 8 Cross-bred Wethers
		No. 1.	No. 2.	No. 3.	No. 4.	No 5.	No. 6.	No. 7.	No 8	
Separate parts of the "offal."	*Stomachs*	2·20	2·21	2·23	2·06	2·60	2·13	2·27	2·21	2·24
	Contents of stomachs	4·15	4·34	5·22	3·83	4·20	3·34	2·27	3·29	3·89
	Caul-fat	4·32	5·54	4·64	6·17	4·45	4·31	5·32	3·72	4·80
	Small intestines and contents	1·16	1·07	1·10	0·93	1·08	1·07	1·21	1·09	1·09
	Large intestines and contents	1·73	1·32	1·99	1·54	1·79	1·49	1·64	1·46	1·62
	Intestinal fat	1·70	2·11	2·04	2·45	1·34	1·10	2·19	1·39	1·79
	Heart and aorta	0·31	0·36	0·31	0·31	0·33	0·34	0·33	0·36	0·33
	Heart-fat	0·58	0·35	0·46	0·46	0·39	0·50	0·29	0·53	0·44
	Lungs and windpipe	0·79	0·86	0·82	0·75	0·87	0·81	0·88	0·76	0·82
	Blood	3·86	3·79	3·21	3·56	4·13	3·65	3·80	3·48	3·68
	Liver	1·27	1·27	1·20	1·28	1·32	1·33	1·31	1·29	1·28
	Gall-bladder and contents	0·06	0·05	0·05	0·05	0·09	0·07	0·07	0·04	0·06
	Pancreas ("sweetbread")	0·10	0·09	0·10	0·09	0·13	0·10	0·08	0·05	0·09
	Milt or spleen	0·13	0·13	0·14	0·13	0·15	0·14	0·17	0·15	0·14
	Bladder	0·04	0·04	0·03	0·04	0·04	0·04	0·03	0·06	0·04
	Head	2·37	2·43	2·51	2·18	2·70	2·50	2·56	2·31	2·45
	Skin and wool * (with feet, &c.)	8·80	10·29	9·89	7·85	11·09	10·37	10·52	9·51	9·83
	Miscellaneous trimmings	0·15	0·15	......	0·07	0·04	......	......	0·07	0·10
	Total "offal" parts	33·72	36·30	35·94	33·75	36·74	33·79	35·23	31·77	34·69
	Carcass	66·55	63·31	61·95	66·08	62·73	65·78	64·25	68·02	64·83
	Loss by evaporation, error in weighing, &c.	—0·27	0·39	2·11	0·17	0·53	0·43	0·52	0·21	0·48
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

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* In the case of these animals killed in December, the wool shorn in the previous spring is not included in the account. For the actual weights of the shorn wool, see Appendix-Table XXV.

APPENDIX.—TABLE LI. Showing the *Percentage Proportion* of the individual Organs, and other separated Parts, in the Fasted Live-weight, of SHEEP.

CLASS XI.—8 Ewe Sheep of *Cross-breed* (Leicester and South Down), about 1¾ year old, in *excessively* Fattened condition. Fattening food—Oilcake, Clover Hay, and Swedish Turnips, under cover, until "moderately" fattened; afterwards, Oilcake, with Grass or Green Clover, or with Meadow-hay and Roots, in the field. Killed at Rothamsted, December 20, 1853.

	Designation of Parts.	8 Very Fat Cross-bred Ewe Sheep (Leicester and South Down).								Means of the 8 Cross-bred Ewes.
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6	No 7.	No. 8.	
Separate parts of the "offal."	Stomachs	2·33	2·62	2·01	2·03	2·54	2·13	2·19	2·49	2·29
	Contents of stomachs	3·25	3·22	3·20	3·22	4·50	2·38	3·53	3·48	3·35
	Caul-fat	5·26	5·31	5·23	4·04	4·82	4·23	5·03	5·27	4·90
	Small intestines and contents	1·31	1·05	1·23	1·34	1·33	1·70	1·42	0·92	1·29
	Large intestines and contents	1·76	1·57	1·71	1·73	1·90	1·62	1·50	2·06	1·73
	Intestinal fat	2·28	2·99	2·18	1·55	2·03	2·29	1·87	1·49	2·08
	Heart and aorta	0·32	0·41	0·33	0·36	0·34	0·37	0·38	0·37	0·36
	Heart-fat	0·62	0·56	0·41	0·31	0·40	0·53	0·38	0·64	0·48
	Lungs and windpipe	0·76	0·92	0·72	1·00	0·73	0·80	0·79	0·89	0·83
	Blood	3·43	3·63	3·74	3·78	3·76	4·88	3·70	4·09	3·88
	Liver	1·27	1·48	1·26	1·44	1·40	1·48	1·44	1·31	1·38
	Gall-bladder and contents	0·05	0·09	0·05	0·10	0·12	0·04	0·06	0·09	0·07
	Pancreas ("sweetbread")	0·07	0·07	0·05	0·08	0·10	0·09	0·07	0·08	0·08
	Milt or spleen	0·14	0·15	0·11	0·15	0·13	0·14	0·13	0·12	0·13
	Bladder	0·03	0·04	0·03	0·04	0·03	0·05	0·03	0·04	0·04
	Head	2·36	2·83	2·44	2·68	2·55	2·58	2·52	2·52	2·56
	Skin and wool * (with feet, &c.)	10·10	10·93	10·27	12·04	12·18	11·22	10·71	10·80	11·03
	Womb	0·09	0·09	0·09	0·10	0·08	0·07	0·07	0·11	0·09
	Miscellaneous trimmings	......	......	......	......	......	......	0·14	......	0·14
	Total "offal" parts	35·43	37·96	35·06	35·99	38·94	36·60	35·96	36·77	36·71
	Carcass	64·83	62·80	65·27	65·11	60·56	66·11	65·28	64·36	64·29
	Loss by evaporation, error in weighing, &c.	—0·26	—0·76	—0·33	—1·10	0·50	—2·71	—1·24	—1·13	—1·00
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

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* In the case of these animals killed in December, the wool shorn in the previous Spring is not included in the calculations. For the *actual weights* of the shorn wool, see Appendix-Table XXVI.

APPENDIX.—TABLE LII Showing the *Percentage Proportion* of the individual Organs, and other separated Parts, in the Fasted Live-weight, of SHEEP

CLASS XII —8 Wether Sheep of *Hampshire Down Breed*, about 1¾ year old, in *excessively* Fattened condition. Fattening food—Oilcake, Clover Hay, and Swedish Turnips, under cover, until "moderately" fattened; afterwards, Oilcake, with Grass or Green Clover, or with Meadow-hay and Roots, in the field. Killed at Rothamsted, December 17, 1851

	Designation of Parts.	8 Very Fat Hampshire Down Sheep								Means of the 8 Hampshire Downs
		No 1.	No 2.	No 3.	No 4	No 5	No 6	No 7	No 8	
Separate parts of the "offal."	Stomachs	2·06	2·07	2·46	2·05	2·14	2·11	2·22	1·76	2·11
	Contents of stomachs	4·13	3·36	3·18	3·95	4·34	3·34	3·67	2·75	3·58
	Caul-fat	8·61	5·05	5·30	5·82	5·57	4·98	7·78	5·01	6·01
	Small intestines and contents	1·16	0·85	1·00	0·88	1·21	1·02	0·61	1·10	0·97
	Large intestines and contents	1·26	0·88	1·51	1·63	1·84	1·54	1·55	1·42	1·45
	Intestinal fat	2·64	2·92	3·40	3·32	2·92	3·13	2·92	3·14	3·05
	Heart and aorta	0·34	0·36	0·36	0·44	0·33	0·31	0·36	0·37	0·35
	Heart-fat	0·26	0·09	0·10	0·20	0·40	0·22	0·33	0·36	0·24
	Lungs and windpipe	0·73	0·80	0·95	0·73	0·81	0·76	0·71	0·85	0·79
	Blood	3·24	3·48	3·40	3·64	3·92	3·84	3·86	3·72	3·64
	Liver	1·09	1·25	1·17	1·24	1·03	1·29	1·08	1·54	1·21
	Gall-bladder and contents	0·06	0·05	0·06	0·07	0·03	0·08	0·09	0·05	0·06
	Pancreas ("sweetbread")	0·16	0·14	0·10	0·13	0·13	0·09	0·15	0·12	0·12
	Milt or spleen	0·13	0·12	0·13	0·12	0·15	0·14	0·13	0·14	0·13
	Bladder	0·02	0·03	0·01	0·03	0·02	0·02	0·03	0·02	0·02
	Head	2·49	2·63	2·51	2·43	2·47	2·33	2·64	2·58	2·51
	Skin and wool * (with feet, &c.)	9·52	11·16	9·65	10·19	10·06	9·70	10·55	9·25	10·00
	Miscellaneous trimmings	......	......	......	......	......	......	......	0·08	0·08
	Total "offal" parts	37·90	35·24	35·29	36·87	37·37	34·90	38·68	34·26	36·32
	Carcass	61·68	63·51	64·12	63·11	62·63	64·56	61·04	65·34	63·25
	Loss by evaporation, error in weighing, &c.	0·42	1·25	0·59	0·02	0·00†	0·54	0·28	0·40	0·43
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

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* In the case of these animals killed in December, the wool shorn in the previous Spring is not included in the calculations. For the actual weights of the shorn wool, see Appendix-Table XXVII.

† In this case the sum of the weights taken for the separated parts exceeded the Fasted Live-weight, and the sum has been taken as the standard, in calculating the Percentages.

APPENDIX.—TABLE LIII. Showing the *Percentage-Proportion* of the individual Organs, and other separated Parts, in the Fasted Live-weight, of SHEEP.

CLASS XIII.—8 Wether Sheep of *Sussex Down Breed*, about 1¾ year old, in *excessively* Fattened condition. Fattening food—Oilcake, Clover Hay, and Swedish Turnips, under cover, until "moderately" fattened; afterwards, Oilcake, with Grass or Green Clover, or with Meadow-hay and Roots, in the field. Killed at Rothamsted, December 17, 1851.

	Designation of Parts.	8 Very Fat Sussex Down Sheep.								Means of the 8 Sussex Downs.
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No 7.	No. 8.	
Separate parts of the "offal."	Stomachs	1·69	1·94	2·02	2·31	2·24	1·95	1·99	2·07	2·03
	Contents of stomachs	3·57	3·92	2·41	3·79	3·51	2·98	2·07	3·82	3·26
	Caul-fat	6·48	6·64	6·65	6·32	5·75	5·32	6·81	4·35	6·04
	Small intestines and contents	1·15	1·21	1·25	1·23	1·40	1·14	1·12	1·00	1·19
	Large intestines and contents	1·33	1·50	1·23	1·12	1·61	1·39	1·30	1·86	1·42
	Intestinal fat	2·71	2·57	3·16	2·14	2·84	3·05	3·57	4·02	3·01
	Heart and aorta	0·35	0·40	0·40	0·36	0·33	0·39	0·41	0·45	0·39
	Heart-fat	0·16	0·22	0·17	0·17	0·24	0·21	0·41	0·28	0·23
	Lungs and windpipe	0·86	0·92	0·89	0·93	0·89	0·89	1·00	0·78	0·89
	Blood	3·77	3·52	3·36	3·76	3·61	3·87	3·69	3·75	3·66
	Liver	1·35	1·47	1·34	1·41	1·47	1·43	1·54	1·38	1·42
	Gall-bladder and contents	0·074	0·037	0·023	0·076	0·058	0·038	0·078	0·076	0·057
	Pancreas ("sweetbread")	0·13	0·13	0·13	0·14	0·12	0·13	0·16	0·14	0·13
	Milt or spleen	0·16	0·18	0·15	0·14	0·16	0·20	0·17	0·19	0·17
	Bladder	0·03	0·02	0·02	0·02	0·02	0·02	0·02	0·02	0·02
	Head	3·04	2·46	2·39	2·57	2·63	2·87	2·61	2·76	2·73
	Skin and wool * (with feet, &c.)	9·84	9·98	10·44	9·60	8·97	9·71	9·35	11·19	9·89
	Total "offal" parts	36·69	37·12	36·54	36·09	35·85	35·59	36·30	38·14	36·54
	Carcass	63·31	62·88	63·37	62·94	64·15	64·41	63·70	61·74	63·31
	Loss by evaporation, error in weighing, &c.	0·00†	0·00†	0·09	0·97	0·00†	0·00†	0·00†	0·12	0·15
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

* In the case of these animals killed in December, the wool shorn in the previous Spring is not included in the calculations. For the *actual weights* of the shorn wool, see Appendix-Table XXVIII.

† In these cases the sum of the weights taken for the separated parts exceeded the Fasted Live-weight, and this *sum* has been taken as the standard in calculating the Percentages.

APPENDIX.—TABLE LIV. Showing the *Percentage Proportion* of the individual

CLASS XIV.—19 Sheep of *Hampshire Down Breed*, divided into 4 Lots, each with different
Experiment 97 days.

	Designation of Parts.	Lot 1.—Food:— Oilcake—in fixed quantity Swedish Turnips—*ad libitum*.					Lot 2.—Food: Oats—in fixed quantity Swedish Turnips—*ad libitum*				
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.	No. 9.	No. 10.
Separate parts of the "offal"	Stomachs	2·28	2·69	2·61	2·82	2·82	2·85	2·81	2·72	2·81	2·91
	Contents of stomachs	5·55	6·77	6·70	9·14	6·70	8·39	9·31	7·65	8·19	8·20
	Caul, intestinal, and heart-fat	8·60	9·51	9·26	7·80	6·20	8·30	8·06	8·51	6·80	6·16
	Small intestines and contents	1·24	1·45	1·13	1·71	2·39	1·13	1·06	1·51	1·23	2·12
	Large intestines and contents	1·03	1·05	1·55	2·00	1·77	1·09	2·15	2·41	2·47	2·28
	Heart and aorta	0·72	0·80	0·76	0·67	0·70	0·88	0·90	0·79	0·89	0·74
	Lungs and windpipe	1·24	1·22	1·10	1·17	1·45	1·33	1·08	1·21	1·31	0·95
	Blood	4·26	3·47	4·27	3·54	3·93	3·80	3·73	4·09	3·81	4·00
	Liver	1·58	1·54	1·71	0·99	1·54	1·30	1·36	1·03	1·42	1·69
	Gall-bladder and contents	0·04	0·06	0·09	0·13	0·03	0·04	0·00	0·18	0·11	0·08
	Head	3·29	3·26	3·19	3·51	3·78	3·10	3·43	3·12	3·91	3·81
	Skin and wool (with feet, &c.)	11·55	11·81	10·63	10·50	13·41	10·19	11·90	10·07	11·61	14·41
Total "offal" parts†		42·32	44·62	43·00	44·16	44·78	43·27	46·71	45·35	44·59	47·61
Carcass		58·94	56·60	55·40	56·61	55·36	59·08	55·17	56·47	57·56	54·24
Loss by evaporation, error in weighing, &c.		−1·26	−1·22	1·60	−0·77	−0·14	−2·35	−1·88	−1·82	−2·15	−1·85
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

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* For particulars of the *feeding* experiment, refer, under the head of "Experiments with Sheep—Series 1,"

† In the case of these animals the Pancreas, Spleen, and Bladder were not weighed.

Organs, and other separated Parts, in the Fasted Live-weight, of SHEEP.

Food*. *Moderately* Fattened. Bred, fed, and killed at Rothamsted.—Period of Feeding
Killed April 11, 1848.

Lot 3.—Food:— Clover Chaff—in fixed quantity. Swedish Turnips—*ad libitum.*					Lot 4.—Food:— Oat-straw Chaff, and Swedish Turnips—each *ad libitum.*				Means of—				
No. 11.	No. 12.	No. 13.	No. 14.	No. 15.	No. 16.	No. 17.	No. 18.	No. 19.	Lot 1.	Lot 2.	Lot 3	Lot 4.	The 19 Sheep
3·17	2·93	3·04	3·17	3·02	2·91	3·07	2·89	2·88	2·64	2·83	3·06	2·94	2·87
13·38	9·35	10·45	10·57	10·63	8·38	8·65	6·90	10·21	6·98	8·34	10·88	8·54	8·69
6·12	7·19	7·82	7·96	7·33	8·96	8·33	9·32	8·32	8·29	7·65	7·28	8·73	7·95
2·88	1·51	1·41	1·53	1·36	1·89	1·04	1·27	2·09	1·58	1·41	1·74	1·70	1·62
3·46	2·69	2·70	2·74	1·81	2·01	2·42	1·86	2·19	1·86	2·20	2·68	2·12	2·22
0·70	0·83	1·11	0·81	0·86	0·85	0·89	0·72	0·64	0·75	0·84	0·88	0·78	0·81
1·23	1·22	1·43	1·30	1·29	1·21	1·40	1·34	1·28	1·24	1·21	1·31	1·31	1·27
4·00	3·70	3·27	3·82	3·81	2·96	3·80	3·75	3·91	3·90	3·91	3·73	3·60	3·70
1·76	1·51	1·82	1·80	1·68	1·50	1·72	1·21	1·65	1·46	1·54	1·71	1·52	1·56
0·02	0·11	0·04	0·03	0·08	0·06	0·00	0·02	0·03	0·07	0·10	0·06	0·05	0·07
2·97	2·15	3·46	3·35	3·45	3·39	3·29	3·72	3·35	3·41	3·53	3·28	3·44	3·41
11·27	10·57	11·07	10·61	11·42	10·85	9·89	12·40	11·51	11·60	11·82	10·90	11·10	11·41
51·05	44·82	47·62	47·78	46·74	44·97	45·49	45·40	48·06	43·78	45·41	47·60	45·98	45·67
50·00	56·99	53·50	53·06	52·76	55·81	55·60	55·78	52·36	56·58	55·60	53·26	54·89	55·33
−1·05	−1·81	−1·12	−0·84	0·50	−0·78	−1·09	−1·18	−0·42	−0·36	−1·01	−0·86	−0·87	−1·00
100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

to Article "Sheep Feeding and Manure," Journal of the Royal Agricultural Society of England, vol. x. part 1.

APPENDIX.—TABLE LV. Showing the *Percentage Proportion* of the individual

CLASS XV.—20 Sheep of *Hampshire Down Breed*, divided into 4 Lots, each with different
Experiment 134 days.

	Designation of Parts.	Lot 1.—Food:— Oilcake in fixed quantity. Clover Chaff *ad libitum*.					Lot 2 Food Linseed in fixed quantity Clover Chaff *ad libitum*				
		No. 1.	No. 2.	No. 3.	No. 4	No. 5.	No. 6.	No. 7.	No. 8	No. 9	No. 10
Separate parts of the "offal."	Stomachs	2·63	3·03	2·88	2·56	2·71	2·55	2·47	2·60	2·33	2·79
	Contents of stomachs	9·32	8·88	9·75	8·55	9·88	5·96	6·58	7·79	7·57	7·30
	Caul-fat	2·94	4·75	4·34	5·42	4·88	6·03	4·98	5·20	4·82	6·12
	Small intestines and contents	1·84	1·85	1·81	1·65	1·52	2·17	1·46	1·77	1·45	1·89
	Large intestines and contents	3·45	2·75	3·34	2·60	2·78	2·13	1·55	3·20	2·38	2·30
	Intestinal fat	1·75	2·44	1·77	1·63	1·57	2·22	1·78	2·39	2·20	1·53
	Heart and aorta	0·40	0·32	0·33	0·40	0·30	0·43	0·42	0·37	0·41	0·40
	Heart-fat	0·28	0·36	0·27	0·46	0·30	0·38	0·39	0·37	0·37	0·45
	Lungs and windpipe	0·80	0·80	0·81	0·86	1·05	0·90	0·87	0·82	0·70	0·99
	Blood	4·45	4·00	3·75	3·92	4·00	3·97	3·97	4·04	3·70	3·60
	Liver	1·28	1·39	1·31	1·32	1·57	1·28	1·10	1·40	1·37	1·30
	Gall-bladder and contents	0·05	0·06	0·03	0·05	0·05	0·08	0·10	0·14	0·06	0·09
	Pancreas ("sweetbread")	0·10	0·12	0·13	0·12	0·05	0·13	0·10	0·15	0·12	0·15
	Milt or spleen	……	0·13	0·10	0·13	0·13	0·13	0·13	0·16	0·14	0·11
	Head	3·23	2·82	3·17	3·28	3·18	3·31	3·22	3·30	3·13	3·42
	Skin and wool (with feet, &c.)	9·80	10·10	11·54	9·60	8·97	8·04	9·40	8·89	9·37	9·45
Total "offal" parts †		41·91	43·80	45·33	42·55	43·61	40·61	38·52	42·59	40·33	41·88
Carcass		58·09	56·21	54·67	57·45	56·39	59·39	61·48	57·41	59·67	58·12
Loss by evaporation, error in weighing, &c.		0·00	—0·01	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

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* For particulars of the *feeding* experiment, refer, under the head of "Experiments with Sheep—Series 2,"

† In the case of these animals the Bladder was not weighed.

Organs, and other separated Parts, in the Fasted Live-weight, of SHEEP.

Food *. *Moderately* Fattened. Bred, fed, and killed at Rothamsted. Period of Feeding
Killed October 17, 1848.

Lot 3. Food:— Barley in fixed quantity. Clover Chaff *ad libitum.*					Lot 4. Food:— Malt in fixed quantity. Clover Chaff *ad libitum.*					Means of				
No. 11.	No. 12.	No. 13.	No. 14.	No. 15.	No. 16.	No. 17.	No. 18.	No. 19.	No. 20.	Lot 1.	Lot 2.	Lot 3.	Lot 4.	The 20 Sheep.
2·94	2·94	3·06	3·01	3·12	2·31	2·56	2·15	2·57	2·90	2·77	2·55	3·07	2·56	2·71
9·35	7·39	10·91	10·02	10·13	8·90	8·23	5·62	7·05	8·90	9·28	7·01	9·62	7·86	8·45
4·26	3·62	4·92	3·79	4·30	3·33	3·34	3·96	3·79	5·60	4·47	5·61	4·18	4·01	4·50
1·19	1·16	1·28	1·31	1·27	1·50	1·51	1·31	1·45	1·31	1·73	1·75	1·31	1·45	1·56
2·59	2·71	3·06	1·96	2·74	2·31	2·17	2·21	2·51	2·08	2·98	2·33	2·61	2·11	2·59
1·62	1·69	1·66	1·70	1·23	2·02	1·97	2·07	1·83	2·07	1·83	2·02	1·58	1·99	1·86
0·31	0·43	0·38	0·38	0·42	0·35	0·41	0·38	0·37	0·39	0·37	0·39	0·38	0·38	0·38
0·49	0·43	0·31	0·42	0·31	0·35	0·39	0·46	0·42	0·39	0·33	0·39	0·33	0·40	0·36
1·12	0·63	0·89	0·78	0·88	0·81	0·96	0·84	0·91	0·91	1·00	0·87	0·80	0·90	0·91
4·21	4·73	3·91	3·75	4·16	4·03	4·12	4·23	4·15	4·28	4·02	3·87	4·16	4·22	4·07
1·27	1·30	1·15	1·27	1·32	1·22	1·28	1·31	1·22	1·56	1·37	1·20	1·26	1·33	1·31
0·04	0·08	0·06	0·13	0·07	0·06	0·05	0·07	0·06	0·08	0·05	0·10	0·08	0·06	0·07
0·12	0·12	0·13	0·12	0·10	0·08	0·12	0·13	0·18	0·15	0·10	0·13	0·12	0·12	0·12
0·10	0·15	0·15	0·11	0·14	0·15	0·13	0·16	0·13	0·14	0·12	0·13	0·14	0·14	0·13
3·20	3·16	3·28	3·35	3·68	3·14	3·52	3·40	3·32	3·20	3·12	3·28	3·33	3·35	3·27
9·21	9·22	9·49	10·58	9·42	8·72	11·12	11·72	10·91	8·08	9·00	9·04	9·59	10·11	9·60
42·08	39·76	44·67	42·71	43·89	39·46	42·18	40·44	41·75	42·76	43·44	40·79	42·62	41·82	42·04
57·92	60·24	56·03	57·29	56·11	60·54	57·82	59·56	58·25	56·54	56·56	59·21	57·52	58·51	57·96
0·00	0·00	−0·70	0·00	0·00	0·00	0·00	0·00	0·00	0·70	0·00	0·00	−0·14	0·14	0·00
100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

Article—"Sheep Feeding and Manure," Journal of the Royal Agricultural Society of England, vol. x. part 1.

APPENDIX.—TABLE LVI. Showing the *Percentage Proportion* of the individual

CLASS XVI.—25 Sheep of *Hampshire Down Breed*, divided into 6 Lots, each with different Food*.

Killed

Designation of Parts.		Lot 1.—Food:— Barley (ground)—in fixed quantity Mangolds—*ad libitum*.				Lot 2. Food: Malt (ground), and Malt-dust in fixed quantity. Mangolds *ad libitum*.				Lot 3 Food Barley (ground and *steeped*) in fixed quantity Mangolds *ad libitum*.			
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.	No. 9.	No. 10.	No. 11.	No. 12.
Separate parts of the "offal."	Stomachs	2·49	2·80	2·80	2·64	2·83	2·77	2·34	2·61	2·65	2·55	2·55	2·55
	Contents of stomachs	3·31	3·32	4·38	3·84	4·21	5·10	4·78	3·40	5·66	4·15	4·17	3·21
	Caul-fat	5·21	3·95	4·43	4·00	4·36	5·30	4·81	5·17	3·12	3·80	5·52	5·53
	Small intestines and contents	1·73	1·88	1·71	1·88	1·46	1·80	1·01	1·10	1·81	1·28	1·38	1·81
	Large intestines and contents	1·64	2·43	2·54	2·12	1·99	2·11	1·76	1·94	2·07	1·54	1·83	1·80
	Intestinal fat	2·20	3·32	2·93	2·73	1·94	2·09	2·80	2·41	2·26	1·71	1·83	2·15
	Heart and aorta	0·40	0·48	0·55	0·38	0·51	0·43	0·41	0·38	0·43	0·43	0·41	0·44
	Heart-fat	0·33	0·35	0·45	0·30	0·61	0·36	0·60	0·60	0·13	0·83	0·43	0·61
	Lungs and windpipe	1·00	1·00	1·30	0·98	0·98	1·11	1·02	1·17	1·08	0·98	0·96	1·19
	Blood	3·01	3·80	4·11	4·34	4·18	3·06	3·89	3·56	4·63	4·10	3·71	4·34
	Liver	1·40	1·50	1·35	1·53	1·52	1·35	1·38	1·24	1·62	1·31	1·30	1·44
	Gall-bladder and contents	0·06	0·03	0·03	0·03	0·06	0·05	0·02	0·08	0·07	0·07	0·04	0·05
	Pancreas ("sweetbread")	0·17	0·20	0·19	0·18	0·18	0·20	0·14	0·19	0·16	0·14	0·21	0·12
	Milt or spleen	0·16	0·13	0·20	0·18	0·18	0·17	0·14	0·11	0·24	0·18	0·20	0·18
	Head	3·38	3·19	3·28	3·11	3·40	3·43	2·90	2·97	3·14	3·14	3·10	3·17
	Skin (with feet, &c.)	7·02	7·63	8·10	7·29	6·36	7·65	7·00	6·84	7·28	7·40	6·90	7·44
	Wool previously shorn	4·80	5·27	4·05	5·87	3·89	5·01	7·20	6·87	5·46	4·52	5·43	5·27
Total "offal" parts†		39·48	40·78	42·55	41·89	38·66	43·21	42·44	41·24	42·11	38·69	40·33	41·60
Carcass		59·68	59·43	56·73	57·24	60·51	54·69	55·35	57·65	57·21	59·50	57·21	57·82
Loss by evaporation, error in weighing, &c.		0·84	−0·21	0·72	0·87	0·83	2·10	2·21	1·11	0·68	1·81	2·46	0·58
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

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* For particulars of the *feeding* experiment, refer, under the head of "Experiments with Sheep—Series 4," to Article—"Sheep Feeding and Manure," Journal o

† In the case of these animals the Bladder was not weighed.

Organs, and other separated Parts, in the Fasted Live-weight, of SHEEP.

Moderately Fattened. Bred, fed, and killed at Rothamsted. Period of Feeding Experiment 70 days.

June 13, 1849.

Lot 4.—Food:— Malt (ground and *steeped*), and Malt-dust—in fixed quantity. Mangolds—*ad libitum*.				Lot 5.—Food:— Malt (ground), and Malt-dust—in fixed quantity. Mangolds—*ad libitum*.					Lot 6.—Food.— Oilcake—in fixed quantity. Mangolds—*ad libitum*.				Means of—						
No. 13.	No. 14.	No. 15.	No. 16.	No. 17.	No. 18.	No. 19.	No. 20.	No. 21.	No. 22.	No 23.	No. 24.	No. 25.	Lot 1.	Lot 2.	Lot 3.	Lot 4.	Lot 5.	Lot 6.	The 25 Sheep.
2·85	2·23	2·67	2·40	2·57	2·51	3·35	2·53	2·93	2·41	2·92	2 52	2·82	2·70	2·64	2·57	2·54	2·78	2·07	2·66
5·74	3·26	4·80	4·85	4·93	6·54	5·48	8·18	7·40	4·31	4·43	4·75	4·88	3·71	4·37	4·45	4·66	6·51	4·59	4·79
4·17	4·81	6·05	4·43	3·88	3·68	4·68	5·26	3·18	5·05	5·02	4·25	4·82	4·63	4·99	4·51	4·87	4·14	4·79	4·63
1·56	0·94	1·29	1·70	1·29	1·67	1·69	2·80	1·67	0·99	1·38	1·06	1·52	1·68	1·45	1·57	1·37	1·82	1·24	1 53
1·94	1·42	1·66	2·05	2·04	2·39	1·44	2·19	2·13	2·25	2·05	1·74	2·27	2·18	2·03	1·81	1·77	2·04	2·08	1·98
2·15	2·36	3·15	2·01	3·06	2·92	1·56	1·76	2·05	2·22	3·01	1·74	1·93	2·82	2·32	2·07	2·42	2·27	2·22	2·35
0·38	0·39	0·42	0·44	0·45	0·42	0·45	0·42	0·30	0·41	0·44	0·45	0·50	0·45	0·43	0·43	0·41	0·43	0·45	0·43
0·55	0·43	0·56	0·48	0·40	0·47	0·45	0·44	0·45	0·52	0·47	0·33	0·32	0·38	0·54	0·58	0·50	0·46	0·41	0 48
1·05	1·01	0·97	0·99	1·22	0·93	1·01	0·86	1·07	0·91	0·90	1·06	1·02	1·11	1·07	1·05	1·00	1·02	0·97	1·04
3·80	3·69	3·92	4·17	3·52	3·58	4·31	4·01	4·18	3·70	3·50	3·95	4·31	4·04	3·90	4·22	3·92	3·92	3·91	3·98
1·32	1·12	1·59	1·30	1·85	1·10	1·42	1·24	1·47	1·44	1·71	1·48	1·74	1·46	1·37	1·42	1·33	1·43	1·59	1·44
0·07	0·08	0·11	0·03	0·07	0·09	0·10	0·10	0·09	0·08	0·10	0·07	……	0·04	0·05	0·06	0·07	0·09	0·08	0 06
0·18	0·15	0·17	0·17	0·16	0·16	0·17	0·11	0·19	0·11	0·18	0·15	0·15	0·19	0·18	0·16	0·17	0·16	0·15	0·17
0·15	0·15	0·20	0·13	0·14	0·12	0·20	0·18	0·18	0·16	0·17	0·17	0·16	0·17	0·15	0 20	0 16	0·16	0·17	0·17
3·21	3·15	3·02	3·10	3·12	3·25	3·17	2·90	3·43	3·09	2·90	3·17	3·41	3·24	3·18	3·15	3·12	3·17	3·14	3·17
7·71	6·65	6·47	7 20	7·21	6·85	8·26	7·51	7·88	7·44	7·02	7·63	8·56	7·51	6·96	7·26	7·01	7·44	7·66	7·31
5·61	5·92	3·84	4·84	7·59	5·76	5·87	4·82	3·60	4·31	5·80	4·55	4·81	4·87	5·76	5·17	5·05	5·55	4·87	5 22
42·53	37·76	40·89	40·29	43·59	42·03	43·61	45·31	42·38	39·49	42·09	39·07	43·22	41·18	41·39	40·68	40·37	43·39	40·99	41 41
55·98	61·80	58·10	58·66	54·64	58·55	55·68	54·77	55·78	58·73	56·78	59·70	55·22	58·27	57·05	57·94	58·63	55·88	57·61	57·50
1·49	0·44	1·01	1·05	1·77	−0·58	0·71	−0·08	1·84	1·78	1·13	1·23	1·56	0·55	1·56	1·38	1·00	0·73	1·40	1·09
100·00	100·00	100·00	100·00	100 00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

Royal Agricultural Society of England, vol. x. part 1. See also, the fuller note at the foot of the Table of the "*Actual Weights*" to which the *Percentages* in this Table refer (Appendix-Table XXXI).

APPENDIX.—TABLE LVII. Showing the *Percentage Proportion* of the individual

CLASS XVII.—14 Sheep of *Hampshire Down Breed,* divided into 3 Lots—Fed respectively, for 63 days, and Mangolds, and Lot 3 with Oilcake and Mangolds. Total period of Feeding Experiment 96 days;

	Designation of Parts.	Lot 1.—Food:— Clover Chaff—in fixed quantity. Swedish Turnips, highly manured, or Mangolds—*ad libitum.* No. 1.	No. 2.	No. 3.	No. 4.	No. 5
Separate parts of the "offal."	Stomachs	3·07	2·26	2·22	2·04	2·88
	Contents of stomachs	6·66	5·75	4·49	5·11	6·69
	Caul-fat	4·70	3·92	4·78	4·53	4·29
	Small intestines and contents	2·35	1·47	1·98	1·81	1·86
	Large intestines and contents	2·58	2·13	2·10	1·98	2·80
	Intestinal fat	1·92	1·94	2·47	2·00	2·10
	Heart and aorta	0·47	0·30	0·37	0·38	0·40
	Heart-fat	0·41	0·46	0·49	0·45	0·48
	Lungs and windpipe	1·24	0·86	0·90	1·03	0·99
	Blood	2·63	3·90	4·10	3·87	4·24
	Liver	1·45	1·34	1·60	1·36	1·46
	Gall-bladder and contents	0·03	......	......	0·04	0·03
	Pancreas ("sweetbread")	0·15	0·10	0·16	0·18	0·17
	Milt or spleen	0·10	0·16	0·16	0·15	0·16
	Head	3·37	3·36	3·01	3·29	3·26
	Skin (with feet, &c.)	7·45	7·20	7·12	6·82	7·04
	Wool previously shorn *	5·04	3·45	5·15	5·63	5·55
Total "offal" parts †		43·71	38·75	41·10	41·27	44·40
Carcass		54·31	59·31	56·65	57·04	53·17
Loss by evaporation, error in weighing, &c.		1·98	1·94	2·25	1·69	2·43
		100·00	100·00	100·00	100·00	100·00

* Wool shorn May 29.

† In the case of these animals the bladder was not weighed.

Organs, and other separated Parts, in the Fasted Live-weight, of SHEEP.

with Clover Chaff, and Turnips, differently manured; then, for 33 days, Lots 1 and 2 with Clover Chaff namely, from March 7 to June 11. Bred, fed, and killed at Rothamsted. Killed June 12, 1849.

Lot 2.—Food:— Clover Chaff—in fixed quantity. Swedish Turnips, manured with Superphosphate of Lime alone, or Mangolds—*ad libitum*.				Lot 3.—Food:— Clover Chaff—in fixed quantity. Swedish Turnips, unmanured—*ad libitum*, or Oilcake—in fixed quantity. Mangolds—*ad libitum*.					Means of—			
No. 6	No. 7.	No. 8.	No. 9	No. 10.	No. 11.	No. 12.	No. 13.	No. 14.	Lot 1.	Lot 2.	Lot 3.	The 14 Sheep
2·85	2·52	3·21	2·76	2·35	2·26	2·62	2·53	2·71	2·61	2·83	2·49	2·63
6·34	5·66	5·53	5·60	5·20	5·59	4·95	4·05	7·49	5·74	5·78	5·47	5·66
3·49	6·64	3·94	3·27	5·31	4·31	4·71	7·59	3·12	4·43	4·33	5·01	4·61
1·70	1·78	1·95	2·08	1·41	2·11	1·98	1·68	2·38	1·89	1·88	1·91	1·89
2·68	2·21	1·90	2·24	1·27	2·13	2·19	1·79	2·05	2·32	2·26	1·88	2·14
1·84	1·82	1·94	2·93	3·01	1·97	1·81	3·70	2·37	2·10	2·13	2·58	2·28
0·31	0·31	0·39	0·17	0·37	0·15	0·44	0·12	0·16	0·10	0·13	0·13	0·12
0·36	0·38	0·27	0·37	0·18	0·43	0·40	0·25	0·30	0·46	0·34	0·30	0·40
1·21	0·94	1·02	1·16	0·81	1·20	1·04	1·11	1·13	1·00	1·08	1·06	1·04
3·71	3·95	4·07	4·20	3·47	4·46	3·87	4·00	4·21	3·75	4·13	4·00	3·95
1·47	1·30	1·37	1·42	1·46	1·58	1·57	1·57	1·56	1·44	1·39	1·55	1·46
0·09	0·06	0·09	……	0·04	0·10	0·09	……	0·04	0·05	0·08	0·07	0·06
0·13	0·19	0·15	0·15	0·17	0·15	0·14	0·11	0·23	0·16	0·16	0·16	0·16
0·18	0·19	0·18	0·17	0·17	0·13	0·15	0·18	0·17	0·16	0·18	0·16	0·17
3·35	3·13	2·96	3·46	2·60	3·12	3·38	3·38	3·68	3·26	3·22	3·23	3·24
7·25	6·99	7·41	7·57	7·40	9·65	8·04	6·58	7·03	7·18	7·31	7·74	7·40
5·86	4·60	4·89	5·65	3·78	3·86	6·09	4·83	4·06	4·90	5·27	4·87	5·02
43·02	42·79	41·87	43·50	39·18	43·50	44·30	43·72	43·80	41·84	42·80	43·00	42·53
54·94	56·25	58·27	54·23	59·55	57·37	54·37	55·22	54·41	56·10	55·92	56·18	56·08
2·04	0·96	−0·14	2·27	0·97	−0·87	1·21	1·06	1·70	2·06	1·28	0·82	1·39
100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

APPENDIX.—Table LIX. Showing the *Percentage Proportion* of the individual

CLASS I.—9 Pigs, divided into 3 Lots, each with rather different Food*.

Designation of Parts.		Lot 1. Food: Bean and Lentil meal in fixed quantity. Bran *ad libitum*.		
		No. 1.	No. 2.	No. 3.
Separate parts of the "offal."	Stomach and contents	2·19	1·48	1·30
	Caul-fat	0·34	0·43	0·39
	Small intestines and contents	1·56	1·91	3·36
	Large intestines and contents	7·53	3·60	5·00
	Intestinal fat, "mudgeon," &c.	0·92	0·76	0·66
	Heart and aorta	0·33	0·35	0·30
	Lungs and windpipe	0·78	1·01	0·98
	Blood	4·41	4·64	3·18
	Liver	1·75	1·39	1·72
	Gall-bladder and contents	0·05	0·12	0·06
	Pancreas ("sweetbread")	0·14	0·24	0·16
	Milt or spleen	0·16	0·20	0·25
	Bladder	0·14	0·05	0·05
	Penis (or uterus)	0·25	0·29	0·18
	Tongue	0·58	0·69	0·48
	Toes	0·12	0·11	0·11
	Miscellaneous trimmings	0·41	0·22	0·11
Total "offal" parts		24·66	17·52	18·29
Carcass (including head and feet)		73·94	81·86	79·59
Loss by evaporation, error in weighing, &c.		1·40	0·62	2·12
		100·00	100·00	100·00

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* For particulars of the *feeding* experiment, refer to Pens 9, 10, and 11, under the head of "Experiments with

Organs, and other separated Parts, in the Fasted Live-weight, of PIGS

Somewhat *under* Fattened—the Food containing a considerable portion of Bran.

Lot 2.—Food.— Indian meal—in fixed quantity. Bran—*ad libitum.*			Lot 3.—Food:— Bean and Lentil meal, and Indian meal—in fixed quantity. Bran—*ad libitum.*			Means of—			
No. 4.	No. 5	No. 6.	No. 7.	No. 8.	No. 9.	Lot 1. 3 Pigs.	Lot 2. 3 Pigs.	Lot 3. 3 Pigs.	The 9 Pigs.
1·37	1·90	1·86	1·91	1·55	1·34	1·60	1·71	1·60	1·66
0·74	0·62	0·48	0·67	0·54	0·15	0·39	0·61	0·55	0·52
2·11	3·72	3·00	2·77	3·16	2·80	3·27	2·95	2·93	3·05
3·04	5·62	4·72	4·59	5·08	5·01	5·38	4·40	4·90	4·91
1·39	0·96	0·80	1·03	0·98	0·69	0·78	1·05	0·90	0·91
0·32	0·18	0·32	0·29	0·27	0·28	0·33	0·27	0·28	0·29
0·84	0·72	0·86	0·89	0·83	0·90	0·93	0·81	0·91	0·88
1·32	4·00	3·43	3·83	3·69	4·20	1·08	3·92	3·91	3·97
1·30	1·61	1·50	1·53	1·39	1·74	1·62	1·47	1·55	1·55
0·07	0·05	0·06	0·09	0·04	0·08	0·08	0·06	0·06	0·07
0·23	0·18	0·16	0·21	0·14	0·14	0·18	0·19	0·16	0·18
0·16	0·12	0·14	0·16	0·13	0·16	0·20	0·14	0·15	0·16
0·10	0·07	0·05	0·10	0·06	0·06	0·08	0·07	0·07	0·07
......	......	0·22	......	......	0·25	0·24	0·22	0·25	0·24
0·63	0·40	0·48	0·48	0·60	0·43	0·58	0·53	0·51	0·54
0·10	0·05	0·08	0·08	0·09	0·11	0·11	0·08	0·09	0·09
0·06	0·32	0·11	0·05	0·10	0·10	0·26	0·16	0·12	0·18
16·78	20·61	18·27	18·78	18·65	18·87	20·16	18·70	18·94	19·27
83·58	78·04	81·41	80·26	82·01	81·27	78·46	81·01	81·18	80·22
−0·36	1·35	0·32	0·96	−0·66	−0·14	1·38	0·29	−0·12	0·51
100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

Pigs—Series 1," Article—"Pig Feeding," Journal of the Royal Agricultural Society of England, vol. xiv. part 2.

APPENDIX.—TABLE LX. Showing the *Percentage Proportion* of the individual

CLASS II.—12 Pigs, divided into 4 Lots, each with rather different Food*. *Moderately*

	Designation of Parts	Lot 1. Food: Bean and Lentil meal *ad libitum.*			Lot 2. Food: Indian meal in fixed quantity Bean and Lentil meal *ad libitum.*		
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.
Separate parts of the "offal."	Stomach and contents	1·49	1·01	1·43	0·75	1·11	0·70
	Caul-fat	0·46	0·70	0·37	0·59	0·40	0·49
	Small intestines and contents	1·77	2·09	2·17	1·73	2·11	2·01
	Large intestines and contents	3·61	3·75	5·16	3·47	4·01	2·76
	Intestinal fat, "mudgeon," &c.	1·74	1·69	1·40	2·16	1·27	1·39
	Heart and aorta	0·25	0·25	0·30	0·21	0·26	0·32
	Lungs and windpipe	0·69	0·67	0·72	0·67	0·61	0·65
	Blood	3·71	3·84	4·27	3·66	4·02	4·16
	Liver	1·85	1·95	1·71	1·61	1·12	1·79
	Gall-bladder and contents	0·08	0·06	0·05	0·05	0·05	0·06
	Pancreas ("sweetbread")	0·10	0·21	0·25	0·31	0·22	0·28
	Milt or spleen	0·15	0·14	0·16	0·12	0·17	0·14
	Bladder	0·06	0·08	0·11	0·09	0·07	0·05
	Penis (or uterus)	0·30	......	......	0·18	....	0·16
	Tongue	0·45	0·45	0·53	0·40	0·41	0·41
	Toes	0·08	0·10	0·08	0·06	0·09	0·08
	Miscellaneous trimmings	0·31	0·32	0·31	0·19	0·30	0·19
Total "offal" parts		17·10	17·31	19·02	16·28	16·53	15·94
Carcass (including head and feet)		83·83	82·48	79·40	82·52	82·50	84·01
Loss by evaporation, error in weighing, &c.		−0·03	0·21	1·58	1·20	0·95	0·02
		100·00	100·00	100·00	100·00	100·00	100·00

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* For particulars of the *feeding* experiment, refer to Pens 1, 2, 3, and 4, under the head of "Experiments with

Organs, and other separated Parts, in the Fasted Live-weight, of PIGS.

Fattened—the Food containing a considerable proportion of Bean and Lentil meal.

Lot 3.—Food:—Bran—in fixed quantity. Bean and Lentil meal—*ad libitum.*			Lot 4.—Food:—Indian meal and Bran—in fixed quantity. Bean and Lentil meal—*ad libitum.*			Means of—				
No 7.	No. 8.	No. 9.	No. 10.	No. 11.	No. 12.	Lot 1. 3 Pigs.	Lot 2. 3 Pigs.	Lot 3. 3 Pigs.	Lot 4. 3 Pigs.	The 12 Pigs.
1·21	1·75	1·45	1·16	1·50	1·69	1·31	0·86	1·47	1·45	1·27
0·60	0·55	0·35	0·46	0·62	0·33	0·51	0·49	0·50	0·47	0·49
2·84	2·57	2·30	2·33	2·09	2·21	2·01	1·95	2·57	2·21	2·19
4·31	4·62	4·62	4·35	4·50	4·60	4·17	3·41	4·55	4·49	4·16
1·03	0·58	1·26	1·16	1·54	0·91	1·61	1·61	0·95	1·20	1·35
0·16	0·33	0·25	0·32	0·27	0·34	0·27	0·27	0·24	0·31	0·27
0·72	0·87	0·63	0·71	1·00	0·84	0·69	0·64	0·71	0·85	0·73
3·70	4·19	4·03	2·99	4·83	5·30	3·91	4·05	3·97	4·38	4·08
1·79	1·52	1·53	1·82	1·96	1·53	1·81	1·61	1·61	1·77	1·71
0·08	0·05	0·03	0·04	0·08	0·04	0·06	0·06	0·05	0·05	0·05
0·24	0·22	0·22	0·20	0·24	0·21	0·19	0·26	0·23	0·22	0·22
0·13	0·16	0·13	0·14	0·17	0·18	0·15	0·14	0·14	0·16	0·15
0·07	0·11	0·10	0·08	0·10	0·12	0·08	0·07	0·09	0·10	0·09
......	......	......	0·13	......	......	0·30	0·17	......	0·13	0·19
0·52	0·54	0·41	0·36	0·46	0·47	0·48	0·42	0·49	0·43	0·46
0·09	0·11	0·10	0·09	0·11	0·11	0·09	0·08	0·10	0·10	0·09
0·63	0·47	0·37	0·20	......	0·32	0·31	0·23	0·49	0·26	0·33
18·12	18·64	17·78	16·54	19·47	19·20	18·01	16·32	18·19	18·58	17·83
80·84	81·61	81·12	83·64	82·71	80·20	81·90	83·02	81·19	82·18	82·07
1·04	−0·25	1·10	−0·18	−2·18	0·60	0·09	0·66	0·62	−0·76	0·10
100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

Pigs—Series 1," Article—"Pig Feeding," Journal of the Royal Agricultural Society of England, vol. xiv. part 2.

APPENDIX.—TABLE LXI. Showing the *Percentage Proportion* of the individua

CLASS III.—15 Pigs, divided into 5 Lots, each with rather different Food*

Designation of Parts.		Lot 1.—Food.—Indian meal—*ad libitum.*			Lot 2. Food: Bean and Lentil meal in fixed quantity. Indian meal *ad libitum.*			Lot 3. Food: Bran in fixed quantity. Indian meal *ad libitum.*		
		No. 1.	No. 2.	No. 3.	No. 1.	No. 5.	No. 6.	No. 7.	No. 8.	No. 9.
Separate parts of the "offal."	Stomach and contents	1·16	1·67	1·02	0·76	0·98	0·86	1·06	1·02	1·00
	Caul-fat	0·61	0·40	0·52	0·55	0·41	0·73	0 53	0·64	0·45
	Small intestines and contents	1·82	1·67	1·97	1 34	1·71	1·56	1·28	1·61	1·75
	Large intestines and contents	2 77	3·15	2·58	3·46	2·01	2·05	4 51	3 95	2·75
	Intestinal fat, "mudgeon," &c.	1·81	1·30	0·83	1·78	1·11	2·21	1·08	1·68	1 43
	Heart and aorta	0·27	0·32	0·32	0·26	0·29	0 25	0·27	0·31	0·29
	Lungs and windpipe	0 72	0·72	0·81	0·62	0 65	0·70	0 51	0·69	0·81
	Blood	3·58	3·61	3·76	3·08	2·91	3·58	3·01	3·62	2·13
	Liver	1·31	1·45	1·42	1·18	1·36	1·31	1·77	1 58	1·41
	Gall-bladder and contents	0 01	0·02	0·07	0 06	0·05	0·06	0·01	0·03	0·05
	Pancreas ("sweetbread")	0 22	0·18	0·27	0 18	0·13	0·25	0·10	0·17	0 26
	Milt or spleen	0·10	0·15	0·13	0·11	0·15	0·11	0·10	0·16	0·12
	Bladder	0·04	0·07	0·05	0·08	0·09	0 06	0·06	0·09	0·10
	Penis (or uterus)	0·21	.. ..	0·25	0·45	.. ..	.. ...	0·21	..	.. ..
	Tongue	0·43	0·45	0·47	0·30	0·11	0·46	0·16	0·58	0·13
	Toes	0·08	0·11	0·08	0·07	0·08	0·09	0·06	0 08	0·08
	Miscellaneous trimmings	0·28	0·31	0 03	0·20	0·60	0·30	0·56	0·13	0·24
Total "offal" parts		15·54	15·76	14·58	15·47	13·87	15 00	15·76	16·37	14·10
Carcass (including head and feet)		85·75	86·04	84·48	83·77	85 61	83·01	83·89	82·91	84·26
Loss by evaporation, error in weighing, &c.		−1·29	−1·80	0·94	0·76	0·52	0·49	0·35	0·72	1·04
		100·00	100 00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

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* For particulars of the *feeding* experiment, refer to Pens 5, 6, 7, 8, and 12, under the head of "Experiments with

Organs, and other separated Parts, in the Fasted Live-weight, of PIGS.

Well Fattened—the Food containing a considerable proportion of Indian-corn Meal.

Lot 4.—Food.— Bean and Lentil meal, and Bran—in fixed quantity. Indian meal—*ad libitum.*			Lot 5. —Food:— Bean and Lentil meal, Indian meal, and Bran—each *ad libitum.*			Means of					
No. 10.	No. 11.	No. 12.	No. 13.	No. 14.	No. 15.	Lot 1. 3 Pigs.	Lot 2. 3 Pigs.	Lot 3. 3 Pigs.	Lot 4. 3 Pigs.	Lot 5. 3 Pigs.	The 15 Pigs.
0·95	1·29	1·20	1·68	1·82	1·16	1·20	0·87	1·02	1·15	1·55	1·18
0·60	0·53	0·64	0·49	0·97	0·38	0·55	0·56	0·54	0·59	0·61	0·57
1·80	1·71	2·22	1·45	2·04	1·32	1·82	1·54	1·56	1·91	1·60	1·69
3·39	4·34	3·19	3·00	3·41	2·78	2·84	3·11	3·73	3·61	3·06	3·28
1·80	1·05	0·74	0·96	1·76	0·87	1·36	1·72	1·40	1·20	1·20	1·37
0·29	0·27	0·25	0·28	0·28	0·24	0·30	0·26	0·29	0·27	0·27	0·27
0·57	0·72	0·68	0·68	0·78	0·50	0·75	0·67	0·68	0·66	0·65	0·68
3·12	3·82	3·63	3·23	3·50	3·12	3·65	3·30	3·20	3·52	3·10	3·43
1·45	1·34	1·30	1·26	1·53	1·38	1·40	1·38	1·50	1·39	1·39	1·43
0·04	0·04	0·10	0·04	0·02	0·04	0·04	0·06	0·04	0·06	0·03	0·05
0·28	0·19	0·18	0·12	0·10	0·10	0·22	0·19	0·20	0·22	0·17	0·20
0·11	0·14	0·13	0·12	0·11	0·12	0·13	0·13	0·13	0·13	0·12	0·13
0·08	0·07	0·05	0·06	0·05	0·11	0·05	0·08	0·08	0·07	0·07	0·07
......	0·22	0·23	0·14	0·21	......	0·23	0·45	0·24	0·22	0·17	0·24
0·39	0·41	0·44	0·56	0·47	0·40	0·45	0·42	0·49	0·41	0·48	0·45
0·07	0·08	0·07	0·08	0·11	0·08	0·09	0·08	0·07	0·07	0·09	0·08
0·42	0·14	0·25	0·28	0·84	0·25	0·21	0·37	0·31	0·27	0·29	0·30
15·36	16·36	15·39	14·43	17·64	13·94	15·38	15·98	15·57	15·78	15·15	15·41
83·87	82·80	83·84	84·94	80·78	85·89	85·42	84·43	83·69	83·50	83·87	84·18
0·77	0·84	0·77	0·63	1·58	0·87	–0·80	0·20	0·74	0·72	0·98	0·41
100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

Pigs—Series 1," Article—"Pig Feeding," Journal of the Royal Agricultural Society of England, vol. xiv. part 2.

APPENDIX.—TABLE LXII. Showing the *Percentage Proportion* of the individual

CLASS IV.—12 Pigs, divided into 4 Lots, according to the Food*. *Moderately* Fattened.

	Designation of Parts.	Lot 1.—Food:— Lentils, and Bran—in fixed quantity. Sugar—*ad libitum.*			Lot 2.—Food:— Lentils, and Bran—in fixed quantity. Starch - *ad libitum.*		
		No. 1.	*No. 2.*	No. 3.	No. 4.	No. 5.	No. 6.
Separate parts of the "offal."	Stomach and contents	0·87	1·08	1·27	1·44	0·93	0·92
	Caul-fat	0·61	0·71	0·51	0·64	0·51	0·19
	Small intestines and contents	1·81	2·19	1·82	2·27	1·06	2·04
	Large intestines and contents	4·70	4·79	4·14	5·86	5·08	6·78
	Intestinal fat, "mudgeon," &c.	0·57	0·65	0·57	0·62	0·44	0·50
	Heart and aorta	0·32	0·35	0·31	0·28	0·30	0·37
	Lungs and windpipe	0 69	0·84	0·91	0·78	0·87	1·06
	Blood	3·12	3 53	3·61	3·83	3 72	4·52
	Liver	1·46	2·01	1·01	1·74	1·51	1·78
	Gall-bladder and contents	0 06	0·00	0·03	0·12	0·11	0·12
	Pancreas ("sweetbread")	0·14	0·23	0·14	0·19	0·18	0·19
	Milt or spleen	0·13	0·15	0·16	0·12	0 13	0·16
	Bladder	0·12	0·06	0·05	0·12	0 05	0·08
	Penis (or uterus)	......	0·17	......	0·19	0·16	0 18
	Tongue	0·67	0·63	0 66	0·62	0·46	0·53
	Toes	.....	......	0 09	0 08	0·08	0·08
	Miscellaneous trimmings	.....	......	......	0·10	0·08	0·08
Total "offal" parts		15·27	17·48	15·88	19·00	16·00	19·07
Carcass (including head and feet)		83·78	81·17	84·24	78·35	82·50	79·55
Loss by evaporation, error in weighing, &c.		0·95	1·35	—0·12	2·65	0·00	0·18
		100·00	100·00	100·00	100·00	100 00	100·00

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* For particulars of the *feeding* experiment, see Article "On the Equivalency of Starch and

Organs, and other separated Parts, in the Fasted Live-weight, of PIGS.

The Food consisted, in considerable proportion, of either Starch or Sugar.

Lot 3.—Food:— Lentils, and Bran—in fixed quantity Sugar, and Starch—*ad libitum.*			Lot 4.- Food: - Lentils, Bran, Sugar, and Starch - each *ad libitum.*			Means of -				
No 7.	No. 8.	No. 9.	No. 10.	No. 11.	No. 12.	Lot 1. 3 Pigs.	Lot 2. 3 Pigs	Lot 3. 3 Pigs.	Lot 4. 3 Pigs.	The 12 Pigs.
1·27	1·19	1·17	0·82	1·39	1·56	1·07	1·10	1·21	1·26	1·16
0·60	0·65	0·53	0·66	0·54	0·67	0·61	0·56	0·59	0·62	0·59
1·82	2·07	2·46	2·01	2·66	2·61	1·94	2·09	2·12	2·45	2·15
4·50	5·44	4·37	4·63	4·40	5·91	4·51	5·91	4·77	4·98	5·05
0·82	0·51	0·73	0·69	0·75	0 51	0 60	0·55	0·70	0·65	0 63
0·32	0·33	0·32	0·29	0·29	0·31	0 33	0·31	0·32	0 30	0·31
0 71	0·72	0 73	0·69	0·78	0 69	0·81	0·90	0·73	0·72	0 79
3·35	3·72	3·06	3·61	3·17	3·71	3·42	4·02	3·38	3·51	3·59
1·62	1·49	1·52	1·65	2 09	1·89	1·69	1·68	1·55	1·88	1·70
0 07	0·09	0·06	0·09	0 10	0·07	0·06	0·12	0·07	0 09	0 08
0·19	0·16	0·14	0·18	0·16	0 21	0·17	0·19	0·16	0·19	0·18
0·15	0 13	0·11	0·12	0·11	0·12	0·15	0·14	0 13	0·12	0·14
0·07	0·10	0·07	0·08	0·08	0·09	0·08	0·08	0·08	0·08	0·08
……	……	……	0·18	0·28	……	0·17	0·18	……	0·23	0·19
0·54	0·54	0·48	0·46	0·44	0·44	0·65	0 53	0·52	0·44	0·53
0·07	0·08	0·10	……	0·08	0·09	0·09	0·08	0·08	0·09	0·08
0·14	0·13	0·16	0·15	0·16	0·04	……	0·09	0·14	0·12	0·12
16·27	17·38	16·01	16·34	17·51	18·08	16·38	18·53	16·55	17·71	17·37
81·77	81·35	82·08	82·24	80·78	79·51	83·06	80·13	81·74	80·84	81·44
1·96	1·27	1·91	1·42	1·71	1·51	0·56	1·34	1·71	1·45	1·19
100·00	100 00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100 00

Sugar in Food"—Report of the British Association for the Advancement of Science for 1854.

APPENDIX.—TABLE LXIII. Showing the *Percentage Proportion* of the individual Organs, and other separated Parts, in the Fasted Live-weight, of PIGS.

CLASS V.—6 Pigs, divided into 2 Lots, each with rather different Food *. Well Fattened. Food comprised a portion of dried Cod-fish.

Designation of Parts.		Lot 1.—Food.—Dried Cod-fish—in fixed quantity. Bran and Indian meal, equal parts—*ad libitum*			Lot 2.—Food.—Dried Cod-fish—in fixed quantity Indian meal—*ad libitum.*			Means of—		
		No 1.	No. 2.	No 3	No 4.	No 5	No 6	Lot 1 3 Pigs	Lot 2 3 Pigs	The 6 Pigs
Separate parts of the "offal."	Stomach and contents	0·99	1·74	0·97	1·03	1·16	1·11	1·23	1·10	1·17
	Caul-fat	0·55	0·44	0·37	0·60	0·63	0·47	0·45	0·57	0·51
	Small intestines and contents	1·61	2·38	1·87	1·43	1·44	1·19	1·96	1·35	1·66
	Large intestines and contents	3·99	3·15	2·60	2·37	2·22	2·22	3·25	2·27	2·76
	Intestinal fat, "mudgeon," &c.	1·19	0·83	1·06	1·04	1·13	0·95	1·02	1·04	1·03
	Heart and aorta	0·24	0·41	0·26	0·19	0·17	0·26	0·30	0·21	0·25
	Lungs and windpipe	0·44	0·55	0·46	0·30	0·98	0·51	0·48	0·66	0·57
	Blood	3·53	3·47	3·12	2·93	2·95	2·65	3·38	2·84	3·11
	Liver	1·34	1·47	1·50	0·99	1·20	1·02	1·44	1·07	1·26
	Gall-bladder and contents	0·08	0·06	0·04	0·06	0·06	0·09	0·06	0·07	0·06
	Pancreas ("sweetbread")	0·18	0·22	0·17	0·19	0·17	0·17	0·19	0·18	0·19
	Milt or spleen	0·12	0·15	0·11	0·12	0·12	0·13	0·13	0·12	0·12
	Bladder	0·07	0·06	0·06	0·05	0·07	0·06	0·06	0·06	0·06
	Penis (or uterus)	0·22	0·21	......	0·19	......	......	0·21	0·19	0·20
	Tongue	0·47	0·51	0·46	0·46	0·37	0·32	0·48	0·39	0·43
	Toes	0·06	0·07	0·07	0·08	0·07	0·06	0·07	0·07	0·07
	Miscellaneous trimmings	0·24	0·14	0·39	0·05	0·23	0·19	0·26	0·16	0·21
Total "offal" parts		15·32	15·86	13·51	12·30	12·97	11·40	14·97	12·35	13·66
Carcass (including head and feet)		84·55	83·70	85·63	87·76	86·67	87·52	84·63	87·32	85·98
Loss by evaporation, error in weighing, &c.		0·13	0·44	0·86	—0·06	0·36	1·08	0·40	0·33	0·36
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

* For particulars of the *feeding experiment*, refer to Pens 1 and 2, under the head of "Experiments with Pigs—Series 3" Article—"Pig Feeding" Journal of the Royal Agricultural Society of England, vol. xiv. part 2.

APPENDIX.—TABLE LXIV. Showing the *Percentage Proportion* of the individual Organs, and other separated Parts, in the Fasted Live-weight, of PIGS.

5 Pigs, divided into 2 Lots, according to condition of Maturity when put to Fatten.

CLASS VI.—Put to Fatten when in Store condition, and fed till only Half-fattened.

CLASS VII.—Put to Fatten when Half-fat, and fed till Moderately Fattened.

	Designation of Parts.	Class 6. Put to Fatten when in Store condition, and fed till only Half-fattened.		Class 7. Put to Fatten when Half-fat, and fed till Moderately fattened.			Means of—		
		No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	Class 6.	Class 7.	The 5 Pigs.
Separate parts of the "offal."	Stomach and contents	2·70	0·92	0·85	1·04	1·07	1·81	0·99	1·32
	Caul-fat	0·52	0·43	0·51	0·53	0·52	0·47	0·52	0·50
	Small intestines and contents	5·00	2·96	2·17	2·97	1·96	3·98	2·36	3·01
	Large intestines and contents	4·44	4·24	2·70	4·21	3·17	4·34	3·38	3·75
	Intestinal fat, "mudgeon," &c.	0·63	0·72	1·29	0·65	0·68	0·67	0·87	0·79
	Heart and aorta	0·30	0·25	0·26	0·30	0·32	0·28	0·29	0·29
	Lungs and windpipe	1·04	0·65	0·85	1·44	0·91	0·85	1·06	0·98
	Blood	2·86	3·21	3·21	3·56	3·34	3·04	3·37	3·24
	Liver	2·22	1·52	1·50	1·84	1·35	1·87	1·56	1·69
	Gall-bladder and contents	0·04	0·07	0·06	0·11	0·09	0·05	0·09	0·07
	Pancreas ("sweetbread")	0·14	0·19	0·20	0·18	0·18	0·17	0·18	0·18
	Milt or spleen	0·21	0·14	0·11	0·12	0·22	0·17	0·15	0·16
	Bladder	0·07	0·06	0·07	0·12	0·10	0·06	0·10	0·08
	Penis (or uterus)	0·17	0·20	0·19	0·29	0·26	0·18	0·24	0·22
	Tongue	0·54	0·43	0·51	0·59	0·42	0·49	0·51	0·50
	Miscellaneous trimmings	0·36	0·33	0·60	0·45	0·15	0·35	0·40	0·38
Total "offal" parts		21·24	16·32	15·08	18·40	14·74	18·78	16·07	17·16
Carcass (including head and feet)		76·25	82·26	84·00	81·66	84·51	79·26	83·39	81·74
Loss by evaporation, error in weighing, &c.		2·51	1·42	0·92	—0·06	0·75	1·96	0·54	1·10
		100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

MEMORANDA, ERRATA, ETC.

[NOTE.—The numerical errors in the Tables, which are noticed in the following list, are the most important that have been detected; but, though it seemed desirable to correct them for purposes of reference, none of them affect the conclusions given in the text. It will be seen, that the larger number of the required corrections are due to but a few original errors in the statement of actual quantities; the results of which, however, ramify into the lines of *Totals*, into the columns of *Means*, into the corresponding *Percentages*, and generally also into more than one Table. A few others, of still less importance, have been observed, to which it is thought unnecessary to call special attention. They occur chiefly in one or two of the Tables of *Percentage Proportion* of Organs or Parts, and in amount are within the range of the second decimal place.]

Page 538, line 4: *for*—"than that of the more moderately fattened animal." *read* than that of the more moderately fattened animal, or than that of either Oxen or Sheep.

Page 556, last sentence of second paragraph: *for*—"Of these, Tables XVII., XVIII., XIX., XX., XXI., and XXII.," &c., *read*—Of these, Tables XVII., XVIII., XIX., XX., XXI., XXII., and XXIII., &c.

Table XVII. p. 558:—

Column 1. *For*—"6 0·5" in line of Contents of stomachs, *read*—4 12·1; and accordingly, *for*—"85 4·9" in line of Total Offal parts, *read*—84 0·5. *For*—"156 10·8" in line of Carcass, *read*—158 3·5. And in accordance with these alterations, *for*—"8 12·3" in line of Loss by evaporation, error in weighing, &c., *read*—8 8·0. [Note—The same amended numbers as here given, should be carried in (in the corresponding lines of course), in the last Column but three, of Appendix-Table XV. p. 595.]

Column 3. *For*—"97 4·7" in line of Contents of stomachs, *read* 95 15·2; *for*—"17 1·8" in line of Small Intestines and contents, *read*—18 1·7; and *for*—"13 4·6" in line of Large Intestines and contents, *read*—13 10·0. [Note—The same amended numbers as here given, should be carried in, in the last Column but one, of Appendix-Table XV. p. 595.]

Column 4. *For*—"98 15·8" in line of Contents of stomachs, *read*—92 12·8; *for*—"16 13·6" in line of Small Intestines and contents, *read*—17 12·0; and *for*—"13 2·3" in line of Large Intestines and contents, *read*—13 7·0. [Note—The same amended numbers as here given, should be carried in, in Column 1, of Table XXIII. p. 564; and in the last Column of Appendix-Table XV. p. 595.]

Column 6. *For*—"1 7·0" in line of Miscellaneous trimmings (=Heart trimmings only, see also Appendix-Table II. p. 581), *read*—3 6·2, as in Appendix-Table XV. p. 595, Column of Bullock No. 13. And accordingly, *for*—"423 10·3" in line of Total Offal parts, *read*—425 9·5; and *for* "10 10·7" in line of Loss by evaporation, error in weighing, &c., *read*—8 11·5.

Column 7. *For*—"33 12·5" in line of Caul Fat, *read*—29 13·0. The difference, 3 15·5 (Heart trimmings, see Appendix-Table III. p. 582) + 5 1·0 (trimmings from the neck) = 9 0·5; which amount, *insert* in line of Miscellaneous trimmings, as in Appendix-Table XV. p. 595, Column of Bullock No. 12. And accordingly, *for*—"460 13·3" in line of Total Offal parts, *read*—465 14·3; and, *for*—"18 12·7" in line of Loss by evaporation, error in weighing, &c., *read*—13 11·7.

Table XVIII. p. 559:—

Column 2. *For*—"152 5·8" in line of Final weight unfasted, *read*—156 0·8. *For*—"0 2·1" in line of Miscellaneous trimmings, *read*—0 3·1; and in accordance with this, *for*—"58 13·2" in line of Total Offal parts, *read*—58 14·2; and *for*—"0 13·0" in line of Loss by evaporation, error in weighing, &c., *read*—0 12·0.

Column 3. *For*—"124 11·4" in line of Original weight, *read*—99 1·8.

Column 4. *For*—"115 1" in line of Original weight, *read*—124 7·2. *For*—"144 13·1" in line of Final weight unfasted, *read*—148 0·1.

Column 5. *Omit* figures—"96 11·4" in line of Original weight. *For*—"6 2·7" in line of Contents of stomachs, *read*—6 7·7; *for*—"4 12·2" in line of Head, *read*—4 11·4; and in accordance with these alterations, *for*—"64 6·4" in line of Total Offal parts, *read*—64 10·6; and *for*—"0 0·1" in line of Loss by evaporation, error in weighing, &c., *read*, —0 4·1. [Note—The same corrections as here given, should be carried in, in the last column, of Appendix-Table XXXIII. p. 625.]

Column 6. *For*—"109 13·5" in line of Original weight, *read*—107 7·2. *For*—"160 4·9" in line of Final weight unfasted, *read*—162 15·4.

Column 9. *For*—"0 8·5" in line of Gall-bladder and contents, *read*—0 0·8; and in accordance with this, *for*—"47 3·6" in line of Total Offal parts, *read*—46 11·9; and *for*—"1 9·3" in line of Loss by evaporation, error in weighing, &c., *read*—2 1·0.

Table XIX. p. 560:—

Column 3. *For*—"245 10·7" in line of Final weight unfasted, *read*—235 10·7.

Column 8. *For*—"222 9·8" in line of Final weight unfasted, *read*—220 1·1.

Table XX. p. 561:—

Column 1. *For*—"2·39" in line of Contents of stomachs, *read*—1·89; and accordingly, *for*—"34·04" in line of Total Offal parts, *read*—33·54. *For*—"62·53" in line of Carcass, *read*—63·13." And in accordance with these alterations, *for*—"3·48" in line of Loss by evaporation, error in weighing, &c., *read*—3·33. [Note—The same amended numbers as here given, should be carried in, in the last Column but three, of Appendix-Table XL. p. 637.]

Column 3. *For*—"8·44" in line of Contents of stomachs, *read*—8·33; *for*—"1·49" in line of Small Intestines and contents, *read*—1·57; and, *for*—"1·18" in line of Large Intestines and contents, *read*—1·21. [Note—The same amended numbers as here given, should be carried in, in the last Column but one, of Appendix-Table XL. p. 637.]

Column 4. *For*—"8·44" in line of Contents of stomachs, *read*—8·34; *for*—"1·52" in line of Small Intestines and contents, *read*—1·60; and *for*—"1·22" in line of Large Intestines and contents, *read*—1·24 [Note—The same amended numbers as here given, should be carried in, in Column 4, of Table XXIII. p. 564; and also in the last Column of Appendix-Table XL. p. 637.]

Last Column. *For*—"2·38" in line of Caul Fat, *read*—2·10; and in line of Miscellaneous trimmings, *insert*—0·64. And in accordance with these alterations, *for*—"32·48" in line of Total Offal parts, *read*—32·84; and *for*—"1·32" in line of Loss by evaporation, error in weighing, &c., *read*—0·96.

Table XXI. p. 562:—

Column 5. *For*—"8·62" in line of Contents of stomachs, *read*—8·85; *for*—"5·81" in line of Caul Fat, *read*—5·18; *for*—"10·84" in line of Skin and Wool, *read*—11·01; and in accordance with these alterations, *for*—"37·71" in line of Total Offal parts, *read*—37·98. *For*—"62·28" in line of Carcass, *read*—61·91; and *for*—"0·01" in line of Loss by evaporation, &c., *read*—0·11.

Column 9 (Half-fat Sheep). *For*—"0·506" in line of Gall-bladder and contents, *read*—0·061; and in accordance with this, *for*—"44·948" in line of Total Offal parts, *read*—44·493; and *for*—"1·506" in line of Loss by evaporation, error in weighing, &c., *read*—1·961.

Page 571, last line of first paragraph: *before* the word—"Carcass" *insert* the words—here reckoned.

Appendix-Table XV. pp. 594—595:—

Mem.: in line of "Bladder, with penis, or womb," the amounts set down to Bullocks Nos. 9, 12, 13 and 14, include both Bladder and Penis; those set down to the other Bullocks refer to Bladder only. In the same line, in the last column but one, *for*—"0 16·7," *read* 1 0·7; which amount is the *sum* of the mean of the Bladder, and of that of the Penis, each taken separately. In the same line, in the last column, the amount represents Bladder only; the Penis, or Womb, being included with the "Loss by evaporation, error in weighing, &c."

Column 2. *For*—"8 3·0" in line of Contents of stomachs, *read*—5 10·2; and in accordance with this, *for*—"87 12·7" in line of Total Offal parts, *read*—85 8·9. *For*—"157 7·5" in line of Carcass, *read*

—160 9·0. And in accordance with these alterations, *for*—"13 7·8" in line of Loss by evaporation, error in weighing, &c, *read*—12 15·1.

Column of Bullock No. 12. *For*—"84 4·0" in line of Contents of stomachs, *read*—77 3 8; *for*—"8 11·7" in line of Small Intestines and contents, *read*—14 10·5; and *for*—"5 2 5" in line of Large Intestines and contents, *read*—6 4 0.

Column of Bullock No. 13. *For*—"100 1·0" in line of Contents of stomachs, *read*—88 0 0; *for*—"8 5·0" in line of Small Intestines and contents, *read*—16 13 0; and *for*—"6 0 0" in line of Large Intestines and contents, *read*—9 9 0.

Appendix-Table XVIII. pp. 600—601:—

Column 6. *For*—"1 7" in line of Loss by evaporation, &c., *read*—0 1 7.

Column 10. *For*—"132 4" in line of Live-weight after fasting, *read*—132 14.

Column 14: in line of Bladder, the amount represents both Bladder and Womb, that one Sheep being an Ewe.

Appendix-Table XXI. p 606. Mem: in Columns 2 and 3, respectively, the amount set down for Bladder includes one Testicle.

Appendix-Table XXVI. p. 613. Column 8. *For*—"0 1 2" in line of Heart-fat, *read*—1 2·0.

Appendix-Table XXX. p. 618. Column 1. *For* "142 2 6" in lines of Final weight, after fasting, and of Live-weight after fasting, *read*—141 2·6. *For*—"83 0" in line of Carcass, *read*—82 0.

Appendix-Table XXXIII. pp 624—625:—

Column 15. *For*—"57 5·6" in line of Total Offal parts, *read*—57 8·6; and in accordance with this, *for*—"—0 12·6" in line of Loss by evaporation, error in weighing, &c., *read*, —0 10·6.

Last column but two. *For*—"4 10·6" in line of Contents of stomachs, *read*—5 2·0; and in accordance with this, *for*—"61 14·7" in line of Total Offal parts, *read*—62 7·0; and *for*—"0 0·7" in line of Loss by evaporation, error in weighing, &c., *read*, —0 7·6.

Appendix-Table XXXVI. pp. 630—631. Column 17. *For*—"144 5·3" in line of Original weight, *read*—148 5·3.

Appendix-Table XL. pp. 636—637:—

Column of Designation of Parts. Omit the words—"Live-weight after fasting" in the bottom line.

Column 2. *For*—"3·17" in line of Contents of stomachs, *read*—2·18; and in accordance with this, *for*—"33·98" in line of Total Offal parts, *read*—32·97. *For*—"60·86" in line of Carcass, *read*—62·05. And in accordance with these alterations, *for*—"5·21" in line of Loss by evaporation, error in weighing, &c., *read*—4 98.

Column of Bullock No. 4. *For*—"0·11" in line of Bladder, &c., *read*—0·04; and in accordance with this, *for*—"41·11" in line of Total Offal parts, *read*—41·04; and *for*—"2·08" in line of Loss by evaporation, error in weighing, &c., *read*—2·15.

Column of Bullock No. 9. Mem.: the amount "0·07" in line of Bladder, &c., includes both Bladder and Penis.

Column of Bullock No. 12. *For*—"5·92" in line of Contents of stomachs, *read*—5·44; *for*—"0·62" in line of Small Intestines and contents, *read*—1·08; *for*—"0·86" in line of Large Intestines and contents, *read*—0·44; *for*—"0·72" in line of Heart-fat, *read*—0·44; and *for*—"0·86" in line of Miscellaneous trimmings, *read*—0 64.

Column of Bullock No. 13. *For*—"8·12" in line of Contents of stomachs, *read*—7·14; *for*—"0·07" in line of Small Intestines and contents, *read*—1·36; and *for*—"0·40" in line of Large Intestines and contents, *read*—0·78. *For*—"0 81" in line of Heart-fat, *read*—0·20; and in accordance with this, *for*—"84·54" in line of Total Offal parts, *read*—84·48; and *for*—"0·71" in line of Loss by evaporation, error in weighing, &c., *read*—0 82.

XXVI. *Experimental Inquiries into the Chemical and other Phenomena of Respiration, and their Modifications by various Physical agencies.*

By EDWARD SMITH, *M.D., LL.B. (Lond.), M.R.C.P., Corresponding Member of the Académie des Sciences et Lettres de Montpellier, and of the Natural History Society of Montreal, Assistant-Physician to the Hospital for Consumption, Brompton, &c. Communicated by Sir* B. C. BRODIE, *Bart., Pres. R.S.*

Received December 16, 1858,—Read January 20, 1859.

NOTWITHSTANDING the number of valuable observations upon this subject which have been made since the publication of the first memoir of LAVOISIER, there is but little which has been conclusively established. Of the causes inducing this, two have paramount importance, viz. the practice of deducing large from small quantities, in reference to a subject in which the quantities are ever varying, and the absence of any method whereby experiments could be repeated so frequently as to trace the changes actually proceeding during the inquiry. I have named these two because it is to correct them that I have directed my own observations.

During the past year I had the honour to transmit to the Royal Society the results of an extended inquiry into the influence of various agents over the quantity of air inspired, a short abstract of which was published in the 'Proceedings' of the Society. Since that period I have carried the inquiry further, and have determined the influence of those agents over the carbonic acid exhaled, as well as over other phenomena of respiration.

The apparatus employed by previous observers for the determination of the quantity of carbonic acid contained in the expired air, has been one of the following description:—

1st. That adopted by PROUT[*], COATHUPE[†], VIERORDT[‡], and BOEKER[§], consisting of a bag or other vessel of known capacity, into which the air was expired during a certain time, and with a noted number of expirations; and a graduated tube, into which a portion of this expired air was passed, and the carbonic acid abstracted by potass, soda, or lime. This was adapted to experiments lasting a few seconds only at a time.

2nd. A box of sufficient capacity to permit a man to be seated in it, and rendered airtight, except at points which permitted the entrance and exit of air in given directions. The analysis of the expired air was made by the aid of potass. This was SCHARLING'S[‖] method, and by it he collected the products of the lungs and skin together, during a period not exceeding 1½ hour. It was not practicable to determine the quantity of air inspired, or to what extent the air was respired more than once; and it was not easy to prove how much carbonic acid remained in the box.

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[*] THOMSON'S Annals of Philosophy, vols. ii. and iv.

[†] Philosophical Magazine, 1839.

[‡] Physiologie des Athmens, &c.

[§] Beiträge zur Heilkunde, &c., 1849.

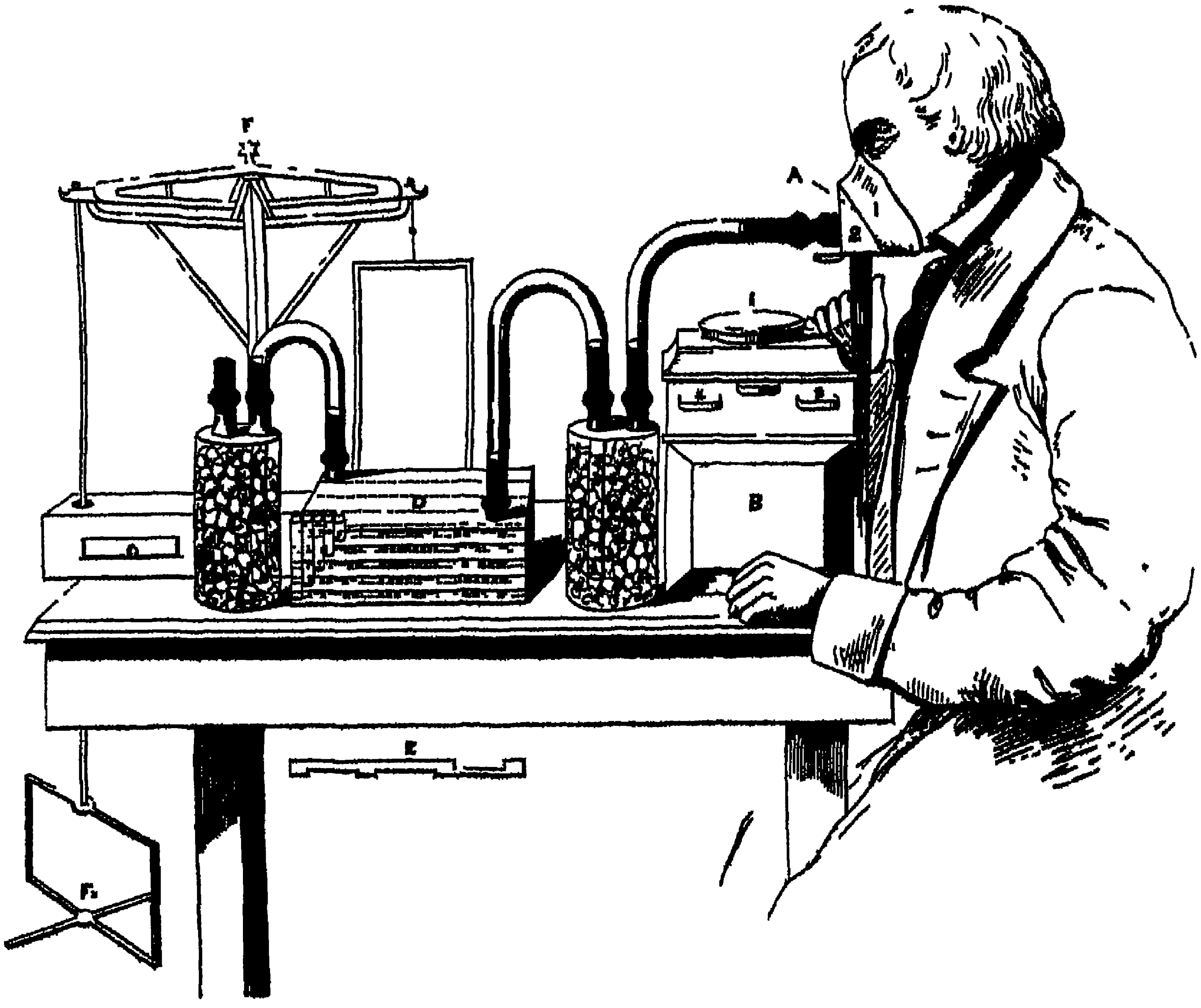
[‖] Annales de Chimie, vol. viii. p. 478.

3rd. The method of ANDRAL and GAVARRET*, by which a mask of the capacity of an ordinary expiration was placed over the whole face, and a continuous current of air made to pass through it into an analytical apparatus without respiratory effort.

As the motor power was a vacuum produced in the receiver of the expired air, the duration of each inquiry was limited to about half an hour; and as all the air admitted to the mask and the receiver did not enter the lungs, it was not possible to determine the volume of air inspired.

Hence it was necessary, in order to pursue any serial and long-continued inquiry, to devise a method which had not been adopted by previous observers.

The apparatus which I prepared for the purpose, after a long series of experiments, has already been presented to the Royal Society, and it differs from any heretofore employed, inasmuch as during the act of expiration, and for any length of time, it abstracts the whole carbonic acid exhaled by the lungs. With it was conjoined a small dry gas-meter, to measure the volume of the inspired air, as described in my former paper. The apparatus and method may be thus briefly described.



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* Annales de Chimie, vol. viii. p 129.

A mask (A) is worn of a capacity only just large enough to receive the nose, lips, and chin, and the apertures of the entrance and exit tubes. The part which sustains the tubes (A 2) and the valves is made of brass, to the free edges of which is soldered sheet-lead of sufficient thickness to remain fixed upon the features when it has been well moulded to them (A 1), and to prevent the entrance of air except through the entrance-tube. Elastic bands are suitably attached to it so as to bind it upon the head, when the hands must be left free for other purposes. The entrance-tube is connected with the spirometer by vulcanized caoutchouc tubing, and during the inspiration of the air the index of the spirometer (B 1) registers the quantity from 1 to 1 million cubic inches. The exit-tube leads to the analytical apparatus, and is connected with it by vulcanized tubing. There are valves suitably arranged to prevent a retrograde current, and so light that they do not offer any important amount of resistance to the current of the expired air.

The analytical apparatus consists of—

1st. A WOULFE's bottle (C), of the capacity of 70 cubic inches, which is filled with pieces of pumice-stone moistened with sulphuric acid. To the bottom of this the current is directed, and in it the vapour is abstracted from the expired air.

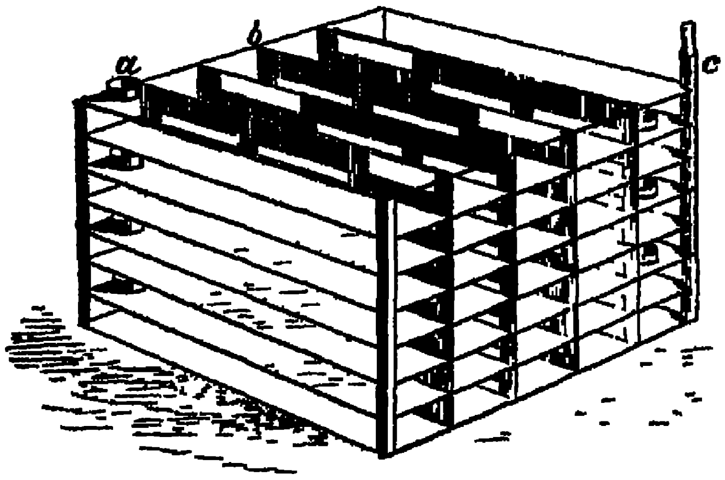
2nd. A gutta-percha box (D), consisting of a series of chambers, each $\frac{5}{8}$ths of an inch in depth, and offering a total superficies of 700 inches. The chambers are imperfectly subdivided by partitions (E) into compartments of 2 inches wide, and so arranged that the column of air must traverse the several compartments in each chamber, and each chamber in succession from the bottom to the top of the box. Hence a column of air, 2 inches wide × $\frac{5}{8}$ths of an inch deep, is directed over an area of 700 inches. This area is occupied by a solution of caustic potash of sp. gr. 1·27, which is introduced into each chamber separately, and which, through fissures in the partitions of the compartments, passes freely over the whole surface of each chamber. By this arrangement all the carbonic acid is abstracted with the rapidity of ordinary expiration; and 80 fluid ounces of the solution was found by experiment to abstract the whole carbonic acid up to 600 grains. (It will be observed that the air is not passed into, but only over caustic potash.)

3rd. A second drying apparatus (C), similar to the first one, to abstract the vapour which had been carried off from the solution of potash.

4th. A test of baryta water, over which the current was made finally to pass.

At the conclusion of each inquiry, the increase in weight of the potash-box (D), and the second drying apparatus (C), gave the amount of carbonic acid abstracted; and this was determined by the aid of one of OERTLING's balances (F), constructed to weigh, with great care, to the $\frac{1}{100}$th of a grain, with 7 lbs. in each pan; but it was not employed to indicate less than the $\frac{1}{10}$th of a grain in the large quantity of carbonic acid collected.

The apertures of the tubes throughout the apparatus had an area equal to that of the trachea, so as to offer the least possible resistance to the current of air; a pressure of $\frac{2}{10}$ths of an inch of a column of water sufficed to move the spirometer, and an adverse pressure of about half that amount was offered to expiration. By a series of experiments, I determined the extent to which the drying and carbonic-acid-abstracting apparatus were perfectly efficient; and care was always taken to keep much within those limits. Several sets of apparatus were at hand, so that by rapidly detaching one and attaching another to the mask (A), a continuous inquiry could be maintained for an indefinite period; and when large quantities of air were required to pass through them (as under the influence of exertion), a double set was used at the same time. By some of these methods I was enabled to make experiments in sleep, as well as in wakefulness, and with walking or other modes of exertion, as well as at rest; also for one or any limited number of minutes, as well as for hours, and even for the whole day, continuously, except during short intervals for meals. When the experiments were not continuous, they could be repeated every ten minutes. Thus was avoided the fallacy attaching to the different composition of different volumes or currents of the expired air by abstracting the carbonic acid from the whole; and the duration of each inquiry permitted me to escape from the errors of the influence of the mind when directed to the respiration, and of inferring large from very small quantities. In addition to this, I have endeavoured to give a more serial and extended character to the investigations than any hitherto recorded.



Drawing of a model in the possession of the Royal Society, showing the interior construction of the potash-box. There is a series of chambers communicating by openings which are defended by a flange, as at *a*, and divided by partitions, as at *b*. At *c* there are two sliding tubes, through which the air is passed into the bottom chamber, and through the lateral openings of which the solution of potash is passed into and out of each chamber separately. The openings are closed during the experiment by turning the inner tube half round its axis.

The barometric pressure, and the temperature with the wet and dry bulb thermometers, were duly recorded. The time was measured by an astronomical clock.

The objects of the inquiry were, the rate of pulsation and respiration, the quantity of air inspired, and the amount of carbonic acid contained in the expired air. The amount of vapour exhaled was also in a few instances determined; but it was found that the addition of this subject would render the inquiry too burdensome, and incapable of repetition with sufficient frequency.

In the following communication I propose to show—

I. The quantity of carbonic acid evolved, and the changes in respiration, with and without exertion, and with and without food, in the twenty-four hours of the day.

II. The variations from day to day, and from season to season.

III. The influence of walking and the treadwheel.

I. QUANTITY OF CARBONIC ACID EVOLVED, AND CHANGES IN RESPIRATION, WITH AND WITHOUT EXERTION, AND WITH AND WITHOUT FOOD, IN THE TWENTY-FOUR HOURS OF THE DAY.

1. *With ordinary food.*

No inquirer has hitherto collected the whole carbonic acid evolved by the lungs in any considerable part of the day, neither has any one made serial experiments at each of the hours of the day. Numerous observers have made isolated experiments at various periods of the day (some have even experimented during the night); and a few have, on certain days, made inquiries at intervals of about an hour for a large part of the day; and from these experiments, varying in number, duration, and interval, the quantity of carbonic acid exhaled by the lungs in the twenty-four hours has been deduced.

Lavoisier and Seguin*, in their memoir of 1789, state that the medium quantity exhaled by the latter per day was 2 lbs. 5 oz. and 4 gros, but in the memoir of 1790 they reduced it to 1 lb. 1 oz. 7 gros and 4 grs. These quantities represent 10 oz. 4 gros and 5 oz. 7 gros of carbon. They do not give the particulars of their computation.

Coathupe† computes that he expired 5·45 oz. avoirdupois of carbon daily.

Valentin and Brunner‡, from inquiries prosecuted on several persons, give 178 grs. of carbon as the quantity excreted per hour.

Andral and Gavarret§, and Scharling‖, from similar inquiries made on a number of persons respectively, state that the quantity of carbon burnt per hour varies from 77·2 grs. to 217·7 grs., and from 80·2 grs. to 154·3 grs.,—quantities equal to 283 grs. and 798 grs., and again to 294 grs. and 566 grs. of carbonic acid per hour.

Vierordt has made a larger number of experiments, and of a more uniform and serial kind than any hitherto published. He found that the medium of five of the highest and five of the lowest experiments gave 7·5 grs. of carbonic acid per minute. The minima were to the maxima as 1 to 2·25.

Barral¶ inferred the quantity of carbon burnt by determining the difference between the quantity contained in the aliment and that found in the excretions. It varied in himself from 8½ oz. to 12 oz. daily at different seasons of the year.

Baron Liebig**, by a similar mode of inquiry upon a large number of Hessian soldiers, found the medium quantity to be 13·9 oz. daily.

I do not think that we are justified in deducing the total quantity of carbon expired in the twenty-four hours from the small quantities determined per minute or per hour as above mentioned, when the authors have declined to do so; for most of the experiments were not intended to embrace all the variations of that period. Andral and Gavarret expressly state that they do not feel warranted in doing so from their experiments. Moreover, no just comparison of the results can be made, since they were obtained under dissimilar and also under unrecorded circumstances; and some of them include the carbon exhaled by the skin also. The following comparison is therefore only to a certain extent true:—

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* Mémoires de l'Académie des Sciences.

† *Loc. cit.*

‡ Lehrbuch der Physiologie.

§ *Loc. cit.*

‖ *Loc. cit.*

¶ Annales de Chimie, 3 sér. vol. xxv.

** Letters on Chemistry.

Observers		Subjects of experiment.	Carbon expired in 24 hours oz.	gros.	
Lavoisier and **Seguin**		One person.	10 5	4 7	French.
Coathupe		One person.	5·45		Avoirdupois.
Valentin and **Brunner**		Several.	9·202		Avoirdupois.
Andral and **Gavarret**		Several.	4·0068 12		Avoirdupois.
Scharling	Skin also	Several.	4·4 8·464		Avoirdupois.
Barral		One person.	8·5 12		
Liebig		Many persons.	13·9		
Vierordt		One person.	6·78		Avoirdupois.

This Table shows that the estimated quantities hitherto recorded are very diverse, and I now proceed to state the result of my own inquiries.

Four series of inquiries have been instituted to determine these facts. In two (see Tables I. and II.) the experiments were continued during ten minutes, at the commencement of each hour, and also of each half-hour immediately following the meals, whilst in the others (see Tables III. and IV.) the inquiry was continuous, that is, the whole carbonic acid was collected during the whole hours of the working day, with very short intervals for meals. Of the two first, one took place on March 12, 1858, in the Hunterian Museum of the Royal College of Surgeons, by the great courtesy of the President and Council of that learned body, beginning at 7 A.M. and ending at midnight; the other in the house of Mr. **Moul**, who, with great devotion to science, gave very efficient aid in this inquiry, continuing from 6½ A.M. until 10 P.M. on May 15.

In the first inquiry the subjects were:—

Myself, æt. 38 years, 6 feet high, 196 lbs. in weight, and a vital capacity of 280 cubic inches.

Mr. **Moul**, æt. 48 years, 5 feet 9½ inches high, 175 lbs. weight.

Dr. **Murie**, æt. 26 years, 5 feet 7½ inches high, 133 lbs. weight, vital capacity 250 cubic inches.

In the second, myself, Mr. **Moul**, and Professor **Frankland**, F.R.S., æt. 33 years, 5 feet 10½ inches high, and 136 lbs. weight.

The two latter inquiries (or those in which the whole carbonic acid was collected) were prosecuted upon myself alone; one on March 22, in the Hunterian Museum, from 6½ A.M. until 9½ P.M., and the other in my own house on August 10, from 7½ A.M. until 11½ P.M., with the exception of 1½ hour in the first, and one hour in the second inquiry for meals at three or four periods of the day. All these inquiries were made in the sitting posture, except during 2½ hours at various times in each of the two last inquiries, when relief was sought in the standing posture, at accurately noted periods. A state of mental and bodily rest was aimed at; but I undertook the weighing, and being alone in the inquiry on August 10, I had also the duty to fill and empty the apparatus; and hence, without removing from my place, there was a small degree of exertion made. In the two former inquiries exertion was taken in the intervals of the examinations, but we sat down some minutes before each experiment was renewed. It is probable that in Dr. MURIE's case there was on a few occasions a little excess in the results, owing to the want of close attention to the latter circumstance. On a review of these experiments, I believe that the two former, viz. those in which the experiment was not continuous, may be taken to represent the state of the system at rest, and the two latter in ordinary but not absolute quietude, that is, in the state in which men remain when slightly engaged but not moving the whole body.

The periods above mentioned include nearly 18 hours, viz. from 6 A.M. to 11 P.M., or from 7 A.M. to 12 P.M. all inclusive; and in order to compare them, I have made such addition by computation to each as will include that period; and I regard the results as indicating the true state of the system during the working day. The particulars of the computation are as follows:—

Computation of the Carbonic Acid expired in eighteen hours from the returns obtained in the four inquiries.

March 12. The inquiry embraced the whole period.

May 15. The inquiry was continued through 16½ hours.

		grs.		Self. grs.	Mr. Moul.	grs.
	16½ hours at	10	per minute (collected) =	9900	at 8·01 =	7950
(6 to 6½ A.M.)	½ hour at	8·85	per minute (computed) =	265·5	at 7·1 =	210·3
(11 to 12)	1 hour at	8·5	per minute (computed) =	510	at 7 =	420
				10675·5		8580·3

In Professor FRANKLAND the average of 15 hours, at 6·66 grs. per minute, was used for the 18 hours = 7192·8 grs.

			grs.
March 22.		15 hours' experiment (collected)	9166
	(6 to 6¾ A.M.)	¾ hour's experiment (computed)	353
	(9¾ to 12)	2¼ hours' experiment (computed)	1125
			10644

			grs.
August 10.		16 hours' experiment (collected)	10780
	(6 to 7½ A.M.)	1½ hour experiment (computed)	627
	(11½ to 12)	½ hour experiment (computed)	300
			11713

Food was taken at 8½ A.M., 1½, 5½ and 8½ P.M., in all the experiments. The breakfast consisted of good tea with sugar and milk, bread, butter, bacon or eggs. The dinner, of beefsteak or mutton chop, with bread, potatoes and water. The tea meal, of tea, with sugar and milk, and bread and butter. The supper of boiled eggs, and bread and butter, with water.

The details of the results of these inquiries are given in the following Tables, Nos. I., II., III., and IV.;—

Table I.

15·432 grains=1 gramme.
0·06103 cubic inch=1 cubic centimetre.

Experiment made in the Hunterian Museum.

Sitting posture. March 12, 1858.	Temperature.		Barometer.	Duration of each experiment ten minutes.											
				Carbonic acid.			Air.			Pulse.			Respiration.		
	Wet.	Dry.		Self.	Dr. Murie.	Mr. Moul.	Self.	Dr. Murie.	Mr. Moul.	Self.	Dr. Murie.	Mr. Moul.	Self.	Dr. Murie.	Mr. Moul.
h m	°	°	inch.	grs. per mn.	grs. per mn.	grs. per mn	c. i. per mn.	c. i. per mn.	c. i. per min	per minute	per minute	per minute.	per minute.	per minute	per minute
7 0 A.M.				6·3	6·14	9·5*	458	304	501	84	65	81	15	16	13·9
8 0 A.M.	46·5	50·2		6·25	6·79	8·15	385	444	502	88	65	82	16	14	14·3
8 30 A.M.	Breakfast:—Tea, bread, and bacon.														
9 0 A.M.	47	50·5	29·77	8·7	9·14	8·22	684	526	467	103	77	98	16	19	14
9 30 A.M.	47	50·8	...	7·9	8·06	11·36	464	490	479	92	78	96	16·2	16	14
10 0 A.M.	47	51·2	...	7·44	8·99	8·95	429	507	533	86	77	100	15·2	16·3	14·3
11 0 A.M.	47·9	52·2	...	8·16	8·21	8·83	...	412	499	88	76	91	15·6	15·8	14
12 0 A.M.	48	53	...	8·92	8·2	9·2	644	493	500	79	72	86	15·3	16·4	14·3
1 0 P.M.	49·2	53·9	...	8·95	7·3	7·65	602	529	462	86	72	84	15	14·2	13·8
1 30 P.M.	Dinner:—Mutton, potato, bread, and water.														
2 0 P.M.	49·8	54	...	8·8	8·06	9·08	578	414	512	92	79	89	15·1	14·6	15
3 0 P.M.	49·9	54·1	...	9·1	9·23	8·6	593	501	581	84	70	85	15·4	16	14·9
4 0 P.M.	49·9	54·1	...	9·49	7·05	8·75	605	596	464	88	82	83	15·3	17·2	14·5
5 0 P.M.	49·9	54·1	...	9·57	7·67	6·76	626	...	402	82	76	82	15·1	15·3	15·3
5 30 P.M.	Tea:—Tea, bread, and butter.														
6 0 P.M.	49·9	54·5	...	9·95	9·25	8·1	667	476	464	84	76	82	15·6	15	15
7 0 P.M.	49·9	54·5	...	9·6	8·45	10·1	710	520	540	...	72	87	16·4	16	15·9
8 0 P.M.	49·9	54	...	9·59	7·83	8·3	595	507	463	76	69	83	15·2	13·9	13·8
8 30 P.M.	Supper:—Eggs, milk, bread, and butter.														
9 0 P.M.	49·9	54	...	9·11	7·87	7·75	582	401	443	76	65	79	15·1	14·8	15·7
10 0 P.M.	49·9	54	...	9·55	7·6	8·35	585	437	480	72	66	82	14·1	14·2	14
11 0 P.M.	49·9	54	...	9·38	7·7	8·25	573	405	432	74	60	79	13·4	13	13·1
12 0 P.M.	49·9	54	...	9·52	7·33	7·5	...	398	440	74	58	79	14·2	14·4	14·8

* An excess probably due to previous exertion.

TABLE II.

Experiment made at Mr. Moul's house.

Sitting posture.	Temperature.		Barometer.	Duration of each experiment ten minutes.														
				Carbonic acid, grains per minute.			Air, cubic inches per minute.			Pulse, per minute.			Respiration, per minute.			Depth of respiration, cubic inches.		
May 15, 1858.	Wet.	Dry.		Self.	Prof. Frankland.	Mr. Moul.	Self.	Prof. Frankland.	Mr. Moul.	Self.	Prof. Frankland.	Mr. Moul.	Self.	Prof. Frankland.	Mr. Moul.	Self.	Prof. Frankland.	Mr. Moul.
h m	°	°	inch.															
6 30 A.M.	57·5	58·5	29·04	8·85	5·15	7·1	491	265	403	65	60	79	12·3	12·3	12·7	39·9	21·5	31·7
7 0 A.M.	59	60	......	8·97	5·43	7·34	464	279	402	64	59	79	12·1	12·5	12·4	38·3	22·3	32·4
8 0 A.M.	60	61	......	8·68	4·58	6·76	470	219	395	63	55	72	11·5	11·2	12·3	40·8	19·7	32·1
8 30 A.M.	Breakfast:—Tea, bacon, eggs, bread, and butter.																	
9 0 A.M.	60·5	61·5	......	10·05	6·01	7·1	551	272	460	74	67	88	12·4	11·8	12·8	44·4	23·1	35·9
9 30 A.M.	60·5	61·5	......	10·41	7·49	8·49	549	375	445	71	63	85	11·6	11·2	12·1	47·3	33·5	36·7
10 5 A.M.	61	62·5	......	10·89	8·32	8·36	560	420	442	74	61	85	12	10·3	12·2	46·8	40·8	36·2
10 35 A.M.	61	62·5	......	11·53	7·47	8·58	579	406	465	76	57	78	11·9	10·9	12·6	48·3	37·2	36·9
11 5 A.M.	62	63	......	11·27	6·23	8·11	478	369	449	71	56	76	11·9	11	12·5	40·1	33·6	35·9
12 0 A.M.	63·5	64·5	......	9·72	6·73	7·16	526	394	406	66	59	74	11·7	12·6	12·2	45*	31·3	33·2
1 0 P.M.	63	64	......	10·01	6·05	7·65	552	366	430	67	55	78	12	11·3	12·6	46	32·4	34·1
1 30 P.M.	Dinner:—Mutton, potato, and water.																	
2 0 P.M.	63·5	65	......	9·4	6·53	7·55	515	359	419	80	63	78	12	11·6	12·6	43·2	31	33·2
2 30 P.M.	63·5	66	......	9·5	6·79	7·58	597	385	391	70	59	78	11·2	11·2	12·8	53·3	32	30·5†
3 0 P.M.	63·5	65	......	9·9	6·45	9·02	517	350	467	73	60	78	11·5	11·3	12·2	45	30·9	38·2
4 0 P.M.	......	......	......	10·82	6·97	8·48	535	401	445	70	59	79	11·1	13	12·6	48·2	28·6	35·4
5 0 P.M.	63	66	......	9·02	6·78	8·05	555	391	420	72	55	78	11·7	12·8	13·1	47·2	30·5	32
5 30 P.M.	Tea:—Tea, bread, and butter.																	
6 10 P.M.	63·5	68	......	9·67	7·0	8·47	590	403	427	73	60	78	11·9	12·6	12·1	49·9	32	35·2
6 35 P.M.	......	......	......	11·49	7·45	8·66	520	423	440	74	58	80	12·1	12·9	11·9	43 ‡	32·8	36·9
7 0 P.M.	......	......	......	11·0	7·75	9·18	533	438	469	73	62	82	11·3	12·9	12·6	47·1§	34	37·2
8 0 P.M.	......	65	......	10·72	7·65	9·35	587	409	456	69	55	78	11	11·7	12	53·3	35	38
9 0 P.M.	......	......	......	9·11	......	8·26	505	......	443	64	......	72	10·8	......	13·3	46·7	......‖	33
10 0 P.M.	......	62	......	8·75	......	7·18	490	......	396	65	......	80	10·4	......	13·8	47·1	......	28·7

* Professor Frankland tested with baryta-water. † Drowsy; full; hot. ‡ Arteries throb; hands swollen § More sombre. ‖ Dr Frankland being very tired had left

TABLE III.

Experiments made in the Hunterian Museum on myself alone.
Continuous inquiry. Carbonic acid weighed every quarter or half hour.

March 22, 1858.	Temperature		Barometer.	Carbonic acid per min	Air per min		Pulse		Rate of respiration	Posture.
	Wet.	Dry.								
h m	°	°	inch.	grs.	cub. in		per min		per min.	
6 45 A.M.	...	...	...	7·93	474	......	77	Little sinking	...	Sitting.
7 0 A.M.	...	...	30·14	7·53	487	......	75	....	15	"
7 15 A.M.	...	...	...	7·66	449	......	75	....	[*]15+	"
7 30 A.M.	...	...	...	7·6	429	Little escape.	...	....	15+	"
7 45 A.M.	...	..	...	7·63	446	......	73	....	15+	"
8 0 A.M.	...	...	...	7·6	445	......	76	....	...	"
8 15 A.M.	...	...	...	7·7	481	......	80	....	15	"
8 30 A.M.	Breakfast:—Tea, cold ham, and bread.									
8 45 A.M.	54·5	58·8	...	10·0	579	Walking from breakfast.	...	....	...	"
9 0 A.M.	...	...	...	11·0	612		89	....	...	"
9 15 A.M.	...			10·2	575	......	88	....	15·5	"
9 30 A.M.		...	...		582	......	92	....	16	"
9 45 A.M.	54·2	58·5	...	9·9	588	......	90	....	16−	"
10 0 A.M.	...	...	...	9·2	573	......	...	....	17	"
10 15 A.M.	...	...	...	10·2	566	......	78	....	16+	"
10 30 A.M.	...	...	...	10·4	609	......	76	....	17+	"
10 45 A.M.	...	...	...	10·3	597	......	...	....	17+	"
11 0 A.M.	...	...	...	10·5	696	......	82	....	19·5	Standing.
11 15 A.M.	...	...	...	10·7	642	......	...	....	19	"
11 30 A.M.	...	...	...	10·3	687	......	76	....	18	"
11 45 A.M.	...	...	...	10·8	533	......	...	....	...	"
12 3 A.M.	...	...	...	11·4	859	Excited.	...	....	...	Sitting.
12 15 A.M.	...	...	...	10·7	605	......	...	....	...	"
12 33 A.M.	...	...	...	10·9	611	Little excited.	76	....	17·5	"
12 45 A.M.	56·8	60·9	...	10·5	589	......	...	....	17	"
1 0 P.M.	...	...	...	10·3	604	......	77	....	17	"
1 15 P.M.	...	...	...	10·9	556	......	72	Little sinking.	17	"
1 30 P.M.	Dinner:—Mutton, bread, potatoes, and water (enjoyed it).									
1 47 P.M.	..	...	...	9·2	582	Walking from dinner.	78	....	16·5	"
2 0 P.M.	58·0	62·0	30·3	9 0	531		80	....	16·5+	"
2 15 P.M.	...	...	...	8·9	604	......	...	....	17	"
2 30 P.M.	...	...	...	8·8	568	......	78	....	17	"
2 46 P.M.	...	...	...	9·0	588	......	...	....	...	"
3 0 P.M.	...	...	...	9·8	605	......	...	....	17+	"
3 15 P.M.	...	...	...	9·6	616	......	78	....	17·5	Standing
3 30 P.M.	...	...	...	9·0	574	......	...	....	17	"
3 45 P.M.	...	...	...	8·3	564	......	90	Little oppressed.	...	"
4 0 P.M.	...	...	...	9·2	580	......	...	....	17	"
4 15 P.M.	...	...	...	8·7	568	......	...	....	...	"
4 30 P.M.	...	...	...	9·0	633	......	...	....	16·5	"
4 46 P.M.	...	...	...	9·5	625	......	...	....	16·5	Sitting.
5 0 P.M.	...	...	...	9·5	576	......	73	....	...	"
5 15 P.M.	...	...	...	8·5	574	......	...	....	16·5	"
5 30 P.M.	Tea:—Tea, bread, and butter.									
5 45 P.M.	...	...	...	10·5	720	Walking from tea.	74	....	18	"
6 0 P.M.	...	...	...	9·7	575		80	....	19	"
6 15 P.M.	...	...	...	11·2	675	......	...	....	18·5	"
6 30 P.M.	...	...	...	10·1	665	......	80	....	...	"
6 45 P.M.	Lost time waiting for lamps.									
7 0 P.M.										
7 15 P.M.	...	...	...	8·58	571	......	...	....	...	Sitting.
7 45 P.M.	68·3	68·0	...	8·9	579	......	72	....	...	"
8 15 P.M.	...	...	...	8·5	578	......	...	....	15	"
8 45 P.M.	...	...	...	8·76	590	......	70	....	14·5	"
9 15 P.M.	...	...	...	7·98	535	......	71	....	15	"

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[*] I talked occasionally during the whole inquiry. The signs + and − indicate that the rate of respiration was a little more or a little less than the figures indicate.

TABLE IV.

Experiment made at my house on myself alone.

Continuous inquiry. Carbonic acid weighed only at the hours stated.

August 16, 1858.	Temperature.		Barometer.	Carbonic acid, grs. per minute.	Carbonic acid, grs. per minute in each hour.	Air, cubic inches.	Pulse, per minute.	Remarks.	Posture.
	Wet.	Dry.							
h m 7 30 A.M.	°	67·5	inch. 29·65	6·96	6·96	475	75	Rain	Sitting.
8 30 A.M.	Breakfast:—Coffee, eggs, bread, and butter (not good one); hurried.								
8 45 A.M.	......	......	......	8·13	9·12	538	79	Perspiring; little excited; very warm	Sitting.
9 30 A.M.	......	69	29·63	12·4	12·4	593	85		
10 30 A.M.	......	70·5	......	10·41	10·42	650	79		
11 30 A.M.	......	71	......	10·96	10·97	578	78	Cooler	Sitting.
12 30 A.M.	......	......	......	10·18	10·91	576	...	....	Sitting.
12 45 A.M.	......	......	......	11·26		584	74	....	Sitting.
1 0 P.M.	......	71·5	29·62	11·04		586	...	Not wanting food	Sitting.
1 30 P.M.	Dinner:—Mutton-pie, potato, and water. Good meal.							Drowsy	Sitting.
1 46 P.M.	......	......	......	10·13	10·59	501	......	....	Standing from 2·40.
2 30 P.M.	......	73	......	10·1	11·3	557	......	Yawning, and oppressed; irritable; bear the experiment badly	Sitting.
3 5 P.M.	......	......	......	12·48		601	86		Legs horizontal from 4·0 to 4·5; standing 4·15 to 4·30.
3 30 P.M.	......	74·5	......	11·89	11·89	579	......		
4 30 P.M.	......	73·3	......	11·62	11·62	547	......	....	Standing ½ hour; sitting 20 minutes; and standing 25 minutes.
5 30 P.M.	Tea:—Tea, bread, and butter.								
5 45 P.M.	......	74·2	......	13·2	13·07	652	......	....	Sitting ½ hour; standing ¼ hour.
6 30 P.M.	......	......	......	11·2	11·21	578	80	....	Sitting ¼ hour; standing ½ hour; sitting ¼ hour.
7 30 P.M.	......	73·8	......	......	12·86	783	......	Lighter and more comfortable	Sitting.
7 46 P.M.	......	......	......	12·74		633	......	Very close; perspiring profusely.	Sitting ½ hour; standing 14 minutes.
8 30 P.M.	Supper:—Milk, oatmeal, and bread.								Sitting.
8 48 P.M.	......	73·8	......	11·69	11·6	609	75	....	Sitting.
9 30 P.M.	......	......	......	12·3	13·28	579	......	....	Sitting ½ hour; standing 10 minutes.
9 50 P.M.	......	......	......	13·7		623	......	....	
10 30 P.M. to 11¼	......	74	......	11·32	11·32	567	73	Perspiring	Sitting.

A. *Carbonic Acid evolved.*

The carbonic acid during eighteen hours, or the period constituting the working day, was as follows in oz. avoirdupois (28·332 grms.), in the order of the inquiries as above mentioned.

	March.		May.		Continuous inquiries.			
					March		August	
	oz.	grms.	oz.	grms.	oz.	grms.	oz.	grms.
Myself	21·6	(611·97)	24·4	(691·3)	24·328	(689·28)	26·776	(758·41)
Mr. Moul . . .	21·205	(600·78)	19·33	(547·65)				
Dr. Murie . . .	19·582	(554 797)						
Prof. Frankland			16·43	(465·49)				

The average results were,—myself, 24·274 oz. (687·7 grms.) (9·77 grs. per minute), a quantity identical with the results of two out of the four inquiries, and Mr. Moul, 20·267 oz. (574·11 grms.); whilst the total average of the eight sets of inquiries was 21·693 oz. (614·6 grms.), or 8·78 grs per minute. This contains a total quantity of 5·917 oz. (167·64 grms.) of carbon.

The quantity evolved during the remaining six hours of the day was determined by two inquiries upon myself alone.

On July 15, whilst scarcely awake, at 1½, 2½, and 6½ A.M., I exhaled 5·7, 5·94, and 6·1 grs. per minute, on the average of one-quarter of an hour at each of those hours. On July 16, during light sleep, at 1 and 3 A.M., the quantities were 4·88 grs. and 4·99 grs. per minute on the same average (Table VIII. July 16). As the latter quantities are notably less than the former, and yet the difference in the state of the system was not great, I think it probable that in profound sleep the quantity would be still less, and probably so low as 4½ grs. per minute. The duration of profound sleep is, however, short; for in a series of hourly inquiries into the rate of pulsation and respiration, both in health and disease, published in the Transactions of the Royal Medical and Chirurgical Society for 1856, I showed that the lowest rate of the respiration was from 1 to 3 A.M.; and in applying the above calculation of the influence of profound sleep I limit its occurrence to those two hours. In estimating the state of the respiration in the hour preceding, and also in the hours succeeding, until the hour when my inquiries in the working day commenced, I take the average between the state in profound sleep and at those preceding and succeeding hours. From these data the result is obtained of 1950 grs. of carbonic acid evolved during those six hours; and this, added to the total average of the working day, gives a total for the twenty-four hours in a state of rest of 26·193 oz. of carbonic acid, or 7·144 oz. of carbon.

The same addition being made to the results obtained from each person, the quantities of carbon exhaled in twenty-four hours, in a state of rest, are as follows in ounces avoirdupois:—

Mr. Moul.	Myself.	Professor Frankland.	Dr. Murie.
Æt. 48. 5 feet 9½ inches high. 173 lbs. weight.	Æt. 39. 6 feet high. 196 lbs. weight. Vital capacity 270 cub. in.	Æt. 33. 5 feet 10½ inches high. 130 lbs. weight.	Æt. 26. 5 feet 7½ inches high. 133 lbs. weight. Vital capacity 250 cub. in.
6·735 oz.	7·85 oz.	5·6 oz.	6·54 oz.

In myself the proportion of carbonic acid evolved per hour during the night of six hours, is to the day as 1 to 1·8,—a proportion which differs somewhat from that which Scharling found with nine hours' rest.

There yet remains for determination the influence of exertion. There are not on record any experiments in reference to the influence of exactly defined degrees of exertion, except that of raising certain weights through a given space in a given time; but Regnault, Vierordt, and others have ascertained in a general way that exertion increases the respiratory changes. I shall again refer to this subject at a future page.

Whilst walking at two miles per hour during three-quarters of an hour, and carrying the spirometer, weighing 7 lbs., I expired 18·1 grs. of carbonic acid per minute, and 25·83 grs. when walking at three miles per hour. These quantities represent 1·85 and 2·64 times that of quietude in the sitting posture. Hence if three hours were spent in walking at two miles per hour, and one hour at three miles per hour, as is probably the case with the non-laborious class (2⅓ hours in the standing posture having been introduced into each of the two continuous inquiries above recorded when at rest), the addition to the daily quantity of carbonic acid evolved will be 2463 grs.; and it is probable that thrice that quantity would be applicable to the really laborious class. It is true that these are probabilities and not demonstrations; but if it be desirable to determine the daily amount of carbonic acid evolved by the community, it can only be effected by dividing them into classes, and determining the precise influence of certain agents during their period of action.

These quantities give the following total average results in the twenty-four hours:—

In quietude	26·193	oz. of carbonic acid =	7·144 oz. of carbon
Non-laborious class . .	31·824	oz. of carbonic acid =	8·68 oz. of carbon
Laborious class . . .	43	oz. of carbonic acid =	11·7 oz. of carbon,

or a general average of 33·67 oz. of carbonic acid and 9·18 oz. of carbon.

As some of the elements in this calculation apply to myself alone, viz. those in reference to sleep and exertion, it may be of interest to determine my average apart from that of those who were conjoined with me in some parts of the inquiry. Thus,

In quietude	28·8	oz. of carbonic acid =	7·85 oz. of carbon
As a member of the non-laborious class	33·43	oz. of carbonic acid =	9·11 oz. of carbon
As a member of the laborious class ..	45·7	oz. of carbonic acid =	12·19 oz. of carbon,

or a total average of 35·97 oz. of carbonic acid, and 9·72 oz. of carbon in the twenty-four hours.

I do not know how to compare these results with those given by previous observers, since the degree of exertion and other circumstances are not recorded by the latter, and hence it is impossible to appreciate the state of the system to which they refer.

It is noticeable how great is the difference amongst men in the amount of carbon which they expire—a fact which has been observed by all inquirers. Thus Professor FRANKLAND and myself differed as 2 to 3, and yet we were nearly of the same age and height, but differed greatly in weight and robustness of constitution. Each person also differs much at various periods—a circumstance noticed by BARRAL. Mr. MOUL varied as 10 to 11, and I at least as 11 to 12. Hence it is safer to adopt comparative rather than absolute quantities in determining the quantity of carbonic acid exhaled per day by any individual. Thus if the amount of carbonic acid exhaled in a state of quietude in the day-time be determined by observations made at various periods of the day in relation to the meal hours, the amount exhaled during six hours of the night may be determined by allowing the amount of 3¼ hours of the day; and in reference to exercise, by allowing 1⅘ hour rest for one hour of walking at two miles per hour, and 2⅗ hours of rest for one hour of walking at three miles per hour.

B. *Quantity of Air inspired.*

In the four series of inquiries now recorded, the quantity of air inspired in cubic inches during the working day was per minute as follows:—

			Continuous inquiries.	
	March. cub. in.	May. cub. in.	March. cub. in.	August. cub. in.
Myself	575	531·5	582·1	583
Mr. Moul . . .	483·1	431·7		
Dr. Murie . . .	464·5			
Professor Frankland		364·4		

The average quantity inspired by myself was 567·7 cubic inches, and by Mr. MOUL 457·4 cubic inches, whilst the average of all the observations was 502 cubic inches per minute.

In order to determine the proportion which the quantity of air inspired bears to the carbonic acid collected (for as I have not measured the expired air I cannot take that as a point of comparison), I insert the following statement of the amount of carbonic acid per minute which was actually collected, with the quantities of air which have just been given. It is as follows:—

					Continuous inquiries.				
	March.		May.		March.		August.		Average.
	Carb. acid. grs.	Air. cub. in.	Carb. acid. grs.	Air. cub. in.	Carb. acid. grs.	Air. cub. in.	Carb. acid. grs.	Air. cub. in.	Carb. acid. grs.
Myself . .	8·75	575	10	531·5	9·5	582·1	10·9	583	9·79
Mr. Moul	8·6	483·1	8·06	431·7					
Dr. Murie	7·93	464·5							
Professor Frankland			6·66	364·4					

Hence the average of the whole was 8·8 grs. per minute, and the proportion was, in myself, Mr. **Moul**, Dr. **Murie** and Professor **Frankland** in order, 1 gr. of carbonic acid to 58 cubic inches, 54·8 cubic inches, 58·5 cubic inches, and 54·7 cubic inches of inspired air, whilst on the whole it was 1 gr. to 56·3 cubic inches. This is a lower proportion than occurs in fasting (the normal state of the system), and much lower than with exertion, in which it increases as the exertion increases. Thus in walking at two miles per hour it was 1 to 44·1 cubic inches, and at three miles per hour 1 to 39·7 cubic inches.

In the above experiments the average temperature with the dry-bulb thermometer was 58°·18, 63°·4, 60°·6, and 72°·2, yielding a total average of 63°·5. The barometric pressure was on the average 29·72, 29·04, 30·13, and 29·63, or a total average of 29·63 inches.

C. *Rate of Respiration.*

This was as follows on the average per minute in each of the three first series of experiments. It was not recorded in the last one.

	March.	May.	Continuous inquiry. March.
Myself	15·2	11·63	16·7
Mr. Moul	14·45	12·6	
Dr. Murie	15·4		
Professor Frankland		11·8	

The total average in myself was 14·54, and in Mr. **Moul** 13·5; whilst that of the whole series was 13·87 per minute.

D. *Depth of Inspiration.*

The average depth of inspiration was in myself, 39·5 cubic inches; Mr. **Moul**, 33·8 cubic inches; Dr. **Murie**, 30 cubic inches; and Professor **Frankland**, 30·9 cubic inches; giving a total average of 33·6 cubic inches.

E. *Rate of Pulsation.*

In the four series of inquiries the rate of pulsation was per minute—

	March.	May.	March.	August.
Myself	83·7	69·6	78·5	78·4
Mr. Moul . . .	85·7	74		
Dr. Murie . . .	71·3			
Professor Frankland		59·1		

giving total averages in each of 76·5, 79·8, 71·3, and 59·1, and on the whole 71·7 per minute.

The factor of the respirations required to render the numbers of respirations and pulsations equal, was, in Mr. **Moul**, 5·72; myself, 5·25; Professor **Frankland**, 5; and Dr. **Murie**, 4·63; variations which are very noticeable and follow the order of age. The total average was 5·15.

I have sought to discover a proportion between the pulsations and respirations together and separately on the one hand, and the quantity of air inspired, or the carbonic acid exhaled, on the other; but by no combinations have I been able to discover it. **Lavoisier** affirmed that the volume of oxygen consumed was "en raison directe du produit des inspirations par les pulsations." I find that half of the product of the respirations and pulsations will very nearly represent the number of cubic inches of air inspired by myself and Professor **Frankland**, and in the whole results combined, but it is excessive when applied to Dr. **Murie** and Mr. **Moul**.

The difference amongst men which has been already noticed in respect to the carbonic acid, is equally observable in all other subjects of inquiry. Thus Professor **Frankland** and myself differed in the quantity of air inspired as 3 to 5, and in pulsation as 6 to 7. Dr. **Murie** exhibited the highest proportion of respirations to pulsations, and Mr. **Moul** the least; the former being the youngest, and the latter the oldest subject of the inquiry.

2. *Without food.*

There have been numerous experiments of long duration made on animals whilst fasting, all of which have proved that in the absence of food the quantity of carbonic acid is reduced. No similar experiments are upon record in reference to Man; but **Lavoisier**, **Scharling**, **Vierordt**, **Valentin** and others, have shown that when the interval between meals is prolonged the same result is obtained.

On July 6 I fasted from 9½ A.M., after a rather small breakfast (12½ hours having elapsed since the previous meal), until midday of July 7, and made an inquiry into the state of the respiration during five minutes at the commencement of each hour, from 1 to 11 P.M. on the 6th, and from 7 to 12 A.M. on the 7th. After having fasted twenty-five hours, I wished to ascertain if starch would produce more respiratory changes when the system was empty of food than under ordinary circumstances, and I took 500 grs. of arrowroot boiled with 10 oz. of water; and having found that the secretions were very alkaline, I took 6 drachms of table vinegar in water in an hour after having taken the starch. The details of this experiment are recorded in Table V. [See opposite page.]

The total quantity of carbonic acid evolved gave an average of exactly 7 grs. per minute. If to this quantity, computed for the eighteen hours of the working day, be added that previously assigned to the six hours of the night, the total quantity of carbonic acid evolved in the twenty-four hours will be 21·78702, containing 5·923 oz. of carbon. The total average of the twenty-four hours was thus 6·61 grs. per minute, and the deficiency from the quantity evolved with food, from the data already given, was 25 per cent., or one-fourth of the larger quantity. The quantity of carbon thus evolved daily in the absence of food is very nearly equal to that contained in 20 oz. of bread, or 7½ oz. of fat.

The quantity of air inspired during the working day, was on the average 367·5 cubic inches per minute. The rate of respiration 10·86, and of pulsation 65·6 per minute. The diminution, when compared with the experiment with food in May, was in the

TABLE V.

Effect of a long fast.

Experiment made at my house upon myself.

Sitting posture.	Temp.		Barom.	Duration of each inquiry five minutes.						
July 1858.	Wet.	Dry.		Carbonic acid.	Air inspired.	Pulse.	Rate of respiration.	Vapour exhaled.	Depth of respiration.	
h m	°	°	inch.	grs. per min.	c.i. per min.	per minute.	per minute.	grs. per min.	cub. inch.	
July 6, 9 30 A.M.	Breakfast:—Coffee, bread, butter, and pickled salmon.									
1 0 P.M.	61·4	63	29·25	7·46	396	71 good	12	2·26	33	
2 0 P.M.	62	63·6	29·25	6·9	369	68 feeble	11·6	2·26	31·8	
3 0 P.M.	62·3	63·8	29·22	7·44	368	……	11·1	1·98	33	
4 0 P.M.	61·6	63·6	29·22	7·12	360	67	11	2	32·7	
5 0 P.M.	62	64·3	29·21	7·04	399	62 very soft	11·4	2·06	33·2	
6 0 P.M.	61·8	63·8	29·21	7·0	264	……	11·6	2·08	31·3	
7 0 P.M.	62·2	64	29·21	7·0	375	63 soft	11·4	2·1	32·9	
8 0 P.M.	61·8	63·8	29·21	6·66	359	60 very soft	10·7	1·8	33·5	
9 0 P.M.	62·2	63	29·21	7·04	360	58 very soft	10·7	1·74	33·6	Yawning; much less craving.
10 0 P.M.	…	…	…	7·06	359	58 very soft	10	1·8	33·6	
11 0 P.M.	61	63	29·22	6·52	341	58 very soft	10·3	…	33	
July 7, 7 0 A.M.	60·5	62	29·32	7·02	369	71	10·4	2·02	34·1	Faintness.
8 0 A.M.	61·2	62·3	…	6·58	357	……	10·6	2	33·6	Faintness; sickly; head throbbing.
9 0 A.M.	61·3	63·5	29·32	7·22	369	66 very soft	10·4	2·2	35·4	
10 0 A.M.	61·3	63	29·32	6·68	373	66	10·2	1·94	36·5	
10 21 A.M.	Arrowroot 500 grs., 10 oz. water.									
10 25 A.M.	…	…	…	6·98	384	75 sharper	10·8	2·06	35·5	
10 43 A.M.	…	…	…	7·22	356	73 feebler	10·6	2·2	33·6	
11 0 A.M.	…	…	…	7·22	348	64	10·4	2	33·4	
11 15 A.M.	…	…	…	7·1	370	67 soft	10·3	…	35·8	
11 19 A.M.	Vinegar 6 drachms, 3 oz. water.									
11 22 A.M.	…	…	…	7·6	376	69 sharper	10·4	…	36·1	
11 38 A.M.	…	…	…	7·08	384	73	10·6	…	36·2	Headache; sickly.
11 50 A.M.	…	…	…	7·52	392	71	10·4	…	36·7	

quantity of air 30 per cent., in the rate of respiration about ·7 per cent., and of pulsation about 6 per cent. Hence the rate of the functions was far less variable than the amount of vital action.

At 11 P.M., or 13½ hours after the last meal, I had felt nothing unpleasant. There was a feeling of great tameness, and the pulse was wavy or jerking and very soft. The respiration was feeble. On going to bed I was very cold, and notwithstanding the addition of blankets, it was some hours before my feet became warm. I slept very fairly, and in the morning, at 7 o'clock, the chief feature was still tameness or inaction. There was, however, headache, accompanied by pulsation, and it was increased on lying down. There was also a disagreeable taste in the mouth and sinking at the stomach and bowels. At 9 A.M., the usual hour for breakfast, there was still greatly lessened nervous, mental, and muscular power, a sickly and fainting feeling at intervals, and throbbing in the head constantly; causing with the nausea a most unpleasant but not constant headache. The pulse was still remarkably soft, feeble, and wavy. There was no marked thirst or craving for food. At 10 A.M. the urine was high coloured, alkaline, with a specific gravity of 1·018. The saliva was very alkaline.

The starch and water filled the pulse temporarily, but rather increased than relieved the depressed state of the system. The acid gave almost instantaneous relief to the headache, but the benefit was only temporary, and it did not diminish materially the alkalinity of the saliva. At a little before the experiment ended I was very low and ill. A cup of tea gave no relief, but bread and butter was of some service. At 12^{h} 10^{m} I began to eat a good dinner, and took a glass of wine; and so soon as I had fairly began to eat the symptoms abated, and at the end of the meal I was quite well and went about my duties. It is important to remark that nothing but nutriment was of any avail. I had no longing for acids, or for anything in particular, except perhaps potatoes.

Vapour exhaled.—In this inquiry I also determined the quantity of vapour exhaled by the lungs; and although the subject has not been referred to in this paper, the uniform state of the system during fasting afforded so good an opportunity to determine it, that I think it right to insert it.

On the average of the whole inquiry, I exhaled 2·02 grs. of watery vapour per minute during the working day, making a total of 5 oz. avoirdupois in that period. On various other occasions with food I found that the quantity exhaled was from 3 grs. to 3·4 grs. per minute, and hence the diminution during the fast was 37 per cent. The quantity of vapour to each 100 cubic inches of expired air was ·548 gr., and hence assuming that the expired air had a temperature of about 98°, it was but little more than half saturated.

II. VARIATIONS FROM DAY TO DAY, AND FROM SEASON TO SEASON.

a. During the Working day.

1. *With ordinary food.*

Variations in the quantity of carbonic acid exhaled at different parts of the day have been affirmed to occur by SEGUIN, PROUT, ALLEN and PEPYS*, VIERORDT and other observers, and they have been ascribed to food, exertion, and sleep. Some of these observers, as **Prout** and **Coathupe**, determined only the per-centage in the expired air; an inquiry of no value in determining the total amount of carbonic acid exhaled. **Lavoisier** states that the quantity differs every moment, and with every person, but each person has a law for himself. **Scharling** states that the proportion of the night to the day varied from as 1 to 1·225 to as 1 to 1·42, whilst **Marchand** regarded the difference as inconsiderable and due simply to quietude. In my experiments the variations during the day were extremely great, so much so that the maximum and minimum quantities of carbonic acid usually differed to the extent of more than half the latter. They were as follows in grains per minute in the four sets of inquiries:—

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* Philosophical Magazine, 1808, p. 242; Philosophical Transactions, 1809, p. 405.

	Min.	Max.		Min.	Max.		Min.	Max.
Myself	6·25 and	9·59	Mr. Moul	6·76 and	11·56	Dr. Murie	6·14 and	9·25
	8·68 and	11·53		6·76 and	9·35	Professor Frankland	4·58 and	8·32
Continuous inquiries.	7·81 and	11						
	6·96 and	13·3						

the total being 6·74 and 10·43 grs. per minute.

These variations were due to food, and were of such a nature that an increase began directly after a meal and progressed to a maximum, after which they declined gradually to a minimum until the following meal.

Generally the maximum quantity was observed in from one to two hours after a meal, and after each meal, but after the breakfast and tea meals it was the greatest, and both were nearly the same. The minima were observed before a meal, and hence there were five in each day, viz. before breakfast, dinner, tea, and supper, and some time after supper; and it is very noticeable that they were nearly the same at each of those periods. Hence there is in a state of quietude in each day a minimum line below which the system does not pass, and also a maximum which it does not exceed, the difference between the two being due to the temporary influence of food. In several of the inquiries, however, as for example in that of Professor **Frankland**, in my first and in Mr. **Moul**'s second experiment, there was not in the afternoon the ordinary amount of diminution in the quantity of carbonic acid exhaled; and as this was exceptional, and we dined before our usual dinner hour, it was probably owing to food having been supplied before the action upon the respiration of the previous quantity had ceased (Plate XXXIII. fig. 1, March 12). There can be no doubt that, with a suitable interval between meals, there are striking alternate elevations and depressions in the line representing the quantity of carbonic acid excreted.

A reference to the above maximum and minimum quantities, and to the variations hour by hour as shewn in the Tables I. II. III. and IV., with the want of uniformity in any of these quantities, proves that any attempt to determine the whole carbonic acid evolved in a day from isolated or irregular observations must be futile. They, however, show that observations taken at the period of the day when the quantity of carbonic acid evolved is at the lowest point, viz. four to five hours after a meal, will be nearly uniform, and that the excess due to food may be approximately arrived at by taking observations at 1½ to 2½ hours after the meal, and deducing the average from those quantities.

2. *Without food.*

The variations in the respiratory phenomena in the working day were so small, that at 2, 4, 5, 6, 7, 9 and 10 o'clock P.M., on July 6, all the quantities were almost identical with those at 7 on the following morning. The maximum and minimum quantities of carbonic acid were 6·52 grs. and 7·44 grs.; of air inspired, 399 cubic inches and 311 cubic inches; rate of respiration 12 and 10·2, and rate of pulsation 71 and 58 per minute —quantities which contrast in a remarkable manner with those previously shown to occur with food. In Plate XXXIII. fig. 1, the variations with and without food are contrasted.

Hence there is great uniformity in the respiratory phenomena during a long fast; but it was noticeable that on the occasions when the carbonic acid would have increased with food, there was a slight decrease without food—a fact corresponding with that which I recorded in the Transactions of the Royal Medical and Chirurgical Society for 1856, in the rate of the functions on five persons of different sexes and ages. A low and uniform state of system is therefore the characteristic condition in a prolonged fast; and it is very like that which occurs with food at the end of the interval between the meals. It has also been shown that in a fast of this duration the quantity of carbonic acid does not progressively diminish, but is the same at the end of twenty-seven hours as it was at the end of 4½ hours.

There is therefore a state of the system which is nearly uniform under all the circumstances of the day, when the body is uninfluenced by exertion or the primary processes of digestion, and which may be called the basal or normal state. From this but little can be taken away; and the additions must be due to exertion or the temporary influence of compound aliments. This I shall subsequently show to be a fact of much interest and importance, since it will be proved that certain substances which, as food, must contain nutriment, do not increase the products of respiration over this basal line; whilst others, including all compound aliments, do produce an increase.

The great uniformity in the state of the system during fasting renders that period particularly fitted for the determination of the numerical relations between the carbonic acid, air inspired, and rate of pulsation and respiration, and also as a basis with which to compare the influence of food and other agents. The law before mentioned in reference to myself was well exemplified in this experiment; for the cubic inches of air inspired are very nearly represented by the half of the product of the pulsations and the respirations. There was 1 gr. of carbonic acid to 52·5 cubic inches of air; which if we consider it as a normal, the proportion with food is less and with exertion is greater.

This experiment also confirms REGNAULT's observation, that the composition of the expired air is uniform during fasting.

In reference to the variations of the day in the quantity of vapour exhaled, the highest amount (2·26 grs. per minute) was obtained in the two first hours of the inquiry, or those nearest to the last meal, and the lowest amount (1·74 gr.) was obtained after 8 P.M. On the following morning the quantity was increased to 2·2 grs. per minute, and it did not exceed that after I had taken the arrowroot and water.

I do not find any observations on record with which I can compare these results.

There yet remain for examination two other sources of variation in the respiratory phenomena, viz. those occurring at short intervals from day to day, and those extending over a lengthened period of months and associated with season. I shall now describe them.

The experiments connected with both heads of the inquiry were made in the investigation of the influence of food, and under precisely the same external circumstances as to period of the day, food, exertion and excitement; for they were all made between 7 and 8 A.M., before breakfast, in the sitting posture, and in the absence of all exertion and mental excitement. They were commenced on March 30, and were continued with regularity by Mr. MOUL and myself until the middle of June, and then by myself alone until the present time.

β. Variations from day to day.

Nearly all observers have found that the absolute quantities recorded on any day did not correspond with those of other days; but as the inquiries were not made under precisely the same external circumstances, it was not possible, with certainty, to assign any cause to which the variation could be attributed. It would not perhaps be unreasonable to presume that, with identity of external circumstances on successive days, the system would not vary in any appreciable degree; but a reference to Plate XXXIV. will show that whilst in not a few instances the quantities obtained were constant, in many others the variation from day to day in the amount of carbonic acid amounted to half a grain, and in a few to even one grain per minute; and that this was not due to any error in the inquiry, may be inferred from the fact that, when a food was under examination which did not vary the respiratory changes, the same numbers were obtained in several successive experiments within from one to two hours. Hence we must search within the system for the explanation, and we shall find reason to believe that the nature of the night's rest, and the amount of nutriment remaining in the system, cause variations in the amount of carbonic acid expired in the early morning, before food has been taken. On many occasions I noted these conditions at the time that I made the experiment. Thus I passed a restless night on April 7, June 1, 16, 18, 23 and 24; had short night's sleep on April 24 and 28; was not well on April 15, 16, May 8, 18, June 14, and

Carbonic acid.	
Increase	Decrease
gr.	gr.
−0·29	−0·6
+0·1	−1·5
+0·8	−0·3
+0·44	+0·18
+0·6	−0·58
+0·58	+0·22
+0·2	−0·36
−0·2	+0·2
+0·68	−1·32
+0·16	+0·16
	−0·26
	−1·12
	−0·67
	−0·27

September 14; and in almost every instance there was a fall in the quantity of carbonic acid on the following morning (see Table). I passed a good night and was very well on April 20, 26, 30, May 19, 20, 24, June 4, 21, July 1, September 19, and November 4, and with equal uniformity the carbonic acid was increased (see Table). These may be appreciated by referring to Plate XXXIV.; and it will also be observed that the changes were extremely well marked on May 18, 19, 20, 31, and June 1. With a large supper of meat there was an increase, but with a late supper with tea or coffee there was a decrease. With much food taken during the preceding day, there was on May 31 an increase so large as to look like an error, whilst on the succeeding day, on which there was an unusual amount of exertion and deficiency of food, the decrease was equally great.

The uniformity in these results is too striking to admit of their being merely coincidences; and whilst the general feeling of the system may not admit of very accurate and minute definition, I believe that the following statement may be relied upon, viz. that with much food on the preceding day, with good and long night's rest, and with a feeling of health and good spirits, the quantity of carbonic acid evolved before breakfast is greater, whilst it is lessened by the contrary conditions. The hour of supper and the nature of the food then taken will also vary the quantity. The mode by which tea and coffee act will be explained in a subsequent paper. I have not recorded many observations in reference to the effects of spirituous liquors; but whenever I had taken them at a late hour, it is recorded that the carbonic acid was lessened, but it is also stated that I had passed a restless night. The conditions favourable to the production of a large amount of carbonic acid in the morning are such as have tended to induce profound and continued sleep, whilst in the adverse conditions there has been an unusual nocturnal activity of the respiration. This is agreeable to *à priori* reasoning; but I do not know that it has been hitherto demonstrated, and by it we may explain much of the remarkable differences in the returns from day to day, and many diversities in the relative quantities evolved by myself and Mr. MOUL.

γ. Variations connected with Season.

BARRAL[*] states that he excreted considerably more carbon in winter than in summer; and a similar observation was made by VIERORDT; but their observations were not made with such regularity and with such identity of external conditions as to eliminate the true effect of season.

Mr. MILNER[†], Surgeon to the Wakefield Prison, in a paper recently read before the British Association, found, in an experience of ten years, and from 40,000 weighings, that the prisoners, on the average, gained in weight from March or April to September, and lost weight during the other months of the year.

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[*] Annales de Chimie, 3 sér. vol. xxv. p. 165.

[†] Sanitary Review, 1858.

The general expression of the results obtained by me is, that the advancing hot season lessened all the vital and mechanical changes of respiration, viz. the quantity of carbonic acid expired and of air inspired, the rate and depth of respiration, the quantity of vapour exhaled, and the cooling of the body. These include lessened muscular and vesicular actions and chemical changes. With the return of the cold season the quantities increased. All the results are delineated in Plate XXXIV., which shows the quantity of air inspired and of carbonic acid expired, with the rate of respiration and pulsation contrasted with the temperature during each month of the year.

The results obtained from myself correspond in all essential particulars with those obtained from Mr. Moul, except that the changes were induced in him more quickly and powerfully. Hence I infer that he was more susceptible to seasonal influences; and this was in conformity with a law which I evolved in former researches at the Hospital for Consumption, and published in the British and Foreign Medical and Chirurgical Review, April 1856, viz. that all who bear heat badly have an excess of all the seasonal effects, for he suffers much from heat, whilst I bear it well. I was thus fortunate in having an example of the two classes of persons. I will first describe my returns, and then those of Mr. Moul.

On the average of the second week in April I inspired 550 cubic inches of air per minute in sixteen inspirations, and expired 8·65 grs. of carbonic acid per minute; but on the average of a whole month, from the middle of July to the middle of August, the quantity of air was reduced to 386·4 cubic inches, of carbonic acid to 7·27 grs., and rate of respiration to 11 per minute. Hence there was a diminution of 30 per cent. in the quantity of air, 32 per cent. in the rate of respiration, and 17 per cent. in the quantity of carbonic acid; so that the proportion of the carbonic acid to the inspired air was greater, although the total amount was lessened. Up to the end of May the quantity of carbonic acid did not fall below 8 grs. per minute, and on many occasions it exceeded 9 grs.; but as June advanced the quantity progressively fell, so that it was often under 8 grs., but never above 8 grs. per minute; and after the 22nd of June, until the middle of August, the quantity was between 7 and 8 grs. Hence the beginning of June was the true period of commencement of seasonal decline in the respiratory changes. The period of the commencement of increase was October, and the increase continued in the succeeding months.

The relation of the carbonic acid expired to the air inspired varied each month as follows, from April to October inclusive:—

April.	May.	June.	July.	August.	Sept.	Oct.
1 gr. to 58 cub. in.	50·7 cub. in.	52 cub. in.	51·5 cub. in.	54·8 cub. in.	56·8 cub. in.	51·4 cub. in.

In the first week of April Mr. Moul inspired 477 cubic inches of air per minute, at a rate of respiration of 16·4 per minute, and with an evolution of 8·28 grs. of carbonic acid per minute; but by the fourth week of that month he had lost more than 20 per cent. of air, 18 per cent. of carbonic acid, and 12 per cent. in the rate of respiration. Thus the proportion of carbonic acid to the air inspired was scarcely changed. After that period the quantities increased from the 8th to the 18th of May; but on the averag there was a further decline in all the respiratory phenomena, until the termination of his experiments at the middle of June. The extreme loss from season on the weekly averages, up to the middle of June, was 27 per cent. of carbonic acid, 27 per cent. of air, and 28 per cent. in the rate,—a degree of uniformity in the phenomena exceedingly striking.

In Mr. Moul the greatest and most sudden changes occurred in the early spring. I had but little change in the spring, but there was a great decline in summer, which continued until the autumn; and although Mr. Moul's experiments ended before midsummer, the progression of his returns give us the right to infer that at the end of summer his quantities would also have been much reduced. Hence there is a close and significant relation between the activity of the vital changes in Man and plants at those two periods, to which common experience has ever attached great importance, viz. the spring and fall with their variations proceeding in yearly cycles.

Having thus briefly stated the changes which occurred, it is now needful to determine to what elements in "season" they are to be attributed. Of these, Temperature and Atmospheric Pressure are two of the most important, and have been investigated by other observers. I have complete records of the former, and incomplete records of the latter.

A. *Temperature.*—The returns prove that the relation between the quantity of carbonic acid evolved and the temperature of the air is an inverse one, as Vierordt and others had established; but they also show that there is no absolute relation between a given temperature and the carbonic acid—a circumstance which has not been hitherto demonstrated.

The effect of the first sudden elevation of temperature in April is strikingly corroborative of the former assertion. Thus from March 31 to April 14, the temperature gently and progressively fell from 56°·5 to 47°·5, and then it suddenly rose 6°·5 in one day, and became so high as 63°·6 in ten days, after which it rapidly fell. Until April 14, with declining temperature, my returns show an increasing quantity of carbonic acid of more than 1 gr. per minute, ending in the large quantity of $9\frac{1}{2}$ grs. per minute. On the contrary, Mr. Moul's returns show a progressive decrease of more than 1 gr. per minute. The Table and Plate XXXIII. figs. 2 & 3, show the coordinated movement of the lines of temperature and carbonic acid during the period of the sudden increase of the former.

Thus in my returns the carbonic acid suddenly decreased 1·3 gr. per minute, and almost as suddenly increased nearly 1 gr. per minute, the former corresponding with the accession and the latter with the decline of temperature. Mr. Moul's decrease was greater, and had its maximum on the day of the maximum temperature, whilst the increase was extremely great and rapid immediately after the temperature began to fall. After this period, to the beginning of June, the curves of temperature and carbonic acid in Mr. Moul's returns are opposed with tolerable uniformity; and on the occurrence of the second elevation of temperature at that period, there was again a sudden and temporary diminution of carbonic acid.

The contrast of the *monthly averages* is of great interest. Thus—

TABLE VI.

Comparison of *Monthly Averages* of Respiratory and Meteorological Phenomena.

1858 and 1859.	Myself.						Mr. Moul.			
	Temp. Dry.	Barom.	Carbonic acid, per min.	Air, per min.	Pulse, per min.	Respirations, per min.	Carbonic acid, per min.	Air, per min.	Pulse, per min.	Respirations, per min.
	°	inch.	grs.	cub. inch.			grs.	cub. inch.		
April ...	54·5	28·84	8·58	498	72·8	14·3	7·18	429	80·3	15·6
May ...	58·1	29·51	8·89	451	68·3	12·4	6·63	384	82·1	12·75
June ...	71·7	29·61	8·19	426	71·1	11·64	6·34	367	79·9	12·6
July ...	65·1	29·48	7·62	393	69·8	11·				
Aug. ...	66·6	29·49	7·15	392	73·3	10·9				
Sept. ...	61·2	29·51	7·13	402	66·6	10·94				
Oct. ...	52·8	29·38	7·67	395	69·8	10·93				
Nov. ...	43·8	29·28	7·86	414	69·1	10·87	See *Note*, page 708.			
Dec. ...	45·2	29·43	8·27	429	67	11·15				
Jan. ...	43·1	29·64	8·35	447	68·8	11·73				
Feb. ...	46·3	29·58	8·2	......	69·2	11·35				
Mar. ...	48·9	29·47	8·25	......	70·9	11·38				

Moreover, the quantity of carbonic acid evolved in different *decades of degrees* of temperature shows that the relation is an inverse one. Thus in the four decades from 40° to 80°, the quantities evolved by Mr. MOUL were in the ascending order—7·58 grs., 7·237 grs., 6·45 grs., and 6·37 grs.,—a series continually decreasing; but the rule is not so well exemplified in myself (for a reason to be presently explained), since the quantities were 8·44 grs., 8·527 grs., 7·841 grs., and 8·29 grs. in their order.

The *weekly averages* show that in myself the rule was maintained in fourteen of twenty-four weeks, and in Mr. MOUL in eight of eleven weeks.

Hence it must be admitted that there is an inverse relation between *seasonal* temperature and the respiratory changes.

But it is evident—and it is a fact of great interest—that there is no uniform relation between the degrees of temperature and the carbonic acid, such as would be necessary to determine the degree of dependence of the one upon the other. This will be proved in two ways: first, by showing the changes which occurred during the long period of months constituting each season; and secondly, the sudden increase of temperature in April.

Mr. MOUL's returns are more uniform than mine, and yet in those the progressive decrease of carbonic acid with increasing decades of temperature is so irregular, as ·343 gr., ·887 gr., and ·08 gr. In mine there was an *increase* in the second and fourth decades. It is also shown in the foregoing Table, that the lowest state of the respiration did not occur with the highest temperature of the season, but after the lapse of two or three months, and the loss of 5° or even 10°, yet with the temperature always exceeding 60°. Further, the following Table shows that the same temperature is attended by the most diverse quantities of carbonic acid; as for example 59°, in which I have returns in five months, the monthly averages are 8·11 grs., 9·13 grs., 7·64 grs., 7·3 grs., and 6·76 grs. in their order; and to take an illustration from Mr. MOUL's returns, at the temperature of 52°, the returns of two months are 9·36 grs. and 6·8 grs.; and at 53°, 8·32 grs. and 6·35 grs. These returns, with others, are found in the following Table, which I think is worthy of close attention.

TABLE VII.

Showing the carbonic acid (in grains per minute) exhaled, on the average of all the observations made at each degree of temperature, in different months, and also the total averages.

Myself.

40°.	41°.	42°.	43°.	44°.	45°.	46°.	47°.	48°.	49°.	50°.	51°.	52°.	53°.	54°.	55°.	56°.	57°.	58°.	59°.	60°.	61°.	62°.	63°.	64°.	65°.	66°.	67°.	68°.	69°.	70°.	71°.	72°.	73°.	74°.	75°.	76°.	77°.	78°.
April	...	...	...	...	...	...	9·46	8·28	8·91	9·1	8·2	7·96	8·34	8·14	...	8·55	9·04	8·68	8·11	8·08	...	8·4	8·4															
May	...	...	...	...	...	...	...	...	...	...	...	9·8	9·24	8·4	8·83	8·8	...	8·44	9·13	8·36	9·32	9·1	8·66															
June	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	..	..	..	...	...	...	7·77	8·26	.	7·9	8·78	8·4	8·2	7·97	8·06	8·66	8·31	8·98	.	8·36	..	8·15
July	...	...	...	...	...	...	...	...	...	...	...	...	...	..	...	. .	.	..	7·64	...	7·91	7·63	.	8·06		8·04	7·23	7·32		...	7·12							
August ..	...	...	...	...	...	...	...	...	...	...	...	..	...	...	. .	...	...	...	...	...	...	...	7·92	7·4	7·26	6·76	6·96	6·8										
September	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	..	..	7·3	6·84	8·22	7·22	...	...	...	7·69	7·42											
October ...	...	...	...	...	...	...	8·06	...	...	...	...	...	...	...	...	7·84	..	7·0	6·76																			
November	8·56	...	...	8·22	...	7·83	...	8·38	...	...	8·06																											
All	8·56	..	...	8·22	...	7·83	8·76	8·33	8·91	9·1	8·13	8·88	8·79	8·27	8·83	8·39	9·04	8·04	7·79	7·76	8·46	8·1	8·07	7·9	7·26	7·59	7·57	7·5	8·2	7·97	7·59	8·66	8·31	8·98	..	8·36	...	8·15
Before July	...	...	...	...	...	...	9·46	8·28	8·91	9·1	8·2	8·88	8·79	8·27	8·83	8·67	9·04	8·56	8·62	8·22	9·32	8·75	8·27	8·26	.	7·9	8·78	8·4	8·2	7·97	8·06	8·66	8·31	8·98	..	8·36	.	8·15
After June	8·56	...	...	8·22	...	7·83	8·06	8·38	...	...	8·06	...	...	...	...	7·84		7·0	7·23	6·84	8·06	7·43	7·92	7·73	7·26	7·49	7·2	7·06	..	.	7·12							
Total Average	Before July 8·88 (All 8·44) After June 8·21									Before July 8·69 (All 8·527) After June 7·43										Before July 8·45 (All 7·841) After June 7·44										Before July 8·35 (All 8·29) After June 7·12								
Mr. Moul.																																						
April	...	...	...	...	...	...	7·7	7·14	7·83	7·54	8·06	9·36	8·32	6·9	...	7·19	8·3	6·91	6·0	5·4	...	6·0	5·12															
May	...	...	...	...	...	...	...	...	...	...	...	6·3	6·35	6·64	6·49	6·46	..	6·83	6·73	7·6	6·76	7·02	6·97															
June	...	...	..	...	...	..	..	...	...	...	...	...	...	...	.	.	...	..		...	...	...	...	.	...	...	6·44	6·3	6·41	6·4	6·38	6·38	6·2	6·52				
All	...	...	...	...	...	...	7·7	7·14	7·83	7·54	8·06	7·63	7·33	6·77	6·49	6·82	8·3	6·87	6·36	6·6	6·76	6·6	6·04	..	...	..	6·44	6·3	6·41	6·4	6·38	6·38	6·2	6·52				
Total Average	(7·58)									(7·237)										(6·45)										(6·37)								

Thus it is proved indisputably that through long periods there is not a close correspondence, but, on the contrary, a wide difference in the relation between the degrees of temperature and the carbonic acid evolved; and that as the season advances towards autumn, with the same temperature or even with a certain decline of it, the carbonic acid progressively falls to a minimum. It is also evident that both before and after the period of change, viz. June, the respiration retains a certain amount of uniformity; and this at two different seasons will differ greatly, whilst it may be that the same temperature is found in both seasons. Thus my average quantities before July and after June respectively, in the four decades of temperature, differed as follows: 8·88 grs. and 8·21 grs., 8·69 grs. and 7·43 grs., 8·45 grs. and 7·44 grs., 8·35 grs. and 7·12 grs.—differences of upwards of 1 gr. of carbonic acid per minute in the three higher decades. Hence there is a want of uniformity in the progression of the decades, as before mentioned, due to the admixture of the results obtained in two seasons; whilst Mr. Moul's returns are more uniform, since they comprehend scarcely more than the early season. Further, it is interesting to know that at each of the seasons the movements of temperature keep in advance of the movements in the carbonic acid quantities.

It now only remains to show that, even with sudden changes of temperature, there is not an exact relation between the temperature and the carbonic acid. The Table and Plate XXXIII. fig. 3, show this, first, in the proportion of carbonic acid to each degree of temperature; and secondly, the amount of diminution of carbonic acid with each increasing degree of temperature.

From the above we learn that as the temperature increased the carbonic acid decreased in an *increasing ratio*, and consequently that the two did not move in parallel lines. This is particularly well marked in my own case, in reference to both modes of inquiry, and in a very striking manner when the temperature exceeded 60°. Mr. Moul's returns in the first series correspond with mine, but in the second they manifest the influence of some other disturbing cause.

Hence it appears that any attempts to find one uniform relation between the carbonic acid and temperature, such as that published by Vierordt and Letellier, are exposed to serious error, and particularly if the inquiries, made upon different persons and in different seasons, are mixed together; and in reference to the influence of sudden changes of temperature, the variations in the carbonic acid in my experiments are greater than ·094 gr. per minute for each degree, as ascertained by Vierordt. It is also quite evident that the changes are due to vital conditions, and not simply to the physical action of heat upon the body and the inspired air.

B. *Barometric pressure.*—The relations between barometric pressure and the carbonic acid expired are much less obvious than those of temperature; and although my returns have a general correspondence with those of Vierordt, in showing that the relation is an inverse one, the rule is liable to many exceptions. Mr. Moul's returns are more amenable to the law than mine, for they continuously declined from month to month, whilst the barometer rose; but in my case the month of May was exceptional. The lowest pressure was, however, in April, when the respiratory phenomena were very active; and both were nearly uniform in July, August, and September, when the respiratory changes were low. To show, however, that there is no invariable relation, it may suffice to mention that with a pressure of 29·51 inches the carbonic acid was 8·19 grs. and 7·13 grs. in May and September respectively.

Thus, after a minute inquiry into the influence of these two agencies, there can be no doubt that the influence of season is far more than can be explained by these meteorological conditions, and that it is one which demands further accurate observation, and particularly in reference to the influence of light as it increases in intensity in the spring. It is, moreover, of the utmost importance to bear in mind that this influence is felt in every action of the system; for as the quantity of carbonic acid varies with the season, so do the quantities evolved under the influence of exertion and food. Hence, in stating the influence of any agent, it would be erroneous to give absolute numbers, under the idea that they would be true at all periods of the year. This is particularly applicable to the next subject of inquiry; for my experiments have shown that the same exertion, as for example walking at a given rate, *produces effects upon the respiration considerably greater in winter than in summer.*

[*Note, April* 1859.—Since the foregoing remarks were written, I have completed the observations during the cycle of the year ending March 31, 1859; and the quantity of carbonic acid evolved, with that of the air inspired and the rate of respiration, have been added to Plate XXXIV.

At the end of October the quantity of carbonic acid evolved had materially increased; and throughout November and December and the beginning of January the increase was progressive, so that the quantity had attained to upwards of 8¼ grs. per minute. Throughout the remaining portion of January, as well as in February and March, the quantity evolved was nearly stationary, and even more uniform than at any other period of the year; but at the end of March it again increased, and on March 31, 1859, was very nearly the same as was recorded on March 31, 1858, viz. 8·25 grs. and 8·51 grs. per minute.

The quantity of air inspired per minute increased from an average of 395 cubic inches in October, to 414, 429, and 447 cubic inches in November, December, and January, in their order.

The rate of respiration in like manner increased on the average from 10·93 per minute in October, to 10·87, 11·15, 11·73, 11·35, and 11·38 in the months of November, December, January, February, and March, and did not reach to the elevation recorded in 1858.

The temperature after October remained, during the winter months, at averages varying from 43°·1 to 48°·9, and exhibited marked perturbations at the end of November and the beginning of January and March.

The monthly averages are appended to Table VI.

The variations observed from day to day were frequently attributable to the causes described in page 702, and tended further to corroborate the deductions then made; and in further confirmation, it is remarkable to notice how frequently the evolution of carbonic acid increased on the Monday, after the rest and retirement of the preceding day.]

III. INFLUENCE OF WALKING AND THE TREADWHEEL.

The only experiments which I have been able to make in reference to exertion are limited to the influence of walking and the treadwheel.

1. *Walking.*

I employed two sets of the apparatus, and attached them to the mouthpiece by a long tube, partly of glass and partly of vulcanized india-rubber, so long as to enable me to walk about 11 yards in each direction. The distance was accurately measured, and the speed regulated with the watch in hand in the most exact manner. I also walked at the same speed for some time before the experiment began, so as to bring the body to the full influence of the increased action. The results are given in detail in Table VIII.

TABLE VIII.

Showing the influence of Sleep and Exertion.

Date.	Hour.	Carbonic acid, per min.	Air, per min.	Pulse, per min.	Rate of respiration, per min.	Temperature.		
						Wet.	Dry.	
	A.M. h m	grs.	cub. in.					
July 15 A.M.	1 30	6·7	......	.....	......	67	69	*Whilst awake*, but after lying down for some hours, and with every approach to *sleep*.
	2 35	5·94	......	66	......	......	......	
	6 45	6·1	......	70	......	......	......	
July 16 A.M.	1 0	4·88	......	......	......	71	71	In *sleep*, but probably not profound.
	3 0	4·99	......	......	.....	......	......	In *sleep*.
	P.M.							
July 15 P.M.	3 30	18·1	709	90	14·5	72·5	75·5	*Walking* at 2 miles per hour.
	4 30	25·83	1028	......	16	......	......	*Walking* at 3 miles per hour.
	P.M.							
Oct. 8 P.M.	4 0	43·36	......	150	22	......	......	*Treadwheel*; 43 steps, ascending 28·65 feet per minute. I was on the wheel ¼ of an hour, and determined the carbonic acid for 3 minutes after 5 minutes, and the last 2 minutes of 15 minutes.
Oct. 9	3 30	42·9	......	150	21	......	......	*Treadwheel.* Determined the carbonic acid during 3 minutes after 5 minutes, and 2 minutes after 10 minutes.
	3 45½	......	1120	......	21	......	......	*Rest after treadwheel.* I sat down directly, and after ½ a minute used the spirometer. Another ½ minute was lost after 3½ minutes. The carbonic acid was collected— During 3 minutes after 4 minutes. During 2 minutes after 10 minutes. During 2 minutes after 13 minutes. 7 minutes.
	3 46½	......	930	......	18	......	......	
	3 47½	......	720	.....	17	......	......	
	3 49	......	680	......	17	......	......	
	3 50	......	590	......	16	......	......	
	3 51	.....	600	......	15	......	......	
	3 52	......	540	......	15	......	......	
	3 53	......	590	......	16	......	......	
	3 54	......	530	......	15	......	......	
	3 55	......	500	......	15	......	......	
	3 56	......	540	......	15	......	......	
	3 57	......	500	......	15	......	......	
	3 58	9·14	560	......	15	......	......	
	3 59		520	102	15	......	......	
Oct. 22	4 0	43·66	......	150	20	......	......	Dined on prison fare at 2½ P.M.; soup and bread. *Treadwheel*, 40 to 44 steps per minute. Carbonic acid taken three times in the last two of each 5 minutes.

On July 15, with a temperature of 75½°, whilst walking at two miles per hour, I expired 18·1 grs. of carbonic acid per minute, with a rate of respiration of 14½, and of pulsation 90 per minute, and with 799 cubic inches of inspired air per minute.

At three miles per hour the carbonic acid was increased to 25·83 grs., the air to 1027·5 cubic inches, and the rate of respiration to 16 per minute. I did not attempt a greater speed; for it required closer attention than I could give to the speed and the apparatus at the same time to maintain a uniform rate of more than three miles per hour.

Thus, taking as a basis of comparison the system at rest and without food, in the sitting posture, at the same hour, viz. 7·44 grs. of carbonic acid per minute, the effect of walking at two miles per hour was 2·5 times as great, and at three miles per hour 3·5 times. There is a curious progression in the relation of carbonic acid to the inspired air under these different conditions, as follows, taking 1 gr. of carbonic acid as the unit of comparison: viz. in rest, sitting, and fasting, 1 : 49·6 cubic inches; walking at two miles per hour, 1 : 44·1 cubic inches; at three miles per hour, 1 : 39·7 cubic inches—a progressive increase in the ratios of nearly one-ninth of the basis quantity.

2. *The Treadwheel.*

The treadwheel is a kind of exertion which consists in lifting the body a given height in a given time, from the steps of a wheel turning downwards, and holding it in a position in which its centre of gravity is anterior to its perpendicular central line. My experiments have been made in many county gaols, but in only that at Coldbath Fields have I determined the quantity of carbonic acid evolved during this kind of exertion. I am under obligation to the Governor and Visiting Justices for much courtesy and assistance.

There are not any similar inquiries on record; but in reference to raising the body through a given space at given times, LAVOISIER has stated the quantity of oxygen which is consumed, and also that the increased rate of pulsation is directly as the weight multiplied by the height, if the effort is an easy one.

The influence of the treadwheel has been determined by three experiments, as recorded in Table VIII. On October 8, with a low normal state of the respiration, I expired 43·36 grs. of carbonic acid per minute whilst upon the wheel; and on October 9 the quantity was 42·9 grs. per minute. On each occasion the pulse was 150 per minute, and the rate of respiration 22 and 21 per minute respectively. On October 22, in the afternoon, after having eaten the prisoners' dinner of brown bread and soup, the quantity was 48·66 grs. per minute, with a pulsation of 150, and respiration 20 per minute. In all the inquiries I worked the wheel fifteen minutes at a time. In the two former I collected the carbonic acid during three minutes, after having been upon the wheel five minutes, and again during the last two of the fifteen minutes; but in the last inquiry I collected it in the last two of each five of the fifteen minutes. Thus the average in the first two is derived from five minutes, and in the last from six minutes. I was unable to use the spirometer as well as the analytical apparatus whilst working the wheel, for I found the breathing too much oppressed by it; and more particularly, since I was compelled to use a double set of the analytical apparatus, on account of the rapidity of the current of the expired air which was forced through it.

After the second inquiry, I determined the state of the respiration in the fifteen minutes constituting the interval of rest. After leaving the wheel I sat down (after the manner of the prisoners), and with a loss of half a minute used the spirometer, and noted the quantity of air inspired per minute. The carbonic acid was also collected during seven minutes; viz. during three minutes after four minutes' rest, two minutes after ten minutes', and two minutes after thirteen minutes' rest. The average quantity of carbonic acid thus obtained was 9·14 grs. per minute; but that does not include the first four minutes of the period of rest, and the following statement shows that then the respiration was fast subsiding to the minimum quantity. From a consideration of the quantity of air inspired, I compute that the quantity of carbonic acid evolved during the first minute of rest was 25 grs., in the second 20 grs., and in the third 15 grs., and hence that the average of the whole fifteen minutes' rest was 11·3 grs. per minute.

The quantity of air inspired in cubic inches per minute, commencing half a minute after the labour had terminated, was as follows, in each minute, in its order:—1120, 930, 720, 680, 590, 600, 540, 590, 580, 560, 540, 500, 560, and 520, yielding an average of 641 cubic inches per minute. It is evident that whilst there is, during the whole interval of rest, an increase in the respiratory changes over that of rest apart from this exertion, the great excess due to the exertion disappears after five minutes' rest.

The remark made already as to the influence of season over the effect of exertion must not be forgotten.

Résumé.

In the foregoing paper it has been shown—

1. That it is possible to collect the whole of the carbonic acid exhaled by the lungs in the twenty-four hours, or for any period, however long or short.

2. The quantity of carbonic acid expired in the twenty-four hours was, on the average of eight sets of inquiries in four adult males when in a state of rest, 20·193 oz. (7·144 oz. carbon), of which 1950 grs. were expired in the six hours of the night. The proportion per minute in the night and the day is 1 to 1·84. In light sleep 4·88 grs. per minute were exhaled, and 4·5 grs. is the estimated amount in profound sleep in July. The quantity of carbon exhaled in the twenty-four hours, by each of the subjects at rest, was 5·6 oz., 6·54 oz., 6·785 oz., and 7·85 oz.

3. The quantity of air inspired during the working day of eighteen hours, was, to the carbonic acid expired, an average of 56·3 cubic inches to each grain.

4. The rate of respiration was on the average 13·87, and of pulsation 71·7. The average depth of inspiration was 33·6 cubic inches.

5. The factor of the respirations to equal the number of the pulsations varied in the different persons, increasing with their age, from 4·63 to 5·72, and on the average was 5·15.

6. Half of the product of the respirations and pulsations very nearly equalled the number of cubic inches of air breathed in several of the cases. No such numerical relation could be found in reference to the carbonic acid.

7. The variations of the carbonic acid in the same day, with food, were so great as an average of 10·43 grs. and 6·74 grs., and extremes of 6·96 grs. and 13·3 grs.; but in a prolonged fast the extreme difference was less than 1 grain.

8. In a prolonged fast of twenty-seven hours, the diminution per cent. from the quantities on a day with food, was 25 of carbonic acid, 30 of air, 37 of vapour, ·7 of rate of respiration, and 6 of rate of pulsation. The loss of carbon in the twenty-four hours is equal to that contained in 20 oz. of bread. The returns were so uniform, that the quantity of carbonic acid expired at the end of twenty-seven hours was the same as at 4½ hours after food.

9. There is a normal or basal line below which the system does not pass in health and wakefulness, and which is tolerably uniform. It is the same in the complete abstinence from food, as at the end of the interval between meals. There is also, when at rest, a higher point, which the system does not exceed, due to food, and it is the highest after breakfast and tea.

10. There is an increase of carbonic acid in the absence of food, at or near to the period when it usually increased with food.

11. There are variations from day to day, in the absence of food and exertion, which are dependent upon the amount of waste and supply of the preceding day and night. *Cæteris paribus*, the more food, rest, and sleep, the greater will be the quantity of carbonic acid evolved on the following morning.

12. There are great variations from season to season, so that, as the hot season advances, all the respiratory phenomena are lessened. The diminution in myself at the middle of August was 30 per cent. of air, 32 per cent. in rate of respiration, and 17 per cent. of carbonic acid. In Mr. MOUL, to the middle of June the diminution was 27 per cent. of air and also of carbonic acid, and 28 per cent. in the rate of respiration.

Spring is the season of the greatest, and the fall, of the least activity of the respiratory and other functions.

13. Temperature and atmospheric pressure only partially explain the effect of season; for with the same temperature, at different seasons, there is great diversity in the carbonic acid expired, and particularly with a medium temperature; as, for example, 59°, with which the quantity of carbonic acid was 9·18 grs. in the spring, and 6·76 grs. in the summer and autumn. The relation of temperature and pressure to the carbonic acid is an inverse one, the former acting in an *increasing ratio* in cases of sudden increase, and with so much power that 2·3 grs. of carbonic acid per minute were lost in seven days.

The period of permanent decline in the carbonic acid is May and June, in different persons, and at the period of minimum quantity there is the greatest uniformity. Temperature moves in advance of the carbonic acid, and particularly as it declines.

14. All respiratory phenomena vary with the season, and hence it is better to adopt comparative rather than absolute quantities—the comparison being with the state of rest, with or without food; but the latter is the most reliable.

15. The effect of walking, compared, 1st, with the average state of rest with food, and 2ndly, with the state of system at rest before the breakfast, and therefore whilst fasting, is as follows:—at two miles per hour, one hour is equal to 1⅘ hour with food, and to 2½ hours without food. At three miles per hour, one hour to 2¾ hours and to 3½ hours. One hour of the labour of the treadwheel, whilst actually working the wheel, is equal to 4½ hours with food, and 6 hours of rest without food. The proportion of carbonic acid to the inspired air is much greater with exertion than at rest; and it increases as the speed, in a progression of one-ninth of the quantity at rest for two and three miles per hour without food.

16. The proportion of the carbonic acid to the inspired air is greater without than with food, and in summer than in winter. It is not absolutely uniform at any time, and it differs in different persons.

Men differ in every subject which has been investigated.

In conclusion, I beg to offer my warmest thanks to Mr. MOUL, to Professors FRANKLAND, PLAYFAIR, SHARPEY, and WILLIAMSON, and other gentlemen who have aided in these and other inquiries.

EXPLANATION OF THE PLATES.

PLATE XXXIII.

Fig. 1. Shows the quantity of carbonic acid evolved per minute, with and without food, in contrast, and also the effect of the supper on March 12, as contrasted with the absence of that meal on May 15. The observations delineated in the morning of the day of fasting in July followed, and did not precede those represented in the afternoon.

Fig. 2. Represents the daily diminution in the quantity of carbonic acid per minute, with sudden increase in temperature in April 1858. The diminution in the quantity of air inspired by Mr. MOUL is also stated on each day.

Fig. 3. Exhibits the same fact in two other aspects, viz. the quantity of carbonic acid in decimals of grains per minute to each degree of temperature, and also the diminution in the expiration of carbonic acid in decimals of grains per minute for each degree of temperature above that which occurred immediately before the increase of temperature took place.

PLATE XXXIV.

Shows the quantity of carbonic acid expired, and of air inspired, per minute, with the depth of inspiration, and the frequency of pulsation and respiration at the same hour, and under the same circumstances, on about 200 mornings in the year, in relation to the temperature, with the wet and dry bulb, and the season of the year. There were two subjects for experiment until the middle of June. The opposition of sudden changes of temperature, and the expiration of carbonic acid, is best seen from April 14 to 28. The progressive seasonal diminution in the expiration of carbonic acid commences at the beginning of June and ends in September, from which period the progressive increase occurs until January, and thenceforward the quantity is shown to be nearly uniform.

XXVII. *Experiments on Respiration.*—Second Communication. *On the Action of Foods upon the Respiration during the primary processes of digestion.* By Edward Smith, *M.D., LL.B. (Lond.), M.R.C.P., Corresponding Member of the Académie des Sciences et Lettres de Montpellier, and of the Natural History Society of Montreal, Assistant-Physician to the Hospital for Consumption, Brompton, &c. Communicated by Sir* B. C. Brodie, *Bart., P.R.S.*

Received January 6,—Read February 10, 1859.

In a paper which I have recently had the honour to forward to the Royal Society, I showed that there are alternate elevations and depressions of all the respiratory phenomena during the day, due to the temporary influence of food, and that the maximum influence of food always occurs in from 1¼ to 2½ hours after the meal. I also proved, from the state of the system in a prolonged fast, as well as from the tolerably uniform state at the end of the interval between the meals, that there is on each day a basal or normal line below which the amount of respiratory action does not ordinarily fall: and I also showed that there is a maximum elevation from food which is tolerably uniform, and which is the most pronounced after the breakfast and tea meals. Hence it appeared that the influence of food is in two degrees:—1st, that which lies between those two lines and exceeds the normal or basal line; and 2nd, that which sustains the system up to the minimum or basal line. The former action is temporary and of short duration, whilst the latter is permanent.

Proceeding from these facts, I have prosecuted a lengthened inquiry into the influence of numerous articles of food over the respiration, when taken separately and not in the combined form in which we take them at meals, and have endeavoured to ascertain what is their maximum effect. As nearly all food tends to sustain and increase the vital actions, and as in the total absence of food for a lengthened period the respiratory changes are sustained to the extent of 75 per cent. of that with food, my inquiries have been almost entirely directed to determine that influence, which acts so as to appear between the maximum and minimum lines just mentioned. Except within very narrow limits, I have not found any substance taken as food or with food which materially lowers that minimum line; and, moreover, I do not know any method whereby it would be possible, in a state of health, to show the action of any substance which acts much below it. Hence my aim has been to show to what extent various substances raise the respiratory changes above or depress them below a basal line, and not merely to state what absolute amount of carbonic acid, for example, was evolved during their action. The former could be referred to no other cause than the food under experiment;

but since, as has been just mentioned, so great a part of the latter would have occurred if the food had been withheld, it is impossible to affirm that it was due to its influence. This has afforded me an inquiry of a well-defined nature, and one which my apparatus permitted me to make with ease; and it is one, moreover, so far as I know, not heretofore pursued.

Nearly all previous inquiries have been made upon the lower animals, and have had for their object to determine the effect upon nutrition of the exclusive use or the entire absence of certain substances, and only a few experiments have been made upon Man. Prout* ascertained that the per-centage of carbonic acid in the expired air was somewhat lessened under the influence of both tea and alcoholic liquors; and Vierordt† found the same result from the latter *without any appreciable difference in the other phenomena of respiration.* Boeker‡ has made the most remarkable inquiries into this subject; and the method adopted by him was to take a considerable quantity of the substance, as sugar, for example, at several periods of the day, in addition to his ordinary food, and afterwards ascertain the state of the excretions and the respiration at several irregular periods. When we recollect how great is the variation in the quantity of carbonic acid evolved at different periods of the same day, and how irregular is the quantity on different days,—how powerful is the effect of exertion, an influence which cannot under ordinary circumstances be the same on any two days,—the disturbing influence of unusually large or often repeated quantities of a substance, and lastly, the unknown influence of other articles of food, varying in quantity and quality,—it is easy to see how difficult it would be for M. Boeker to eliminate the normal and true effect of the article under inquiry. He found that sugar, coffee, and alcohol lessened the respiratory changes, and that in their principles of action they are identical; but he then inquires how it is, if this be so, that they act as antidotes to each other? It will be shown that my results do not accord with his. Mr. Milner§, Surgeon to the Government Prison at Wakefield, in a paper which he recently read before the British Association, states that he had proved by experiment that tea given to the prisoners, whether in lieu of, or in addition to their ordinary food, caused them to lose weight at an increasing rate,—a fact which supports my results on the action of that substance.

My experiments have been made chiefly upon myself, æt. 39, and upon Mr. Moul, æt. 48 (a gentleman whose devotion to the inquiry in the interests of science is beyond all praise), but in a few instances upon others also. The plan pursued was as follows:—

A quantity of the substance under inquiry, not greatly different from that ordinarily taken by mankind, was administered apart from any other food. The experiments were nearly all made from 7 to 9 A.M., before breakfast; but some were made at 2 P.M., after the influence of the breakfast had passed over. All were made in the sitting posture, and in the absence of all exertion and mental and bodily excitement. Hence we isolated the influence of the substance, and made one long series of experiments under precisely similar and normal conditions. We sat down at least a quarter of an hour before taking the first observation, or that which showed the state of the system before the substance under inquiry was taken, and which was the basal state with which the subsequent effects of the substance were compared, and upon the accuracy of which the truthfulness of the results mainly depended. We then ate the substance in question, and in from 3 to 10 minutes afterwards made our first inquiry into its effects; and repeated the inquiry every 12 or 15 minutes (as frequently as we could complete each experiment), until the maximum effect had passed; the same conditions as to posture and quietude being maintained unbroken throughout the whole inquiry. We did not continue the experiment beyond our usual breakfast hour; for, in accordance with a statement made in my former paper, we found that an increase in the respiratory phenomena occurred at the meal hour before any food had been taken. Hence it was neither necessary nor desirable to continue the inquiry much beyond two hours. Each of these experiments lasted five minutes, but in several of them the inquiry was continuous, and the whole of the carbonic acid evolved during the action of the substance was collected.

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* Thomson's 'Annals of Philosophy.'

† Physiologie des Athmens.

‡ Beiträge zur Heilkunde, &c., 1849.

§ Sanitary Review, 1858.

In recording the results, we made use of the total quantities; but in abstracting them for the purposes of this communication, I have compared them with the basis quantity, and have given the amount of increase or decrease only. This was necessary, both on account of the observation made above, and because the total quantities varied with the season.

The apparatus employed was that described in my previous paper*, consisting of an instrument to measure the quantity of inspired air, and an analytical apparatus to abstract the vapour and carbonic acid from the whole of the expired air. The force of inspiration and expiration was in a few instances determined by a bent tube with a column of water attached to the mask; and the temperature of the expired air was determined by the aid of a small thermometer inserted at right angles into a box-wood tube, 1¼ inch in length, and protected by valves, as described in my paper published in the 'Proceedings' of the Society for 1857. The temperature with the wet and dry bulb, and the barometric pressure, were recorded, as was also the state of the weather.

The foods which have been the subjects of inquiry are very numerous, and of the finest quality; and some of them, as tea, sugar, milk, and alcohols, were subjected to very numerous and varied experiments. As the results have shown that it would be inconvenient to arrange them solely according to their chemical constitution, I purpose to describe them under the following heads:—

1. *The Starch Series*, viz. arrowroot, arrowroot and butter, arrowroot and sugar, commercial starch, wheat starch, gluten, bread, oatmeal, rice, rice and butter, potato, gum.
2. *The Fat Series*, viz. butter, olive oil, cod-liver oil.
3. *Sugars*, viz. cane-sugar, cane-sugar and butter, cane-sugar with acids and alkalies, grape-sugar, milk-sugar.

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* Page 682 of this volume.

4. *The Milk Series*, viz. new (cows') milk, skimmed milk, casein, casein and lactic acid, lactic acid, sugar of milk and lactic acid, cream.
5. *Alcohols*, viz. spirits of wine, brandy, whisky, gin, rum, sherry and port wine, stout, ale.
6. *The Tea Series*, viz. tea, green and black, hot and cold, in different quantities, and with acids and alkalies; coffee, coffee leaves, chicory, cocoa.
7. *Certain other nitrogenous substances*, viz. gelatin, albumen, fibrine, almond-emulsion*.

The immediate object of inquiry was the effect of these various substances over the carbonic acid and vapour exhaled, the quantity of air inhaled, and the rate of pulsation and respiration. The inquiry as to the amount of vapour exhaled was chiefly pursued in the experiments on alcohols.

I shall describe the effect of these various substances in the order now given, and in order to avoid repetition shall reserve some general observations until the end of the paper. The whole of the detailed results, forming many sheets of tables accompanying this paper, are delineated on Plates XXXV. and XXXVI.

1. *The Starch Series.*

The general expression of the results obtained, is that starch does not excite the evolution of carbonic acid, whilst the ordinary foods, containing starch with other substances, as sugar and gluten, are powerful and enduring respiratory excitants.

Arrowroot.—June 19 (7 experiments†). The purest starch which I could obtain was arrowroot. The effect of 500 grs., well cooked with water, was an average increase of only ·154 gr. of carbonic acid per minute in myself, and ·208 gr. in Mr. Moul (Plate XXXV. fig. 3). On another occasion, September 6 (6 exps.), 750 grs. gave me an average decrease of ·08 gr. per minute. In the first inquiry there was an average decrease in the air inspired of 8 cubic inches per minute in myself, and an increase of 10 cubic inches in Mr. Moul, whilst in the last I had an increase of 5 cubic inches per minute. The rate of respiration was reduced in all the inquiries, whilst that of pulsation was decreased in Mr. Moul 3 per minute in the first, and increased in myself 3 per minute in the last.

When taken after a long fast, as described in my former paper, page 696, 500 grs. gave a maximum increase of ·45 gr. of carbonic acid per minute.

Starch, &c.—June 14 (10 exps.). Starch, imperfectly washed from 4 oz. of wheat flour, and therefore associated with other substances, gave a maximum increase of 2·1 grs. of carbonic acid per minute in myself, and 44 cubic inches of air inspired. There was a decrease in the rate of respiration, and a very slight increase in the rate of pulsation. When the starch had been much better washed, June 24 (7 exps.), Plate XXXV. fig. 6, the increase in the carbonic acid was ·52 gr., and 14 cubic inches of air inspired.

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* It will be borne in mind that in this series of inquiries the amount of hydrogen consumed was not determined.

† This expression means that 7 observations were made upon each of us (or 14 on both of us) at regular intervals during the continuance of this experiment.

Bread.—April 13 (6 exps.), Plate XXXV. fig. 7. 4 oz. of white home-made bread gave maxima of increase in the carbonic acid of 1·48 gr. and 2·4 grs. per minute to myself and Mr. Moul respectively, and on another occasion, April 23 (7 exps.), of 1 gr. and 2·2 grs. respectively. The quantity of air was increased on the two occasions 60 cubic inches and 20 cubic inches per minute in myself, and 96 cubic inches and 118 cubic inches per minute in Mr. Moul. The rate of respiration was not increased in myself or in Mr. Moul, and in the latter the pulsation fell somewhat. The effect was very enduring.

Oatmeal.—April 26 (7 exps.), Plate XXXV. fig. 10, July 9 (8 exps.), fig. 11. 4 oz. of good Scotch oatmeal, made into pudding with water, gave me a maximum increase of carbonic acid, on two occasions, of 1·63 gr. and ·93 gr. per minute, and to Mr. Moul of 1·32 gr. per minute. The volume of air inspired by myself was increased 55 cubic inches and 19 cubic inches, but Mr. Moul had a less increase. The respirations were lessened, and the pulsations were not increased. The effect was very enduring.

Rice.—April 28 (8 exps.), May 13 (8 exps.), Plate XXXV. fig. 9. 4 oz. of the best rice, well cooked, gave me maxima of increase in the carbonic acid of 1·9 gr. and 1·44 gr. per minute, and Mr. Moul of 1·15 gr. and 1·94 gr. per minute. I had a maximum increase in the volume of air of 54 cubic inches and 40 cubic inches per minute, and Mr. Moul of 44 cubic inches and 96 cubic inches per minute. There was generally a diminution in the rate of respiration, and an increase in that of pulsation. The effect was very enduring.

Potato.—May 17 (8 exps.), Plate XXXV. fig. 12, July 21 (6 exps.), fig. 13. 8 oz. of very good cooked potato, with 3 oz. of cold water, gave a maximum increase of carbonic acid in myself of 1·27 gr., on two occasions, and 52 cubic inches and 20 cubic inches in the air inspired. The rate of respiration was in both instances reduced, as was also that of pulsation, but in a less degree. The potatoes were old in the first and new in the second inquiry.

Commercial Starch.—June 18 (6 exps.). 500 grs. of commercial starch gave me a maximum increase in carbonic acid of ·8 gr. per minute, and of air 22 cubic inches per minute, whilst the rate of both respiration and pulsation was slightly increased.

Gluten.—June 11 (8 exps.), June 16 (8 exps.), Plate XXXV. fig. 8. The gluten washed out of 4 oz. of fine wheat flour gave me a maximum increase in carbonic acid of ·84 and ·74 gr. per minute, and in air of 8 cubic inches and 11 cubic inches per minute. The rate of respiration was decreased very slightly, whilst that of pulsation was increased on one occasion and decreased on another.

Arrowroot and Butter.—June 21 (6 exps.), Plate XXXV. fig. 4. 1 oz. of fresh butter, added to 500 grs. of arrowroot, gave me an average increase in carbonic acid of ·17 gr. per minute, and a maximum increase of ·4 gr. per minute. The quantity of air inspired was somewhat lessened, as was also the rate of respiration. The pulsation was increased 4 per minute.

Rice and Butter.—May 14 (8 exps.), 1 oz. of butter, with 4 oz. of rice, gave to myself and Mr. MOUL an average increase of ·84 gr. and 1·25 gr. of carbonic acid, and a maximum increase of ·94 gr. and 1·9 gr. per minute respectively. The quantity of air was increased 14 cubic inches and 22 cubic inches. The rate of respiration was reduced, and to the extent of 2·6 per minute, whilst that of pulsation was increased 9 and 6 per minute.

Arrowroot and Sugar.—September 13 (7 exps.), Plate XXXV. fig. 5. 250 grs. of sugar, added to 500 grs. of arrowroot, gave a maximum increase of 1 gr. of carbonic acid, and 20 cubic inches of air per minute in forty minutes. The rate of respiration was progressively reduced, whilst that of pulsation was temporarily increased.

Hence it appears—

1. That pure starch exerts but an insignificant influence over the increase of carbonic acid and over pulsation, and that after a fast of twenty-four hours the influence is but very slightly increased.

2. That wheat flour, oatmeal, and rice have a great and very similar influence over respiration, both in degree and duration; but the latter differs from the others in increasing the rate of pulsation. In each of them there remained an increase of 1 gr. of carbonic acid per minute after the lapse of two hours.

3. That potato differs from the foregoing less in the amount of its maximum influence than in the shorter duration of its influence, the subsidence from the maximum being very rapid. New and old potatoes had the same influence.

4. The foregoing invariably reduced the rate, and thereby increased the depth of respiration.

5. The proportion of carbonic acid to the inspired air was increased by these substances to the greatest extent in rice, and the least in potato. The maximum increase of air and carbonic acid almost invariably occurred together.

6. The maximum influence was obtained in from $1\frac{1}{2}$ to 2 hours, and it was the same in wheat, oatmeal, and rice, and nearly the same in potato.

7. The addition of fat to starchy foods did not increase, but rather lessened the influence of the latter. It however increased the influence over pulsation.

8. Gluten exerts a considerable influence, but much less than that of bread, oatmeal, and rice. Its effects were very uniform on two occasions. Its maximum influence was produced in forty minutes, and the whole effect ended in about two hours.

9. Arrowroot, alone, gave no sense of satisfaction (all our experiments were made whilst fasting), but, on the contrary, there was in about one hour an unnatural sensation of sinking in the stomach and small intestines.

With bread there was a sweet taste in 14 to 20 minutes; and with bread, rice, and potato sour eructations in 35 minutes. The inspirations were forcible.

2. *Fats.*

Fat, like starch, does not excite the respiration.

Butter.—June 25 (7 exps.), Plate XXXV. fig. 2. 500 grs. of butter gave me an average increase of carbonic acid of only ·05 gr., and a maximum of ·34 gr. per minute. There was a maximum decrease of 33 cubic inches in the air inspired, and a progressive decrease in the rate of both respiration and pulsation. On another occasion, June 28 (7 exps.), there was an average decrease of ·11 gr. of carbonic acid per minute, with decrease in the rate of respiration and the quantity of air inspired.

Olive Oil.—February 22 (8 exps.). 1 oz. of olive oil gave me an average increase of ·11 gr. of carbonic acid, and a maximum of ·55 gr. per minute. The quantity of air was not materially varied, but there was a diminution in the rate of respiration.

Cod-liver Oil.—March 8 (6 exps.), Plate XXXV. fig. 1. 1 oz. of cod-liver oil gave me an average decrease of carbonic acid of ·27 gr., and a maximum decrease of 18 cubic inches of air per minute. The rate of respiration was decreased 1 per minute. On another occasion, June 24 (8 exps.), the average increase of carbonic acid was ·03 gr. per minute, with a maximum decrease of 19 cubic inches in the inspired air. There was also a diminution in the rate of respiration, and an increase in that of pulsation.

Hence—

1. The general tendency of these fats was to lessen the above-mentioned respiratory changes, but the variation, whether above or below the basis, was not great.

2. The rate of respiration was always lessened, whilst that of pulsation was usually increased. The quantity of air was lessened disproportionately to the carbonic acid, and thereby the proportion of the latter was increased.

3. Both cod-liver oil and butter produced a soothing effect, whilst with all the fats the respiratory efforts were feeble, and in about 20 to 30 minutes there was a slight sense of oppression at the heart.

I am of the class of persons who are fond of fat, whilst Mr. MOUL dislikes it.

3. *Sugars.*

Sugar, unlike starch and fat, powerfully excites the respiratory changes. I have made a very extensive series of inquiries into its influence.

Cane-sugar.—April 14 (6 exps.), Plate XXXV. fig. 23; May 1 (7 exps.), fig. 22. The maximum increase in the carbonic acid, due to 1½ oz. of white sugar dissolved in water, was 2·18 grs. per minute in myself on two occasions, and 1·5 gr. and 2·71 grs. per minute in Mr. MOUL. There was an increase of about 1 gr. per minute within 10 minutes, and the maximum increase was attained in about 25 minutes, after which there was a rapid subsidence. The whole effect did not usually pass away in less than two hours. The maximum increase in the air inspired was per minute 74 cubic inches and 111 cubic inches in myself, and 30 cubic inches and 100 cubic inches in Mr. MOUL. The rate of respiration was lessened, and the depth was increased 8 cubic inches and 6·6 cubic inches in myself, 8 cubic inches and 12·2 cubic inches in Mr. MOUL. The rate of pulsation was lessened.

August 25 (6 exps.).—The effect upon Mr. REID, Surgeon to the Canterbury Hospital, who made an experiment for the first time, was a maximum increase of 1·9 gr. of carbonic acid in 25 minutes, and after the expiration of 70 minutes the increase was still 1·42 gr. per minute. The pulse was fuller and more frequent.

May 10 (13 exps.), Plate XXXV. fig. 25.—½ an oz. of sugar, taken at intervals of one hour, gave me a maximum increase of carbonic acid, on each occasion, of 1·3 gr., and of air of 26 cubic inches, 25 cubic inches, and 11 cubic inches per minute over the first basis quantity. The depth of inspiration was increased on each occasion, and the pulsation was somewhat decreased. On the second occasion the sugar was eaten dry; and as an uneasy sensation followed, water was drunk in one hour afterwards, and was followed by a renewed increase in the carbonic acid evolved.

On another occasion, May 18 (9 exps.), fig. 24, ½ an oz. of sugar, with 3 oz. of water, taken twice, with an interval of 1 hour, gave me a maximum increase in carbonic acid of 2·18 grs. per minute in 23 minutes in the first, and ·52 gr. per minute in 23½ minutes in the second experiment, whilst in Mr. Moul the increase was ·86 gr. and 2·14 grs. per minute in the same period. The maximum increase in the quantity of air inspired at the same period was 51 cubic inches and 29 cubic inches per minute in myself, and 58 cubic inches and 52 cubic inches per minute in Mr. Moul. The depth of inspiration was considerably increased in both persons.

May 21 (19 exps.).—500 grs. of white sugar, dissolved in 6 oz. of cold water, were taken, and the whole of the carbonic acid evolved afterwards was collected and weighed every half hour. In the first half hour the increase was 1·57 gr. per minute, and in the second half hour ·58 gr. per minute; whilst the quantity evolved at 95 minutes, when the inquiry ended, was ·12 gr. per minute above the basis quantity. The increase in the carbonic acid, in the first hour, was thus 57·5 grs. The quantity of air inspired was registered every 5 minutes. The maximum increase was 64 cubic inches in 35 minutes, and the quantity was reduced to the basis in 85 minutes. The average increase was 19·3 cubic inches per minute in the first half hour, 30·6 cubic inches per minute in the second half hour, and 8·3 cubic inches per minute in the third half hour. The rate of respiration was scarcely changed, and hence the depth of inspiration varied as the quantity of air inspired.

Cane-sugar and Vinegar.—June 29 (7 exps.), Plate XXXV. fig. 26. 750 grs. of white sugar, with 6 drachms of good vinegar and 7 oz. of water, gave a maximum increase per minute of carbonic acid of 3·3 grs. in 20 minutes. The average increase in 1½ hour was 1·24 gr. per minute; the maximum increase in the air inspired was 79 cubic inches in 20 minutes, and in the depth of inspiration 4·1 cubic inches. The rate of respiration was increased slightly, and that of pulsation 5 per minute.

October 4 (6 exps.).—The repetition of this experiment in October (but with raw sugar) gave a less increase, and at a later period.

June 24 (7 exps.).—With 30 grs. of citric acid instead of the vinegar, the maximum increase in carbonic acid in myself was 1·74 gr., and in Mr. Moul 2·25 grs. per minute. The increase of air inspired was 47 cubic inches and 20 cubic inches per minute.

July 5 (8 exps.), Plate XXXV. fig. 27.—750 grs. of white sugar, with 40 ♏ of

Liquor Potassæ in 12 oz. of water, gave me a maximum increase in carbonic acid of 2·13 grs. per minute, and after 100 minutes the increase was more than 1 gr. per minute. The maximum increase in the air inspired was 165 cubic inches per minute, in a little less than one hour. The rate of respiration was scarcely altered, but that of pulsation was lessened 7 per minute in 1¼ hour.

Cane-sugar and Butter.—July 1 (8 exps.), Plate XXXV. fig. 28. 750 grs. of white sugar, with 500 grs. of butter, and without water, gave a maximum increase of only 1·3 gr. of carbonic acid per minute, of air 48 cubic inches per minute, and in depth of inspiration 3·8 cubic inches. The pulsation was slightly increased.

Grape-sugar.—July 2 (7 exps.), Plate XXXV. fig. 29. 500 grs. of grape-sugar, dissolved in 10 oz. of hot water, gave a maximum increase in carbonic acid of 1·04 gr. per minute in 55 minutes; whilst on another occasion, July 8 (7 exps.), 750 grs. gave an increase of 1·1 gr. per minute in 50 minutes. The maximum increase in the quantity of air inspired per minute was 8 cubic inches and 23 cubic inches. The rate of respiration was lessened, as was also that of pulsation, in the first experiment.

Milk-sugar.—June 12 (7 exps.), Plate XXXV. fig. 20. 250 grs. of sugar of milk, with 6 oz. of hot water, produced a maximum increase in carbonic acid per minute of 1·62 gr. in myself, and ·66 gr. in Mr. MOUL. The volume of air inspired was increased 24 cubic inches and 29 cubic inches per minute. The rate of pulsation fell, in Mr. MOUL, 5 per minute, and that of respiration 1·5 per minute, whilst in myself the subsidence was to a less extent.

Hence it is proved—

1. That sugar in every form is a powerful respiratory excitant.

2. The action is almost immediate, and it rapidly rushes up to a maximum, but the whole effect does not disappear in less than from 1½ to 2 hours.

3. When taken dry the effect is lessened, and the subsequent addition of water increases its action.

4. *In some conditions* the addition of an acid increased the degree, but lessened the duration, of its influence, whilst that of an alkali had the contrary effect. The variation in these results is in accordance with the variation which I have found in the action of acids and alkalies alone. The contrast in the experiments on June 29 and July 5 is very striking.

5. The addition of fat lessened its action.

6. Milk-sugar has less influence than cane-sugar, and grape-sugar has less influence than either.

7. The rate of respiration was always lessened, except when acid was added to the sugar, whilst that of pulsation was sometimes increased, and particularly with acid and fat; but when an alkali was added, it was decreased considerably.

The depth of inspiration was always largely increased.

8. There was a sense of great ease and depth in both inspiration and expiration. In 5 minutes there was sometimes a sour taste; and although there was commonly at first

a comfortable feeling, in 1½ hour there was usually craving for food,—a circumstance very different from that which occurs with ordinary *foods*.

The urine in one instance (May 10) had a celery odour.

9. With both sugar and starch, but especially with the former, there was a very unusual condensation of vapour in the mask.

10. The proportion of carbonic acid to the air inspired was always increased, but it was the most so with the sugar and potass, when the proportion of 1 gr. of carbonic acid to 52 cubic inches of air was increased to as 1 : 44 cubic inches at the period of maximum increase of the air and the carbonic acid.

I like sugar moderately, but Mr. Moul dislikes it.

4. *The Milk Series.*

Milk, a highly complex body, both as a whole, and in nearly all its elements, excites the respiratory function.

New Milk.—April 3 (5 exps.), Plate XXXV. fig. 14, April 8 (5 exps.). One pint of cold good new cows' milk gave me a maximum increase of 2·26 grs. of carbonic acid per minute on one occasion, and 1·9 gr. per minute on another. With the latter, the maximum increase in Mr. Moul was ·94 gr. per minute. The maximum increase in the inspired air was 96 cubic inches per minute in myself, and 22 cubic inches per minute in Mr. Moul. The rate of both respiration and pulsation was increased in myself, but not in Mr. Moul.

Skimmed Milk.—June 9 (5 exps.), Plate XXXV. fig. 15. One pint of good skimmed milk gave me a maximum increase of carbonic acid per minute of ·84 gr., and Mr. Moul of ·54 gr. per minute. The volume of air was increased 21 cubic inches per minute in myself, but there was a decrease in Mr. Moul. The rate of respiration was decreased 1 per minute in both, and that of pulsation 8 per minute in Mr. Moul.

Casein.—June 10 (7 exps.), Plate XXXV. fig. 17, June 11 (7 exps.). The casein of 1 pint of good skimmed milk, well washed with water, gave me, on two occasions, a maximum increase in carbonic acid of 1·34 gr. and ·92 gr. per minute; but Mr. Moul had a decrease, and the maxima of decrease were ·38 gr. and ·6 gr. per minute. My maximum increase in the volume of air inspired was 28 cubic inches in the first, but there was no increase in the second experiment, whilst in Mr. Moul the maximum decrease was 26 cubic inches and 28 cubic inches per minute. My rate of respiration and pulsation was nearly unchanged, but Mr. Moul had a decrease in both.

Lactic Acid.—June 7 (7 exps.), Plate XXXV. fig. 19. 40 ♏ of lactic acid in 8 oz. of water, gave me a maximum increase in carbonic acid of ·42 gr., and Mr. Moul a maximum decrease of ·8 gr. per minute. There was a slight decrease in the quantity of air inspired, and a decrease in the rates of both respiration and pulsation, but the diminution in that of respiration was greatest in myself, and that of pulsation in Mr. Moul.

Casein, and Lactic Acid.—June 14 (8 exps.), Plate XXXV. fig. 18. The casein of 1 pint of skimmed milk and 40 ♏ of lactic acid with water, gave me a maximum increase of ·91 gr. of carbonic acid and 19 cubic inches of air per minute. The rate of respiration slightly increased at first and decreased afterwards, and that of pulsation slightly increased.

Milk-sugar.—June 12 (7 exps.), Plate XXXV. fig. 20. I have already stated the effect of sugar of milk.

Milk-sugar and Lactic Acid.—June 15 (8 exps.), Plate XXXV. fig. 21. 250 grs. of sugar of milk and 40 ♏ of lactic acid, with 8 oz. of hot water, gave me a maximum increase in carbonic acid of 1·18 gr., and in air of 22 cubic inches per minute. The rate of respiration fell slightly, whilst that of pulsation was scarcely changed.

Cream.—June 22 (6 exps.), Plate XXXV. fig. 16. 2 oz. of good fresh cream produced in myself an average increase of ·24 gr., and a maximum increase of ·48 gr. of carbonic acid per minute; whilst in Mr. Moul there was a maximum decrease of ·58 gr. per minute.

June 23 (8 exps.).—2½ oz. gave me an average increase of ·29 gr., and a maximum of ·64 gr. of carbonic acid per minute. The volume of air was increased in myself 33 cubic inches and 19 cubic inches per minute, but in Mr. Moul it was lessened 42 cubic inches per minute. My rate of respiration was slightly increased in the first experiment, and Mr. Moul's fell 1 per minute. My rate of pulsation was somewhat increased.

The effect of rum and milk will be described in the alcohol series.

These experiments show—

1. How different the effects of milk are upon different persons, and that there is a relation between this variation and the enjoyment of the food. I am fond of milk, but Mr. Moul states that neither he nor any of his family can take cheese or milk. As the difference in reference to the influence of milk and its components is very striking, I have tabulated the results for their more ready appreciation.

Thus the increase or decrease of carbonic acid in grains per minute was—

	New milk.	Skimmed milk.	Casein.	Sugar of milk.	Cream.	Lactic acid.
	grs.	gr.	gr.	gr.	gr.	gr.
Myself …………	$+\left\{\begin{matrix}2\cdot26\\1\cdot98\end{matrix}\right\}$	+0·84	$+\left\{\begin{matrix}1\cdot34\\0\cdot72\end{matrix}\right\}$	+1·62	$+\left\{\begin{matrix}0\cdot48\\0\cdot64\end{matrix}\right\}$	+0·42
Mr. Moul ………	$+\left\{\begin{matrix}0\cdot28\\0\cdot94\end{matrix}\right\}$	+0·54	$-\left\{\begin{matrix}0\cdot38\\0\cdot6\end{matrix}\right\}$	+0·66	−0·58	−0·8

2. That there is no element in the milk (except the acid in some conditions) which is not excito-respiratory.

3. That no artificial combination of the component parts of milk produces the effect upon the respiration which follows the use of the natural combination.

4. That new milk has greater influence over the respiration than skimmed milk, and that the cream has greater influence than was found to exist in butter.

5. The rate of respiration and pulsation was increased by new milk and by cream in myself, whilst it was lessened by skimmed milk and lactic acid, and was unchanged by casein.

6. The proportion of carbonic acid to the inspired air was slightly increased.

5. *Alcohols.*

Alcohol, and substances containing much alcohol, disturb the respiration rather than influence it uniformly in any direction. Certain members of the class increase, whilst others decrease the activity of that function, probably according to the elements other than alcohol of which they are composed.

Alcohol.—March 9 (8 exps.). 11 drachms of Spt. Vini (76 per cent. of pure alcohol) in 6 oz. of cold water, caused in myself an average increase of ·18 gr. of carbonic acid, and a maximum of ·46 gr. per minute; and on another occasion, May 6 (7 exps., Plate XXXVI. fig. 44), an average increase of ·8 gr. and a maximum of 1·64 gr. per minute. Mr. MOUL on the latter occasion had no average change, but a maximum decrease of ·72 gr., and an increase of ·4 gr. per minute. The quantity of air was increased 17 cubic inches and 53 cubic inches per minute in myself, and 26 cubic inches per minute in Mr. MOUL. The rate of respiration declined throughout in the first, but in the second experiment there was an increase in both of us, and a subsequent decrease in myself.

Dec. 21 (5 exps.), fig. 45.—I took ½ an oz. of alcohol, spec. grav. ·858 (76 per cent.), with 2 oz. of cold water, every quarter of an hour for three times, after which (as also in the experiment on March 9) I was nearly unconscious. The carbonic acid was increased ·74 gr. per minute, and the air 37 cubic inches per minute. The rate of respiration was scarcely changed, but that of pulsation rose. In 70 minutes from the first dose the quantities fell to the basis. The vapour exhaled from the lungs increased from 3·12 grs. to 3·76 grs. per minute.

Whisky.—April 6 (6 exps.), Plate XXXVI. fig. 46. 1½ oz. of whisky, spec. grav. ·936 (42 per cent.), in 6 oz. of cold water, caused an average decrease of carbonic acid in myself of ·33 gr., and a maximum of ·7 gr. per minute. The rate of respiration was unchanged, but that of pulsation fell 7 per minute.

April 29 (5 exps.).—2 oz. of the finest whisky, spec. grav. ·875 (69 per cent.), bottled more than twenty years, with water, taken after an experiment on the influence of the vapour of wine, gave an average increase of ·29 gr. of carbonic acid in myself, and ·22 gr. in Mr. MOUL; with maxima of decrease of ·73 gr., and of increase of 1·53 gr. in myself, and a maximum increase of ·78 gr. in Mr. MOUL. My quantity of air was at first increased and then lessened, whilst it varied in Mr. MOUL. The rate of respiration fell, and that of pulsation rose in both. Since the basis quantity in this experiment was due to the inhalation of wine, it is possible that the comparative results thus given may not be quite normal.

May 4 (6 exps.), fig. 47.—2 oz. of the whisky which was used in the first experiment, spec. grav. ·928 (45·5 per cent.), caused an average decrease in carbonic acid of ·57 gr., and a maximum of 1 gr. per minute, whilst in Mr. MOUL the average increase was ·29 gr., and the maximum ·66 gr. per minute. The rate of respiration declined considerably in myself, and, with one exception, in Mr. MOUL also. The rate of pulsation fell in

Mr. Moul 8 per minute. The quantity of air was reduced in myself 52 cubic inches per minute.

Brandy.—April 20 (6 exps.). 1½ oz. of excellent brandy, with water, caused in me an average decrease of ·2 gr. of carbonic acid, and a maximum of ·38 gr. per minute.

April 24 (6 exps., Plate XXXVI. fig. 52).—2 oz. gave me an average decrease of ·38 gr., and a maximum of ·71 gr. per minute, and to Mr. Moul ·02 gr. and ·62 gr. respectively; but in him there was, on two occasions, an increase of ·4 gr. per minute. The quantity of air fell 42 cubic inches and 37 cubic inches per minute in myself, and 34 cubic inches per minute in Mr. Moul. My rate of respiration and pulsation fell on both occasions, whilst in Mr. Moul the former varied and the latter fell a little.

Gin.—April 27 (6 exps.), Plate XXXVI. fig. 48. 2 oz. of fine old British gin, with 6 oz. of cold water, gave me a maximum decrease of 1·52 gr. of carbonic acid in half an hour, and an average decrease of ·65 gr. per minute in 86 minutes. In Mr. Moul the average decrease was ·2 gr., and the maximum ·4 gr. per minute. The volume of air was reduced 56 cubic inches in myself, but it was unchanged in Mr. Moul. The rate of respiration fell in myself, but it varied much in Mr. Moul; whilst that of pulsation fell 6 per minute in myself, and was unchanged in Mr. Moul.

Dec. 21 (4 exps.).—The same quantity of newer and probably inferior gin, taken in the afternoon, gave me a maximum decrease of ·46 gr. of carbonic acid and 43 cubic inches of air per minute, the diminution ending in three-quarters of an hour. The rate of respiration slightly fell, whilst that of pulsation rose 5 per minute. There was a decrease in the amount of vapour exhaled from the lungs, from 3·12 grs. to 2·7 grs. per minute (·699 gr. and ·667 gr. to 100 cubic inches).

Rum.—April 8 (6 exps.). 1½ oz. of Navy rum, spec. grav. ·900 (58 per cent.), with water, taken in the afternoon, gave me a maximum increase in carbonic acid of ·78 gr., and an average increase of ·26 gr. per minute, whilst Mr. Moul had a maximum increase of ·7 gr. per minute. The volume of air was increased 18 cubic inches and decreased 49 cubic inches in myself, and in Mr. Moul it was decreased 80 cubic inches per minute. The rate of respiration fell, and chiefly in Mr. Moul; whilst that of pulsation rose in myself and fell in Mr. Moul. On another occasion, April 10 (7 exps., Plate XXXVI. fig. 49), in the morning, the increase of carbonic acid was much greater, viz. a maximum of 1·24 gr. in myself, and 2·14 grs. per minute in Mr. Moul. The quantity of air inspired varied in myself, but in Mr. Moul it was increased 82 cubic inches per minute. My rate of respiration fell, and that of Mr. Moul fell greatly, and his rate of pulsation fell also. The depth of inspiration was increased in both of us.

April 30 (7 exps.).—2 oz. of very fine old rum, spec. grav. ·875 (69 per cent.), taken in the morning, gave to me a maximum increase in carbonic acid of only ·1 gr., but to Mr. Moul of 1·5 gr. per minute. There was a maximum increase in the quantity of air of 11 cubic inches in myself, and 40 cubic inches in Mr. Moul per minute. The rate of respiration fell towards the end of the inquiry in myself, and at an earlier period in Mr. Moul. His pulsation also fell a little, whilst mine rose.

December 27 (5 exps.), Plate XXXVI. fig. 50.—½ an oz. of only moderately good rum, which had been used in the experiments on inhalation, and so weak as ·944 spec. grav. (37·5 per cent.), with 2 oz. of cold water, taken every quarter of an hour, gave the following increase in the quantity of carbonic acid evolved after each dose:—·14 gr., ·43 gr., ·2 gr., and ·66 gr., showing a continuous increase. The maximum increase in the quantity of air inspired was 49 cubic inches after the last dose, and the proportion of the carbonic acid to the inspired air was somewhat reduced in consequence of the disproportionate increase of the latter. The rate of respiration was slightly increased with the first dose, and as slightly decreased with the subsequent ones, whilst the rate of pulsation was also slightly increased. The amount of vapour exhaled per minute in the expired air varied from a decrease of ·3 gr. and ·2 gr. after the second and third doses, when the respiration felt feeble and there was sighing, to an increase of ·04 gr. and ·06 gr. after the first and fourth doses. There was, however, a progressive decrease to each 100 cubic inches of inspired air, as follows:—·742 gr. (basis), ·7 gr., ·67 gr., ·65 gr., and ·67 gr., after succeeding doses.

Rum and Milk.—April 12 (8 exps.), Plate XXXVI. fig. 51. 1½ oz. of Navy rum, with 1 pint of good new milk, produced in myself and Mr. MOUL an average increase in the carbonic acid of ·73 gr. and ·65 gr. per minute, whilst the maxima of increase were ·9 gr. and 1·38 gr. per minute respectively. The quantity of air was increased 25 cubic inches and 18 cubic inches per minute, with a diminution also of 42 cubic inches in the latter (Mr. MOUL). The rate of respiration declined, and very much so in Mr. MOUL, whilst that of pulsation was increased. The depth of respiration was increased in both.

Sherry Wine.—April 2 (5 exps.), May 5 (5 exps.), May 6 (7 exps.), Plate XXXVI. fig. 53. 3 oz. of tolerably good sherry wine, alone, produced in myself, on two occasions, an average increase of ·19 gr. and ·3 gr., and on another occasion an average decrease of ·32 gr. of carbonic acid per minute. The maxima of increase were ·36 gr. and ·44 gr., and of decrease ·52 gr. per minute on the several occasions. In Mr. MOUL the average decrease was ·32 gr., and increase ·926 gr. and ·21 gr. per minute; with maxima of decrease ·9 gr., and of increase 1·44 gr. and ·82 gr. per minute. The quantity of air was lessened in myself in all the inquiries, but chiefly in the first one, and least in the last; whilst in Mr. MOUL it varied in the two former and was reduced in the last. The rate of respiration in both fell, and chiefly so in the first experiment; whilst that of pulsation rose in myself, and was unchanged in Mr. MOUL.

Inhalation of Alcohols.

I have made several experiments with a view to determine the influence of the volatile matters of wines and spirits when inhaled, but the uneasy feeling in the lungs which followed each experiment made me fear the effect of a too prolonged series. I placed 3 or 4 oz. of the fluid for examination in a WOULFE's bottle, and inspired air at the ordinary temperature which had passed directly over it, taking care to shake the fluid frequently

Port Wine.—April 29 (4 exps.), Plate XXXVI. fig. 54. The wine was exceedingly fine and old, and the vapour was inspired during 10 minutes, on four occasions, within 70 minutes. The carbonic acid was reduced on the average ·53 gr. in myself, and ·42 gr. in Mr. Moul per minute, with maxima of decrease of ·87 gr. and ·58 gr. per minute respectively. The maximum decrease in the inspired air was 56 cubic inches in myself, and 36 cubic inches in Mr. Moul per minute. The rate of respiration was increased in Mr. Moul, but it was ultimately decreased in myself. A moderately pungent sensation was perceived in the pharynx, and particularly when the wine was shaken. The loss in weight was 40 grs., and the spec. grav. increased from 1·004 to 1·005.

Sherry Wine.—December 15. The bouquet was good but not persistent, and it produced but little effect upon the throat. The effect of my inhaling it during 10 minutes was to reduce the carbonic acid ·12 gr., and the air 8 cubic inches per minute. The vapour exhaled was increased from 3 grs. to 3·66 grs. per minute (·707 gr. to ·88 gr. to 100 cubic inches).

Alcohol.—March 30. Spt. Vini, inhaled 15 minutes, caused during the last 5 minutes a decrease in the carbonic acid of ·34 gr., and in the air of 11 cubic inches per minute. The quantity of vapour exhaled was increased from 3·22 grs. to 4·04 grs. per minute (·755 gr. to ·973 gr. to 100 cubic inches). On another occasion (December 6), the alcohol being inhaled during 10 minutes, there was no variation either in the carbonic acid or air, and the vapour exhaled was increased from 3·3 grs. to 3·9 grs. per minute (·78 gr. to ·91 gr. to 100 cubic inches). The alcohol lost 21 grs. in weight. In both experiments the pulse became fuller.

Gin (British).—December 13. Inhalation of gin during 10 minutes caused a decrease in the carbonic acid of ·5 gr. per minute, whilst there was an increase in the volume of air inspired of 18 cubic inches per minute. The vapour exhaled was increased from 3 grs. per minute to 3·51 grs. per minute (·746 gr. to ·885 gr. to 100 cubic inches).

Rum.—December 4, December 13. Rum, inhaled during 10 minutes on one occasion, and 15 minutes on another, gave a decrease in carbonic acid of ·56 gr. and ·12 gr., and in air an increase of 12 cubic inches per minute. The pulse became much fuller. The vapour exhaled increased from 3·24 grs. to 3·89 grs. per minute (·799 gr. and ·962 gr. to 100 cubic inches), and from 3·12 grs. to 4·4 grs. per minute (·699 gr. to ·043 gr. to 100 cubic inches). In the first experiment the rum lost 17 grs. in weight in ten minutes.

I now proceed to describe the effect of ale and porter.

Stout.—May 7 (7 exps.), Plate XXXVI. fig. 55. 10 oz. of good stout gave to myself and Mr. Moul an average increase in the carbonic acid of ·83 gr. and ·81 gr., and maxima of 1·56 gr. and 1·02 gr. respectively per minute. The air was increased 41 cubic inches and 46 cubic inches per minute. The rate of respiration was a little increased, and varied somewhat in myself, whilst it fell in Mr. Moul. The rate of pulsation was increased 4 and 7 per minute. The depth of inspiration was increased 1·6 cubic inch. On another occasion, May 11 (6 exps.), the maximum increase in carbonic acid was 1·16 gr. and ·98 gr.

TABLE showing the amount of variation in the quantity of carbonic acid and vapour exhaled, and of the air inhaled, as also in the rate of respiration and pulsation, &c. in the various experiments on alcohols previously recorded.

The sign + signifies increase and — decrease.

	Myself.									Mr. Moul.							
	Variation in the carbonic acid expired per minute.		Variation in the air inspired per minute.		Variation in the vapour expired per min.	Variation in the rate per minute of		Variation in the depth of inspiration.	Fluid lost.	Variation in the carbonic acid expired per minute.		Variation in the air inspired per minute.		Variation in the vapour expired per min.	Variation in the rate per minute of		Variation in the depth of inspiration.
	Average.	Maximum.	Average.	Maximum.		Respiration.	Pulse.			Average.	Maximum.	Average.	Maximum.		Respiration.	Pulse.	
	gr.	gr.		cub. inch.	gr.			cub. in.	grs.	gr.	grs.		cub. inch.				
Alcohol	+ ·18	+ ·46	...	+47	......	—											
	+ ·8	+1·64	...	+53	......	+ —	...	...	...	....	+ ·4 — ·72	...	+26	...	+		
		+ ·74	...	+87	+ ·64	None.	... +	...	...								
Whisky	— ·33	— ·7	...	......	......	None.	—7										
	+ ·29	— ·73 +1·53	... ...	+ —	......	—	+	...	...	+ ·22	+ ·78	...	Varied.	...	—	+	
	— ·57	—1	...	—52	......	—(Much.)	...	...	...	+ ·29	+ ·66	...	......	...	—	—8	
Brandy	— ·2	— ·38	...	—42	......	—	...	...	...	......	......	...	......	...	Varied.		
	— ·38	— ·71	...	—37	......	—	...	...	...	— ·02	— ·62 + ·4	...	—34	...	Varied.	—	
Gin	— ·65	—1·52	...	—56	......	—	—6	...	...	— ·2	— ·4	.	0	...	Varied.	None.	
		— ·46	...	—43	— ·42	—(Little.)	+5										
Rum	+ ·26	+ ·78	...	+13 —49	......	—	+	...	...	......	+ ·7	...	—80	...	—	—	
		+1·24	...	Varied.	......	—	...	+	...	......	+2·14	.	+82	...	—(Much.)	—	+
		+ ·1	...	+11	......	—	+	...	...	......	+1·5	...	+40	...	—	—	
	+ ·35	+ ·66	...	+49	......	—	+3	—									
Rum and Milk ...	+ ·73	+ ·9	...	+25	......	—	...	+	...	+ ·65	+1·3	...	+18 —42	...	—(Much.)	+	+
Sherry	+ ·19	+ ·36	...	+Much.	......	—	+	...	...	— ·32	— ·9	...	Varied.	...	—	None.	
	+ ·3	+ ·44	...	—	......	—	+	...	...	+ ·926	+1·44	...	Varied.	...	—	None.	
	— ·32	— ·52	...	Least.	......	—	+	...	...	+ ·21	— ·52	.	—	...	—	None.	
Vapours.																	
Port Wine ...	— ·53	— ·87	...	—56	......	—	...	...	40	— ·42	— ·58	...	—36	...	+		
Sherry	— ·12	......	...	— 8	+ ·66												
Alcohol	— ·34	......	...	—11	+ ·92	...	Fuller.										
	0	......	...	......	+ ·6	...	Fuller.	...	21								
Gin	— ·5	......	...	+18	+ ·51												
Rum	— ·56	......	...	......	+ ·65	...	...	...	17								
	— ·12	......	...	+12	—1·29	...	Much fuller.										
Stout	+ ·83	—1·56	...	+41	......	—Varied.	—4 —7	—1·5	...	— ·51	+1·02	...	—46	...	—		
		—1·16	...	0	......	—	—	+	...	......	+ ·98		+18	...			+
Ale..........	— ·6	+1·4	...	—60	......	None.	None.	...	...	+ ·27	+ ·36		—	...	+		

per minute. There was no increase in the quantity of air in myself, but there was a maximum increase of 18 cubic inches in Mr. MOUL. The rate of respiration fell a little, and that of pulsation rose. The depth of inspiration was much increased.

Ale.—May 10 (6 exps.), Plate XXXVI. fig. 56. 11 oz. of good old home-brewed Hertfordshire ale, a little acid, gave an average increase in carbonic acid of ·6 gr. and ·27 gr., and maxima of 1·4 gr. and ·36 gr. to myself and Mr. MOUL respectively. The volume of air inspired was increased 60 cubic inches in myself, but there was a diminution in Mr. MOUL. The rate of respiration and pulsation scarcely varied in myself, and that of respiration was at first increased in Mr. MOUL.

As there was much variation in the action of the substances classed under this head, I have thrown the results now given into the Table on the opposite page, with a view to the more ready comprehension of the numerous details.

Abstract of the Effects of Alcohols.

From the foregoing we may learn—

1. That the presence of alcohol, being one amongst many elements, and that one varying greatly in quantity, is an insufficient ground for classification, and does not give a common action to the members of this class.

2. The *direct* action of pure alcohol was much more to increase than to lessen the respiratory changes, and sometimes the former effect was well pronounced. Small doses repeated had a more uniform and persistent effect than would have followed the administration of the whole at once. The *indirect* action, as, for example, in lessening the appetite for food, and the mode of its action, I have not investigated.

3. Brandy, whisky, and gin, and particularly the latter, almost always lessened the respiratory changes recorded, whilst rum as commonly increased them. Rum and milk had a very pronounced and persistent action, and there was no effect upon the sensorium. Ale and porter always increased them, whilst sherry wine lessened the quantity of air inspired, but slightly increased the carbonic acid evolved.

4. The volatile elements of alcohol, gin, rum, and sherry and port wine, when inhaled, lessened the quantity of carbonic acid exhaled, and usually lessened the quantity of air inhaled. The effect of fine old port wine was very decided and uniform; and it is known that wines and spirits improve in aroma and become weaker in alcohol by age. The excito-respiratory action of rum is probably not due to its volatile elements.

5. The quantity of vapour exhaled from the lungs was increased during the inhalation of the volatile elements of wines and spirits, without the quantity of air having increased. When gin was drank, the quantity of vapour in the expired air was lessened, whilst it was increased under the influence of alcohol, in about the same degree as during the inhalation of that substance. Hence the exhalation of vapour and carbonic acid are not parallel acts.

6. The rate of respiration was in almost all instances lessened in both of us, whilst that of pulsation was as constantly increased in myself, but not in Mr. MOUL.

7. The relation between the quantity of carbonic acid expired and the volume of air inspired, was usually increased at the period of maximum influence.

8. The variation in the results was greater than the statement of the average and maximum effects indicates, as may be seen in the Tables and Plates.

The *general effects upon the system* of these substances may be thus epitomized.—

1. There is not an exact correspondence in time and intensity of the effects upon consciousness, sensibility, and respiration, and their principal influence is not upon the respiratory function. They *disturb* the vital actions.

2. There were two sets of effects in each of the inquiries on spirits

A. The early effects, consisting of—

Lessened consciousness, with cloudiness, swimming or giddiness, beginning in less than 10 minutes, and increasing during about 30 minutes.

Lessened sensibility to light, sound, and touch.

Wavy or buzzing sensation passing through the whole body, and a semi-cataleptic state, in which there was indisposition to move any part of the body from the then existing position.

These occurred at the same period as:

Lessened voluntary muscular power and control, with sensation of stiffness and hanging of the upper lip, and stiffness of the face and forehead, beginning in 8 minutes, and continuing about 15 minutes. The dartos was relaxed, and the erector penis and the sphincter of the bladder were rendered less effective. The action of the heart and arteries was increased, as was that of the muscles of inspiration, with a sensation of sudden and *forcible* action, to a greater degree than the quantity of air inspired accounted for. There was certainly a difference in the effect upon the muscles subject to, and not subject to volition.

Lessened transpiration of vapour from the lungs during ¾ to 1 hour, with dryness of the skin (as if it had been induced by an east wind), and particularly with rum. Increased arterial action near to the surface in 8 minutes, with heat, tingling and swelling of the skin, and a dry state of the whole mouth, with whisky; and dryness, redness, and soreness of the tip of the tongue with rum.

Pleasant dreaminess and talkativeness, particularly with rum, in 13 to 15 minutes.

B. The later effects.

Taciturnity in from 18 to 80 minutes, followed by depression and a miserable feeling in from 60 to 90 minutes.

Sensation of cold often occurred suddenly and apart from the temperature of the air in about 50 minutes.

The principal influence over consciousness and sensibility was often lessened suddenly, and the effects of the alcohol nearly disappeared at the following periods: 71 to 73 minutes with alcohol; 48 to 120 minutes with rum; 60 to 84 minutes with whisky; 46 to 80 minutes with brandy, and 68 minutes with gin.

6. *The Tea Series.*

The members of this class are nearly all powerful respiratory excitants. The inquiries have been very varied and extensive. The tea was exceedingly good and pure.

Tea.—April 2 (6 exps.), Plate XXXVI. fig. 32. 100 grs. of black tea gave to myself and Mr. Moul a maximum increase of carbonic acid of ·87 gr. and 1·72 gr. per minute in 50 and 71 minutes. The average increase was ·73 gr. and 1 gr. per minute. The quantity of air was increased 71 cubic inches and 68 cubic inches per minute, and the depth of inspiration 4 cubic inches in each of us. The rate of respiration was increased and that of pulsation decreased, except at the first observation, after the tea had been taken.

April 7 (5 exps.), April 17 (5 exps.), Plate XXXVI. fig. 31, April 19 (5 exps.).—50 grs. of black tea gave the following results to myself, Mr. Moul, Mr. Moul's son (æt. 16), and Professor Frankland on different occasions:—a maximum increase of carbonic acid of 1·08 gr. in myself; 1·38 gr., 2·58 grs. and 1·6 gr. in Mr. Moul; 2 grs. in Mr. Moul's son, and ·69 gr. per minute in Professor Frankland. The maximum increase in the quantity of air inspired was in the same order,—34 cubic inches, 39 cubic inches, 50 cubic inches, 72 cubic inches, 95 cubic inches, 26 cubic inches per minute. My rate of pulsation was lessened, and that of respiration was scarcely changed. The rate of both was lessened in Mr. Moul, whilst in his son that of respiration was increased and of pulsation decreased.

May 26 (7 exps.).—100 grs. of green tea, drunk when cold, gave to myself and Mr. Moul maxima of increase of carbonic acid of ·9 gr. and 2·58 grs. per minute, with average increase of ·44 gr. and 1·57 gr. per minute. The quantity of air was not increased in myself, but it was increased 120 cubic inches per minute in Mr. Moul. The rate of respiration declined so much as 1·2 and 2 per minute, and in Mr. Moul that of pulsation also slightly declined. The depth of inspiration was increased 8 cubic inches and 10 cubic inches.

August 26 (4 exps.).—The same quantity given to Mr. Hoffman, Surgeon to the Margate Infirmary, caused a maximum increase of ·64 gr. of carbonic acid per minute and 50 cubic inches of air per minute in one hour. The pulsation declined after the first observation.

May 24 (10 exps.), Plate XXXVI. fig. 35.—25 grs. of green tea, drunk when cold several hours after it had been infused, and repeated every quarter of an hour for five times, gave Mr. Moul an average increase in carbonic acid of 1·2 gr., and a maximum of 1·8 gr. per minute. The maximum increase in the quantity of air inspired was 66 cubic inches per minute after the fifth dose. The total increase of carbonic acid, as deduced from the ten observations, was 193 grs.; but at the end of the inquiry, in two hours and thirty-four minutes, there was still an increase of ·9 gr. of carbonic acid and 86 cubic inches of air per minute. The rate of respiration declined 1·6 per minute, whilst that of pulsation was scarcely changed. The depth of inspiration was increased 10·6 cubic inches, and was much the greatest after the fourth dose.

June 8 (13 exps.), fig. 34.—Mr. Moul took 150 grs. of black tea infused in one pint of water, and the whole carbonic acid exhaled by him during the inquiry was collected and weighed every five minutes. The basis quantity was again reached after sixty-five minutes. On the whole average the increase in the carbonic acid was ·79 gr., with a maximum of 1·24 gr. per minute. Thus the excess of the carbonic acid exhaled over the basis quantity was 51·35 grs., or only one-fourth of that resulting from a smaller quantity when divided and taken at intervals. The maximum increase in the quantity of air inspired was 92 cubic inches per minute, and the greatest increase in both the carbonic acid and the air occurred in the first half of the period of inquiry. The rate of respiration was increased in the first three examinations.

May 22 (7 exps.), Plate XXXVI. fig. 33.—I took 100 grs. of black tea and collected all the carbonic acid exhaled under its influence, and weighed all that collected in the first hour, and then each quarter of an hour for three times, and finally in five minutes, when the basis was reached and the inquiry ended. The average increase in the carbonic acid was ·64 gr. and the maximum 1·23 gr. per minute, and thus the total increase in 110 minutes was 70·40 grs. The quantity of air was increased on the average 17·5 cubic inches, and the maximum increase was 77 cubic inches per minute at the end of the first hour. The pulse fell and the rate of respiration slightly increased, whilst the depth of inspiration increased nearly 5 cubic inches.

Tea, Milk, and Sugar.—June 5 (6 exps.), Plate XXXVI. fig. 30. 50 grs. of black tea, taken with milk and sugar, gave us maxima of increase in the carbonic acid of 2·96 grs. and 2·56 grs. per minute. The maximum increase in the quantity of air was 78 cubic inches per minute. There was also increased rate both of respiration and pulsation.

Tea and Citric Acid.—July 9 (7 exps.), fig. 36, July 13 (5 exps.), July 15 (5 exps.), July 13 P.M. (3 exps.), July 12 (5 exps.). 100 grs. of black tea, taken with 30 grs. of citric acid, gave me maxima of increase of carbonic acid in three experiments of 1·86 gr., 1·11 gr., and 1·14 gr. per minute; on another occasion it was only ·88 gr. per minute in Mr. MOUL. The increase in the quantity of air inspired was 72 cubic inches, 60·5 cubic inches, and 72 cubic inches per minute in myself, and 6 cubic inches per minute in Mr. MOUL; but in him there was also a maximum decrease of 18 cubic inches per minute. The rate of respiration and pulsation was always considerably increased in myself, but not in Mr. MOUL.

When taken in the afternoon I had no increase of the carbonic acid, and the effects much more resembled those observed in Mr. MOUL in his experiment before breakfast.

Tea, with Soda and Potash.—July 10 (6 exps.), fig. 37. 100 grs. of black tea, with 50 grs. each of carbonate of soda and potash, with 8 oz. of water, gave me a maximum increase in carbonic acid of 1·08 gr. per minute, and in air 39 cubic inches per minute. The rate of respiration scarcely varied, but that of pulsation was a little increased.

Tea, with Caustic Alkali.—July 22 (5 exps.), fig. 38. 100 grs. of black tea, with 40 ♏ of Liq. Potassæ and 7 oz. of water, gave no increase in the carbonic acid. There was an increase of 27 cubic inches of air. The rate of respiration was unchanged, but that of pulsation fell.

On many occasions we took a cup of tea with a view to relieve the system after experiments with other substances; and the results have much interest, as showing their uniform direction and the rapidity of the action of tea. We both took milk with it, but I alone took sugar.

April 24.—After having taken brandy, tea gave to myself and Mr. Moul in fifteen minutes an increase of carbonic acid of ·62 gr. and 2·58 grs., and of air 25 cubic inches and 108 cubic inches per minute.

May 16.—After cocoa had increased the respiration in myself, but scarcely in Mr. Moul, tea gave us an increase in carbonic acid of ·2 gr. and 2·2 grs., and in air of 9 cubic inches and 78 cubic inches per minute.

May 28.—After an experiment on rice, 50 grs. of tea infused one minute, gave Mr. Moul an increase in carbonic acid of 1 gr., and in air of 17 cubic inches per minute.

May 29.—After having taken whisky, the same quantity of tea infused two minutes, gave us in ten minutes an increase in carbonic acid of ·96 gr. and 1·32 gr. of carbonic acid, and 50 cubic inches and 56 cubic inches of air per minute.

May 30.—After an experiment on rum, tea caused a small decrease in my carbonic acid with a slight increase of air; but Mr. Moul had in twenty-five minutes an increase of ·92 gr. of carbonic acid and 48 cubic inches of air per minute.

June 21.—After an experiment on arrowroot and butter, I had again a small decrease in the carbonic acid, whilst Mr. Moul had an increase of 2·12 grs. and 98 cubic inches of air per minute. Neither substance agreed with us.

Coffee.—April 6 (4 exps.), April 9 (6 exps.), April 15 (6 exps.), Plate XXXVI. fig. 39, May 3 (7 exps.), fig. 40. ½ an oz. of good coffee gave to myself and Mr. Moul, on three occasions, the following increase per minute:—Carbonic acid, ·98 gr. and 1·02 gr., 9 gr. and ·4 gr., 1·16 gr. and 2·54 grs.; air, 36 cubic inches and 14 cubic inches, 40 cubic inches and 34 cubic inches, 35 cubic inches and 84 cubic inches. ¾ of an oz. gave us an increase per minute of carbonic acid, 1·08 gr. and ·82 gr.; and of air, 28 cubic inches and 54 cubic inches per minute. The rate of respiration, but not of pulsation, was increased in myself, whilst in Mr. Moul the increased rate of both functions was much less than in myself in the first experiment; and in the others there was no increase. The increase in the depth of inspiration was not great.

April 27.—A cup of coffee with milk, after an experiment on gin, gave neither of us any increase of carbonic acid or air within a short period.

May 6.—¾ of an oz. of coffee with milk gave us, after an experiment on wine, an increase in carbonic acid of ·68 gr. and 1·68 gr., and of air 64 cubic inches and 86 cubic inches per minute in thirty minutes.

Chicory.—May 17 (7 exps.), Plate XXXVI. fig. 42. ½ an oz. of chicory with 8 oz. of boiling water, gave to myself and Mr. Moul a maximum increase in carbonic acid of 1·17 gr. and ·66 gr. per minute, whilst the quantity of air inspired was increased 27 cubic inches and 42 cubic inches per minute. The rate of respiration and pulsation fell considerably in Mr. Moul, whilst in myself the latter was lessened 1 per minute and the former slightly increased. The depth of inspiration was increased 4 cubic inches and 6·2 cubic inches.

Cocoa.—April 16 (6 exps.), Plate XXXVI. fig. 43, April 20 (6 exps.). 1 oz. of good cocoa, well boiled in 11 oz. of water, gave me on two occasions a maximum increase in carbonic acid of 1·92 gr. and 1·1 gr. per minute, whilst Mr. Moul had on one occasion a maximum increase of ·64 gr. There was a maximum increase of air of 27 cubic inches and 61 cubic inches per minute in myself, with an increase of 91 cubic inches in Mr. Moul. The rate of respiration and pulsation was nearly unaffected in myself, but it fell in Mr. Moul. The depth of inspiration was increased 3·6 cubic inches and 5·7 cubic inches in myself.

Coffee leaves.—April 16 (6 exps.), Plate XXXVI. fig. 41, April 30 (5 exps.). Mr. Hanbury of Plough Court kindly furnished me with a specimen of coffee leaves, from which a beverage is made in Sumatra. ½ an oz. infused in 10 oz. of boiling water gave me on two occasions a maximum decrease in carbonic acid of ·84 gr. and ·89 gr. per minute, and Mr. Moul had a decrease of 1·42 gr. per minute. There was a maximum decrease in the quantity of air inspired of 25 cubic inches and 51 cubic inches per minute in myself, and 160 cubic inches in Mr. Moul. The rate of pulsation and respiration, and the depth of inspiration, all fell in both of the inquiries.

The foregoing experiments prove—

1. That tea, coffee, chicory and cocoa are respiratory excitants, whilst coffee leaves depress the respiratory function.

2. The uniformity in the direction of the results is exceedingly striking, whilst the degree of influence is to a certain extent variable.

3. Tea is the most powerful, then coffee and cocoa, and lastly, chicory.

4. The rate of respiration was sometimes a little increased and at others a little decreased, but the depth of inspiration was always largely increased. The rate of pulsation was usually slightly increased.

5. With the addition of an acid the effect was somewhat lessened, and the rate of both functions was increased to a greater degree than with tea alone.

6. The addition of an alkali also lessened the effect of tea, and a fixed alkali totally destroyed its influence.

7. The action of acids and alkalies varies with the state of the system and in different persons.

8. The addition of sugar and milk in the ordinary way increased the effect.

9. Small doses of tea, frequently repeated, have much greater effect than the total quantity taken at once.

10. Cold tea, and tea infused and kept twenty-four hours, has as much effect as when hot and recently made.

11. Green tea has somewhat more influence than black tea, and particularly in lessening the rate and increasing the depth of respiration.

12. The proportion of the carbonic acid to the quantity of air inspired was always increased at the period of maximum influence.

13. Mr. Moul experienced much greater effect from tea than myself. He is exceedingly fond of tea, is not fond of coffee, and dislikes acids, and in the above experiments the results corresponded.

14. The influence of both tea and coffee is exerted almost immediately, viz. in five minutes, and the maximum is attained in from twenty-five to sixty minutes. The duration varies from one to two hours. In all these particulars there is a variation in different persons.

15. With tea we frequently found nausea in ten minutes, and sometimes to a very unpleasant degree, but it left in ten or fifteen minutes. There was also a soothing or narcotic effect at first on several occasions, and when it had been taken with an alkali this effect was continued to the end; whilst on the other hand the influence was more stimulating with the acid. There was great freedom of inspiration, and sometimes of expiration also, in about forty to seventy minutes, and with this there was a feeling of lightness and clearness. The pulse was always soft, and the skin moist or soft.

16. With coffee there was no nausea or soothing; the pulse was sometimes feeble, and the pulsation in the head and hands more perceptible. There was often an uncomfortable sensation in the small intestines and forcing at the rectum, and not unfrequently a sense of constriction about the diaphragm in from sixteen to forty minutes. There was more action upon the kidneys than with tea. The skin was often hot and dry.

17. Coffee leaves caused the hands to be hot in seven minutes when 1 oz. had been taken, and in thirty-five minutes a purring sensation occurred similar to that with alcohol, and a not unpleasant feeling of listlessness. The effect was narcotic in thirty-seven minutes.

7. *Some other Nitrogenous Substances.*

Albumen.—April 9 (8 exps.). Two good-sized boiled eggs gave Mr. Moul's son an average increase in carbonic acid of ·27 gr., and a maximum of ·88 gr. per minute. The maximum increase in the quantity of air inspired was 17 cubic inches per minute. On another occasion, April 21 (6 exps.), Plate XXXVI. fig. 57, my increase in carbonic acid was an average of ·45 gr. and a maximum of 1·12 gr. per minute, whilst in Mr. Moul they were ·18 gr. and 38 gr. per minute, respectively. I had a decrease in the quantity of air, whilst Mr. Moul had an increase of 38 cubic inches per minute. My rate of respiration scarcely varied, but Mr. Moul's fell, as did also his pulsation in a slight degree.

Gelatin.—May 8 (6 exps.). The effect of 120 grs. of pure dry isinglass prepared with 12 oz. of water, gave to myself and Mr. Moul an average increase in carbonic acid of ·43 and ·14 gr., and maxima of ·84 gr. and 66 gr. per minute respectively. On another occasion, May 21 (7 exps.), 100 grs. of dry commercial isinglass gave to myself and Mr. Moul an average increase in carbonic acid of ·14 gr. and ·26 gr., and maxima of ·76 gr. and ·92 gr. per minute respectively. I had no increase of air in the first experiment, but there was a maximum increase of 14 cubic inches per minute in the second, whilst the increase in Mr. MOUL was 70 cubic inches and 28 cubic inches per minute. The rate of respiration was reduced in both, and the respirations were free and deep. Mr. MOUL is fond of jelly.

Almond-emulsion.—September 16 (6 exps.). 1000 grs. of almonds made into an emulsion with 8 oz. of water, gave me an average decrease in carbonic acid of ·17 gr., and in air of 18 cubic inches per minute. The rate of both pulsation and respiration fell, and there was a semi-narcotic effect in thirty-five minutes. The hands were hot and congested.

Lean flesh.—April 23 (7 exps.). 6 oz. of raw (4½ oz. cooked) excellent lean beefsteak, gave to myself and Mr. MOUL a maximum increase of ·7 gr. and ·2 gr. per minute, but I had no average increase. The maximum increase in the air inspired was 19 cubic inches in myself, whilst there was a decrease in Mr. MOUL. The rate of respiration was reduced.

Fish.—July 5 (8 exps.). 8 oz. of very fine, well cooked salmon, gave me a maximum increase of ·84 gr. of carbonic acid per minute in sixty-five minutes, but there was no average increase. There was a maximum increase of air of 15 cubic inches, with a slight fall in the rate of both respiration and pulsation.

Hence albumen, fibrine, and gelatin exert an influence in exciting the respiratory function—fibrine in the least, and gelatin in the greatest degree. Almond-emulsion, although so powerful a ferment, is not a respiratory excitant.

Conclusion.

Having now described the action of each of the substances mentioned in the list, I proceed to offer a few general remarks upon the results obtained.

1. *It is evident that foods may be fitly divided into two classes*, viz. those which excite certain respiratory changes (excito-respiratory), and those which do not.

The excito-respiratory are nitrogenous foods, milk and its components, sugars, rum, beer, stout, the cereals, and potato.

The non-exciters are starch, fat, certain alcoholic compounds, the volatile elements of wines and spirits, and coffee leaves.

2. Of the hydrocarbons, sugar must be regarded apart from starch and fat; the former being destructive and the latter conservative of material in the system. Alcohols are allied to both, but chiefly to the latter.

The very similar and powerful action of the cereals, and the uniform and powerful action of milk—substances upon which the life of man chiefly depends—are remarkable, whilst the very feeble excito-respiratory action of pure starch is in accordance with its exceptional use.

No sufficiently distinctive action between fat and starch has been demonstrated, but certain differences have been shown, such as that fat, as compared with starch, less excites the respiration, does not increase the action of starch, increases pulsation somewhat, has a [illegible] influence, and gives a sensation of satisfaction.

[illegible] Nearly all nitrogenous foods are "excito-respiratory" in various degrees, and they comprehend nearly all the members of the class. This power is not in a definite proportion to the quantity of nitrogen contained by them, and sugar, which is powerfully "excito-respiratory," is destitute of nitrogen. Probably all compound foods containing sugar or gluten, or both, are "excito-respiratory." The principal ferments are "excito-respiratory."

4. Respiratory excitants have a temporary action; but the action of most of them commences very quickly, and attains its maximum within one hour.

5. The most powerful respiratory excitants are tea and sugar; then coffee, rum, milk, cocoa, ales, and chicory; then casein and gluten, and lastly, gelatin and albumen. The amount of action was not in uniform proportion to their quantity. Compound aliments, as the cereals containing several of these substances, have an action greater than that of any of their elements.

6. Most respiratory excitants, as tea, coffee, gluten and casein, cause an increase in the evolution of carbon greater than the quantity which they supply, whilst others, as sugar, supply more than they evolve in this excess, that is, above the basis. No substance containing a large amount of carbon evolves more than a small portion of that carbon in the temporary action occurring above the basis line, and hence a large portion remains unaccounted for by these experiments.

7. The source of the carbon evolved, whether *directly* from the food recently taken, or *indirectly* from increased action induced in the tissues, or from the more rapid disengagement of that contained in the blood, has not been determined; but it has been shown in reference to the rapidity and amount of action of foods,—

α. That the increase in the evolution of carbonic acid with some of the respiratory excitants is considerable in from three to eight minutes after the introduction of the substance into the stomach, and increases regularly and quickly to a maximum, and then often declines rapidly, as is well shown by tea and sugar.[*] In others, as casein, the action is more tardy.

β. That the effect of alcohols upon the sensorium was often perceived in four minutes after they had been swallowed; and the effect of the inhalation of alcohols over the chemical and physical changes was immediate.

γ. That alkalies usually lessen and prolong their action. That fat and the absence of fluid lessened the action of sugar, whilst acids often increased its action.

δ. That small quantities, often repeated, were more efficacious than one large dose.

ε. Also that the whole of the carbonic acid evolved under the influence of tea, coffee, and gluten, could not have been derived from those substances.

It is remarkable that starch and fat, which constitute the chief supply of carbon to the system, scarcely increase the respiratory changes beyond the amount in which they are found in the absence of food. This may help in the elucidation of their intermediate transformations. Fat never, and lactic acid seldom, increases the respiration above the minimum line. Grape-sugar is a less powerful respiratory excitant than any other kind of sugar; but I have not determined its influence when given in the doses in which it may be presumed to be given to the respiration as the result of chemical transformations within the body.

Starch and fat maintain the respiration to the minimum line, and only under the influence of exertion or of some other respiratory excitant is that amount much exceeded.

8. Pulsation and respiration, carbonic acid and vapour in the expired air, do not respectively move in parallel lines. There is a close but not absolutely uniform relation between the quantity of air inspired and the carbonic acid expired. There was almost always a parallel movement between them.

9. Very generally there was an increase in the quantity of carbonic acid in relation to the air inspired.

10. The depth of inspiration was almost always increased; and never with food was increased quantity of air inspired, or of carbonic acid expired, due to increased rate of respiration. The rate usually declined and the depth increased.

11. In reference to alcohols, it must be remarked—

α. That alcohol alone was not used in the whole of PROUT'S and VIERORDT'S experiments, but various substances containing alcohol were taken by the former, and white wine by the latter.

β. There is the utmost variation in the composition and quality of the members of this class, so that in reference to wines and spirits, and perhaps alcohol, it would be impossible to obtain two precisely similar specimens from different supplies.

γ. There is great variation in the habits of men, and therefore of inquirers in reference to their use. COATHUPE states that he took one pint of wine at dinner, and occasionally a glass of weak brandy and water at night. PROUT states "that the quantity I am in the habit of taking is very small." Mr. MOUL and myself scarcely ever take spirits; and I rarely take wine or ale, but Mr. MOUL takes three glasses of good wine daily.

δ. Our experiments were made when fasting.

Hence there are many causes for discrepancy in the results of different observers. It is in accord with common observation that different members of this class have different effects; and it is not the practice in medicine to substitute alcohol for wine, brandy, or ale in the proportion in which it is found in those substances. The preference of rum to gin, or other spirits, for the use of the navies of all countries is probably, at the present day, based more upon their different action than upon their relative cost.

12. Tea also varies much in quality, and the effect of different weights of it varies much also. These circumstances have not been recorded in connexion with previous inquiries.

Its powerful, uniform, and rapid excito-respiratory action without increasing pulsation and without supplying much carbon, renders it worthy of being more highly regarded as a medicinal agent. Its efficient action when cold, and after having been infused for many hours, and even days, and its accumulative influence with repeated doses, increase its value.

It is probable that coffee leaves have valuable medicinal properties. The contrast in the effect of tea, and such alcohols as brandy, gin, and whisky, upon the respiration, consciousness, sensibility, muscles, skin, and mucous membranes and pulsation, and the power of regulating the heat of the body, is very striking, and shows that the two classes of substances are applicable to very different conditions.

13. The different action of milk and its components upon Mr. Moul and myself is instructive; and in general these experiments have shown that there is a direct relation between the idiosyncrasies of individuals in the enjoyment of certain articles of food and their effect upon the system, or in other words, that they are not merely prejudices, but have a relation to the state of the body. I enjoy every kind of food, whilst Mr. Moul dislikes many, but in such a manner that, with the avoidance of certain members of a class, as respiratory excitants, he had an unusual relish for others of the *same* class.

As some of the results at which I have arrived are not in accordance with those obtained by some other observers, I have felt it to be a duty to carefully reconsider my own labours, and to make myself familiar with the methods adopted by those with whom I differ; and after doing so, I find no reason to distrust the truthfulness of my own observations.

As Dr. Prout, forty-five years ago, simply sought to determine the per-centage of carbonic acid in the expired air, without being apparently aware that that would not give the total quantity of carbonic acid evolved in any given period, whilst I have determined the latter only, our results cannot be compared; but as I think that he is not always correct, even in reference to the per-centage amount of carbonic acid, I venture to ask attention to the following circumstances:—

He compares the results obtained with standard quantities for each hour, most of which are "only the result of estimation;" and when they were observed quantities, they were derived from a limited number of inquiries upon himself, and evidently without a due appreciation of the varying effects due to the meals and the duration of the intervals between the meals. He states that the results were not in accordance with those of Mr. Brande, and were quite unexpected by him. There were also remarkable oscillations; and when the alcohols had induced yawning, the quantity of carbonic acid was found to be much above the standard.

In one experiment only was alcohol taken, whilst wine in different quantities, and porter, with and without food, were taken in other experiments.

He also states that the inspirations and expirations were somewhat deeper than natural, and that "the results obtained are evidently to be understood as measures of the *capability* of the organs of respiration to form carbonic acid at any given time, and not as measures of the *quantity* of it *formed* in a given time."

In reference to M. Boeker's experiments, I cannot but attach weight to some of the objections made by me at the commencement of this paper; and although I cannot fully explain the cause of the discrepancy in our results, I think that the decided and uniform action of sugar, tea, and coffee, in my experiments, affords indisputable evidence of their truthfulness. He classes sugar, coffee, and alcohol together; and although the class of alcohols is a difficult one for investigation, and consists of substances which are known to vary in their action, the truthfulness of my results in reference to two of the three, in opposition to those obtained by M. Boeker, is presumptive of the truthfulness of the third.

Explanation of the Plates.

PLATE XXXV.

Represents the effect of numerous articles of food when taken in moderate doses alone, fasting, and under precisely the same circumstances, upon the carbonic acid expired, the air inspired, the depth of inspiration, and the rate of pulsation and respiration. They are arranged in four series: viz. Fats, figs. 1 and 2; the Starch Series, figs. 3 to 13; the Milk Series, figs. 14 to 21; and the Sugar Series, figs. 22 to 29.

PLATE XXXVI.

Represents similar inquiries in reference to articles arranged in the Tea Series, figs. 30 to 43, and in the Alcohol Series, figs. 44 to 56; and also the effect of Albumen.

In the construction of these two Plates the *absolute quantities* have not been delineated, but only the increase or decrease from the quantities ascertained immediately before the food was taken. These latter quantities are regarded as basal quantities, and they are stated in figures near the basal line of each object of inquiry in each experiment. Thus in fig. 47, "S. 8·33" and "M. 7·08" placed under the basal line of the carbonic acid, show that before the tea was taken Dr. Smith expired 8·33 grains, and Mr. Moul 7·08 grains of carbonic acid per minute; and the direction of the curves shows how far the quantity at each inquiry exceeded the basal quantity. The period at which each inquiry was commenced after the food had been taken is stated in hours and minutes at the head of each column. Each inquiry was continued during five minutes after the period just mentioned; but in some experiments it was continuous for one hour, and followed by others at intervals, as in fig. 33, or was continued without intermission until the end, but the quantities recorded every five minutes, as in fig. 34. In figs. 24, 25, 35, 45, and 50, the dose of the food was repeated more or less frequently during the experiment, and then the figures at the head of each column show the period when the inquiry was made after the repetition of each dose.

The temperature and the height of the barometer are recorded at the head of each [illegible];

XXVIII. *Supplement to Mr.* MACQUORN RANKINE'S *Paper "On the Thermodynamic Theory of Steam-engines with dry saturated Steam, and its application to practice*."*

Received September 3,—Read December 8, 1859.

THE following additional information respecting the steam-ships referred to in the examples may be interesting, although it does not strictly belong to the special subject of the paper.

EXAMPLE I.—Paddle-steamer 'Admiral,' built by Mr. JAMES R. NAPIER; engines made by Messrs. RANDOLPH, ELDER and Co.; draught 7 feet 6 inches; length 210 feet; breadth 32 feet; displacement 820 tons; speed with 774 indicated horse-power 11·9 nautical miles an hour. Effective work in driving the ship, about 604 horse-power.

Available heat expended per hour in foot-pounds per indicated horse-power,

$$\frac{1,980,000}{\text{efficiency of steam}}=\frac{1,980,000}{0·123}=16,100,000.$$

Coal burned per indicated horse-power per hour, 2·97 lbs.

Available heat of combustion of one pound of coal,

$$\frac{16,100,000}{2·97}=5,420,000 \text{ foot-lbs.}$$

The total heat of combustion of one pound of the coal employed being roughly estimated at 10,000,000 foot-pounds, it appears that the efficiency of the furnace and boiler was about 0·542.

The boilers were improved marine boilers of ordinary proportions.

EXAMPLE II.—Screw-steamer 'Thetis,' built by Messrs. C. SCOTT and Co.; the engine made by Messrs. ROWAN and Co.

Available heat expended per hour in foot-lbs. per indicated horse-power,

$$\frac{1,980,000}{\text{efficiency of steam}}=\frac{1,980,000}{0·192}=10,312,500.$$

Coal burned per indicated horse-power per hour, during an experiment of one hour's duration, 1·02 lb.

Available heat of combustion of one pound of the coal employed,

$$\frac{10,312,500}{1·02}=10,110,000 \text{ foot-pounds.}$$

The coal used was of very good quality; and its total heat of combustion per pound is estimated at 11,560,000 foot-pounds. Hence the *efficiency of the furnace and boiler was*

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* Philosophical Transactions, Part I. 1859, p. 177; Proceedings of the Royal Society, January 1859.

$$\frac{10,110,000}{11,560,000}=0·88.$$

In this case the short duration of the experiment on the consumption of coal, which was interrupted by a fog, makes the result less satisfactory than it would have been if the experiment had been continued, as intended, for several hours. The engine and boiler were of a kind invented some years ago by Mr. CRADDOCK; the boiler consisting chiefly of a sort of cage of vertical water-tubes enclosing each fire-grate. The heating surface was about nine times as great, relatively to the fuel burned, as it is in ordinary marine boilers.

EXAMPLE III.—Paddle-steamer 'Callao,' built by Messrs. JOHN REID and Co.; the engines by Messrs. RANDOLPH, ELDER and Co.; displacement 1100 tons; speed with 1176 indicated horse-power 12·05 nautical miles per hour.

Available heat expended per hour in foot-pounds per indicated horse-power,

$$\frac{1,980,000}{\text{efficiency of steam}}=\frac{1,980,000}{0·14}=14,143,000 \text{ foot-lbs.}$$

Coal burned per indicated horse-power per hour, 2·67 lbs.

Available heat of combustion of 1 lb. of coal,

$$\frac{14,143,000}{2·67}=5,300,000;$$

being nearly the same as in Example I.

The boilers in this case were, like those in Case I., improved marine boilers of ordinary proportions. In such boilers, 5,400,000 foot-lbs. may be considered a fair estimate of the available heat of combustion of good ordinary steam-coal.

XXIX. *On the Deflection of the Plumb-line in India, caused by the Attraction of the Himmalaya Mountains and of the elevated regions beyond; and its modification by the compensating effect of a Deficiency of Matter below the Mountain Mass. By the Venerable* JOHN HENRY PRATT, *M.A., Archdeacon of Calcutta. Communicated by Professor* STOKES, *Sec. R.S.*

Received October 25,—Read November 25, 1858.

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§ 1. *Reference to a former Paper on this subject.*

1. Two notices* which appeared last year in the Journal of the Astronomical Society on my Paper on Himmalayan Attraction, written at the Cape of Good Hope in 1854, and published by the Royal Society the year following, have called my attention again to this subject. Those who read that paper will remember, that it consisted of two parts; the first a calculation of the amount of deflection of the plumb-line, caused by the Mountain Mass in India, at the principal stations of the northern part of the Great Indian Arc; and the second, the effect which the application of these deflections, as corrections to the astronomical amplitudes, would have upon the calculated ellipticity of the Indian Arc. The results I arrived at are much greater than were anticipated. The author of the communications to the Astronomical Society proposes to test the truth of

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* By Lieut. TENNANT, Bengal Engineers, and First Assistant in the Great Trigonometrical Survey of India.

I am indebted to Mr. TENNANT for having detected a numerical error in page 98 of my paper. By going through the calculations, in page 98, it will be seen that

$$\text{for } a=-0{\cdot}0089737-0{\cdot}0051426u+0{\cdot}0016881v,$$

$$\text{we must read } a=-0{\cdot}0019208+0{\cdot}0059576u-0{\cdot}0014564v.$$

This will change the value of $a(1+\alpha)$ in the next line but one. These corrections have no effect, however, upon the results of my paper. When the paper was written, I was far away from all means of employing a computer, as is usual in such cases, to verify the long numerical calculations, not one-tenth of which appears in what is printed. In the last section of the present communication I have given a revise of such parts of the former one as need correction.

my results, by comparing the curvature thus deduced with the curvature of other arcs on the continent of India. But this proceeds upon the gratuitous hypothesis, and one which for geological reasons is most likely not true, that the earth is at present an exact spheroid of revolution; *i. e.* that all meridians are ellipses, and indeed the same ellipses, and that every arc of longitude is circular. There are only two ways of avoiding the conclusion regarding the curvature of the Indian Arc to which I came in my paper of 1855; either by showing that my data and reasoning are wrong, or by pointing out that some other cause is in operation, which either in whole or in part counteracts the effect of the Himmalayan Mass. My calculation has been before the public three years; and, though some small numerical errors have been detected, they are not of sufficient importance to affect the result; and the data I have every reason for believing to be correctly taken, as the Surveyor-General—who first called my attention to the subject in 1852, as an unsolved difficulty in the operations of the Great Trigonometrical Survey of India—has been requested to forward to me any corrections which may appear to him to be advisable, and none have been sent*. There remains, then, only the resource of looking for some counteracting cause to compensate for the large disturbance produced by the Himmalayas and the regions beyond.

2. The Astronomer Royal, in a paper published in the Transactions for 1855, suggested that immediately beneath the mountain-mass there was most probably a deficiency of matter, which would produce, as it were, a negative attraction, and so counteract the effect on the plumb-line. This hypothesis appears, however, to be

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* The whole Mountain Region (which I call the Enclosed Space, see par. 8) I divide into two portions by a circular arc of about 850 miles radius described about Kaliana, the northern station of the Great Arc, as centre. The portion of the mountain country within that arc I have called, in my former Paper, the *Known Region*, because the heights are all readily obtained from the Survey Maps. The remaining portion I designate the *Doubtful Region*, because the heights cannot possibly be so well determined. My chief source of information for the Doubtful Region is HUMBOLDT's 'Aspects of Nature.' The Doubtful Region is, in superficial extent, about sixteen times as large as the Known Region; but, as I show in my former Paper, being more distant, produces nothing like a corresponding effect on the stations of the Arc.

By the use of the Tables in paragraphs 8 and 11 it may be shown without much difficulty, that the three Deflections 27″·073, 12″·017, 6″·790 at the three principal stations would be reduced only to 21″·033, 9″·937, 5″·010, if the whole of the Doubtful Region beyond a radius of about 700 miles from the northern station were left out of the reckoning. If the *whole* Doubtful Region were considered to be a dead flat and to have no effect at all—an hypothesis clearly impossible—the Deflections would still be reduced only to 12″·972, 8″·219, 1″·336. In those three cases the corrections to the amplitudes would be

15″·926 and 5″·257 if the Doubtful Region be as I have taken it;
14″·696 and 4″·927 if all beyond 700 miles from Kaliana be left out or annihilated;
9″·753 and 1″·883 if the whole Doubtful Region be supposed non-existent.

The errors to be accounted for (if the ellipticity of the Indian Arc be what Colonel EVEREST assumes it to be, viz. the *mean*) are 5″·236 and −3″·789. This extravagant, and indeed impossible hypothesis, then, of the non-existence of the vast Mountain Region beyond the Himmalaya crest, will not account for these errors. Much less, then, will any *mere correction* of the heights of this Doubtful Region. A solution, if there be one, must be sought for in another direction.

untenable for three reasons:—(1) It supposes the thickness of the earth's solid crust to be considerably smaller than that assigned by the only satisfactory physical calculations made on the subject—those by Mr. HOPKINS of Cambridge. He considers the thickness to be about 800 or 1000 miles at least. (2) It assumes that this thin crust is lighter than the fluid on which it is supposed to rest. But we should expect that in becoming solid from the fluid state, it would contract by loss of heat and become heavier. (3) The same reasoning by which Mr. AIRY makes it appear that every protuberance outside this thin crust must be accompanied by a protuberance inside, down into the fluid mass, would equally prove that wherever there was a hollow, as in deep seas, in the outward surface, there must be one also in the inner surface of the crust corresponding to it; thus leading to a law of varying thickness which no process of cooling could have produced.

§ 2. *Hypothesis of Deficiency of Matter adopted in this Paper.*

3. It is nevertheless to this source—I mean a Deficiency of Matter below—that we must look, I feel fully assured, for a compensating cause, if any is to be found. My present object is to propose another hypothesis regarding deficiency of matter below the mountain-mass, as first suggested by Mr. AIRY; and to reduce my hypothesis to the test of calculation.

I will here observe, that all the more laborious numerical calculations in this Paper have been performed for me—as I could not possibly find leisure for the work—by a practised Computer of the Great Trigonometrical Survey Office in Calcutta, obligingly recommended to me by the Deputy Surveyor-General. In the course of the present paper the attraction of the Mountain Mass, in the direction of the meridian, at the three principal stations of the Great Arc—Kaliana, Kalianpur, and Damargida—are again calculated, and a test so far afforded of the general accuracy of my former results.

4. I will now state the hypothesis on which my present calculation proceeds. At the time when the earth had just ceased to be wholly fluid, the form must have been a perfect spheroid, with no mountains and valleys nor ocean-hollows. As the crust formed, and grew continually thicker, contractions and expansions may have taken place in any of its parts, so as to depress and elevate the corresponding portions of the surface. If these changes took place chiefly in a vertical direction, then at any epoch a vertical line drawn down to a sufficient depth from any place in the surface will pass through a mass of matter which has remained the same in amount all through the changes. By the process of expansion the mountains have been forced up, and the mass thus raised above the level has produced a corresponding *attenuation* of matter below. This attenuation is most likely very trifling, as it probably exists through a great depth. Whether this cause will produce a sufficient amount of compensation can be determined only by submitting it to calculation, which I proceed to do.

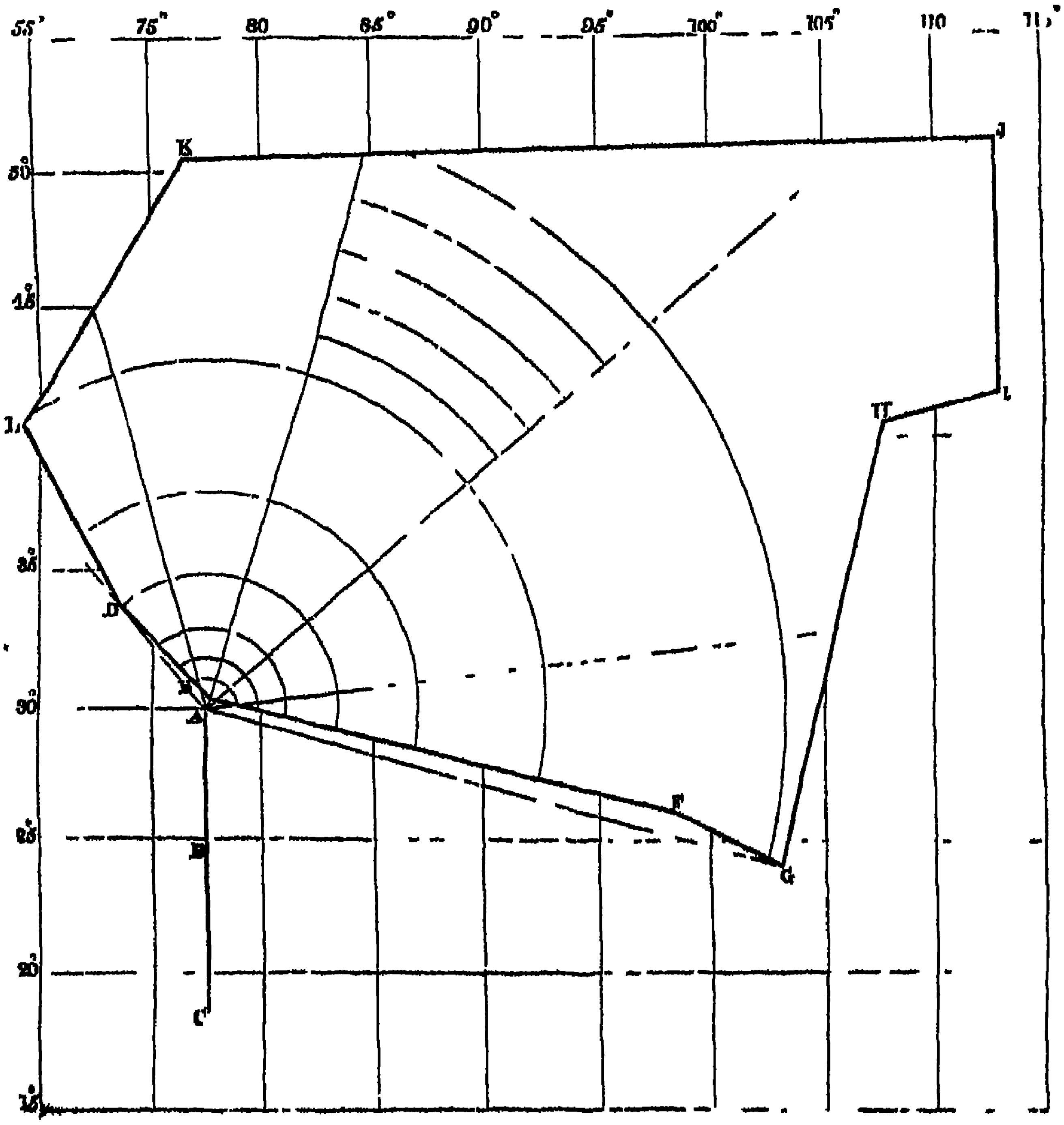
§ 3. *Summary of data and of some of the results of the former Paper, necessary for the present calculation.*

5. Before proceeding it is necessary here to gather into one view such of the results of my former Paper* as will be required for the present calculation.

The Great Arc under consideration is in longitude 77° 42′. The three principal stations of this arc, the latitudes of which have been observed astronomically, are A, or Kaliana, in latitude 29° 30′ 48″; B, or Kalianpur, 24° 7′ 11″; and C, or Damargida, 18° 3′ 15″. The lengths of the two arcs, Kaliana—Kalianpur, and Kalianpur—Damargida, as determined by the Survey, are 326859·52 and 367154·37 fathoms.

6. In order to effect the calculation of the attraction on the three stations, the mass is conceived to be divided into smaller masses in the following manner, taking the station A for example. From the point A (fig. 1) great circles are drawn meeting in

Fig. 1



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* Philosophical Transactions, 1855, pp. 53–100.

the antipodes of A, dividing the superficial mass into so many lunes of attracting matter. Circles are then drawn with their centres in the vertical line through A, at distances increasing according to a peculiar law, which, for a name, I call the Law of Dissection, dividing the whole surface into a number of four-sided "compartments*," which have this property, that the attractions of the masses standing on the several compartments of a lune thus formed are exactly the same when the heights of the masses are the same; or, which amounts to the same thing, since the heights are all small compared with the distances from the station, the attractions of the masses standing upon the several compartments are in proportion to their average heights.

7. It is then proved (p. 65) that if β be the angular width of the compartment, and h the average height above the level of A, the deflection of the plumb-line at A, in the direction of the meridian, caused by the mass standing on this compartment,

$$=1''\cdot 1392\, h \sin \tfrac{1}{2}\beta \cos \mathrm{Az}.$$

Hence the deflection caused by the whole mass

$$=1''\cdot 1392\, \Sigma . h \sin \tfrac{1}{2}\beta \cos \mathrm{Az}, \quad \ldots\ldots\ldots \quad (1.)$$

Az being the azimuth, reckoned from the north, of the middle line of the lune.

8. By an examination of the physical geography of the earth's surface, I next show that the only mass which can affect the plumb-line lies within a space DEFGHIJKLD, which I call the Enclosed Space, and then describe: the extreme diagonal lengths are each about 2000 miles.

Tables are next formed—of which a summary is given in the following page—containing all the data regarding the average heights of the masses on the several compartments; and the formula (1.) brings out the following results:—

	At A.	At B.	At C.
Deflection of the plumb-line in the meridian . .	$27''\cdot 858$	$11''\cdot 968$	$6''\cdot 909$

Hence the differences of deflections, or the errors in the astronomical amplitudes, are

$$15''\cdot 885 \text{ and } 5''\cdot 059.$$

After applying these corrections, I proceed to calculate the ellipticity of the Indian Arc. With a view to make the results available for any other calculated attractions—a

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* The horizontal attraction of the mass standing on any one of these compartments is shown to equal (see p. 62 of former Paper)

$$\rho \sin \frac{1}{2}\beta \frac{\phi \cos^2\left(\frac{1}{2}\alpha+\frac{1}{4}\phi\right)}{\sin\left(\frac{1}{2}\alpha+\frac{1}{4}\phi\right)} h,$$

where α and $\alpha+\phi$ are the angular distances of the nearer and further sides of the compartment from A. The Law of Dissection, connecting ϕ with α, is this,

$$\frac{\phi \cos^2\left(\frac{1}{2}\alpha+\frac{1}{4}\phi\right)}{\sin\left(\frac{1}{2}\alpha+\frac{1}{4}\phi\right)}=\frac{21}{4},$$

ϕ being assumed equal to $\frac{1}{10}\alpha$ when these angles are very small.

TABLE of the average HEIGHTS (in feet) of the masses in the several compartments above the level of Kaliana.

No. of compartment	Distance of middle of compartment from station.	For Station A (Kaliana).					For Station B (Kalianpur).					For Station C (Damargid.		
		Lune I.	Lune II.	Lune III	Lune IV.	Lune V.	Lune I	Lune II	Lune III	Lune IV.	Lune V.	Lune I	Lune II	Lune I
1	0·787	......	......	......	600	......	......	......	......	......	......	......	......	......
2	0·866	......	......	500	600	......	......	......	......	......	......	......	......	......
3	0·949	......	......	600	1,200	300	......	......	......	......	......	......	......	......
4	1·048	......	500	1,500	800	300	......	......	......	......	......	......	......	......
5	1·153	300	800	1,500	1,500	300	......	......	......	......	......	......	......	......
6	1·268	500	1,000	1,200	1,600	300	......	......	......	......	......	......	......	......
7	1·395	800	2,000	600	1,600	800	......	......	......	......	......	......	......	......
8	1·534	1,300	3,000	2,000	1,800	1,000	......	......	......	......	......	......	......	......
9	1·687	2,300	3,000	2,500	2,500	1,500	......	......	......	......	......	......	......	......
10	1·856	1,800	5,000	3,000	6,000	2,000	......	......	......	......	......	......	......	......
11	2·042	300	2,500	5,500	7,000	2,500	......	......	......	......	......	......	......	......
12	2·247	800	4,000	7,000	7,000	3,000	......	......	......	......	......	......	......	......
13	2·472	800	4,000	8,500	9,000	2,500	......	......	......	......	......	......	......	......
14	2·719	800	5,000	9,500	9,000	2,500	......	......	......	......	......	......	......	......
15	2·990	800	5,500	9,000	9,000	2,500	......	......	......	......	......	......	......	......
16	3·289	800	6,000	9,500	9,000	2,500	......	......	......	......	......	......	......	......
17	3·616	1,300	7,000	9,500	9,500	2,500	......	......	......	......	......	......	......	......
18	3·980	1,800	7,500	9,500	9,500	3,000	......	......	......	......	......	......	......	......
19	4·378	2,800	8,300	9,500	9,500	3,000	......	......	......	......	......	......	......	......
20	4·813	4,800	9,500	9,500	9,500	3,000	......	......	......	......	......	......	......	......
21	5·298	7,000	8,700	8,500	7,000	3,000	......	......	......	......	......	......	......	......
22	5·828	8,000	8,200	8,000	9,000	3,750	......	......	500	......	......	......	......	......
23	6·408	9,000	7,700	7,500	8,600	4,500	......	500	2,500	......	......	......	......	......
24	7·054	9,000	7,200	7,000	8,100	5,250	......	2,500	7,000	......	......	......	......	......
25	7·707	8,700	6,800	6,500	7,600	6,000	......	4,500	9,000	......	......	......	......	......
26	8·533	8,400	6,300	6,000	7,100	6,730	......	6,500	9,500	......	......	......	......	......
27	9·386	8,100	5,800	5,500	6,700	7,500	1,200	8,200	9,000	4,000	......	......	......	......
28	10·324	7,900	5,400	5,000	6,200	8,250	6,000	9,500	8,250	5,700	......	......	......	......
29	11·360	7,600	4,900	4,500	5,700	9,000	9,000	9,000	7,500	7,300	......	......	......	......
30	12·506	7,300	4,400	4,000	5,200	9,000	9,000	8,200	6,750	9,000	......	600	......	......
31	13·770	7,000	3,900	3,500	4,800	8,250	8,450	7,400	6,000	9,000	......	4,000	6,000	......
32	15·211	......	3,450	3,000	4,300	7,500	7,850	6,600	5,250	8,150	5,000	7,300	9,000	......
33	16·80	......	3,000	2,500	3,850	6,750	7,250	5,750	4,500	7,300	7,000	9,500	7,950	7,0
34	18·51	......	2,500	2,000	3,400	6,000	6,700	4,900	3,750	6,450	9,000	9,000	6,850	9,0
35	20·40	......	2,000	1,500	2,900	......	......	4,100	3,000	5,550	8,250	7,900	5,800	8,1
36	22·50	......	......	1,500	2,400	......	......	3,300	2 250	4,700	7,500	6,850	4,700	7,2
37	24·83	......	......	3,500	1,900	......	......	2,500	1,500	3,850	......	5,750	3,650	6,4
38	27·43	......	......	......	1,500	......	......	......	2,000	3,000	......	4,650	2,550	5,5
39	30·31	......	......	......	1,500	......	......	......	3,000	......	......	3,600	1,500	......
40	33·55	......	......	......	4,000	......	......	......	......	......	......	2,500	2,000	......
Totals in feet		110,000	154,850	180,400	207,950	125,000	55,450	83,450	91,250	74,000	36,750	61,650	50,000	43,2
Totals in miles		20·833	29·328	34·167	39·385	23·673	10·502	15·805	17·282	14·015	6·960	11·676	9·470	8·1

precaution I now feel the value of—I multiply them by arbitrary coefficients $1-u$ and $1-v$, making them

$$15''\cdot885\,(1-u) \text{ and } 5''\cdot059\,(1-v).$$

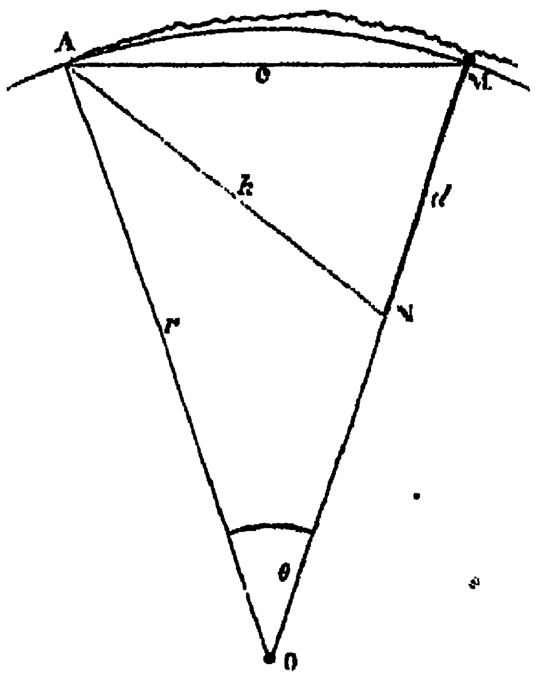
The ellipticity and the semi-axis major then are

$$\left.\begin{aligned} \epsilon &= 0\cdot002346 + 0\cdot003693u - 0\cdot001046\,v \\ a &= 2088274\cdot3 + 0\cdot0059576u - 0\cdot0014564v \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

These are the only results which it is necessary here to recapitulate.

§ 4. *Calculation of the Effect of Deficiency of Matter, according to the assumed hypothesis.*

Fig. 2.



9. I now proceed to calculate the effect of the attenuation caused below by the upheaval of the mountains. I shall suppose that the masses standing upon the several compartments are concentrated into the middle points of the meridian-diameters of the compartments. This may be done without sensible error, as shown in the note*. And next that these masses are distributed into vertical bars or prisms, all to the same depth d, each bar or prism being of uniform density in itself, but containing the whole amount of matter belonging to the compartment from the centre of which it is drawn down. The difference between the attractions of the mass thus distributed downwards, and of the mass as it lies in the Himmalayas, will give the horizontal attraction on the plumb-line at the three stations A, B, C, on the hypothesis of the attenuation I have described.

10. Let M (fig. 2) be one of these masses, concentrated in the middle of its compartment: MN the bar or prism into which it is distributed. Join AM, AN, AO, NO. Let $AM=c$, $AN=k$, $AO=r$, $MN=d$, $\angle AOM=\theta$. It is a property easily proved, that the attraction of the prism MN on A in the direction $AM=\frac{M}{c.k}$. But the attraction of M on A in the same direction $=\frac{M}{c^2}$. The ratio of these is $\frac{c}{k}$. In this ratio, therefore, must the attractions of the masses on the compartments, as deduced in my former Paper (and re-calculated in this), be reduced, to obtain from them the attractions of the same masses when distributed into vertical prisms.

∴ Deflection along the middle line of any lune, caused by the mass on the compartment when distributed down to a depth d, becomes

$$=1''\cdot1392\,h\frac{c}{k}\sin\frac{1}{2}\beta;$$

$$\therefore\ \text{deflection in the meridian}=1''\cdot1392\,h\frac{c}{k}\sin\frac{1}{2}\beta\cos\text{Az}.$$

---

* Area of compartment $=r^2\beta\left(\cos\alpha-\cos(\alpha+\phi)\right)$, r=rad. of earth.

Mass on the compartment $=r^2\beta\rho\left(\cos\alpha-\cos(\alpha+\phi)\right)h=4r^2\beta\rho h\sin\frac{\phi}{2}\sin\left(\frac{\alpha}{2}+\frac{\phi}{4}\right)\cos\left(\frac{\alpha}{2}+\frac{\phi}{4}\right)$,

$$\therefore\ \frac{\text{attraction of the mass, concentrated in the centre}}{\text{ditto (from last note)}}=\frac{\frac{1}{2}\beta}{\sin\frac{1}{2}\beta}\ \frac{\sin\frac{1}{2}\phi}{\frac{1}{2}\phi}=1+\frac{\beta^2-\phi^2}{24}+\ldots$$

The largest value of this is when $\beta=\frac{30}{180}\pi=\frac{11}{21}$, and ϕ is evanescent, when it is about $\frac{1}{100}$th part greater than unity.

If Σ be a symbol representing the sum of the quantities $h\sin\frac{1}{2}\beta\cos \mathrm{Az}$ for the five lunes for any given value of $\frac{c}{k}$, and $\Sigma.h\sin\frac{1}{2}\beta\cos\mathrm{Az}=\mathrm{S}$,

$\therefore$ Deflection in the meridian, for these five compartments, $=1''\cdot1392\,\frac{c}{k}\,\mathrm{S}$. . (3.)

11. We must first obtain the values of S. The width β of the several lunes into which the Enclosed Space is divided, and the azimuth of the middle lines, are as follows (see par. 39 of my former Paper):—for A, widths, 24°, 30°, 30°, 30°, 20°, azimuths, 27°, 0°, 30°, 60°, 85°; for B, widths, 8°, 30°, 30°, 30°, 10°, azimuths, 19°, 0°, 30°, 60°, 80°; for C, widths, 30°, 30°, 20°, azimuths, 0°, 30°, 55°. Hence the values of $\sin\frac{1}{2}\beta\cos\mathrm{Az}$ for the several lunes are—

for A, 0·1852, 0·2588, 0·2241, 0·1294, 0·0151;
for B, 0·0660, 0·2588, 0·2241, 0·1294, 0 0151;
for C, 0·2588, 0·2241, 0·0996.

By these numbers must the heights in the Table of Heights (page 750) be multiplied; they must then be added together from left to right, and the totals, given in the Table in page 753, will be the aggregates for the several distances from the station, that is, the values of S. The grand totals at the foot of the Table, reduced to miles and multiplied by $1''\cdot1392$, give the total meridian deflections at the three stations caused by the mountain-mass as it now lies at the surface. These differ but slightly from the results of my last Paper, as recapitulated in par. 22 of the present communication*.

12. The values of $\frac{c}{k}$ have now to be determined, in order to carry out the calculation of formula (3.). By fig. 2 we have

$$k^2=c^2+d^2-2cd\sin\tfrac{1}{2}\theta,\qquad\left(c=2a\sin\tfrac{1}{2}\theta\right)$$

$$=4a^2\sin^2\tfrac{1}{2}\theta+d^2-4ad\sin^2\tfrac{1}{2}\theta=d^2+4a^2\sin^2\tfrac{\theta}{2}\left(1-\tfrac{d}{a}\right).$$

Put $$\frac{2a}{d}\sqrt{1-\frac{d}{a}}\sin\tfrac{1}{2}\theta=\tan\psi,\qquad\therefore\ k^2=d^2\sec^2\psi,$$

$$\therefore\ \frac{c}{k}=\frac{2a}{d}\cos\psi\sin\tfrac{1}{2}\theta.$$

These formulæ give

$$\log\frac{2a}{d}\sqrt{1-\frac{d}{a}}+\log\sin\tfrac{1}{2}\theta=\log\tan\psi\ .\ .\ .\ .\ .\ .\ .\ .\ (4.)$$

$$\log\frac{2a}{d}+\log\cos\psi+\log\sin\tfrac{1}{2}\theta-20=\log\frac{c}{k}.\ .\ .\ .\ .\ .\ .\ .\ (5.)$$

---

* To test the accuracy of these results, the totals of the thirteen columns of the Table in page 750 have been multiplied by the thirteen constants above given, and then the three grand totals taken for the three stations. They are found to accord exactly with the three totals of the Table in page 753.

TABLE OF DEFLECTIONS in Meridian (*mass all at the surface*) (from formula 3).

No. of compartment.	Values of $S=\Sigma.h\sin\frac{1}{2}\beta\cos Az.$		
	Station A. Kaliana.	Station B. Kalianpur.	Station C. Damargida.
1	77·64	………	………
2	189·69	………	………
3	294·27	………	………
4	573·60	………	………
5	797·38	………	………
6	831·89	………	………
7	1019·34	………	………
8	1713·38	………	………
9	2108·76	………	………
10	3106·26	………	………
11	2878·66	………	………
12	3703·16	………	………
13	4290·56	………	………
14	4773·46	………	………
15	4790·81	………	………
16	5032·26	………	………
17	5448·36	………	………
18	5677·91	………	………
19	6070·15	………	………
20	6751·11	………	………
21	6403·91	………	………
22	6617·79	112·05	………
23	6521·10	689·65	………
24	6225·27	2215·70	………
25	5901·77	3181·50	………
26	5551·39	3811·15	………
27	5213·94	4735·86	………
28	4907·95	5441·00	………
29	4557·57	5548·57	………
30	4195·86	5393·44	155·28
31	3835·76	4982·02	2379·80
32	2234·83	4532·81	3906·14
33	1936·77	4025·37	4937·39
34	1625·76	3521·23	4760·69
35	1229·01	2576·13	4151·06
36	646·71	2079·69	3547·65
37	1030·21	1481·34	2943·01
38	194·10	836·40	2322·17
39	194·10	672·30	1267·83
40	517·60	………	1095·20
In feet ………………	129670·05	55836·21	31466·22
In miles ………………	24·559	10·575	5·960
Multiplying by $1''\cdot1392$, DEFLECTIONS=$27''\cdot978$		$12''\cdot047$	$6''\cdot790$

The Tables in the following pages (which I call the Four TABLES OF REDUCTION), give the values of $\frac{c}{k}$ for four values of the depth, viz. $d=100$, 300, 500, and 1000 miles. For these values of d, the two constants $\log\frac{2a}{d}\sqrt{1-\frac{d}{a}}$ and $\log\frac{2a}{d}$ are respectively

Four TABLES OF REDUCTION, calculated by formulæ (4.) and (5.).

TABLE I.—Depth = 100 miles.

No of the compartment.	$\frac{1}{2}\theta$.	$\log\sin\frac{1}{2}\theta$.	$\log\sin\frac{1}{2}\theta$ $+1{\cdot}8975923$ $-\log\tan\psi$.	$\log\cos\psi$.	$\log\sin\frac{1}{2}\theta$ $+\log\cos\psi$ $+1{\cdot}9030900-20$ $\log\frac{c}{k}$.
1	0·393 or 0° 23′ 35″	7·8363279	9·7339202	9·9440904	1 6835083
2	0·433 or 0 25 59	7·8784168	9·7760091	9·9337984	1·7152952
3	0·474 or 0 28 26	7·9175489	9 8151412	9·9228096	1·7434485
4	0·524 or 0 31 26	7·9611105	9·8587028	9·9088382	1·7730387
5	0·576 or 0 34 34	8·0023763	9·8999686	9 8937914	1·7992677
6	0·634 or 0 38 2	8·0438816	9·9414739	9·8767823	1·8237539
7	0·697 or 0 41 49	8·0850648	9·9826571	9·8579831	1·8461379
8	0·767 or 0 46 1	8·1266283	10·0242206	9·8370370	1·8667553
9	0·843 or 0 50 35	8·1677179	10·0653102	9·8143838	1·8851917
10	0·928 or 0 55 41	8·2094324	10·1070247	9·7894454	1·9021678
11	1·021 or 1 1 16	8·2509274	10·1485197	9·7627636	1·9167810
12	1·123 or 1 7 23	8·2922508	10·1898431	9·7314460	1·9297868
13	1·236 or 1 14 10	8·3339012	10·2314935	9·7042486	1·9412398
14	1·359 or 1 21 26	8·3744877	10·2720800	9·6733584	1·9509361
15	1·495 or 1 29 42	8·4164693	10·3140616	9·6400279	1·9595872
16	1·644 or 1 38 38	8·4576902	10·3552825	9·6060825	1·9668627
17	1·808 or 1 48 29	8·4990171	10·3966094	9·5709770	1·9730841
18	1·990 or 1 59 24	8·5406431	10·4382354	9·5346689	1·9784020
19	2·189 or 2 11 20	8·5819954	10·4795877	9·4977795	1·9828649
20	2·406 or 2 24 22	8·6230654	10·5206577	9·4601449	1·9869003
21	2·649 or 2 38 56	8·6647864	10·5623787	9·4219106	1·9897870
22	2·914 or 2 54 50	8·7061631	10·6037554	9·3831792	1·9924323
23	3·204 or 3 12 14	8·7473285	10·6449208	9·3442061	1·9946246
24	3·527 or 3 31 37	8·7890017	10·6865940	9·3044808	1·9965725
25	3·853 or 3 51 11	8·8273552	10·7249475	9·2674791	1·9979243
26	4·266 or 4 15 58	8·8715081	10·7691004	9·2246997	1·9992978
27	4·698 or 4 41 35	8·9128474	10·8104397	9·1844141	0·0003515
28	5·162 or 5 9 43	8·9541030	10·8516953	9·1440456	0·0012386
29	5·680 or 5 40 48	8·9955141	10·8931064	9·1033699	0·0019740
30	6·253 or 6 15 11	9·0871072	10·9346995	9·0623859	0 0025831
31	6·885 or 6 53 6	9·0787356	10·9763279	9·0212643	0·0030899
32	7·605 or 7 36 18	9·1217006	11·0192929	8·9787284	0·0035190
33	8·40 or 8 24 0	9·1645998	11·0621921	8·9361833	0·0038731
34	9·25 or 9 15 0	9·2061309	11·1037232	8·8949442	0·0041651
35	10·20 or 10 12 0	9·2481811	11·1457734	8·8531198	0·0043909
36	11·25 or 11 15 0	9·2902357	11·1878280	8·8112590	0·0045847
37	12·41 or 12 24 36	9·3322481	11·2298404	8·7694070	0·0047451
38	13·71 or 13 42 36	9·3747625	11·2723548	8·7270362	0·0048887
39	15·15 or 15 9 0	9·4172174	11·3148097	8·6846817	0·0049891
40	16·77 or 16 46 12	9·4601918	11·3577841	8·6417987	0·0050805

Four TABLES OF REDUCTION, calculated by formulæ (4.) and (5.).

TABLE II.—Depth =300 miles. TABLE III.—Depth =500 miles.

No. of the compartment.	$\log\sin\frac{1}{2}\theta$ $+1·4090396$ $=\log\tan\psi$.	$\log\cos\psi$.	$\log\sin\frac{1}{2}\theta$ $+\log\cos\psi$ $+1·1259687-20$ $=\log\frac{c}{k}$.	$\log\sin\frac{1}{2}\theta$ $+1·1751210$ $=\log\tan\psi$.	$\log\cos\psi$.	$\log\sin\frac{1}{2}\theta$ $+\log\cos\psi$ $+1·2011200-20$ $=\log\frac{c}{k}$.
1	9·2453674	9·9933804	1·2556770	9·0114519	9·9977228	$\bar{1}$·0381707
2	9·2874563	9·9919899	1·2963754	9·0535408	9·9972390	1·0797758
3	9·3265884	9·9904426	1·3339604	9·0926729	9·9966980	1·1183669
4	9·3701500	9·9883756	$\bar{1}$·3754548	9·1362345	9·9959712	1·1612017
5	9·4114158	9·9860193	1·4143643	9·1775003	9·9951373	1·1016336
6	9·4529211	9·9831858	1·4530361	9·2190056	9·9941268	1·2421984
7	9·4941043	9·9798329	$\bar{1}$·4908664	9·2601888	9·9929198	$\bar{1}$·2821046
8	9·5356678	9·9758083	1·5284053	9·3017523	9·9914554	1·3222037
9	9·5767574	9·9710902	1·5647768	9·3428419	9·9897169	$\bar{1}$·3615548
10	9·6184719	9·9654319	$\bar{1}$·6008330	9·3845564	9·9893680	1·4029204
11	9·6599669	9·9588048	1·6357009	9·4260514	9 9850772	1·4101316
12	9·7012903	9·9510786	$\bar{1}$·6692981	9·4673748	9·9820752	1·4781460
13	9·7429407	9·9420082	1·7018781	9·5090252	9·9784675	1·5161887
14	9·7835272	9·9317918	1·7322512	9·5196117	9·9742951	1·5529028
15	9·8255088	9·9196563	1·7620943	9·5915933	9·9691849	1·5897742
16	9·8667297	9·9060534	1·7897123	9·6328142	9·9632611	1·6250713
17	9·9080566	9·8906270	1·8156128	9·6741411	9·9562887	1·6594258
18	9·9476826	9·8731890	1·8398008	9·7157671	9·9480801	1·6928432
19	9·9910349	9·8339217	1·8618858	9·7571194	9·9386035	1·7247189
20	10·0321049	9·8328399	$\bar{1}$·8818740	9·7981894	9·9277437	$\bar{1}$·7549201
21	10·0738259	9·8094513	$\bar{1}$·9002064	9·8399104	9·9150995	1·7840059
22	10·1152026	9·7843319	$\bar{1}$·9164637	9·8812871	9·9008277	$\bar{1}$·8111108
23	10·1563680	9·7575188	$\bar{1}$·9308160	9·9224525	9·8848149	1·8362634
24	10·1980412	9·7286294	$\bar{1}$·9435998	9·9641257	9·8666831	$\bar{1}$·8598048
25	10·2363947	9·7005925	$\bar{1}$·9539164	10·0024792	9·8482392	$\bar{1}$·8797144
26	10·2805476	9·6667435	$\bar{1}$·9642203	10·0466321	9·8249809	1·9005490
27	10·3218869	9·6336708	$\bar{1}$·9724869	10·0879714	9·8010778	1·9180452
28	10·3631425	9·5994871	$\bar{1}$·9795588	10·1292270	9·7753959	$\bar{1}$·9336189
29	10·4045536	9·5641182	$\bar{1}$·9856010	10·1706381	9·7478206	$\bar{1}$·9474547
30	10·4461467	9·5276703	$\bar{1}$·9907462	10·2122312	9·7184113	$\bar{1}$·9590385
31	10·4877751	9·4903904	$\bar{1}$·9950947	10·2538596	9·6873971	$\bar{1}$·9702527
32	10·5307401	9·4511894	$\bar{1}$·9988587	10·2968246	9·6538752	$\bar{1}$·9796958
33	10·5736898	9·4174137	0·0019822	10·3397238	9·6190275	$\bar{1}$·9877473
34	10·6151704	9·3724169	0·0045165	10 3812549	9·5841387	$\bar{1}$·9943896
35	10·6572206	9·3315394	0·0056892	10·4233051	9·5477953	0·0000964
36	10·6992752	9·2922210	0·0084254	10·4653597	9·5105573	0·0049130
37	10·7412876	9·2516772	0·0098940	10·5073721	9·4725938	0·0089619
38	10·7838020	9·2104015	0·0111327	10·5498865	9·4335103	0·0123928
39	10·8262569	9·1689547	0·0121408	10·5923414	9·3939077	0·0152451
40	10·8692313	9·1268419	0·0130024	10·6353158	9·3533447	0·0176565

Four TABLES OF REDUCTION, calculated by formulæ (4) and (5).

TABLE IV.—Depth =1000 miles.

No. of the compartment.	$\frac{1}{2}\theta$.	$\log\sin\frac{1}{2}\theta$.	$\log\sin\frac{1}{2}\theta$ $+0·8106207$ $=\log\tan\psi$.	$\log\cos\psi$.	$\log\sin\frac{1}{2}\theta$ $+\log\cos\psi$ $+0·9030900-20$ $-\log\frac{c}{\lambda}$.
1	0·393 or 0° 23′ 35″	7·8363279	8·6769486	9·9995100	$\bar{2}$·7389279
2	0·433 or 0 25 59	7·8784168	8·7190375	9·9994054	$\bar{2}$·7809122
3	0·474 or 0 28 26	7·9175489	8·7581696	9·9992882	2·8209271
4	0·524 or 0 31 26	7·9611105	8·8017312	9·9991303	$\bar{2}$·8633308
5	0·576 or 0 34 34	8·0023763	8·8429970	9·9989487	$\bar{2}$·9044150
6	0·634 or 0 38 2	8·0438816	8·8845023	9·9987280	$\bar{2}$·9456996
7	0·697 oa 0 41 49	8·0850648	8·9256855	9·9984634	2·9866182
8	0·767 or 0 46 1	8·1266283	8·9672490	9·9981406	1·0278589
9	0·843 or 0 50 35	8·1677179	9·0083386	9·9977552	1·0685631
10	0·928 or 0 55 41	8·2094324	9·0500531	9 9972826	1·1098050
11	1·021 or 1 1 16	8·2509274	9·0915481	9·9967147	1·1507321
12	1·123 or 1 7 23	8·2922508	9·1328715	9·9960323	1·1913731
13	1·236 or 1 14 10	8·3339012	9·1745219	9·9952026	1·2321938
14	1·359 or 1 21 26	8·3744877	9·2151084	9·9942299	1·2718076
15	1·495 or 1 29 42	8·4164693	9·2570900	9·9930188	$\bar{1}$·3125781
16	1·644 or 1 38 38	8·4576902	9·2983109	9·9915873	$\bar{1}$·3523675
17	1·808 or 1 48 29	8·4990171	9·3396378	9·9898643	1·3919714
18	1·990 or 1 59 24	8·5406431	9·3812638	9·9877819	1·4315150
19	2·189 or 2 11 20	8·5819954	9·4226161	9·9853037	$\bar{1}$·4703891
20	2·406 or 2 24 22	8·6230654	9·4636861	9·9823649	$\bar{1}$·5085203
21	2·649 or 2 38 56	8·6647864	9·5054071	9·9788067	1·5466831
22	2·914 or 2 54 50	8·7061631	9·5467838	9·9746090	$\bar{1}$·5838621
23	3·204 or 3 12 14	8·7473285	9·5879492	9·9696639	$\bar{1}$·6200824
24	3·527 or 3 31 37	8·7890017	9·6296224	9·9637547	$\bar{1}$·6558464
25	3·853 or 3 51 11	8·8273552	9·6679759	9·9574003	$\bar{1}$·6878455
26	4·266 or 4 15 58	8·8715081	9·7121288	9·9488494	1·7234475
27	4·693 or 4 41 35	8·9128474	9·7534681	9·9394965	$\bar{1}$·7554339
28	5·162 or 5 9 43	8·9541030	9·7947237	9·9287188	$\bar{1}$·7859118
29	5·680 or 5 40 48	8·9955141	9·8361348	9·9163146	$\bar{1}$·8149187
30	6·253 or 6 15 11	9·0371072	9·8777279	9·9021263	$\bar{1}$·8423235
31	6·885 or 6 53 6	9·0787356	9·9193563	9·8860837	$\bar{1}$·8679093
32	7·605 or 7 36 18	9·1217006	9·9623213	9 8674166	$\bar{1}$·8922072
33	8·40 or 8 24 0	9·1645998	10·0052205	9·8468584	$\bar{1}$·9145482
34	9·25 or 9 15 0	9·2061309	10·0467516	9·8248529	$\bar{1}$·9340738
35	10·20 or 10 12 0	9·2481811	10 0888018	9·8005771	$\bar{1}$·9518482
36	11·25 or 11 15 0	9·2902357	10·1308564	9·7743449	$\bar{1}$·9676706
37	12·41 or 12 24 36	9·3322481	10·1728688	9·7402853	$\bar{1}$·9816234
38	13·71 or 13 42 36	9·3747625	10·2153852	9·7161171	$\bar{1}$·9939696
39	15·15 or 15 9 0	9·4172174	10·2578381	9·6848585	0·0046609
40	16·77 or 16 46 12	9·4601918	10·3008125	9·6506910	0·0139728

TABLE OF DEFLECTIONS, for Station A (*Kaliana*), when the mountain-mass is distributed to the several depths specified.

No. of the compartment.	Depth = 100 miles.			Depth = 300 miles.		Depth = 500 miles.		Depth = 1000 miles.	
	log S.	$\log \frac{c}{k}$ S.	$\frac{c}{k}$ S.	$\log \frac{c}{k}$ S.	$\frac{c}{k}$ S.	$\log \frac{c}{k}$ S.	$\frac{c}{k}$ S.	$\log \frac{c}{k}$ S.	$\frac{c}{k}$ S.
1	1·8900855	1·5735938	37·462	1·1457625	13·988	0·9282562	8·477	0·6290134	4·256
2	2·2780444	1·9933396	98·478	1·5744198	37·533	1·3578202	22·794	1·0589566	11·454
3	2·4687460	2·2121945	163·002	1·8027064	63·490	1·5871129	38·647	1·2896731	19·484
4	2·7586091	2·5316478	340·132	2·1340639	136·164	1·9198108	83·140	1·6219399	41·874
5	2·9016653	2·7009330	502·265	2·3160296	207·028	2·0032989	100·762	1·8060803	63·984
6	2·9200659	2·7438198	554·396	2·3731020	236·103	2·1621943	145·276	1·9657655	92·420
7	3·0083191	2·8544570	715·248	2·4991855	315·635	2·2904237	195·175	1·9949373	98·841
8	3·2338537	3·1006090	1260·692	2·7622590	578·441	2·5560574	359·797	2·2617126	182·689
9	3·3240271	3·2091188	1618·522	2·8888039	774·112	2·6855819	484·822	2·3925902	246·940
10	3·4922377	3·3947055	2481·450	3·0930707	1238·998	2·8951581	785·521	2·6020427	399·984
11	3·4591903	3·3759713	2376·683	3·0948912	1244·203	2·8993149	793·076	2·6099224	407·308
12	3·5685724	3·4983592	3150·360	3·2378705	1729·300	3·0470184	1114·346	2·7599455	575·368
13	3·6325039	3·5737437	3747·518	3·3343820	2159·643	3·1489926	1409·265	2·8640977	732·314
14	3·6788333	3·6297694	4263·530	3·4110845	2576·822	3·2317361	1705·046	2·9506409	892·567
15	3·6804089	3·6399961	4365·120	3·4425032	2770·150	3·2701831	1862·873	2·9929870	983·982
16	3·7017631	3·6686258	4661·519	3·4914754	3100·811	3·3268344	2122·435	3·0541306	1132·741
17	3·7362658	3·7093499	5120·942	3·5518786	3563·515	3·3956916	2487·090	3·1282372	1343·498
18	3·7541885	3·7325905	5424·447	3·5939893	3926·353	3·4470317	2799 186	3·1857035	1533·570
19	3·7831994	3·7660643	5835·315	3·6450852	4416·571	3·5079183	3220·463	3·2535885	1793·034
20	3·8293752	3·8162755	6550·515	3·7112492	5143·387	3·5843043	3839·761	3·3378955	2177·186
21	3·8064452	3·7962322	6255·070	3·7066516	5089·225	3·5904511	3894·494	3·3531283	2254·905
22	3·8207130	3·8131453	6503·470	3·7371767	5459·800	3·6318238	4283·747	3·4045751	2538·488
23	3·8143209	3·8089455	6440·884	3·7451369	5560·795	3·6505843	4472·850	3·4344033	2718·963
24	3·7941586	3·7907311	6176·339	3·7377584	5467·118	3·6539634	4507·787	3·4500050	2818·416
25	3·7709822	3·7689065	5873·630	3·7248986	5307·605	3·6506966	4474·006	3·4588277	2876·257
26	3·7444017	3·7436995	5542·420	3·7086220	5112·366	3·6449507	4415·203	3·4678492	2936·630
27	3·7171660	3·7175175	5218·151	3·6896529	4893·876	3·6352112	4317·290	3·4725999	2968·930
28	3·6909007	3·6921393	4921·973	3·6704595	4682·303	3·6245196	4212 303	3·4768125	2997·868
29	3·6587334	3·6607076	4578·335	3·6443344	4408·942	3·6061881	4038·203	3·4736521	2976·131
30	3·6228210	3·6254041	4220·890	3·6135672	4107·402	3·5824595	3823·486	3·4651445	2918·400
31	3·5838514	3·5869413	3863·145	3·5789461	3792·680	3·5541041	3581·825	3·4517607	2829·833
32	3·3492445	3·3527635	2253·012	3·3481032	2228·965	3·3289403	2132·751	3·2414517	1743·620
33	3·2870781	3·2909512	1936·725	3·2890603	1945·630	3·2748254	1882·984	3·2016263	1590·840
34	3·2110564	3·2152215	1641·427	3·2155729	1642·755	3·2054460	1604·893	3·1451302	1396·787
35	3·0895554	3·0939463	1241·500	3·0952446	1245·216	3·0896518	1229 283	3·0414036	1100·028
36	2·8107096	2·8152943	653·573	2·8191350	659·379	2·8156226	654·068	2·7783802	600·317
37	3·0129257	3·0176708	1041·525	3·0228197	1053·925	3·0218876	1051·690	2·9945491	987·527
38	2·2880255	2·2929142	196·297	2·2991582	199·140	2·3004183	199·718	2·2819951	191·423
39	2·2880255	2·2930146	196·343	2·3001663	199·603	2·3032706	201·034	2·2926864	196·194
40	2·7139948	2·7190748	523·691	2·7269967	533·331	2·7316508	539·077	2·7279671	534·524
		In feet ……	122545·996	………	97822·301	………	79994·644	………	51909·575
		In miles……	23·209	………	18·527	………	14·980	………	9·831
Multiply by 1″·1392, DEFLECTIONS=26″·440					=21″·106		=17″·066		=11″·199

TABLE OF DEFLECTIONS for Station B (*Kalianpur*) under the same circumstances.

No. of the compartment.	log S.	Depth =100 miles.		Depth =300 miles.		Depth =500 miles.		Depth =1000 miles.	
		log $\frac{c}{k}$ S.	$\frac{c}{k}$ S.	log $\frac{c}{k}$ S.	$\frac{c}{k}$ S.	log $\frac{c}{k}$ S.	$\frac{c}{k}$ S.	log $\frac{c}{k}$ S	$\frac{c}{k}$ S.
22	2·0494119	2·0418442	110·114	1·9658756	92·433	2·8605227	725·307	1·6332740	42·981
23	2·8386287	2·8332533	681·167	2·7694447	588·091	2·6748921	473·034	2·4587111	287·548
24	3·3455110	3·3420835	2198·283	3·2891108	1945·856	3·2053158	1604·412	3·0013574	1003·130
25	3·5026319	3·5005562	3166·330	3·4565483	2861·200	3·3823463	2411·827	3·1904774	1550·520
26	3·5810559	3·5803537	3804·991	3·5452762	3509·750	3·4816049	3031·132	3·3045034	2016·060
27	3·6753989	3·6757504	4739·694	3·6478858	4445·144	3·5934441	3921·427	3·4308328	2696·701
28	3·7356791	3·7369177	5456·544	3·7152379	5190·842	3·6692980	4669·796	3·5215909	3323·463
29	3·7441811	3·7461551	5573·848	3·7297821	5367·625	3·6916358	4916·271	3·5590998	3623·262
30	3·7318654	3·7344485	5425·609	3·7226126	5279·740	3·6915039	4914·778	3·5741889	3751·361
31	3·6974044	3·7004943	5017·580	3·6924991	4926·053	3·6676571	4652·186	3·5653137	3675·477
32	3·6563681	3·6598871	4569·693	3·6552268	4520·920	3·6360639	4325·775	3·5485753	3536·513
33	3·6048058	3·6086789	4061·429	3·6067880	4043 785	3·5925531	3913·390	3·5193540	3306·390
34	3·5466938	3·5508589	3596·325	3·5512103	3558·036	3·5410834	3476·029	3·4807676	3025·294
35	3·4109670	3·4153579	2602·303	3·4166562	2610·095	3·4110634	2576·697	3·3628152	2305·766
36	3·3179997	3·3225844	2101·766	3·3264251	2120·435	3·3229127	2103·355	3·2856703	1930·502
37	3·1706547	3·1753998	1497·614	2·1805487	1515·475	3·1796166	1512·225	3·1522781	1419·967
38	2·9224140	2·9274031	846·064	2·9335467	858·117	2 9397959	870·554	2·9163836	824·866
39	2·8275631	2·8326436	680·211	2·8397039	691·359	2·8478887	704·512	3·8322240	679·554
40	......	......	......	......	......	....	.. .	......	......
		In feet	56129·565	......	54124·956	......	50802·707	......	38999·355
		In miles ...	10·631	......	10·251	......	9·6217	......	7·3862
Multiply by 1″·1392, Deflections=12″·111					=11″·678		=9′·622		=7″·386

TABLE OF DEFLECTIONS for Station C (*Damorgida*) under the same circumstances.

30	2·1911155	2·1936986	156·206	2·1818617	152·006	2·1507540	141·499	2·0334390	108·004
31	3·3765405	3·3796304	2396·792	3·3716352	2 53·072	3·3467932	2222·252	3·2444498	1755·698
32	3·5917478	3·5952668	3937·920	3·5906065	3895·888	3·5714436	3727·722	3·4839550	3017·580
33	3·6934978	3·6973709	4981·623	3·6954800	4959·981	3·6812451	4800·043	3·6080460	4055·515
34	3·6776695	3·6818346	4806·562	3·6521860	4810·453	3·6720591	4699·580	3·6117433	4090·190
35	3·6181590	3·6225499	4193·242	3·6238482	4205·800	3 6182554	4151·982	3·5700072	3715·413
36	3·5499407	3·5545254	3585·300	3·5583661	3617·145	3·5548537	3558·010	3·5176113	3293·149
37	3·4687918	3·4735369	2975·340	3·4786858	3010·827	3·4777537	3004·372	3·4504152	2821·080
38	3·3658939	3·3707826	2348·455	3·3770266	2382·470	3·3782867	2389·388	3·3598635	2290·148
39	3·1030610	3·1080501	1282·479	3·1152018	1303·772	3·1183061	1313·125	3·1077219	1281·510
40	3·0394934	3·0445739	1108·087	3 0524958	1128·485	3·0571499	1140·643	3·0534662	1131·009
		In feet	31772·006	......	31819·899	......	31178·616	.. ...	27559·296
		In miles ...	6·017	......	6·027	.. ..	5·905	.....	5·2196
Multiply by 1″·1392, Deflections= 6″·855					=6″·866		=6′·670		=5′·220

1·8975923 and 1·9030900, 1·4090396 and 1·4259687, 1·1751240 and 1·2041200, 0·8406207 and 0·9030900. The values of θ are obtained from the first column in the Table of Heights (page 750), or from page 67 of my former Paper.

The values of $\frac{c}{k}$S for the three stations, for the various examples of depth, are now easily obtained by adding the logarithms of the last columns of the Four Tables of Reduction to the logarithms of S in the Table in page 753, and finding the natural numbers. The results are given for each of the three stations A, B, C, for all four cases of depth, in the four next Tables, which I call the Four TABLES OF DEFLECTIONS in the meridian; the totals at the foot of these Four Tables give the final results, viz. the Deflections under the various suppositions of depth. These I now proceed to discuss.

§ 5. *Discussion of the Deflections under the various cases of depth, and of the effect upon the Ellipticity of the Indian Arc.*

13. The Tables thus calculated furnish the following results:—

	At Kaliana.	At Kalianpur.	At Damargida.
Deflection in meridian, caused by the mass of the Himmalayas and the mountain region beyond	27″·978	12″·047	6″·790
Ditto, by same mass distributed through a depth of . . . . 100 miles	26·440	12·111	6·855
Ditto, ... 300 miles	21·106	11·678	6·866
Ditto, ... 500 miles	17·066	9·622	6·670
Ditto, ... 1000 miles	11·199	7·386	5·220

By subtracting each of the last four lines from the first line, we have the following results:—

	At Kaliana.	At Kalianpur.	At Damargida.
Deflection in meridian, caused by the mass of the Himmalayas and of the mountain region beyond	27″·978	12″·047	6″·790
Ditto, modified by the supposed attenuation of matter extending down to a depth of 100 miles	1·588	−0·064	−0·065
Ditto, ... 300 miles	6·872	0·369	−0·076
Ditto, ... 500 miles	10·912	2·425	0·120
Ditto, ... 1000 miles	16·779	4·661	1·570

It will be seen how much the Deflections are reduced by this hypothesis, especially in the case where the attenuation extends through only 100 miles. In fact, in this case the upheaval of the mountains and the consequent attenuation below produce a slight deviation the other way at the two further stations. The success of the hypothesis may therefore, thus far, be considered to be established, although it remains an hypothesis still; and we must always be in uncertainty, not as to its answering this end, but as to its being true in nature. The existence of the mountain-mass is a fact indisputable. Not so the compensating cause, which is simply conjectural as to its existence, and altogether uncertain as to its extent, if it exist. We have no certain and independent method of determining this; nor of ascertaining, even if the hypothesis be valid, how far down the attenuation extends, or what law it follows.

14. I will now determine the effect of these several results upon the Ellipticity of the Indian Arc.

The corrections to be applied to the astronomical amplitudes of the two arcs Kaliana—Kalianpur and Kalianpur—Damargida, are determined by taking the differences of the numbers in each line of the Deflections above given. Hence we get—

	Kaliana to Kalianpur	Kalianpur to Damargida
Corrections to be added to the astronomical amplitudes, in consequence of the attraction of the mass of the Himmalayas and of the region beyond	15″·931	5″·257
Ditto, from the same cause, modified by the supposed attenuation below, extending down to a depth of 100 miles	1·602	0·001
Ditto, ... 300 miles	6·503	0·445
Ditto, ... 500 miles	8·487	2·305
Ditto, ... 1000 miles	12·118	3·091

Putting the arbitrary expressions $15''\cdot885\,(1-u)$ and $5''\cdot050\,(1-v)$, taken from par. 8, for these pairs of corrections in succession, we find the following corresponding values of u and v:—

$u=-0\cdot0028$ and $v=-0\cdot0391$
$+0\cdot8992$ $+0\cdot9998$
$0\cdot5906$ $0\cdot9120$
$0\cdot4057$ $0\cdot5444$
$0\cdot2371$ $0\cdot3890$

By substituting these, in succession, in the formula (2) of par. 8 for ϵ, the ellipticity, we have the following values:—

$\epsilon=0\cdot002377$ or $\frac{1}{421}$, when no Deficiency of Matter exists.

$0\cdot004621$ or $\frac{1}{216}$, when Deficiency of Matter extends down 100 miles.

$0\cdot003578$ or $\frac{1}{280}$, when Deficiency of Matter extends down 300 miles.

$0\cdot003497$ or $\frac{1}{286}$, when Deficiency of Matter extends down 500 miles.

$0\cdot002815$ or $\frac{1}{355}$, when Deficiency of Matter extends down 1000 miles.

The value of the semi-axis major may similarly be found. These values of the ellipticity differ in every instance from the mean value, $\frac{1}{300}$. They seem, however, to point out, that if the attenuation extend down to between 500 and 1000 miles the ellipticity would come out equal to the mean value. In this case the residual mountain attraction is considerable. Thus, then, it appears, that although the hypothesis of Deficiency of Matter, if it extend to no greater depth than 100 miles or so, will very much multiply the effect of the Himmalayas and the mountain-region beyond on the plumb-line, the result shows that in computing the Indian Arc there is little ground for working with the mean ellipticity, as is done in the Great Trigonometrical Survey.

15. Nor are the peculiarities of the Great Trigonometrical Survey explained by the hypothesis of Deficiency of Matter below. None of the pairs of errors in the astronomical amplitudes given in par. 14, nor any which can be interpolated, coincide with the errors in the amplitudes detected by Colonel EVEREST in the Great Arc*. According to him, astronomical observations make the amplitude of the arc Kaliana—Kalianpur too small by $5''\cdot236$, and that of the arc Kalianpur—Damargida too large by $3''\cdot789$. This latter error indicates the existence of some local cause of disturbance in the neighbourhood of Damargida. That such local causes may exist without the presence of so enormous a mass as that of the Himmalayas, I have shown in a Paper on the English Arc published in the 'Transactions' for 1856. For example, in the course of the calculations of that Paper the following instance occurs (p. 46, in the Table). A tabular mass only about 600 feet high, and measuring thirty-seven miles by forty-six, will cause a deflection of $3''\cdot172$ at a station three miles from its longer side, and about one-third from one end of it and two-thirds from the other end. Colonel EVEREST himself enters into an elaborate calculation in his earlier quarto volume (of 1830) to find the local attraction at the Station Takal K'hera, which had been previously fixed upon as one of the principal stations of the Great Arc; but was afterwards abandoned, notwithstanding all the time and labour which had been bestowed upon calculations in connexion with it; and that because the deflection $5''\cdot098$ was found to result from the attraction of an extensive table-land commencing as far as twenty miles off, and rising to only about 1600 feet above the level of the station. Colonel LAMBTON had considered the table-land to be too distant to have any effect; but calculation proved the reverse.

16. In fact, as it appears to me, while the absolute necessity of attending to local attraction is strongly illustrated by the investigations I have given in this and my previous Papers, the utter hopelessness of attaining to any certain results seems to be as clearly demonstrated. Here are causes, not only visible in mountain-masses and table-lands and deep neighbouring oceans which affect the vertical, and which can be calculated if sufficient labour be bestowed; but also invisible and hidden, as important as the others, of which we know absolutely nothing, except that they may exist and that we have no power of proving or disproving their existence. Were it not for these disturbing causes, the comparison of the parts even of the same arc ought to bring out its ellipticity (if the meridian *be* elliptical) with considerable precision. The approximation can

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* See p. clxxvii of his 4to volume published in 1847.

be carried to any degree of exactness by the formulæ. The measures of the Survey give almost exact results. The lengths of the arcs measured are known to a wonderful degree of exactness. But the smallest disturbing causes affecting the plumb-line introduce an element of doubt and uncertainty, through the astronomical amplitudes, which vitiates the whole. This is unhappily the case with the Great Indian Arc. The errors brought to light in Colonel EVEREST's volume (of 1817), viz. $5''\cdot236$ in one portion and $-3''\cdot789$ in another, are too important not to be strictly and numerically accounted for. The cause *must* lie in the determination of the vertical. The deflection of the plumb-line caused by the attraction of the vast Mountain Region on the north will not account for these errors: it would make them very much larger (and would not explain the negative sign), as I showed in my Paper in 1855. No hypothesis of deficiency of matter below, which we can conceive, will remove the anomaly. The disturbing cause must lie elsewhere; perhaps in the immediate neighbourhood of Kalianpur and Damargida, which should be most carefully surveyed for the purpose of detecting it; or it may be hidden beneath the surface, and arise from sources which we cannot possibly examine and calculate, because of our ignorance of their position and magnitude. [This will be illustrated in the following paragraph.] Some would recommend that we ignore the above errors altogether, and put them to the account of these uncalculated causes. They would reverse the problem, and make the errors the measure of the disturbing cause. But unfortunately this will not do; for it proceeds upon the *assumption* that the meridian is elliptical, and moreover that its ellipticity is known and is equal to the mean ellipticity of the whole earth; which is, in short, begging the question at issue*.

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* The degree of influence which errors in the amplitude and ellipticity may have in the mapping of the country may be learnt from the following approximate calculation.

Let A be the length of the arc, λ the amplitude, a the semi-axis major, μ the latitude of the middle point, ϵ the ellipticity of the arc, m the mean radius vector, then $m=a\left(1-\frac{1}{2}\epsilon\right)$. also

$$\frac{A}{\lambda}=a\left\{1-\frac{1}{2}\epsilon(1+3\cos 2\mu)\right\}=m\left\{1-\frac{3}{2}\epsilon\cos 2\mu\right\}\quad .\ .\ .\qquad (\alpha)$$

Suppose that the quantities used in this formula are incorrect, and that they ought to be $A+dA$, $\lambda+d\lambda$, $m+dm$, $\epsilon+d\epsilon$; the corrections are connected by the following equation:—

$$\frac{dA}{A}=\frac{d\lambda}{\lambda}+\frac{dm}{m}-\frac{3}{2}\cos 2\mu\ d\epsilon.$$

Now in the arc between Kaliana and Kalianpur, A is 871 miles $\left(\text{calculated on the supposition that the ellipticity}=\frac{1}{300}\right)$, λ is 19417, $\cos 2\mu=0\cdot59295$. I shall suppose that the *mean* radius m is correct, and therefore $dm=0$;

$$\therefore\quad \frac{dA}{A}=\frac{d\lambda}{\lambda}-0\cdot889d\epsilon;$$

$$\therefore\quad dA=\frac{A}{\lambda}(d\lambda-17270d\epsilon)=\frac{1}{52\cdot3}(d\lambda-17270d\epsilon)\ \text{miles}.\ .\ .\quad .\ .\ .\ (\beta.)$$

[illegible] this formula $d\lambda$ must be put = to the deflection in seconds, with the *negative sign*, for the following

I proceed now to make use of the calculations of this Paper to show how some idea may be formed of the effect, which the possible and not improbable existence of extensive tracts within the mass of the earth where the density is, though perhaps very slightly, either greater or less than the density of those parts as required by the fluid theory of equilibrium.

§ 6. *General conclusions regarding the effect of a defect or excess of density in any part of the mass of the earth.*

17. It is quite possible that extensive tracts may exist in the interior of the earth's mass throughout which the density is somewhat less or somewhat greater than the density which the fluid theory would require for those parts, and upon which the theoretical direction of gravity is determined. I can conceive of a vast tract beneath in which the development of local heat had by long action expanded the material of the mass, and compressed it in a region beyond where no sufficient heat was developed to counteract this effect. Suppose this took place chiefly in a direction parallel to the

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reason. A in formula (α.), the length of the arc between Kaliana and Kalianpur, is known accurately by the Survey: and, values of m and e being assumed, λ is calculated by the formula. But λ comes out *larger* than the *observed* value, the difference arising from mountain attraction. Now $\lambda+d\lambda$ is used in formula (β.) to find how much A is affected by taking the *observed*, and not the calculated, value of λ. Hence $d\lambda$ must be negative. Let l be the number of seconds in the deflection; then $d\lambda=-l$, and

$$dA=-\frac{1}{52\cdot3}(l+17270de)=1\cdot1-\frac{l}{52\cdot3}-330\left(de+\frac{1}{300}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\gamma.)$$

This formula agrees with the statement of Colonel EVEREST in p. clxxvii of the Preface of his volume of 1847: for by putting $de=0$ (as he calculates with the mean ellipticity), and $l=5''\cdot23$ the error he gives, $dA=-\frac{1}{10}$ mile$=-88$ fathoms, the error in the arc which he deduces. If then a place between Kaliana and Kalianpur be laid down on the map, first by reckoning from Kaliana and then from Kalianpur, the two spots will not coincide, but will be separated by $\frac{1}{10}$th of a mile. Colonel EVEREST avoided this by distributing the error among all the stations of the arc, and slightly altering all the latitudes accordingly. But the natural way of correcting it, as it appears to me, would have been to find an ellipticity which would have reduced the error to zero, and to have worked with *that* and not with the mean. In doing this, it would also be necessary, as shown in the calculations of these Papers, to correct for mountain attraction.

As a further illustration of the use of formula (γ.), and to show the influence of errors on the mapping part of the Survey, suppose that the deflections, after all, owing to the compensating cause, make l only $0''\cdot528$, and that the curvature of the arc is measured by the ellipticity $\frac{1}{400}$ (which is not impossible), then $de+\frac{1}{300}=\frac{1}{400}$, and $dA=1\cdot1-0\cdot01-0\cdot825=0\cdot26$, and the error in the places would be more than a quarter of a mile, and in the opposite direction.

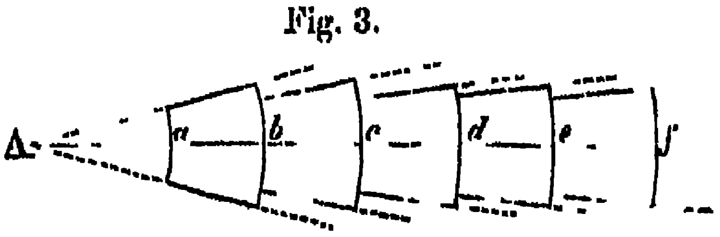
If the pairs of values of l and de taken from par. 14 are put in formula (γ.) they will not reduce dA to zero, because they are mixed up with the data of the other arc, and are not correct, as is evident from the calculations of this Paper, as local causes of disturbance near Kalianpur or Damargida, or both, are not taken account of.

surface, so as to raise no considerable mountain mass. The effect on the plumb-line at a station over the place where the expanded and compressed portions meet would nevertheless be considerable.

I will proceed to calculate the effect on the plumb-line of a large given space (the space I use is 4 millions of cubic miles, *i. e.* half a cube of 200 miles each way, the thickness being 100 miles and always vertical), situated at different depths and distances from the station where the plumb-line is, and of a density equal to $\frac{1}{100}$th part of the density of the part of the earth where its middle point lies.

Let *Af* (fig. 3) be the meridian through A. Draw upon it five four-sided figures bounded by parts of great circles diverging from A, and also by circles of which A is the centre: and let their dimensions be so chosen, that the meridian length of each is 200 miles (=2° 52′ 40″) and the area of each equals a square of 200 miles each way. It will result from this, that the angular distances of the sides of the spaces from A will be 4°, 6° 53′, 9° 46′, 12° 38′, 15° 31′, 18° 23′. By comparing these with the angular distances of the "compartments" by which I have before divided the meridian, it will be seen that the five "spaces" comprise the following whole and fractional parts of the "compartments:"—

Fig. 3.



Space *ab* includes $\frac{1}{2}$ of 18th, whole of 19th to 23rd, $\frac{1}{4}$ of 24th compartment.
Space *bc* includes $\frac{3}{4}$ of 24th, whole of 25th and 26th, $\frac{11}{12}$ of 27th compartment.
Space *cd* includes $\frac{1}{12}$ of 27th, whole of 28th and 29th, $\frac{6}{10}$ of 30th compartment.
Space *de* includes $\frac{4}{10}$ of 30th, whole of 31st , $\frac{2}{5}$ of 32nd compartment.
Space *ef* includes $\frac{3}{5}$ of 32nd . . . . $\frac{3}{7}$ of 33rd compartment.

The angular distances of the middle points of the five spaces from A are 5° 26′, 8° 19′, 11° 12′, 14° 4′, 16° 57′: and therefore the chords of these angles (rad.=4000 miles), or the values of *c* for the middles of the five spaces, are

379, 581, 781, 980, 1173 miles.

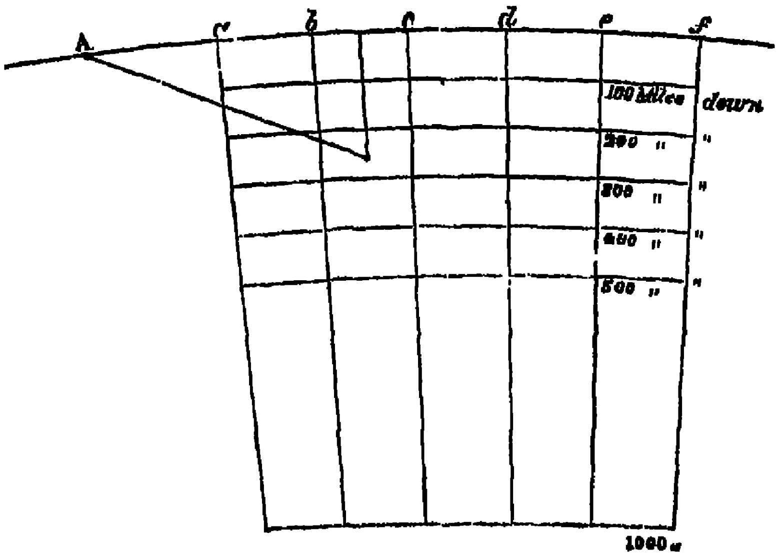
The angles of the lunes of which the five spaces are portions are found from the expression

$$\beta=\frac{\text{area of space}\times 180^\circ}{2\pi r^2 \sin 1^\circ 26' 20'' \sin (\text{angular distance of middle point from A})}.$$

The results are β=30° 7′, 19° 42′, 14° 41′, 11° 45′, 9° 47′.

18. Suppose now that each of the five spaces is covered by a mass one mile in uniform height; and that in each case this mass is then distributed uniformly to the three depths in succession, 100, 300, and 500 miles, as represented in fig. 4, which is a vertical section. I propose to find the deflection caused at A by the mass under these several circumstances. For this purpose we must use formula (3.) of paragraph 10. The calculation is much simplified, as *h* is always the same, and =1, and Az=0. The values of

Fig. 4.



log $\left(1''\cdot1392 \sin\frac{1}{2}\beta\right)$ for the five spaces are as in the margin. The values of $\frac{c}{k}$ belonging to each space must next be gathered out from the first, second, and third of the Tables of Reduction (see pages 754, 755), which give the values of log $\frac{c}{k}$. The natural numbers are formed into the following Table, which gives the values of $\frac{c}{k}$ for such of the compartments (from the 18th to the 33rd) as are comprised in the five spaces. We must now apportion these values of $\frac{c}{k}$ to the several spaces to which they appertain, take their sum, and multiply by $1''\cdot1392 \sin\frac{1}{2}\beta$, and we shall have the Deflections produced by the mass of one mile high and on a base equal to a square of 200 miles each way, when diffused downwards to the several depths of 100, 300, and 500 miles. The following Table will explain itself:—

$\bar{1}\cdot4712930$
$\bar{1}\cdot2897722$
$\bar{1}\cdot1625924$
$\bar{1}\cdot0661096$
$\bar{2}\cdot9866678$

No. of compartment.	Values of $\frac{c}{k}$ gathered from the 1st, 2nd, and 3rd of Tables of Reduction (in par. 12) for following depths:—		
	100 miles.	300 miles.	500 miles.
18	0·95149	0·69152	0·49300
19	0·96131	0·72759	0·53054
20	0·97029	0·76186	0·56876
21	0·97676	0·79471	0·60814
22	0·98273	0·82502	0·64731
23	0·98770	0·85274	0·68590
24	0·99214	0·87821	0·72411
25	0·99523	0·89932	0·75808
26	0·99838	0·92092	0·79533
27	1·00081	0·93861	0·82803
28	1·00286	0·95402	0·85826
29	1·00456	0·96739	0·88604
30	1·00597	0·97892	0·91125
31	1·00714	0·98877	0·93380
32	1·00814	0·99738	0·95432
33	1·00896	1·00457	0·97218

Table, giving the Deflections caused by a Mass, which at the surface is 1 mile high and 40,000 square miles in base, diffused downwards to the following depths; viz.—

No of compartment.	Compartments, in whole or in part, comprised in the spaces	100 miles.	200 miles (by interpolation)	300 miles	400 miles (by interpolation).	500 miles
18	Half	0·47574		0·34576		0·24650
19	whole					
20	whole					
21	whole } space *ab*	4·87879		3·96192		3·04065
22	whole					
23	whole					
24	$\frac{1}{1}$th	0·24803		0·21955		0·18103
		5·60256 log=0·7483865 $\bar{1}$·4712930 0·2196795 Deflect.=1″·658	1″·499	4·52723 log=0·6558326 $\bar{1}$·4712930 0·1271258 1″·340	1″·183	3·46818 log=0·5401015 1·4712930 0·0113945 1″·027
24	$\frac{3}{1}$ths	0·74411		0·65566		0·54308
25	whole } space *bc*	1·99361		1·82024		1·55341
26	whole					
27	$\frac{12}{11}$ths	0·92383		0·86641		0·76434
		3·66155 log=0·5636649 $\bar{1}$·2897722 $\bar{1}$·8534371 0″·714	0″·689	3·34531 log=0·5244361 $\bar{1}$·2897722 1·8142086 0″·652	0″·614	2·86083 log=0·4561920 1·2897722 1·7462642 0″·558
27	$\frac{1}{13}$th	0·07698		0·07220		0·06369
28	whole } space *cd* …	2·00742		1·92141		1·74430
29	whole					
30	$\frac{6}{10}$ths	0·60358		0·58735		0·54675
		2·68798 log=0·4294260 $\bar{1}$·1625924 $\bar{1}$·5920184 0″·391	0″·384	2·58096 log=0·4117812 $\bar{1}$·1625924 $\bar{1}$·5743736 0″·375	0″·363	2·35474 log=0·3719130 $\bar{1}$·1625924 $\bar{1}$·5345354 0″·342
30	$\frac{4}{10}$ths	0·40239		0·39157		0·36450
31	whole } space *de* …	1·00714		0·98877		0·93380
32	$\frac{4}{10}$ths	0·40326		0·39895		0·38173
		1·81279 log=0·2583474 $\bar{1}$·0661096 1·3244570 0″·212	0″·210	1·77929 log=0·2502467 1·0661096 $\bar{1}$·3163563 0″·207	0″·208	1·68003 log=0·2253170 $\bar{1}$·0661096 $\bar{1}$·2914266 0″·196
32	$\frac{6}{10}$ths } space *ef* …	0·60488		0·59843		0·57259
33	$\frac{3}{7}$ths	0·43241		0·43053		0·41665
		1·03729 log=0·0159000 $\bar{2}$·9866678 $\bar{1}$·0025678 0″·101	0″·100	1·02896 log=0·0123985 $\bar{2}$·9866678 $\bar{2}$·9990663 0″·099	0″·098	0·98924 log=$\bar{1}$·9953017 2·9860678 2·9819695 0″·096

19. This Table furnishes the following results:—

	Distance of middle point from A, along the chord.				
	379 miles.	581 miles.	781 miles.	980 miles.	1173 miles.
Deflections, caused by the mass distributed downwards through a depth of ... 100 miles	1″·658	0″·714	0″·391	0″·212	0″·101
Ditto .... 200 miles	1·499	0·680	0·384	0·210	0·100
Ditto .... 300 miles	1·340	0·652	0·375	0·207	0·099
Ditto .... 400 miles	1·183	0·614	0·363	0·203	0·098
Ditto .... 500 miles	1·027	0·558	0·342	0·196	0·096

Multiply these successive lines of numbers by 1, 2, 3, 4, 5, and we shall have the Deflections caused by masses having the same volumes as above, but all having the same density, viz. that of the first, *i. e.* $\frac{1}{100}$th part of the density of the materials of the surface. The numbers then become—

1″·658	0″·714	0″·391	0″·212	0″·101
2·998	1·378	0·768	0·420	0·200
4·020	1·956	1·125	0·621	0·297
4·732	2·456	1·452	0·812	0·392
5·135	2·790	1·710	0·980	0·480

Now subtract each line from the line below it, and substitute the four lines thus formed for the last four above, and we have—

	Distance of the middle from A, measured along the chord.				
	379 miles.	581 miles.	781 miles.	980 miles.	1173 miles.
Deflections, caused by a semi-cubic mass, 200 miles in each horizontal side, and 100 miles deep, density = $\frac{1}{100}$th of the density of the surface, and the depth of its centre = 50 miles	1″·658	0″·714	0″·391	0″·212	0″·101
Ditto .... 150 miles	1·340	0·664	0·377	0·208	0·099
Ditto .... 250 miles	1·022	0·578	0·357	0·201	0·097
Ditto .... 350 miles	0·712	0·500	0·327	0·191	0·095
Ditto .... 450 miles	0·403	0·334	0·258	0·168	0·088

The several volumes, the lower down they are, will, owing to the converging of the vertical lines, be somewhat contracted, and the densities slightly increased in a corresponding degree. If we suppose, therefore, the spaces to be correspondingly enlarged in the horizontal direction the volumes will be all the same, and the densities all the same; viz. $\frac{1}{100}$th part of the density of the surface, or of granite. But in order to compare them with the densities of those parts of the earth's interior where they are situated, we should increase their densities in proportion; and this will increase the deflection in a corresponding degree. The following is the usual law of density assumed from the fluid theory, D being the density of the surface, r = radius of the earth:—

$$\text{Density at depth } d \text{ miles} = \frac{r}{r-d} D \sin\left(\frac{5\pi}{6}\,\frac{r-d}{r}\right),$$

from which I gather the following results:—

Ratio of density at depth 50 miles to D = 1·170
Ratio of density at depth 150 miles to D = 1·210
Ratio of density at depth 250 miles to D = 1·353
Ratio of density at depth 350 miles to D = 1·498
Ratio of density at depth 450 miles to D = 1·616

20. Multiply the lines in the last series of Deflections by these numbers, and we have our final results, as follows:—

TABLE OF DEFLECTIONS, caused by an excess or defect of matter throughout a semi-cubic space of 4 millions of cubic miles, the mean density of the excess or defect being $\frac{1}{100}$th part of the density of the earth at the depth of the centre of the cubic space.

Depth of the centre of the semi-cubic space.	Distance of the middle point of the space from A, measured along the chord to the surface.				
	379 miles.	581 miles.	781 miles.	980 miles.	1173 miles.
50 miles	1″·940	0″·835	0″·457	0″·248	0″·118
150 miles	1·621	0·803	0·456	0·252	0·120
250 miles	1·383	0·782	0·483	0·272	0·131
350 miles	1·067	0·749	0·490	0·286	0·142
450 miles	0·663	0·713	0·425	0·277	0·145

The effect of this calculation is to show how much uncertainty must always attend the exact determination of the true vertical, a thing which is absolutely essential in the calculation of the curvature of the several portions of the earth's surface. It will be observed that the supposed defect or excess of density has been assumed to be only $\frac{1}{100}$th part of the density of the earth where the hidden cause may lie. But a much larger fraction might have been chosen. Rocks at the surface of the earth, even of the same kind, differ considerably in their density according to the specimens examined. The following are examples:—

Basalt	varies through $\frac{2}{9}$ths of its mean density.
Chalk	varies through $\frac{1}{6}$th of its mean density.
Coal	varies through $\frac{1}{4}$th of its mean density.
Dolomite	varies through $\frac{1}{9}$th of its mean density.
Felspar	varies through $\frac{2}{19}$ths of its mean density.
Granite	varies through $\frac{1}{8}$th of its mean density.
Gypsum	varies through $\frac{1}{5}$th of its mean density.
Hornblende	varies through $\frac{1}{6}$th of its mean density.
Hornstone	varies through $\frac{1}{18}$th of its mean density.
Limestone	varies through $\frac{2}{9}$ths of its mean density.

These show how probable it is that the several portions of the earth's interior, although preserving roughly the general average of density, according to their position, as required by the fluid theory, may nevertheless vary sufficiently to disturb the position of the plumb-line most materially. The space I have selected as the basis of my calculation is no doubt extensive; but I hardly think too extensive. I have seen the same kind of rock (gneiss) prevail for hundreds of miles in the Himmalaya Mountains; and can see no reason why a space as large as I have chosen, 200 miles square parallel to the surface and 100 miles deep, may not exist beneath the surface, having a density, too, differing much more than $\frac{1}{100}$th part from the proper density of its locality.

§ 7. Appendix, *containing a revise of some parts of my former Paper.*

21. The results of this Paper may appear in some respects to render the calculations of my former communication now unnecessary, as it is here shown that it is not improbable that a compensating cause exists sufficient to counteract the effect of the Mountain Mass, at any rate to a considerable degree. But it must be observed that the demonstration of the sufficiency of this cause, should it exist in nature, rests altogether upon the process of dissection of the mass and the calculations consequent thereon, given at large in that Paper. The conclusions also in the present communication regarding the effect of wide-spread, though minute, defect or excess of density below, rest upon those former calculations. I have thought it well, therefore, to take this opportunity of revising some parts into which errors have crept, as intimated in the Note to par. 1.

22. The corrections of the Deflections in meridian, produced by the Mountain Mass as it exists on the surface, at the three principal stations, have already been given. They are but trifling. I have, since completing this new calculation, gone over the former one and detected the several minute errors, so as to account for every figure of discrepancy*. This is highly satisfactory, especially when the troublesomeness of the calculation is considered.

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* The errors have crept in among the calculations at the foot of the Six Tables of my former Paper (pp. 73–83). The details are as follows. The values of H $\sin\frac{1}{2}\beta\cos$ Az. as there given, and the necessary corrections, are as below:—

	Lune I.	Lune II.	Lune III.	Lune IV.	Lune V.	Totals.
For Station A. (miles)	0·772	3·656	4·262	2·602	0·095	
Errors ...	......	+0·001	−0·001			
	3·009	3·933	3·395	2·493	0·235	
Errors ...	+0·078	+0·001	......	+0·001	+0·027	
						24·559 miles.
For Station B.	0·015	1·600	1·211			
Errors ...	......	−0·046	−0·001			
	0·677	2·536	2·643	1·813	0·011	
Errors ...	+0·001	......	+0·020	+0·001	+0·094	
						10·575 miles.
For Station C.	1·173					
Errors ...	−0·124					
	1·973	2·103	0·816			
Errors...	......	+0·019				
						5·960 miles.

These three final quantities are precisely the same as at the foot of the Table in page 753.

In my former Paper I brought out the Deflections	27″·853, 11″·968, 6″·909
The correct values are now shown to be	27″·978, 12″·047, 6″·790

Determination of the Mass of the whole Mountain-region of the Enclosed Space.

23. The calculation on this subject needs also revision. The direct way of determining the volume of this mass is to find its average height and multiply it by the area of the Enclosed Space. First, then, I will find this area. By examining the diagram in par. 6 (fig. 1), it will be seen that by re-arranging some of the portions furthest from A the area of the Enclosed Space is equivalent to the part of a lune about 132° wide and stretching from A to the end of the 35th compartment, the angular distance of which is 21° 24′. Hence, by a known trigonometrical formula,—

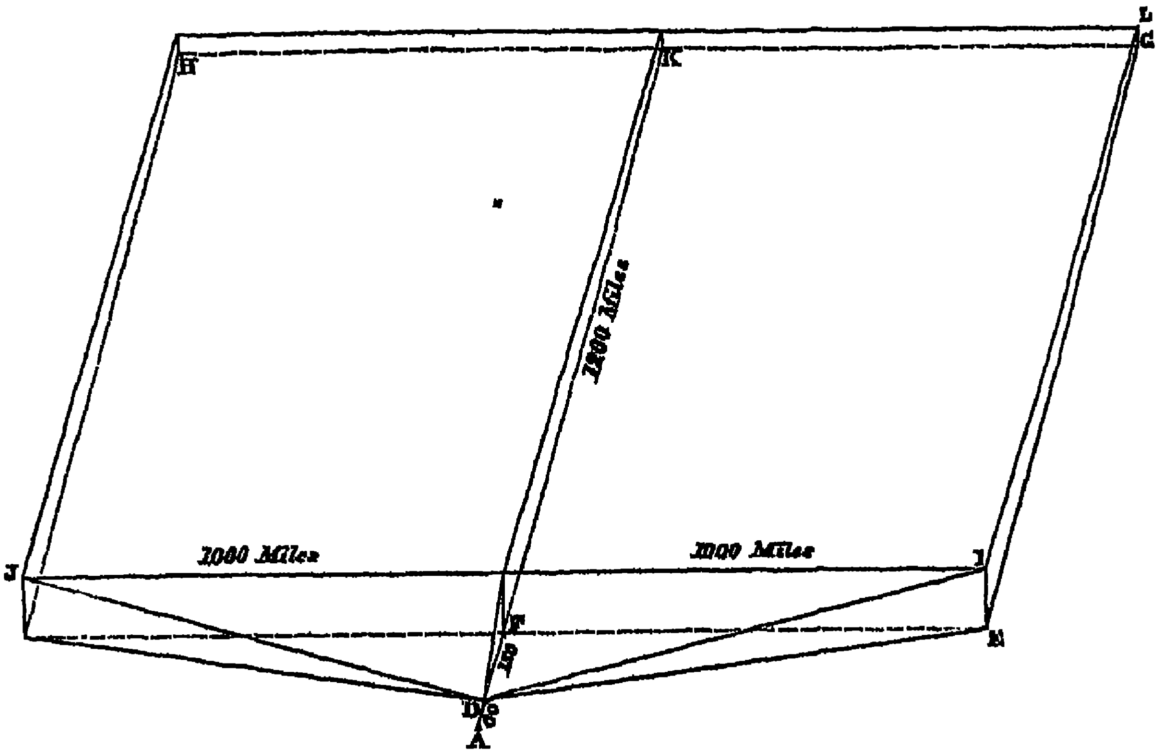
$$\text{Area of Enclosed Space} = \text{area of this portion of the lune}$$
$$=\frac{132}{180}\pi(1-\cos 21^\circ\, 24')r^2=2,559,162 \text{ square miles.}$$

24. We must now find the average height of the mass standing on this space. This is not at first sight an easy matter. We cannot obtain it by taking the average of the heights given in the Table of Heights in page 750, because the bases on which these heights stand are of such unequal extents, that an undue advantage would be given to those which stand upon the smaller bases. In my former Paper I overcame the difficulty by finding how much I must cut down the mass in order to reduce the attraction to zero; this quantity I considered to be the average height. It is obvious, that, as a general rule, this would not be true. It so happens, however, that for a mass shaped as the mass under consideration is, it is true. This I did not show in my former Paper, although I made use of the property. I propose now to supply the deficiency. The method I shall pursue is this. I take a geometrical figure which sufficiently well represents the actual mass in general form, but one of which the attraction upon A can be accurately calculated. I then show, in the case of this figure, that the average height obtained in the direct way by geometry, and also by the method of attractions, is the same. I infer, therefore, that it is so in the case of the actual mountain mass. The reason of this coincidence it is not difficult to see. The highest part of the mass is much nearer to A than the middle of the mass is. Suppose the highest parts had been about the centre; then in levelling these down so as to form the table-land which would have the average height, equal portions would have to be brought to equal distances from the middle towards A and opposite to A. The latter transfer would add more to the attraction than the other would detract from it; and therefore the average volume would not be the average mass measured by the attraction. But as the higher parts of our mass are much nearer A than the centre, it is obvious that an exact compensation is possible. The calculation shows that it is real.

25. The accompanying diagram (fig. 5) represents the geometrical figure I have alluded to. It is drawn upon a scale, except that the vertical heights are exaggerated sixty times. A is the station Kaliana, D the point north-north-east where the mass begins, and thence

shelves up in a pyramidal form to the ridge IJ, 2000 miles long and 210 miles from A, and rising to an elevation above A of 1·5 mile; whence it shelves down again to GH, 1200 miles further off, where it abruptly terminates at a height of half a mile above the level of A. That this is a fair representative of the mass with reference to its effect on A, will be seen in the course of the investigation in this,—that its area, its volume, and its attraction on A, as well as its principal heights, are all the same as in the case in nature. The following are the measures: AF or $a=210$ miles; FE or $c=1000$ miles; FK or $e=1200$ miles; DF or $b=150$ miles; EI or $h=1\cdot5$ mile; GL or $h'=0\cdot5$ mile.

Fig. 5.



The area of the base of this figure $=2000(1200+75)=2,550,000$ square miles. The volume $=\frac{1}{2}(h+h')2000\times1200+\frac{1}{3}h\times2000\times150=2,550,000$ cubic miles. The average height of the whole figure $=\frac{\frac{1}{2}(h+h')2000\times1200+\frac{2}{3}h\times2000\times75}{2000\times1200+2000\times75}=1$ mile.

It is not difficult to prove by the Integral Calculus, that the Attraction of this mass on A in the direction AD, its density being half the mean density of the earth,

$$=\frac{g}{16760}\left\{\left\{h+\frac{a}{e}(h+h')\right\}\log_e\left[\frac{\sqrt{(a+e)^2+c^2}-c}{\sqrt{a^2+c^2}-c}\,\frac{a}{a+e}\right]-\frac{c}{e}(h-h')\log_e\left[\frac{\sqrt{(a+e)^2+c^2}+a+e}{\sqrt{a^2+c^2}+a}\right]\right.$$

$$\left.+h\frac{a-b}{c}\,\frac{\sqrt{1+A^2}-B}{B^3}-h\frac{a-b}{b}\,\frac{2B^2-1}{B^3}\log_e\left[\frac{A+\sqrt{1+A^2}}{\frac{b}{c}+B}\right]+h\frac{a-b}{b}\log_e\left[\frac{\sqrt{1+A^2}+\frac{c}{a-b}B}{\frac{a}{a-b}B}\right]\right\}$$

where $A=\frac{ab+c^2}{c(a-b)}$, $B^2=1+\frac{b^2}{c^2}$.

Also the attraction of a tabular mass on the same base and of height k

$$=\frac{g.k}{16760}\left\{\log_e\left[\frac{\sqrt{(a+e)^2+c^2}-c}{\sqrt{a^2+c^2}-c}\,\frac{a}{a+e}\right]+\frac{1}{B^3}\log_e\left[\frac{A+\sqrt{1+A^2}}{\frac{b}{c}+B}\right]-\log_e\left[\frac{\sqrt{1+A^2}+\frac{c}{a-b}B}{\frac{a}{a-b}B}\right]\right\}.$$

Putting the numerical values in these formulæ, we obtain

$$A=17\cdot175,\ \sqrt{1+A^2}=17\cdot204,\ B=1\cdot011;$$

and the final results are as follows:—

$$\text{Attraction on } A=0\cdot0001557\,g=\tan(32'')\,g.$$

Hence the Deflection of the plumb-line$=32''$, or, in the direction of the meridian, $=32''\cos 30^\circ=27''\cdot7$, 30° being about the azimuth of the line along which the whole attraction on A acts (see my former Paper). This deflection very nearly coincides with the deflection calculated in this Paper, which gives a further testimony that the actual mass, in reference to its effect on A, is fairly represented by our geometrical figure.

The last formula gives, Attraction of tabular mass on the same base $=0\cdot0001563\,g\,.\,k$. That this may be the same as the attraction of the geometrical figure, we must have

$$0\cdot0001563\,k=0\cdot0001557,\ \text{or}\ k=0\cdot996\ \text{mile}.$$

But I have already shown that the average height of the geometrical figure is one mile, and is therefore almost exactly the same as this deduced from attraction. Hence the principle of determining the average height in this manner, in the case of a mass formed like the Himmalayas and the Mountain region beyond, is correct; and I shall proceed to use it to determine the average height more precisely.

26. If all the numbers in the five columns which appertain to Station A in the Table of Heights (page 750) were replaced by 1000, and the process were gone through by which the first column (that for A) in the Table of page 758 is formed, we should have the meridian deflection caused by a tabular mass standing on the enclosed space and 1000 feet in height above the level of A. This process leads to the following result. As the five numbers in each line of the Table from left to right are to be multiplied by the same five numbers, viz. by 0·1852, 0·2588, 0·2241, 0·1294, 0·0151, we may add up the columns first and then afterwards multiply the aggregates by these constants.

The aggregates are	27,000	32,000	36,000	40,000	32,000 feet.
Multiply these by	0·1852	0·2588	0·2241	0·1294	0·0151
The products are	5000·4	8281·6	8067·6	5176·0	483·2 feet.
The same reduced to miles .	0·947	1·568	1·517	0·980	0·092 mile.

The grand total $=5\cdot104$ miles, and multiplied by $1''\cdot1392=5''\cdot814$.

This is the meridian deflection at A caused by a mass 1000 feet high.

Hence the average height of the whole mass

$$=\frac{27''\cdot978\times1000}{5''\cdot814}=4812\ \text{feet}=0\cdot911\ \text{mile}.$$

And the volume of the whole mass $=$area of Enclosed Space $\times$ average height

$$=2{,}550{,}000\times0\cdot911=2{,}323{,}050\ \text{cubic miles}.$$

Also the mass, taking the density ρ equal to half the mean density of the earth,

$$=\frac{3}{8\pi}\,\frac{2{,}323{,}050}{64{,}000{,}000{,}000}\times\text{Mass of the Earth}=\frac{1}{230{,}895}\text{th of Mass of the Earth}.$$

27. As I have not again calculated in this Paper the *total* deflections of the plumb-line at the three stations A, B, C, but only the *meridian* deflections, I cannot revise the positions of the three points where the whole mass must be collected that it may produce the same effects as in nature. This is of no importance. The principle devised for interpolating the deflection at any intermediate station between A and C, by means of the property of a curve, still holds good: and the amount of meridian deflection may be represented nearly by the expression $\frac{111''\cdot 7}{l-L+4}$, l and L being the latitudes of Kaliana and of the intermediate station on the arc between its extremities where the deflection is sought. It makes the deflection at the southern extremity about one-fifteenth too large; but at the northern and middle principal stations it gives it correctly*.

28. In the last page of my former Paper I compare the curvature of the Indian Arc under several hypotheses by means of the formula,

Height of the middle point of an arc of which the amplitude is λ, above the chord of the arc,

$$=\frac{1}{8}a.\lambda^2\left\{1-\epsilon\left(\frac{1}{2}+\frac{3}{2}\cos 2\mu\right)\right\},$$

μ being the latitude of the middle point, a the radius of the earth, and λ sufficiently small to allow λ^4 to be neglected. This formula is correct. But I should not have left it in terms of λ, the amplitude, but of s, the length of the arc; since λ is not the same, whereas s is, in the three cases to which the formula is applied. This change will make the height above the chord

$$=\frac{s^2}{8a}\left\{1+\epsilon\left(\frac{1}{2}+\frac{3}{2}\cos 2\mu\right)\right\}=20(1+1\cdot 512\epsilon) \text{ miles},$$

which is the same as before, except in the sign of ϵ. The consequence of this is, that the arc is *flatter* when attraction is taken account of, and is *more curved* when it is neglected, than the mean curvature.

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* The law of the inverse chord will naturally deviate from the truth, and give too large a value, as we recede from the Himmalayas, for the following reason. The Himmalayan Mass has been shown to produce the same effect as a comparatively slender uniform prism of great length running nearly east and west. Now the attraction of such a prism on a point opposite to its middle, equals its mass divided by the product of the point's distances from the middle and from either extremity of the prism. Hence when the distance from the middle, compared with the prism's length, is small, the attraction will vary most nearly as the inverse distance; but as the distance increases, the law evidently tends towards that of the inverse *square*, which it ultimately attains when the distance is very great compared with the length of the prism. This sufficiently accounts for the actual deflection at Damargida being somewhat smaller than that given by the formula, and tends therefore to confirm the general calculation.

Calcutta, September 1, 1858.

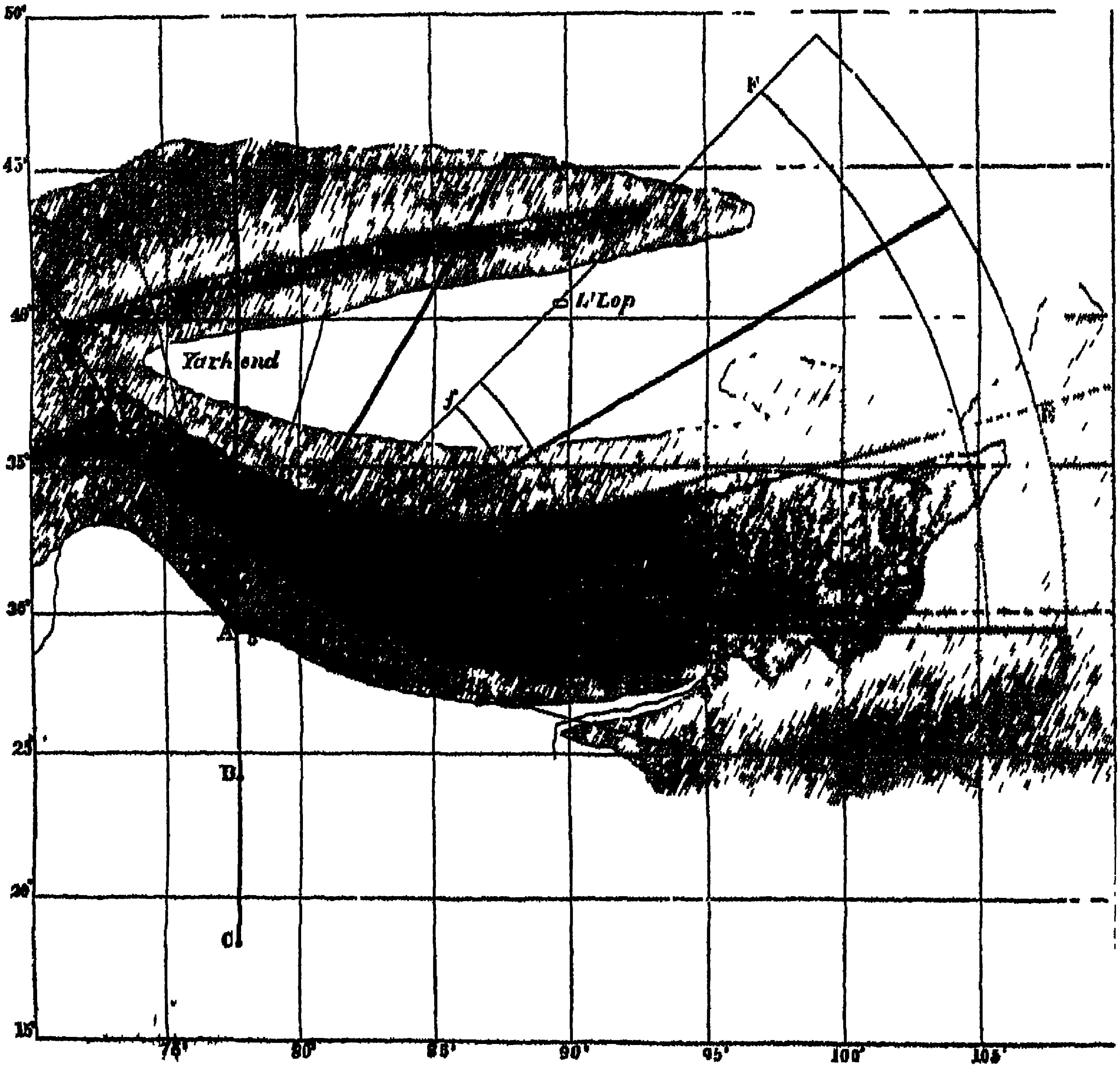
Received February 21, 1859.

Postscript on Himmalayan Attraction.

1. Since transmitting in September last my Paper "On the Deflection of the Plumb-line in India caused by the Attraction of the Himmalaya Mountains," &c., I have had the advantage of seeing the pages of a very interesting and valuable work by Major R. Strachey, now passing through the press, upon the Physical Geography of the Himmalayas; and being allowed to make use of them, I gladly avail myself of the important information they contain to add a Postscript to my former communications.

2. The new information now obtained gives a view of the regions which lie beyond the Himmalayan crest, differing in some respects from that which I gathered from the sources of scanty data which I have hitherto been able to consult. The results of Major Strachey's investigations, combined with those of his brother Captain H. Strachey, are represented in the following diagram. A, B, and C (as in my former Papers) are the

Fig. 6.



three principal stations of the northern portion of the Indian Arc of Meridian. Kalian

(29° 30′ 48″ lat.), Kalianpur (24° 7′ 11″), and Damargida (18° 3′ 15″). The parts of the diagram left white in the neighbourhood* of the range are the plains; the lightly shaded parts are the mountain slopes, with very varied surface, by degrees attaining the height of the celebrated plateau from which the Indus and the Brahmaputra take their rise and flow towards the Ocean on different sides of India; the darkly shaded parts are this plateau or table-land.

"The loftiest points known on the surface of the earth are to be found along the southern border of the Table-land among the mountains of the Himmalayan Slope; one of them having been measured the height of which exceeds 28,000 feet, while peaks of 20,000 feet abound along the entire chain. The plains of India which skirt the foot of its southern face for an extent of 1500 miles, nowhere have an elevation exceeding 1200 feet above the sea, the average being much less; and we have every reason to suppose that the northern plateau of Yarkend and Khotan, like the country around Bukhara, lies at a very small elevation, probably not more than 1000 or 2000 feet above the sea, while the surface, as we know, descends on the borders of the Caspian to 80 feet below that level" (pp. 4, 5). Major STRACHEY thinks that "none of the numerous ranges commonly marked on our maps of Tibet have any special definite existence as mountain chains, apart from the general mass of the Table-land, and that this country should not be considered to lie as if in the interval between the two so-called chains of the Himmalaya and Kouenlun, but that it is in reality the summit of a great protuberance above the general level of the earth's surface, of which the supposed Kouenlun and Himmalaya are nothing more than the north and south faces, while the other ranges are but corrugations of the table-land more or less strongly marked" (pp. 5, 6). The two rivers, Indus and Brahmaputra, "maintain a course along the length of the summit of the Table-land, and receive, as they proceed, the drainage of its entire breadth; with the exception, first, of an occasional strip along its southern edge, from which the water passes off more or less directly to the north through the Himmalaya; and, secondly, of some parts chiefly found in the northern half of the Table-land, from which the water has no escape, but is collected in lakes in depressions on its very summit. The waters thus accumulated in these two streams are at length discharged by two openings in the Himmalaya Slope through the plains of Hindostan into the Indian Ocean. None of the drainage of the table-land, so far as we know, passes in the opposite direction through the northern slope; and the area that discharges itself southward at points intermediate between the debouche of the Indus and Brahmaputra is, with one exception, that of the Sutlej, comparatively insignificant. The waters of the northern slope itself exclusively flow down to the plains of Yarkend; while in like manner those of the southern slope, with the drainage of the exceptional area along the southern border of the table-land running off to the south, traverse the Himmalaya more or less directly, and constitute such rivers as the Jumna, Ganges Proper, &c., and other main tributaries of the Indus, Ganges, and Brahmaputra" (pp. 32, 33).

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* Down towards B and C the country becomes hilly; but not sufficiently so to affect my results.

Major STRACHEY, after his brother, calls the northern boundary of the table-land the Turkish Watershed, and the southern the Himmalayan Watershed (p. 33).

"The average elevation of the crest of the Indian watershed most probably exceeds 18,000 feet. In a comparatively few points only its continuity is broken, and it allows the passage of rivers that rise on its northern flank; but at all other points its summit must be crossed in entering Tibet from the south. The passes over it are frequently more than 18,000 feet above the sea; and, except where it is broken through, I know of no point to the east of Kashmir where it can be surmounted under 16,400 feet" (p. 51).

"The summit of the table-land, though deeply corrugated with valleys and mountains in detail, is in its general relief laid out horizontally, at a height little inferior to that of its southern scarp" (p. 52).

The plain along the upper course of the Sutlej "lies immediately to the north of the British provinces of Kumaon and Gurhwal, and is about 120 miles in length, its breadth varying from 15 to 60 miles. Its surface, to the eye a perfect flat, varies in elevation from 16,000 feet along its outer edges on the south-west and north-east, to about 15,000 feet in its more central parts, where it is cut through by the river Sutlej which flows at the bottom of a stupendous ravine, furrowed out of the alluvial matter of which the plain is composed to a depth of 2000 or 3000 feet, and at its west end even more" (p. 53). This will account for the statement on the Survey Map (as noticed at p. 75 of my Paper of 1855), at the point where the Sutlej leaves the table-land, that the height of its bed is only 10,792 feet. This I have taken in my former calculation as the *greatest* height of any of the compartments into which I divide the surface; this, therefore, the researches of Major STRACHEY show to be much under the mark.

On a careful consideration of all the data, Captain H. STRACHEY estimates the mean elevation of the table-land between the Himmalayan and Turkish watersheds, and to the west of the ridge between the sources of the Indus and Brahmaputra, to be 15,000 feet (p. 56).

3. A comparison of Major STRACHEY'S map, copied in part in the diagram above, with the diagram I have given in former Papers of the "Enclosed Space*," will show that much attracting matter which, from HUMBOLDT'S account, I supposed to exist, in Major STRACHEY'S description does not appear, at any rate not in so important a degree. Lest, therefore, it should be thought that, my data being in some respects wrong, my results are altogether vitiated, I have examined the effect of these new measures, and I find that the increased height given to the plateau compensates for the removal of any attracting mass higher north which I had supposed to exist.

4. I do not intend to enter anew into a lengthened calculation of the deflection of the plumb-line, with a view, as before, to obtain an exact result, because my object will be equally answered by taking a simpler course. My object in these various communications has been, first, to give an easy method of determining the amount of attraction

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* Philosophical Transactions, 1855, p. 76.

and deflection, when the heights are known—this has been done in the Paper of 1855; secondly, to point out, from such trustworthy data as I could procure, that the amount of deflection is so great as to render it absolutely necessary to allow for it in finding the astronomical amplitude for geodetic operations; and, lastly, to suggest that such surveys and calculations should be made as to make it possible to determine the amount with a sufficient degree of precision.

The only calculation I propose now to make, is to show that the deflection caused by this Table-land *alone*, as laid down by Major STRACHEY, produces an error double the error brought to light by the Survey, in which mountain attraction is neglected and the ellipticity of the Indian arc is assumed to be the mean ellipticity of the whole earth. Much greater, then, will be the discrepancy, as I might easily show were it worth while, when the attraction of all the slopes—especially the parts nearest to A—is added.

5. Through A I have drawn a straight line AD, and have marked off certain divisions which indicate the Law of Dissection according to which the attracting mass is to be divided. From A several fainter lines are drawn dividing the attracting mass into lunes of 30° width: the dark lines bisect these lunes, one of them being in the meridian of A. Now if through the points of division of AD circles be drawn about A, they and the lunes will divide the surface into a number of four-sided compartments, like EF and *ef*: and the law of this dissection is so chosen, that if the height of the attracting matter on EF and *ef* were the same, the attraction of these two partial masses on A along the dark mid-line of the lune would be the same: this may also be expressed by saying, that the attraction of the mass on any compartment thus formed is proportional to the height of that mass. In a former Paper the following formula has been proved:—

Meridian deflection of plumb-line at A, caused by the attraction of a mass of height h miles standing on any compartment,

$$h \sin 15° \times 1''{\cdot}139 \cos \text{azimuth of mid-line of the lune.}$$

6. I propose now to apply this formula to find the deflections caused at the three stations by the attraction of the Table-land alone. The height of the Table-land above Kaliana I will take to be $2\frac{2}{3}$ miles; that is, about 14,000 feet, or 15,000 feet above the sea. Then the above coefficient becomes $2\frac{2}{3} \times {\cdot}2588 \times 1''{\cdot}139 = 0''{\cdot}786$; and

$$\text{Merid. Deflection} = 0''{\cdot}786 \cos \text{azimuth.}$$

The calculation is rendered easy by the heights of all the compartments being the same; and the only difficulty is in finding how many compartments in each lune lie on the Table-land. This is done in the following manner:—A strip of paper is laid on the diagram along AD, and the divisions of AD marked upon it. This scale is then laid along the mid-line of each lune, and the number of divisions (and therefore the number of the compartments) which fall on the Table-land in that lune easily read off. In some instances the Table-land only partially fills a compartment; in that case compensation is made by diminishing the number of the compartments, in the ratio of the deficiency to the whole space of the compartment. This is indicated in the following Table by an *asterisk.* Lune V. is omitted in station A because it can have but very little effect in the meridian at A.

	Station A.				Station B.			Station C.	
	Lunes I.	II.	III.	IV.	Lunes II.	III.	IV.	Lunes II.	III.
No. of compartments on Table-land...	8* *or* 4	8	9	12	5	5	10* *or* 7·5	2	5* *or* 4
Cos azimuth	·866	1	·866	·5	1	·866	·5	1	·866
No. of equivalent compartments if placed in the meridian	3·464	8	7·794	6	5	4·33	3·75	2	3·464
Sums of these............................	25·258				13·08			5·464	
Multiply by 0″·786 and the Deflections are	19″·85				10″·28			4″·29	

Hence the errors produced in the astronomical amplitudes will be 9″·57 and 5″·99; which much exceed the errors 5″·236 and −3″·791 brought to light by the Survey, and will far more so when the attraction of the nearer parts is also taken into account.

7. The new information regarding the nature of the country immediately north of the Himmalayas, does not, it thus appears, relieve this subject of its difficulties: and no geodetic calculations can be of service in the problem of the Figure of the Earth, nor indeed in mapping the country with extreme precision, till these perplexities are removed, by the deflection being found and allowed for.

Calcutta, January 14, 1859.

XXX. *On the Influence of the Ocean on the Plumb-line in India.* *By the Venerable* J. H. PRATT, *M.A.*, *Archdeacon of Calcutta.* *Communicated by Professor* STOKES, *Sec. R.S.*

Received December 7, 1858,—Read January 6, 1859.

§ 1. *Preliminary remarks.*

1. THE present paper is a sequel to two former communications to the Society on the effect of Mountain Attraction on the Plumb-line in India. In the first of these (communicated in 1855) the deflection of the plumb-line caused by the Mountain Mass north of Hindostan is calculated; and in the second (forwarded during the present year) the effect of a small excess or defect of density prevailing through extensive tracts of the earth's mass is found; with a view to determine whether any compensating cause can possibly exist below, to counteract the large amount of deflection caused by the superficial mass lying above the sea-level. A survey of the causes of disturbance of the plumb-line cannot be complete without taking into consideration the influence of the ocean. To approximate to this is the object of the present Paper.

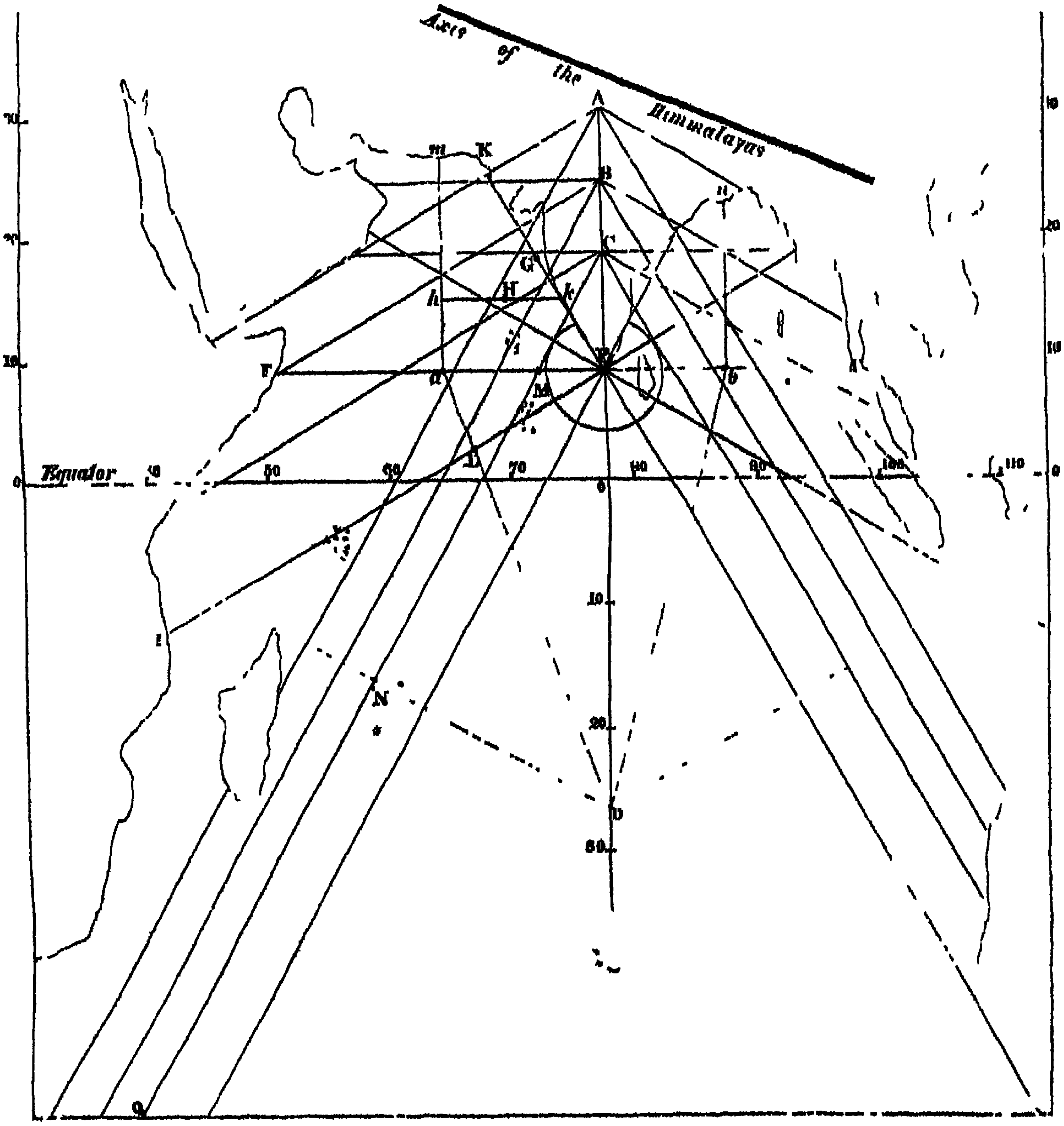
2. The geographical position of Hindostan is somewhat remarkable. The highest mountain-ground in the world lies to the north of it; and an unbroken expanse of ocean extends from its shores down to the neighbourhood of the south pole. The excess of matter presented by the first causes a deflection of the plumb-line towards the north, decreasing in amount as we travel southwards. The deficiency of matter arising from the second causes a deflection of the plumb-line also towards the north, but decreasing in amount as we travel northwards. The consequence is, that while these two causes conspire to increase the deflection at the different stations, their combined action tends to reduce in amount the errors the mountain attraction causes in the amplitudes. But the attraction of the mountains northwards, and the deficiency of attraction of the ocean southwards, which last is in fact equivalent to a repulsive force northwards, combine to produce another effect upon the measures of the Survey besides the deflection of the plumb-line, an effect of some importance. They have a sensible influence in changing the sea-level, so as to make the level of Karachi—to which a great longitudinal chain of triangles is brought down from Kalianpur in the centre of India—many feet higher than the level at Punnœ, near Cape Comorin, the south extremity of the Great Arc of meridian.

3. If the density of the sea were the same as that of the superficial stratum of rock, there would be no disturbing force. The deficiency of matter arising from the smaller density of the ocean, produces, therefore, a northerly repulsive force exactly equal to the attraction of a mass of the same form in superficial extent and depth, but having a density equal to the excess of the density of rock over that of sea-water.

§ 2. *Hypothesis regarding the form of the bottom of the sea.*

4. To calculate this force, it is necessary, in our ignorance of the form of the bottom of the sea, to resort to hypothesis. In doing so it should be remembered, that as we want to find the aggregate effect at certain places comparatively near each other, being all in India, and not to find the law in passing from one point to another over the whole surface of the ocean, an *average* representation of its form will be quite as good for our purpose as an accurate knowledge of all its variations of depth.

Fig 1



The hypothesis I assume I will now describe. Let the preceding diagram (fig. 1) represent an outline of the ocean and bounding countries under consideration. Let A, B, C (as in my former Papers) be Kaliana, Kalianpur, and Damagida, the three terminal stations of the two higher portions of the Great Arc: P is Punnœ, the station close to Cape Comorin. Draw EPF east and west, meeting Africa in E and the Malay Peninsula in F. I call the depths at the middle points, viz. at a and b, of these lines at the opening of the Arabian Sea and Bay of Bengal into the Indian Ocean, a and b: and I assume that the bottom slopes up gradually from these points in lines due north to m and n, near Karachi and the Sandheads of Calcutta; and that the bottom shelves down to these lines from every part of the shore. Thus to find the depth at any point H, we must draw hHk at right angles to am; first find the depth at h in terms of a, from the measured ratio of mh to ma; then find the depth at H in terms of that at h, and so in terms of a, from the measured ratio of Hk to hk. In this way the depth at all points in the Arabian Sea and the Bay of Bengal may be found in terms of a and b. I next assume that at a point D, 36° south of Cape Comorin and nearly midway between Madagascar and Australia, the depth is D; and that the bottom slopes down gradually to D from all sides—from the bottom at FP and PE, from the shores on the east and west, and from the neighbourhood of the south pole. It will soon be seen that any uncertainty regarding the depth down towards the south pole is comparatively unimportant. From this description it will be seen that the depth along any line PI may be found as follows:—The bottom slopes down to L from both shores: the depth at L is known in terms of a and D from the ratio of aL to LD. Again, the depths along a line CMNQ are found thus. The depth at M is found in terms of a from the slope of Pa. From M to N the bottom slopes to a depth which is found in terms of D by the relative distance of N from Madagascar, the nearest shore, and D. By this contrivance the depths of all parts of the ocean under examination can be found in terms of one or more of the arbitrary depths a, b, D: and by rightly choosing these quantities I think we shall have a very good average representation of the volume and general form of the ocean, as far as its effect on the stations in India require a knowledge of it. The effect of the ocean to the right of the eastern peninsula and archipelago I have omitted, as it would be about the same for all the stations.

§ 3. *Calculation of the attractions and deflections.*

5. The method of calculating the attraction is precisely the same as that adopted in my Paper of 1855 for the Mountains. Thus for the station P. Through P imagine great circles drawn (not given in the diagram to prevent confusion) at 30° apart. These will divide the surface into lunes reaching to the antipodes. The lines drawn in the diagram are the middle or dividing lines of these lunes, along which the attraction acts. Around P describe a circle with radius $=5°$; and beyond this other circles with the radii noted in the Table in page 782. The property of this law of dissection is, that the attractions of the masses of the several compartments into which this process divides the surface, are proportional to their average depths; or if $d_1\, d_2\, d_3 \ldots$ be the depths, the attractions of the masses on P are as $d_1\, d_2\, d_3 \ldots$. The attraction of each mass* $=\frac{4}{21}\varrho \sin 15° \times$ depth. Now the density of sea water $=1{\cdot}028 \times$ dens. of distilled water.

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* See p. 64 of Paper of 1855.

	Radii of circles		Radii of circles
1	5°·0	16	21·4
2	5·5	17	23·6
3	6·1	18	26·1
4	6·7	19	28·8
5	7·4	20	31·8
6	8·1	21	35·3
7	8·9	22	38·9
8	9·8	23	43·1
9	10·8	24	48·1
10	11·9	25	53·9
11	13·1	26	60·7
12	14·4	27	68·9
13	16·0	28	79·3
14	17·6	29	93·3
15	19·4	30	116·7
		The next is beyond the antipodes.	

Also mean density of the earth=5·66 times that of distilled water. Also mean density of the earth $=\frac{3}{4\pi}\frac{g}{r}$, g=gravity, r=4000 miles. The density of Schehallien, which is a fair representative of the density of rock at the surface, =2·75 × dens. distilled water;

$$\therefore \rho = \text{excess of density of rock above that of sea-water}$$
$$= \frac{1\cdot722}{5\cdot66} \text{ mean dens. of earth} = \frac{3}{4\pi}\,\frac{1722}{5660}\,\frac{g}{r};$$

also sin 15°=0·2588,

$$\therefore \text{ attraction of mass in any compartment} = 0\cdot000000079 \times \text{depth} \times g$$
$$= \tan(0''\cdot185)\ \text{depth} \times g,$$
$$\therefore \text{ deflection caused by this} = 0''\cdot185 \text{ depth},$$

as is easily seen by the resolution of forces.

Hence the deflection caused by the whole of any one lune is found by adding together all the depths of the compartments, and multiplying the sum by the cosine and sine of the azimuth, reckoned from the south to the left, to find the North and East force produced by the deficiency of matter in the ocean. These added together for all the lunes will give the total effect of the portions of the surface beyond the first circle.

6. In parts where the compartments are broken into by the shores, or the lunes must be narrowed in consequence of the land, I introduce the correction by reducing the depths in proportion, and leaving the constant multiplier 0″·185 unchanged. Thus too in the third lune to the west, the middle line lies along the shore, and therefore half the lune is on land, and the depth along the middle line, being zero all along, does not represent the average depth of the compartments. I therefore take the depths along the middle line of the half lune, that is, along a line making an angle 7° 30′ with the shore; and besides this I take half only of the result, as the width of the lune is diminished in that ratio. Again, the most important lune, that which has its middle line due south, needs consideration. The middle line lies at the greatest depth in all the compartments. The depths of the middle of the compartments will therefore be all larger than the average depth of the compartments. The average depth of the compartment in the

latitude of D, it is easily seen, is about $\frac{5}{6}$ D. This, therefore, I shall adopt as the depth at D for all lunes which have their middle line in APD.

7. I have not yet considered the portion within the first circle. It is easy to show by the Integral Calculus, that the attraction of a small horizontal pyramid, of depth d at its vertical base, $=\varrho\delta\theta.d$, irrespectively of its length, $\delta\theta$ being the width. Hence, by integration, $2\varrho\sin 15^\circ.d$ is the attraction for the whole width of the lune. And the consequent deflection caused by this portion of the ocean $=\frac{21}{2}\cdot\frac{4}{21}\varrho\sin 15^\circ.d=0''\cdot185\times\frac{21}{2}d$. Hence the deflection caused by the whole lune

$$=0''\cdot185\left(\frac{21}{2}d+d_1+d_2+d_3+\cdots\cdots\right).$$

8. We are now prepared to enter into Tables the deflections caused by the several lunes. The process I follow is partly graphic. I take a strip of paper and mark it, to be used as a scale. After the zero-point the first mark is at 5° distance, this being the radius of the first circle about P bounding the compartments. The distances of the remaining circles, as noted in the Table in page 782, are next marked on the scale. It will be observed that the distances of the circles from each other are at first very small, but they afterward rapidly increase, and the 29th compartment terminates at 116° from P, and the next stretches beyond the antipodes. The rapid increase of the dimensions of the compartments in the southern ocean renders it unimportant to know the depths of those regions with any degree of accuracy, as I have already noticed.

The scale thus formed is laid upon the diagram (fig. 1), along the middle lines of the several lunes, the zero-point being at P. The values of $d\ d_1\ d_2\ d_3\ldots$ are then, without much difficulty, determined from the diagram for P, and similarly for other stations, as noted down in the six Tables which follow. The deflections are resolved north and east.

Of the six stations I have chosen (for reasons which will appear), one has no name, as there is no known place there. I therefore call it "Near-Goa." It is half-way between Punnœ and Karachi.

TABLE I.—Deflections at Kaliana.

Depths	Central Lune.	First Lune.		Second Lune.	
		East.	West.	East.	West.
d	0	0	0	0	0
$2\frac{1}{3}d$	0	0	0	0	0
d_1	....	....	....	....	....
d_2	....	....	....	....	....
d_3	....	....	....	....	....
d_4	....	....	....	....	....
d_5	....	....	....	....	....
d_6	....	....	....	....	....
d_7	....	....	....	....	....
d_8	....	....	Nine depths from 0 to a (=4·5 a)	....	....
d_9	....	....		3 from 0 to $\frac{1}{27}b$ (=·05 b)	3 from 0 to $\frac{1}{6}a$ (=·3 a)
d_{10}	....	Five from 0 to $\frac{2}{3}b$ (=1·67 b)			
d_{11}	....				
d_{12}	....			Six depths from $\frac{1}{27}b$ to 0 (=·11 b)	Six from $\frac{1}{6}a$ to 0 (=·6a)
d_{13}	....				
d_{14}	....				
d_{15}	....	Nine depths from $\frac{2}{3}b$ to $\frac{1}{5}$D (=3 b+·9 D)			
d_{16}	Nine depths from 0 to $\frac{5}{8}$D (=3·7 D)				
d_{17}			Seven depths from a to $\frac{1}{10}$D (=·35 D)		
d_{18}				....	....
d_{19}				....	....
d_{20}				....	....
d_{21}				....	....
d_{22}				....	....
d_{23}				....	....
d_{24}		Seven depths from $\frac{1}{5}$D to 0 (=·7 D)	Three from $\frac{1}{10}$D to $\frac{1}{5}$D (=·45 D)	....	....
d_{25}	Six depths from $\frac{5}{8}$D to 0 (=2·5 D)			....	....
d_{26}				....	....
d_{27}			Four from $\frac{1}{5}$D to 0 (=·4 D)	....	....
d_{28}				....	....
d_{29}				....	....
d_{30}				....	....
Totals ... Deflectns	6·2 D 1″·15 D	1·6 D+4·67 b 0″·30 D+0·86 b	1·2 D+4·5 a 0″·22 D+0·83 a	·16 b 0″·03 b	·9 a 0″·17 a
Sums ... Cos azim. Def. *N.* ...	1″·15 D 1″·15 D	0″·52 D+0″·86 b+0″·83 a ·866 0″·45 D+0″·74 b+0″·72 a		0″·03 b+0″·17 a ·500 0″·02 b+0″·08 a	
Diff. ...... Sin azim. Def. *E.* ...	0 0 0	—0″·08 D—0″·86 b+0″·83 a ·500 —0″·04 D—0″·43 b+0″·41 a		—0″·03 b+0″·17 a ·866 —0″·03 b+0″·15 a	

Total Deflection at Kaliana, *North*, 1″·60 D+0″·76 b+0″·80 a.

Total Deflection at Kaliana, *East*, —0″·04 D—0″·46 b+0″·56 a.

TABLE II. Deflections at Kalianpur.

<table>
<tr><th rowspan="2">Depths.</th><th rowspan="2">Central Lune.</th><th colspan="2">First Lune.</th><th colspan="2">Second Lune.</th><th colspan="2">Third Lune.</th></tr>
<tr><th>East.</th><th>West.</th><th>East</th><th>West.</th><th>East.</th><th>West.</th></tr>
<tr><td>d</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>$2\frac{1}{2}d$</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>d_1</td><td>......</td><td>......</td><td>......</td><td>......</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_2</td><td>......</td><td>......</td><td>......</td><td>......</td><td rowspan="10">Ten depths from 0 to $\frac{10}{17}a$ (=2·94a)</td><td>......</td><td>......</td></tr>
<tr><td>d_3</td><td>......</td><td>......</td><td>......</td><td rowspan="8">Eight depths from 0 to $\frac{8}{27}b$ (=1·19b)</td><td>......</td><td>......</td></tr>
<tr><td>d_4</td><td>......</td><td>......</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_5</td><td>......</td><td>......</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_6</td><td>......</td><td rowspan="8">Eight depths from 0 to $\frac{6}{7}b$ (=3·43 b)</td><td rowspan="8">Eight depths from 0 $\frac{17}{27}a$ (=2·52 a)</td><td>......</td><td>......</td></tr>
<tr><td>d_7</td><td>......</td><td>......</td><td rowspan="4">4 from 0 to $\frac{3}{17}a$ (=·23 a)</td></tr>
<tr><td>d_8</td><td>......</td><td>......</td></tr>
<tr><td>d_9</td><td>......</td><td>......</td></tr>
<tr><td>d_{10}</td><td>......</td><td>......</td></tr>
<tr><td>d_{11}</td><td>......</td><td rowspan="6">Six depths from $\frac{8}{27}b$ to 0 (=·89 b)</td><td>......</td><td rowspan="4">4 from $\frac{3}{17}a$ to 0 (=·23 a)</td></tr>
<tr><td>d_{12}</td><td rowspan="12">Twelve depths from 0 to $\frac{5}{6}$ D (=5 D)</td><td rowspan="9">Nine depths from $\frac{10}{17}a$ to 0 (=2·65a)</td><td>......</td></tr>
<tr><td>d_{13}</td><td>......</td></tr>
<tr><td>d_{14}</td><td rowspan="9">Nine depths from $\frac{6}{7}b$ to $\frac{4}{7}$ D (=3·85 b+1·29 D)</td><td rowspan="9">Nine depths from $\frac{17}{27}a$ to $\frac{3}{16}$ D (=2·83 a+0·81 D)</td><td>......</td></tr>
<tr><td>d_{15}</td><td>......</td><td>......</td></tr>
<tr><td>d_{16}</td><td>......</td><td>......</td></tr>
<tr><td>d_{17}</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_{18}</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_{19}</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_{20}</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_{21}</td><td>......</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_{22}</td><td>......</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_{23}</td><td rowspan="8">Eight depths from $\frac{4}{7}$ D to 0 (=1·14 D)</td><td rowspan="8">Eight depths from $\frac{3}{16}$ D to 0 (=·75 D)</td><td>......</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_{24}</td><td rowspan="7">Seven depths from $\frac{5}{6}$ D to 0 (=2·92 D)</td><td>......</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_{25}</td><td>......</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_{26}</td><td>......</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_{27}</td><td>......</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_{28}</td><td>......</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_{29}</td><td>......</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>d_{30}</td><td>......</td><td>......</td><td>......</td><td>......</td></tr>
<tr><td>Totals......</td><td>7·92 D</td><td>2·43 D
+7·28 b</td><td>1·56 D
5·35 a</td><td>2·08 b</td><td>5·59 a</td><td>......</td><td>·46 a</td></tr>
<tr><td>Deflections</td><td>1″·47 D</td><td>0″·45 D
+1″·35 b</td><td>0″·29 D
+0″·99 a</td><td>0″·38 b</td><td>1″·03 a</td><td>0</td><td>0″·09 a</td></tr>
<tr><td>Sums ......</td><td>1″·47 D</td><td colspan="2">0″·74 D+1″·35 b+0″·99 a</td><td colspan="2">0″·38 b+1″·03 a</td><td colspan="2">0″·09 a</td></tr>
<tr><td>Cos azim.</td><td>1</td><td colspan="2">·866</td><td colspan="2">·500</td><td colspan="2">0</td></tr>
<tr><td>Def. N. ...</td><td>1″·47 D</td><td colspan="2">0″·64 D+0″·90 b+0″·87 a</td><td colspan="2">0″·19 b+0″·81 a</td><td colspan="2">0</td></tr>
<tr><td>Diff. ......</td><td>0</td><td colspan="2">−0″·16 D−1″·35 b+0″·99 a</td><td colspan="2">−0″·38 b+1″·03 a</td><td colspan="2">0″·09 a</td></tr>
<tr><td>Sin azim.</td><td>0</td><td colspan="2">·500</td><td colspan="2">·866</td><td colspan="2">1</td></tr>
<tr><td>Def. E. ...</td><td>0</td><td colspan="2">−0″·08 D−0″·67 b+0″·50 a</td><td colspan="2">−0″·33 b+0″89 a</td><td colspan="2">0″·09 a</td></tr>
</table>

Total Deflection at Kalianpur, *North*, 2″·17 D+1″·09 *b*+1″·68 *a*.

Total Deflection at Kalianpur, *East*, −0″·08 D−1″·0 *b*+1″·48 *a*.

TABLE III. Deflections at Damargida.

Depths.	Central Lune.	First Lune.		Second Lune.		Third Lune.		Fourth Lune.	
		East.	West.	East.	West.	East.	West.	East.	West.
d	0	0	0	0	0	0	0	0	0
$2\frac{1}{4}d$	0	0	0	0	0	0	0	0	0
d_1	....	......	....	Nine depths from 0 to $\frac{7}{27}b$ (=1·17 b)	Eleven depths from 0 to $\frac{15}{17}a$ (=4·85 a)	Eight depths from 0 to $\frac{8}{27}b$ (=1·11 b)	Ten depths from 0 to $\frac{3}{7}a$ (=2·14 a)	......	Eleven depths from 0 to $\frac{1}{17}a$ (=·33 a)
d_2	....	......	Seven depths from 0 to $\frac{6}{13}a$ (=1·62 a)					......	
d_3	....	......						......	
d_4	....	......						......	
d_5	....	......						......	
d_6	....	......						......	
d_7	....	Fifteen depths from 0 to $\frac{1}{3}$ D (=2·5 D)						......	
d_8	Fifteen depths from 0 to $\frac{5}{6}$ D (=6·25 D)							......	
d_9			Thirteen depths from $\frac{6}{13}a$ to $\frac{9}{32}$ D (=3 a+1·83 D)			4 from $\frac{8}{27}b$ to 0 (=0·59 b)		......	
d_{10}				Seven depths from $\frac{7}{27}b$ to 0 (=0·91 b)				......	
d_{11}							4 from $\frac{3}{7}a$ to 0 (=0·85 a)	......	
d_{12}					Ten depths from $\frac{15}{17}a$ to 0 (4·41 a)			......	3 from $\frac{1}{17}a$ to 0 (=·09 a)
d_{13}								......	
d_{14}						......		......	
d_{15}						......	......	......	......
d_{16}						......	......	......	......
d_{17}						......	......	......	......
d_{18}				......		......	......	......	......
d_{19}				......		......	......	......	......
d_{20}				......		......	......	......	......
d_{21}				......		......	......	......	......
d_{22}		Nine depths from $\frac{1}{3}$ D to 0 (=1·5 D)	Nine depths from $\frac{9}{32}$ D to 0 (=1·27 D)	......	......	......	......	......	......
d_{23}	Eight depths from $\frac{5}{6}$ D to 0 (=3·33 D)			......	......	......	......	......	......
d_{24}				......	......	......	......	......	......
d_{25}				......	......	......	......	......	......
d_{26}				......	......	......	......	......	......
d_{27}				......	......	......	......	......	......
d_{28}				......	......	......	......	......	......
d_{29}				......	......	......	......	......	......
d_{30}				......	......	......	......	......	......
Totals ...	9·58 D	4 D	3·1 D +4·62 a	2·08 b	9·26 a	1·7 b	2·99 a	0	0·42 a
Deflect^{ns}	1″·77 D	0″·74 D	0″·57 D +0·85 a	0″·38 b	1″·71 a	0″·31 b	0″·56 a	0	0″·08 a
Sums ...	1″·77 D	1″·31 D+0″·85 a		0″·38 b+1″·71 a		0″·31 b+0″·56 a		0″·08 a	
Cos azim.	1	·866		·500		0		−·500	
Def. *N*....	1″·77 D	1″·13 D+0″·74 a		0″·19 b+0″·85 a		0		−0″·04 a	
Diff.	0	−0″·17 D+0″·85 a		−0″·38 b+1″·71 a		−0″·31 b+0″·56 a		0″·08 a	
Sin azim.	0	·500		·866		1		·866	
Def. *E*....	0	−0″·08 D+0″·42 a		−0″·33 b+1″·48 a		−0″·31 b+0″·56 a		0″·06 a	

Total Deflection at Damargida, *North*, 2″·90 D+0″·19 b+1″·55 a.

Total Deflection at Damargida, *East*, −0″·08 D−0″·64 b+2″·52 a.

TABLE IV.—Deflections at Punnæ.

Depths	Central Lune.	First Lune.		Second Lune.		Third Lune.		Fourth Lune.		Fifth Lune.	
		East.	West.	East.	West.	East.	West.	East.	West.	East.	West.
d	$\frac{5}{12}$D	$\frac{1}{12}$D	$\frac{3}{32}$D	$\frac{3}{44}$D$+\frac{19}{44}b$	$\frac{4}{55}$D$+\frac{18}{55}a$	$\frac{1}{2}b$	$\frac{10}{27}a$	$\frac{3}{13}b$	$\frac{4}{21}a$	$\frac{1}{28}b$	$\frac{1}{40}a$
$\frac{3}{21}d$	1·25 D	0·87 D	0·98 D	0·72 D +4·53 b	0·76 D +3·62 a	5·25 b	3·89 a	2·43 b	2 a	0·37 b	0·26 a
d_1 d_2 d_3 d_4 d_5 d_6 d_7 d_8 d_9 d_{10} d_{11} d_{12} d_{13} d_{14} d_{15} d_{16} d_{17} d_{18} d_{19} d_{20} d_{21} d_{22} d_{23} d_{24} d_{25} d_{26} d_{27} d_{28} d_{29} d_{30}	Twenty depths from $\frac{5}{12}$D to $\frac{5}{6}$D (=9·52 D) Ten depths from $\frac{5}{6}$D to 0 (=4·17 D)	Nineteen depths from $\frac{1}{12}$D to $\frac{1}{2}$D (=5·54 D) Eleven depths from $\frac{1}{2}$D to 0 (=2·75 D)	Nineteen depths from $\frac{3}{32}$D to $\frac{7}{16}$D (=5·05 D) Eleven depths from $\frac{7}{16}$D to 0 (=2·41 D)	Seven from $\frac{3}{44}$D$+\frac{19}{44}b$ to $\frac{3}{22}$D$+\frac{19}{22}b$ (=·72 D +4·53 b) Fifteen depths from $\frac{3}{22}$D$+\frac{19}{22}b$ to 0 (=1·02 D +6·48 b) …… …… …… …… …… …… …… ……	Ten depths from $\frac{4}{55}$D$+\frac{18}{55}a$ to $\frac{2}{11}$D$+\frac{9}{11}a$ (=1·27 D +5·73 a) Twelve depths from $\frac{2}{11}$D$+\frac{9}{11}a$ to 0 (=2D+9 a) …… …… …… …… …… …… …… ……	Eight depths from $\frac{1}{2}b$ to b (=6 b) Six from b to 0 (=3 b) …… …… …… …… …… …… …… …… …… …… …… …… …… …… …… ……	Ten depths from $\frac{10}{27}a$ to a (=6·85a) Eight depths from a to 0 (=4 a) …… …… …… …… …… …… …… …… …… …… …… ……	Nine depths from $\frac{3}{13}b$ to $\frac{5}{9}b$ (=3·54b) 4 from $\frac{5}{9}b$ to 0 (=1·11b) …… …… …… …… …… …… …… …… …… …… …… …… …… …… …… …… ……	Twelve depths from $\frac{4}{21}a$ to $\frac{4}{7}a$ (=4·57a) 3 from $\frac{4}{7}a$ to 0 (=0·86a) …… …… …… …… …… …… …… …… …… …… …… …… …… …… ……	Six from $\frac{1}{28}b$ to $\frac{1}{18}b$ (=·27 b) 4 from $\frac{1}{18}b$ to 0 (=0·11b) ……	Ten depths from $\frac{1}{40}a$ to $\frac{1}{14}a$ (= ·48 a) 4 from $\frac{1}{14}a$ to 0 (=0·14a) …… …… …… …… …… …… …… …… …… …… …… …… …… …… …… ……
Totals ...	15·54 D	9·16 D	8·44 D	2·46 D +15·54 b	4·03 D +14·73 a	14·25 b	14·74 a	7·08 b	7·43 a	0·75 b	0·88 a
Deflectns	2″·81 D	1″·69 D	1″·56 D	0″·46 D +2·87 b	0″·75 D 2·73 a	2″·64 b	2″·73 a	1″·31 b	1″·37 a	0″·14 b	0″·16 a
Sums ... Cos azim. Def.*N.* ...	2″·81 D 1 2″·81 D	3″·25 D 0·866 2″·81 D		1″·21 D+2″·87b+2″·73a 0·500 0″·60 D+1″·43b+1″·36a		2″·64 b+2″·73 a 0 0		1″·31 b+1″·37 a −0·500 −0″·65 b− 0″·68 a		0″·14 b+0″·16 a −0·866 −0″·12 b−0″·13 a	
Diff. ...... Sin azim. Def.*E.* ...	0 0 0	−0″·13 D 0·500 −0″·06 D		0″·29 D−2″·87b+2″·73a 0·866 0″·25 D−2″·49b+2″·36a		−2″·64 b+2″·73 a 1 −2″·64 b+2″·73 a		−1″·31 b+1″·37 a 0·866 −1″·13b+1″·19 a		−0″·14b+0″·16 a 0·500 −0″·07 b+0″·08a	

Total Deflection at Punnæ, *North*, =6″·22 D+0″·66 b+0″·55 a.

Total Deflection at Punnæ, *East*, =0″·19 D−6″·33 b+6″·36 a.

TABLE V.—Deflections at Near-Goa.

Depths.	Central Lune.	First Lune.		Second Lune.		Third Lune.		Fourth Lune.		Fifth Lune.	
		East.	West.	East.	West.	East.	West.	East.	West.	East.	West.
d	$\frac{5}{17}a$	$\frac{1}{12}a$	$\frac{7}{14}a$	0	$\frac{7}{13}a$	0	$\frac{5}{16}a$	0	$\frac{1}{9}a$	0	$\frac{1}{10}a$
$\frac{21}{2}d$	3·09 a	0·25 a	3·06 a	0	4·38 a	0	3·28 a	0	1·17 a	0	0·26 a
d_1	Six depths from $\frac{5}{17}a$ to $\frac{10}{19}a$ (=2·45a)	Seven from $\frac{1}{12}a$ to $\frac{1}{21}a$ (=·26 a)	Seven depths from $\frac{7}{18}a$ to $\frac{7}{9}a$ (=4·08a)	......	Six depths from $\frac{5}{14}a$ to $\frac{3}{4}a$ (=3·5 a)	......	Five from $\frac{5}{16}a$ to $\frac{1}{4}a$ (=3·59a)	......	Seven depths from $\frac{1}{9}a$ to $\frac{1}{5}a$ (=1·09a)	......	Five from $\frac{1}{10}a$ to $\frac{1}{40}a$ (=·19 a)
d_2				......		......		......		......	
d_3				......		......		......		......	
d_4				......		......		......		......	
d_5				......		......		......		......	
d_6				Eight depths from 0 to b (=4 b)		......	Seven depths from $\frac{1}{4}a$ to 0 (=1·75a)	......		......	4 from $\frac{1}{40}a$ to 0 (=·1 a)
d_7	Sixteen depths from $\frac{10}{19}a$ to $\frac{5}{6}$D (=6·67D +4·74 a)				Fourteen depths from $\frac{3}{4}a$ to 0 (=5·25a)	......		......		......	
d_8		Fifteen depths from $\frac{1}{21}a$ to $\frac{1}{2}$D (=·36 a +3·75D)	Thirteen depths from $\frac{7}{9}a$ to $\frac{1}{7}$D (=5·05 a +·93 D)			Five from 0 to $\frac{8}{27}b$ (=·94 b)		......	Five from $\frac{1}{5}a$ to 0 (=·5 a)	......	
d_9								......		......	
d_{10}								......		......	......
d_{11}								......		......	......
d_{12}								......		......	......
d_{13}						3 from $\frac{8}{27}b$ to 0 (=·44 b)	......	......	......	......	......
d_{14}				Six depths from b to 0 (=3 b)			......	......	......	......	......
d_{15}							......	......	......	......	......
d_{16}						......	......	......	......	......	......
d_{17}						......	......	......	......	......	......
d_{18}						......	......	......	......	......	......
d_{19}						......	......	......	......	......	......
d_{20}				......		......	......	......	......	......	......
d_{21}			Ten depths from $\frac{1}{7}$D to 0 (=·71 D)	......	......	......	......	......	......	......	......
d_{22}				......	......	......	......	......	......	......	......
d_{23}	Eight from $\frac{5}{6}$D to 0 (=3·33 D)	Eight from $\frac{1}{2}$D to 0 (=2 D)		......	......	......	......	......	......	......	......
d_{24}				......	......	......	......	......	......	......	......
d_{25}				......	......	......	......	......	......	......	......
d_{26}				......	......	......	......	......	......	......	......
d_{27}				......	......	......	......	......	......	......	......
d_{28}				......	......	......	......	......	......	......	......
d_{29}				......	......	......	......	......	......	......	......
d_{30}				......	......	......	......	......	......	......	......
Totals ...	10 D +10·28 a	5·75 D +0·86 a	1·64 D +12·19a	7 b	13·13 a	1·38 b	8·62 a	0	2·76 a	0	·55 a
Deflectns	1″·85 D +1·90 a	1″·06 D +0·16 a	0″·30 D +2·69 a	1″·30 b	2″·43 a	0″·26 b	1″·59 a	0	0″·57 a	0	0″·10 a
Sums ...	1″·85 D +1·90 a	1″·36 D+2″·85 a		1″·30 b+2″·43 a		0″·26 b+1″·59 a		0″·57 a		0″·10 a	
Cos azim.	1	·866		·500		0		−·500		−·866	
Def. *N*....	1″·85 D +1·90 a	1″·18 D+2″·47 a		0″·65 b+1″·22 a		0		−0″·28 a		−0″·09 a	
Diff.......	0	−0″·76 D+2″·53 a		−1″·30 b+2″·43 a		−0″·26 b+1″·59 a		0″·57 a		0″·10 a	
Sin azim.	0	·500		·866		1		·866		·500	
Def. *E*....	0	−0″·38 D+0″·75 a		−1″·13 b+2″·10 a		−0″·26 b+1″·59 a		0″·49 a		0″·05 a	

Total Deflection at Near-Goa, *North*, = 3″·03 D+0″·65 b+5″·22 a.

Total Deflection at Near-Goa, *East*, = −0″·38 D−1″·39 b+4″·98 a.

TABLE VI.—Deflections at Karachi.

Depths.	Central Lune.	First Lune.		Second Lune.	
		East.	West.	East.	West.
d	$\frac{3}{7}a$	$\frac{1}{88}a$	$\frac{1}{10}a$	0	0
$2\frac{1}{2}d$	$3a$	$0{\cdot}12a$	$1{\cdot}05a$	0	0
d_1	Thirteen depths from $\frac{3}{7}a$ to a $(=8{\cdot}36a)$	Fifteen depths from $\frac{1}{88}a$ to $\frac{1}{11}a$ $(={\cdot}77a)$	Seven depths from $\frac{1}{10}a$ to 0 $(={\cdot}35a)$	......	......
d_2				......	......
d_3				......	......
d_4				......	......
d_5				......	......
d_6				......	......
d_7				......	......
d_8			......	......	......
d_9			......	......	......
d_{10}			......	......	......
d_{11}			......	......	......
d_{12}			......	......	......
d_{13}			......	......	......
d_{14}	Eleven depths from a to $\frac{7}{12}$ D $(=5{\cdot}5a+6{\cdot}42\,\mathrm{D})$		......	Five from 0 to $\frac{1}{2}b$ $(=1{\cdot}25b)$	......
d_{15}			......		......
d_{16}		Nine depths from $\frac{1}{11}a$ to $\frac{1}{2}$ D $(={\cdot}41a+2{\cdot}25\,\mathrm{D})$	......		......
d_{17}			......		......
d_{18}			......		......
d_{19}			......	2 from $\frac{1}{2}b$ to 0 $(={\cdot}5b)$	......
d_{20}			......		......
d_{21}			......	......	......
d_{22}			......	......	......
d_{23}			......	......	......
d_{24}			......	......	......
d_{25}	Six from $\frac{7}{12}$ D to 0 $(=1{\cdot}75\,\mathrm{D})$	Six from $\frac{1}{2}$ D to 0 $(=1{\cdot}5\,\mathrm{D})$	......	......	......
d_{26}			......	......	......
d_{27}			......	......	......
d_{28}			......	......	......
d_{29}			......	......	......
d_{30}			......	......	......
Totals ...	$8{\cdot}17\,\mathrm{D}+16{\cdot}86a$	$3{\cdot}75\,\mathrm{D}+1{\cdot}30a$	$1{\cdot}40a$	$1{\cdot}75b$	0
Deflectns	$1''{\cdot}51\,\mathrm{D}+3''{\cdot}12a$	$0''{\cdot}69\,\mathrm{D}+0''{\cdot}24a$	$0''{\cdot}26a$	$0''{\cdot}32b$	0
Sums ...	$1''{\cdot}51\,\mathrm{D}+3''{\cdot}12a$	$0''{\cdot}69\,\mathrm{D}+0''{\cdot}50a$		$0''{\cdot}32b$	
Cos azim.	1	${\cdot}866$		${\cdot}500$	
Def. *N*....	$1''{\cdot}51\,\mathrm{D}+3''{\cdot}12a$	$0''{\cdot}60\,\mathrm{D}+0''{\cdot}43a$		$0{\cdot}16b$	
Diff.	0	$-0''{\cdot}69\,\mathrm{D}+0''{\cdot}02a$		$-0''{\cdot}32b$	
Sin azim.	0	${\cdot}500$		${\cdot}866$	
Def. *E*....	0	$-0''{\cdot}35\,\mathrm{D}-0''{\cdot}01a$		$-0''{\cdot}28b$	

Total Deflection at Karachi, *North*, $2''{\cdot}11\,\mathrm{D}+0''{\cdot}16b+3''{\cdot}55a$.

Total Deflection at Karachi, *East*, $-0''{\cdot}35\,\mathrm{D}-0''{\cdot}28b-0''{\cdot}01a$.

9. The results gathered from these Tables are—

	North.	*East.*
Deflection at Kaliana . .	1″·60 D+0″·76 b+0″·80 a,	−0″·01 D−0″·16 b+0″·56 a.
Deflection at Kalianpur .	2·17 D+1·09 b+1·68 a,	−0·08 D−1·00 b+1·18 a.
Deflection at Damargida .	2·90 D+0·19 b+1·55 a,	−0·08 D−0·64 b+2·52 a.
Deflection at Punnæ . .	6·22 D+0·66 b+0·55 a,	0·19 D−6·33 b+0·36 a.
Deflection at Near-Goa .	3·03 D+0·65 b+5·22 a,	−0·38 D−1·39 b+4·98 a.
Deflection at Karachi . .	2·11 D+0·16 b+3·55 a,	−0·35 D−0·28 b−0·01 a.

When the values of D, a, and b are known, these formulæ will give the deflections.

There have been no deep-sea soundings in the Indian or Southern Oceans. The results, therefore, at which I arrive in what follows must be looked upon as demonstrating that the effect of the ocean on the plumb-line is of importance, rather than as determining its amount.

10. The widths of the openings of the Arabian Sea and Bay of Bengal are as 4 to 3. In this ratio I shall take a to b. Also D I shall suppose 3 times a, and therefore 4 times b. The formulæ then become—

At Kaliana . .	deflection North 2″·06 D,	deflection East 0″·03 D.
At Kalianpur .	deflection North 3·00 D,	deflection East 0·16 D.
At Damargida .	deflection North 3·47 D,	deflection East 0·60 D.
At Punnæ . .	deflection North 6·57 D,	deflection East 0·73 D.
At Near-Goa .	deflection North 4·93 D,	deflection East 0·93 D.
At Karachi .	deflection North 3·33 D,	deflection East −0·42 D.

11. It appears from the chart of the North Atlantic in Lieut. MAURY's interesting work on the 'Physical Geography of the Sea,' that between North Africa and the West India Islands—a width about three-fourths of that of the Indian Ocean, where I have placed the depth D—many depths have been measured between 3 and 4 miles. If, then, we make D=3, which will hardly be too much, the formulæ become—

At Kaliana . .	deflection North 6″·18,	deflection East 0″·09.
At Kalianpur .	deflection North 9·00,	deflection East 0·48.
At Damargida .	deflection North 10·44,	deflection East 1·80.
At Punnæ . .	deflection North 19·71,	deflection East 2·19.
At Near-Goa .	deflection North 18·88,	deflection East 2·79.
At Karachi . .	deflection North 9·99,	deflection West 1·26.

These at any rate serve to show that no calculations of the Figure of the Earth derived from the measurement of arcs in India can safely be depended upon, for great exactness, if the effect of the ocean is neglected.

§ 4. *The effect of the Ocean on the Ellipticity of the Great Arc of Meridian in India.*

12. The results in the last paragraph increase the amplitudes of the two arcs Kaliana—Kalianpur and Kalianpur—Damargida by the quantities $2''\cdot82$ and $1''\cdot44$. In former communications I have shown that the effect of mountain attraction is to decrease these amplitudes by $14''\cdot896$ and $5''\cdot257$. The effect, then, of the ocean is to reduce these to $12''\cdot88$ and $4''\cdot12$.

In my Paper of 1855 I obtain the following formula for calculating the effect of errors in amplitude on the ellipticity. If $15''\cdot885(1-u)$ and $5''\cdot059(1-v)$ be corrections to be added to the astronomical amplitudes of these two northern portions of the arc, then the ellipticity of the arc

$$=0\cdot002346+0\cdot003693u-0\cdot001046v.$$

Put, therefore, $15\cdot885(1-u)=12\cdot88$ and $5\cdot059(1-v)=4\cdot12$;

$$\therefore \qquad u=0\cdot18918 \text{ and } v=0\cdot18561,$$

and

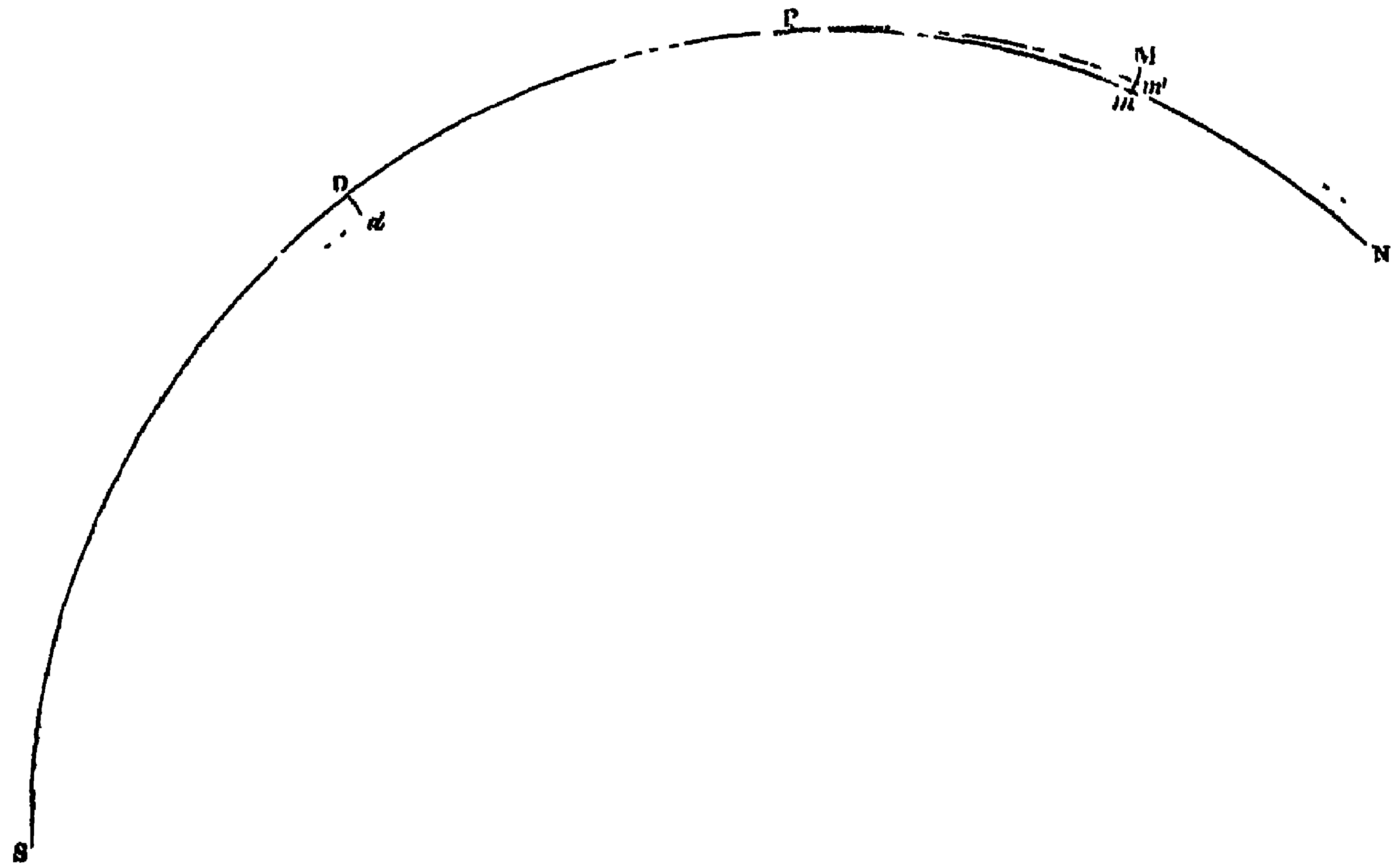
$$\text{ellipticity}=0\cdot003614=\frac{1}{276\cdot7}.$$

This is nearer to the mean value of the ellipticity than the hypothesis of deficiency of matter below will make it, if the deficiency extend no deeper than 300 miles below the surface (see my last Paper). The allowance, therefore, for the effect of the ocean brings back the curvature of the Indian Arc, as deduced by the comparison of the computed and observed amplitudes (the latter being corrected for the effect of mountain and ocean attraction), more nearly to the mean curvature, and thus far acts in the right direction. I may also observe, that the increase of amplitude between Damargida and Punnæ deduced from the last paragraph, taken in combination with the decrease which the mountains produce (as far as we can infer this latter amount by supposing the law of attraction between A and C to extend to P), is not very different from the error which Colonel EVEREST deduced (p. clxxvii of his volume of 1847) for that portion of the arc.

13. The effect of the mountain and ocean attractions is shown in the accompanying diagram (fig. 2). SDPmN is the meridian line in the longitude of Cape Comorin or Punnæ (P), on the supposition that the ocean is of the same density as superficial rock, and that the mountain mass is all removed. D is 30° from P, and Dd= the depth which measures the deficiency of matter arising from the inferior density of sea-water = about 1·878 miles. From d the curves dP and dS slope up to Punnæ and the neighbourhood of the south pole, so as to make the change of depth vary in arithmetical progression. The curve dP produced through P reaches a height at M (25° from P, and where the axis of the Himmalayas crosses the meridian) $=\text{M}m=1\cdot878\times25\div36=1\cdot3$ mile. The average height of the whole mountain mass (in the Enclosed Space) is only about one mile; also the greatest height between Punnæ and the beginning of the Himmalayas, which is about half-way, does not exceed half a mile; and about Kaliana it is only one-fifth of a mile. Hence the curve PM decidedly lies *above* the general curve of Asia from Punnæ towards the north pole; and the curvature of the actual meridian line is *greater* than the average curvature. This is the result to which the ellipticity deduced

in the last paragraph has also brought us. Thus while the effect of mountain attraction *alone* (as the diagram shows, and my former calculations have shown) would bring

Fig. 2.



out a curvature flatter than the mean, when the effect of the ocean is also considered, the curve deduced is more convex; and, as the calculation of the last paragraph shows, is nearer the mean. The Figure of the Earth is, however, more distorted from the mean than if one only of these causes acted. For although the *curvature* of the portion of the meridian under consideration is not so much affected, the *position* of this portion has, so to say, a tilt northwards and a slope southwards, so as to break the continuity of the ellipse; and at all the stations of the arc in India the plumb-line hangs more northerly from the operation of both the mountains and the ocean, than if one of these causes only existed.

§ 5. *Change of the Sea-level produced by the Mountains and the Ocean.*

14. Both the positive attraction of the mountains and the deficiency of attraction of the ocean have the effect of raising the sea-level at Karachi, near the mouths of the Indus in latitude 25°, above that at Cape Comorin. If the difference of level is at all sensible, it is important to calculate it, as the Great Survey has brought down two of its chains of triangles to these two places on the coast.

In order to estimate the effect of these disturbing forces upon the sea-level, it is necessary to find their value along some line joining the two places, and to substitute them in the equation of fluid equilibrium. The line I choose is the straight line joining Cape Comorin and Karachi, which runs about 30° west of north. I have calculated the effect of the ocean on the point half-way, which I have called "Near-Goa," as well as upon Punnœ and Karachi, the two extremities, with a view to obtain the approximate law of the force along this line. The formulæ of par. 9 show that the chief part of the effect arises from the portions of the sea which depend on D, down in the south at a considerable distance from Hindostan. The law, therefore, along the above line may be obtained approximately by assuming a formula in powers of λ, or the difference of latitude of any point on the line and Punnœ. The value of this difference is 8°·5 at "Near-Goa," and 17° at Karachi. The east and west parts of the force I neglect, as not modifying the force northwards at all materially. In this way I obtain—

$$\text{Deflection}=19''{\cdot}71-0''{\cdot}585\lambda+0''{\cdot}00084\lambda^2.$$

This, as may be seen by substitution, gives the deflections of par. 11 at the three stations, Punnœ, Near-Goa, and Karachi. It is easily seen, by the resolution of forces, that the ratio of the force producing this deflection to gravity equals the tangent of this deflection. Hence, calling the force W,

$$\frac{W}{g}=0{\cdot}000095556839-0{\cdot}000002836162\lambda+0{\cdot}000000004072\lambda^2,$$

λ being expressed in degrees and parts of a degree.

15. We shall soon require the integral of this force multiplied by the element of its direction, taken between λ=0 and λ=17. I will therefore calculate it at once.

$$\begin{aligned}\int\frac{W}{g}\,d\lambda&=0{\cdot}000095556839\lambda-0{\cdot}000001418081\lambda^2+0{\cdot}000000001357\lambda^3\\&=0{\cdot}0012215 \text{ in parts of a degree}\\&=448{\cdot}25 \text{ feet.}\end{aligned}$$

This, it will be soon seen, is the rise of the sea-level at Karachi above that at Cape Comorin in consequence of the deficiency of attraction of the ocean, on the hypothesis as to depths which I have assumed.

16. I will now consider the force arising from mountain attraction. It has been proved (in my Paper of 1855) that the Himmalayan Mass attracts points between A and C (in the diagram, fig. 1) as if it were a dense prism of great length and small transverse dimensions running about W.N.W and E.S.E., and cutting the meridian of the Great Arc at about 8° 30′ north of Kaliana. The law of attraction for places between Kaliana and Damargida varies as the distance from this line inversely. For distances from this line greater than Damargida this law needs modification, for the following reason. A prism attracts a point opposite its middle with a force varying inversely as the product of the distances of the point from the middle and from either of the extremities. Hence when the point is not far from the prism in comparison with the prism's length, the force will vary nearly as the inverse distance; but when at a considerable distance, it will vary more nearly as the inverse square.

The point where the line joining Punnœ and Karachi cuts the 20° parallel of longitude is about equally distant with Damargida from the above fixed line (or axis) of the Himmalayas. I shall therefore take the deflection there to be the same as at Damargida, viz. $6''\cdot 8$ sec 22° 30′ or $7''\cdot 36$, $6''\cdot 8$ being the *meridian* deflection at that station. Above this point up to Karachi I shall suppose the deflection to vary as the inverse distance, and below it down to Punnœ as the inverse square. This will give results rather under, than over the mark; for it makes the deflections at Punnœ and Karachi only $2''\cdot 65$ and $9''\cdot 15$, as will be seen below.

Let b and u be the distances from the fixed line of Damargida and of any point on the line between Punnœ and Karachi, of which λ is the number of degrees of latitude above Punnœ;

$$\therefore \quad b = \text{arc } 15^\circ \cos 22^\circ\ 30', \quad u = \text{arc } 25^\circ \cos 22^\circ\ 30' - \lambda \sec 30^\circ \sin 37^\circ\ 30'.$$

Hence from Punnœ to the 20° latitude, the deflection

$$= 7''\cdot 36 \left(\frac{15 \cos 22^\circ\ 30'}{25 \cos 22^\circ\ 30' - \lambda \sec 30^\circ \sin 37^\circ\ 30'}\right)^2$$

$$= \frac{2860''\cdot 6}{(32\cdot 86 - \lambda)^2}. \quad \text{At Punnœ this } = 2''\cdot 65.$$

Calling M_1 the force producing this deflection,

$$\frac{M_1}{g} = \frac{\tan 2860''\cdot 6}{(32\cdot 86 - \lambda)^2} = \frac{0\cdot 0138666}{(32\cdot 86 - \lambda)^2}.$$

17. Also since $du = -d\lambda . \sec 30^\circ \sin 37^\circ\ 31' = -0\cdot 702937 d\lambda$

$$\int -\frac{M_1}{g}\,du = 0\cdot 0007473 \left\{\frac{1}{20\cdot 86} - \frac{1}{32\cdot 86}\right\} = 0\cdot 0001706$$

the limits of λ being 0 and 12. This is in parts of a degree: in feet it $=15\cdot 88$. This, it will be seen, is the rise of the point between Punnœ and Karachi above Punnœ in consequence of mountain attraction.

18. From the point in latitude 20° up to Karachi, the deflection

$$= 7''\cdot 36\, \frac{15 \cos 22^\circ\ 30'}{25 \cos 22^\circ\ 30' - \lambda \sec 30^\circ \sin 37^\circ\ 30} = \frac{145''\cdot 1}{32\cdot 86 - \lambda}.$$

At Karachi this $=9''\cdot 15$. The force producing the deflection being called M_2,

$$\frac{M_2}{g} = \frac{\tan 145''\cdot 1}{32\cdot 86 - \lambda} = \frac{0\cdot 00070298}{32\cdot 86 - \lambda}.$$

19. Also $\displaystyle\int -\frac{M_2}{g}\,du = \frac{0\cdot 00049415}{\cdot 434} \log \frac{20\cdot 86}{15\cdot 86} = 0\cdot 0001855$ in parts of a degree $=50\cdot 44$ feet.

This is the rise of the sea-level at Karachi above the point in latitude 20°, in consequence of mountain attraction. This, as well as the similar statements in pars. 15 and 17, I now proceed to prove.

20. The equation to the surface of a fluid mass acted on by forces XYZ at the point xyz is,

$$\text{constant} = \int (X dx + Y dy + Z dz).$$

In the case of the ocean the forces are the centrifugal force, the attraction of the general mass of the earth, and these three disturbing forces W, M_1 and M_2 which I have been calculating. Let ω be the angular velocity of the earth round its axis, θ the latitude of any point of the surface, r its distance from the earth's centre, a the semi-axis major of the mean meridian. Then $\omega^2.x$ and $\omega^2.y$ is the centrifugal force parallel to x and y, z being the earth's axis, and $\frac{1}{2}\omega^2a^2\cos^2\theta$ is the corresponding part of the above equation. Let V be the potential for the earth's mass, supposed a perfect spheroid of equilibrium differing little from a sphere; E the earth's mass. Then V differs from $\frac{E}{r}$ only by a small variable quantity depending upon the ellipticity: let it equal $\frac{E}{r}(1+U)$. Substituting these and the three disturbing forces, the equation of the surface now becomes

$$\text{const} = \frac{E}{r}(1+U)+\frac{1}{2}\omega^2a^2\cos^2\theta+\int W d\lambda-\int M_1 du-\int M_2 du$$

between the several limits, as already explained, or

$$\text{const} = \frac{E}{r}(1+U)+\frac{1}{2}\omega^2a^2\cos^2\theta+L.g;$$

$$\therefore \quad \text{const} = \frac{a}{r}\left(1+U+\frac{\omega^2a^2\cos^2\theta}{2E}\right)+L.\frac{ag}{E}.$$

But $$1=\frac{a}{r}(1-\epsilon\sin^2\theta)=\frac{a}{r}(1-\epsilon+\epsilon\cos^2\theta)$$

is the equation to the surface, ϵ being the ellipticity, when there is no disturbing force. Hence the equation in the present case is

$$1=\frac{a}{r}(1-\epsilon\sin^2\theta)+L.\frac{ag}{E};$$

$$\therefore \quad r=a(1-\epsilon\sin^2\theta)+L, \text{ as } g=\frac{E}{a^2}.$$

Let ψ be the angle through which the normal to the surface is thrown backwards. Now the tangent of the angle between r and the normal

$$=\frac{1}{r}\frac{dr}{d\theta}=-\epsilon\sin 2\theta+\frac{1}{a}\frac{dL}{d\theta},$$

$$\therefore \quad \psi=\frac{1}{a}\frac{dL}{d\theta},$$

and $ds=rd\theta=ad\theta$ being an element of the arc of the surface, the Elevation of the surface of the sea in passing northwards $=\int\psi ds=L$, between the limits $\lambda=0$ and $\lambda=17$. Hence by adding together the values deduced in pars. 15, 17, 19 to obtain L, we have

Elevation of sea-level at Karachi above that of Cape Comorin

$$=448{\cdot}25+15{\cdot}88+50{\cdot}44=514{\cdot}57 \text{ feet.}$$

This will alter all heights which depend upon Karachi, but is not of sufficient importance to affect the horizontal measures of the Survey.

21. This calculation shows, perhaps, the greatest extent to which the sea-level along

our shores can be affected, as there is no part of the world where the disturbing causes can be more influential. Had these lateral forces been capable of drawing up the sea more, and changing the level from the mean more entirely, we should have no means of detecting large protuberances or extensive hollows, that is large departures from the spheroidal form either in excess or defect. The sea is our only standard of measurement, to which the form can be referred; and were these local departures from the mean figure capable of drawing the sea-level to a greater conformity with themselves, than the above calculation shows they are able to do, we might despair of ever obtaining an accurate knowledge of the form of the several parts of the surface, however much they differed from the spheroid. This problem, as it is, is sufficiently beset with difficulties. It is therefore satisfactory that this is not added to their number.

Calcutta, October 25, 1858.

XXXI. *On the Properties of Electro-deposited Antimony* (continued). *By* G. GORE, *Esq.* *Communicated by Professor* TYNDALL, *F.R.S.*

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a. *Heat of discharge not due to cohesive action.*

59. I HAVE already shown to some extent that the change observed in electro-deposited antimony is not an instance of cohesive force exercised between the particles of the metal, because extensive fractures may occur in the mass (16. 17.) without producing or materially influencing the change; and the cohesion of the particles may be indefinitely reduced by triturating the metal to a very fine powder (30. 31. 32.) without a corresponding reduction taking place in the amount of heat evolved. Heat also, which decreases the cohesive force, increases the molecular energy (28.).

60. To test the question further, I took a quantity of small pieces of unchanged antimony, averaging about $\frac{1}{4}$th of an inch square and $\frac{1}{16}$th of an inch thick, divided it into two portions, reduced one portion to a very fine powder in water at 45° FAHR., dried it in the open air, and finally with a gentle heat. I then placed 300 grains of the powder in a short test-tube closed by a cork, and surrounded by 400 grains of distilled water contained in a small glass beaker covered with cotton-wool; and when the whole had acquired a temperature of 53°·2 FAHR., I induced the change by means of a heated wire, and agitated the water until a thermometer in it indicated a maximum temperature of 72°·5 FAHR. = a rise of 19·3 degrees. The powder was then removed, and a perfectly similar experiment made with 300 grains of the unpulverized fragments, in which case the temperature of 400 grains of water rose from 52°·8 FAHR. to 72°·2 FAHR. = a rise of 19·4 degrees.

61. I also took a freshly formed bar† of the unchanged substance, perfectly free from grey antimony, $\frac{1}{8}$th of an inch thick and $1\frac{1}{4}$ inch long, weighing 278·92 grains; and the same weight of a mixture of small fragments and particles of similar metal, and made two similar experiments to those described; the weight of water in each experiment being 403·78 grains, and the specific heat-value of the immersed portions of the thermometer and test-tube being =14·60 grains of water. With the *bar* the temperature of the water, &c. rose 20·05 FAHR. degrees; and with the *fragments* and *particles* it rose 20·00 degrees. The bar lost 9·80 grains, or 3·51 per cent. of expelled vapour, 9·72 grains of which was found condensed in the tube in the form of a thick buttery substance, evidently terchloride of antimony.

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* Subsequently revised and condensed by the author.

† *Bars* are formed by depositing the substance upon narrow strips of sheet silver, the edges of which are enclosed by vertical borders of white wax (see 96.).

62. And as a final settlement of this question, I took a bar weighing 277·16 grains, and the same weight of very finely powdered active antimony formed at the same period, and made two more experiments, the weight of water in each instance being 401·14 grains, and the specific heat-value of the thermometer and tube being = 14·6 grains of water. With the *bar*, the water rose in temperature from 59°·3 to 78°·4 Fahr., and with the *powder* it rose from 58°·9 to 78°·0 Fahr., equal in each case to a rise of 19·1 Fahr. degrees. The loss of weight in the bar was 9·12 grains, or 3·39 per cent., and the amount of condensed vapour found in the tube was 9·26 grains.

63. After making a small deduction for loss of heat by particles which may have undergone the change during the process of grinding, it is evident that the amount of heat set free by the powdered substance is not sensibly different from that evolved by the substance in the coherent massive state, and therefore the force exercised in the peculiar change of antimony is not that of cohesion.

64. The only method by which I have yet been able approximately to determine the amount of changed antimony contained in the powder, or the extent to which the powder has lost its peculiar thermic power, has been to measure the amount of heat evolved by the discharge of a given weight of it in a calorimeter.

b. *Heat of discharge not due to alteration of specific heat.*

65. It may readily be supposed that a portion of the evolved heat may be due to diminution of specific heat attendant upon the increase of specific gravity which occurs during the sudden change (46.); I therefore made several determinations of the specific heat of unchanged and changed specimens in an apparatus similar to that used by Regnault for specific heats. The specimen to be examined was placed in a small copper wire basket, suspended by a horsehair, and raised by means of small pulleys into a vertical cylindrical air-chamber, surrounded by a vessel of hot water, and retained there in contact with the bulb of a thermometer until it had acquired a perfectly uniform and stationary temperature (generally about 160° or 180° Fahr.) by careful regulation of the heat. Meanwhile a known quantity of distilled water (usually an equal weight to the antimony) in a thin glass vessel covered externally with cotton-wool, and containing a thermometer, was brought to a uniform and stationary temperature (generally about 60° Fahr.) by surrounding it by non-conducting screens. At that moment the vessel of water was placed close beneath the air-chamber, and the basket lowered in an instant into the water by means of the horsehair and pulleys. The vessel of water was immediately removed and closed, and the basket agitated continually by means of a hooked glass rod passing through a hole in the cover until the thermometer indicated a maximum temperature, which occurred in about two minutes. Corrections in the calculation were made in each instance for the amount of heat evolved by the basket, and for that absorbed by the immersed portion of the thermometer.

66. Seven determinations were made of the specific heat of two unchanged bars, each weighing upwards of 250 grains, and the following numbers were obtained, in the

order given,—0·06405, 0·06261, 0·06204, 0·06372, 0·06329, 0·06240, and 0·06377; mean 0·06312. The discharging point of perfectly similar bars was 205°·5 Fahr., and the highest temperature to which these were raised was 185°·3 Fahr.; nevertheless on several occasions during their presence in the air-bath at the higher temperatures I observed that they evolved heat; and after the experiments, I found that they had entirely lost their heating property: their weights and specific gravities were, however, but very slightly different from those found before the specific heat determinations, but their fractured surfaces were much less bright and of a lighter colour than those of unchanged antimony. From these experiments it appears that the discharge of heat by electro-deposited antimony is not necessarily attended by alteration of the specific heat of the substance; because whilst the substance was *gradually* discharging the whole of its peculiar heat, its specific heat did not sensibly diminish, as may be observed by examining the numbers obtained.

67. An extra determination was made with two similar bars, weighing together upwards of 480 grains; raising their temperature only to 118°·0 Fahr., the number obtained was 0·0637; they still possessed the heating property, evidently because they had not been subjected to a sufficiently high temperature to produce even a gradual discharge. The specific heat of the solution in which the bars were formed was found by the method of cooling to be =0·0550.

68. Some determinations of the specific heat of antimony which had been *suddenly* discharged were also made. With two bars which had been changed in air at 60° Fahr., and weighing together about 730 grains, the following successive numbers were obtained, —0·0533, 0·0545, 0·0549, 0·0547; mean =0·0543. And with thin pieces discharged in the hot-air bath, and weighing nearly 740 grains, the numbers 0·0522 and 0·0533 were obtained. The results were checked by separate determinations of the specific heat of purified antimony in the ordinary state of aggregation. The manifest reason why the specific heat of the *suddenly* changed bars was less than that of the *gradually* changed ones, was because they had by the *sudden* discharge been subjected to a very much higher temperature, and had therefore lost a larger proportion of their enclosed liquid, which is a substance of greater specific heat than metallic antimony. A similar reason explains why the thin pieces suddenly discharged in a heated atmosphere had less specific heat than the bars suddenly discharged in cool air. In all cases, the higher the temperature the substance had been subjected to in the process of discharge, —either by being suddenly instead of gradually discharged, or by being suddenly discharged in a non-conducting heated medium instead of in cool air (or in cold water or mercury, for example), or by discharging a large bulk instead of a small quantity,—the greater was the loss of enclosed liquid, increase of specific gravity, and decrease of specific heat.

c. *Temperature at which the sudden discharge occurs.*

69. A number of experiments were made to ascertain the temperature at which the substance discharges its heat, in the following manner:—two perfectly similar specimens

were taken; one was suspended in the air-bath of the calorimeter and heated with moderate speed until the discharge occurred, to determine *approximately* its discharging point; the second was then tried, raising the temperature of the bath with increasing slowness until within ten to four degrees of the discharging point; the bath being then kept stationary at that temperature, the metal began to evolve its heat, raising its own temperature above that of the bath, slowly at first, and then more quickly, until the sudden discharge took place. In some cases, where the quantity of metal operated upon was small, and especially if the temperature of the surrounding bath was a few degrees too low, the heat was evolved *gradually* without any sudden discharge.

	grains.			° Fahr.	
1. Coarse granules (freshly formed)	350·00	in a fine brass gauze basket.	Discharged suddenly at	208·3	and lost 4·07 per cent. in vapour.
2. Bars and thin pieces (stale)	600·00	in a thin copper bucket	Discharged suddenly at	205·5	
3. Thin pieces (stale)	572·27	in a thin copper bucket......	Discharged suddenly at	198·0	
4. Thin pieces (freshly formed)......	96·16	in a thin copper bucket.. ...	Discharged suddenly at	196·5	
5. Bars (formed one day)	249·78	in a fine brass gauze basket	Discharged suddenly at	196·0	and lost 4·70 per cent. in vapour.
6. Bars (recently deposited)	605·00	in a thin copper bucket......	Discharged suddenly at	181·0	
7. Lumps (recently deposited)	445·24	in a thin copper bucket......	Discharged suddenly at	183·1	
8. Powder (stale)........................	307·77	in a closed brass vessel	Discharged suddenly at	178·0	
9. Powder, but had been kept in a warm place	105·70	in a closed brass vessel	Discharged gradually during two minutes at	172·0	
10. Powder, but had been kept in a warm place	300·00	in a closed brass vessel	Discharged suddenly but feebly at	171·0	
11. Fine powder	300·00	in a thin copper bucket......	Discharged suddenly at	166·0	

In a former experiment (35.), with less perfectly amorphous metal, a much higher temperature of discharge was observed, partly because a bath of mercury was employed which conveyed the evolved heat more rapidly away.

70. It is probable that the real temperature of sudden discharge is above 212°·0 Fahr., because the temperature observed in each of the above experiments was only that of the *external surface* of the basket; the *centre* of the mass or heap of antimony was probably at a much higher temperature. It also appears that the discharge is not limited to a particular temperature, but commences between 170° and 190° Fahr., and gradually increases in rapidity to some point above 212° Fahr., when it attains its maximum, and becomes sudden.

71. Small thin pieces of the substance formed at the same period, and in the same liquid as those of Nos. 4 and 6, were heated a few minutes in water at 209° Fahr., and in air at 208° Fahr. without losing their heating power; but by immersing them in boiling water during twenty minutes, nearly the whole of the heating power was removed without loss of weight in the pieces. Two portions of 20 grains each were also placed, one in a fine brass wire-gauze vessel, and the other in a water-tight copper bucket, and both immersed in boiling water during thirty minutes: at the end of that time the heating power of each had disappeared; the former had gained 0·11 grain in weight (probably by absorption of water), and the weight of the latter had remained unaltered.

72. The thermic power may be exhausted either gradually or suddenly, and the conditions of gradual or sudden discharge appear to be these:—if the amount of cooling

influences is great in relation to the amount of heat to be discharged the action is gradual, but if it is small the action is sudden.

d. *Total amount of heat evolved by the change.*

73. The following determinations were made of the total amount of heat evolved during the change. In each instance the metal was contained in a fine brass gauze basket with a lid, placed in a glass tube immersed in the water of a small calorimeter, and its heat was discharged by contact of a small heated wire momentarily introduced through a hole in the cork of the tube and in the lid. The water was constantly stirred, and it occupied about nine minutes in attaining its maximum temperature. The calculations include the heat absorbed by the basket, the immersed portions of the tube and thermometer, the water, and the metallic mass; reckoning the terchloride of antimony and actual metal *together*, i. e. as one mass, possessing a mean specific heat of 0·06312 (see 66.).

	grains.		Evolved sufficient heat to raise its own weight of antimony, of specific heat 0·0508.		grains.		
1.	300·0	of coarse granules, deposited 1 day	666·27 Fahr. degrees;	and lost	10·81	of expelled vapour =3·6	per cent.
2.	350·0	of coarse granules, deposited 2 days	662·38 Fahr. degrees;	and lost	12·74	of expelled vapour =3·64	per cent.
3.	350·0	of coarse granules, deposited 2 days	664·64 Fahr. degrees;	and lost	12·52	of expelled vapour =3·57	per cent.
4.	350·0	of coarse granules, deposited 2 days	657·95 Fahr. degrees;	and lost	12·00	of expelled vapour =3·42	per cent.
5.	350·0	of coarse granules, deposited 3 days	620·20 Fahr. degrees,	and lost	12·46	of expelled vapour =3·56	per cent.
6.	300·0	of thin pieces, deposited 1 day ……	705·80 Fahr. degrees;	and lost	11·13	of expelled vapour =3·71	per cent.
7.	300·0	of fine powder, deposited 6 weeks …	602·84 Fahr. degrees;	and lost	2·33	of expelled vapour =0·776	per cent.
8.	300·0	of thin pieces, deposited 6 hours……	651·78 Fahr. degrees;	and lost	11·26	of expelled vapour =3·75	per cent.
9.	272·42	a bar, deposited 1 day………………	628·70 Fahr. degrees;	and lost	7·56	of expelled vapour =2·43	per cent.
10.	400·0	of thin pieces, deposited 3 hours……	655·35 Fahr. degrees;	and lost	15·49	of expelled vapour =3·87	per cent.
11.	251·92	a bar ………………………………	633·06 Fahr. degrees;	and lost	10·8	of expelled vapour =4·28	per cent.

The coarse granules used in experiments 1 to 5 were formed by depositing the substance upon coarse copper wire-gauze, and bending the coated gauze backwards and forwards in cold water, then sifting and drying the product. The metal of No. 7 had been previously exposed for two hours in the air-bath to a uniform temperature of 152° Fahr., and during that time had lost 0·05 grain in weight.

74. A bar of active antimony rapidly deposited in a nearly pure solution of acid hydrochlorate of terchloride of antimony, taken from the liquid, and discharged the same day in the calorimeter at 65°·5 Fahr., evolved sufficient heat to raise the temperature of its own weight of ordinary antimony (specific heat =0·0508) 624·65 Fahr. degrees. The following are the specific heat-values in water of the materials and apparatus:—

Antimony ……	356·27 grs.	(sp. heat =0·06312)=	22·4877	grs. of water	Raised from 65°·5 Fahr. to 79°·0 Fahr. =13·5 degrees.
Brass basket ………………		(sp. heat =0·0939)=	5·0696	grs. of water	
Immersed part of glass tube		(sp. heat =0·1770)=	28·718	grs. of water	
Mercury of thermometer …		(sp. heat =0·0330)=	6·882	grs. of water	
Water………………………………………			774·820	grs.	
			837·4278	grs. of water	

The discharge in this instance was quite instantaneous, and shattered the bar into minute fragments. The loss of weight by evolved vapour was 5·90 grains =1·68 per cent. In several cases, when the substance has been rapidly deposited and discharged within a short period, the action has been instantaneous, and the metal has been shattered into minute particles (see 25.).

e. *The evolution of vapour not a* cause *but an* effect *of the heat.*

75. The evolution of vapour and consequent loss of weight by the discharge appears to be entirely dependent upon the temperature which the substance acquires during the discharge. 1st. If the substance acquires a very high temperature, such as when it is discharged in a hot-air bath, its loss of weight is considerable; but if it does not acquire a very high temperature, as when it is gradually discharged by suspending it in a hot-air bath at about 150° or 170° FAHR., or in a hot-water bath at 212° FAHR., its loss of weight is very small. The following experiments will illustrate this statement:—

Three perfectly similar bars, taken the previous day from the depositing liquid, were separately discharged in the hot-air bath: the first was gradually raised in temperature during one hour and forty minutes from 60° to 196° FAHR., it then suddenly discharged with considerable force; the second was maintained at a temperature varying from 170° to 178° FAHR. during two hours and eleven minutes, it then suddenly discharged its heat with less force; and the third was kept at a temperature varying from 170° to 177° FAHR. during nine hours and twelve minutes, and (after the heat appeared to be all discharged) during the last two hours at 180° FAHR., and finally at 186° FAHR.; it evolved its heat gradually without any sudden discharge, and was found to have entirely lost its thermic power. The following are the weights of the bars, their specific gravities, their losses of vapour, &c.:—

Before the experiment.

grains.
No. 1. 249·78.
No. 2. 254·65. Specific gravity 5·811 at 66° FAHR.
No. 3. 299·65. Specific gravity 5·825 at 63°·3 FAHR.

After the experiment.

grains. grains.
No. 1. 238·02 lost 11·76=4·7 per cent.
No. 2. 248·68 lost 5·95=2·34 per cent.
No. 3. 297·41 lost 2·24=0·747 per cent. Specific gravity 5·858 at 62°·6 FAHR.

Also two perfectly similar bars, the first weighing 251·92 grains, and the second weighing 256·18 grains, were deprived of their energy; the first *suddenly* in the air-bath of the calorimeter at 60° FAHR., and the second *gradually* by suspending it in boiling water during six hours. The first one lost 10·8 grains =4·28 per cent., and the second lost [illegible] grain =0·28 per cent.; and another bar, weighing 256·18 grains, was immersed [illegible] the water heated to boiling in thirty minutes, and kept boiling during six hours: the bar was then taken out; it had lost the whole of its heating power, and weighed 275·6 grains = a loss of 0·26 per cent. in weight.

And 2nd. If two pieces of active antimony are suddenly discharged, and from any cause the temperatures they acquire are different, the amounts of vapour they evolve are also different; thus a quantity of thin fragments was divided into equal parts of 180 grains each; the first was placed in a fine brass wire-gauze basket with a wire-gauze lid, and raised quickly to a temperature of 163°·0 Fahr. in the air-bath, and then at once discharged in the open air at 60° Fahr., by momentary contact of a heated wire; and the second portion, placed in the same basket, was surrounded by ice and cooled to 34° Fahr., and similarly discharged in the air at 60° Fahr.; the former evolved its heat instantaneously, and lost 8·85 grains, or 4·916 per cent. in weight; and the latter discharged feebly and much less rapidly, and lost 5·40 grains, or 3·00 per cent. It appears from these various experiments that the evolution of vapour is not a cause nor even a necessary consequence of the change, but only occurs when the temperature of the substance during the discharge is considerably raised; and it makes no difference whether the requisite temperature is acquired by means of internal heat or by heat from external sources.

76. In all instances, and these were very numerous, the heat from the discharge alone was insufficient to expel more than three-fourths of the contained liquid, and in those instances, whether of sudden or gradual discharge, the remainder, making a total in each case of upwards of *six* per cent., was always readily expelled by fusing the metal in a test-tube.

f. *Influence of temperature upon the amount of heat evolved.*

77. Two similar bars were taken, No. 1 weighing 257·21 grains, and No. 2 weighing 245·90 grains: the first was placed in the wire-gauze basket and cooled to 34°·0 Fahr., then at once transferred to the calorimeter (which was at 57°·6 Fahr.), and instantly discharged; the second was also placed in a similar basket and heated to 157°·15 Fahr. in the air-bath; the basket and bar were then instantly lowered into the calorimeter (which was at 57°·75 Fahr.), and at once discharged; the calorimeter of the former attained a maximum temperature of 67°·35 Fahr. in nine minutes, and of the latter a maximum of 69°·4 Fahr. in the same period. The following are the various specific heat-values in water of the substances, &c.:—

No. 1.

Antimony...... 257·21 grains (sp. heat = 0·06812) = 16·235 grains of water
Brass basket .... (sp. heat = 0·0989) = 6·242 grains of water
22·477 grains of water
} Raised from 34°·0 to 67°·35 Fahr. = 33·35 degrees.

Immersed part of glass tube of calorimeter (sp. heat = 0·1770) = 28·718 grs. of water
Mercury of thermometer.......................(sp. heat = 0·033) = 2·070 grs. of water
Portion of stem of thermometer...............(sp. heat = 0·1770) = 3·600 grs. of water
Water of calorimeter ....743·764 grs.
778·152 grs. of water
} Raised from 57°·6 to 67°·35 Fahr. = 9·75 degrees.

The thermic discharge in this case was rather feeble and slow, and the loss of vapour was 7·79 grains = 3·02 per cent. The amount of heat evolved by the bar was sufficient to raise the temperature of its own weight of ordinary antimony (of specific heat 0·0508) 638·022 Fahr. degrees.

No. 2.

Antimony	215·90 grains	(sp heat=0·06312)=	15·521 grains of water	Fell from 157°·15 to 69°·4 Fahr.=87·75 degrees
Brass basket		(sp heat=0·0939)=	6·346 grains of water	
			21·867 grains of water	

Immersed part of glass tube of calorimeter	(sp heat=0·1770)=	28·718 grs. of water	Rose from 57°·75 to 69·4 Fahr. =11·65 degrees
Mercury of thermometer	(sp heat=0·033)=	2·070 grs of water	
Portion of stem of thermometer	(sp heat=0·1770)=	3·600 grs of water	
Water of calorimeter	.	773·272 grs	
		807·660 grs. of water	

The discharge in this instance was instantaneous and powerful, and the loss of vapour was 13·06 grains=5·311 per cent. The amount of heat evolved by the bar was sufficient to raise the temperature of its own weight of ordinary antimony 732·829 Fahr. degrees. It appears from these experiments that the amount of heat evolved by the peculiar change of antimony varies with the temperature at which the discharge occurs, and is greater at high temperatures than at low ones, similar to the specific heat of substances. These experiments also confirm the previous ones (75.) with regard to the absence of relation of the amount of evolved heat to that of evolved vapour; if the amount of heat varied with the amount of vapour, it would, in the second of these experiments, have been sufficient to have raised the temperature of the given weight of ordinary antimony 1069·64 Fahr. degrees, instead of 732·829 degrees.

g. *Chemical composition of the active substance.*

78. The deposited metal has been repeatedly found by chemical analysis to contain, in addition to terchloride of antimony and hydrochloric acid, small portions of nearly all the impurities which existed in the anode and in the depositing liquid. The small portions of gas and carbon observed in former experiments of fusion (53. 54.), appeared to be due to enclosed traces of tartar-emetic decomposed by the heat.

79. To exclude the possibility of the peculiar property of electro-deposited antimony being dependent upon the presence of impurities, I purified antimony and teroxide of antimony for the anode and solution by several methods. 1st. Some very pure tartar-emetic was projected, in small portions at a time, into a red-hot assay crucible; and the resulting mass of antimonide of potassium removed when cool, triturated with distilled water until all the potash and carbon were removed, and after drying fused with a little cyanide of potassium. 2nd. Some of the best quality of commercial antimony was very finely pulverized and boiled with pure sulphuric acid, the cooled crystalline sulphate washed first with cold and then with hot distilled water until all traces of iron disap-

peared, then heated with a weak solution of ammonia, and finally washed with distilled water until all traces of sulphuric acid were removed, and thoroughly dried at about 300° Fahr. Some of the oxide thus formed was reduced to the metallic state by fusion with cyanide of potassium. 3rd. Best commercial antimony, also electro-deposited grey antimony, was very finely pulverized, the powder boiled nearly to dryness with pure nitric acid, the mass well washed with distilled water, and the process with the acid and water repeated several times to remove iron, copper, and lead; the dried residue was then heated to redness as long as vapours were evolved, and reduced by fusion with twice its weight of cyanide of potassium. And 4th. A hot solution of pure tartar-emetic was precipitated by solution of ammonia, and the resulting teroxide washed with abundance of hot distilled water and dried.

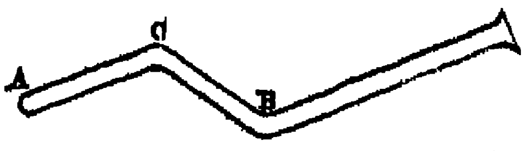
80. Several depositing liquids were prepared by dissolving the purified oxide in pure hydrochloric acid to about two-thirds' or three-fourths' saturation; using the purified metal as anodes, and sheets of platinum as cathodes. The oxide and metal prepared by the second method were used much more extensively than either of the others. With each depositing liquid an abundant deposit of bright black antimony was readily obtained in the usual manner, and the deposited metal possessed the usual structure and peculiar heating power, and evolved the usual vapour during the change.

81. Analyses of two different specimens of the pure unchanged substance deposited from a solution composed of 3 parts of pure hydrochloric acid saturated with pure oxide of antimony and then 1 part more of the acid added, yielded the following results. The first specimen was deposited more rapidly than the second.

No. 1.			No. 2.		
Sb . . .	93·36		Sb . . .	93·51	
Sb, Cl^3 . .	5·98	= 6·44*.	Sb, Cl^3 . .	6·03	= 6·24.
H, Cl . .	0·46		H, Cl . .	0·21	
	99·80			99·75	

A trace of water contained in them was not estimated.

82. The method of analysis was as follows:—a tube of German glass, nearly half an inch in diameter and 12 inches long, closed at one end, was bent to the annexed figure; the closed end was heated, and 200 grains of unchanged antimony, in small fragments, was introduced, a single fragment at a time, to discharge the heat safely; the metal was then gradually heated to fusion and collected in A, whilst the semisolid distillate collected in B; the two were then separated by fusing the tube at C. The distillate was dissolved in a solution of tartaric acid, and the amount of terchloride of



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* It is worthy of remark, that grey antimony deposited in the crystalline state from the tartaric acid solution (2.), contains only a minute quantity of enclosed liquid, similar to the exclusion of impurities from substances generally by crystallization.

antimony contained in it determined as sulphide by sulphuretted hydrogen, and after separating the excess of sulphuretted hydrogen from the filtrate by adding solution of sulphate of sesquioxide of iron and filtering, the total amount of chlorine was determined as chloride of silver; the amount of chlorine due to the terchloride found was then subtracted, and the remainder estimated as hydrochloric acid.

h. *Is the peculiar change of antimony necessarily attended by a chemical change?*

83. To ascertain whether during the act of *gradual* discharge the acid terchloride of antimony contained in the substance passed from a state of stronger combination with the metal to a state of weaker combination, I took 200 grains of unchanged antimony powder, placed it in equal portions in two small flasks, added to each 500 grains of distilled water; immersed one of the flasks in boiling water during thirty minutes, to discharge the heating power of its metal, and then replaced the evaporated water; added 50 grains of tartaric acid and 100·35 grains of hydrochloric acid to the contents of each flask, and allowed both vessels to remain at 60° FAHR. during two days. The two liquids were then separately filtered, the contents of the filters washed with equal quantities of a solution of tartaric acid, the washed residues dried, the filtrates precipitated by sulphuretted hydrogen, and the precipitates washed and dried. The contents of the heated flask yielded 93·15 grains of metallic residue possessing no heating power, and 4·20 grains of precipitated tersulphide of antimony: the contents of the other flask gave 97·24 grains of residue possessing the heating power, and 1·77 grain of tersulphide. I have also found on other occasions, when the powdered active substance had lost its peculiar property with extreme slowness (see 86.), and its weight and chemical composition had remained unaltered, that the terchloride of antimony contained in it was much more completely and easily extracted by means of solvents than whilst the powder retained its heating power. No appearance of *mechanical mixture* of terchloride of antimony with metallic antimony could be detected in changed powder by means of a powerful microscope.

84. From these observations it appears, that during the act even of gradual discharge, the terchloride of antimony changes from a state of more intimate to a state of less intimate union with the metal; but whether that change consists of diminished *chemical*, or simply diminished *mechanical* union, I have been unable to ascertain.

i. *Can the terchloride of antimony be separated without destroying the heating property?*

85. Several portions of the unchanged powder were digested in strong hydrochloric acid, solution of tartaric acid, and in strongly alkaline liquids for lengthened periods of time, at the atmospheric temperature, with an expectation of extracting at least the greater portion of the terchloride and retaining the whole of the heating power; but it was found that in every instance the heating property gradually diminished, and in about six or seven months entirely disappeared. The loss of heating power was not alone due (if at all, see 86.) to the abstraction of the terchloride by the solvents, because

it occurred in those samples from which only about one-half of the terchloride had been dissolved, as well as in those from which the whole had been extracted (see also 52.).

j. *Does electro-deposited antimony gradually change?*

86. Other portions of each of the samples of powder used in these experiments (85.) were enclosed in well-stoppered bottles, and kept during the same period (seven months) in a cool place not exposed to sunshine; in each instance the heating power entirely disappeared, and the powder became somewhat oxidized and acquired a slightly acid odour from partial separation of the terchloride; several other samples of equal age behaved similarly. Dr. Percy has also obtained similar results by placing the active powder in a capsule over strong sulphuric acid for several months. Some active powder, exposed to strong daylight during two months, still retained a heating power. Coarse granules of the active substance lost a large portion of their power in seven months: and solid bars, nearly a quarter of an inch thick, which had been formed several months before, and had been more exposed to the atmosphere, were also found to be slightly but distinctly weakened.

87. From these and other observations, it is certain that electro-deposited antimony undergoes a gradual change or decomposition even at 60° Fahr., attended by a diminished union between the terchloride of antimony and metal, and gradual and complete loss of its peculiar heating power, similar to its change in heated air at 160° Fahr., or in boiling water (83.), but much more slowly. It also appears that the substance *in a finely divided state* loses its heating power with much greater rapidity than when in a *massive form*; in the former case it requires only a few months, while in the latter case it requires at least several years (26.); but whether this difference is due to the difference of cohesion or to the difference of amount of exposed surface of the substance, I have not ascertained.

k. *Miscellaneous observations.*

88. A small quantity of mercury in a gutta-percha capsule, was made the cathode in the tartaric acid solution (2.), and a deposit of grey antimony formed upon it; a peculiar appearance and disappearance of metallic films upon the surface of the mercury occurred on discontinuing and renewing the electric current, apparently caused by deposition of films of antimony and absorption of the films by the mercury; this was repeated many times until accumulation of antimony in the mercury interfered. By passing a continuous feeble electric current into the mercury, a compound of that metal and antimony, of a pasty consistence, was formed, which gradually spread (in an arborescent form interspersed by warty excrescences) by capillary action all over the inner and outer surfaces of the small capsule, and transferred the remaining fluid mercury over the edge of the capsule into the depositing vessel. If ordinary chloride of antimony was substituted for the tartaric acid liquid, the whole of the deposited black bright antimony remained unabsorbed upon the surface of the mercury, and exhibited the cohesive action (17.) strongly, as long as the deposit was sufficiently thin.

89. In addition to the liquid already mentioned (2.) for obtaining grey crystalline antimony, the grey metal may be readily obtained by partly saturating hydrofluoric acid with tartar-emetic or oxide of antimony, and acting upon the liquid in the usual manner. Also by placing fragments of antimony with $2\frac{1}{2}$ parts of hydrofluoric acid in a gutta-percha cup, and adding to them in small portions at a time about one part of nitric acid; and when the metal has ceased to dissolve, electrolysing the solution by a battery of moderate power. Each of these liquids yields crystalline antimony freely, but in neither instance has the metal the peculiar heating property.

90. The best liquid I have yet found for forming bars, &c. of the active variety, is composed of one avoirdupois ounce of teroxide of antimony (or of oxychloride of antimony) dissolved in five or six ounces by measure of pure hydrochloric acid of sp. gr. 1·12. Bars are formed by depositing the substance upon narrow strips of sheet silver, the edges of which are enclosed by vertical borders of white wax. To remove safely the bars from the moulds within which they are deposited, the latter are placed (after being well washed with water and the aid of a soft brush) upon a surface of gutta percha in ice-cold water, and the walls of wax carefully removed in portions at a time. Inconvenient projections of the metal are removed by means of a small pair of nippers in *minute* portions at a time, with the greatest caution, in the cold water. *Cutting* action with this tool is apt to discharge the heat of the metal, whilst *slow lever* action will frequently remove projections one-twelfth of an inch thick with safety.

l. *Recapitulation.*

91. From the experiments and observations recorded in this paper, it appears—that the heat evolved by electro-deposited antimony is not due to cohesive action (63.), nor to alteration of the specific heat of the substance (66.):—the temperature at which the sudden discharge occurs varies from 170° to upwards of 212° FAHR. (69. 70.):—the total amount of heat evolved by electro-deposited antimony is usually sufficient to raise the temperature of an equal weight of ordinary antimony about 650 FAHR. degrees (73.):—the evolution of vapour of terchloride of antimony is not a *cause* but an *effect* of the heat (75):—the amount of heat evolved is greater at high temperatures than at low ones (77.), similar to the *specific* heat of substances:—the active substance consists of about 93·5 per cent. of antimony, 6·0 per cent. of terchloride of antimony, 0·8 per cent. of hydrochloric acid, and a trace of water (81.); it is also liable to contain traces of nearly all the impurities of the anode and of the depositing liquid (78.):—the discharge of heat is always attended by a diminution of attraction between the metal and its associated terchloride of antimony (84.):—the terchloride of antimony cannot apparently be extracted without destroying the heating property of the substance (85.):—and by lapse of time the active substance invariably loses its heating power, especially if it is in a state of minute division (87.).

XXXII. *On the* MEGATHERIUM (Megatherium Americanum, CUVIER *and* BLUMENBACH).
Part V.—*Bones of the Posterior Extremities.*

By Professor OWEN, *F.R.S.*,
Superintendent of the Natural History Departments in the British Museum.

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IN the description of the bones of the hind limbs of the Megatherium I commence with the ilium, as being the homotype or correlative of the scapula in the fore limb. The ischium, which is the homotype of the coracoid, is confluent with the ilium, as the coracoid is with the scapula: the pubis, which is the homotype of the clavicle, is confluent with both the ilium and ischium. All the three bones on both sides become confluent with the sacrum, and form therewith a single compound bone—the pelvis—which is the largest known amongst recent or extinct terrestrial Mammals, and gives the most striking feature to the skeleton of the present gigantic extinct Sloth.

As a like progressive ossification brings to pass a similar state of the pelvis in the small existing Sloths, the limits of the primitive bony constituents of that of the Megatherium can only be determined by analogy with the pelvis of its modern congeners, studied in individuals at a period of immaturity. This I have had the opportunity of doing in the young of the *Bradypus tridactylus*, prior to the completion of the coalescence of the several bones.

The five vertebræ composing the sacral part of the pelvis of the Megatherium have been described in a former memoir*, treating of the vertebral column to which they belong.

The iliac bones (Plate XXXVII., *aa*, and Philosophical Transactions, 1855, Plate XXIII.), as they extend from their place of anchylosis with the sacrum, expand in depth and breadth; their anterior plane is directed forward, being almost vertical and at right angles with the axis of the spine. Each ilium, after contributing its share to the acetabulum (*a*), rapidly contracts to an obtuse point bent downward and outward. The two bones, in a front view, resemble a pair of broad outstretched wings at the sides of the fore part of the sacrum. The anterior surface is slightly concave, but is undulated, with many sharp ridges that have penetrated between the fasciculi of the muscles thereto attached. The 'labrum,' or upper and outer convex border of the ilium, is unusually thick and rugged; the under concave border is also rugged, but is thin, and in some parts sharp. On the inner side of the acetabulum there is a well-defined, raised and very rough, oblong surface (*p*) for the insertion of the tendon of a powerful 'psoas' muscle.

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* "On the Megatherium," Philosophical Transactions, Part II. 1851, p. 378.

The outer surface of the ilium is slightly concave near the sacrum, and is then convex in the direction of its longest diameter, which is from within outwards; in other directions it is nearly flat: its surface is much broken by numerous intermuscular ridges.

The pubis (Plate XXXVII. 61) is very slender where it forms the anterior border of the 'foramen ovale' (*o*), but expands at its extremities, and especially where it coalesces with the ischium to form the produced and pointed 'symphysis.' The extent of this symphysis is 10 inches in a straight line.

The pelvis, as in the Sloths and other *Bruta*, shows the conversion of the ischiadic notches (*i*) into foramina by the anchylosis of the ischia with the posterior sacral vertebræ (*s*). Each ischium, 63, as it extends from its confluence with the sacrum, expands into a broad smooth plate of bone, bent outward and forward, then contracting as it converges inward towards its fellow, to combine with the pubic bones at the symphysis. The hinder border, forming the tuberosity, 63, is thick and rugged; and two or three perforations here indicate the original line of its separation from the sacrum. The part of the ischium which joins the pubis on the sacral side of the foramen ovale presents on its inner surface the usual oblique channel leading to that foramen.

Among the existing species of the Order BRUTA the Sloths alone resemble the Megatherium in the expansion of the iliac bones, but this is much less in comparison with the length of the trunk; the iliac expansion is relatively greater in the Megatherium than in the Elephant, and is associated with a much greater proportionate size of the whole pelvis and of its cavity or channel. The extreme breadth of the pelvis of a large Asiatic Elephant is 3 feet 8 inches, whilst in the Megatherium it is upwards of 5 feet.

The pelvis which CUVIER was led to suspect, from the defective condition of its fore part in the Madrid skeleton, to be naturally open anteriorly, as in the *Myrmecophaga didactyla*, is closed anteriorly, as in the Sloths, by a 'symphysis pubis' of short extent.

The acetabulum (Plate XXXVII. fig. 2) presents a full oval shape, with the lower margin bisected by a narrow and deep 'Haversian' groove, which extends, slightly expanding and becoming more shallow, to near the bottom of the cavity; the outer division of the lower border of the groove is most produced. The large and deep acetabula look downward and a little outward. One diameter of the hemispheroid cavity is 8 inches, the other diameter is 7 inches; it therefore presents a plane surface of [illegible] square inches, which, multiplied by 15, with the barometer at 30 inches, gives about [illegible] pounds atmospheric pressure upon the hip-joint of the Megatherium.

The size and strength of the ordinary processes of the pelvis, the breadth of the rough labrum of the iliac bones, and the numerous and well-defined intermuscular crests, indicate the unusual size and vigour of the muscular masses which proceeded from the pelvis in different directions to act upon the trunk and fore limbs and upon the hind limbs and tail. They lead to the conviction that the resistance which demanded such forces for its overcoming must have have been of a very different nature and degree from any that now opposes itself to the labours of the existing vegetable feeders when engaged in supplying their daily wants; whence it may be inferred that the exertion of such forces was associated with equally peculiar habits in the megatherioid animals.

The femur of the Megatherium (Plate XXXVIII. fig. 1) is one of the most massive limb-bones in the Mammalian class; from its proportions it might rank with the 'flat' instead of the 'long' bones, but that its thickness would rather bring it into the category in which the carpal or tarsal bones are placed according to the old anatomical character derived from shape.

The head (Plate XXXIX. fig. 1) would be a smooth hemisphere, but that the antero-posterior diameter somewhat exceeds the transverse one. Its surface is unimpressed, and its periphery uninterrupted, save by a small entering notch at the middle of its back part, into which possibly some ligament like the '*ligamentum teres*' may have been implanted. The neck of the femur is short and ill-defined; the upper contour passes from the head to the summit of the great trochanter (Plate XXXVIII. *t*), which is on the same level, in a slightly concave line; the inner contour of the bone descends with a somewhat deeper concavity from the lower periphery of the head to the shaft. This is flattened from before backward, and presents a slightly oblique twist, the head and the outer condyle being on a plane anterior to the trochanter and inner condyle. The great trochanter presents a broad rugged surface, flattened obliquely from above downward and inward, divided into two somewhat flattened facets by a transverse ridge arching upward. The lower facet contracts as it descends, and is continued into the strong outer ridge which descends to the external condyle. The thick rough border of the rest of the trochanter stands out beyond the contiguous parts of the femur; least so at the upper, and most at the back part of the process, where the border defines outwardly a small but deep trochanterian fossa (ib. *f*). This fossa is bounded below by a small tuberosity. The small trochanter is represented by a rough ridge, 6 inches long, 2 inches broad, occupying the middle third of the shaft a little anterior to its inner border. A few shorter longitudinal ridges occur on the fore part of the shaft between the upper end of the small, and the upper and fore part of the great trochanter. The rest of the anterior surface of the femur is smooth, concave lengthwise, slightly convex transversely.

The back part of the femur presents a small low tuberosity below its middle part, near the inner border; the triangular surface between this tuberosity, the head, and the great trochanter, is smooth and flat. The contour of the back part of the femur, from the head to the outer condyle, is convex; that from the trochanter to the inner condyle is concave; the lower half of the back part of the shaft is convex transversely. The lower end of the femur (Plate XXXIX. fig. 2) presents two articular surfaces, the inner one (ib. *i*) being that of the internal condyle, the outer one being the combined ectocondyloid (*e*) and rotular (*r*) surfaces. The latter is extensive, and describes a semicircle from before backward, but is narrow from side to side; in this direction the rotular portion is slightly concave; its limits are indicated by a notch on the inner side: the condyloid portion is slightly convex transversely, in which direction the extent is scarcely 2½ inches. The entocondyloid surface (*i*) is of nearly twice that breadth; is of a reniform shape, and is convex in every direction. The intercondyloid channel is roughened by decussating ligamentous impressions and ridges, with some vascular pits. Its narrowest part is anterior, and measures an inch and a half across; posteriorly it is 3 inches across. The intercondyloid border of the inner condyle is sharply defined and projects below the level of the canal, that of the outer condyle is mostly on a level with the canal. The surface of the canal meets the hind surface of the femur at almost a right angle, the intervening ridge being rounded off. The fore part of the inner condyle stops short of the fore part of the shaft; the back part of the condyle projects 1 inch beyond the back part of the shaft. The process or prominence above the inner condyle (*eminentia entocondyloidea*, Plate XXXVIII. *ic*) forms an obtuse angle, the lower side of which is rough, rather flattened, and expands as it descends towards the articular part of the condyle. The outer condyle does not project so far back as the inner one: its supracondyloid prominence (*ec*) is larger, of a similar angular form, but is bent forward as well as outward.

In the Elephant, Mastodon, and Diprotodon, the shaft of the femur is flattened from before backward; but the length of the femur so far exceeds its breadth, that, strong as the thigh-bones of these quadrupeds are and well-proportioned to the weight they had to sustain, they appear weak and even slender when placed by the side of the femur of the Megatherium. The Rhinoceros, which has the thigh-bone relatively broader and flatter than in the proboscidian pachyderms, differs more markedly from the Megatherium in the presence of the third trochanter.

The shaft of the femur is more or less flattened in all the species of *Bruta*; but the Orycteropus and Armadillos, in which this character is conspicuous, differ, like the Rhinoceros, from the Megatherium in having the third trochanter. This process is not present in the Pangolins (*Manis*), Anteaters (*Myrmecophaga*) or Sloths (*Bradypus*); but in all these genera the femur is relatively longer and more slender than in the Megatherium, and only the Sloths amongst existing Mammals, not marine, repeat the remarkable megatherioid character of the absence of a medullary cavity in the shaft of the femur.

The general characters of the femur of the Megatherium are most closely repeated in that of the Mylodon[*] and Scelidotherium[†]. In these genera, however, instead of the shallow notch, there is a deep and prolonged fossa for the ligamentum teres on the middle of the hind border of the head of the bone. The post-trochanterian depression is relatively larger in the Mylodon, the small trochanter is relatively less and higher placed; the whole femur is longer in proportion to its breadth, and the distal expansion is relatively less. Here, also, the articular surfaces offer a well-marked character of distinction; the rotular articular surface is continuous with that of both condyles, and unites them anteriorly.

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[*] Owen 'On the *Mylodon robustus*,' 4to. p. 111. pl. 17.

[†] Ib. 'Fossil Mammalia' of the 'Voyage of the Beagle,' 4to. pl. 25. fig. 5.

From the cast of a distal epiphysis, transmitted by Dr. Harlan to the Museum of the Royal College of Surgeons, London, as of the Megalonyx, it would appear that this extinct megatherioid offered a third modification of the knee-joint, the rotular surface being distinct from those of both condyles. Thus the knee-joint of the Mylodon must have had one large synovial capsule, that of the Megatherium two, and that of the Megalonyx three such sacs.

The patella (Plate XXXVIII. fig. 2, and Plate XVII. *66'*, Philosophical Transactions, 1855) is a strong, thick, subtrihedral conical bone, with the base rounded, and also the angle between the two rough outer sides. In the natural position of the bone the base is uppermost, and chiefly composed of a strong tuberosity coarsely and irregularly striated: at the lower half of the outer surface the striæ have a longitudinal and subparallel direction, giving that part of the bone the appearance of an ossified fibrous ligament. On the inner and broadest side the articular surface occupies the upper two-thirds: it is less distinctly divided by a median longitudinal rising into two channels than in the Mylodon, being more nearly level and uniform. The non-articular surface below the joint is irregularly grooved and perforated by vascular canals.

The fabella*, or post-tibial sesamoid bone (Plate XVII. *67'*, Philosophical Transactions, 1855), is a smaller subhemispheric bone, with a circular, slightly concave articular surface, which was applied to part of the outer condyle of the femur: the rest of the surface is rough and fibrous, indicative of the imbedding of the bone in a flexor tendon of the leg.

The tibia and fibula (Plate XVII. *66*, *67*, *tom. cit.*) become anchylosed together at both extremities in the Megatherium: they are both short, and the tibia presents massive proportions corresponding with those of the femur. Its proximal end (Plate XXXIX. fig. 3) presents two distinct and well-marked articular surfaces; the inner one (*i*) is concave, the outer one (*e*) is convex: the extent of these surfaces corresponds with the breadth of the articular part of the outer and inner femoral condyles respectively. The back part of the outer facet which bends downward affords an articulation (*f*) to the fabella. The rough interspace between the articular surfaces is a little concave transversely, and convex from before backward; its breadth equals that of the outer surface: it developes no intercondyloid process for crucial ligaments.

The fore part of the proximal end of the tibia presents a large irregularly triangular rugged protuberance for the ligamentum patellæ; the back part, below the outer condyle, developes a smaller but more prominent rugged process, the apex of which overhangs the upper part of the interosseous space: between this 'post-tibial' process and the rough inner border of the bone there is a deep and wide longitudinal channel inclining a little obliquely to the interosseous space. The shaft of the tibia gradually contracts to its middle, and as gradually expands to its distal end. It is subcompressed from before backward; is smoother behind than in front: there is a longitudinal channel on each side the back part of the lower end of the tibia, which forms a convexity between them.

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* Owen, 'Archetype of the Vertebrate Skeleton,' 8vo. p. 190.

The anterior surface is divided by a ridge extending obliquely from the rotular protuberance to the inner malleolus: the surface on the fibular side of this ridge is smoother than the other, which seems to have been wholly given up to musculo-tendinous attachments. The inner malleolus (Plate XXXIX. fig. 4, *m*) is a slight expansion below the confluence of the inner and oblique anterior ridges: it does not project below the level of the distal articular surface. This is deeply concave, and is divided into two facets by the deeper hemispherical excavation near its inner side for the reception of the inner protuberance of the astragalus: this excavation (*e*) gives to the larger and shallower facet (*i*) a full crescentic figure. The smooth surface is sometimes continued upon, sometimes interrupted by, a narrow tract from the vertical surface upon the malleolar end of the fibula, which surface is applied to the outer facet of the astragalus. There is a small orifice for a medullary artery at the middle of the back part of the tibia, but it does not open into any medullary cavity: the bone is cancellous throughout.

The fibula is thickest at its upper end, where it has a trihedral form; the outer surface is convex and rough, the inner and hinder surfaces are concave and smooth, meeting at a sharp interosseous border directed obliquely backward: this border is slightly thickened and produced at its middle part, where the shaft of the fibula is compressed: it augments in thickness and resumes its trihedral shape as it descends, and terminates in a moderately produced outer malleolus (*f*) with a very rugged surface, except where it articulates with the astragalus.

In the Mylodon, as well as in the Megalonyx and Scelidotherium, the tibia and fibula continue separate, a fact affecting the value of the evidence which CUVIER deduced from their anchylosed condition in the Megatherium in favour of its affinities to the Armadillos, to which this structure is peculiar amongst existing Mammals. But, since it is known only in the order *Bruta*, it forms an interesting additional proof of the essential relations of the huge extinct animal under description to that now anomalous group of Mammals. The tibia of the Mylodon is proportionally shorter and thicker than in the Megatherium: the outer articular surface at the upper end is of a subcircular form and slightly concave: in the Megalonyx it is convex, but in a less degree than in the Megatherium. The lower articular surface in both Mylodon, Scelidotherium, and Megalonyx, presents the additional facet for the fibula. The hemispheric excavation on the inner side of the distal articulation is relatively larger in the Mylodon than in the Megatherium. This excavation, with the concomitant protuberance of the astragalus, is peculiar to the great extinct Sloth-like quadrupeds; in which so secure an interlocking of the foot with the leg bespeaks some habits peculiar to them, connected with the requirement of unusual resistance in the foot to the forces acting upon it from the leg and thigh.

In the series of existing animals Man presents the plantigrade foot in which the weight of the body presses most nearly upon the crown of the tarsal arch; but, in the Megatherium, owing to the length of the heel and the shortness of the toes, the leg transmits the superincumbent weight nearly upon the middle of the foot. So singularly shaped and adjusted, however, are the tarsal bones in the Megatherium, that the tibia articulates with the side instead of the summit of the tarsus, so that the whole foot is turned inward and rests upon its outer edge instead of its sole (Plate XLI. fig. 1, and Plate XVII. *cl*, *a*, Philosophical Transactions, 1855).

The number of tarsal bones is reduced to six, through the absence of the entocuneiform*; but the astragalus, and especially the calcaneum, are developed to a great size.

The astragalus (Plates XL. & XLI. fig. 1, *a*) is of a peculiarly irregular form: if the foot be placed with the sole flat on the ground, as in Plate XL., the chief articular surface (*a*) for the tibia looks inward, and the small fibular facet, at right angles therewith, is uppermost. The extensive surface by which it articulates with the bones of the leg is divided into three parts, the planes of which are at right angles to each other. The middle and largest division (*a*), answering to the outer ridge of the trochlear surface in the common form of astragalus, is here expanded into a broad reniform smooth tract, horizontal in the ordinary position of the Megatherium's foot (Plate XLI. fig. 1, *a*), almost flat from before backward, convex from side to side. This surface is continued over the outer edge upon the outer side of the bone, in a triangular form, with the apex rounded off, to form the facet (*o*) for the fibula. The surface answering to the main part of the trochlea and its inner malleolar facet in ordinary astragali, is here reduced to a small triangular convexity (*ib.* & Plate XL. fig. 1, *i*), forming the third and internal division of the surface, and supported on what appears to be an obtuse pyramidal process from the inner and lower part of the bone. This convexity is wedged into the deeper excavation on the inner part of the tibial articular surface, and forms a kind of pivot on which the foot worked.

The under or calcaneal side of the astragalus slopes from behind downward and forward, and is divided by an oblique groove, about an inch broad, into two facets; the outer and posterior one is for the calcaneum exclusively; the inner and anterior one is for the calcaneum, but is continuous with the surface for the naviculare. The outer calcaneal surface is ovate, concave lengthwise, convex across its posterior broader end, nearly flat at its anterior end. The inner calcaneal surface is of much less extent and is nearly flat; its anterior end suddenly bends forward and upward to be continued into the outer convex part of the navicular surface, which surface is divided into this convex portion and a contracted subcircular concavity. The lower part of the convex facet rests upon an articular concave surface on the cuboides.

Only a small proportion of the outer surface of the astragalus is non-articular, and this is chiefly on the outer, or, in the megatherian position of the foot, the upper surface (Plate XLI. fig. 1). The groove dividing the calcaneal surfaces begins at their lower part and passes backward and inward, with slight terminal bends, which give it the form of the italic *ſ*. [illegible] by a ridge from a wider channel, excavating the under part of the inner process; from which a third channel extends backward, dividing part of the inner calcaneal facet from the naviculo-cuboid surface. Both this channel and the calcaneal one are perforated by conspicuous vascular canals.

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* [illegible]

The calcaneum (Plates XL. & XLI. fig. 1, *b*) is a long irregular pyramidal bone, with an obtuse apex: the base is obliquely truncated, forms the fore part of the bone, and supports two articular surfaces (Plate XLI. fig. 2): the upper and outer surface (*as*) is ovate, concave where that on the astragalus is convex, and *vice versâ*: the oblique channel which divides this astragalar surface from the astragalo-cuboidal surface (*as'*) is as deep as in the astragalus, and, like it, is perforated by vascular canals. The slightly concave triangular inner astragalar facet (*as'*) is continued below, at an obtuse angle, into the semielliptic slightly concave surface (*cb*) for the os cuboides. External to the astragalar facet a strong vertically extended tuberosity (Plate XLI. fig. 1, *t*) forms the fore part of a wide and deep tendinal canal (*u*); a narrow and feeble ridge bounds it behind; a second and more shallow groove (*v*) succeeds; and, behind this, is a wide and deep vascular perforation (*w*). The under part of the calcaneum is very rugged and flat. The obtuse hinder extremity of the heel-bone shows by its sculpturing and the outstanding osseous spiculæ, the force with which the attached cable-like 'tendo Achillis' must have acted on so unusually produced a calcaneal lever.

The os naviculare (Plates XL. & XLI. fig. 1, *c*) is a transversely oblong bone, compressed from before backward. Its posterior surface is occupied by the articulation for the astragalus, which is equally divided into an inner concavity and an outer convexity, the latter approaching the conical form. From the lower part of this is continued at a right angle a small flat triangular surface for the cuboides (Plate XL. fig. 3, *cb''*). The upper non-articular surface is narrowest at its middle, and is developed into a low oblong tuberosity on each side. On the fore part of the bone (Plate XL. fig. 3) is the articular surface, divided into the narrow oblong tract (*cm*), with two slight convexities, for the mesocuneiforme (*ib.* fig. 1, *f*), and into the almost flat triangular tract (fig. 3, *ci*) for the ectocuneiforme (fig. 1, *e*). The latter surface (fig. 3, *ci*) is divided by a very narrow non-articular tract from the cuboidal surface (fig. 3, *cb''*). The inner (tibial) part of the anterior surface is non-articular, and extends beyond the mesocuneiform articulation to where the entocuneiforme would have been, had that bone existed in the tarsus of the Megatherium.

The mesocuneiforme (Plates XL. & XLI. fig. 1, *f*) is a laterally compressed reniform bone, with a thick rough convex inferior border: the inner (tibial) surface (Plate XL. fig. 1, *f*) is irregular and flattened: the similarly modified outer (fibular) surface (Plate XL. fig. 2, *f*) is varied by a flat elliptic articular facet (*iii*) near its upper part, for a similar surface on the side of the metatarsal of the 'digitus medius.' The back part of the bone is chiefly occupied by the narrow surface for the naviculare. The fore part of the bone is obtuse and rough. Not a vestige of the toe (digitus secundus), usually supported by the mesocuneiforme, is developed in the Megatherium.

The ectocuneiforme (Plates XL. & XLI. fig. 1, *e*) presents the normal wedge-like figure of the tarsal cuneiform bones. It is flattened from before backward; with its thick base (*e*) rough and convex. The posterior surface presents the almost flat, slightly concave articular surface for the naviculare. The outer border or surface is rough and tuberous; the inner one less rough and flat. The anterior convex facet (Plate XL. fig. 2, *iii*) for the base of the metacarpal does not extend to the upper or under borders of the bone. The articulation with the contiguous cuneiforme, as with the cuboides, is by syndesmosis.

The cuboides (Plate XLI. fig. 1, *d*) presents an articular surface divided into three facets on its upper, or tibial, and back part; the anterior and smallest facet is for the naviculare (*c*), the middle and largest for the astragalus (*a*), and the posterior for the calcaneum (*tb*): the latter surface is at almost a right angle with the astragalar one, and looks backward. On the under, or fibular, and fore part of the bone is the articular surface for the two outer metatarsals; that for the fourth toe (Plate XL. fig. 2, *iv*) being concave transversely and slightly convex lengthwise; that for the fifth toe (ib. *v*) being uniformly but very slightly convex.

A broad non-articular surface, rough and with two oblique low ridges on the upper and outer part of the bone, divides the back from the front articular surface; a narrower non-articular tract, but produced into a strong obtuse ridge, divides the same surfaces on the inner or under side of the bone. The fore part of the bone is produced into an angular process (Plate XL. fig. 2, *p*), which forms the inner part of the articular channel for the fourth metatarsal. The under part of the bone is impressed by the broad tendinal groove continued from that which impresses the outer part of the calcaneum.

The above-described composition of the tarsus of the Megatherium has been deduced from the study of three entire specimens of the bones of the hind foot; and it demonstrates that the digits of that foot were but three in number, and that they answered to the 'third,' 'fourth,' and 'fifth' of the pentadactyle type. Not a rudiment of the 'second' exists, and every vestige of the first, together with the cuneiform bone supporting it, is absent. There are no little bones missing on the inner side of the 'mesocuneiforme,' as Dr. PANDER conjectured might be the case in the Madrid skeleton; and there is no 'os cuneiforme' for the hallux ('grand doit du pied'), as CUVIER supposed.

Metatarsus.—The metatarsal of the 'third' toe (Plates XL. & XLI. fig. 1, m_{iii}) resembles rather an 'os cuneiforme,' by reason of its extreme shortness, or fore-and-aft compression. Its upper non-articular surface is the broadest, and is rough and convex, like that of the ectocuneiforme. The proximal or posterior surface for that cuneiform bone is triangular and slightly concave (Plate XLI. fig. 3, *ci*); the bone is prolonged into a tuberous process beneath it, forming a lever of advantage for the insertion of a flexor tendon. On the inner side of the bone, at its upper part, is the small flat surface (*cm*) adapted to that on the mesocuneiforme; on the outer side of the bone is a larger surface, partly convex, partly concave, for articulating with the side of the base of the fourth metacarpal (Plate XLI. fig. 1, m_{iv}); the rest of the outer surface is very irregular, as if honeycombed. The distal or anterior articular surface presents a vertical median prominence, passing into a partly flat surface internally, and into a vertically concave surface externally. The lower part of the vertical prominence is most produced (*ib.* fig. 2, *p*).

The fourth metatarsal (Plate XLI. fig. 1, m_4) has its base compressed laterally, and expanded vertically; but the distal end is still more produced in that direction, so as to be almost hammer-shaped. The vertically extended articular surface at the base of this metatarsal is narrow, convex transversely, and adapted to the channel in the cuboides (Plate XL. fig. 2, *p iv*). On the inner or tibial side of the base is the articular surface, concave posteriorly, less convex anteriorly, for the interlocking joint with the 'third' metatarsal. In front and below this articular surface is a very rough honeycombed tract of bone for firm syndesmotic junction with the similarly modified surface of the third metatarsal.

On the outer or fibular side of the base are two articular surfaces for the fifth metatarsal: the hinder and smaller one is flat, and is continued, at right angles, with the basal surface; the larger surface is subcircular and rather undulating: in front and below these surfaces is a narrow rugged tract for ligamentous junction with the fifth metatarsal. The part of the shaft of the fourth metatarsal which stands out free is smooth upon its inner and upper sides, is traversed by a wide oblique tendinal groove below, and is rather rough and irregular externally.

The vertically produced distal surface presents a large rough protuberance at its upper part (Plate XL. fig. 1, *iv*), and three protuberances at its under part, of which the innermost is the most produced, the outermost the least. The articular surface resembles in character that on the end of the third metatarsal, but is relatively smaller; it consists of a longitudinal median prominence, continuous with a smooth narrow tract on the inner side, on which surface articular cartilage and synovial membrane seem to have existed at only a small part at its lower end. On the outer side, the smooth tract is limited to the upper half of the prominence; this is slightly concave vertically or lengthwise. It is a form of articulation calculated rather for firm and unyielding junction than for flexibility.

The outermost metatarsal (Plate XLI. fig. 1, m_5), answering to the fifth of the pentadactyle foot, is the longest, but from its more backward articulation with the tarsus it does not reach so far forward as the fourth.

The proximal end is a free rough tuberosity, somewhat more than an inch anterior to which, on the tibial side, is the oval slightly concave articular surface adapted to the cuboides (Plate XL. fig. 2, *v*), which surface is continuous with the short transverse flattened surface for the fourth metatarsal; and, in advance of this, is the larger, subcircular, slightly undulated surface for the corresponding surface on the fourth metatarsal. The second surface is on the middle of the inner surface of the fifth metatarsal, and is bordered in front and beneath by the very rough honeycombed surface for ligamentous junction with the adjoining metatarsal. This combination of synovial with syndesmotic joints admits of that degree of slight elastic movement of the very firmly attached fourth and fifth metatarsals, which must have facilitated the heavy tread of the ponderous quadruped, and have alleviated the effect of the enormous pressure on the two gradatorial toes. The whole outer and under part of the non-articular surface of the fifth metatarsal is strongly sculptured by irregular ridges, tubercles, grooves, and foramina, indicative of the hoof-like callosity of the outer border of the sole in which it was chiefly imbedded. The upper and inner non-articular surfaces are comparatively smooth. The inner side is flat, and is traversed by an oblique shallow (tendinal?) groove. On the distal surface are two small articular facets; the inner one, which is the best marked, is subcircular, about 8 lines in diameter.

Phalanges.—The bones of this class are unusually reduced in number in the foot of the Megatherium, even admitting the accuracy of the figures of the hind foot of the Madrid skeleton, in which two stunted phalanges appear to terminate both the fourth and fifth toes. For, as the great unguiculate toe, like that of the fore-foot, has only two moveable phalanges, the total number of these bones is but six, not exceeding that of the tarsal bones of the same foot.

The phalanx (Plate XLI. figs. 1 & 3, $_{1\,\&\,2}$) of the innermost toe (*iii*), answering to the third of the pentadactyle foot, represents, as in the corresponding digit of the fore foot, the proximal and middle phalanges connate. The compound bone (*ib.* fig. 3, $_{1\,\&\,2}$) is shaped like a wedge with an oblique edge, deeply notched. It has articular surfaces not only on its proximal and distal ends, but upon its upper or dorsal surface. The two former surfaces, which are the 'ends' of the bone in ordinary phalanges, here form the 'sides' of the wedge: the sides of the phalanx are the 'margins' of the wedge: the dorsal surface forms the base, the plantar or under surface is represented by the two processes of the cleft apex or edge of the wedge. The proximal articular surface presents a longitudinal channel, convex vertically, concave transversely, from which a flat surface extends from nearly the whole of the tibial side, and a convex surface from the upper part of the fibular side; the whole articular surface being the counterpart of that on the metatarsal (ib. *m*), with which the present remarkable bone is, by this interlocking joint, firmly united. The lower prominence of the median rising of the distal joint of the metatarsal (*p*) protrudes through the lower notch in the phalanx (*n*). A yielding, elastic, slightly-sliding movement was all that could take place between these bones. The complex distal articulation is adapted to an equally restricted junction with the enormous terminal phalanx ($_3$); it consists of four distinct articular surfaces, two on the anterior and two on the upper part of the connate phalanges ($_1$ & $_2$). The internal of the two distal surfaces is the largest and is slightly concave, the external one, near the upper and outer angle of the bone, is slightly convex; they are divided by a rough tract, indicating strong ligamentous union with the claw-phalanx, of half an inch in breadth; and they cover the smaller proportion of the distal surface of the bone. Below the outer articular surface there is a protuberance, with a smooth but non-synovial surface in the present bone, which is adapted to a definite smooth surface upon the claw-phalanx. The two upper surfaces are subcircular, each about half an inch in diameter, very slightly convex, and about [illegible] lines apart.

The great terminal phalanx of the present toe (Plates XL. & XLI. *iii*, $_3$) is modified, like its homotype on the fore foot, for the firm fixation and development of a powerful claw, the hollow base of which covered the bony core, and was encompassed by a bony sheath of 5 inches in length.

The base of the phalanx, whence both core and sheath extend forward, is a surface of a long, narrow, vertically elliptical form, with the upper third produced backward so as to overhang or cover the confluent supporting phalanges; and the lower third sloping forward at nearly the same angle with the vertical middle third, but terminated by a backwardly projecting ridge. There are two articular surfaces on the middle division, two on the upper division, and one on the lower division of the base. The innermost of the middle articulations is slightly convex, the outermost as slightly concave; they both look directly backward. The lower articular surface is flat and subcircular; it is on the hinder and outer angle of the lower third of the base, and looks downward and a little backward; the two small surfaces on the upper third of the base are subcircular and flat, and look downward: these different articular surfaces are counterparts of those on the distal and upper parts of the connate phalanges. On each side of the lower third of the base of the claw-phalanx is a large canal, leading forward to the interspace between the bony core and sheath of the claw, and giving passage to the vessels and nerves of the formative matrix of that instrument. The tibial side of the sheath (Plate XL. fig. 1, $_3$) is gently convex; the fibular side (Plate XLI. fig. 3, $_3$) is flatter: a similar modification affects the claw-core, the narrow basal part of which is divided by sharp borders from the sides. The point of the core projects about 3 inches in advance of the lower border of the sheath: its form, broken in the specimen, is restored in dotted outline in the figures.

The distal articular surface of the fourth metatarsal shows that it supported a phalanx so articulated with it as to have no movements of flexion or extension, and only a slight degree of bending from side to side. The illustrations of the Madrid skeleton, especially plate 5, fig. 5, show that this phalanx (Plate XLI. fig. 1, *iv*, $_1$) was very short, and that it supported a second phalanx (ib. *iv*, $_2$) of a subhemispheric form, terminated obtusely.

The distal articular surface of the fifth metatarsal indicates that it supported a phalanx smaller than the proximal one of the fourth toe; and the figure, above cited, of the Madrid skeleton shows such a phalanx (ib. *v*, $_1$), and also a second small stunted hemispheric phalanx (ib. *v*, $_2$)*; and this, from the analogy of the Mylodon, is most probably the true structure.

If we contemplate the bones of this singularly constructed foot in their natural co-adaptation, the same relation of the osseous masonry to the transference of pressure from the leg to the outer border of the foot will be appreciated, as has been pointed out in the fore limb. The broad surface of the astragalus—the great keystone of the tarso-metatarsal arch (Plate XLI. fig. 1, *a*)—transmits the superincumbent weight in two chief directions, backward upon the massive heel-bone (*b*), forward upon the metatarsus. By the naviculare (*c*) it is transmitted through the ectocuneiforme (*e*) and the produced outer angle of the base of the mid-metatarsal (m_3) to the fourth (m_4), and thence to the fifth metatarsal (m_5). The cuboides (*d*), receiving the weight from both astragalus and naviculare, transmits it by its produced fore part to the base of the fourth metatarsal; and partly by that medium, but chiefly by direct articulation, to the side of the base of the fifth metatarsal. The tendency in the cuboides to yield under this pressure and slip back, is resisted by the abutment of the calcaneum (*b*, *t*) against its back part.

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* In the original memoir, by Bru and Garriga, on the Skeleton of the Megatherium at Madrid, the metatarsus and toes of the hind foot are said to agree with those of the fore foot, except that there is only one toe with a claw, instead of three ("Tambien se advierte que en este hay solo un *Dedo* con uña, quando en la *Mano* se registran tres; en lo demas convienen en un todo," p. 16). In the description of the outermost (fourth developed) toe of the fore foot it is stated that it has two phalanges, and that there is nothing to be remarked except that they are rounded ("Los dos *Falanges* del *quarto* que se reconocen en J. y L. no tienen cosa que advertir mas que son casi redondas," p. 18).

Comparison of the Bones of the Hind Foot.

No known recent Mammal offers in its astragalus any repetition of the peculiarities of that bone in the Megatherium. In the Anteaters and Armadillos the upper surface of the astragalus has the usual configuration, and is received into a deep tibio-peroneal mortice. In the Sloths the outer part of the astragalus is excavated by a deep cell, in which the pivot-shaped end of the fibula rotates: the inner side is applied, as usual, against a malleolar process of the tibia. There is an analogy in the pivoted part of the articulation, but the process and the cavity are on reverse parts of the ankle-joint, as compared with the Megatherium.

The convex protuberance or pivot on the inner half of the tibial surface of the astragalus is common to all the extinct Megatherioids hitherto discovered, but it is associated with well-marked modifications of the bone in each genus, and with minor differences in different species. In the Mylodon* the calcaneal surface is single, and is continuous with the navicular one; no part is insulated by a bisecting groove, as in the Megatherium: the middle division (*a*) of the upper articular surface is less convex; the upper half of the navicular surface is flat, instead of being concave.

The astragalus of the Scelidotherium† agrees with that of the Mylodon in the less depth of the middle division of the upper surface, and the more open angle at which it joins the inner convexity; it agrees with that of the Megatherium in the division of the calcaneal surface; but it differs from both in the presence of two deep concavities upon the naviculo-cuboid surface, the portion to which the cuboides articulates being concave instead of convex.

As the modification of the calcaneal surface of the astragalus governs that of the co-adapted surface in the calcaneum, this bone, in the Mylodon, is distinguished by the uninterrupted continuity of the articulation presented to the astragalus, with which that for the cuboides is continuous; the posterior part of the astragalar surface is less convex than in the Megatherium: the posterior prolongation of the calcaneum is relatively shorter and is less pointed: the posterior wall of the great outer tendinal groove is more developed in the Mylodon. In the Scelidotherium the posterior termination of the calcaneum is broader, and terminated by a less angular convexity than in the Mylodon; but in the separation of the posterior from the anterior part of the astragalar surface it agrees with the Megatherium and with the Sloths. These, of all recent *Bruta*, most resemble the Megatherioids in the posterior prolongation of the calcaneum, and more especially the Megalonyx in the compression and distal expansion of that lever; but, in the Megalonyx, this expansion in the vertical direction is extreme, and gives the heel-bone more the form of an os ilium than of a tarsal bone.

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* "Description of the Skeleton of the *Mylodon robustus*," 4to, 1842, p. 117. pls. 21, 22, & 23. figs. 1 & 2.

† "Fossil Mammalia," Voyage of the Beagle, 4to, pl. 25.

In the os cuboides of the Mylodon the two surfaces for the fourth and fifth metatarsals are nearly on the same plane; in the Megatherium they are nearly at a right angle. In the Scelidotherium the surface by which the cuboides joins the astragalus is convex, instead of concave, as in the Megatherium and Mylodon. In the latter genus the surface on the os naviculare for the mesocuneiform bone is relatively broader than in the Megatherium; and the upper division of the astragalar surface is flat, instead of being convex.

With respect to the digits, it is significant of the true affinities of the Megatherium to find that the Sloths alone amongst the existing *Bruta* show a suppression of certain toes: in all the other existing genera of the order the hind foot retains the typical number of digits.

The Armadillos, including the *Chlamyphorus*, resemble the Megatherium in the terminal confluence of the tibia and fibula; but these bones articulate in the *Dasypodidæ* in the normal way with the astragalus, and the foot is planted firmly on the ground by its flat surface. The hallux is not only present in the *Chlamyphorus*, but is longer than the fifth toe. All the five toes are armed with claws; and anomalous as the three-toed hind feet of the Sloths may appear, they have more real resemblance with the feet of the Megatherioids than have those of any of the Armadillo family. The great extinct *Glyptodon* agrees with the *Chlamyphorus* in the pentadactyle condition of the hind foot, the os naviculare having three facets for as many cuneiform bones, the innermost of which supports a well-developed hallux. This toe is abortive in existing Sloths, and is represented by a small compressed bone, consisting apparently of the connate ento-cuneiform and metatarsal: it retains its distinctness in the Unau, as the similar representative of the second toe does in the Megatherium; and the meso- and ecto-cuneiform bones are also distinct; but in the Ai each cuneiform bone is confluent with its metatarsal, with each other, with the naviculare, and with the cuboides.

In the Megatherium, the mutilation of the foot has commenced on its outer side by the removal of the ungual phalanx from the fifth and fourth toes; in the Sloths the mutilation is restricted to the fifth toe, but is carried further, viz. to the removal of all the phalanges and the reduction of size of the metatarsal: nevertheless, the Sloths alone, amongst existing *Bruta*, manifest in any degree the characteristic megatherioid condition of the extreme toes of the pentadactyle series: and the minor modifications of this character, differentiating the existing from the extinct phyllophagous *Bruta*, are such as intelligibly relate to the different powers and habits of creatures so different in size and mode of progression.

The mutilation of the two outer toes in the Megatherium is accompanied by modifications which adapt them to the important office of the support and progression of the body on level ground: in the scansorial Sloths, the three middle digits being equally developed for prehensile purposes, and none being needed for walking, one toe on the outer and one on the inner side of the foot is reduced to the metatarsal basis, and is concealed beneath the skin. In the Megatherium the hallux and its cuneiform bone are wanting: the second toe is represented by its cuneiform bone, with, perhaps, a connate rudiment of a metatarsal: only the third toe can be compared, by its size and the claw it supports, with the condition of the three unguiculate toes in the Sloths; and, like those of the Ai, it has but two moveable phalanges. The reduction of the claws to one in the hind foot of the Megatherium, its enormous size and strength, its secure attachment to the phalanx, and the solidity of the articulations of the constituent bones of the whole toe, relate to the contrasted mode of obtaining the leafy food in the colossal extinct Sloth as compared with the diminutive climbing species which still exists. If the hind foot were put by the Megatherium to the preliminary work of exposing and loosening the roots of the tree about to be prostrated, its efficiency for that purpose would be greater by having but one claw, than if it had two or three: for the single claw, like a pickaxe, would clear away the soil from the interstices of the root-ramifications, the more easily by not being associated with a second contiguous claw, impeding such operation by striking upon the root itself.

Physiological Summary.

To sum up the results of the foregoing descriptions of the known fossilized parts of the extinct giant of the order *Bruta*, as indicative of its habits and affinities. The teeth agree in number, kind, mode of implantation, and growth, with those of the Sloth (*Bradypus*), and their structure is a modification of that peculiar to the same genus. All the modifications of the skull relating to the act of mastication, especially the large and complex malar bones, repeat the peculiarities presented by the existing Sloths. There are the same hemispheric depressions for the hyoid bone in the Megatherium as in the Sloth. In the number of cervical vertebræ, the Megatherium, like the Two-toed Sloth, agrees with the Mammalia generally *. In the accessory articular surfaces afforded by the anapophyses and parapophyses of the hinder dorsal and lumbar vertebræ†, the Megatherium resembles the Anteaters (*Myrmecophaga*‡); but it does not resemble the Armadillos (*Dasypus*§) in having long metapophyses, the peculiar development of which in those loricated *Bruta* has a direct relation to the support of their bony dermal armour. In the mesozygapophyses of the middle dorsal vertebræ*, the Megatherium is peculiar amongst Mammalia. In the small extent of the produced and pointed symphysis pubis it resembles the Sloths; and in the junction of both ilium and ischium with the sacrum, it manifests a character common to the Edentate order; but in the expanse and massiveness of the iliac bones, it can only be compared with other extinct members of its own peculiar family of phyllophagous *Bruta*. The habits of the Megathere necessitating a strong and powerful tail, we find this resembling in its bony structure that of other *Bruta* with a similar appendage, especially in the independency of the two hæmapophyses of the first caudal, a character which obtains in the Anteaters† and in some Armadillos; but this is no evidence of direct affinity to either of those families: the habits of the small arboreal Sloths render their eminently prehensile limbs sufficient for their required movements, and the tail is wanting. Had that appendage been proportionally as large as in the Megatherium, we cannot suppose that the caudal vertebræ would have materially differed from those of other *Bruta*.

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* Philosophical Transactions, [illegible], Plate XVII. 6.

† Ibid. Plate XXVI. *a* and *p*.

‡ Ibid. 1851, Plate XLIX. fig. 20.

§ Ibid. Plate XLIX. figs. 18, 19.

In the coalescence of the anterior vertebral ribs with the bony sternal ribs, the Megatherium resembles the Sloths. This essential affinity is still more marked in the peculiarities of the scapula and of the carpus. In the *Myrmecophaga jubata*, the scaphoid is distinct: in the *Manis* it coalesces with the lunare: in the *Dasypus gigas* the trapezoides is anchylosed to the second metacarpal: in the *Das. sexcinctus* it has coalesced with the trapezium. Not any of these characteristics are manifested by the Megatherium: its carpus repeats the peculiarities of that in the Sloths, viz. the reduction of the number of carpal bones to seven by the coalescence of the scaphoid with the trapezium. The first digit (pollex), which is retained in the Anteaters and Armadillos, is obsolete in the Megatherium, as in the Sloths and Orycteropus: three digits are fully developed and armed with claws, as in the *Bradypus tridactylus*; and the fifth, though incomplete in the Megatherium, is better developed, because it was required in the ponderous terrestrial Sloth for its progression on level ground. In no existing ground-dwelling member of the *Bruta* is the fifth digit deprived of its ungual phalanx, as in the Megatherium. The bones of the fore foot of that extinct animal are thus seen to be modified mainly after the type of the *Bradypodidæ*.

The long bones of all the limbs are devoid of medullary cavities, as in the Sloths. The [illegible] the ligamentum teres, as in the Sloths. The fibula is anchylosed to the tibia [illegible] Megatherium, as in *Dasypus*; but this is not the case in the closely-allied extinct Megatherioids called Mylodon, Megalonyx, and Scelidotherium, a fact which diminishes the force of the argument which Cuvier deduced from the coalesced condition of the bones in the Megatherium in favour of its affinities to the Armadillos. The semi-inverted but firm interlocking articulation of the hind foot to the leg shows the peculiarities of that joint in the Sloths exaggerated, and departs further from its characteristics in other *Bruta*. In all the existing members of the order, save the

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[*] Philosophical Transactions, 1855, Plate XIX. *ms.*

[†] Ibid. 1851, Plate LIII. fig. 30.

Sloths, the hind foot is pentadactyle, and four of the toes have a long claw, even in the little arboreal *Myrmecophaga didactyla*: the departure by degradation from the pentadactyle type is a peculiar characteristic of the Sloth-tribe in the order. It is carried further in the same direction in the Megatherium and other great extinct terrestrial Sloths.

Guided by the general rule that animals having the same kind of dentition have the same kind of food, I conclude that the Megatherium must have subsisted, like the Sloths, on the foliage of trees; but that the greater size and strength of the jaws and teeth, and the double-ridged grinding surface of the molars in the Megatherium, adapted it to bruise the smaller branches as well as the leaves, and thus to approximate its food to that of the Elephants and Mastodons. The Elephant and the Giraffe are specially modified to obtain their leafy food; the one being provided with a proboscis, and the entire frame of the lofty ruminant adapting it to browse on branches above the reach of its largest congeners. If the Megatherium possessed, as CUVIER conjectured, a proboscis, it cannot, judging from the suborbital foramina, have exceeded in size that of the Tapir, and could only have operated upon branches brought near the mouth. Of the use of such a proboscis in obtaining nutritious roots, on the Cuvierian hypothesis that such formed the sustenance of the Megatherium, it is not easy to speculate: the Hog's snout might be supposed to be more serviceable in obtaining those buried parts of vegetables; but no trace of the prenasal bone exists in the skull of the Megatherium. A short proboscis might be useful in rending off the branches of a tree when prostrated and within reach of the low and broad-bodied Megatherium, but this office has been provided for by the organization of the tongue, of which, both the hyoid skeleton by its strength and articulation, and the foramina for the muscular nerves by their unusual area, attest the great size and power.

As regards the limbs, the Megatherium differs from the Giraffe and Elephant in the unguiculate character of certain of its toes, in the power of rotating the bones of the fore arm, in the corresponding development of supinator and entocondyloid ridges on the humerus, and in the possession of complete clavicles. These bones are requisite to give due strength and stability to the shoulder-joint for varied actions of the fore arm, as in grasping, climbing, and burrowing. But they are not essential to scansorial or fossorial quadrupeds: the Bear and the Badger have not a trace of clavicles, and merely rudiments of these bones exist in the Rabbit and the Fox. We must seek, therefore, in the other parts of the organization of the Megatherium, for a clue to the nature of the actions by which it obtained its food. In habitual burrowers the claws can be extended in the same plane as the palm, and they are broader than they are deep. In the Megatherium the depth of the claw-phalanx exceeds its breadth, especially in the large one of the middle finger; and not any of the claws can be extended into a line with the metacarpus, but they are all more or less bent inward and downward. Thus, although they might be used for occasional acts of scratching up the soil, they are better adapted for grasping; and the whole structure of the fore foot militates against the hypothesis of Pander and D'Alton[*], that the Megatherium was a burrowing animal.

The same structure equally shows that it was not, as Dr. Lund[†] supposes, a scansorial quadruped; for, in the degree in which the foot departs from the structure of that of the existing Sloths, it is unfitted for climbing; and the outer digit is modified, after the ungulate type, for the exclusive office of supporting the body in ordinary terrestrial progression. It may be inferred from the diminished curvature and length, and from the increased strength and the inequality of the claws, especially the disproportionately large size of that weapon of the middle digit, that the fore foot of the Megatherium was occasionally applied by the short and strong fore limb in the act of digging: but its analogy to that of the Anteaters teaches that the fossorial actions were limited to the removal of the surface-soil, in order to expose something there concealed, and not for the purpose of burrowing. Such an instrument would be equally effective in the disturbance of roots and of ants; it is, however, still better adapted for grasping than for delving. But to whatever task the partially unguiculate hand of the Megatherium might have been applied, the bones of the wrist, fore arm, arm, and shoulder, attest the prodigious force which would be brought to bear upon its execution. The general organization of the anterior extremity of the Megatherium is incompatible with its being a strictly scansorial or exclusively fossorial animal, and its teeth and jaws decidedly negative the idea of its having fed upon insects; the two extremes in regard to the length of the jaws are presented by the phyllophagous and myrmecophagous members of the order *Bruta*, and the Megatherium in the shortness of its face agrees with the Sloths.

Proceeding, then, to other parts of the skeleton for the solution of the question as to how the Megatherium obtained its leafy food, it may be remarked that the pelvis and hind limbs of the strictly burrowing animal, *e. g.* the Mole, are remarkably slender and feeble, and they offer no notable development in the Rabbit, the Orycterope, or other less powerful excavators. In the climbing animals, as, *e. g.*, the Sloths and Orangs, the hind legs are much shorter than the fore legs; and even in those Quadrumana in which the prehensile tail is superadded to the sacrum, the pelvis is not remarkable for its size or the expansion of its iliac bones. But, in the Megatherium, the extraordinary bulk and massive proportions of the pelvis and hind limbs arrest the attention of the least curious beholder, and became to the physiologist eminently suggestive of the peculiar powers and habits of the animal. The enormous pelvis was the centre whence muscular masses of unwonted force diverged to act upon the trunk, the tail, and the hind legs, and also by the 'latissimus dorsi' on the fore limbs. The fore foot being adapted for scratching as well as for grasping, may have been employed in removing the earth from the roots of the tree and detaching them from the soil: but the hind foot, which, like a pickaxe, had but one strong perforating and digging pointed weapon, was more probably the instrument mainly employed in removing the earth from the ramifications of the root. The fore limbs, terminated each by three claws, appear to have been more especially adapted for grasping the trunk of a tree; and the forces concentrated upon them from the broad posterior basis of the body must have cooperated with them in the labour, for which they are so amply organized, of uprooting and prostrating the tree. To give due resistance and stability to the pelvis, the bones of the hind legs are extraordinarily and massively developed, and the strong and powerful tail must have concurred with the two hind legs in forming a tripod, as a firm foundation for the vast pelvis, thus providing adequate resistance to the forces acting and re-acting from and upon that great osseous centre. The large processes and capacious spinal canal indicate the strength of the muscles which surrounded the tail, and the vast mass of nervous fibre from which those muscles derived their energy. The natural co-adaptation of the articular surfaces shows that the ordinary inflection of the end of the tail was backward, as in a *cauda fulciens*, not forward, as in a *cauda prehensilis*. Dr. LUND's hypothesis, therefore, that the Megatherium was a climber and had a prehensile tail, is destroyed by the now known structure of that part.

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[*] Das Riesen-Faulthier, &c. fol. 1821, p. 16.

[†] Blik paa Brasiliens Dyreverden för sidste Jordomvæltning, af Dr. Lund, 4to. Kjöbenhavn, 1838, p. 21.

But viewing the pelvis of the Megatherium as being the fixed centre towards which the fore legs and fore part of the body were drawn in the gigantic leaf-eater's efforts to uprend the tree that bore its sustenance, the colossal proportions of its hind extremities and tail lose all their anomaly, and appear in just harmony with the robust claviculate and unguiculate fore limbs with which they combined their forces in the Herculean labour.

Finally, with reference to the hypothesis of the German authors and artists* of the gradual degeneration of the ancient Megatherioids of South America into the modern Sloths, it may be admitted that the general results of the labours of the anatomist in the restoration of extinct species, viewed in relation to their existing representatives of the different continents and islands, have been such as might naturally suggest the idea that the races of animals had deteriorated in point of size. Thus the palmated Megaceros is contrasted, by its superior bulk, with the Fallow-deer, and the great Cave-bear with the actual Brown Bear of Europe. The huge Diprotodon and Nototherium afford a similar contrast with the Kangaroos and Koalas of Australia, as do the towering Dinornis and Palapteryx with the small Apteryx of New Zealand. But the comparatively diminutive aboriginal animals of South America, Australia, and New Zealand, which are the nearest allies of the gigantic extinct species respectively characteristic of such tracts of dry land, are specifically distinct, and usually by characters so well marked as to require a subgeneric division, and such as no known outward influences have been observed to produce by progressive alteration of structure. Moreover, as in England, for example, our Moles, Water-voles, Weasels, Foxes, and Badgers, are of the same species as those that coexisted with the Mammoth, Tichorhine Rhinoceros, Cave Hyæna, Bear, &c.; so likewise the remains of small Sloths and Armadillos are found associated with the Megatherium

* Pander and D'Alton, *loc. cit.*

and Glyptodon in South America; the fossil remains of ordinary Kangaroos and Wombats occur together with those of gigantic herbivorous Marsupials in Australia; and there is similar evidence that the Apteryx coexisted with the Dinornis in New Zealand. I have been led, therefore, to offer the following suggestions as more applicable to, or explanatory of, the phenomena than LAMARCK's progressive hypothesis of the origin of species by transmutation*, or BUFFON's retrograde hypothesis by degradation†.

"In proportion to the bulk of an animal is the difficulty of the contest which, as a living being, it has to maintain against the surrounding influences which are ever tending to dissolve the vital bond and subjugate the organized matter to the ordinary chemical and physical forces. Any changes, therefore, in the external circumstances in which a species may have been adapted to exist, will militate against that existence in probably a geometrical ratio to the bulk of such species. If a dry season be gradually prolonged, the large Mammal will suffer from the drought sooner than the small one; if such alteration of climate affect the quantity of vegetable food, the bulky Herbivore will first feel the effect of the stinted nourishment; if new enemies are introduced, the large and conspicuous quadruped or bird will fall a prey, whilst the smaller species might conceal themselves and escape. Smaller quadrupeds are usually more prolific than larger ones. The actual presence, therefore, of small species of animals in countries where the larger species of the same natural families formerly existed, is not to be ascribed to any gradual diminution of the size of such larger animals, but is the result of circumstances which may be illustrated by the fable of the 'oak and the reed;' the small animals have bent and accommodated themselves to changes under which the larger species have succumbed‡."

EXPLANATION OF THE PLATES.

PLATE XXXVII.

Fig. 1. Side view of the pelvis: one-fifth the natural size.
Fig. 2. The acetabulum. (By permission of the President and Council, from the specimen presented by Sir WOODBINE PARISH, K.H., to the Museum, of the Royal College of Surgeons.)

PLATE XXXVIII.

Fig. 1. Back view of the left femur.
Fig. 2. Back view of the left patella.

One-third the natural size.

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* Philosophie Zoologique, 8vo. 1809, tom. i. ch. vii.

† Histoire Naturelle, 4to. tom. iv. (1766), "Dégénération des Animaux," p. 311.

‡ "On the genus *Dinornis*" (part 4), Transactions of the Zoological Society, vol. iv. p. 15.

PLATE XXXIX.

Fig. 1. Proximal end of the femur.
Fig. 2. Distal end of the femur.
Fig. 3. Proximal end of the tibia.
Fig. 4. Distal ends of the anchylosed tibia and fibula.
One-third the natural size.

PLATE XL.

Fig. 1. Tibial side of the bones of the foot.
Fig. 2. Front view of the two cuneiform and the cuboid bones.
Fig. 3. Front view of the navicular bone, and parts of the astragalus and calcaneum.
One-half the natural size.

PLATE XLI.

Fig. 1. Upper view of the bones of the foot.
Fig. 2. Front view of the calcaneum.
Fig. 3. Fibular side of the bones of the great unguiculate toe.
One-half the natural size.

The letters and figures on the several Plates are explained in the text.

XXXIII. *Conductibility of Mercury and Amalgams. By* F. CRACE-CALVERT, *Esq.*, *F.C.S.*, *Mem. Roy. Acad. of Turin, &c. &c., and* RICHARD JOHNSON, *Esq.*, *F.C.S.*, *Mem. Phil. Soc. of Manchester. Communicated by Professor* STOKES, *Sec. R.S.*

Received April 14,—Read May 19, 1859.

WE have the honour to lay before the Royal Society the second part of our paper on the Relative Powers of Metals, Alloys, and Amalgams to conduct Heat. Having in our former paper described our experiments upon metals and their alloys, we now give the results obtained with mercury and amalgams.

The method followed in the investigations described in this paper is the same as that detailed in our former one. When the amalgams were solid, we melted and cast them in square bars, and filed them down until they were 1 c. m. square and 6 c. m. long; but when the amalgam was fluid, we introduced it into the small iron box (see former paper, Philosophical Transactions for 1858, p. 354) and determined its conducting power.

Before stating the results obtained with amalgams made of pure metals in equivalent and multiple quantities, we wish to draw attention to the remarkable manner in which heat is conducted by mercury.

But before entering into the details of our experiments, it is necessary that we should state that, having completed our researches some time since, we forwarded the results to the Junior Secretary, Professor G. G. STOKES, for presentation, when he kindly made to us the remark that mercury might be a worse conductor than we had found it to be, and that the means adopted by us were not sufficient to prevent the mass of mercury in the little iron box becoming heated through currents; and he suggested that we should tilt our apparatus, and ascertain what would be the influence of various angles on the conductibility of mercury as determined by our method.

By following out this suggestion, we were led to the interesting discovery that mercury is the worst conducting metal known, when the heat is so applied as to prevent the mass becoming heated by currents.

To attain this object, we filled our little iron box with pure mercury, and having ascertained by its weight that it was quite full, we introduced 1 cub. cent. of it into each of the vulcanized caoutchouc vessels; we then poured 50 cub. cent. of cold water, and waited until it had arrived at the temperature of the atmosphere of the laboratory. The larger vessel was in its turn filled with 200 cub. cent. of water at 90°. The apparatus was so arranged that the large vessel, or the source of heat, was placed perpendicularly over the small one. The temperature of the large one was maintained at 90° during one quarter of an hour by a small jet of steam brought into it (for further details see the first part of these researches, page 350 of the Philosophical Transactions for 1858), when we obtained the following results:—

	Temperature of the 75 cub. cent at beginning of experiment	Temperature of 75 cub cent. at end of 15 minutes	Conductibility found reduced to 50 cub cent. water	Mean	Silver 1000
Mercury vertical...	14°·8 12·6	16°·0 13·7	1·80 1·65	1·7	54

We also tilted the apparatus and gave it gradually different angles, and the conductibility of heat by mercury gradually decreased as the angle increased, showing the following results:—

		Silver 1000.
At a slight angle	13·5	423
Angle slightly increased . .	7·3	229
Angle still more increased .	6·9	216
Considerable angle . . .	5·1	160

Lastly, the apparatus was used as in our former experiments, the little box being placed in a horizontal position, and the results agreed with those already published; for we obtained

	Mean.		Silver 1000.
21·6 21·8 21·5	21·63	or	679

There cannot therefore be a doubt that the supposed good conductibility of heat by mercury arose from not taking into account that mercury being a fluid, its facility to conduct heat was owing to currents. The same may be said of water; for we have observed, as is already known, that it presents a complete barrier to the conduction of heat when the source of heat is applied at the upper part of a column of water.

Thus in our experiments we have found that the temperature of the water in the lower vessel did not rise one-tenth of a degree during the quarter of an hour that the water in the upper vessel was maintained at 90° C.

The bad conductibility of heat by fluids when all currents are prevented in their mass, appears to us difficult to explain by the theories of undulation or radiation; for we cannot understand why the imponderable fluid caloric should not travel equally well between the molecules in whatever way the source of heat is applied, or why the undulations should not be as rapid, nay, more rapid in a fluid than in a solid. All these difficulties disappear if we adopt the views of Mr. J. P. JOULE, F.R.S., which are, that heat is conveyed in bodies by the vibrations of the solid molecules composing them.

The remarkably low conducting power of mercury presents another point of interest, as it establishes a further analogy between heat and electricity. The ratio of conductibility of these two agents by mercury, as compared with that of silver, shows such close relations when examined under the same volumes, that they deserve especial notice. Thus—

	Heat.	Electricity.
Silver	100·00	100·00
Mercury	5·33	2·12

On the Conductibility of Solid and Semi-solid Amalgams, or in which exists an excess of the Amalgamated Metal.

The amalgams belonging to this series were prepared with equivalent quantities of pure metals, and their conductibility for heat confirms the figure (54) which we now publish as representing the conductibility of mercury, silver being 1000. In fact the observed conductibility of heat by these amalgams agrees perfectly with the theoretical quantities, whilst there exists a great difference between them when we adopt 677 as representing the conductibility of mercury. The calculated numbers are obtained, as in the former paper (p. 358), by multiplying the conducting powers of the respective metals by the weights, and dividing by the sum of the weights.

Amalgams of Tin.

Formula of amalgams, and per-centages.	Exterior temperature.	Temperature of the 50 cub. cent. of water before experiment.	Temperature of the 50 cub. cent. of water after 15 minutes, from 5 to 5 minutes.			Found.	Mean.	Calculated, mercury being 1·7.
$HgSn_2$ Hg 45·88 Sn 54·12	15·0 14·8	14·5 14·6	18·0 18·2	20·8 20·9	23·1 23·3	8·6 8·7	8·65	8·11
$HgSn_3$ Hg 36·18 Sn 63·82	15·0 15·0	15·1 15·5	18·9 19·4	21·9 22·3	24·5 25·0	9·4 9·5	9·45	9·2
$HgSn_4$ Hg 29·84 Sn 70·16	14·0 15·0	13·1 14·9	16·9 18·8	20·1 21·9	22·8 24·5	9·7 9 6	9·65	9·95
$HgSn_5$ Hg 25·38 Sn 74·62	16·0 16·0	14·9 15·11	19·1 19·4	22·5 22·8	25·2 25·4	10·8 10·3	10·6	10·5

Amalgams of Zinc.

Formula of amalgams, and per-centages.	Exterior temperature.	Temperature of the 50 cub. cent. of water before experiment.	Temperature of the 50 cub. cent. of water after 15 minutes, from 5 to 5 minutes.			Found.	Mean.	Calculated, mercury being 1·7.
$HgZn_2$ Hg 60·63 Zn 39·37	......	16·0 15·3	19·6 18·9	22·8 22·2	25·6 25·1	9·6 9·8	9·70	8·97
$HgZn_3$ Hg 54·70 Zn 45·30	......	16·1 17·1	19·7 20·9	23·4 24·5	26·6 27·5	10·5 10·4	10·45	10·05
$HgZn_4$ Hg 43·50 Zn 56·50	......	14·8 17·5	18·4 21·0	22·1 25·2	25·4 28·4	11·1 10·9	11·00	12·08
$HgZn_5$ Hg 38·11 Zn 61·89	......	17·1 16·2	22·6 21·7	27·2 26·3	31·0 30·2	13·9 14·0	13·95	13·05

Amalgams of Bismuth.

Formula of amalgams, and per-centages.	Exterior temperature.	Temperature of the 50 cub. cent. of water before experiment.	Temperature of the 50 cub. cent. of water after 15 minutes, from 5 to 5 minutes.			Found.	Mean.	Calculated, mercury being 1·7.
$HgBi_2$ Hg 31·82 Bi 68·18	15·5 15·5	15·2 15·3	16·9 17·1	18·4 18·5	19·8 20·0	2·1 2·2	2·15	1·87
$HgBi_3$ Hg 23·86 Bi 76·14	15·0 15·0	14·8 14·7	15·7 15·6	16·6 16·6	17·4 17·3	2·6 2·6	2·6	1·89
$HgBi_4$ Hg 19·03 Bi 80·97	14·5 14·8	14·9 15·0	15·8 15·8	16·7 16·6	17·5 17·5	2·6 2·5	2·55	1·90
$HgBi_5$ Hg 15·82 Bi 84·18	13·0 13·5	13·6 13·4	14·35 14·25	15·2 15·1	15·9 15·8	2·3 2·4	2·35	1·91

On Amalgams which contain an excess of Mercury.

These amalgams, also prepared in equivalent quantities, were all fluid, owing to the circumstance that the proportions per cent. of mercury predominated over those of tin, zinc, and bismuth. The conduction of heat by these amalgams was therefore determined in the small iron box placed perpendicularly, and the source of heat applied at the upper part of the column; and this mode of operating has led us to observe the curious and interesting fact, that all this class of amalgams have the same, or nearly the same, conducting power, viz. from 1·9 to 2·3, although the proportions of tin vary in them from 10·52 to 22·98; those of zinc from 6·09 to 13·97; and those of bismuth from 17·55 to 34·73.

All the results obtained with these amalgams being within the limits of 1·9 to 2·3, we think it useless to give the details of the experiments.

Conductibility of Compound Bars.

In our former paper we described some experiments which we had made with bars composed of small cubes of copper soldered alternately with cubes of zinc, tin, and lead, having 1 c. m. of surface, and we showed that such compound bars conducted heat as indicated by theory. Since then we have pursued our researches, and have found that when compound bars are made of cubes of copper and bismuth, or of copper and antimony, then they conduct heat no longer in relation with the calculated numbers, as for instance—

		Found.	Calculated.
No. 1 bar	3 cubes of Copper, 1 cub. cent. 3 cubes of Antimony, 1 cub. cent.	9·45	17·32
No. 2 bar	3 cubes of Copper, 1 cub. cent. 3 cubes of Bismuth, 1 cub. cent.	3·40	13·25*

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* *Added during printing.*—These numbers were calculated in the same manner as in the case of

It will no doubt be remembered that we also experimented upon bars which, instead of being composed of cubes, were made of two longitudinal bars of copper, juxtaposed and soldered to two other longitudinal bars of either tin, zinc, or lead, and that all these bars conducted heat as if they had been entirely composed of pure copper, and had not contained half their bulk of tin, zinc, or lead. We have made, since those results were published, a great number of experiments with the hope of throwing some light on the above interesting fact, but we regret to say without success. We have, however, noticed a result which deserves to be recorded; it is, that a bar composed of 2 of bismuth and 2 of antimony, juxtaposed and soldered together, is the only one which conducts heat in the same ratio as indicated by theory; for example,—

$$\text{No. 3 bar}\left\{\begin{array}{l}\text{2 bars of Bismuth}\\ \text{2 bars of Antimony}\end{array}\right\}\quad \begin{array}{c}\text{Found.}\\ 3{\cdot}90\end{array}\qquad \begin{array}{c}\text{Calculated}\\ 3{\cdot}63\end{array}$$

We have also ascertained that the fine film of solder existing between the blades exerts no influence whatever on the proportion of heat conducted by the compound bars, for we have—

$$\text{No. 4 bar soldered}\left\{\begin{array}{l}\text{2 of Copper}\\ \text{2 of Zinc}\end{array}\right\}\text{gives } 26{\cdot}85.$$

$$\begin{array}{l}\text{No. 5 bar, in which gutta percha was employed}\\ \quad\text{to keep together the four small blades}\quad .\quad .\end{array}\left\{\begin{array}{l}\text{2 of Copper}\\ \text{2 of Zinc}\end{array}\right\}\text{gives } 26{\cdot}35.$$

---

alloys (see page 833). It has been pointed out to us that in the case of the bars composed of different metals placed end to end, the theory of the conduction of heat leads to the following simple result:—the resistance of the whole bar, multiplied by its length, is equal to the sum of the specific resistances of the separate metals multiplied by their respective lengths, the resistance being measured by the reciprocal of the conductibility. This gives for the conductibilities of the bars Nos. 1 and 2, the reciprocal of the mean of the reciprocals of the conductibilities of the two component metals. Taking the numbers given in the former paper, copper (rolled) 26·95, antimony (mean of two) 6·485, bismuth 1·95, we thus find for bar No. 1, 10·45, for No. 2, 3·04, which do not greatly differ from the numbers given by experiment.

XXXIV. *On the Construction of Life-Tables, illustrated by a New Life-Table of the Healthy Districts of England.* *By* W. Farr, *Esq., M.D., F.R.S.*

Received March 17,—Read April 7, 1859.

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THE Transactions of the Royal Society contain the first Life-Table. It was constructed by HALLEY, who discovered its remarkable properties, and illustrated some of its applications. The Breslau observations did not supply HALLEY with the data to frame an accurate Table, for reasons which will be immediately apparent; but the conception is full of ingenuity, and the form is one of the great inventions which adorn the annals of the Royal Society.

Tables have since been made correctly representing the vitality of certain classes of the population; and the form has been extended so as to facilitate the solution of various questions.

In deducing the English Life-Tables from the National Returns, I have had occasion to try various methods of construction; and I now propose to describe briefly the nature of the Life-Table, to lay down a simple method of construction, to describe an extension of its form, and to illustrate this by a new Table representing the vitality of the healthiest part of the population of England.

The Life-Table is an instrument of investigation; it may be called a *biometer*, for it gives the exact measure of the duration of life under given circumstances. Such a Table has to be constructed for each district and for each profession, to determine their degrees of salubrity. To multiply these constructions, then, it is necessary to lay down rules, which, while they involve a minimum amount of arithmetical labour, will yield results as correct as can be obtained in the present state of our observations.

1. GENERAL DESCRIPTION OF A LIFE-TABLE. (See Table C, p. 870.)

A Life-Table represents a *generation of men* passing through *time*; and time under this aspect, dating from birth, is called age. In the first column of a Life-Table *age* is expressed in *years*, commencing at 0 (birth), and proceeding to 100 or 110 years, the extreme limit of observed life-time.

If we could trace a given number of children, say 100,000, from the date of birth, and write the numbers down that die in the first year, living therefore less than one year, against 0 in the Table, and on succeeding lines the numbers that die in the second, third, and every subsequent year of age until the whole generation had passed away, these numbers would form a *Table of Mortality*, showing at what ages 100,000 lives become extinct.

Again, if the 100,000 children were followed, and the numbers living on the first, on the second, and on every subsequent birthday until none was left, the column of numbers would constitute a *Table of Survivorship*. So if of 100,000 children born at a given point of time, the numbers dying (d_x) in each subsequent year were written in one column, and the numbers surviving (l_x) at the end of each year in another column, the two primary columns of the Life-Table would be formed.

It is evident that if one of these columns is known the other may be immediately deduced from it; for if of 100,000 children born 10,295 die in the first year of age, 3005 in the second year of age, it follows that the numbers living at the end of one year must be 89,705, at the end of two years 86,700. Upon adding the column (d_x) from the bottom up to the number against any age (x), the sum will represent the whole of the numbers *dying after that age*; and consequently the numbers *living at that age*, as shown in the collateral column (l_x).

The 100,000 children born at the same moment, and counted *annually* to determine the numbers *living* at *the end of every year*, would by our Table completely pass away in less than 107 years. If another generation of 100,000, born a year afterwards, were followed, the numbers dying in the various years of age would not be very different, the circumstances remaining the same; and the numbers of those entering each year of age would vary inconsiderably from those of the first series. If 100,000 children again were born at annual intervals, and were subject to an invariable law of mortality, they would form a community of which the numbers living at each age would be represented by the sucessive numbers (l_x) in the Life-Table. The sum of these numbers, by the new Table of Healthy Districts, would be 4,951,908. The births are here assumed to take place simultaneously at annual intervals; immediately before the births, therefore, in such a community its population would be 4,851,908, to which it would fall progressively from 4,951,908 by 100,000 successive deaths in the year. The average number constantly living would be some number between 4,951,908 and 4,851,908; and it would be very nearly the mean of these limiting numbers.

In the ordinary course of nature, the births in a community take place in remittent succession; and if it is assumed that the 100,000 births occur at equal intervals over every year, it is evident that at any given date a certain number will be found living at all the intermediate points of age between 0 to 1 year, 1 to 2, 2 to 3, and all the remaining years of age. The population in the above instance would be found by enumeration to be nearly 4,899,665.

The annual *births* would be 100,000 in such a community. The annual deaths would also be 100,000; and by taking out the deaths at each year of age, from the parish registers of a single year, the second column (d_x) of the Life-Table would be found. By adding this column of deaths up and entering the sum of the numbers year by year against every year of age (x), the third column (l_x) of the Life-Table would be obtained; for it has been already shown that the numbers attaining any age x are equal to the numbers dying at that age, and all the subsequent ages. From the registers of the deaths, a Table of the numbers of the *population living* in a parish *so constituted* could be immediately determined without any enumeration. Its deviations from the truth would be accidental; and they would be set right by taking the mean of many years. So also from a simultaneous enumeration of the *numbers living in each year of age*, the two columns d_x and l_x of the life could be constructed without reference to any registry of the deaths at different ages.

The *mean age at death* in such a community would express the mean lifetime, or the expectation of life at birth; and the product of the number expressing the annual births multiplied into the mean age at death would give the numbers of the population.

The facts which a Life-Table expresses in numbers may be represented by the lines of a figure; age (x) being indicated by the abscissas measured from 0, the *numbers living* (l) at each age by the ordinates of a curve line, and the numbers living between any two ages by the plane surface within the two ordinates, the curve line, and the corresponding portion of the abscissa. The relative numbers living at the ages 20 and 21 are seen in the two lines of Plate XLII. fig. 1, over the ages 20 and 21; if the deaths in the intervening year all occurred immediately after the age 20 was attained, the numbers living would also be represented by the parallelogram having its two sides equal to the ordinate over 21, and for its base the portion of the abscissa between 20 and 21; but if all the deaths occurred only the instant before the age 21 was attained, the height of the parallelogram would be represented by the ordinate over the age of 20. The deaths occur at intervals between the two ages, so the numbers living, and the *lifetime* which is passed between the two ages, are correctly represented by the curvilinear area.

The deaths in each year of age are called the *decrements of life*. They are represented by the differences in the lengths of the successive ordinates. Thus by cutting off a small portion of the ordinate at the age 20, the ordinate at the age 21 is obtained; this small portion, shown in Plate XLII., represents the decrement of life in that year of age. It will be observed that the decrements vary at every year of age; and this is more evident when they are exhibited on the larger scale of Plate XLII. fig. 2. The decrement in the first year is large; in the first five years the decrements of life are considerable; at the age of 10 to 15 they fall to their minimum; slowly increase to the age of 56; increase more rapidly until the maximum is attained at the age of 75; then decline gradually to 85, and after that more rapidly until every life is extinct at the age 107 by this Table.

II. PRINCIPLES OF CONSTRUCTION. THE FUNDAMENTAL COLUMN l.

The conditions of the hypothesis upon which the preceding reasoning rests are never precisely realized in nature; in the first place the number of births fluctuates, increases, or decreases from year to year, and the deaths fluctuate still more; rarely equalling the births in number. Immigration and emigration interfere. Under those circumstances, Tables such as those which HALLEY, PRICE and others made from the observations on the *deaths alone* are never accurate, and require correction to give approximate results. If it be assumed that the law of mortality remains invariable, and that migration does not interfere, then the nature of the correction to be applied to a Table framed from the deaths alone will become immediately apparent by an example. The births increase in England. Let the annual births in a portion of the community be doubled in sixty years, thus be 50,000 in 1796, and 100,000 in 1856; then the deaths of persons of the age of 60 in 1856 must be doubled to obtain the deaths which would have happened at that age if the annual births sixty years before these deaths had been 100,000. If the births have been accurately registered, formulæ for correcting the ordinary Table drawn up from the deaths at different ages will be suggested by the above considerations.

I now proceed to describe another method which has been adopted in framing the Table C, and is applicable wherever (1) the number of annual births, (2) the numbers of the population living at definite periods of age, (3) the deaths at the corresponding ages during a certain number of years, in any community are ascertained by observation. This method is not open to the previous objections.

The aim is to obtain equations which will describe the curve lines (Plate XLII. fig. 1) of the Life-Table, in the most direct way; and these equations may be deduced from the determined rate of mortality at certain intervals of age.

The relative numbers living at two ages, 20 and 21, can evidently be found from an equation which expresses the relation of the average numbers living and dying between those ages during a given time. This can be determined very nearly; for although the ages of the living are not ascertained with exact precision at the census, still by taking all the numbers living at the ages 15, 16, 17 years up to 24 and under 25, together, the aggregate represents very nearly the numbers living in that decenniad of life. The deaths at the same ages are obtained with at least equal accuracy from the registers of deaths. By this process, and by extending the observations over five or more years, a number of facts is obtained sufficiently great to yield average results; and it may be assumed that the ratio of the living at the ages 15—25* to the dying in a year at the same ages 15—25 represents the annual rate of mortality at the exact age 20. So also the mortality rate at the ages 30, 40, 50 and other ages may be determined. As observations grow more exact, and the facts are multiplied, the intervals of age may be diminished to 5 years, and ultimately to 1 year.

In determining the *rate* of *mortality*, a given number of persons living a year is considered equivalent to twice that number living half a year, or to half the number living two years.

Thus if nd represent the deaths in n years out of a number amounting on an average to P during the same years, then $\frac{nd}{nP}=m=$ the rate of mortality, or the proportions of death in a *year* (always taken as the unit of time) out of *one year* of *lifetime*. It is found from all the observations hitherto made on a large scale, that the rate of mortality varies at every interval of age; but at the same age it may for the present purpose be considered invariable under similar circumstances.

m_x therefore varies in every moment of age; but I have employed it to express the mean annual rate of mortality during the year following the year of age x, $\therefore \frac{d_x}{P_x}=m_x$, where d_x indicates the deaths, P_x the year of lifetime, after the year of age x. The m_x is the expression of the force of the causes that induce death, of the death-force, *vis mortalis*; and its reciprocal $\frac{1}{m_x}=u_x$ measures the forces that sustain life, the *vis vitalis*.

The vital force under natural circumstances may by one hypothesis be sufficient to sustain a whole generation alive for seventy or eighty years, and then suddenly collapse. The Life-Table, if this hypothesis were true, would be represented by the *parallelogram* in which the curve of the Life-Table is inscribed (Plate XLII. fig. 1).

By the hypothesis of DEMOIVRE† the rate of mortality is such, that at the age of 20 one in 66 living at the beginning dies before the end of the year, leaving 65, 64, 63, 62, 61 to enter on each year of age until at the age of 86 all are dead.

Upon this hypothesis the relative numbers living up to the age 86 form an arithmetical progression: and the deaths in the equal times are equal out of the diminishing numbers living. The rate of mortality increases on this hypothesis as age advances in the same ratio as $n-\frac{1}{2}:1$; where n is the difference between the actual age x and 86. It is called the complement of life. The Life-Table, upon this hypothesis, has equal decrements, and might be represented on Plate XLII. fig. 1, by drawing a diagonal line through the parallelogram. Its deviation from the true curve on this scale is evident; but it is also evident that a series of straight lines, which would nearly represent the true curve, may be drawn from point to point of all the ordinates.

If the causes of death act with equal intensity at all ages, they may be represented by any simple external cause, destroying an equal *proportion* of the numbers living in equal intervals of time. Thus, if 1600 men were distributed equally over ground where they were exposed to certain dangers represented by successive discharges of musketry which at every discharge shot down one-half of the numbers remaining, they would be reduced successively from 1600 to 800, to 400, to 200, to 100, to 50, and so on *ad infinitum*, if a fraction of a living man could be conceived: the numbers living at each year of age in a Life-Table would not decrease at *these rates*, but they *would decrease* at a constant rate if the dangers at every stage of life remained *constant* and equally *great*. The numbers of the living at successive ages would be in geometrical progression, and would be represented by the ordinates of the logarithmic curve.

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* By this 15 and *under* 25 years of age is understood, and so in all similar cases.

† See Treatise of Annuities on Lives, Preface to 2nd Edition.

The law of mortality can only be derived from observation, and it is found to be less simple than either of these hypotheses implies. It can, however, be represented nearly by equations at different periods of age. Upon inspecting Table A (p. 864), it will be seen that at the age 55—65, which may be represented by the exact age 60, the mortality is such, that 2162 women die in a year out of a number equal to 100,000 living a year; and the mortality, which is the ratio of the dying to the living in a unit of time, here set down as a year, is therefore $m = \cdot 02162$. Again, the mortality at the age of 70 is ·04992; at the age of 80 it is ·11866, and at the age of 90 it is ·26711. The mortality increases rapidly, and is more than doubled every ten years. The four numbers differ little from the terms of a geometrical progression, the logarithms of which have a constant difference. Let the rate at which the mortality increases be r, and $r^{10} = 2\cdot 3116$, and the first term (m) be ·02177; then a series of numbers will be formed differing little from those which express the value of m at decennial intervals of age.

Values of m at the precise age x.—*Females.*

Age (x).	60.	70.	80.	90.
By observation . . .	·02162	·04992	·11866	·26711
By hypothesis . . .	·02177	·05033	·11633	·26891

Note.—It may be assumed that m at 60 is the mean value of m in its range from $m_{54\frac{1}{2}}$ to $m_{64\frac{1}{2}}$; and so in other cases.

The *annual rate* of the increase of m from the age of 55 to 95 is $r = 1\cdot 0874$; and if m is the mortality at any age after 55, then $m_z = mr^z =$ the mortality at z years after the age at which m is taken. The common logarithm of r is $= \lambda r = \cdot 03639$.

The mortality (m) of males at corresponding ages is higher than the mortality of females; but the rate of increase as age advances is nearly the same.

The value of m for females at the age of 20 is ·00765, and the mortality increases at the rate of nearly one-seventh part every ten years. The exact value of r is $1\cdot 0149$, and $\lambda r = \cdot 006423$.

Values of m.—*Females.*

Age.	20.	30.	40.	50.
By observation . . .	·00765	·00894	·00998	·01192
By hypothesis . . .	·00760	·00882	·01022	·01185

By these observations in the healthy districts the mortality (m) of men at the ages 15 to 45 is lower than the mortality of women at the same ages; yet during that period the rate of increase r is nearly the same for the two sexes. From the age of 40 to 50, and 50 to 60, the mortality of males increases at a rate intermediate between the rates of manhood and mature age.

Females.

Limits of ages.				
15 to 55	or	20 to 50	$r=1{\cdot}0149$	$\lambda r={\cdot}00642$
55 to 95	or	60 to 90	$r=1{\cdot}0874$	$\lambda r={\cdot}03639$

Males.

15 to 45	or	20 to 40	$r=1{\cdot}0148$	$\lambda r={\cdot}00640$
55 to 95	or	60 to 90	$r=1{\cdot}0874$	$\lambda r={\cdot}03640$

The subjoined Table exhibits the series of values for m derived from the hypothesis of two constant rates, and from direct observation. The values of r for females may be evidently applied to males in every period, except in the ten years of age, 40 to 50.

Mortality (m) of *males* and *females*, (1) derived from observation, and (2) from the *hypothesis* that m increases at the preceding rates.

Precise age.	ANNUAL MORTALITY to 100 constantly living at each age (m).			
	Males.		Females.	
	By observation.	By hypothesis.	By observation.	By hypothesis.
20	·691	·696	·765	·760
30	·818	·807	·894	·882
40	·928	·935	·998	1·022
50	1·273	1·083	1·192	1·185
60	2·294	2·329	2·162	2·177
70	5·486	5·385	4·992	5·033
80	12·817	12·451	11·866	11·633
90	28·350	28·785	26·711	26·891
100	40·000?	66·550?	45·000?	62·160?

The observations on the numbers living and dying of the age of 95 and upwards are exceedingly uncertain; and it is probable that many of the persons believed to be 100, are really persons five or ten years younger; so that these values of m, by the hypothetical method, are probably as correct as the direct numbers.

I shall now notice briefly the application of this hypothesis, first suggested by Mr. GOMPERTZ, and applied by him to the interpolation of the Northampton and other Tables*. Mr. EDMONDS, in 1832, extended the "Theory," and applied it to the construction of three Life-Tables†. He gave an elegant formula, similar in principle to that of Mr. GOMPERTZ, from which the curve of a Life-Table can be deduced, upon the above hypothesis.

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[*] Philosophical Transactions, 1825, paper by B. GOMPERTZ, Esq., F.R.S.

[†] Life-Tables founded upon the discovery of a Numerical Law regulating the existence of every Human Being, &c., by T. R. EDMONDS, B.A., 1832.

In the equation $\frac{s}{t}=v$, where s indicates space, t time, v velocity, the units of measure must be fixed before numbers can be inserted in the general expression; and then v will express, in the measure that has been applied to space, the number of such units of space described in *one* unit of time. Here v is a ratio; it is the rate at which the body moves: and in the same manner m, in the equation $\frac{d}{l}=m$, is the *rate of dying*, that is, as I shall express it, the *mortality*; or it is the ratio of the dying to the living in a given unit of time, the time during which the deaths occur being of precisely the same duration as the time during which the living are under observation,

l (living during 1 year) : d (dying during a year) :: 1 (year of life) : m.

If for l the number 100,000 is substituted, it is assumed that immediately a death occurs another life is substituted; and as the time is a year, then 760 will represent the value of d at the age 20, according to the preceding Table; $\therefore m=\cdot 00760$. If the *time*, instead of *one year*, be the *thousandth part* of one year, then $m=\cdot 0000076$; and if the time be infinitely short, m will be infinitely small: m is a ratio; the quantity of life existing during the time is represented by 1, and the quantity of life destroyed by a fraction, m. Whether the life inheres in the first organic molecule after conception, in the infant, or in the man, the vital action has a certain force of continuance, which is constantly varying; and the amount of this *force* that is *extinguished* at a given instant of time will be represented by the force of mortality, namely, by m at that instant. Then let the age $x=z+a$, where a represents the number of years up to the age at which a given rate (r) of increase of m begins; then $z=x-a$. And the mortality at any instant of age, in an instant of time at the end of z years or parts of years, will be mr^z. Now let y represent the living at that precise age; then the decrement of y in an infinitely short time will be $-dy=ymr^z dz$; the dy being negative as it is taken in a direction opposite to that in which the ordinate y of the curve is assumed to be drawn. Transferring y to the other side of the equation, this becomes $-\frac{dy}{y}=mr^z dz$; and integrating both sides, we have ($\lambda_{,}y$ being put for the hyperbolic logarithm of y, and $\lambda_{,}c$ for the difference between the constants of the two integrals)—

$$\lambda_{,}c-\lambda_{,}y=\lambda_{,}\frac{c}{y}=\frac{mr^z}{\lambda_{,}r}; \quad \ldots\ldots\ldots \quad (1.)$$

$$\therefore \quad \lambda_{,}y=\lambda_{,}c-\frac{mr^z}{\lambda_{,}r}, \quad \ldots\ldots\ldots \quad (2.)$$

and

$$\lambda_{,}c=\lambda_{,}y+\frac{mr^z}{\lambda_{,}r}. \quad \ldots\ldots\ldots \quad (3.)$$

When z is made zero, let $y=1$; then $\lambda_{,}y$ will also disappear, and $\lambda_{,}c=\frac{m}{\lambda_{,}r}$. Upon substituting this value of $\lambda_{,}c$ in equation (2.), it becomes

$$\lambda_{,}y=\frac{m}{\lambda_{,}r}-\frac{mr^z}{\lambda_{,}r}=\frac{m}{\lambda_{,}r}(1-r^z). \quad \ldots\ldots\ldots \quad (4.)$$

Upon passing to the numbers, equation (4.) becomes

$y=\varepsilon^{\frac{m}{\lambda_\iota r}(1-r^z)}$ = the value of y (taken as 1 at the origin) at the end of z years.

Let λ denote the common logarithm with the base 10; then $\lambda_\iota y=\frac{\lambda y}{k}$, where k is the modulus of the common system of logarithms; as also

$$\lambda_\iota o=\frac{km}{\lambda r}, \quad \text{and} \quad \frac{mr^z}{\lambda_\iota r}=\frac{kmr^z}{\lambda r}.$$

Equation (2.) becomes, after the required substitutions,

$$\frac{\lambda y}{k}=\frac{km}{\lambda r}-\frac{kmr^z}{\lambda r}$$

and $$\lambda y=\frac{k^2m}{\lambda r}(1-r^z); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.)$$

so the equation becomes finally $$y=10^{\frac{k^2m}{\lambda r}(1-r^z)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.)$$

This is the form given by Mr. EDMONDS, and is convenient for use.

By making z successively 1, 2, 3, ... up to any number less than the number of years of age within which r remains constant, the number l, being known, the number living at any other age within that range will be obtained by multiplying l_x by the corresponding value of y. Thus, if y_{10} is the value of y when $z=10$ in equation (6.); then putting l_{20} for the numbers living at the age 20, the living at the age 30 will be $y_{10}\times l_{20}=l_{30}$.

This hypothesis does not express the facts deduced from the observations exactly. If m_x could be expressed exactly over more than 20 years by $m_x=m_0r^x$, the first differences (δ^1) of the logarithms in the series following would in a certain number of cases be equal.

FEMALES in HEALTHY DISTRICTS of England.

Precise age.	Annual rate of mortality.	Logarithms of the annual mortality.	First decennial differences of λm_x.	Second decennial differences of λm_x.
x.	m^*.	λm.	δ^1.	δ^2.
20	·00765	$\bar{3}$·8835	·0677	—·0197
30	·00894	$\bar{3}$·9512	·0480	·0290
40	·00998	$\bar{3}$·9992	·0770	·1817
50	·01192	$\bar{2}$·0762	·2587	·1047
60	·02162	$\bar{2}$·3349	·3634	·0126
70	·04992	$\bar{2}$·6983	·3760	—·0236
80	·11866	$\bar{1}$·0743	·3524	—·1259
90	·26711	$\bar{1}$·4267	·2265	
100	·45000	$\bar{1}$·6532		

* Here, at the age 20, m is the mean mortality that rules over the age $19\frac{1}{2}$ to $20\frac{1}{2}$ years of exact time.

The inequalities in the second differences vary in every separate class of observations; but there is generally a tendency in the first and in the second differences to increase, over a certain extent of the series. The error of the hypothesis is slight if the rate of increase (r), of which λ ·00677 is the logarithm in the case in hand, is only assumed to remain uniform for the ten years 20 to 30, or for the one year 20 to 21. Now let the number living at the age 20 be represented by l_{20}, and the number living at the age 21 by l_{21}; then put $\frac{l_{21}}{l_{20}}=p_{20}$. Here it is evident that if l_{20} and p_{20} be known, l_{21} is determined immediately by the equation $l_{21}=l_{20}\times p_{20}$. But p_{20} is the value of y in the equation $y_1=10^{\frac{k^2m}{\lambda r}(1-r^z)}$, when z is put $=1$. Taking the numbers from Table A., we have $m=·00765$ at the precise age $20=(19\frac{1}{2}+20\frac{1}{2})\frac{1}{2}$; and $\lambda m=\bar{3}·8835130$; $\lambda r=·0067728$; and $\therefore$ $r=1·015717$; k is put for the modulus of the common logarithms, $\therefore$ $\lambda k^2=\bar{1}·2755686$; $k(\lambda r)$ is the complement of the *logarithm* of (λr).

λk^2	$\bar{1}·2755686$
λm	$\bar{3}·8835130$
$k(\lambda r)$	$2·1692317$
$\lambda(1-r)$	$\bar{2}·1963697$
$-·0033472$	$\bar{3}·5246830$
$\bar{1}·9966528$	

As the factor $(1-r)$ is negative it makes the exponent of 10 negative, and upon taking the complement of this the logarithm of y is found to be $\bar{1}·9966528$. This is also the logarithm of $p_{20}=·99232$; and it enables us to pass, in the construction of a Life-Table, from the living at the age of 20 to the living at 21. If we obtain the several values p_x at every year of age, the whole of the Life-Table can be constructed.

It will be found that p_x is always a fraction, and it does not differ very much from $1-m_x$. But while m_x* shows the *deaths* in a year out of a *unit* of *life* (which may consist of any *number* of individual *lives* constantly kept up), p_x shows how much out of a *unit of the same life* at the beginning of a year, the dead not being replaced, *survives a year* after the age x; and $1-p_x$ is the amount of loss which occurs in the same year out of a unit of life at its commencement. Thus, as $p_{20}=·99232$, it follows that $1-p_{20}=·00768$. In the same year of age 20 to 21 the mortality is $m_{20}=·00771$, or ·00003 more than $(1-p_{20})$. If the unit of life is made 100,000 living at the age 20, then 99232 will survive, and 768 will die in the ensuing year of age. But if it is assumed that the deaths take place at equal intervals, it may also be assumed that the number of lives (100,000) being constantly sustained, the accessions of 768 new lives take place at equal intervals, consequently that they are under observation half a year on an average, giving the equivalent of $\frac{768}{2}=384$ years of lifetime at the age 20 to 21;

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* m serves to indicate the mean mortality in the year following the exact age x.

now out of this number (384) at that age *three* die when the mortality is m_{20}. This accounts for the difference of ·00768 and ·00771; the former occurring in a year out of a unit of life of which the waste is not replaced.

From these considerations it may be inferred that if m_x is known, p_x may be deduced from it upon the hypothesis of equal decrements through the year by the formula $p_x=\frac{1-\frac{1}{2}m_x}{1+\frac{1}{2}m_x}=\frac{2-m_x}{2+m_x}$. Thus m_{20} being ·0077072, we have $\frac{\cdot 9961464}{1\cdot 0038536}=\cdot 99232$ *, as before. The λp_{20} by the previous method is $\bar{1}\cdot 9966528$, and by this method it is the same. By either of the methods the value of p_x may be deduced for the subsequent ages, and $p_{20}, p_{30}, p_{40} \cdots\cdots p_{90}, p_{100}$ will be obtained. These values are here given, and it will be seen that the results by the two methods are nearly identical at all ages, except the two last, when the observations themselves become less exact.

Females.

Age (x).	$\lambda p_x=\lambda y_1=10^{\frac{c m}{\lambda r}(1-r)}$.	$\lambda p_x=\lambda\left(\frac{1-\frac{1}{2}m}{1+\frac{1}{2}m}\right)$.
20	$\bar{1}$·9966528	$\bar{1}$·9966527
30	·9960967	·9960967
40	·9956263	·9956264
50	·9946669	·9946676
60	·9902049	·9902073
70	·9773538	·9773557
80	·9463182	·9462643
90	·8809176	·8801776

It will be observed that the fraction $p=\frac{1-\frac{1}{2}m}{1+\frac{1}{2}m}$ approximates to $1-m$ as m becomes less; for upon developing it into a series, $p=1-m+\frac{1}{2}m^2-\frac{1}{4}m^3+\frac{1}{8}m^4\cdots\cdot$ And taking m infinitely small, the terms after the two first may be neglected.

The values of $m_0, m_1 \ldots\ldots m_4$ may be obtained by the method already described. But it rarely happens that the population living at each year of age is accurately enumerated at the Census; and besides inaccuracies of statement, the numbers living at each of the early years of age fluctuate considerably, so that the numbers of children living of each year of age in 1851 do not represent the average numbers living of those ages in the five years 1849 to 1853, for instance.

The following method is less exceptionable. It may be assumed *for this purpose* (1) that the births registered in the year 1848 represent the births in that year; (2) that the births are equally distributed over the years in which they occur, and consequently

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* m at the precise age 20 is nearly ·00765. The increase in this mortality from the age 20 to $20\frac{1}{2}$, the middle of the year of age 20 to 21 is obtained by adding $\frac{1}{2}\lambda r$, as above given, to $\lambda m_{19\frac{1}{2}}$, that is, to the log of $(m_{19\frac{1}{2}}+m_{20\frac{1}{2}})\frac{1}{2}$; $\therefore m_{20}=\cdot 0077072$

$$\begin{array}{rl} \lambda m_{19\frac{1}{2}} & \bar{3}\cdot 8835130 \\ \frac{1}{2}\lambda r & 0\cdot 0033864 \\ \hline \lambda m_{20} & \bar{3}\cdot 8868994 \end{array}$$

(3) that the *mean date* of all *the births* in the two years 1848, 1849 was immediately before January 1, 1849. The *half* of the births in those two years will consequently represent pretty accurately the number of births out of which the deaths of children *under one year* of age happened in the year 1849. And the deaths and survivors can be followed by this method year by year, as is evident in the annexed scheme:—

Age
0 ½ (births 1848, 1849)=mean annual births of which the mean date is January 1, 1849.
minus deaths under age 1 in 1849
1 =surviving on January 1, 1850.
minus deaths age (1 to 2) in 1850
2 =surviving on January 1, 1851.
minus deaths age (2 to 3) in 1851
3 =surviving on January 1, 1852.
minus deaths age (3 to 4) in 1852
4 =surviving on January 1, 1853.
minus deaths age (4 to 5) in 1853
5 =surviving on January 1, 1854.

By commencing with the mean number of births in the years 1849, 1850, and deducting the deaths, a similar series may be obtained; and thus a succession of similar series may be deduced, the mean of which will supply the ordinary series l_0, l_1, l_2, l_3, l_4, l_5 of a Life-Table.

These series are liable to various disturbances. If all the births are not registered, the *rate* of mortality is overstated. If all the deaths are not registered, or if the children are carried off as emigrants, the decrements of life are understated. The annual number of births fluctuates, and now increases in England; they are in excess also in the early months of the year. Several of the disturbances are slight, and some of them are in opposite directions. The results can also be, and have been, checked by the results of the other method. The value of m_7 and m_{12} are deduced by dividing the annual deaths at the ages 5 to 10 and 10 to 15 by the mean population at those ages. The interpolation of the series λp_x from λp_3 to λp_{20} succeeds; taking λp_3, λp_7, λp_{12}, and λp_{20} as the fixed points of the series, and λp_{12} being adjusted to allow for the turn of the curve.

The Tables A, B, and C supply the data from which the Life-Table of Healthy English Districts was deduced. One or two arithmetical examples of the application of the method adopted in the earlier ages are also supplied.

III. INTERPOLATION.

We have therefore determined the values of λp_x at certain ages. The values of λp_x at the intervening ages may be determined by changing the value of r, and making x successively 1, 2......10 in the formula (p. 846). They may also be interpolated for every year of age by the method of finite differences; and upon the whole this method is preferable to any other. The logarithms of p_x are required; and to them it will be convenient to apply the interpolation directly. Any number of differences beyond four becomes cumbersome, and it will be therefore sufficient to give the general formula, which can be employed in deriving the first of either four or three orders of differences.

Investigation of Formulæ—Intervals equal.

Let any numbers of a series be so related that u_n, the nth from the first, u_0, is determined by the equation (1.)—

$$u_n=u_0\pm\frac{n}{1}\delta^1\underset{+}{+}\frac{n(n-1)}{1.2}\delta^2\pm\frac{n(n-1)(n-2)}{1.2.3}\delta^3\underset{+}{+}\frac{n(n-1)(n-2)(n-3)}{1.2.3.4}\delta^4. \quad . \quad . \quad (1.)$$

δ^1, δ^2, δ^3, and δ^4*, the first differences of the four orders, are unknown; they can all be determined from any five values of u_n. Now let n be successively $1x$, $2x$, $3x$, $4x$; then the coefficients of u_0, u_{1x}, u_{2x}, u_{3x}, u_{4x} can be found, to give the values of δ^1, δ^2, δ^3, and δ^4 in four equations. But when x is ten or more the coefficients become large, and the numerical calculation laborious. It is therefore well to obtain the numerical values of δ^4, δ^3, δ^2, δ^1 in succession. Thus if the series is ascending or descending, the following are convenient forms. The upper rows of signs are used in the *ascending*, the lower rows in the *descending* series:—

$$\delta^4=\frac{\underset{+}{+}u_{4x}\underset{-}{-}4u_{3x}\underset{+}{+}6u_{2x}\underset{-}{-}4u_x\underset{+}{+}u_0}{x^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

$$\delta^3=\frac{\pm u_{3x}\mp 3u_{2x}\pm 3u_x\mp u_0}{x^3}\mp\frac{3}{2}(x-1)\delta^4. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.)$$

$$\delta^2=\frac{\underset{+}{+}u_{2x}\underset{-}{-}2u_x\underset{+}{+}u_0}{x^2}\mp(x-1)\delta^3\underset{-}{-}\frac{(7x^2-18x+11)}{12}\delta^4. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

$$\delta^1=\frac{\pm u_x\mp u_0}{x}\mp\frac{x-1}{2}\delta^2\underset{-}{-}\frac{(x^2-3x+2)}{6}\delta^3\mp\frac{(x^3-6x^2+11x-6)}{24}\delta^4. \quad . \quad . \quad . \quad (5.)$$

It is necessary to be careful in deducing the successive values of δ from the values preceding; and before commencing their use their accuracy should be tested by inserting them in the checking equation,

$$u_{4x}=u_0\pm\frac{4x}{1}\delta^1\underset{+}{+}\frac{4x(4x-1)}{1.2}\delta^2\pm\frac{4x(4x-1)(4x-2)}{1.2.3}\delta^3\underset{+}{+}\frac{4x(4x-1)(4x-2)(4x-3)}{1.2.3.4}\delta^4. \quad . \quad . \quad (6.)$$

x may be any number. If only four terms are given, δ^3 is assumed to be constant; and δ^4 being 0, all the terms into which it enters disappear. The above formulæ, if this is borne in mind, are applicable when δ^4, δ^3, or δ^2 are assumed to be constant, and serve therefore to supply the differences when there are one, two, three, or four orders by the most expeditious method.

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* It will be borne in mind that these imply first differences, or δ^1u_0, δ^2u_0, δ^3u_0, δ^4u_0.

In constructing the Life-Table, x was made 10 from the age of 20, and on inserting the numbers, the equations (2, 3, 4, 5, 6) became

$$\delta^4=\frac{\substack{+\\+}u_{40}\substack{-\\-}4u_{30}\substack{+\\+}6u_{20}\substack{-\\-}4u_{10}\substack{+\\+}u_0}{10{,}000}. \qquad (7.)$$

$$\delta^3=\frac{\substack{+\\-}u_{30}\substack{-\\+}3u_{20}\substack{+\\-}3u_{10}\substack{-\\+}u_0}{1000}\substack{-\\+}13\tfrac{1}{2}\delta^4. \qquad (8.)$$

$$\delta^2=\frac{\substack{+\\+}u_{20}\substack{-\\-}2u_{10}\substack{+\\+}u_0}{100}\substack{-\\+}9\delta^3\substack{-\\-}44\tfrac{1}{4}\delta^4. \qquad (9.)$$

$$\delta^1=\frac{\substack{+\\-}u_{10}\substack{-\\+}u_0}{10}\substack{-\\+}4\tfrac{1}{2}\delta^2\substack{-\\-}12\delta^3\substack{-\\+}21\delta^4. \qquad (10.)$$

The checking equation is

$$u_{40}=\substack{+\\+}u_0\substack{+\\-}40\delta^1\substack{+\\+}780\delta^2\substack{+\\-}9880\delta^3\substack{+\\+}91390\delta^4. \qquad (11.)$$

If three orders of differences are used, the checking equation is

$$u_{30}=\substack{+\\+}u_0\substack{+\\-}30\delta^1\substack{+\\+}435\delta^2\substack{+\\-}4060\delta^3. \qquad (12.)$$

After adding or subtracting any constant to or from a series of numbers, the differences remain the same; and if consecutive terms are multiplied or divided by the same factor, the differences are multiplied or divided by that factor. Thus $(b+a)-(c+a)=b-c$, and $ab-ac=a(b-c)$. Advantage is taken of these properties to reduce any one of the terms in the equations to *zero*.

Thus let the logarithms to be interpolated be the following—values of p_{20}, p_{30}, p_{40}, and p_{50}, taken from the column headed *males*, Table B; then they may, among other ways, be interpolated as follows:—

As $\bar{1}\cdot9969724$ is the contracted expression of $(\cdot9969724-1)$, we have

Age				
20	$\bar{1}\cdot9969724=-\cdot0030276$	(1) Multiplying each term by 10,000,000, that is, striking out the decimal point and the two adjoining ciphers, and (2) then subtracting from each 30,276, the values of $u_x=\lambda p_x$ to be operated on become	$u_0=-00000$
30	$\bar{1}\cdot9964260=-\cdot0035740$		$u_{10}=-\ 5464$
40	$\bar{1}\cdot9959051=-\cdot0040949$		$u_{20}=-10673$
50	$\bar{1}\cdot9943048=-\cdot0056952$		$u_{30}=-26676$

By inserting these values with their negative signs in the equations, and taking the upper signs, the three differences are found; that is,

$$\delta^3=-11\cdot049:\quad \delta^2=101\cdot991;\quad \text{and } \delta^1=-872\cdot7715.$$

The differences are now divided by 10,000,000, that is, ciphers are added to their left-hand side, so that the above decimal point may be moved seven places in that direction, and the operation may be thus commenced. By adding the differences successively to each other and to $\lambda p_{20}=\bar{1}\cdot9969724$, the successive values are found of λp_{21}, λp_{22}, λp_{23} .... λp_{50} up to and including λp_{58} for males, where the series joins naturally the subsequent series, commencing at λp_{59}.

δ^3.	δ^2.	δ^1	λp_t.
—·000,0011,0490	·000,0101,9910	—·000,0872,7715	$\bar{1}$·996,9724,0000
(constant)	·000,0090,9420	—·000,0770,7805	$\bar{1}$·996,8951,2285
		—·000,0679,8385	$\bar{1}$·996,8180,4180
			1·996,7500,6095

In the actual operation the δ^3 is *subtracted* from δ^2, δ^2 from δ^1, and δ^1 from λp_t; it is therefore convenient to substitute for their present values the complements of δ^3 and δ^1, as thus all the series become additive.

As $\lambda l_{20}+\lambda p_{20}=\lambda l_{21}$, and $\lambda l_{21}+\lambda p_{21}=\lambda l_{22}$, and generally $\lambda l_t+\lambda p_t=\lambda l_{t+1}$, it is evident that the λp_t is the *first difference* of the series λl_t; and the whole series, λl_t, from λl_{20} to λl_{59}, may be formed as in the subjoined example, where δ^2 becomes δ^3, δ^1 becomes δ^2, and so on.

Healthy Districts.—Males.

δ^4 (constant)

9·999,9988,9510

Age	δ^3.	δ^2.	$\delta^1=\lambda p_t$.	$u_t=\lambda l_t$.
20	0·000,0101,9910	9·999,9127,2285	9·996,9724,0000	4·584,1951,2769
21	0·000,0090,9420	9·999,9229,2195	9·996,8851,2285	4·581,1675,2769
22	0·000,0079,8930	9·999,9320,1615	9·996,8080,4480	4·578,0526,5054
23			9·996,7400,6095	4·574,8606,9534
24				4·571,6007,5629

Note.—The four last figures in the decimal portion of the series λp_t and in λl_t may in practice be omitted.

The corresponding values of λp_t in the column headed Females, Table B, are interpolated in the same way. And the λp_{60}, λp_{70}, λp_{80}, and λp_{90} are interpolated by the same methods, the series being continued backwards to λp_{57} and forwards to λp_{108}; the actual observations of age after the age of 90 furnishing results less reliable than those thus obtained, which bring a generation of 100,000 to their last end in 107 years. The successive values of λp_t in the period from the age of 3 to the age of 19 inclusive, are derived from λp_3, λp_7, λp_{12}, and λp_{20}, which represent u_0, u_4, u_9, and u_{17}. As the terms of the series are here at unequal distances, the first differences cannot be derived from the preceding formulæ. The δ can in this and similar cases be derived from the proper equations by substituting figures for letters. But three literal equations supply formulæ for finding the three first differences from any four terms of series of the kind which have been discussed: u_0, which has a troublesome coefficient, can always be reduced to *zero*, and is therefore omitted. The first given term being u_0, let the second u_x be the xth from u_0, and u_y be the yth, u_z the zth from u_0. Here $x<y<z$. Then the following equations give the differences*:—

$$\delta^3=\frac{6\left\{(y-x)\frac{u_z}{z}-(z-x)\frac{u_y}{y}+(z-y)\frac{u_x}{x}\right\}}{(y-x)\{(z-1)(z-2)-(y-1)(y-2)\}-(z-y)\{(y-1)(y-2)-(x-1)(x-2)\}}. \quad (13.)$$

$$\delta^2=\frac{2}{y-x}\left\{\frac{u_y}{y}-\frac{u_x}{x}-\{(y-1)(y-2)-(x-1)(x-2)\}\frac{\delta^3}{6}\right\} \quad \text{. . . . .} \quad (14.)$$

$$\delta^1=\frac{u_x}{x}-(x-1)\frac{\delta^2}{2}-(x-1)(x-2)\frac{\delta^3}{6}. \quad \text{. . . . .} \quad (15.)$$

By making $y=2x$, and $z=3x$, these equations assume the same forms as equations (3.), (4.), (5.), with the term δ^4 struck out.

Putting $x=4$, $y=9$, and $z=17$, the three preceding equations become those which were actually used in constructing the series p_3 to p_{19}: u_0 is reduced to zero and is not used.

$$\delta^3=\frac{45u_{17}-221u_9+306u_4}{13260}. \quad \text{. . . . .} \quad (16.)$$

$$\delta^2=\frac{4u_9-9u_4-300\delta^3}{90}. \quad \text{. . . . .} \quad (17.)$$

$$\delta^1=\frac{u_4-6\delta^2-4\delta^3}{4}. \quad \text{. . . . .} \quad (18.)$$

Checking equation.

$$u_{17}=u_0+17\delta^1+136\delta^2+680\delta^3. \quad \text{. . . . .} \quad (19.)$$

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* A useful Table in applying the above formulæ.

x.	$(x-1)(x-2)$.	x.	$(x-1)(x-2)$.	x.	$(x-1)(x-2)$.	x.	$(x-1)(x-2)$.
20	342	30	812	40	1482	50	2352
21	380	31	870	41	1560	51	2450
22	420	32	930	42	1640	52	2550
23	462	33	992	43	1722	53	2652
24	506	34	1056	44	1806	54	2756
25	552	35	1122	45	1892	55	2862
26	600	36	1190	46	1980	56	2970
27	650	37	1260	47	2070	57	3080
28	702	38	1332	48	2162	58	3192
29	756	39	1406	49	2256	59	3306

Table of first differences in the Life-Table of Healthy Districts of England.

Age x.	λl_x.	$\lambda p_x = \delta^1$.	δ^2.	δ^3.	δ^4.
Males.					
3	4·631,5849,0000	9·993,2422,0000	0·001,2416,1260,934	9·999,8012,4393,666	0·000,0141,9648,567
20	4·584,1951,2769	9·996,9724,0000	9·999,9127,2285	0·000,0101,9910	9·999,9988,9510
59	4·403,7768,0454	9·990,6137,0980	9·998,9756,9020	9·999,9704,0800	9·999,9843,1520
60	4·394,3905,1434	9·989,5894,0000	9·998,9460,9820	9·999,9547,5320	9·999,9843,1520
Note.—The last series p_x was carried backwards from λp_{60} to λp_{59}.					
Females.					
3	4·623,2586,0000	9·993,2928,0000	0·001,2164,1598,794	9·999,7874,2556,561	0·000,0170,1566,365
20	4·570,6868,3846	9 996,6528,0000	9·999,9241,5455	0·000,0060,2930	9·999,9991,2530
57	4·405,2189,6826	9·992,9332,3725	9·999,0836,2675	0·000,0123,2100	9·999,9838,1950
60	4·381,2818,8126	9·990,2049,0000	9·999,0720,4825	9·999,9637,7950	9·999,9838,1950
Note.—The last series p_x was carried backwards from λp_{60} to λp_{57}.					

A series of the form $v^x l_x + v^{x+1} l_{x+1} + v^{x+2} l_{x+2}$ is required in rendering the Life-Table applicable to the solution of questions in Annuities and Life Insurance.

The logarithms of the series are obtained by making the first term of the new series, $\lambda(v^x l_x)$, and the first term of the first order of differences $\lambda(vp_x) = \lambda v + \lambda p_x = \delta^1$, the δ^2, δ^3 and δ^4 of the original series remaining unchanged. Taking the interest of money at 3 per cent. $v = \frac{1}{1·03}$; and $\lambda v = \bar{1}·9871627,758$.

The derivation of the new series from this value of λv, and from the above Table (males), is shown in the annexed example. Any value of v^x may be introduced in the same way.

$$\delta^4 = 9·9999988,951$$

Age.	δ^3.	δ^2.	$\lambda(vp_x) = \delta^1$.	$u_0 = \lambda(l_x v^x)$.
20	0·0000101,991	9·9999127,2285	9·9841351,7530	4·3274506
	·0000090,942	·9999229,2195	·9840478,9815	·3115858
		·9999320,1615	·9839708,2010	·2956337
			·9839028,3625	·2796045
				·2635074

In describing the first English Life-Table, I ventured to express the belief that the chances of life may ultimately be calculated by Mr. BABBAGE's machine*. Mr. BABBAGE's conception has been realized in the original and ingeniously constructed machine of the Messrs. SCHEUTZ, which was favourably reported upon by a committee of the Royal Society. The first differences to be inserted in the machine can be immediately deduced from those given above; and we may hope ere long to see the logarithms of Life-Tables, for single and for joint lives, printed from types cast in moulds stamped by the machine now in the course of construction by the Messrs. DONKIN, for Her Majesty's Government, at the instance of the Registrar-General.

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* Letter to the Registrar-General, in Appendix (p. 352) to his Fifth Annual Report, year 1843.

IV. CONSTRUCTION OF THE COLUMNS d_x, l_x, L_x, P_x, Q_x, Y_x, AND NOTICES OF SOME OF THEIR PRACTICAL APPLICATIONS.

The series l_x has been constructed; and from that series others are deduced to complete the Life-Table, consisting now of six columns.

(1.) $d_x = l_x - l_{x+1}$ = number of deaths in the year of age following, out of l_x alive at the age x. By taking x successively at 0, 1, 2, 3, ... to the last age in the Table, the numbers *dying* in every year of age are obtained. The numbers dying of the age x and under the age l_{x+n} are immediately derived from the column l_x; as (2.) $l_x - l_{x+n} = d_x + d_{x+1} \dots d_{x+n-1}$. When $x+n > \omega$ = the oldest age in the Table, $l_x = d_x + d_{x+1} \dots + d_\omega$.

(3.) $L_x = l_x + l_{x+1} \dots\dots + l_\omega$. The series is formed by the successive addition of the series l_x, from l_ω upwards.

(3 *a*.) $L_x - L_{x+n} = L_{x|n} = l_x + l_{x+1} \dots + l_{x+n-1}$.

(4.) $\left.\begin{array}{l} P_x = l_{x+1} + \frac{1}{2} d_x \\ P_x = l_x - \frac{1}{2} d_x \end{array}\right\}$ and (5) $P_x = \dfrac{l_x + l_{x+1}}{2}$.

$P_{x+1} = l_{x+1} - \frac{1}{2} d_{x+1} = l_{x+2} + \frac{1}{2} d_{x+1}$.

The series in column P_x is constructed from the two columns l_x and d_x, or from the single column l_x, as $2P_x = l_x + l_{x+1}$; and ∴ $P_x = \dfrac{l_x + l_{x+1}}{2}$, ∴ $l_x = 2P_x - l_{x+1}$; so, conversely, the series l_x can be constructed from the series P_x. The P_x is assumed to represent the population, as expressed by the Life-Table, living at the age x and under the age $x+1$. Thus P_{20} = the population of the age 20 and under 21 years.

By substituting the successive values of P_x in the equation (5*a*), $P_x + P_{x+1} \dots P_{x+n}$, we have $\frac{1}{2} l_x + l_{x+1} \dots + l_{x+n} + \frac{1}{2} l_{x+n+1}$.

(6.) $Q_x = P_x + P_{x+1} + P_{x+2} \dots P_{x+n-1} + P_{x+n} \dots + P_\omega \dots\dots$

$Q_{x+n} = P_{x+n} + P_{x+n+1} + P_{x+n+2} \dots + P_\omega$.

(7.) ∴ $Q_x - Q_{x+n} = Q_{x|n} = P_x + P_{x+1} + P_{x+2} \dots P_{x+n-1}$. The column Q_x is constructed by adding up the column P_x, and transferring the successive sums to the column Q_x.

By substituting for the series P_x its values in l_x, we have

(8.) $Q_x = \frac{1}{2} l_x + l_{x+1} + l_{x+2} \dots + l_\omega$.

And by again substituting for the series l_x its corresponding values in d_x, we have

(9.) $Q_x = \frac{1}{2} d_x + 1\frac{1}{2} d_{x+1} + 2\frac{1}{2} d_{x+2} \dots + (\omega + \frac{1}{2}) d_\omega$.

(10.) Thus Q_x is equal to the numbers dying in each year of age after the age x, multiplied by the time (expressed in years and fractions of a year) that they have respectively lived over that age; and if $x=0$, then $Q_0 = \frac{1}{2} d_0 + 1\frac{1}{2} d_1 + 2\frac{1}{2} d_2 \dots (n + \frac{1}{2}) d_{x+n}$, when $(x+n)$ becomes $> \omega$.

(11.) This column Q_x represents, therefore, two distinct orders of facts: it represents the sum of the number of years that will be lived after the age x by the l_x persons then living, and ∴ $\dfrac{Q_x}{l_x}$ = the mean after-lifetime; of which $\dfrac{Q_{x|n}}{l_x}$ will be enjoyed before the age $x+n$ is attained, and $\dfrac{Q_{x+n}}{l_x}$ after the age $x+n$ is attained. At birth the mean after-lifetime is $\dfrac{Q_0}{l_0}$, the unit here being one year of individual life.

(12.) Q_x also represents the sum of the numbers of men or women living at all ages over the age x, out of Q_0 living at all ages, as Q_x is in all cases the sum of the numbers living in each year of age, represented by the series P_x. The unit is here an individual man.

(13.) Thus, on referring to Plate XLII. fig. 1, the lifetime of 100,000 children born simultaneously may be represented by 100,000 parallel lines, drawn from AB horizontally in the direction of CD until they cut the curved line BC. And Q_0 is the sum of these lines expressed in the linear units of the scale on the line AC; so $\frac{Q_0}{l_0}=\frac{Q_0}{100,000}=\frac{4,899,665}{100,000}=48\cdot99665$; the mean length of those lines = the number of years of mean lifetime.

It will be observed that in this Table, instead of 100,000 lines, these lines are thrown into 106 groups, each comprising the variable number of lines terminating in each of 106 intervals numbered on the line AC, and representing years of age. And in these short intervals it is assumed that the mean length of the lines terminating in the eleventh interval (10 to 11) is represented by $10\frac{1}{2}$, and so on.

The relative numbers of persons living simultaneously at each interval of age will also be represented in the same Plate, fig. 1, by 106 successive vertical lines, raised from nearly the centre of each interval between the ordinates on the line AC, and measured in units of which the line AB contains 100,000. The same lines bound the figure representing the two orders of facts; and the numerical units expressing the aggregate length of the vertical lines equal in amount the units expressing the aggregate length of the horizontal lines expressed in the horizontal units.

(14.) I will now explain briefly the nature of the column Y_x, which I have added to the Life-Table*. The Life-Table (column P_x) exhibits a representative population, such as would be constituted by separating every year 100,000 births as they occurred,

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[*] See paper in Appendix to Registrar-General's Sixth Annual Report, pp. 544–552.

Extract from the Registrar-General's Sixth Annual Report (1845), p. 528.

"*Note.*—HALLEY's Table (1693) contained the column P. JOHN SMART made 1000 "born" the basis of his Table (1738), and introduced the columns d and l. SIMPSON adopted SMART's form of Table, which was followed by KERSSEBOOM (1738), DEPARCIEUX (1746), PRICE (1773), and MILNE (1815). The columns S_y, y and Δy in DUVILLARD's 'Loi de Mortalité (en France) dans l'état naturel †,' correspond with the columns L, l, d in the new Table. The S_y added by DUVILLARD is our L and BARRETT's column B; DUVILLARD's short Table (p. 128) has the four columns d, l, P, Q for quinquennial or decennial ages, and the 'expectation of life.' [illegible]'s Table II. is an expansion of the column Q of DUVILLARD's short Table, and is that column for each year of age. In a recent report on the Bengal Military Fund, Mr. DAVIES has a Table (1) containing columns corresponding with the d, l, L, P, Q of the English Table, the 'Mortality per cent.,' and the 'Expectation of Life' at each age ‡."

I have in this paper employed d, l, L, instead of C, D, N, which have been formerly used by me and others, and should still be used where the factor [illegible] is introduced.

[†] Influence de la Petite Vérole, p. 161.

[‡] See the note (A), p. 558.

and keeping them apart in a separate community, subject to a definite law of mortality. Any population living in the tabular proportions at each year of age may, for the sake of distinction, be called a normally constituted population.

The ages of the population represented by the Life-Table amount, in the aggregate, to Y_0 years; it is the aggregate *number of years which they have already lived*, and, singularly enough, it is also, if the law of mortality remain constant, *the number of years which they will live*. Thus Q_0 persons in such a population have lived on an average $\frac{Y_0}{Q_0}$ years; *that is their* MEAN AGE, and it is also their mean *after-lifetime*. Y_x is the number of years that Q_x persons have lived *over the age x*; and the mean age of such persons is $x+\frac{Y_x}{Q_x}$; their after-lifetime is $\frac{Y_x}{Q_x}$.

The series Y_x is formed by successively adding up a series of the form $\frac{1}{2}(Q_x+Q_{x+1})$, commencing at $x+1=\omega=$ the oldest age in the Table.

(15.) $\therefore\ Y_0=\frac{1}{2}Q_0+Q_1+Q_2\ldots+Q_\omega,$

$Y_x=\frac{1}{2}Q_x+Q_{x+1}+Q_{x+2}\ldots+Q_\omega.$

By substituting for Q_0, for Q_1, for Q_2, and so on, their values in P_x, it will be found that

(16.) $Y_0=\frac{1}{2}P_0+1\frac{1}{2}P_1+2\frac{1}{2}P_2+3\frac{1}{2}P_3\ldots\ldots+(n+\frac{1}{2})P_n\ldots\ldots+(\omega+\frac{1}{2})P_\omega.$

(17.) But the mean age of the persons (P_0) of the age of 0 and under 1 is nearly $\frac{1}{2}$; and so the series $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{1}{2}$, $6\frac{1}{2}$ .... $(n+\frac{1}{2})$ expresses nearly the mean age of all the persons in the first (P_0), second (P_1), third (P_2), and $(n+1)$th (P_n) years of age, and so for all other ages; consequently the sum of the series (16) Y_0 is the sum of the ages of all the persons living contemporaneously, as they are represented in the Life-Table.

In like manner it is shown that

(18.) $$Y_x=\tfrac{1}{2}P_x+(1+\tfrac{1}{2})P_{x+1}+(2+\tfrac{1}{2})P_{x+2}\ldots\ldots\ldots+(\omega+\tfrac{1}{2}-x)P_\omega$$

is the sum of the number of years that the Q_x persons in the Table have lived over the age x. They have all lived x years; and consequently $x+\frac{Y_x}{Q_x}$ gives their average age precisely as $\frac{Y_0}{Q_0}$ gives the average age of the whole community.

(19.) It has been shown that Q_x expresses the number of years that l_x persons will live; in the same manner it may be shown that Q_{x+1} expresses the number of years that l_{x+1} persons will live; $\therefore$ (l_x+l_{x+1}) persons will live (Q_x+Q_{x+1}) years, $\therefore$ $\frac{1}{2}(l_x+l_{x+1})=P_x$ persons will live $\frac{1}{2}(Q_x+Q_{x+1})$ years. And the same may be demonstrated for each successive value of x.

But the sum of the series P_x is $Q_x=$ the number of persons living of all ages. And the sum of the series $\frac{1}{2}(Q_x+Q_{x+1})$ is $Y_x=$ the number of years that Q_x persons will live; $\therefore$ $\frac{Y_x}{Q_x}=$ the *mean after-lifetime* of all the persons living simultaneously of the age x and upwards. Thus by the Table D, 4,899,665 persons are living contemporaneously; their mean age is $\frac{Y_0}{Q_0}=\frac{166209701}{4899665}=33{\cdot}92$ years; and they will live on an average 33·92 years.

(20.) The Life-Table serves to determine the value of Life Annuities, the value of policies, and the premiums of insurance.

This is effected by introducing a new unit, such as £1, 1 franc, 1 dollar, or any other monetary unit. Thus if £1 is payable at each death, the series d_x will show the number of pounds falling due in each year of age; so if £1 is payable by each person on attaining the age x, and each subsequent year of age, the series l_x shows the number of *pounds* payable every year by the l_x persons; and N_x will be the number of pounds payable in the whole course of life after the age x: thus $\frac{N_x £1}{l_x}=$ the AVERAGE AMOUNT of an annuity of £1 payable on each life at and after the age x. The money-unit may be introduced into the other columns; and $\frac{Y_x}{Q_x} \cdot £1$ would show the AVERAGE AMOUNT payable under an annuity of £1 on each of Q_x lives. The *present value* of these future payments can always be determined by assuming a given rate of interest. The estimates thus obtained are also always read subject to the qualification that by hypothesis the *Life-Table* is based on a law of mortality actually to rule for a definite time in the population to which it is applied. The probability of the hypothesis is not here in question.

Under the same circumstances masses of mankind appear to experience, at the same ages, the same rates of mortality. Consequently if for several years d_x persons have died annually on an average out of l_x persons living at the beginning of the year, other things being equal, the probability that the same number will die out of l_x persons in a year to come is greater than any other that can be named, and the fraction expressing that probability is $\frac{d_x}{l_x}$. We know that d_x expressing the numbers dying in a year, l_{x+1} must express the numbers surviving as $l_{x+1}+d_x=l_x$. The chances may be represented by l_x balls; l_{x+1} *white* balls in an urn will represent the chances of living, d_x *black* balls in the same urn will represent the chances of dying. Now let each of l_x persons pay the sum z for a ticket, and each person that draws a *white ball* be entitled to £1. Before the drawing commences the value of each ticket is $\frac{l_{x+1}}{l_x}$; for l_x (the total chances) : l_{x+1} (the chances in favour of winning on one ticket) :: 1 : $\frac{l_{x+1}}{l_x}=z$.

Put $l_x=30{,}007$, and $l_{x+1}=29{,}647$; then $\frac{l_{x+1}\,.\,£1}{l_x}=\frac{29{,}647\,.\,£1}{30{,}007}=£{\cdot}98802$. The amount of money to be paid on l_{x+1} white balls is £29,647, and $£{\cdot}9802 \times 30{,}007=z\,.\,l_x=£29{,}647$.

In like manner it may be shown that if £1 is paid to each person who draws a *black* ball, the value of each ticket is $\frac{d\ £1}{l_x}=y£1$; for $y\,.\,l_x\,.\,£1=d_x£1$, and £1 is to be paid on each of d_x tickets.

Should £1 be paid alike to those who draw white balls and to those who draw black balls, the value of a ticket will be equal to the sum of the two fractions expressing the several probabilities, namely,

$$\frac{l_{x+1}\,.\,£1}{l_x}+\frac{d_x£1}{l_x}=z+y=\frac{l_{x+1}+d_x}{l_x}£1=\frac{l_x}{l_x}£1=£1.$$

As one or other of the two kinds of balls *must by hypothesis be drawn*, and £1 is paid for each ball, the receipt of the £1 is certain: certainty is thus in all cases expressed by *unity*.

If every ball as it was drawn were replaced in the urn, although in 30,007 trials *white balls* were not actually drawn 29,647 times, black balls 360 times, still $\frac{29,647}{30,007}$ would express the probability of drawing a white ball, and the value of £1 contingent on that event, more accurately than any other fraction that could be named.

Again, if an urn contained by hypothesis an indefinite number of balls, out of which 29,647 white balls and 360 black balls were drawn and then replaced, the probability of again drawing a white ball on trial, and the value of £1 contingent on that event, would be expressed more accurately by $\frac{29,648}{30,009}$* than by any other fraction that could be named; past experience being by hypothesis the only means we have here of judging of the future.

Thus a Life-Table applicable to the case furnishes the fractions to determine the value of any sums of money dependent on the life or death of a given person, or a certain number of given persons in a given time.

The probability of living two years expressed by the fraction $\frac{l_{x+2}}{l_x}=\frac{l_x-(d_x+d_{x+1})}{l_x}$, is less than the probability of living one year.

Making n any number of years and fractional parts of years, the fraction $\frac{l_{x+n}}{l_x}$ will invariably express the probability of living n years after the age x. As n approaches zero the fraction will approximate to 1, the symbol of certainty; thus a person is more likely to live a day than a year, a minute than a day. As n increases l_{x+n} diminishes in value; and when $x+n$ expresses a year after the age ω in the Life-Table, $l_{\omega+1}$ is by hypothesis zero, $\therefore \frac{l_{\omega+1}}{l_x}=\frac{0}{l_x}=0$. The chance of living so long is expressed in this case by zero, the chance of dying in the time by 1, the symbol of certainty.

(21.) l_{x+n} expresses the number of chances in favour of surviving n years, and l_x-l_{x+n} the number of chances of dying in the same time, the sum of the two together (l_x) expressing the total number of chances. Thus the fraction $\left(\frac{l_{x+n}}{l_x}\right)$ expressing the probability of living a given time ranges from 1 to 0, and $\frac{l_x-l_{x+n}}{l_x}=1-\frac{l_{x+n}}{l_x}$, or the chance of dying in a given time also ranges from 1 to 0 as n varies. When the two fractions are equal $\frac{l_{x+n}}{l_x}=\frac{l_x-l_{x+n}}{l_x}$, then $l_{x+n}=l_x-l_{x+n}$, and $2l_{x+n}=l_x$, $\therefore l_{x+n}=\frac{l_x}{2}$.

To verify the equations, an age $x+n$ must be chosen at which l_{x+n} is exactly equal to $\frac{1}{2}l_x$. Thus by the Life-Table of healthy districts 100,000 children born alive are reduced to 50,851 in 58 years, and to 49,895 in 59 years; so the chances are rather in favour of their living 58 years, as they are 50,851 to 49,149; upon the other hand, the chances of their living 59 years (49,895) are less than the chances 50,105 of their dying before attaining that age. Upon trial it will be found that the chances of living to and the chances of dying before $58\frac{851}{956}$ years$=58+\frac{50{,}851-50{,}000}{d_{58}}=58+\frac{851}{956}$ years, or about $58\frac{8}{9}$ years are nearly equal; hence this is called the *probable lifetime*, or *vie probable* by French writers, for $\frac{l_{58\frac{851}{956}}}{l_0}=\frac{1}{2}$. At the age 20 the probable lifetime is $47\frac{1588}{1638}$, nearly 48 years. The probable lifetime at every age is immediately seen by inspection.

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* The addition of 1 to the numerator, and of 2 to the denominator, may be neglected, when, as in this case, the numbers are large.

(22) V THE THREEFOLD LIFE-TABLE—PERSONS, MALES, FEMALES.

The Life-Table is threefold. A Table having the six columns is made for males; another Table is separately made for females. The several columns of the two Tables incorporated together form the Table of persons which has 100,000, and may have any other number for its basis. The basis of the Male Table in the illustration is 51,125, while the basis of the Female Table is 48,875. In that proportion males and females were born in the districts. Under this arrangement the number of contemporaneous males and females living at each age in columns l_x is shown: thus 38,388 males and 37,212 females attain the age of 20; 17,145 males attain the age of 70, and 17,133 females attain the same age; at all ages under 71 the number of males exceeds the females; at the age of 71 and upwards the females exceed the males in number: and upon referring to the columns d_x, it will be seen that the males die off in greater numbers than females after the age of 42. The age after the second year at which the greatest number of deaths occurs is 75 in males, 76 in females.

These numbers all refer to the Life-Table for healthy districts.

Some of the other properties of the Life-Tables, admitting of innumerable applications in the solution of social phenomena, will appear in the following formulæ, which will be found useful in practice.

VI. USEFUL FORMULÆ.

The following formulæ will facilitate the use of the Life-Table. The figures must be taken from the Tables of Persons, of males or females, applicable to the case. The formulæ are general, and are applicable to any other Life-Table.

(23.) $\frac{d_x}{P_x}=m_x=$the rate of mortality in the year of age following the precise age x.

(24.) $\frac{d_x}{l_x}=\frac{l_x-l_{x+1}}{l_x}=1-\frac{l_{x+1}}{l_x}=$the probability that a person A of the age x, in average health, will die in the following year.

(25.) $\frac{l_{x+1}}{l_x}=p_x=\frac{l_x-d_x}{l_x}=1-\frac{d_x}{l_x}=$the probability that A, a person of the age x, will live a year; $1-p_x=$the probability that A, age x, *will die in the year following*, as certainty of life$=1$;

(26.) $\frac{l_x - l_{x+n}}{l_x}$=the probability that A, age x, will die in the next n years.

(27.) $\frac{l_{x+n}}{l_x}$=the probability that A, of age x, will live n years.

(28.) Put $\frac{l_x}{2} = l_{x+n}$; and when l_{x+n} is taken at such an age as to fulfil the conditions of the equation, then n is the *probable lifetime*=*vie probable*=the time that it is an even chance a person of the age x will live.

(29.) $\frac{Q_x}{l_x} = A_x$=the mean *after lifetime*, or as it is often called, the *expectation of life*—an incorrect expression, which is rather applicable to the probable lifetime.

Note.—Upon DEMOIVRE's hypothesis, the *probable lifetime*, that is the time that a person may fairly expect to live, his *expectation*, was the same as the mean after lifetime.

(30.) $G_x = x + A_x$=the mean age at death of persons who have already lived exactly x years.

(31.) $S = c\frac{Q_{x|n}}{l_x}$=the number of members of any Society between the ages x and $x+n$, which will be permanently sustained by c... annual admissions at the age x.

(32.) $c = \frac{Sl_x}{Q_{x|n}}$=annual recruits of the Society (S).

(33.) $\frac{Sl_{x+n}}{Q_{x|n}}$=annual members leaving the Society (S) on attaining the age $x+n$.

(34.) $\frac{Sl_{x|n}}{Q_{x|n}}$=annual deaths in such a Society (S).

(35.) $S\frac{Q_{x+n}}{Q_{x|n}}$=the aggregate number of persons living, who have left such a Society, as pensioners or otherwise.

In the following formulæ it is assumed that the population is normally constituted.

(36.) $\frac{Y_x}{Q_x} = A'_x$=the mean after lifetime of all persons of the *age* x *and upwards*.

(37.) $\frac{Y_x - Y_{x+n}}{Q_x - Q_{x+n}} = \frac{Y_{x|n}}{Q_{x|n}}$=the mean after lifetime of all persons of the age of x and under the age of $x+n$.

(38.) $c.\frac{Y_{x|n}}{Q_{x|n}}$=the number of persons of which a Society will *ultimately consist*, recruited by c annual additions of members in the tabular proportions between the age x and $x+n$.

(39.) $c\frac{Y_{x|n} - Y_{x+m|n}}{Q_{x|m}}$=the number of persons to which a Society joined by c persons of the tabular ages x and under $x+m$ would amount in n years. When $x+n > \omega$; this formula will be reduced to the same form as equation (38.). And when $x+m$, as well as $x+n > \omega$, the equation becomes the same as (36.).

VII. LIFE-TABLE OF THE SIXTY-THREE HEALTHIEST ENGLISH DISTRICTS

Upon inquiry it was found that in many districts of England the mortality of the population did not exceed the rate of 17 annual deaths to 1000 living.

For the sake of convenience these were called healthy districts, consisting of sixty-four, or nearly a tenth part of the total registration districts of England and Wales, and inhabited by nearly a million of people: sixty-three of these districts have been taken as the basis of the new Life-Table, constructed according to the methods previously described.

It will be seen that these districts, generally conterminous with Poor Law Unions, are distributed over the various parts of the country. They comprise—*Hendon* (with Harrow*) (17), *Lewisham* (17), and *Bromley* (17) in the neighbourhood of London; *Hambledon* (16), *Dorking* (17), *Reigate* (16), and *Godstone* (17) on the southern slope of the Surrey hills; *East Ashford* (17) in East Kent, *Blean* (including Herne Bay) (17) between Canterbury and the sea; ten districts of Sussex—*Battle* (16) near Hastings, *Eastbourne* around Beachy Head (15), *Hailsham* (17), *Uckfield* (17), *East Grinstead* (17), *Cuckfield* (16), *Steyning* near *Brighton* (16), *Petworth* (17), *Worthing* (17), and *Midhurst* (17); seven districts of Hampshire—the *Isle of Wight* separated from the mainland by the sea (17), *Lymington* (17), *Christchurch* (16), *Ringwood* (17), *New Forest* (17), *Catherington* (17), and *Alresford* (17); *Wokingham* (17), and *Easthampstead* (16) in Berkshire, south of the Thames; *Ongar* (17) in Essex, east of Epping Forest; *Mutford* (17), including Lowestoft on the Suffolk coast; *Henstead* (17), south of Norwich; *Kingsbridge* (17), on the south coast of Devon; *Okehampton* (16); *Crediton* (17), *Barnstaple* (17), *Torrington* (17), *Bideford* (17), *Holsworthy* (16), stretching from the centre over Dartmouth and Exmoor, along the coast of the Bristol Channel; *Stratton* (17), *Camelford* (17), and *Launceston* (17), in the adjacent parts of Cornwall, and further south *St. Columb* (17); *Williton* (17) in Somerset, also on the Bristol Channel; *Winchcomb* (17), to the east of Cheltenham, and the Cotswold Hills around the sources of the Thames; *Kings Norton* (17) in Worcestershire, adjoining Birmingham; *Melton Mowbray* (17) in Leicestershire; *Southwell* (17) about Sherwood Forest, in the centre of Nottinghamshire; *Garstang* (16) in Lancashire, looking northward over Lancaster Bay; *Easingwold* (17) in the North Riding of Yorkshire, *Guisborough* (16) on the eastern coast north of Whitby; then follow five border districts of Northumberland on the southern face of the Cheviot Hills:—*Belford* (17), *Glendale* (15), *Rothbury* (15), *Bellingham* (17), *Haltwhistle* (16) (is omitted in the Table); *Longtown* (17) and *Brampton* (17) on the border, and *Bootle* (16) on the coast of Cumberland, the *East Ward* (17) of Westmoreland, *Haverfordwest* (17), on the western point of South Wales; *Builth* (16), *Corwen* (17), *Pwllheli* (17) on Carnarvon Bay, and *Anglesey* (17) complete the list. These districts, and others nearly equally healthy, have been thus described:—

"Such is the variety of the soil of England, that tested by the rates of mortality, the children reared out of a given number born, the longevity of the inhabitants, the freedom from common epidemics, or the immunity from cholera, Healthy Districts are found in nearly every county. Large tracts of country are, however, so much healthier than the rest, that they may be justly called Salubrious Fields; and it is remarkable that here the finest races of animals are bred. The north districts of Northumberland around the beautiful Cheviot Hills, covered with grasses, ferns, wild thyme,—extending from the region of the heaths to the rich cultivated land at their bases, touching each other, or intersected by narrow valleys; the districts extending from the Tees over the North and East Ridings of York to Leicestershire, Herefordshire, and parts of Shropshire; some of the districts of Gloucestershire about the Cotswold Hills; parts of Wales; North Devon, including Dartmoor and Exmoor; the Surrey and Sussex hills with the Southdowns,—have given names to the best breeds of sheep, fowls, cattle, and horses in the kingdom." * * * * * *

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* The annual deaths to 1000 living of all ages inserted in parentheses are deduced from returns of the living at the censuses 1841 and 1851, and the deaths registered in the ten years 1841 to 1850. See Registrar-General's Sixteenth Report, pp. 141–158.

"The dry and most inland are not always the healthiest regions of the country. The salubrious fields are sometimes watered by running streams, and diversified by lakes; the dew is abundant; they are often veiled, not by infectious fogs, but by mists drawn from the sky as it breathes over them; the mountains rise above, the ocean rolls at the distance below them, as on the coast of Sussex, North Devon, the western region of Wales, extending under Snowdon and Cader Idris in a vast amphitheatre round Cardigan Bay; the lake land and moors of the North, rising between the Irish Sea and the German Ocean. The land is sometimes heathy, but may be covered by the sweetest herbage and bees feeding on the flowers: the cereal grains, the hop, the timber, are often of the finest quality; the animals are healthy, the native breeds are vigorous, and those fine varieties are produced at intervals, which men of the genius of BAKEWELL, ELLMAN, TOMKINS, COLLING, and O'KELLY make the permanent stock of the country. Industry and the army receive their best recruits from the population; while they get their worst from the people of the low parts of sickly towns. Agriculture has reclaimed many unhealthy districts on the plains, so that a considerable extent of the cultivated land is now in a state of comparative salubrity; and vast systems of drainage have subdued the noxious fens, although carried out less efficiently than is desirable, and interfered with by milldams on the rivers, descending like the Nene from the inland high lands*."

The sanitary condition of the people in these districts is, however, still in many respects defective.

CONCLUSION.

HALLEY first pointed out the financial applications of the Life-Table, and first calculated the values of life annuities. That branch of science, in the various forms of life insurance, has since received great developments. The new Table shows that the duration of life, among large classes of the population, by no means in unexceptionable sanitary conditions, exceeds the term of the ordinary Tables, and proves that life annuities cannot be sold advantageously by offices, or by the Government, to large classes of lives for less than the values deducible from the new Table.

A new branch of science has been developed since HALLEY's day,—it is the science of Public Health. And here a new application of the Life-Table is found.

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* Report to the Registrar-General on Cholera, pp. xcv, xcvi.

It is probable, upon physiological grounds, that man goes through all the phases of his natural development in a hundred years; and that the period of active life seldom extends beyond eighty years. But this is a very indefinite measure, as the rates of mortality, in all the intermediate ages, are left undetermined after it has been ascertained in what proportions men attain the extreme limits.

Generations of men, under all circumstances, die at all ages; but the proportions vary indefinitely under different conditions from a slight tribute to death each year, down to the point of extermination by pestilence. If we ascertain at what rate a generation of men dies away under the least unfavourable existing circumstances, we obtain a standard by which the loss of life, under other circumstances, is measured; and this I have endeavoured to determine in the Life-Table of English Healthy Districts. And recollecting that the science of public health was almost inaugurated in England by a former President of this Society*, who encouraged and crowned the sanitary discoveries of Captain COOK, I feel assured that it will receive with favour this imperfect attempt to supply sanitary inquirers with a scientific instrument.

In a subsequent paper I hope to be able to lay before the Society the mortality by different kinds of diseases at each age, as they have been deduced from the same series of observations.

HEALTHY DISTRICTS.

TABLE A.—Population, 1851. Deaths in the five years 1849 to 1853. Average Annual Mortality per cent., and Logarithms of the Mortality.

Ages.	Population.			Deaths.			Average annual mortality to 100 living (m)			Logarithms of the mortality (λm).		
	Persons	Males.	Females.	Persons	Males	Females.	Persons	Males.	Females.	Persons	Males.	Females.
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.
All ages ……	996773	493525	503248	87345	43736	43609	1·753	1·772	1·733	$\bar{2}$·2436718	$\bar{2}$·2485599	$\bar{2}$·2388240
Under 5 ……	130635	65700	64935	26361	14282	12079	4·036	4·348	3·720	$\bar{2}$·6059323	$\bar{2}$·6382536	$\bar{2}$·5705821
5— ……	122406	61733	60673	4209	2080	2129	·688	·674	·702	$\bar{3}$·8374062	$\bar{3}$·8285759	$\bar{3}$·8462102
10— ……	110412	56651	53761	2377	1087	1290	·431	·384	·480	$\bar{3}$·6340429	$\bar{3}$·5840519	$\bar{3}$·6811523
15— ……	181339	90066	91273	6603	3113	3490	·728	·691	·765	$\bar{3}$·8622801	$\bar{3}$·8396482	$\bar{3}$·8835130
25— ……	136892	65422	71470	5869	2675	3194	·857	·818	·894	$\bar{3}$·9332160	$\bar{3}$·9126300	$\bar{3}$·9512411
35— ……	108056	52734	55322	5208	2447	2761	·964	·928	·998	$\bar{3}$·9840521	$\bar{3}$·9675733	$\bar{3}$·9991985
45— ……	85244	42383	42861	5252	2698	2554	1·232	1·273	1·192	$\bar{2}$·0906909	$\bar{2}$·1048802	$\bar{2}$·0761886
55— ……	62857	31105	31752	7001	3568	3433	2·228	2·294	2·162	$\bar{2}$·3478365	$\bar{2}$·3606246	$\bar{2}$·3349327
65— ……	39453	18860	20593	10313	5173	5140	5·228	5·486	4·992	$\bar{2}$·7183350	$\bar{2}$·7392308	$\bar{2}$·6982734
75— ……	16737	7718	9019	10297	4946	5351	12·304	12·817	11·866	$\bar{1}$·0900631	$\bar{1}$·1077793	$\bar{1}$·0743066
85— ……	2614	1097	1517	3581	1555	2026	27·399	28·350	26·711	$\bar{1}$·4377287	$\bar{1}$·4525536	$\bar{1}$·4266838
95 and upwards	128	56	72	274	112	162	42·813	40·000	45·000	$\bar{1}$·6315706	$\bar{1}$·6020600	$\bar{1}$·6532125

Note.—The ages at death of 146 persons, viz. 128 males and 28 females, were not stated; in calculating the mortality they have been distributed proportionally over the several ages in the Table. The Table may be read thus: 136,892 persons, of whom 65,422 were males, 71,470 were females at the age of 25 and under 35, were enumerated in 1851; at the same ages, 5869, 2675 males and 3194 females, died in the five years 1849 to 1853; consequently the annual rates of mortality per cent. were ·857, ·818, and ·894.

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* Sir JOHN PRINGLE.

Number of Deaths at five periods of Age in the Healthy Districts, in 1848 to 1855.

Years.	Ages.														
	Persons.					Males.					Females.				
	0.	1.	2.	3.	4.	0.	1.	2.	3.	4.	0.	1.	2.	3.	4.
1848.	2935	832	458	371	312	1678	442	244	204	162	1257	390	214	167	150
1849.	2932	858	541	427	292	1637	452	263	207	154	1295	406	278	220	138
1850.	2969	859	466	331	301	1676	453	231	164	144	1293	406	235	167	157
1851.	3185	932	543	341	288	1769	502	274	179	148	1416	430	269	162	140
1852.	3405	860	567	389	297	1913	446	273	206	140	1492	414	294	183	157
1853.	3370	946	554	376	287	1888	514	293	179	137	1482	432	261	197	150
1854.	3404	1047	601	386	311	1903	539	317	197	165	1501	508	284	189	146
1855.	3350	907	533	445	297	1948	483	257	230	156	1402	424	276	215	141

Number of Births in Sixty-three Healthy Districts of England, 1848 to 1855.

Years.	Persons.	Males.	Females.
1848	28679	14756	13923
1849	29128	14751	14377
1850	29699	15176	14523
1851	30163	15465	14698
1852	30370	15557	14813
1853	29214	15010	14204

Age.	Males.
	2)29,507=births in 1848 and 1849
0	14,754=births on January 1, 1849
1	13,117=living on January 1, 1850
2	12,664=living on January 1, 1851
3	12,390=living on January 1, 1852
4	12,184=living on January 1, 1853
5	12,047=living on January 1, 1854

Age.	Males.
0	1637=deaths in 1849
1	453=deaths in 1850
2	274=deaths in 1851
3	206=deaths in 1852
4	137=deaths in 1853

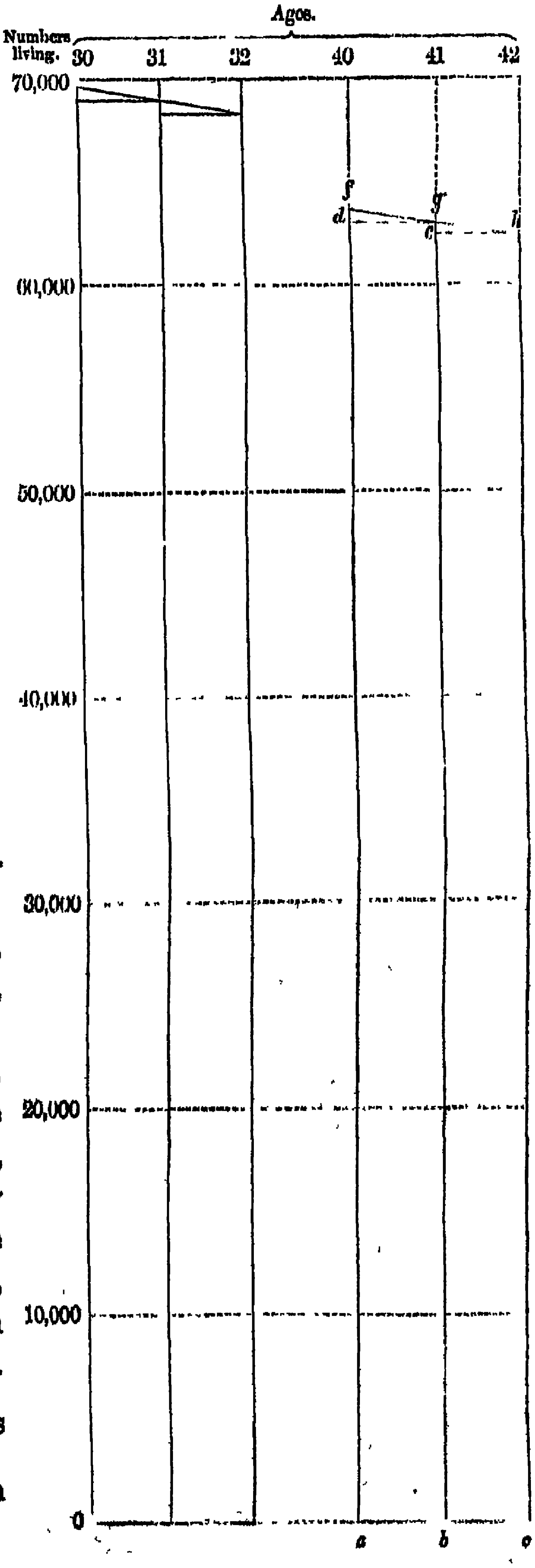
TABLE B.—The several values of λp_x on which the Life-Table of Healthy Districts is based: also the corresponding values of p_x and $(1-p_x)$

Age x	λp_x =logarithms of the probability of living one year after the age x.		p_x =probability of *living* a year		$(1-p_x)$ probability of *dying* in a year	
	Males.	Females	Males	Females.	Males.	Females.
0	$\bar{1}$·9480215	$\bar{1}$·9577796	·88720	·90736	·11280	·09261
1	$\bar{1}$·9844929	$\bar{1}$·9859276	·96492	·96812	·03508	·03188
2	$\bar{1}$·9904341	$\bar{1}$·9904679	·97821	·97829	·02179	·02171
3	$\bar{1}$·9932422	$\bar{1}$·9932928	·98456	·98467	·01544	·01533
7	$\bar{1}$·9970729	$\bar{1}$·9969512	·99328	·99300	·00672	·00700
12	$\bar{1}$·9984539	$\bar{1}$·9980197	·99645	·99545	·00355	·00455
20	$\bar{1}$·9969724	$\bar{1}$·9966528	·99305	·99232	·00695	·00768
30	$\bar{1}$·9964260	$\bar{1}$·9960967	·99180	·99105	·00820	·00895
40	$\bar{1}$·9959051	$\bar{1}$·9956263	·99062	·98998	·00938	·01002
50	$\bar{1}$·9943048	$\bar{1}$·9946669	·98697	·98780	·01303	·01220
60	$\bar{1}$·9895894	$\bar{1}$·9902049	·97631	·97770	·02369	·02230
70	$\bar{1}$·9751357	$\bar{1}$·9773538	·94436	·94919	·05564	·05081
80	$\bar{1}$·9420680	$\bar{1}$·9463182	·87512	·88373	·12488	·11627
90	$\bar{1}$·8747315	$\bar{1}$·8809176	·74943	·76018	·25057	·23982

Note.—Age x is in this Table the precise age. Age 12 is applied frequently to all persons of the age of 12 and under the age of 13; but in this Table it applies only to persons of the precise age of 12 years, neither more nor less. The λp_7 was in both cases derived from the formula $\left(\frac{2-m}{2+m}\right)$. The λp_{12}, deduced from this formula, is for males $\bar{1}$ 9983497, and for females $\bar{1}$ 9979153; which may be regarded either as the constant or the mean values of λp_{10}, λp_{11}, λp_{12}, λp_{13}, and λp_{14}; but as these are the terminations of an ascending and a descending series, it is probable, and quite in conformity with other observations, that one, two, or more of these values will exceed the mean value. The logarithms of p_{12} adopted are given above; and the two arithmetical means of the five logarithms, λp_{10}, λp_{11}, λp_{12}, λp_{13}, and λp_{14}, resulting from the interpolation, are $\bar{1}$·9983688 for males, and $\bar{1}$·9979435 for females.

The values of λp_{20}, λp_{30} .... are derived from the formula $y_x = 10^{\frac{k\lambda m}{\lambda r}(1-r^x)}$.

NOTE ON THE TWO HYPOTHESES.



Let b be the decrement of the ordinate y in a unit of time, then the decrement Δy of the ordinate in the time x, represented by the abscissa, will be $\Delta y = -bx$, on DEMOIVRE'S hypothesis; and as it is always proportional to the time, it will be in an infinitely short time $dy = -bdx$.

Passing to the integral $y = c - bx$. And if $y = a$ at the origin when $x = 0$, $c = a$, $\therefore y = a - bx$. And if $b = 1$, then $y = a - x$. This evidently represents very closely short portions of the Life-Table curve; and the smaller x is taken, the nearer is the approximation to the corresponding value of y.

Again, let Δy be the decrement of the ordinate y in the indefinite time Δx represented by the abscissa; and let the mortality (m) represented by the ratio of the area $abfg$ to the area dfg be $\frac{d_0}{P_0} = m_0$. Let also m_0 increase at the rate r in a unit of time, so that $\frac{gch}{bcgh} = \frac{d_1}{P_1} = m_1 = m_0 r$, and generally within given limits $m_0 r^x = m_x$; then $\Delta y = -y m_x \Delta x$ nearly, Δx being any small portion of time.

The error increases as the time Δx is extended, from the circumstance that on the one hand m_x varies by hypothesis momentarily, and that y, from which the varying proportional part is taken, constantly grows shorter. But by passing to the limit and making the time dx infinitely short, m_x and y during that infinitely short time may be considered constant, and $dy = -y m_x dx$ will be the true decrement. Substituting $m_0 r^x$ for m_x, the equation becomes $dy = -y m_0 r^x dx$, from which the value of y can be derived, as before shown. For $\frac{dy}{y} = -m_0 r^x dx$, and integrating both sides $\lambda_\epsilon y = \lambda_\epsilon c - \frac{m_0 r^x}{\lambda_\epsilon r}$. Here λ_ϵ stands for the logarithm having ϵ for its base.

At the origin of the curve, when $x = 0$, let $y = 1$, and then $\lambda_\epsilon c = \frac{m_0}{\lambda_\epsilon r}$. Now substituting this value for $\lambda_{,}c$, we have $\lambda_{,}y=\frac{m_0}{\lambda_{,}r}-\frac{m_0r^x}{\lambda_{,}r}$, $\therefore \lambda_{,}y=\frac{m_0}{\lambda_{,}r}(1-r^x)$; and passing to the number, $y=\varepsilon^{\frac{m_0}{\lambda_{,}r}(1-r^x)}$. Putting k for the modulus of the common logarithm (λ) having 10 for its base, we have $\lambda_{,}y=\frac{\lambda y}{k}$, and $\lambda_{,}r=\frac{\lambda r}{k}$, $\therefore \frac{\lambda y}{k}=\frac{km}{\lambda r}(1-r^x)$; or passing to the number, $y=10^{\frac{k^2m}{\lambda_{,}r}(1-r^x)}$.

Upon the one hypothesis, out of a generation of men an *equal quantity of life** is destroyed in equal times, out of diminishing quantities in existence, the *proportion* that perishes of the residual life constantly *increasing*.

Upon the other hypothesis, a *decreasing proportion* of the residual life is destroyed from birth down to the age of puberty; in the after ages, a *proportion increasing* at different rates is destroyed in equal times. The *quantity* of life *destroyed* in equal times may be the same, or different upon this hypothesis. And in very short intervals of age the differences between *the quantities* of *life destroyed* may be so inconsiderable, that they may be neglected.

The two hypotheses may be illustrated. Assume that at every beat of the heart an equal quantity of vital force on an average is consumed in excess of that produced; or if this does not happen at distant ages, assume that it happens during two consecutive years, two consecutive days, two consecutive pulses of a generation of men, and is represented by the deaths in the two intervals; this will give an idea of the first hypothesis.

The second hypothesis will be represented by assuming that, in addition to the existing force, a certain amount of vital force is produced, while a certain amount is also destroyed at every beat of the heart; the quantity destroyed exceeding the quantity produced in a diminishing ratio, and then in an increasing ratio; the proportional part destroyed being for this purpose always represented by the proportional number of hearts beating to the number of hearts ceasing to beat at every instant of age, among a generation of men. The respirations, the sensations, the secretions, nutrition, and all the vital acts may be conceived like the heart to influence the continuance of the vital force; implying here simply the force which sustains life.

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* The quality or the intensity of life at different ages is purposely left out of consideration.

June 15, 1859.

TABLE B1.—LIFE-TABLE OF HEALTHY ENGLISH DISTRICTS.

Logarithms of the Numbers of Males and Females living at each year of age.

λl_x				λl_x			
Age. x.	Males.	Age. x.	Females.	Age. x.	Males.	Age. x.	Females.
0	4·7086364	0	4·6890835	55	4·4351998	55	4·4177773
1	4·6566579	1	4·6468631	56	4·4279511	56	4·4116015
2	4·6411508	2	4·6327907	57	4·4203212	57	4·4052190
3	4·6315849	3	4·6233586	58	4·4122719	58	4·3981522
4	4·6248271	4	4·6165511	59	4·4037768	59	4·3901691
5	4·6193109	5	4·6110606	60	4·3943905	60	4·3812819
6	4·6148376	6	4·6065737	61	4·3839799	61	4·3714868
7	4·6112225	7	4·6028950	62	4·3725154	62	4·3607637
8	4·6082954	8	4·5998162	63	4·3599518	63	4·3490765
9	4·6059001	9	4·5972658	64	4·3462281	64	4·3363727
10	4·6038946	10	4·5950091	65	4·3312678	65	4·3225837
11	4·6021511	11	4·5929197	66	4·3149786	66	4·3076249
12	4·6005560	12	4·5909763	67	4·2972528	67	4·2913951
13	4·5990100	13	4·5889960	68	4·2779668	68	4·2737774
14	4·5974279	14	4·5869326	69	4·2569811	69	4·2546384
15	4·5957387	15	4·5847269	70	4·2341118	70	4·2338287
16	4·5938855	16	4·5823368	71	4·2092775	71	4·2111835
17	4·5918259	17	4·5797373	72	4·1822024	72	4·1865180
18	4·5895314	18	4·5769202	73	4·1527146	73	4·1596372
19	4·5869878	19	4·5738947	74	4·1205968	74	4·1303269
20	4·5841951	20	4·5706868	75	4·0856157	75	4·0983537
21	4·5811675	21	4·5673396	76	4·0475328	76	4·0634711
22	4·5780527	22	4·5639166	77	4·0060831	77	4·0254[illegible]
23	4·5748607	23	4·5604237	78	3·9609[illegible]77	78	3·9839[illegible]
24	4·5716008	24	4·5568665	79	3·9118198	79	3·9386819
25	4·5682808	25	4·5532198	80	3·8585083	80	3·8893831
26	4·5649078	26	4·5495779	81	3·8005763	81	3·8357013
27	4·5614874	27	4·5458546	82	3·7377111	82	3·7772929
28	4·5580344	28	4·5420830	83	3·6695542	83	3·7137979
29	4·5545223	29	4·5382656	84	3·5967348	84	3·6448105
30	4·5509835	30	4·5344046	85	3·5158541	85	3·5700281
31	4·5474095	31	4·5305013	86	3·4295159	86	3·4889534
32	4·5438005	32	4·5265566	87	3·3363962	87	3·4011904
33	4·5401557	33	4·5225708	88	3·2357583	88	3·3063992
34	4·5364730	34	4·5185435	89	3·1274500	89	3·2038228
35	4·5327494	35	4·5144739	90	3·0109034	90	3·0932880
36	4·5289808	36	4·5103606	91	2·8856349	91	2·9742056
37	4·5251620	37	4·5062016	92	2·7511463	92	2·8460701
38	4·5212864	38	4·5019942	93	2·6069196	93	2·7083590
39	4·5173467	39	4·4977353	94	2·4524273	94	2·5605372
40	4·5133342	40	4·4934212	95	2·2871223	95	2·4020479
41	4·5092398	41	4·4890475	96	2·1104426	96	2·2323219
42	4·5050512	42	4·4846093	97	1·9218108	97	2·0507729
43	4·5007570	43	4·4801012	98	1·7206337	98	1·8567982
44	4·4963465	44	4·4755172	99	1·5063024	99	1·6497793
45	4·4918029	45	4·4708506	100	1·2781926	100	1·4290811
46	4·4871119	46	4·4660943	101	1·0356640	101	1·1940536
47	4·4822570	47	4·4612404	102	0·7780608	102	0·9440265
48	4·4772210	48	4·4562807	103	0·5047118	103	0·6783194
49	4·4719852	49	4·4512061	104	0·2149296	104	0·3962318
50	4·4665301	50	4·4460074	105	9·9080117	105	0·0970476
51	4·4608349	51	4·4406748	106	9·5882396	106	9·7800351
52	4·4548778	52	4·4351962	107	9·2398792	107	9·4444160
53	4·4486358	53	4·4295620	108	8·8771808	108	9·0895160
54	4·4420848	54	4·4237598	109	8·4943792	109	8·7144646

The above Tables were calculated and stereoglyphed by SCHEUTZ's Calculating Machine at the General Register Office, Somerset House. The impression was made by the machine on *papier maché* in the dry state. Sheet lead received the impressions in the original invention. The use of *papier maché* was suggested by Mr. W. MATTHEWS, Overseer in the Firm of Messrs. TAYLOR and FRANCIS. In the wet state, as it is used by stereotype founders, *papier maché* did not however succeed; but after several trials, it was found that dry *papier maché*, black-leaded, supplies a good mould for the stereotype metal.

TABLE C.—HEALTHY DISTRICTS

Age.	Living at each age (l_x).			Dying in each year of age (d_x)			Age
x	Persons.	Males	Females	Persons	Males	Females	x
0	100000	51125	48875	10295	5767	4528	0
1	89705	45358	44347	3005	1591	1414	1
2	86700	43767	42933	1885	953	932	2
3	84815	42814	42001	1305	661	644	3
4	83510	42153	41357	1051	532	519	4
5	82459	41621	40838	847	427	420	5
6	81612	41194	40418	682	341	341	6
7	80930	40853	40077	555	275	280	7
8	80375	40578	39797	459	223	236	8
9	79916	40355	39561	391	186	205	9
10	79525	40169	39356	347	161	186	10
11	79178	40008	39170	324	146	178	11
12	78854	39862	38992	319	142	177	12
13	78535	39720	38815	328	144	184	13
14	78207	39576	38631	350	154	196	14
15	77857	39422	38435	379	168	211	15
16	77478	39254	38224	414	186	228	16
17	77064	39068	37996	451	205	246	17
18	76613	38863	37750	489	227	262	18
19	76124	38636	37488	524	248	276	19
20	75600	38388	37212	552	267	285	20
21	75048	38121	36927	562	272	290	21
22	74486	37849	36637	571	277	294	22
23	73915	37572	36343	577	281	296	23
24	73338	37291	36047	583	284	299	24
25	72755	37007	35748	588	287	301	25
26	72167	36720	35447	591	288	303	26
27	71576	36432	35144	593	289	304	27
28	70983	36143	34840	595	290	305	28
29	70388	35853	34535	596	291	305	29
30	69792	35562	34230	598	292	306	30
31	69194	35270	33924	599	292	307	31
32	68595	34978	33617	599	292	307	32
33	67996	34686	33310	601	293	308	33
34	67395	34393	33002	601	293	308	34
35	66794	34100	32694	603	295	308	35
36	66191	33805	32386	604	296	308	36
37	65587	33509	32078	608	298	310	37
38	64979	33211	31768	610	300	310	38
39	64369	32911	31458	613	302	311	39
40	63756	32609	31147	618	306	312	40
41	63138	32303	30835	623	310	313	41
42	62515	31993	30522	630	315	315	42
43	61885	31678	30207	638	320	318	43
44	61247	31358	29889	645	326	319	44
45	60602	31032	29570	656	334	322	45
46	59946	30698	29248	666	341	325	46
47	59280	30357	28923	679	350	329	47
48	58601	30007	28594	692	360	332	48
49	57909	29647	28262	706	370	336	49

50	57203	29277	27926	722	381	341	50
51	56481	28896	27585	740	394	346	51
52	55741	28502	27239	758	407	351	52
53	54983	28095	26888	777	420	357	53
54	54206	27675	26531	798	435	363	54
55	53408	27240	26168	820	451	369	55
56	52588	26789	25799	843	467	376	56
57	51745	26322	25423	894	483	411	57
58	50851	25839	25012	956	501	455	58
59	49895	25338	24557	1040	542	498	59
60	48855	24796	24059	1123	587	536	60
61	47732	24209	23523	1205	631	574	61
62	46527	23578	22949	1281	672	609	62
63	45246	22906	22340	1356	712	644	63
64	43890	22194	21696	1430	752	678	64
65	42460	21442	21018	1501	789	712	65
66	40959	20653	20306	1571	826	745	66
67	39388	19827	19561	1638	861	777	67
68	37750	18966	18784	1705	895	810	68
69	36045	18071	17974	1767	926	841	69
70	34278	17145	17133	1825	954	871	70
71	32453	16191	16262	1876	978	898	71
72	30577	15213	15364	1921	999	922	72
73	28656	14214	14442	1955	1013	942	73
74	26701	13201	13500	1980	1022	958	74
75	24721	12179	12542	1991	1023	968	75
76	22730	11156	11574	1987	1016	971	76
77	20743	10140	10603	1966	1000	966	77
78	18777	9140	9637	1931	977	954	78
79	16846	8163	8683	1875	943	932	79
80	14971	7220	7751	1803	902	901	80
81	13168	6318	6850	1713	851	862	81
82	11455	5467	5988	1608	794	814	82
83	9847	4673	5174	1491	731	760	83
84	8356	3942	4414	1360	662	698	84
85	6996	3280	3716	1224	591	633	85
86	5772	2689	3083	1084	520	564	86
87	4688	2169	2519	943	448	495	87
88	3745	1721	2024	805	380	425	88
89	2940	1341	1599	675	316	359	89
90	2265	1025	1240	555	257	298	90
91	1710	768	942	444	204	240	91
92	1266	564	702	350	159	191	92
93	916	405	511	269	122	147	93
94	647	283	364	201	89	112	94
95	446	194	252	146	65	81	95
96	300	129	171	104	45	59	96
97	196	84	112	71	31	40	97
98	125	53	72	48	21	27	98
99	77	32	45	31	13	18	99
100	46	19	27	19	8	11	100
101	27	11	16	12	5	7	101
102	15	6	9	7	3	4	102
103	8	3	5	4	1	3	103
104	4	2	2	2	1	1	104
105	2	1	1	1	1	....	105
106	1	..	1	1	....	1	106

TABLE D.—HEALTHY DISTRICTS. PERSONS.

	Dying in each year of age 0—1, 1—2, to 105—106	Born and living at each age	Sum of the numbers born and living at each age (x) from x to the last age in the Table	Population, or the living in each year of age 0 to 1, 1 to 2, &c	(1) Sum of the living, and of the living of every age (x) and upwards to the last age in the Table, also (2) the years which the persons (l_x) will live	(1) The years which the persons at the age (x) and upwards will live, also (2) the years which they have lived over x	Age
		Σd_x	Σl_x	$\frac{1}{2}(l_x+l_{x+1})=l_{x+1}+\frac{1}{2}d_x$	ΣP_x	$\Sigma\frac{1}{2}(Q_x+Q_{x+1})=Y_{x+1}+(Q_{x+1}+\frac{1}{2}P_x)$	
	d_x	l_x	L_x	P_x	Q_x	Y_x	x
0	10295	100000	4951908	92611	4899665	166209701	0
1	3005	89705	4851908	88202	4807054	161356341	1
2	1885	86700	4762203	85758	4718852	156593388	2
3	1305	84815	4675503	84162	4633094	151917415	3
4	1051	83510	4590688	82985	4548932	147326402	4
5	847	82459	4507178	82036	4465947	142818963	5
6	682	81612	4424719	81270	4383911	138394034	6
7	555	80930	4343107	80653	4302641	134050757	7
8	459	80375	4262177	80145	4221988	129788443	8
9	391	79916	4181802	79721	4141843	125606527	9
10	347	79525	4101886	79352	4062122	121504545	10
11	324	79178	4022361	79016	3982770	117482099	11
12	319	78854	3943183	78694	3903754	113538837	12
13	328	78535	3864329	78371	3825060	109674430	13
14	350	78207	3785794	78032	3746689	105888556	14
15	379	77857	3707587	77668	3668657	102180882	15
16	414	77478	3629730	77271	3590989	98551059	16
17	451	77064	3552252	76838	3513718	94998706	17
18	489	76613	3475188	76369	3436880	91523407	18
19	524	76124	3398575	75862	3360511	88124711	19
20	552	75600	3322451	75323	3284649	84802131	20
21	562	75048	3246851	74767	3209326	81555144	21
22	571	74486	3171803	74201	3134559	78383202	22
23	577	73915	3097317	73626	3060358	75285743	23
24	583	73338	3023402	73047	2986732	72262198	24
25	588	72755	2950064	72461	2913685	69311989	25
26	591	72167	2877309	71872	2841224	66434535	26
27	593	71576	2805142	71279	2769352	63629247	27
28	595	70983	2733566	70685	2698073	60895535	28
29	596	70388	2662583	70091	2627388	58232804	29
30	598	69792	2592195	69493	2557297	55640462	30
31	599	69194	2522403	68894	2487804	53117911	31
32	599	68595	2453209	68296	2418910	50664554	32
33	601	67996	2384614	67695	2350614	48279792	33
34	601	67395	2316618	67095	2282919	45963025	34
35	603	66794	2249223	66492	2215824	43713654	35
36	604	66191	2182429	65889	2149332	41531076	36
37	608	65587	2116238	65283	2083443	39414688	37
38	610	64979	2050651	64674	2018160	37363887	38
39	613	64369	1985672	64062	1953486	35378064	39
40	618	63756	1921303	63447	1889424	33456609	40
41	623	63138	1857547	62827	1825977	31598909	41
42	630	62515	1794409	62200	1763150	29804345	42
43	638	61885	1731894	61566	1700950	28072295	43
44	645	61247	1670009	60925	1639384	26402128	44
45	656	60602	1608762	60274	1578459	24793206	45
46	666	59946	1548160	59612	1518185	23244885	46
47	679	59280	1488214	58941	1458573	21756505	47
48	692	58601	1428934	58255	1399632	20327403	48
49	706	57909	1370333	57556	1341377	18956899	49

50	722	57203	1312424	56842	1283821	17644300	50
51	740	56481	1255221	56111	1226979	16388899	51
52	758	55741	1198740	55362	1170868	15189976	52
53	777	54983	1142999	54594	1115506	14046789	53
54	798	54206	1088016	53808	1060912	12958580	54
55	820	53408	1033810	52997	1007104	11924572	55
56	843	52588	980402	52167	954107	10943966	56
57	894	51745	927814	51298	901940	10015943	57
58	956	50851	876069	50373	850642	9139652	58
59	1040	49895	825218	49375	800269	8314197	59
60	1123	48855	775323	48293	750894	7538615	60
61	1205	47732	726468	47130	702601	6811867	61
62	1281	46527	678736	45887	655471	6132831	62
63	1356	45246	632209	44568	609584	5500304	63
64	1430	43890	586963	43175	565016	4913004	64
65	1501	42460	543073	41709	521841	4369575	65
66	1571	40959	500613	40173	480132	3868589	66
67	1638	39388	459654	38570	439959	3408544	67
68	1705	37750	420266	36897	401389	2987869	68
69	1767	36045	382516	35161	364492	2604929	69
70	1825	34278	346471	33366	329331	2258017	70
71	1876	32453	312193	31515	295965	1945369	71
72	1921	30577	279740	29617	264450	1665162	72
73	1955	28656	249163	27678	234833	1415520	73
74	1980	26701	220507	25711	207155	1194527	74
75	1991	24721	193806	23726	181444	1000227	75
76	1987	22730	169085	21736	157718	830646	76
77	1966	20743	146355	19760	135982	683796	77
78	1931	18777	125612	17811	116222	557694	78
79	1875	16846	106835	15909	98411	450377	79
80	1803	14971	89989	14070	82502	359921	80
81	1713	13168	75018	12311	68432	284454	81
82	1608	11455	61850	10651	56121	222178	82
83	1491	9847	50395	9102	45470	171382	83
84	1360	8356	40548	7676	36368	130463	84
85	1224	6996	32192	6383	28692	97933	85
86	1084	5772	25196	5230	22309	72432	86
87	943	4688	19424	4217	17079	52739	87
88	805	3745	14736	3342	12862	37768	88
89	675	2940	10991	2603	9520	26577	89
90	555	2265	8051	1988	6917	18358	90
91	444	1710	5786	1488	4929	12436	91
92	350	1266	4076	1090	3441	8251	92
93	269	916	2810	782	2351	5355	93
94	201	647	1894	547	1569	3395	94
95	146	446	1247	372	1022	2099	95
96	104	300	801	249	650	1263	96
97	71	196	501	160	401	737	97
98	48	125	305	101	241	416	98
99	31	77	180	61	140	226	99
100	19	46	103	37	79	116	100
101	12	27	57	21	42	56	101
102	7	15	30	11	21	24	102
103	4	8	15	7	10	9	103
104	2	4	7	2	3	3	104
105	1	2	3	1	1	..	105
106	1	1	1	..		..	106

Age.	Dying in each year of age 0—1, 1—2, to 104—105.	Born and living at each age.	Sum of the numbers born and living at each age (x) from x to the last age in the Table.	Population, or the living in each year of age 0 to 1, 1 to 2, &c.	(1) Sum of the living, and of the living of every age (x) and upwards to the last age in the Table, also (2) the years which the males (l_x) will live.	(1) The years which the males at the age (x) and upwards will live, also (2) the years which they have lived over x.	Age.
		Σd_x	Σl_x	$\frac{1}{2}(l_x+l_{x+1})=l_{x+1}+\frac{1}{2}d_x$	ΣP_x	$\Sigma\frac{1}{2}(Q_x+Q_{x+1})$ $=Y_{x+1}+(Q_{x+1}+\frac{1}{2}P_x)$.	
x	d_x	l_x	L_x	P_x	Q_x	Y_x	x
0	5767	51125	2509635	46915*	2482745	84008921	0
1	1591	45358	2458510	44562	2435830	81549633	1
2	953	43767	2413152	43291	2391268	79136084	2
3	661	42814	2369385	42483	2347977	76766462	3
4	532	42153	2326571	41887	2305494	74439726	4
5	427	41621	2284418	41408	2263607	72155176	5
6	341	41194	2242797	41023	2222199	69912273	6
7	275	40853	2201603	40716	2181176	67710585	7
8	223	40578	2160750	40466	2140460	65549767	8
9	186	40355	2120172	40262	2099994	63429540	9
10	161	40169	2079817	40089	2059732	61349677	10
11	146	40008	2039648	39935	2019643	59309990	11
12	142	39862	1999640	39791	1979708	57310314	12
13	144	39720	1959778	39648	1939917	55350502	13
14	154	39576	1920058	39499	1900269	53430409	14
15	168	39422	1880482	39338	1860770	51549889	15
16	186	39254	1841060	39161	1821432	49708788	16
17	205	39068	1801806	38965	1782271	47906937	17
18	227	38863	1762738	38750	1743306	46144148	18
19	248	38636	1723875	38512	1704556	44420217	19
20	267	38388	1685239	38254	1666044	42734917	20
21	272	38121	1646851	37985	1627790	41088000	21
22	277	37849	1608730	37711	1589805	39479203	22
23	281	37572	1570881	37431	1552094	37908253	23
24	284	37291	1533309	37149	1514663	36374875	24
25	287	37007	1496018	36864	1477514	34878786	25
26	288	36720	1459011	36576	1440650	33419704	26
27	289	36432	1422291	36287	1404074	31997342	27
28	290	36143	1385859	35998	1367787	30611412	28
29	291	35853	1349716	35708	1331789	29261624	29
30	292	35562	1313863	35416	1296081	27947689	30
31	292	35270	1278301	35124	1260665	26669316	31
32	292	34978	1243031	34832	1225541	25426213	32
33	293	34686	1208053	34539	1190709	24218088	33
34	293	34393	1173367	34247	1156170	23044648	34
35	295	34100	1138974	33952	1121923	21905602	35
36	296	33805	1104874	33657	1087971	20800655	36
37	298	33509	1071069	33360	1054314	19729512	37
38	300	33211	1037560	33061	1020954	18691878	38
39	302	32911	1004349	32760	987893	17687455	39
40	306	32609	971438	32456	955133	16715942	40
41	310	32303	938829	32148	922677	15777037	41
42	315	31993	906526	31836	890529	14870134	42
43	320	31678	874533	31518	858693	13995823	43
44	326	31358	842855	31195	827175	13152889	44
45	334	31032	811497	30865	795980	12341311	45
46	341	30698	780465	30527	765115	11560764	46
47	350	30357	749767	30182	734588	10810912	47
48	360	30007	719410	29827	704406	10091415	48
49	370	29647	689403	29462	674579	9401923	49

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50	381	29277	659756	29087	645117	8742075	50
51	394	28896	630479	28699	616030	8111501	51
52	407	28502	601583	28298	587331	7509821	52
53	420	28095	573081	27885	559033	6936639	53
54	435	27675	544986	27458	531148	6391548	54
55	451	27240	517311	27014	503690	5874129	55
56	467	26789	490071	26556	476676	5383946	56
57	483	26322	463282	26080	450120	4920548	57
58	502	25839	436960	25589	424040	4483468	58
59	542	25338	411121	25067	398451	4072223	59
60	587	24796	385783	24502	373384	3686305	60
61	631	24209	360987	23894	348882	3325172	61
62	672	23578	336778	23242	324988	2988237	62
63	712	22906	313200	22550	301746	2674870	63
64	752	22194	290294	21818	279196	2384399	64
65	789	21442	268100	21047	257378	2116112	65
66	826	20653	246658	20240	236331	1869258	66
67	861	19827	226005	19397	216091	1643047	67
68	895	18966	206178	18518	196694	1436654	68
69	926	18071	187212	17608	178176	1249219	69
70	954	17145	169141	16668	160568	1079847	70
71	978	16191	151996	15702	143900	927613	71
72	999	15213	135805	14714	128198	791564	72
73	1013	14214	120592	13707	113484	670723	73
74	1022	13201	106378	12690	99777	564093	74
75	1023	12179	93177	11668	87087	470661	75
76	1016	11156	80998	10648	75419	389408	76
77	1000	10140	69842	9640	64771	319313	77
78	977	9140	59702	8651	55131	259362	78
79	943	8163	50562	7692	46480	208556	79
80	902	7220	42399	6769	38788	165922	80
81	851	6318	35179	5892	32019	130519	81
82	794	5467	28861	5070	26127	101446	82
83	731	4673	23394	4308	21057	77854	83
84	662	3942	18721	3611	16749	58951	84
85	591	3280	14779	2984	13138	44007	85
86	520	2689	11499	2429	10154	32361	86
87	448	2169	8810	1945	7725	23422	87
88	380	1721	6641	1531	5780	16669	88
89	316	1341	4920	1183	4249	11655	89
90	257	1025	3579	897	3066	7997	90
91	204	768	2554	666	2169	5380	91
92	159	564	1786	484	1503	3544	92
93	122	405	1222	344	1019	2283	93
94	89	283	817	239	675	1436	94
95	65	194	534	161	436	880	95
96	45	129	340	107	275	525	96
97	31	84	211	68	168	303	97
98	21	53	127	43	100	169	98
99	13	32	74	25	57	91	99
100	8	19	42	15	32	46	100
101	5	11	23	9	17	22	101
102	3	6	12	4	8	9	102
103	1	3	6	3	4	3	103
104	1	2	3	1	1	1	104
105	1	1	1	...	...	...	105
106	...	...	...	...	...	...	106

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* P_0 is $\frac{1}{2}(l_0+l_1)\times(\cdot9725)$. The factor ·9725 has been introduced, as the number living in the first year is less than the arithmetical mean of those born and surviving a year.

TABLE F.—HEALTHY DISTRICTS. FEMALES.

[illegible]	[illegible] each year of [illegible]—2, [illegible]—106.	Born and living at each age.	Sum of the numbers born and living at each age (x) from x to the last age in the Table.	Population, or the living in each year of age 0 to 1, 1 to 2, &c.	(1) Sum of the living, and of the living of every age (x) and upwards to the last age in the Table; also (2) the years which the females (l_x) will live.	(1) The years which the females at the age (x) and upwards will live; also (2) the years which they have lived over x.	Age.
		Σd_x.	Σl_x.	$\frac{1}{2}(l_x+l_{x+1})=l_{x+1}+\frac{1}{2}d_x$	ΣP_x.	$\Sigma\frac{1}{2}(Q_x+Q_{x+1})$ $=Y_{x+1}+(Q_{x+1}+\frac{1}{2}P_x)$.	
	d_x.	l_x.	L_x.	P_x.	Q_x.	Y_x.	x.
[illegible]	4528	48875	2442273	45696†	2416920	82200780	0
[illegible]	1414	44347	2393398	43640	2371224	79806708	1
[illegible]	932	42933	2349051	42467	2327584	77457304	2
[illegible]	644	42001	2306118	41679	2285117	75150953	3
[illegible]	519	41357	2264117	41098	2243438	72886676	4
[illegible]	[illegible]	40838	2222760	40628	2202340	70663787	5
[illegible]	[illegible]	40418	2181922	40247	2161712	68481761	6
[illegible]	[illegible]	40077	2141504	39937	2121465	66340172	7
[illegible]	[illegible]	39797	2101427	39679	2081528	64238676	8
[illegible]	[illegible]	39561	2061630	39459	2041849	62176987	9
10	[illegible]	39356	2022069	39263	2002390	60154868	10
11	[illegible]	39170	1982713	39081	1963127	58172109	11
12	[illegible]	38992	1943543	38903	1924046	56228523	12
13	[illegible]	38815	1904551	38723	1885143	54323928	13
14	[illegible]	38631	1865736	38533	1846420	52458147	14
15	[illegible]	38435	1827105	38330	1807887	50630993	15
16	[illegible]	38224	1788670	38110	1769557	48842271	16
17	246	37996	1750446	37873	1731447	47091769	17
18	[illegible]	37750	1712450	37619	1693574	45379259	18
19	276	37488	1674700	37350	1655955	43704494	19
20	285	37212	1637212	37069	1618605	42067214	20
21	290	36927	1600000	36782	1581536	40467144	21
22	294	36637	1563073	36490	1544754	38903999	22
23	296	36343	1526436	36195	1508264	37377490	23
24	299	36047	1490093	35898	1472069	35887323	24
25	301	35748	1454046	35597	1436171	34433203	25
26	303	35447	1418298	35296	1400574	33014831	26
27	304	35144	1382851	34992	1365278	31631905	27
28	305	34840	1347707	34687	1330286	30284123	28
29	305	34535	1312867	34383	1295599	28971180	29
30	306	34230	1278332	34077	1261216	27692773	30
31	307	33924	1244102	33770	1227139	26448595	31
32	307	33617	1210178	33464	1193369	25238341	32
33	308	33310	1176561	33156	1159905	24061704	33
34	308	33002	1143251	32848	1126749	22918377	34
35	308	32694	1110249	32540	1093901	21808052	35
36	308	32386	1077555	32232	1061361	20730421	36
37	310	32078	1045169	31923	1029129	19685176	37
38	310	31768	1013091	31613	997206	18672009	38
39	311	31458	981323	31302	965593	17690609	39
40	312	31147	949865	30991	934291	16740667	40
41	313	30835	918718	30679	903300	15821872	41
42	315	30522	887883	30364	872621	14933911	42
43	318	30207	857361	30048	842257	14076472	43
44	319	29889	827154	29730	812209	13249239	44
45	322	29570	797265	29409	782479	12451895	45
46	325	29248	767695	29085	753070	11684121	46
47	329	28923	738447	28759	723985	10945593	47
48	332	28594	709524	28428	695226	10235988	48
49	336	28262	680930	28094	666798	9554976	49

50	341	27926	652668	27755	638704	8902225	50
51	346	27585	624742	27412	610949	8277398	51
52	351	27239	597157	27064	583537	7680155	52
53	357	26888	569918	26709	556473	7110150	53
54	363	26531	543030	26350	529764	6567032	54
55	369	26168	516499	25983	503414	6050443	55
56	376	25799	490331	25611	477431	5560020	56
57	411	25423	464532	25218	451820	5095395	57
58	455	25012	439109	24784	426602	4656184	58
59	498	24557	414097	24308	401818	4241974	59
60	536	24059	389540	23791	377510	3852310	60
61	574	23523	365481	23236	353719	3486695	61
62	609	22949	341958	22645	330483	3144594	62
63	644	22340	319009	22018	307838	2825434	63
64	678	21696	296669	21357	285820	2528605	64
65	712	21018	274973	20662	264463	2253463	65
66	745	20306	253955	19933	243801	1999331	66
67	777	19561	233649	19173	223868	1765497	67
68	810	18784	214088	18379	204695	1551215	68
69	841	17974	195304	17553	186316	1355710	69
70	871	17133	177330	16698	168763	1178170	70
71	898	16262	160197	15813	152065	1017756	71
72	922	15364	143935	14903	136252	873598	72
73	942	14442	128571	13971	121349	744797	73
74	958	13500	114129	13021	107378	630434	74
75	968	12542	100629	12058	94357	529566	75
76	971	11574	88087	11088	82299	441238	76
77	966	10603	76513	10120	71211	364483	77
78	954	9637	65910	9160	61091	298332	78
79	932	8683	56273	8217	51931	241821	79
80	901	7751	47590	7301	43714	193999	80
81	862	6850	39839	6419	36413	153935	81
82	814	5988	32989	5581	29994	120732	82
83	760	5174	27001	4794	24413	93528	83
84	698	4414	21827	4065	19619	71512	84
85	633	3716	17413	3399	15554	53926	85
86	564	3083	13697	2801	12155	40071	86
87	495	2519	10614	2272	9354	29317	87
88	425	2024	8095	1811	7082	21099	88
89	359	1599	6071	1420	5271	14922	89
90	298	1240	4472	1091	3851	10361	90
91	240	942	3232	822	2760	7056	91
92	191	702	2290	606	1938	4707	92
93	147	511	1588	438	1332	3072	93
94	112	364	1077	308	894	1959	94
95	81	252	713	211	586	1219	95
96	59	171	461	142	375	738	96
97	40	112	290	92	233	434	97
98	27	72	178	58	141	247	98
99	18	45	106	36	85	135	99
100	11	27	61	22	47	70	100
101	7	16	34	12	25	34	101
102	4	9	18	7	13	15	102
103	3	5	9	4	6	6	103
104	1	2	4	1	2	2	104
105	1*	1*	2	1	1	..	105
106	1*	1*	1	..	..	..	106

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* The values of l_{104}, l_{105}, and l_{106}, decimally carried out, are 2·490, 1·250, and 0·603; and their differences are 1·240, 0·647, and 0·325. The apparent anomaly that no death happens between the ages 105 and 106, arises from the omission of decimals.

† P_0 is $\frac{1}{2}(l_0+l_1)\times(\cdot 98037)$. The factor ·98037 has been introduced, as the number living in the first year is less than the arithmetical mean of those born and surviving a year.

TABLE G.—HEALTHY DISTRICTS LIFE-TABLE.

The MEAN AFTER-LIFETIME (or the *Expectation of Life*) at the age x, and at the age x and upwards; also the MEAN AGES of the LIVING and the MEAN AGES AT DEATH. (Constructed from Tables D, E, F.)

	PERSONS.				
Age (or past Lifetime).	Mean After-lifetime of Persons of the Age x.	Mean After-lifetime of Persons of the Age x and *upwards*.	Mean Age of Persons living of the Age x and *upwards*.	Mean Age at Death: Of Persons actually living at the Age x.	Mean Age at Death: Of Persons actually living at the Age x and *upwards*.
x.	$A_x=\frac{Q_x}{D_x}$.	$A'_x=\frac{Y_x}{Q_x}$.	$x+A'_x$.	$x+A_x$.	$x+2A'_x$.
0	49 00	33 92	33 92	49 00	67·84
5	54·16	31·98	36 98	59 16	68·96
10	51·08	29·91	39 91	61 08	69·82
15	47·12	27·85	42·85	62·12	70·70
20	43·45	25·82	45·82	63 45	71·64
25	40·05	23·79	48·79	65 05	72·58
30	36·64	21·76	51·76	66 64	73·52
35	33·17	19·73	54·73	68 17	74·46
40	29·64	17·71	57·71	69·64	75·42
45	26·05	15·71	60 71	71·05	76 42
50	22·44	13·74	63 74	72·44	77·48
55	18·86	11·84	66 84	73 86	78·68
60	15·37	10·04	70·04	75·37	80·08
65	12·29	8·37	73·37	77·29	81·74
70	9 61	6·86	76·86	79·61	83·72
75	7·34	5·51	80·51	82·34	86·02
80	5·51	4·36	84 36	85·51	88·72
85	4·10	3·41	88·41	89·10	91·82
90	3·05	2·65	92·65	93 05	95·30
95	2·29	2·05	97·05	97 29	99·10
100	1·72	1·47	101·47	101 72	102·94

Age (or past-Lifetime).	MALES.		FEMALES.	
	Mean After-lifetime of Males of the Age x.	Mean Age at Death of Males actually living at the Age x.	Mean After-lifetime of Females of the Age x.	Mean Age at Death of Females actually living at the Age x.
x.	$A_x=\frac{Q_x}{D_x}$.	$x+A_x$.	$A_x=\frac{Q_x}{D_x}$.	$x+A_x$.
0	48·56	48·56	49 45	49·45
5	54·39	59·39	53·93	58·93
10	51·28	61·28	50 88	60·88
15	47·20	62·20	47·04	62·04
20	43·40	63·40	43·50	63·50
25	39·93	64·93	40·18	65·18
30	36·45	66·45	36 85	66 85
35	32·90	67·90	33·46	68·46
40	29·29	69 29	30·00	70·00
45	25·65	70 65	26·46	71·46
50	22·03	72·03	22 87	72 87
55	18·49	73 49	19·24	74·24
60	15·06	75 06	15·69	75 69
65	12 00	77·00	12·58	77·58
70	9·37	79·37	9 85	79·85
75	7·15	82 15	7·52	82·52
80	5·37	85·37	5 64	85·64
85	4·01	89 01	4·19	89·19
90	2·99	92·99	3·11	93·11
95	2·25	97·25	2·32	97·32
100	1·69	101 69	1·75	101·75

The Table may be read thus:—Persons in the Healthy Districts of England of the precise age 20 will live on an average 43·45 years; while persons of the age 20 and *upwards*, living in a normally constituted population of the same character, will live on an average 25·82 years. The mean age of persons of the age 20 and *upwards* is 45·82 years; the mean age at death of persons living at the precise age 20 will be 63·45; while the mean age at death of persons actually living at the age x and *upwards* will be 71 64 years.

XXXV. *Photo-chemical Researches.*—Part IV. *By* Robert Bunsen, *Professor of Chemistry at the University of Heidelberg, and* Henry Enfield Roscoe, *B.A., Ph.D., Professor of Chemistry at Owens College, Manchester.*

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The measureless store of energy which Nature has amassed in the sun's body flows in an unceasing current as solar rays throughout the universe.

The labour expended on the earth's surface in the maintenance of the animal and vegetable creation, and in the production of geological change, is derived, almost exclusively, from this source.

Those of the sun's rays which vibrate most slowly, and form the red portion of the solar spectrum, including the rays visible and invisible which surround them, give rise by their absorption, more especially to the thermic actions observed on the surface of the earth, and in both the fluid zones which as ocean and atmosphere encircle the solid crust of our planet. These rays constitute the sources of heat which, in those grand processes of distillation and atmospheric deposit, have effected those vast transformations of the earth's crust, by the study of which we obtain some idea of the immensity of the sun's action exerted during geological ages upon our globe.

Of a totally different kind, on a scale less magnificent, but not less important, are the effects mainly produced by the more highly refrangible and more rapidly vibrating portions of the solar rays. These rays exert the most marked influence upon the chemical changes on which the vegetable world depends; and are therefore of the greatest importance as regards the character and geographical distribution of organic nature.

Although the atmospheric phenomena regulating the amount and distribution of the chemical action of light on the earth's surface have not as yet been systematized to the same extent as the thermic, electrical, and magnetic phenomena of meteorology, the reason is not so much that their importance has been overlooked, as that the difficulties which surround an exact investigation of the subject have up to the present time proved insurmountable. If in the following research we have attempted to clear a path towards this new field of meteorological inquiry, we trust that these acknowledged difficulties may in some degree prove an excuse for the many failings which our investigation presents.

All experiments on the above subject can be of no value until some method has been discovered for expressing the chemical action of light in terms of some general and comparable measure. To this point, therefore, we were first obliged to direct our attention.

I. COMPARATIVE AND ABSOLUTE MEASUREMENT OF THE CHEMICAL RAYS.

In all photometric measurements, whether made with the eye, or by means of thermic, thermo-electric, or chemical agencies, the great difficulty has been to obtain a convenient constant source of light.

The flames of ordinary candles or lamps are subject to variation in their optical and chemical illuminating power, such as to prevent their being employed when any great degree of accuracy is required. The intensity of the light evolved by a wire heated to whiteness between the poles of a constant battery is also extremely variable, as an imperceptible alteration in the quantity of circulating electricity effects a considerable change in the luminosity of the incandescent wire. These difficulties are, however, overcome when a flame is employed which is fed by a current of gas issuing with uniform velocity. Olefiant gas cannot be advantageously used for the production of such normal flames, not only on account of the difficulty of preparing large quantities of this gas in a state of purity, but also because on combustion it suffers very complicated decompositions. Carbonic oxide, on the other hand, burns without elimination of carbon, does not undergo other secondary decompositions, and forms one product of combustion, namely, carbonic acid. In other respects too, especially on account of the considerable chemical action which it effects, as also from the ease with which it can be prepared in a state of purity from the salts of formic acid, the flame of carbonic oxide is particularly well adapted for absolute photo-chemical measurements.

The amount of light radiating from a flame does not merely depend on the mass and constitution of the illuminating material, but is often even more influenced by the particular circumstances under which the combustion is effected. We have therefore been obliged to determine, more accurately than we had previously done*, the conditions most favourable to the constant evolution of light from flames of various kinds. The flames formed by the combustion in free air of gases issuing from small openings under a pressure of even only a few millimetres of water, are subject to such changes in form and temperature, arising from the production of lateral currents of air, that they cannot be employed as constant sources of light. When, however, the pressure under which the gas issues differs but very slightly from that of the atmosphere, the flame assumes a perfectly constant form. In the following experiments the rate at which the gas issued from the burner during the combustion of the carbonic oxide, which formed our constant source of light, was only 129·9 millimetres in the second. By means of the formula for the rate of issue of gases, not regarding the friction, which in this case is very small, we have calculated the amount of the difference of pressure which forces the gas out, and found it to be equal to a column of water of 0·001 millimetre in height. When the gas issues under so slight a pressure, and burns from an opening of several millimetres in diameter, placed in the blackened tin box which we have formerly described†, the flame assumes the form of a very obtuse cone and burns perfectly steadily.

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* Philosophical Transactions, 1857, p. 874.

† Ibid. p. 875.

The flame of carbonic oxide, upon which all our measurements are founded, issued from a circular platinum burner 7 millimetres in diameter. The normal velocity with which the gas issued, was taken to be such that 5 cubic centimetres of gas, at 0° C. and 0·76 pressure, passed through the burner in 1 second of time. As, however, it was practically impossible so to regulate the supply of gas that it should issue exactly at the required rate, we were obliged to determine, first of all, the relation existing between the supply of gas and the luminosity of the flame. This determination we have made, by means of photo-chemical measurement, for velocities of issue lying near the standard rate of 5 cubic centimetres per second. The carbonic oxide used for these experiments was prepared by the action of sulphuric acid on pure formiate of soda, and after being washed with potash, was collected in a large gasometer, previously calibrated, in which the volume of gas could at any moment be read off, and the temperature and pressure observed by means of a thermometer and manometer. The insolation-vessel of our chemical photometer was placed at a distance of 176 millims. from the flame. The gas contained in the gasometer under a constant pressure of $0^{m}\cdot8$ of mercury, passed through a very narrow stopcock into the wide tube on which the burner was placed, so that when the gas issued from the burner the excess of pressure over that of the atmosphere, measured by a small manometer, amounted to a column of water a very small fraction of a millimetre in height.

The actual observations which are thrown together in the annexed Table, were made in the following manner. One observer at fixed intervals read off the chemical action of the flame as seen on the index of the instrument, whilst another noted the volume of gas contained in the gasometer before and after the experiment, as also the temperature and pressure.

TABLE I.

I.	II.	III.	IV.	V.	VI.	VII. *g*.	VIII. *w*.
			m				
1	3	5342·7 4274·2	0·8230 0·8244	20°·0 C. 20·0 C.	41·8	5·950	13·932
2	6	3907·3 2237·9	0·8254 0·8280	20·0 C. 20·0 C.	56·7	4·678	9·450
3	5	6596·7 5439·5	0·8179 0·8195	19·5 C. 19·5 C.	34·0	3·839	6·799
4	10	8600·8 6762·1	0·8199 0·8219	20·0 C. 20·0 C.	41·5	3·053	4·152

Column I. gives the number of the experiment; column II. the duration of the experiment in minutes; column III. the volume of gas, in cubic centimetres, contained in the gasometer before and after the experiment; columns IV. and V. the observed pressure and temperature; column VI. the chemical action effected during the experiment, read off on the scale of the instrument; column VII. the volume (*g*) of carbonic oxide, reduced to 0° and 0·76, which issued in one second; and, lastly, in column VIII. the chemical action (*w*) effected in one minute expressed in divisions of the photometer-scale.

The observed action (w) does not, however, express the total action of the carbonic oxide flame, for between the burner and the sensitive gas several screens were placed. The light had to pass, in the first place, through the water-screen, then through two plates of mica, and also through the glass sides of the insolation-vessel before it reached the mixture of chlorine and hydrogen. Hence in order to find what the action would have been if the light had not been diminished by passing through these screens, we must multiply these values of w by a constant dependent on the absorbent character of these media. This constant, which we will call K, is made up of three parts:—

1. The diminution which the light suffers in passing through the mica-plates $=$L.
2. The diminution the light suffers by passing through the water-screen $=$M.
3. The light lost by reflexion from the surfaces of the insolation-vessel $=$N.

The value of L is easily found from the coefficient of reflexion for mica, which we have already determined *. From the experimental number $\rho=0{\cdot}1011$, the value of $L=1{\cdot}450$ is found from the equation $L=\frac{1+3\rho}{1-\rho}$. We determined the value of M by observing, twelve times successively, the action effected in 30 seconds by a constant gas flame, the water-screen being alternately placed before the flame and removed. The following Table contains the results of these observations:—

TABLE II.

No	Action without screen.	Action with screen.
1	12·69	9·25
2	12·85	9·59
3	12·46	9·81
4	12·31	9·59
5	13·47	9·00
Mean ...	12·76	9·45

By dividing the numbers in the second column by the corresponding numbers in the third, the mean value of $M=1{\cdot}351$ is obtained.

The value of N is arrived at by help of the following considerations:—Let $A\,A_1$ (fig. 1, Plate XLIII.) represent the surface of the insolation-vessel nearest the flame, and $B\,B_1$ that furthest from the flame. The space between those two surfaces being supposed to be filled with sensitive gas, let 1 represent the intensity of the light falling perpendicularly on the insolation-vessel; and let $r=\frac{2\rho}{1+\rho}$ represent the amount of light reflected from both surfaces of the side $A\,A_1$ of the insolation-vessel, when ρ is the coefficient of reflexion for glass and air; and let $\gamma=10^{-\alpha h}$ represent the amount of light remaining after passing from A to $B=h$, when α represents the coefficient of extinction of the chlorine and hydrogen gas; and lastly, let $d=1-\frac{2\rho}{1+\rho}$ represent the

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* Philosophical Transactions, 1857, p. 609.

intensity of the original unit amount of light after having passed through the side $A A_2$ of the insolation-vessel. For the sake of clearness, we have represented the rays of light, which are in reality reflected perpendicularly from the surface $B B_1$, by the lines AB, BA_1, A_1B_1, &c. At these several points the unit quantity of light falling on the insolation-vessel has the following intensities:—

At A in the line AB the intensity is d.
At B in the line BA the intensity is $d\gamma$.
At B in the line BA_1 the intensity is $d\gamma r$.
At A_1 in the line BA_1 the intensity is $d\gamma^2 r$.
At A_1 in the line A_1B_1 the intensity is $d\gamma^2 r^2$.
At B_1 in the line A_1B_1 the intensity is $d\gamma^3 r^2$.
At B_1 in the line B_1A_2 the intensity is $d\gamma^3 r^3$.
At A_2 in the line B_1A_2 the intensity is $d\gamma^4 r^3$.
&c.

Hence the chemical action proportional to these various intensities is found to be

$d\beta$ in the line AB.
$d\gamma r\beta$ in the line BA_1.
$d\gamma^2 r^2\beta$ in the line A_1B_1.
$d\gamma^3 r^3\beta$ in the line B_1A_2.

The sum of these actions is, however,

$$d\beta(1+\gamma r+\gamma^2 r^2+\gamma^3 r^3+\cdots\cdots)=\frac{d\beta}{1-\gamma r},$$

or, as $d=1-r$, we have $\frac{1-r}{1-\gamma r}\cdot\beta$.

We can therefore eliminate the error arising from the infinite number of reflexions which the light suffers in the interior of the insolation-vessel, by multiplying the action, as read off on the scale of the instrument, by the factor

$$\frac{1-\gamma r}{1-r}=N. \qquad (1.)$$

The numbers thus obtained represent the amount of photo-chemical action which would have been observed if the sensitive gas had not been surrounded by a reflecting medium.

In the third part of our researches* we determined the value of $\rho=0{\cdot}0509$ and $\alpha=0{\cdot}00427$ for the flame of coal-gas; $\alpha_1=0{\cdot}0136$ for the diffuse cloudless zenith light of morning; and $\alpha_2=0{\cdot}0174$ for the cloudless zenith light of evening. The depth of the insolation-vessel was $h=9{\cdot}4$ millims. When these values are substituted in formula (1.),

N is found to be equal to 1·010 for a coal-gas flame;
N is found to be equal to 1·028 for morning zenith light of an unclouded sky;
N is found to be equal to 1·034 for evening zenith light of an unclouded sky;

and if we take the mean of these closely agreeing numbers, $N=1{\cdot}024$, as representing the loss by reflexion of the flame of carbonic oxide, we have, as the value of the product L, M, N,

$$K=2{\cdot}005.$$

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* Philosophical Transactions, 1857, p. 605, &c.

The numbers contained in column VIII. of Table I. must therefore be multiplied by this constant, 2·005, in order to obtain the action (w_1) which the flame would have produced if no loss of light from mica-plates, water-screen, or by reflexion from the insolation-vessel had occurred. The following numbers give the values of w_1, the normal action effected when the gas issued at the corresponding rates g:—

g.	w_1.
5·950	27·86
4·673	18·00
3·839	13·60
3·053	8·30

If the values of A, B, and C are calculated from the above values of g and w_1 in the following series of powers of $(5-g)$,

$$w_1=A+B(5-g)+C(5-g)^2+\cdots,$$

it is seen that the value of C is so small, that the third term $C(5-g)^2$ may be struck out. The values of A and B, calculated according to the method of least squares, give the following simple equation,

$$w_1=21\cdot34\{1-0\cdot3153(5-g)\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

From this it is seen that, although the rates at which the gas issues differ considerably, the increase in the action effected by the flame is, within the observed limits, directly proportional to the increase in the rate of issue of the gas. The straight line (fig. 2, Plate XLIV.) shows how closely the observations agree with this assumption. The abscissæ represent the volumes (in cub. cent.) of carbonic oxide, measured at 0° and $0^{m}\cdot76$, which escaped from the burner in one second; and the ordinates give the corresponding chemical actions observed in divisions of the instrument in one minute, when the distance from the burner to the insolation-vessel was 176 millims. The observations are denoted in the figure by black dots on or near the line.

By means of the foregoing equation (2.) it is easy to calculate, from the chemical action observed for any particular rate of issue of the gas lying between 3 and 6 cub. cent. per second, what the chemical action would have been if the gas had issued at the rate $g=5$ instead of at the observed rate, other circumstances remaining of course the same.

The standard flame which we employ to compare all the sources of light we have to measure may be thus defined:—"*The standard flame is that produced by the free combustion in atmospheric air of carbonic oxide gas issuing at the rate of* 5 *cub. cent. per second (reduced to* 0° *and* $0^{m}\cdot76$) *from a circular platinum burner* 7 *millims. in diameter, under a pressure differing but very slightly from that of the atmosphere.*"

By means of the standard flame we are enabled to reduce the indications of different instruments to the same unit of luminous intensity, and thus to render them comparable. For this purpose we define the photometric unit for the chemically active rays to be that amount of action which is produced in one minute by a standard flame placed at a distance of 1 metre; and we determine experimentally for each instrument the number of such units (n) which correspond to one division on the observation-tube. By multiplying the observed number of divisions (t) by the number of photometric units (n) equal to one division, the observations are reduced to a comparable standard, whose unit is taken to be the amount of action which a standard flame at the distance of one metre effects on the normal mixture of chlorine and hydrogen in one minute, when the depth of insolated gas is so small that the alteration of the value of the coefficient of absorption for different sources of light becomes inappreciable.

We call this unit a chemical unit of light, and ten thousand of these units, one chemical degree of light.

In the following experiments we have always employed the same insolation-vessel, but two observation-tubes. On one of the observation-tubes, or scales, which we shall henceforward call "the narrow," or "No. 1 scale," one division corresponded to $n=0·6612$; on the wide, or "No. 2. scale," one division corresponded to $n=2·249$. One division on the narrow scale had a capacity of 0·7642 cubic millimetres; one division on the wide scale contained 2·598 cubic millimetres.

A few examples will best explain the use of this measure of light.

1. By *Chemical Illumination* we signify that amount of chemically active light which falls perpendicularly on a plane surface. If the insolation-vessel of our instrument be supposed to form a part of this plane, the number of divisions on the scale which the index has passed in one minute, multiplied by n, expresses the number of units of light to which that plane is illuminated. As an example of this kind, we will determine, for two different sources of light, the distance at which they must be placed, in order that a given surface may be illuminated with one degree of light. We chose as the sources of light flames of carbonic oxide and coal-gas, which were each fed with gas at the rate of 4·105 cub. cent., at 0° and $0^{m}·76$ per second. The carbonic oxide issued from the large platinum burner, the coal-gas from an ordinary burner, and both gases were expelled from the burners under a pressure but very slightly exceeding that of the atmosphere. The rays from both flames had to pass through two plates of mica and a water-screen; the narrow scale, No. 1, was employed in these experiments. The carbonic oxide flame, placed at a distance of $0^{m}·176$ from the insolation-vessel, produced an action of 7·68 divisions in one minute. This corresponds to $Kn.7·68=10·15$ degrees of light. As the amount of illumination is inversely proportional to the square of the distance from the source of light to the illuminated surface, the distance (r) at which the source of light must be placed in order that the surface may be illuminated one degree, is found to be

$$r=\sqrt{0·176^{2}.10·15}=0·5607 \text{ metre.}$$

The flame of coal-gas, burnt under the same conditions, gave at a distance of $0^{m}·216$ from the insolation-vessel an action of 13·98 divisions in the minute, corresponding to $K.n.13·98=18·48$ degrees of light. This flame accordingly effects a chemical illumination of one degree at a distance equal to

$$r=\sqrt{0·216^{2}\times 18·48}=0·9287 \text{ metre.}$$

In like manner it is found that when the carbonic oxide flame is removed to a distance of $0^{\mathrm{m}}\cdot3349$, and the coal-gas flame to a distance of $0^{\mathrm{m}}\cdot4309$, the surface is chemically illuminated by 5 units of light.

2. *The chemical illuminating power*, or the chemical intensity of various sources of light which may be considered as luminous points, is measured by the chemical action effected by the rays emanating from such sources in equal times and at equal distances. As, however, in the case of such sources of light, the chemical illuminating power is directly proportional to the square of the distances at which an equal amount of chemical illumination is obtained, and as the standard flame produces an illumination of one unit of light at a distance of 1 metre, we have, in order to compare the chemical illuminating power of any luminous source of the kind described, with the unit of chemical light of the standard flame, only to take the square of the distance at which the luminous source in question produces the illumination of one unit of light. If we carry out this calculation for the three flames already mentioned, we find that the chemical intensities of the standard flame, the carbonic oxide flame of the above dimensions, and the coal-gas flame, are respectively represented by the numbers

1, 0·718, and 1·972.

It is of some interest to compare the chemical with the visual* illuminating power of these different flames. For this purpose we have compared the visual illumination produced by the carbonic oxide flame burning under the prescribed conditions, and issuing at the rate of 6·032 cub. cent. per second, with that of the flame of coal-gas above described. Although these photometric measurements could not be made very exactly, owing to the different colour of the two flames, we were nevertheless able to assure ourselves that a coal-gas flame, burning at the rate of 4·105 per second, produced a visual illumination of *at least* 150 times as great an intensity as the carbonic oxide flame, burning at the rate of 6·032 cub. cent.

The chemical illuminating powers of flames of carbonic oxide issuing at the rate of 5 cub. cent., 6·032 cub. cent., and 4·105 cub. cent. in the second, are found from formula (2.) to be in the relation of 21·34, 28·28, and 15·32. As in one and the same source of light the chemical and visual intensities are proportional, the visual intensities of these three flames are to the visual intensity of the coal-gas flame, issuing at the rate of 4·105 cub. cent., as 21·34, 28·28, and 15·32, to 42·42. Hence we have

	Visual intensity.	Chemical intensity.
Flame of carbonic oxide issuing at rate of 5 cub. cent. per second	1·000	1·000
Flame of carbonic oxide issuing at rate of 4·105 cub. cent. per second . .	0·718	0·718
Flame of coal-gas issuing at rate of 4·105 cub. cent. per second . .	198·800	1·972

From this we see that the visual illuminating power of the above coal-gas flame is nearly 200 times as large as that of the normal carbonic oxide flame, whereas the chemical illuminating power of the same is not double that of the normal flame; and hence

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[*] By "visual" illuminating power we intend to signify the luminous intensity as measured by its effect on the eye.

we conclude that, in the case of different luminous sources, the amount of light received by the eye in no degree serves as a measure of the chemical action which the light can effect.

3. *Chemical Brightness.*—The amount of light measured photo-chemically which falls perpendicularly from a luminous surface upon a physical point, divided by the apparent magnitude of the surface, we designate as the *intrinsic chemical brightness.* As unit of apparent magnitude, we select the thousandth part of a hemisphere; and as unit of intrinsic brightness, we select the amount of light which must proceed from this thousandth part in order to communicate an illumination of 1 degree of light to a physical point placed in the centre of the hemisphere.

In order to measure the amount of the chemical brightness of a surface, all we need is to allow the light proceeding from this surface to fall through a circular opening upon the insolation-vessel, and to measure in degrees of light the chemical illumination thus effected. Let l signify the observed number of light-degrees, d the diameter of the circular opening, r its distance from the insolation-vessel: we then have $\frac{2\pi r^2}{1000}$ as the thousandth part of a hemisphere whose radius is r, and $2\pi r^2 . 2\sin^2\frac{\theta}{2}$ as the portion of this hemisphere occupied by the circular opening, when the value of θ is obtained from the equation $\sin\theta=\frac{d}{2r}$. The portion of the hemisphere cut out by the circular opening, is to the thousandth part of the hemisphere in the proportion of $2\sin^2\frac{\theta}{2}$ to $\frac{1}{1000}$. Hence we obtain the intrinsic brightness H expressed in the above units from the following equation,—

$$H=\frac{l}{2000\sin^2\frac{\theta}{2}}, \quad \ldots \ldots \ldots \ldots \quad (3.)$$

in which l signifies the number of observed degrees of light.

As an example of such a measurement, we select a comparison of various sized circular portions of the zenith of a cloudless sky. The elements for this calculation are obtained from the following experiment:—Outside the window of our dark room (*aa*, fig. 3) was placed a mirror (*b*), inclined at an angle of 45° towards the horizon; by means of this mirror the light from the zenith could be reflected through a horizontal tube (*c*, fig. 3, Plate XLIII.) upon the insolation-vessel (*i*) in the dark room. On the end of the tube nearest the mirror, and outside the window, circular diaphragms were placed, the diameters (*d*) of which are given in column I. of Table III. The distance of the insolation-vessel from this opening was, as is seen in column II., for all the experiments $2^{m}\cdot225$. Before the insolation-vessel, a screen (*g*, fig. 3, Plate XLIII.) containing two plates of mica was placed. The chemical action (*w*), effected in one minute, obtained from the mean of six observations made with the scale No. 2, is found in column III. The number of degrees of light (*l*) seen in column IV., which illuminate the insolation-vessel, is found from the equation $N.L.R.\kappa.w=l$, in which the following constant values occur:—

R*=1·777 for the loss of light by the reflexion from mirror.
L =1·450 for the loss of light from the mica-plates.
N =1·031 for the loss of light from the glass sides of insolation-vessel.
n =2·249 for the reduction of the observed divisions on scale No. 2 to degrees of light.

Column V. gives the required mean brightness calculated from formula (3.); column VI. contains the magnitude of the observed surface compared with that of the hemisphere taken as unity.

TABLE III.

No of experiment	I. *d*	II *r*.	III *w*	IV. *l*	V H.	VI
	m	m				
1.	0·0590	2·225	10·89	65·1	0·741	0·0000878
2.	0·0359	2·225	4·04	24·1	0·744	0·0000325
3.	0·0590	2·225	10·59	63·3	0·720	0·0000878
4.	0·0530	2·225	8·71	52·0	0·733	0·0000708
5.	0·0590	2·225	10·60	63·3	0·721	0·0000878
6.	0·0359	2·225	3·89	23·2	0·715	0·0000325

From these numbers it is seen that, within the limits of experimental error, the portions of sky near the zenith are equally bright; hence that the chemical action effected by portions of zenith-sky not exceeding 0·00009 of the total heavens, may be taken to be directly proportional to their apparent magnitudes, when the sun does not materially alter its position.

The determinations hitherto considered all depend upon the comparison of the light which has to be measured with that of a constant luminous source. In many cases, however, it is of importance to express the chemical action of light, not only comparatively in *units of light* or in *degrees of light*, but also *absolutely*, that is, in *units of time and space*. Such an absolute measure can be obtained from the observations of our instrument when the following data are given:—

v=the volume of hydrochloric acid, measured at 0° C. and 0m·76, formed by the unit of light;

h=the thickness of sensitive gas (reduced to 0° and 0·76) through which the light passed;

q=the surface-area of the insolated gas;

α=the coefficient of extinction of the chlorine and hydrogen for the light employed;

l=the number of observed units of light in the time t.

When the values of these quantities are known, the volume of hydrochloric acid which would be formed in the time t, by the rays falling perpendicularly on the unit of surface, if $h=\infty$, or if the light had been completely extinguished in passing through an indefinitely extended atmosphere of dry chlorine and hydrogen gas, is found from the equation

$$V=\frac{v}{q}\cdot\frac{l}{1-10^{-\alpha h}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

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* The value of R was determined by a special experiment with candle-light.

As an example of such a measurement, we cite the experiment made with the coal-gas flame, detailed in the Philosophical Transactions for 1857, page 379. This flame gave an action of 14·2 divisions of scale No. 2 in one minute at a distance of $0^{m}\cdot216$ at a temperature of $22^{\circ}\cdot7$ C., and under $0^{m}\cdot753$ barometric pressure. From the capacity of the observation-tube, the volume of hydrochloric acid produced by one unit of light is found to be

$$v=0\cdot001155 \text{ cubic centimetre.}$$

The exposed area of the insolation-vessel was

$$q=3\cdot3 \text{ square centimetres.}$$

The thickness of gas (h) through which the light passed is obtained from the thickness of the insolation-vessel $d=0\cdot94$ centimetre, by help of the following equation:—

$$h=\frac{d(\mathrm{P}-p)}{(1+0\cdot00366t)0\cdot76},$$

in which P represents the barometric pressure, and p the tension of aqueous vapour at the temperature t of the experiment. By calculation h is found to be $h=0\cdot837$. According to our former experiments*, the reciprocal of the coefficient of extinction for gas-light in pure chlorine and hydrogen, at 0° C. and 0·76, is $\frac{1}{23\cdot4}$ centimetre. As between flame and insolation-vessel two plates of mica and a water-screen were placed, the number of units of light (l) corresponding to the observed alteration on the scale is obtained by multiplying this alteration by the factors $k=2\cdot005$ and $n=0\cdot6612$. When this is carried out a value $l=18\cdot56$ is obtained, which, when substituted in formula (4.), gives

$$\mathrm{V}=0\cdot08204 \text{ cubic centimetre.}$$

We thus see that the chemical rays which fall on a surface of one square centimetre from the flame of coal-gas of above dimensions, placed at a distance of 21·6 centimetres, produce in one minute the combination of a column of hydrochloric acid of 0·08204 centimetre in height over the whole area of this square centimetre, under the supposition that the chemical action of the rays had been completely expended in passing through a layer of sensitive gas of indefinite thickness. If we imagine the flame to be placed in the centre, and the insolation-vessel at the surface of a sphere whose radius is equal to the distance of the flame from the insolation-vessel, $r=21\cdot6$ c. m.; the amount of hydrochloric acid formed in one minute is V cubic centimetre on every square centimetre of surface, or on the whole surface of the sphere

$$4\pi r^{2}\mathrm{V}=481\cdot1 \text{ cubic centimetres.}$$

As the rate of issue of the coal-gas was 4·105 cub.cent. per second, the quantity of coal-gas which was burnt during the time in which 481·1 cub. cent. of hydrochloric acid was formed is 246·8 cub. cent.

The result of this calculation may be thus expressed: when one cubic centimetre of coal-gas burns in the above-mentioned manner, such an amount of chemical rays is evolved, that 1·95 cubic centimetre of hydrogen and chlorine can be thereby combined as hydrochloric acid.

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* Philosophical Transactions, 1857, p. 616.

The mean composition by volume of the coal-gas burnt in the flame is, however,—

Hydrogen	41·42
Marsh-gas	39·49
Carbonic oxide	5·97
Elayl	4·57
Ditetryl	3·25
Nitrogen	5·10
Carbonic acid	0·20
	100·00

In the combustion of one cubic centimetre of this gas, an amount of heat is evolved sufficient to raise 1 grm. of water from 0° to 6°·8 C.; one cubic centimetre of hydrogen, on the other hand, on combining with chlorine, evolves heat enough to raise 1 grm. of water from 0° to 2°·08 C.; hence we may conclude that for every thermal unit evolved by the union of the components of the coal-gas flame in question with the oxygen of the air, there are only 0·30 thermal units evolved from the chemical combination effected by this flame upon an infinitely extended atmosphere of chlorine and hydrogen.

When it is required to represent in absolute measure the amount of light which falls during a time t on the unit of surface, it is most convenient to express the photo-chemical action of the rays falling on such a surface according to formula (4.), as a column of hydrochloric acid at 0° C. and 0·76, which would have been produced if the light had passed in a parallel direction through an unlimited atmosphere of chlorine and hydrogen. This light, measured in metres, we propose, for the sake of abbreviation, to call a *Light-metre*.

The chemical action of the solar rays is best expressed in these light-metres. The intensity of the chemical illuminating power which a unit of the earth's surface receives either directly from the sun, from the diffuse light of a cloudless sky, or from clouds, can in this way be represented by the height of a column of gas, which, as we shall see in the sequel, when the atmosphere is clear, rapidly increases as the sun rises above the horizon until it reaches the meridian, as rapidly diminishing when the sun has passed that point; and rising and falling under the influence of passing clouds, forms, as it were, a wave accompanying the clouds in their motions, now ascending to a great elevation when bright white clouds reflect much light, and now sinking to a lower and permanent level when the sky is covered with grey clouds or obscured by mist. The mean daily, monthly, or annual height of this column of gas, dependent upon the latitude and longitude, regulates the chemical climate of a place, and points the way to relations for the chemical action of the solar rays, which in the thermic actions are already represented by Isothermals, Isotherals, Isocheimals, and Isonomals.

II. CHEMICAL ACTION OF DIFFUSE DAYLIGHT.

It would be extremely difficult to measure directly, by means of our chemical photometer, the amount of light which a point of the earth's surface receives from the whole atmosphere by dispersion. Experiments of this kind can of course only be made in the open air, either at an elevated situation with an extended horizon, or in the middle of a large plain removed from all objects which could obstruct or reflect light. When, in addition to this, we remember that the instrument with which the observations are made is so sensitive that a violent explosion occurs when a feeble ray of sunlight falls upon the apparatus, it will not appear surprising that we endeavoured to obtain the wished-for result in another manner. For this purpose we determined *chemically* in absolute measure, the action of the rays falling from a measured portion of cloudless sky situated near the zenith, and then compared the *visual* luminosity of this same portion of zenith-sky with that of the total heavens.

In order to be able to make this comparison, we had to carry out a somewhat lengthy series of experiments, which, as they form the basis of our present measurements, we must consider in detail.

As the light from certain zones of sky varying with the sun's altitude is polarized, and may therefore disappear on reflexion, it is impossible, in these determinations of the visual luminosity of the atmosphere, to employ any arrangement of mirrors. It appeared at first sight, that instead of reflecting the light it would be most convenient to illuminate one side of a sheet of white paper, first with the light falling from a measured portion of the zenith, and afterwards with the light from the whole heavens, determining in each case, at the other side of the paper, by means of a photometric arrangement, the amount of visual illumination. As, however, this method of observation can only give satisfactory results when the illumination of one side of the paper is proportional to the intensity of the rays which fall on the other side with very various angles of incidence, it was necessary, first of all, to determine whether these conditions were sufficiently fulfilled under the circumstances of the experiment.

The following arrangement was made in order to determine this point. A, fig. 4, Plate XLIII. represents a blackened tube closed at one end (*a*) by the paper about to be examined. On a table in front of the tube (A), a large divided quadrant (B) is so placed that the centre of the circle rests on the table directly below the centre of the disk of paper at *a*. In connexion with the quadrant is a moveable scale (C) divided into millimetres, and upon this scale the standard burner is placed, so that the rays proceeding from the flame can be made to fall upon the paper (*a*) at any given angle of incidence. Inside the tube at *e* is fixed a diaphragm of drawing-paper, the centre of which had been rendered transparent by a small piece of stearic acid. When this diaphragm is illuminated from D by a small constant flame, the stearine spot, as seen through the eye-tube (*b*), appears in a certain position of the lamp (*g*), to be white on a black ground; if the lamp be now brought nearer, at a certain point the spot disappears; and when the flame is brought still nearer, the spot is again seen, but now appears black upon a white ground. In order to obtain the wished-for proof, the flame (g) is first so placed that the rays fall perpendicularly on the paper screen (a), and then moved over the graduated circle (B), so that the rays fall upon the screen with the angle of incidence φ, care being in each case taken that the lamp is moved along the scale (C) until the spot of stearine disappears, so that in both positions of the flame the illumination of the side of the screen (e) next the eyepiece should be the same. When the distances have been found at which the flame (g) must be placed in the two positions, in order that the *back* of the paper screen should be illuminated to the same extent, it is easy to calculate the amount of light which falls on the *front* of the screen, and which, in order to answer the required purpose, ought to be equal to that seen through the tube (b). If φ and φ_1 represent the complements of the two angles of incidence, measured on the divided quadrant, with which the light fell upon the paper screen, and $r\,r_1$ the distances between the screen and flame at which the stearine spot vanished, then the following equation is true, provided the assumptions made at the commencement are correct:—

$$\frac{\sin\varphi . r_1^2}{\sin\varphi_1 . r^2}=1.$$

Experiment gave the following numbers:—

TABLE IV.

	Writing-paper.			Drawing-paper.	Filter-paper
φ	90°	90°	90°	90°	90°
r	250	262	203	146	300
φ_1	56° 15′	33° 45′	22° 30′	33° 45′	33° 45′
r_1	205	167	81	103	186
$\frac{\sin\varphi . r_1^2}{\sin\varphi_1 . r^2}$	0·83	0·73	0·42	0·89	0·69

It is seen that the numbers in the lowest horizontal column differ widely from 1, and that no simple relation exists between the amount of light falling on the front and that issuing from the back of the screen. It now only remained to be seen whether a better result is obtained when the diaphragm with the stearine spot is itself directly illuminated. For this purpose the experiments were repeated, with the sole difference that the diaphragm e was placed at the end of the tube at a instead of the paper screen. With this arrangement the following numbers were obtained:—

TABLE V.

φ	90°	90°	90°	90°	90°
r	225	199	178	220	224
φ_1	22° 30′	33° 45′	45° 0′	67° 30′	78° 45′
r_1	140	150	150	214	221
$\frac{\sin\varphi . r_1^2}{\sin\varphi_1 . r^2}$	1·01	1·02	1·00	1·02	0·993

The numbers in the lowest division are all so nearly equal to 1, that the slight differences [illegible] certainty be ascribed to the unavoidable errors of observation.

From these experiments we see that a paper diaphragm, with a transparent spot of stearic acid, can be most advantageously employed for the photometric measurement of rays of light falling upon a point at very various angles of incidence.

In order to determine the intensity of the sky's brightness with ease and exactitude, we have employed the photometer depicted in fig. 5, Plate XLIII. *a*, fig. 5, represents the paper diaphragm with the spot of stearine, which closes the vertical tube (*b*), and is in the same plane as the horizontal plate (*c*) fastened on to the tube (*b*). This tube (*b*) is coloured in the inside white, and into it is soldered the narrow tube (*d*), blackened inside, and placed at such an angle below the diaphragm (*a*), that on looking through it the stearine spot on the paper is plainly seen. The observations are more conveniently made when a small blackened box (*e*), containing a mirror and carrying an eyepiece (*f*), is placed on the end of the tube (*b*), and thus the change of shade in the stearine diaphragm noted. The tube *b* is fixed into the top of the tin case *hi*, one side of which is open, and the lower end of the tube *b* is closed by a divided circle, half of which, as is seen in fig. 6, *a a*, Plate XLIII., has been removed. This semicircular opening can be partially or totally closed at pleasure by a moveable disk of metal, and the angle thus left open read off on a divided circle. Supposing that a constant source of light be placed in the case *hi*, the lower surface of the stearine diaphragm receives a quantity of light which is determined by the divided scale of the semicircular opening at the bottom of the tube. If we suppose that the semicircular arc is opened to its full extent (180°), and that the amount of light which is thrown on the lower surface of the diaphragm is larger than that which falls on the upper surface, the spot of stearine will appear dark on a bright ground. If the opening be now gradually closed, the spot will be observed to become lighter, until at a certain position of the divided circle the spot entirely disappears, and a homogeneous white surface is seen. If the size of the open sector be still diminished, the spot again appears, but this time it is bright on a dark ground. When the divided arc has been placed in such a position that for a given illumination (A) the spot of stearine disappears, any other source of light placed in *a*, which under like conditions does not cause the spot to appear, is equal in intensity to A.

For the purpose of measuring the diffuse light of day, the diaphragms are best made of white drawing-paper, of about double or treble the thickness of ordinary writing-paper. When the whole of the paper, with the exception of a small ring, is saturated with stearic acid, the observations are more exact than when a paper with merely a spot of stearic acid is made use of. In order to prepare such a diaphragm, the drawing-paper, with a piece of filter-paper under it, is laid upon a warm metallic plate, and a piece of fused stearic acid (a stearine candle answers the purpose when the pure acid is not at hand) gradually rubbed into the heated paper, so that a small circle in the centre of the paper is the only part not saturated with the stearine. When the plate is cool, a small piece of the acid is laid in the centre of the circle, and the plate gently warmed until the stearic acid has sunk into the pores of the paper, so that only a small ring not saturated with acid remains, the breadth of which should be about 1 to 2 millimetres, and the diameter from 5 to 6 millims.

We have already mentioned that we were only able to measure the chemical action produced by a small circle of sky at the zenith. In order to be able to determine the action effected by the light from the whole visible heavens, it was in the first place necessary to compare, by means of the photometer just described, the amount of light which a point on the earth's surface receives from the whole sky, with that which the same point receives from a measured circular portion of sky at the zenith. As the quantity of light which emanates from the measured portion of sky in the zenith is from 300 to 800 times less than that reflected from the whole sky, and as with this photometer we are unable to measure such large differences in the amount of light, it was necessary to employ some known fraction of the total diffuse light of day. For this purpose a hollow hemispherical metallic cap (fig. 7, Plate XLIII.) was made use of. This cap, which is painted black, can be placed concentric with the diaphragm (*a*, fig. 4), on the plate *cc*, and is perforated at regular distances with 184 equal-sized holes, the diameter of which was accurately determined. In order to measure the light from a circle of sky at the zenith of known magnitude, the blackened tube (fig. 6, Plate XLIII.) was employed; this tube is furnished at its upper end with a semicircular moveable disk, similar to the one already described as placed at the lower end of the photometer-tube (*b*, fig. 5), and can be fixed on the plate *cc*, so that the ring of the paper diaphragm (*a*) falls exactly below the centre of the divided semicircle.

The observations are commenced by placing the blackened cap on the plate over the diaphragm; the diffuse light falls through the holes upon the diaphragm, and the divided disk in the tube (*b*) is moved until the ring on the paper, seen through the eye-piece *b*, disappears. The amount of light which under these circumstances is reflected from the sky upon the diaphragm we will call 1. Let the total surface of the hemispherical cap be Q_1, and that portion which has been removed by the holes be Q; then the amount of light which would have fallen on the diaphragm from the whole sky if the cap had not been present is equal to $\frac{Q_1}{Q}$. If the cap be removed, and in its stead the tube (fig. 6) placed on the plate, without otherwise altering the arrangement, and then the divided circle on the *upper* end of the tube moved until the ring disappears, the diaphragm (*a*) is again illuminated with the intensity 1. Let q represent the fraction of the total hemisphere from which the light falls through the open sector of the tube upon the diaphragm, and let q_1 represent the area of a given circle of sky at the zenith expressed in the same units as q; the amount of light which would fall upon the diaphragm from the whole circle of sky q_1 is $\frac{q_1}{q}$; and hence the relation between the light from the whole heavens and that from the circle of sky q_1 is $\frac{Q_1}{Q} \cdot \frac{q}{q_1}$. If, therefore, we know the amount of chemical action (w) which the light reflected from a circle of sky q_1 at the [illegible] on a given point, the action (W) which the light from the whole sky would produce on the same point is obtained by the equation

$$W=\frac{Q_1}{Q}.\frac{q}{q_1}.w.$$

The relation $\frac{Q_1}{Q}$ is easily found by help of the following considerations. If r be the radius of the hemispherical cap, we have $Q_1=2\pi r^2$; and if d is the diameter of the hole, its area is $4\pi r^2\sin^2\frac{\theta}{2}$, when $\sin\theta=\frac{d}{2r}$; and if there are n holes,

$$Q=n.4\pi r^2\sin^2\frac{\theta}{2}, \quad \text{or} \quad \frac{Q_1}{Q}=\frac{1}{2n.\sin^2\frac{\theta}{2}}.$$

In the apparatus above described the values of these constants were as follows:—$n=184$; $d=0{\cdot}775$ millim.; $r=77{\cdot}5$ millims.; hence we obtain $\frac{Q_1}{Q}=436$. The value of q is determined in the following manner:—Let d_1 be the diameter of the circle through the open sector of which the light falls on the diaphragm; let r_1 be the distance from any point in the circumference of this circle to the centre of the paper diaphragm; and lastly, let ψ be the number of degrees to which the sector was opened; we have then

$$q=\frac{\psi}{360}.4\pi.\sin^2\frac{\theta_1}{2},$$

when $\frac{d_1}{2r_1}=\sin\theta_1$, and the area of the whole visible sky $=2\pi$. If we suppose that q_1 has an arbitrary value of the $\frac{1}{1000}$th of the hemisphere, that is, if $q_1=\frac{2\pi}{1000}$, we have $\frac{q}{q_1}=\frac{50}{9}\psi.\sin^2\frac{\theta_1}{2}$. The values of those constants in our instrument are, $d_1=45{\cdot}5$ millims.; $r_1=190{\cdot}6$ millims.; hence $\frac{q}{q_1}=0{\cdot}0197\,\psi$. The amount of light (I) which a point on the earth's surface receives from the whole sky, expressed in units of the amount which is reflected from a portion of sky at the zenith equal in area to $\frac{1}{1000}$th part of the whole, is found from the equation

$$I=\frac{Q_1}{Q}.\frac{q}{q_1}\psi=8{\cdot}58\,\psi.$$

In order to obtain the exact value of I certain precautions must be taken, which if neglected would give rise to very considerable errors. It is of primary importance to notice, when two observations have to be compared together, that the amount of light which the white surface of the inside of the photometer receives, and which is reflected on to the lower side of the diaphragm, remains perfectly constant. If the inside of the photometer is illuminated by the reflected light from houses, hills, or other neighbouring objects, a passing shadow may render the observations useless. Even when a portion of the light which should fall into the photometer is by chance obstructed by the hand whilst altering the opening of the divided circle, an error of considerable magnitude is introduced. In order to avoid irregularities of this kind, it was found most convenient to expose the white sides of the photometer to the diffuse light of the cloudless sky, avoiding altogether other reflected or direct sunlight. It is self-evident that no direct sunshine must be allowed to fall during the experiment, either upon the opening of the tube (fig. 6), or upon the hollow metallic cap (fig. 7). This is best prevented by holding a round sheet of pasteboard fixed on a pole between the sun and the photometer, at a distance of about 3 metres from the latter, so that the amount of diffuse light cut off is inappreciable.

As the relative amount of illumination of the sky is materially altered by the presence of the smallest cloud, it is only when the heavens are perfectly clear that any general conclusion can be drawn from these observations. Perfectly cloudless days are, however, so uncommon in our latitudes, that on an average not more than six or eight occur at Heidelberg in a year*. We must hence consider it fortunate that we were able to complete a series of hourly observations from sunrise to sunset upon a day during the whole of which the blue sky was not obstructed by the smallest particle of cloud.

A series of these observations was made on June 6th, 1858, at the summit of the Geisberg, a hill in the neighbourhood of Heidelberg, the height of which is 376 metres above the sea, and 105 metres above the river Neckar. On the highest point of the wooded hill, a platform, of about 40 feet in height, had been built, rising above the tops of the trees; and on this platform, which commanded a perfectly free horizon on every side, the visual illumination of the heavens was determined. During the course of the observations, which lasted from sunrise to near sunset, a strong breeze blew from the eastward, and the air was so clear, that the Haardt Mountains, situated at a distance of twenty miles, were seen not only in distinct contour, but in relief.

The amount of the illumination of the sky was determined in the following manner: the hollow cap was first fixed on the photometer, shaded from direct sunlight by the paper screen, and the divided circle so placed that the diaphragm-ring disappeared; the time was then noted and the hollow cap quickly removed. The tube (fig. 6, Plate XLIII.) was next fixed on the photometer, and the divided circle attached to the tube moved until the diaphragm-ring disappeared, when the time was again observed; the mean of these two observations was taken as the time of the experiment. The opening (ψ) of the divided circle, measured in degrees and tenths of degrees, when multiplied by 8·58, gives, as we have already shown, the amount of light (I) which the diaphragm-ring receives from the *whole sky*, when the unit amount of light is represented by the amount which falls upon the same diaphragm-ring during the same time from a portion of zenith-sky equal to $\frac{1}{1000}$th part of the total visible heavens.

The following Table gives the mean of four independent series of observations made on the 6th of June, 1858. The first vertical column contains the numbers of the observations; the second the times of the experiment reduced to true solar time; the third the values of ψ; and lastly, the fourth contains the required values of I calculated from the values of ψ.

* According to observations of Padre Secchi, extending over a period of eight years, not more than [illegible] cloudless days occur in Rome in the course of a year.

TABLE VI.

I.	II.	III. ψ.	IV. I.
	h m		
1	5 38 A.M.	88·8	762
2	6 8 A.M.	86·0	738
3	7 25 A.M.	74·2	637
4	8 52 A.M.	56·0	481
5	9 32 A.M.	50·4	432
6	10 9 A.M.	47·2	405
7	10 43 A.M.	42·0	360
8	11 26 A.M.	37·8	324
9	0 1 P.M.	35·8	307
10	0 32 P.M.	38·4	330
11	0 57 P.M.	42·0	360
12	1 24 P.M.	47·4	407
13	2 4 P.M.	50·0	429
14	2 38 P.M.	53·0	455
15	3 21 P.M.	60·8	522
16	3 57 P.M.	68·8	590
17	4 42 P.M.	76·0	652
18	5 20 P.M.	79·6	683
19	6 2 P.M.	88·0	755

The amounts of light which fall upon the diaphragm-ring and are arranged in column IV. of the foregoing Table, in no way represent the mean brightness of the whole sky *as compared with the respective brightnesses of the zenith*, for it is only the zenith rays which fall perpendicularly upon the unit of surface; the rays from other points of the sky with a greater zenith-distance fall upon the diaphragm with a larger angle of incidence. This relation between the luminosity of the zenith and the mean luminosity of the whole sky, could be easily found if, instead of the perforated cap used, another were employed in which the number or area of the holes was in the inverse proportion of the cosines of their distance from the vertical.

The amount of chemical illumination which a horizontal unit on the earth's surface receives from the whole heavens, depends upon the height of the sun above the horizon, and upon the constitution of the atmosphere. If, when the sky is cloudless, the atmosphere undergoes considerable change in its transparency, a long series of determinations made at different times of the year would be necessary before we could hope to arrive at any general expression for the atmospheric extinction and dispersion of the chemical rays. Fortunately, however, we learn from Professor SEIDEL's classical research on the Brightness of the Fixed Stars*, that the atmospheric extinction varies so slightly when the sky is cloudless that the differences may be altogether neglected. In a lecture given at Munich†, SEIDEL says, "It has fortunately been proved, although one could scarcely have expected it, that the variations in the amount of transparency of the air on different nights may, by care in the selection of nights, be reduced to within very narrow limits."

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* Abhandl. der Kön. Bayer. Akad. d. Wiss. 2te Classe, Bd. vi. Abth. 3.

† Wissenschaftliche Vorträge, gehalten zu München im Winter, 1858, p. 301. Braunschweig bei Vieweg und Sohn.

Hence we think that we are justified in setting aside the alterations in the extinction of the cloudless sky, and in considering the chemical illumination of the earth's surface as being merely a function of the sun's zenith-distance. We are at the same time perfectly well aware that the constants contained in our formulæ, derived as they are from a comparatively small number of observations, cannot lay claim to any very great degree of accuracy. Still we believe, and we hope in the sequel to show, that the amount of reliability which our experiments present is sufficient to permit us to deduce therefrom the empirical laws according to which the chemical energy proceeding from the sun is diffused over the earth's surface when the sky is unclouded.

The following formula gives the value of the zenith distance (ϕ) for the observed times of experiment:

$$\cos\phi = \cos\delta \,.\, \cos t \cos p + \sin\delta \,.\, \sin p \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5.)$$

where δ signifies the sun's declination on the day of the experiment (+22° 30′ 30″); p gives the latitude of the place (49° 24′ 25″), and t represents the sun's hour-angle.

In the following Table the observations of June 6, 1858, are arranged with the corresponding zenith-distance of the sun. The first column contains the times of observation in true solar time; the second gives the sun's hour-angle corresponding to the time of observation; the third contains the corresponding zenith-distances of the sun; and the fourth shows the amount of light which the unit of area receives from the whole sky, compared with that which the same area receives at the same time from a portion of sky at the zenith equal in area to the $\frac{1}{1000}$th part of the whole heavens.

TABLE VII.

Number of experiment	I Time of observation	II t hour-angle of the sun	III ϕ zenith distance of the sun	IV I.
	h m	° ′	° ′	
1	5 38 A.M.	95 30	76 25	762
2	6 8 A.M.	88 0	71 44	738
3	7 25 A.M.	68 45	59 19	637
4	8 52 A.M.	47 0	45 24	481
5	9 32 A.M.	37 0	39 28	432
6	10 9 A.M.	27 45	34 31	405
7	10 43 A.M.	19 15	30 45	360
8	11 26 A.M.	8 30	27 34	324
9	0 1 P.M.	0 15	26 47	307
10	0 32 P.M.	8 0	27 29	330
11	0 57 P.M.	14 15	29 1	360
12	1 24 P.M.	21 0	31 27	407
13	2 4 P.M.	31 0	36 11	429
14	2 38 P.M.	39 30	40 54	455
15	3 21 P.M.	50 15	47 26	522
16	3 57 P.M.	59 15	53 10	590
17	4 42 P.M.	70 30	60 28	652
18	5 20 P.M.	80 0	66 38	683
19	6 2 P.M.	90 30	73 16	755

The several values of I in the 4th column can be expressed, as exactly as the observational errors allow, as functions of the zenith-distance ϕ by help of the simple equation

$$I=77{\cdot}0+9{\cdot}275\,\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6.)$$

By means of this equation we have calculated and thrown together in the following Table, the luminosity of the whole sky for zenith distances varying from 20° to 90°.

TABLE VIII.

ϕ.	l.	ϕ.	l.	ϕ.	l	ϕ.	l.	ϕ.	l.	ϕ.	l.	ϕ.	l.	ϕ.	l.	ϕ.	l.
20°	263	28°	337	36°	411	44°	485	52°	559	60°	634	68°	708	76°	782	84°	856
21	272	29	346	37	420	45	494	53	569	61	643	69	717	77	791	85	865
22	281	30	355	38	430	46	504	54	578	62	652	70	726	78	801	86	875
23	290	31	365	39	439	47	513	55	587	63	661	71	736	79	810	87	884
24	300	32	374	40	448	48	522	56	597	64	671	72	745	80	819	88	893
25	309	33	383	41	457	49	532	57	606	65	680	73	754	81	828	89	902
26	318	34	392	42	467	50	541	58	615	66	689	74	764	82	838	90	912
27	327	35	402	43	476	51	550	59	624	67	699	75	773	83	847		

The curve (fig. 8, Plate XLIV.) represents the relation between the values of ϕ as abscissæ and those of l as ordinates. The points about the curve marked with a cross represent the observations made in the morning; those marked by a dot show the brightness of the afternoon sky. From these it is seen that the illumination effected by the diffuse daylight for the same zenith-distance of the sun before and after noon remains nearly constant, although at these times of day the temperature and humidity of the air must differ widely.

From Table VIII. we can find, for any given zenith distance of the sun, the relation between the amount of light, optically (or visually) measured and taken as unity, which falls from a certain spherical surface of sky in the zenith and that (also optically measured) falling from the whole sky. The amount of light, chemically measured, which falls from this same surface of zenith-sky, multiplied by the amount of optically measured light which the whole sky gives, must therefore give the chemical action which the whole sky would produce on a horizontal unit of surface. According to our former definition, however, the amount of light falling from a circle of zenith-sky upon the unit of area is the chemical brightness of that piece of zenith-sky. We have carried out, according to the method already described, a series of determinations of this chemical brightness at various times and at different zenith-distances of the sun. Some of these observations were made on the 18th of October, 1856, others on July 23rd and August 5th, 1858. During all the experiments the sky was perfectly cloudless, and the gas had reached the maximum degree of sensibility. The observations are contained in the following Table (IX.).

TABLE IX.

Number of experiment	True solar time		Sun's zenith-distance	Action in scale-divisions	Chemical brightness in units of light of a piece of zenith-sky $\frac{1}{1000}$th of the whole sky
		h m	° ′		
1	1856, 18 Oct.	6 46 A.M	90 4	0·000	0·0
2		6 51 A.M.	89 17	0·63	23·2
3		6 55 A.M.	88 39	0·88	32·5
4		7 3 A.M.	87 25	1·92	71·1
5		7 7 A.M.	86 48	2·19	81·0
6		7 17 A.M.	85 16	3·15	116·3
7		7 23 A.M.	84 22	3·42	126·5
8		7 41 A.M.	81 41	4·81	177·6
9		8 5 A.M.	78 14	6·01	222·2
10		8 11 A.M.	77 25	6·40	236·8
11		8 33 A.M.	74 27	7·82	289·2
12		9 8 A.M	70 7	8·56	316·6
13		3 53 P.M.	77 58	5·44	200·8
14	1858, 23 July.	7 30 A.M.	60 21	6·94	485·8
15		7 52 A.M.	56 47	7·93	555·1
16		8 52 A.M.	47 21	10·06	704·2
17		9 46 A.M.	39 36	11·82	827·4
18		10 15 A.M.	36 0	13·04	912·8
19		10 48 A.M.	32 37	13·72	960·4
20		11 20 A.M.	30 22	15·26	1068·2
21		12 0 A.M.	29 18	16·04	1122·8
22	1858, 5 Aug.	7 46 A.M.	60 1	7·59	531·3
23		8 11 A.M.	56 1	8·65	605·4
24		8 51 A.M.	49 52	9·79	685·3
25		9 17 A.M.	46 5	10·32	722·4

In experiments 1 to 13 the following data were employed for the calculation of the brightness, expressed in units of light, found in the 5th column: R=1·777, L=1·45, N=1·031, $n=2\cdot249$, $d=0^{m}\cdot0530$, $r=1^{m}\cdot474$. In the other experiments these values remained constant, except $d=0^{m}\cdot059$, and $r=2^{m}\cdot260$.

The chemical brightness (w) of the circle of zenith-sky equal in area to $\frac{1}{1000}$th part of the whole heavens, as seen in the last column of the preceding Table, can be represented as a function of the corresponding zenith-distance of the sun (φ), with sufficient accuracy by help of the following equation,

$$w=1182\cdot7-13\cdot857+\frac{8884\cdot9}{\varphi}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7.)$$

in which the numbers are calculated from all the observations according to the method of least squares. From this equation Table X. is calculated. It gives for every zenith-distance of the sun from 20° to 90° the amount of light, expressed in light-units, which falls perpendicularly upon a unit of area on the earth's surface, from a portion of zenith-sky equal to $\frac{1}{1000}$th part of the whole heavens.

TABLE X.

Zenith-distance. ϕ.	Chemical brightness in light-units. w.	Zenith-distance. ϕ.	Chemical brightness in light-units. w.	Zenith-distance. ϕ.	Chemical brightness in light-units. w.	Zenith-distance. ϕ.	Chemical brightness in light-units. w.	Zenith-distance. ϕ.	Chemical brightness in light-units. w.	Zenith-distance. ϕ.	Chemical brightness in light-units. w.
20°	1350·0	32°	1017·2	44°	775·1	56°	565·7	68°	371·5	80°	184·8
21	1315·0	33	994·9	45	756·8	57	549·0	69	355·8	81	170·4
22	1281·9	34	973·0	46	738·7	58	532·5	70	340·1	82	155·0
23	1250·6	35	951·7	47	720·8	59	515·9	71	324·3	83	139·7
24	1220·5	36	930·9	48	703·0	60	499·6	72	308·9	84	125·5
25	1192·0	37	910·4	49	685·3	61	483·5	73	293·4	85	110·3
26	1164·4	38	890·1	50	667·8	62	467·2	74	277·7	86	95·0
27	1137·7	39	870·2	51	650·4	63	451·1	75	262·2	87	79·8
28	1112·2	40	850·6	52	633·4	64	435·1	76	246·6	88	64·6
29	1087·4	41	831·5	53	616·3	65	419·2	77	231·1	89	49·5
30	1063·4	42	812·5	54	599·2	66	403·3	78	216·7	90	34·4
31	1039·9	43	793·6	55	582·3	67	387·2	79	201·2		

The curve (fig. 9, Plate XLV.) shows how closely the actual observations approach the calculated values. The simple points about the curve represent the observations made on July 28, 1858; the small circles denote the experiments of August 5, 1858; and the crosses those of October 18, 1856. As these observations were made in different years and at different times of the day and year, we may conclude from the near approach of the single points to the calculated curve, that in this case, as in that of the determination of visible illumination, alterations in the humidity and temperature of the air have but a very slight influence upon the dispersion of light in the atmosphere when free from clouds or mist; and that, therefore, the chemical actions effected at equal hour-angles on each side of the sun's culminating point, may, without risk of appreciable error, be taken as equal.

After this investigation we may now pass to the determination of the chemical action which the whole visible heaven effects upon a horizontal unit area of earth's surface at various zenith-distances of the sun. The numbers marked l in Table VIII. give the relation of the light falling on a unit of area from the whole sky, visually measured, to that falling from a given portion of zenith-sky, also measured visually. In Table X. we find the chemical actions produced by the same portion of zenith-sky. Now, as we have formerly shown that in light from the same source the chemical is proportional to the visual action, we have only to multiply together the numbers in each Table corresponding to the same zenith-distance, in order to have the required chemical action of the whole heavens. The action (w_1) thus obtained can be expressed in a form adapted for calculation, as a function of the sun's zenith-distance, according to the following formula,

$$w_1 = a + b \cos\varphi + c \cos^2\varphi + \ldots, \quad \ldots \ldots \ldots \quad (8.)$$

in which the fourth term can be neglected. The values of the coefficients a, b, and c have been calculated, according to the method of the least squares, from the following eight values of φ and w_1:—

TABLE XI.

φ.	w_1.
90	31370
80	151400
70	246800
60	316700
50	361300
40	381100
35	382500
30	377500

Hence we obtain

$$w_1=27760+808490\cos\varphi-459960\cos^2\varphi, \quad \ldots \quad \ldots \quad (9.)$$

by help of which the following Table (XII.) is calculated. The chemical action (w_1) effected by the diffuse light from the whole sky is here expressed, not in units of light, but in degrees of light, each of which is equal to 10,000 units of light.

TABLE XII.

φ.	w_1.	φ	w_1.	φ.	w_1.	φ.	w_1	φ.	w_1.	φ	w_1.
31	38·29	41	37·60	51	35·45	61	31·17	71	24·22	81	14·30
32	38·26	42	37·47	52	35·11	62	30·60	72	23·37	82	13·15
33	38·24	43	37·31	53	34·78	63	30·00	73	22·48	83	11·95
34	38·21	44	37·14	54	34·40	64	29·38	74	21·56	84	10·72
35	38·14	45	36·96	55	34·02	65	28·73	75	20·62	85	9·47
36	38·08	46	36·74	56	33·61	66	28·06	76	19·64	86	8·19
37	38·01	47	36·53	57	33·17	67	27·34	77	18·64	87	6·88
38	37·93	48	36·30	58	32·72	68	26·61	78	17·60	88	5·54
39	37·85	49	36·02	59	32·22	69	25·84	79	16·53	89	4·17
40	37·72	50	35·75	60	31·70	70	25·05	80	15·43	90	2·77

These degrees of light (w_1) express the chemical action which the diffuse light from the whole sky effects during one minute upon the horizontal unit of area on the earth's surface when the corresponding zenith-distance of the sun is φ. The curve (fig. 10, Plate LV.) represents the relation of this chemical action to the sun's zenith-distance, marked as abscissæ. The points on the figure denote the values of Table XI., from which the curve is derived; their approximation to the line shows how closely the results obtained by calculation agree with the observed numbers.

By means of formula (5.), $\cos\varphi=\cos\delta\cos p\cos t+\sin\delta\sin p$, already given, and either formula (9.) or Table XII., the illuminating effect of the cloudless sky at a given locality and at a given time, can be calculated. As an example of this method, we give the results of the calculation of the chemical illuminating effect produced by the cloudless sky during the various hours of the day on the vernal equinox at Cairo, Naples, Heidelberg, Manchester, Petersburg, Reykiavik, and Melville Island.

TABLE XIII.

h m	Melville Island.	Reykiavik.	Petersburg.	Manchester.	Heidelberg.	Naples.	Cairo.
6 A.M. or 6 P.M.	2·77	2·77	2·77	2·77	2·77	2·77	2·77
7 A.M. or 5 P.M.	8·06	11·32	12·49	14·19	15·09	16·84	18·59
8 A.M. or 4 P.M.	12·61	18·22	20·13	22·81	24·21	26·77	29·15
9 A.M. or 3 P.M.	16·20	23·34	25·64	28·72	30·24	32·87	35·03
10 A.M. or 2 P.M.	18·78	26·76	29·20	32·30	33·74	35·80	37·58
11 A.M. or 1 P.M.	20·32	28·67	31·14	34·10	35·43	37·20	38·23
12 A.M.	20·83	29·30	31·74	34·67	35·91	37·49	38·30

The curves (fig. 11, Plate XLVI.) drawn from these numbers show graphically the hourly variation in the chemical diffused light on the equinox at the several localities.

We may now proceed to the consideration of the chemical actions produced by diffuse light upon the horizontal unit area of earth's surface during a longer period of time.

Let w_1 signify, as before, the action effected by the light in one minute, when the sun's hour-angle is t; and let W be the action during a period in which the hour-angle increases from $t_{\prime}$ to $t_{\prime\prime}$, then we have

$$W=\frac{12\times 60}{\pi}\int_{t_{\prime}}^{t_{\prime\prime}} w_1 dt,$$

where the unit in which the hour-angle is expressed is the angle whose arc is equal to the radius. We have, however,

$$w_1=a+b\cos\varphi+c\cos^2\varphi,$$

and

$$\cos\varphi=\sin\delta\sin p+\cos\delta\cos p\cos t;$$

hence when $\sin\delta\sin p=\alpha$, and $\cos\delta\cos p=\beta$,

$$W=\frac{12\times 60}{\pi}\left[\left\{\left(a+b\alpha+c\left(\alpha^2+\frac{\beta^2}{2}\right)\right)(t_{\prime\prime}-t_{\prime})+(b\beta+2c\alpha\beta)(\sin t_{\prime\prime}-\sin t_{\prime})\right.\right.$$
$$\left.\left.+\tfrac{1}{4}c\beta^2(\sin 2t_{\prime\prime}-\sin 2t_{\prime})\right\}\right] \quad . \quad . \quad . \quad (10.)$$

This expression is somewhat simplified, if the action for the whole day, from sunrise to sunset, be calculated; for then $t_{\prime}=-t_{\prime\prime}$, and

$$\cos t_{\prime\prime}=-\tan\delta\tan p, \text{ or }=-\frac{\alpha}{\beta};$$

and hence

$$W=\frac{24\times 60}{\pi}\left\{\left(a+b\alpha+c\left(\alpha^2+\frac{\beta^2}{2}\right)\right)t_{\prime\prime}+\left(b\beta+\frac{3}{2}c\alpha\beta\right)\sin t_{\prime\prime}\right\}.$$

If the calculation be made for the time of the equinoxes, the equation becomes much more simple; for in this case

$$\delta=0, \quad \alpha=0, \quad \beta=\cos p, \quad t_{\prime\prime}=\frac{\pi}{2},$$

or
$$W=12\times 60\left(a+\frac{2}{\pi}b\cos p+\tfrac{1}{2}c\cos^2 p\right);$$

or substituting the values of a, b, and c,

$$W=1998{\cdot}7+37058\cos p-16559\cos^2 p.$$

By applying this formula for the places given in Table XIII., we obtain the following numbers as representing the chemical action, expressed in degrees of light, which, at the time of the equinoxes, the whole heaven effects on the unit of area in each of these places, from sunrise to sunset:—

TABLE XIII*b*.

Melville Island	10590
Reykiavik	15020
St. Petersburg	16410
Manchester	18220
Heidelberg	19100
Naples	20550
Cairo	21670

We need scarcely remark that these numbers, as well as the data from which they are obtained, only hold good for skies free from cloud and mist, and for situations not too far distant from the sea's level. In places which lie nearer to the higher portions of the atmosphere, it is evident that the conditions will be totally different. The law according to which the atmospheric diffused light varies with the height above the sea, might in like manner be determined empirically, but such a series of experiments entail an expenditure of time and money which we have unfortunately been unable to afford.

The chemical action of diffused sunlight becomes extremely irregular when either clouds or mists intercept the clear blue sky. The chemically active light, which under these circumstances reaches the earth's surface, varies as irregularly as do the other meteorological phenomena dependent on the thermic and hygrometric state of the atmosphere. Without at present entering into an explanation of a method by which, taking cognizance of these circumstances, photo-chemical determinations may be made, we will here give one example to show the variable nature of the diffuse light from a clouded sky. The following Table contains the results of a series of observations made at Heidelberg on October 5, 1856. The columns marked I. give the time of observation; those marked II. the amount of light expressed in units of light radiated from a portion of sky at the zenith equal to the $\frac{1}{1000}$th part of the whole sky.

TABLE XIV.

I.	II.	I.	II.	I.	II.	I.	II.	I.	II.
h m		h m		h m		h m		h m	
7 24 A.M.	617	9 12 A.M.	1537	10 47 A.M.	1881	11 38 A.M.	1650	2 28 P.M.	765
7 [illegible] A.M.	670	9 15 A.M.	1469	10 53 A.M.	1731	0 4 P.M.	1660	3 20 P.M.	370
8 [illegible]	968	9 30 A.M.	1686	11 0 A.M.	1881	1 1 P.M.	1494	3 27 P.M.	450
8 [illegible]	1165	9 50 A.M.	1797	11 6 A.M.	1921	1 7 P.M.	1475	3 33 P.M.	445
8 [illegible]	[illegible]	9 56 A.M.	1631	11 31 A.M.	2274	2 11 P.M.	675	4 16 P.M.	304
9 0 A.M.	[illegible]	10 2 A.M.	1934	11 37 A.M.	2231	2 22 P.M.	886	4 41 P.M.	188

In order to give a clear idea of the daily changes of chemical luminosity in such a piece of sky over which clouds passed, we have represented the observations by a curve (A, fig. 12, Plate XLVI.), in which the times are marked as abscissæ, and the chemical actions expressed in units of light as ordinates. For the sake of comparison we have calculated the action which would have taken place on that day if the sky had been perfectly cloudless, and we have represented this action by the second curve, B. From 7^h A.M. to nearly noon the sky was covered with a layer of cloud of varying thickness, through which the sun but slightly shone. In the afternoon the clouds gradually disappeared, so that after 2 o'clock only light detached clouds passed over the zenith, and after about half-past 3 the sky became cloudless. These alterations are clearly seen on the curve, *which shows that a thin film of cloud increased the quantity of light* reflected from the portion of sky under observation to beyond four times the amount reflected from the same portion of sky when cloudless; and that the chemical action before noon rose much more rapidly, and was more irregular than the action in the afternoon when the clouds were gradually disappearing. It is seen too that, at $2^h\ 22^m$ and at $3^h\ 30^m$, the action attained two maxima limits, owing to the passage at those times of a bright white cloud through the zenith.

These observations prove that the presence of a thin film of cloud increases the amount of chemical illuminating effect in the most striking manner; we have, however, also convinced ourselves, by a large number of experiments, that a thicker layer of clouds, such as those which precede rain or thunder-storms, or dense mists or fogs, may so diminish the chemical light reflected from the portion of heavens under examination, *that our instrument is unable to measure its amount.* The clouds act, therefore, not merely as collectors of atmospheric moisture, they also serve as mighty reflectors of light, regulating the supply of chemical rays, which are as necessary to the growth and well-being of plants as the warmth or moisture of the earth and air.

After these considerations on the diffuse daylight, we may pass on to the measurement of the chemical action effected by the direct solar rays.

III. CHEMICAL ACTION OF THE DIRECT SUNLIGHT.

In order to obtain data for the determination of the theory of this action, the volume of hydrochloric acid was measured, which was produced each minute in our instrument upon a cloudless day and at various zenith-distances of the sun, when the direct sunlight, diminished to a known extent, was allowed to fall perpendicularly over the whole surface of our insolation-vessel. For this purpose we directed the solar rays, by means of a SILBERMANN's heliostat, through a fine opening in a thin plate, and allowed the sun's image thus formed so to fall upon the insolation-vessel that every portion of the sensitive mixture of chlorine and hydrogen was equally exposed. The position in which this takes place is easily found by holding a piece of paper behind the apparatus, and then moving the instrument until the shadow of the insolation-vessel appears exactly in the middle of the sun's image. The fine hole in the plate must be very carefully bored, as its area has to be accurately determined. We found that the best way of obtaining a perfectly circular opening was to lay a thin piece of copper foil upon a thick sheet of tin or lead, and to bore a small hole through the copper with a fine needle, afterwards rubbing the foil upon a moistened hone to separate all irregularities, and boring carefully with the needle, until, under the microscope, the edge of the hole was seen to be sharp and circular.

The diameter of the fine opening was determined by measurement with a micrometer, and the distance from the insolation-vessel to the point on the shutter of our dark room, where the opening was placed, was also determined. If we suppose that this distance be called b, and the diameter of the opening d, the apparent diameter (λ) of the opening, as seen from the insolation-vessel, is found from the formula

$$\tan \tfrac{1}{2}\lambda=\frac{d}{2b}.$$

The square of this apparent diameter (λ) is to the square of the apparent diameter of the sun (λ_1), as the observed action is to the action which the sun would have produced if it had shone direct upon the insolation-vessel. In order to obtain the total action proceeding from the sun, it is necessary to multiply the observed action by the factor

$$\Lambda=\frac{\lambda_1^2}{\lambda^2}.$$

The factor for the transformation of the direct observations into units of light is, as in the former calculations,

$$n=2\cdot249.$$

The factor N, by help of which the experimental results are made independent of reflexion in the insolation-vessel, is found from the formula (1.),

$$N=\frac{1-\gamma r}{1-r},$$

when $\frac{2\varrho}{1+\varrho}=r$ and $10^{-\alpha h}=\gamma$, and when ϱ signifies the coefficient of reflexion from glass into air, h the thickness of exposed sensitive gas in the insolation-vessel, and α the coefficient of extinction of direct sunlight for the chlorine and hydrogen mixture. As ϱ and h were known from our former experiments, we had only to determine the value of α; this was accomplished in the following way. Between the small opening in the shutter and the insolation-vessel, we placed one of our transparent cylinders* filled with air, and then measured the amount of action (w_1) effected by the direct sunlight falling on the instrument. The empty cylinder was next replaced by a similar one filled, the usual precautions being adopted, with the mixture of chlorine and hydrogen, and the action (w) again determined. After removal of this cylinder, the former one containing air was replaced [illegible] the action once more measured, and the mean of the three times of observation [illegible] the first and third observations, with the cylinder containing air, the action (w_1) was calculated which the sun's image would have effected at the mean time of observation if no chlorine and hydrogen mixture had been present. If we call h_1 the thickness of chlorine and hydrogen, reduced to 0° and $0^{m}\cdot76$, contained in the transparent cylinder, and if we neglect, as we can do without appreciable error, the reflexion from the interior of the transparent cylinder, the coefficient of extinction (α) is obtained from the equation*

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* Philosophical Transactions, 1856, p. 357.

$$\alpha=\frac{1}{h_1}\log\left(\frac{w_1}{w}\right).$$

The following Table gives the values of w and w_1 as found by experiment for the corresponding zenith-distances (ϕ) of the sun, and the values of α calculated therefrom. The thickness of the layer of chlorine and hydrogen collected at 22° C. and $0^{m}\cdot757$ was 32·3 millims., giving at 0° and $0^{m}\cdot76$ a thickness of 29·8 millims.

TABLE XV.

ϕ.	w.	w_1.	α.
61 28	3·41	14·03	0·0190
58 26	3·72	15·45	0·0191
50 20	4·35	18·40	0·0194

The mean of these numbers gives as the value of the coefficient of extinction for direct sunlight in the chlorine and hydrogen mixture $\alpha=0\cdot01923=\frac{1}{52}$ millim. The value of N is then found to be, according to formula (1.), by help of this number,

$$N=1\cdot036.$$

The experiments still require very considerable correction. The sunlight was reflected into our dark room by means of a SILBERMANN'S heliostat. This was so placed that its mirror of speculum metal lay in one horizontal plane with the opening in the window-shutter. In one series of experiments, the rays, after having passed through the opening, were again reflected by means of a steel mirror; in the other experiments the rays fell without a second reflexion, in the same horizontal plane upon the insolation-vessel. In the case of two reflexions, a portion of the light is lost by absorption and polarization in quantities varying with the angles of incidence, and the angles at which the planes of polarization cut one another. This variable loss of light must be brought into calculation, and it is obtained by help of the following considerations.

A ray of light is reflected from a steel mirror in the angle i (fig. 13, Plate XLIII.); suppose that this ray when falling on the mirror has the intensity 1, and suppose that the reflected ray has the intensity p^2 when the ray is polarized parallel to the plane of incidence, and the intensity s^2 when the ray is polarized at right angles to this plane. In POGGENDORFF'S 'Annalen'† are found two tables of JAMIN'S, in one of which are found the values of p, and the other those of s expressed as functions of i.

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* Philosophical Transactions, 1857, p. 608.

† Ergänzungs, Band ii. 1848, p. 445.

When $i=i_1$, then $p=p_1$, and $s=s_1$.

When $i=i_2$, then $p=p_2$, and $s=s_2$.

Suppose that a ray of light be reflected from both the mirrors 1 and 2 (fig. 14, Plate XLIII.), of which the first is of speculum metal and the second of steel, with the angles of reflexion i_1 and i_2; and suppose that the angle which the two planes of reflexion make with each other be β. The incident ray has the intensity 1, and consists of white light; the intensity of the twice reflected ray is then

$$S=\tfrac{1}{2}\{p_1^2(p_2^2\cos^2\beta+s_2^2\sin^2\beta)+s_1^2(p_2^2\sin^2\beta+s_2^2\cos^2\beta).$$

The observed chemical action, therefore, divided by S, gives the action which would have been observed if the sunlight had fallen directly without previous reflexion on the insolation-vessel. In order to calculate S, the values of i_1, i_2, and β must be determined for each observation. The second angle of incidence (i_2) was measured once for all.

In order to calculate the angle of incidence (i_1) varying with the position of the sun, we conceive the point of reflexion of the ray on the mirror of the heliostat to be the centre of a sphere, from which centre lines are drawn to the sun, to the earth's pole, to the south point of the horizon, and in the direction of the reflected ray. Let the four points produced by these lines cutting the sphere be represented by $SPMR_1$, fig. 15, Plate XLIII. The angle SR_1 is twice the angle of incidence of the ray; it is found from the spherical triangle PSR_1. In this triangle we know the side $PS=90-\delta$, when δ is the sun's declination. The sides R_1P and the angle R_1PS are thus found. In the spherical triangle PMR_1 the side MR_1, the azimuth of the opening in the shutter, directly measured, is known; the angle PMR_1 is a right angle, and $MP=180-p$ when p is the latitude of the place.

Hence

$$\cos PR_1=-\cos MR_1 \cos p$$

and

$$\sin MPR_1=\frac{\sin MR_1}{\sin PR_1}.$$

The angle SPR_1 is, however, $=MPR_1+t$, when t signifies the hour-angle of the sun, positive before noon. Hence we have for R_1S, or for the angle $2i_1$ which the incident rays make with the ray reflected from the heliostat mirror,

$$\cos R_1S=\cos 2i_1=\cos PR_1 \cos(90-\delta)+\sin PR_1 \sin(90-\delta)\cos(R_1PM+t).$$

The angle β is thus obtained: SR_1, fig. 15, is the first plane of reflexion, R_1R_2 is the second plane of reflexion; hence

$$\beta=PR_1M-PR_1S, \quad \text{and} \quad \sin PR_1S=\frac{\sin PS \sin SPR_1}{\sin SR_1}.$$

When there is only one reflexion, we have for the value of S simply

$$S=\tfrac{1}{2}(p_1^2+s_1^2).$$

If we consider all these corrections together, we find that the action (W_0) expressed in degrees of light, which the sun shining freely upon the insolation-vessel would have produced if no disturbing influences were acting, is obtained from the direct observations of our instrument (w_0) by means of the following formula:—

$$W_0=\frac{n_1 N \Lambda}{10000 S} w_0. \qquad (11.)$$

The subjoined Table (XVI.) gives a series of observations made, as just described, on the 3rd of August, 1857, and on the 14th and 15th of September, 1858, the sky being perfectly cloudless and the gas having reached the maximum degree of sensibility. In the series of observations made on August 3, 1857, in which two mirrors were employed, namely, the heliostat mirror of speculum metal and another mirror of steel, the data for the calculation of Λ and S were as follows:—

$\lambda=17''{\cdot}34$; $\lambda_1=1894''{\cdot}6$; $i_2=72° 50'$; $MR_1=45° 7'$; $\delta=+17° 29'$; $p=49° 24'$.

In the experiments made on September 14 and 15, 1858, in which only one mirror was used, the data were

$\lambda=21''{\cdot}22$; $\lambda_1=1912''$; $MR_1=73° 44'$; $\delta=+3° 26'$ for the 14th, and
$\delta=+3° 4'$ for the 15th; $p=49° 24'$.

TABLE XVI.

I. True time.	II. Sun's zenith-distance.	III. Observed action, w_0.	IV. W_0.	V. S.
August 3, 1857. Barometer=0m·7560.				
h m	° ′			
7 59	57 35	8·70	63·13	0·384
8 42	50 51	12·11	89·21	0·378
9 14	46 3	12·56	92·96	0·376
September 14, 1858. Barometer=0m·7550.				
8 1	68 34	8·84	26·23	0·631
September 15, 1858. Barometer=0m·7562.				
7 9	76 30	1·52	5·54	0·637
7 26	73 49	4·22	15·50	0·633
7 40	71 37	6·09	22·43	0·631
8 0	68 34	7·56	27·85	0·631
8 7	67 30	8·38	33·87	0·631
8 26	64 42	12·48	45·85	0·633
8 54	60 48	17·09	62·59	0·634
9 14	58 11	18·51	67·61	0·636

The increase in the chemical action on diminution of the sun's zenith distance, as seen in the numbers of the second and fourth columns, arises from the extinction of the chemical rays effected by passage through the atmosphere. In order to obtain from these observations the law of this extinction, we may, without sensible error, disregard the curvature of the earth's surface, and consider the atmosphere as a horizontal layer. As the extinction in one and the same mass of substance of different densities remains the same, the question becomes still simpler if we consider the atmosphere to be of equal density throughout and measured at $0^{m}\cdot76$ and 0° C.; in the following we shall therefore suppose the existence of such an ideal atmosphere. If we represent the chemical action of a solar ray before entering such an atmosphere by A, and its action when it has passed through a layer of atmosphere of the thickness l, by W_0, we find from the preceding considerations,

$$W_0 = A10^{-\alpha l},$$

when $\frac{1}{\alpha}$ represents the depth of atmosphere through which the ray has to travel until the chemical action produced is reduced from its original amount A to $\frac{1}{10}$th of that amount. The value of l is determined by the height of this ideal atmosphere and the zenith-distance of the sun.

Let us suppose that $c_{\prime}$, fig. 16, Plate XLIII., represents the place where the chemical action is measured, situated under the ideal atmosphere (L) measured at 0° and $0^{m}\cdot76$; and let $c_{\prime}b$ represent the direction of the zenith, $c_{\prime}a$ that of the sun; then $bca = \varphi$, *i. e.* the sun's zenith-distance, $c_{\prime}b = h$ the perpendicular height of the atmosphere, and $ac_{\prime} = l$ the depth of atmosphere traversed by the ray. We have then $l = \frac{h}{\cos\varphi}$, and

$$W_0 = A10^{-\frac{\alpha h}{\cos\varphi}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12.)$$

If the values of A and αh are calculated from the observations of August 3, 1857, and September 14 and 15, 1858, by the method of least squares, we obtain $A = 318\cdot3$, $\alpha h = 0\cdot3596$. The mean barometric pressure in the three experiments was $0^{m}\cdot7557 = P_0$. The perpendicular height (h) to which the atmosphere at the time of observation would have extended, if its density had throughout been that corresponding to $0^{m}\cdot7557$ and 0° C., is easily found from the specific gravities of air and mercury. Taking REGNAULT'S number 0·000095084 as representing the relation of the density of air and mercury, we have for our experiment—

$$h = \frac{0\cdot7557}{0\cdot000095084} = 7947 \text{ metres.}$$

This number, substituted in $\alpha h = 0\cdot3596$, gives for α the value 0·00004525. The sun's rays must therefore pass through a column of air at 0° and $0^{m}\cdot76$ of $\frac{1}{0\cdot00004525}$ $= 22100$ metres in length in order to reduce the chemical action to $\frac{1}{10}$th of its original amount. If we call P_0 the barometric pressure observed in the experiments from which α and A were found, the action which would have been observed under another pressure P, and under the zenith distance φ, is found from the equation

$$W_1 = A10^{-\frac{\alpha h P}{\cos\varphi P_0}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13.)$$

Substituting the experimental values of A, α, and P,

$$W_1 = 318\cdot3 \times 10^{-\frac{0\cdot4758 P}{\cos\varphi}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14.)$$

In the following Table (XVII.) the numbers calculated by means of this formula are compared with the action actually observed:—

TABLE XVII.

No. of the experiment.	Sun's zenith-distance.	Calculated chemical illumination in degrees of light.	Observed chemical illumination in degrees of light.
1	57° 35′	67·9	63·1
2	50 51	85·8	89·2
3	46 8	96·4	93·0
4	68 34	33·1	26·2
5	76 30	9·2	5·5
6	73 49	16·8	15·5
7	71 37	24·5	22·4
8	68 34	33·1	27·9
9	67 30	36·6	38·9
10	64 42	47·9	45·9
11	60 48	58·3	62·6
12	58 11	66·2	67·6

The probable error of the observations is

$$0\cdot0745\sqrt{\frac{\Sigma v^2}{12-1}}=\pm2\cdot7 \text{ degrees of light.}$$

This mean error must be regarded as extremely small when we consider,—1st, the great difficulty experienced in keeping the mixture of chlorine and hydrogen at the maximum degree of sensibility during the whole range of experiments; 2ndly, the numerous reductions by means of which all the disturbing influences must be eliminated; and 3rdly, the slight alterations in the extinction of the light which local changes in the air's transparency may effect.

Formula (14.) shows that the sun's rays before their entrance into the earth's atmosphere would effect an illumination of 318·3=L degrees of light. If the rays producing this degree of illumination passed through an infinitely extended atmosphere of chlorine and hydrogen until they were completely extinguished, the absolute height (H) of the column of hydrochloric acid, measured at 0° and $0^{m}\cdot76$, thus produced in one minute, is found from formula (4.), already given on page 888,

$$H=\frac{V}{q}\cdot\frac{1}{1-10^{-\alpha l}}\cdot L.$$

In the experiment from which the number 318·3 is obtained, one degree of light effected a combination per minute of V=0·00001155 cubic metre of hydrochloric acid measured at 0° and $0^{m}\cdot76$. According to our former experiments, the coefficient of extinction of sunlight for chlorine and hydrogen is $\alpha=\frac{1}{0\cdot052}$ metre. The internal area of the insolation-vessel was $q=0\cdot00033$ square metre. The internal diameter of the same, $d=0\cdot0094$ metre. The gas in the insolation-vessel was contained, during the observation, under a pressure of [illegible] and at a temperature of 18° C., for which the tension of aqueous vapour is $p=0^{m}\cdot0154$. Hence the value of h is found to be

$$h=0\cdot00858 \text{ metre.}$$

By substitution of this number in the last formula, we have

$$H=35\cdot3 \text{ metres.}$$

From this we see that the sun's rays, if they fell on the earth's surface without being weakened by passing through the atmosphere, would exert an action represented by 35·3 light-metres; that is, they would effect in one minute a combination, on a surface upon which they fall perpendicularly, of a column of hydrochloric acid 35·3 metres in height, assuming that the rays are extinguished by passing through an infinitely extended column of the sensitive gas.

By help of formula (14.) we find, moreover,—

That the sun's rays, after they have passed in a perpendicular direction through the atmosphere to the sea's level under a mean pressure of 0·76 metre, only effect an action of 14·4 light-metres, or that under these conditions, nearly two-thirds of their chemical activity has been lost by extinction and dispersion in the atmosphere.

If we assume that the mean distance of the sun to the earth is $r=20682329$ geographical miles, and if we imagine the sun in the centre of a sphere whose radius $=r$, the surface of this sphere is $4\pi r^2$. The light radiated from the sun upon this sphere would, if all the light were extinguished in the chlorine and hydrogen gas, produce in each minute a layer of hydrochloric acid of 35·3 metres, or 0·004756 geographical mile in height. Hence it follows,—

That the light which the sun radiates into space during each minute of time, represents a chemical energy, by means of which more than twenty-five and a half billions of cubic miles of chlorine and hydrogen may be combined to form hydrochloric acid.

In a similar way the chemical action has been calculated which the sun's rays, undiminished by atmospheric extinction, produce at the surface of each of the eight chief members of our planetary system. The second column of the following Table (XVIII.) contains the mean distances of these planets from the sun; the third column shows the chemical action represented in degrees of light effected by the sun's rays on the planet whose name is found in column I.; and in column IV. this same action is given in light-metres.

TABLE XVIII.

I.	II.	III.	IV.
			Light-metres.
Mercury	0·387	2125·6	235·4
Venus	0·723	608·9	67·5
The Earth	1·000	318·3	35·3
Mars	[illegible]	137·1	15·2
Jupiter	5·203	11·8	1·2
Saturn	9·539	3·5	0·4
Uranus	19·183	1·0	0·1
Neptune........	30·040	0·4	0·04

From these numbers it is seen how lavish Nature has been in her distribution of chemical energy throughout the universe. The Earth receives but an infinitely small portion of this radiation, whilst Saturn and the more remote planets obtain so much less, that on these bodies the existence of any organic life at all similar to that enjoyed on our globe must be impossible.

After these considerations touching the magnitude of the total chemical energy radiated from the sun, we may proceed to consider the conditions under which the small portion reaching the surface of our earth is distributed.

Compared with the thermic actions of the sun's rays, we immediately see an important difference. The heat, produced in the first instance by the absorption of the sun's rays, is so irregularly distributed by radiation and by oceanic and atmospheric currents over the earth's surface, that it rarely, if ever, is completely transformed at the time and at the place of its formation into real labour; so that the simple law, according to which the thermic climate of a place is dependent upon its elevation above the sea's level and the mean height of the sun, becomes completely obscured. The photo-chemical climate, on the other hand, is not subject to any such sweeping irregularities; for the chemical action which the sun effects upon a place, varying with its elevation above the sea and its geographical position, cannot be made to act at any point on the earth's surface other than the one on which the rays directly fall. Hence the diffusion and arrangement of the photo-chemical energy follows a much more simple law than that exhibited in the distribution of heat on the earth's surface.

By means of formula (14.) the amount of chemical energy effected by the sun's rays at a given time when the atmosphere is cloudless, can be calculated for any place whose geographical position is known, and for any height above the level of the sea.

In the following Table (XIX.) is found the chemical action expressed in degrees of light which the direct sun's rays effect at heights represented by barometric pressures of $0^{\mathrm{m}}{\cdot}8$ to $0^{\mathrm{m}}{\cdot}05$, with varying sun's zenith-distances from 90° to 0°. The highest horizontal division gives the zenith-distances, the first vertical column the atmospheric pressures, and the remaining columns the corresponding chemical actions. The curves (fig. 17, Plate XLVII.) show graphically the dependence of the chemical illumination upon the barometric pressure. The abscissæ denote sun's zenith-distances, the ordinates the chemical action in light-degrees effected at these zenith-distances. With each curve the corresponding barometric pressure is given. From this we see how unequal, under otherwise similar circumstances, the chemical illumination of low-land and high-land must be, and how this illumination increases as the distance above the sea becomes higher in a greater ratio than corresponds to the diminution of atmospheric pressure.

TABLE XIX.

	0°.	10°	20°	30°	40°	50°	60°	70°.	80°	90°
m 0·80	132·5	130·7	125·2	115·7	102·3	81·4	55·2	24·5	2·1	0·0
0·75	139·8	138·2	132·7	123·2	108·9	88·1	61·5	28·8	2·8	0·0
0·70	147·8	146·1	140·7	131·3	116·9	96·5	68·7	33·8	3·8	0·0
0·65	156·2	154·5	146·6	139·6	125·7	105·2	76·6	29·2	5·3	0·0
0·60	165·0	163·3	158·1	149·0	135·0	114·5	85·5	46·6	7·2	0·0
0·55	174·2	172·6	167·6	158·7	145·0	124·7	95·4	54·7	9·9	0·0
0·50	184·1	182·5	177·7	169·0	155·7	135·7	106·4	64·2	13·6	0·0
0·45	194·4	193·0	188·4	180·1	167·2	147·8	118·8	75·3	18·6	0·0
0·40	205·3	204·0	199·6	191·9	179·7	161·0	132·5	88·4	25·5	0·0
0·35	216·9	215·7	211·6	204·4	193·0	175·3	147·8	103·8	35·0	0·0
0·30	229·1	228·0	224·4	217·8	207·2	190·9	165·0	121·7	48·0	0·0
0·25	241·9	241·0	237·8	231·9	222·6	207·9	184·1	142·9	65·8	0·0
0·20	255·7	254·8	252·1	247·2	239·1	226·3	205·3	167·7	90·1	0·0
0·15	269·5	269·4	267·3	263·2	256·8	246·5	229·1	196·9	123·5	0·0
0·10	285·2	284·7	283·2	280·5	275·8	268·4	255·7	231·1	169·3	0·0
0·05	301·3	301·0	300·2	298·7	296·3	292·2	285·2	271·1	232·1	0·0
0·00	318·3	318·3	318·3	318·3	318·3	318·3	318·3	318·3	318·3	0·0

From the curves we also see that the variations in the illumination appear most strikingly the lower the sun is above the horizon. When, for instance, at Reykiavik, under a barometric pressure of $0^{\mathrm{m}}{\cdot}770$, the sun is 10° above the horizon, the ground is illuminated by direct sunlight to 2·5 degrees of light; if the barometer sinks 10 millims., the illumination rises to 2·9 degrees of light. On the highest point of the neighbouring crater of Hecla, upon which one of us observed a barometric pressure of $0^{\mathrm{m}}{\cdot}6290$ on the 26th of July, 1846, the illumination from direct sunlight must rise to 6·1, and on the summit of Dwalaghiri to at least 90 degrees of light. At the time when the sun has nearly reached the zenith in the latitude of the Himalayas, the amount of the direct sunlight which falls on the valleys of the Thibetian high-lands where grain is cultivated, is nearly one and a half time as large as that falling on the neighbouring low-lands of Hindostan. This difference increases in so rapid a ratio with increasing zenith-distance, that when the sun is 45° removed from the zenith, the direct solar rays on the high table-land of Thibet give more than twice the chemical action of those falling on the plains of India.

From these few examples it is seen how rapidly the chemical intensity of the sunshine increases with considerable elevation above the sea's level. Yet the variations effected by reason of these changes are small when compared with the differences brought about by alterations in the latitude of the place of observation. In order to give an idea of these variations, we have calculated the chemical action which the sun's rays effect under a barometric pressure of $0^{\mathrm{m}}{\cdot}76$ at the spring equinox during every hour of the day, on a horizontal unit of area situated at the points on the earth's surface for which the chemical illumination of the whole diffuse light has already been determined. The results of this calculation are contained in Table XX. The formula

$$W_0 = 318{\cdot}3 \times 10^{-\frac{0{\cdot}4758\,P}{\cos\phi}}$$

gives the chemical illumination for the unit of area upon which the sun's rays fall perpendicularly. In order that the supposed surface may be considered to be in the plane of the horizon and not at right angles to the incident ray, the values of W_0 have been multiplied by cos φ.

TABLE XX.

	Melville Island.	Reykiavik.	Petersburg.	Manchester.	Heidelberg.	Naples.	Cairo.
h h							
6 A.M. or 6 P.M. ...	0·00	0·00	0·00	0·00	0·00	0·00	0·00
7 A.M. or 5 P.M. ...	0·00	0·02	0·07	0·22	0·38	0·89	1·74
8 A.M. or 4 P.M. ...	0·07	1·53	2·88	5·85	8·02	13·31	20·12
9 A.M. or 3 P.M. ...	0·67	6·62	10·74	18·71	23·99	35·88	50·01
10 A.M. or 2 P.M. ...	1·86	13·27	20·26	32·91	40·94	58·46	78·61
11 A.M. or 1 P.M. ...	3·02	18·60	27·55	43·34	53·19	74·37	98·33
12 A.M.	3·51	20·60	30·26	47·15	57·62	80·07	105·3

The curves (fig. 18, Plate XLVIII.) represent the illuminations at the various places and times given in the foregoing Table (XX.). The ordinates represent the chemical actions, expressed in degrees of light, effected at the hours measured on the abscissæ for the places named on the curves. If we compare these curves with those of fig. 11, Plate XLVI. representing the diffuse light, the singular fact becomes apparent, that at the time of the vernal equinox the chemical actions which the direct sunlight effects from the north pole to below the latitude of Petersburg is, during the whole day, *less* than that effected by the total diffuse daylight; and that even in lower latitudes, down to the equator, the same phenomenon is observed, if not for the whole, yet for a portion of the day.

A further comparison of formulæ (9.) and (14.), shows that not only at the vernal equinox, but at all times and in all places where the sun rises more than 20° 56′ above the horizon, from sunrise till it attains a given height above the horizon, the chemical action effected by the diffuse daylight exceeds that of the direct sunlight; and that as the sun gradually rises, a point is reached at which both sunlight and diffuse daylight produce exactly the same amount of chemical action, whilst beyond this point the effect of the sunshine is most powerful. This "*phase of equal chemical illumination*" between daylight and sunshine, occurs twice daily, and represents a certain zenith-distance of the sun (φ), which may be found by a method of approximation from the formula

$$2{\cdot}776+80{\cdot}849\cos\varphi-45{\cdot}996\cos^2\varphi=\frac{318{\cdot}3}{\text{num log}=\frac{0{\cdot}4758P}{\cos\varphi}}$$

where the expression to the right denotes the chemical action effected by the sunlight at the pressure P (formula 14.), and that on the left hand gives the action proceeding from the whole heavens (formula 9.).

If we assume that the barometric pressure was $0^{m}{\cdot}76$, that the plane of the surface illuminated by the sunlight was placed perpendicular to the incident rays, and that the area illuminated by the daylight lay in the plane of the horizon, we find, on calculation, that the illumination of the sunshine is equal to that from the daylight when the distance of the sun from the zenith is 71° 12′.

These daily phases of equal chemical intensity can be determined by allowing the diffuse daylight and the direct sunlight to fall separately on two pieces of the same photographic paper, and by observing the time at which the same degree of colour is brought about on both pieces. From the correspondence of these observed times with those calculated according to the foregoing method, a good idea can be formed of the degree of reliability which our experiments, and the theory based thereon, warrant.

On February 21st and 22nd, and March 7th and 11th, 1859, the sky was perfectly cloudless from sunrise to sunset, and hence these days were well-adapted for the purpose of the experiments. The observations were made in a darkened space under the roof of the Heidelberg laboratory by two persons, one of whom, outside the roof, allowed the whole diffuse light of day, excluding the direct sunshine, to act upon the prepared paper, whilst the other observer allowed the direct sun's rays to fall inside the darkened space for an equal length of time perpendicularly upon another portion of the same paper. The sunlight passed through a hole in the roof, on to a blackened box with a circular opening of 1 inch in diameter. In order to preserve the paper always at right angles to the incident rays, it was fixed on a piece of board held in the hand, upon which an upright pin was fastened, and the board was so placed that the shadow of the pin on the paper was never thrown to one side or the other.

Allowing for the actual height of the barometer observed, $0^{m}\cdot764$, the zenith-distance of the sun, at which the chemical illumination from the direct sunlight is equal to that from diffuse light, is found to be 71° 4′. The sun reached this point on February 21, 1859, at $9^{h}\ 7^{m}$ A.M., and at $2^{h}\ 53^{m}$ P.M. The experiments on this day began at $11^{h}\ 30^{m}$ A.M., when the blackening effected by the sunlight was much more considerable than that produced by daylight; at a later hour this difference between the actions diminished; at $3^{h}\ 1^{m}$ scarcely any difference could be observed, and at $3^{h}\ 16^{m}$ the point of equal colour was passed, as the paper exposed to the sunlight was at this time considerably less coloured than that exposed to diffuse daylight. The phase required lies therefore between $3^{h}\ 1^{m}$ and $3^{h}\ 16^{m}$; according to calculation, it should have occurred at $2^{h}\ 53^{m}$. Another determination, made on the following day, showed that the phenomenon occurred in the afternoon between $3^{h}\ 23^{m}$ and $3^{h}\ 39^{m}$, instead of at $2^{h}\ 56^{m}$; and in the morning between $8^{h}\ 30^{m}$ and $8^{h}\ 43^{m}$, instead of at $9^{h}\ 4^{m}$. On March 7, 1859, under a barometric pressure of $0^{m}\cdot752$, the phase occurred in the afternoon between $4^{h}\ 17^{m}$ and $4^{h}\ 27^{m}$, when, according to calculation, it should have occurred at $3^{h}\ 33^{m}$. On March 11, 1859, the sky was intensely blue; the phase of equal colour appeared, under a pressure of $0^{m}\cdot764$, in the morning at $7^{h}\ 42^{m}$ instead of $8^{h}\ 22^{m}$. From these experiments, it is not only seen that these phases which the theory requires actually occur, but also that the agreement between the observed and calculated time of appearance of these phases is very close. The point at which the phenomenon is observed in the morning is indeed about forty-five minutes earlier than the calculated point, whilst in the afternoon the calculated is about thirty minutes earlier than the observed point. These differences are, however, satisfactorily explained by the fact, that, in the situation at which the observations were made, the horizon towards north, west, and east is not free, owing to the hills of the Neckar Valley which rise 12° above the horizon, in consequence of which a deviation in the direction observed must occur.

We pass now to the consideration of the total chemical effect which the direct sunlight, varying with the time of day, produces on a unit of area lying in the plane of the horizon during a given time.

As has been already shown, the formula

$$W_1 = \cos\varphi \times 318{\cdot}3 \times 10^{-\frac{0{\cdot}4758\,P}{\cos\varphi}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15.)$$

gives the chemical effect (W_1) which the sun's rays exert on a horizontal unit of surface in the space of one minute. For the further calculation, it is, however, simpler to express W_1 by a series of powers of the sun's zenith-distance,

$$W_1 = a\cos^2\varphi + b\cos^3\varphi + c.\cos^4\varphi + \ldots$$

By help of formula (15.) we have calculated the value of W_1 for every 10° of zenith-distance between 0° and 90°, and then by the method of least squares found the values of the coefficients a, b, and c. The calculation gives

$$W_1 = -31{\cdot}99\cos^2\varphi + 417{\cdot}6\cos^3\varphi - 248{\cdot}7\cos^4\varphi. \quad . \quad . \quad . \quad . \quad . \quad (16.)$$

The accompanying numbers show that the differences between the values of W_1, calculated according to the two formulæ (15.) and (16.), are so inconsiderable, as to fall within the unavoidable errors of observation.

According to formula 15.		According to formula 16.
138·4	0°	136·9
134·6	10	133·8
123·3	20	124·3
105·4	30	107·3
82·2	40	83·3
56·0	50	55·2
30·1	60	29·6
9·5	70	9·6
0·5	80	1·0
0·0	90	0·0

We will now calculate the amount of the chemical rays which fall during a given time and at a given situation upon a unit of surface in the plane of the horizon.

The sun's zenith-distance is connected with the true solar time of a place by the equation

$$\cos\varphi = \cos\delta\cos p\cos t + \sin\delta\sin p,$$

when δ represents the sun's declination, p the latitude of the place, and t the sun's hour-angle. If, for the sake of shortness, we call $\sin\delta\sin p = \alpha$, and $\cos\delta\cos p = \beta$, and develope formula (16.) in a series of powers of $\cos t$, we have

$$\begin{aligned} W_1 = {} & a(\alpha^2 + 2\alpha\beta \cos t + \beta^2 \cos^2 t) \\ & + b(\alpha^3 + 3\alpha^2\beta \cos t + 3\alpha\beta^2 \cos^2 t + \beta^3 \cos^3 t) \\ & + c(\alpha^4 + 4\alpha^3\beta \cos t + 6\alpha^2\beta^2 \cos^2 t + 4\alpha\beta^3 \cos^3 t + \beta^4 \cos^4 t). \end{aligned}$$

The integral $\int W_1 dt$ has now to be determined. For this purpose we consider

$$\begin{aligned} & \int dt = t \\ & \int \cos t dt = \sin t \\ & \int \cos^2 t dt = \frac{\cos t \sin t}{2} + \tfrac{1}{2} t \\ & \int \cos^3 t dt = \frac{\cos^2 t \sin t}{3} + \tfrac{2}{3} \sin t \\ & \int \cos^4 t dt = \frac{\cos^3 t \sin t}{4} + \tfrac{3}{8} \cos t \sin t + \tfrac{3}{8} t. \end{aligned}$$

For the equinox $\alpha = 0$ and $\beta = \cos p$; if the action for the whole day is required, the integration must be taken from

$$t = -\frac{\pi}{2} \text{ to } t = +\frac{\pi}{2};$$

for these limits we have—

$$\begin{aligned} & \int dt = \pi \\ & \int \cos t dt = 2 \\ & \int \cos^2 t dt = \frac{\pi}{2} \\ & \int \cos^3 t dt = \tfrac{4}{3} \\ & \int \cos^4 t dt = \tfrac{3}{8}\pi; \end{aligned}$$

also

$$\int w dt = \frac{\pi}{2} a \cos^2 p + \tfrac{4}{3} b \cos^3 p + \tfrac{3}{8} \pi \cos^4 p,$$

and

$$W = 12 \times 60 \left(\frac{a}{2} \cos^2 p + \frac{4}{3\pi} b \cos^3 p + \tfrac{3}{8} c \cos^4 p \right);$$

on substitution of the values of a, b, and c,

$$W = -11520 \cos^2 p + 127600 \cos^3 p - 67140 \cos^4 p.$$

Calculated according to this formula, the following Table (XXI.) contains the total amount of sunlight, expressed in degrees of light, falling during the day at the vernal equinox upon the horizontal unit of area at the previously mentioned places. Division I. contains the names of the places for which the calculation has been made; division II. the latitudes of these places; division III. the chemical degrees of light to which the horizontal area is illuminated by sunshine alone; division IV. the degrees of light, already given in Table XIII. b, to which the same area is illuminated by the total diffuse daylight; division V. the chemical illumination effected by both the sun and diffuse light together; and lastly, division VI. the height of the column of hydrochloric acid (at 0° and $0^{m}\cdot76$), calculated according to formula (4.), which the total illumination of sun and daylight is able to combine during this one day.

TABLE XXI.

I.	II.	III.	IV.	V.	VI.
					metres.
Melville Island	74° 47′ N.L.	1196	10590	11790	1306
Reykiavik	64 8 N.L.	5964	15020	20980	2324
Petersburg	59 56 N.L.	8927	16410	25340	2806
Manchester..............	53 20 N.L.	14520	18220	32740	3625
Heidelberg	49 24 N.L.	18240	19100	37340	4136
Naples....................	40 52 N.L.	26640	20550	47190	5226
Cairo	30 2 N.L.	36440	21670	58110	6437
Bombay	19 0 N.L.	43820			
Ceylon	10 0 N.L.	47530			
Borneo	0 0 N.L.	48940			

From the numbers in division V. it is seen that the total chemical energy which the earth receives simultaneously from the whole heavens and from the sun, varies comparatively slightly with the latitude of the place. At Cairo the total chemical action is about five times, and at Heidelberg about twice as large as it is on Melville Island, which is only about 15° distant from the north pole. Notwithstanding this small difference, the highest position of the sun above the horizon on the day in question at these three places is very different; viz. at Melville Island, 15° 13′; at Heidelberg, 40° 30′; and at Cairo, 59° 58′. The explanation of this remarkable relation is to be found in the large dispersive power of the atmosphere, which acts as a regulator of the photochemical processes occurring on the earth's surface, modifying and lessening the great differences in the chemical illumination produced by the direct sunlight. This is plainly seen on reference to the numbers in divisions III. and IV. The numbers representing the chemical activity developed by sunlight alone in Melville Island, Heidelberg, and Cairo, are nearly in the relation of 1:15·3:30·5, whereas the chemical actions brought about by diffuse daylight at these places, when the amount of direct sunlight on Melville Island is taken as unity, are as 8·9:16:18·1.

A comparison of columns III. and IV. show moreover that, singularly enough, the amount of chemical energy effected by the diffuse daylight is, down to the latitude of Heidelberg, larger than that produced by the direct sunlight. At Heidelberg the action from both sources is nearly equal; at Petersburg that of the diffuse daylight is almost double of that of the sunlight; and at Melville Island the effect produced by the former source is nearly ten times as large as that effected by the latter.

IV. THE PHOTO-CHEMICAL ACTION OF THE SUN COMPARED WITH THAT OF A TERRESTRIAL SOURCE OF LIGHT.

We have made a few experiments for the purpose of comparing the light evolved from the sun's body with that liberated from a terrestrial source. The most suitable light for this object, appeared to be the intense illumination produced by the combustion of magnesium wire in the air. In the following experiments we have employed a piece of magnesium wire, the radius of which, measured under a microscope, was $r=0·1485$ mm.; this wire was pressed out of a steel press by the method employed by one of us for the preparation of potassium and sodium wire, and applied by Dr. MATTHIESSEN to the metals of the alkaline earths. When the end of such a wire is raised in temperature, it takes fire and burns on regularly throughout its entire length, leaving a coherent thread of magnesia.

In order to compare the light thus evolved with that of the sun, it was necessary to know the length of wire which remained incandescent during the combustion. This cannot be measured directly, because a surface of a few square millimetres appears during the ignition like a ball of fire as large as a nut. If the enormous intensity of the light be weakened by transmission through dark coloured glasses, the edges of the wire can be more distinctly seen; but, owing to the rapidity with which the combustion proceeds, even in this way no exact measurement is possible. The length of the incandescent piece of wire may, however, be obtained, with sufficient accuracy, in the following way:—Between 30 and 40 millimetres of wire of the above thickness were held in front of a photometric stearinized diaphragm illuminated by a gas-flame burning in the tin box described in our former papers, and then burnt at such a distance from the diaphragm that the ring just disappeared. If, now, all the arrangements remaining the same, shorter pieces of wire of exactly known length are burnt, the ring always disappears until the length of the wire taken is less than the part which during the combustion is kept incandescent. From this time forward the ring appears plainly, and is seen to be black on a white ground. Two experiments made in this way showed that pieces of wire above 10 millims. in length rendered the ring invisible, whilst with pieces shorter than 10 millims. the ring was seen to be dark on light ground. The length of wire kept continually incandescent during the combustion is therefore $h=10$ millims. in length. The wire burning at a distance of 2440 millims. from the insolation-vessel of our photometer, effected an action of 181·7 units of light, or of 0·01817 degree of light per minute. The light producing this action emanated from a cylinder of magnesium, having a radius 0·1485 millim. and a height of 10 millims. The surface of this half cylinder gives out as much light as an equally intensely illuminated rectangle whose base is equal to the diameter, and whose height is the same as the height of the cylinder. This rectangle had, in our experiment, the area $2rh=2·07$ square millimetres, or a circular area having a radius of 0·9725 millim. radius. If this surface had been placed at a distance of 208·7 millims. from the insolation-vessel, instead of 2440 millims., the action, instead of being 0·01817 degree of light, would have been

$$\frac{0·01817 \times 2440^2}{208·7^2} = 2·482 \text{ degrees of light.}$$

This distance (208·7) is, however, that at which the luminous circular area of burning magnesium seen from the insolation-vessel has the apparent magnitude of the sun's disk. As the sun, before its rays suffered diminution by passing through the atmosphere, would effect upon the insolation-vessel an illumination of 318·3 degrees of light (seen from formula 14.), the chemical brightness, or the chemical activity of the rays proceeding from the sun's surface is $\frac{318·3}{2·482}$, or 128·2 times larger than that of the rays evolved from magnesium wire burnt in the way described. By help of the same formula (14.) it is seen that the chemical brightness of our incandescent magnesium wire is equal to that of the sun when it stands about 9° 53′ above the horizon. A burning surface of magnesium wire, which, seen from a point at the sea's level, has an apparent magnitude equal to that of the sun, effects on that point the same chemical action as the sun would do when shining from a cloudless sky at a height of 9° 53′ above the horizon. If, for instance, such a surface had a diameter of 1 metre, the chemical action which it would produce at a distance of 107 metres is the same as the sun would effect when shining perpendicularly on any object from an elevation of 9° 53′ above the horizon.

As a matter of interest we have, in contradistinction to the *chemical*, compared the *visual* brightness of these two sources of light, *i. e.* the brightness as measured by the eye.

For this purpose the direct rays of the sun at 12 o'clock at noon on the 13th of November, 1858, when the sky was cloudless, were reflected by means of a mirror of black glass through a circular opening of 0·899 millim. diameter on to the stearinized diaphragm of the photometer, and then the gas-flame of the photometer was so arranged that the diaphragm ring disappeared. In order to counterbalance the different colour of gas- and sun-light, a piece of pale blue glass was placed between the flame and the diaphragm.

Let us suppose that the intensity of the light when the ring disappears is I. Let S be the intensity which the unit amount of sunlight possesses after the one reflexion, g the apparent area which the hole through which the sun passes appears to have when seen from the diaphragm, and g_1 the apparent area of the sun's disk; then the following fraction gives the intensity of the sun's direct rays falling perpendicularly upon the diaphragm,

$$\frac{Ig_1}{Sg}.$$

In order to compare with this, the intensity of the burning magnesium, the wire was burnt before the diaphragm of the photometer, at such a distance that the ring, as in the case of illumination with the sun, disappeared. If we call g_2 the apparent area of the incandescent surface of the wire as seen from the photometer ring, the burning portion of the wire will illuminate the diaphragm, when its apparent area is equal to that of the sun, with the intensity

$$\frac{Ig_1}{g_2}.$$

The visual brightness (G) of the sun, compared with that of a burning magnesium wire, is therefore

$$G=\frac{g_2}{Sg}.$$

If the radius of the opening through which the sun shines be r; the distance of the opening from the photometer diaphragm d; the radius of the incandescent surface of magnesium considered as circular r_1; and the distance of this surface from the diaphragm d_1, the brightness of the sun compared with that of the magnesium is

$$G=\frac{d^2r_1^2}{Sd_1^2r^2}:$$

S is found from the formula

$$S=\frac{\sin^2(\varphi-\varphi')}{\sin^2(\varphi+\varphi')}+\frac{\tan^2(\varphi-\varphi')}{\tan^2(\varphi+\varphi')},$$

when φ signifies the angle of incidence and φ' the angle of refraction of the ray. φ_1 is calculated from the refractive index of the mirror $=1\cdot55=\frac{\sin\varphi}{\sin\varphi_1}$, when the angle of incidence (φ) is known. The value of φ is obtained from the azimuth (A) of the opening through which the rays pass; from the sun's declination (δ) on the day of experiment; from the time of observation (t), and from the latitude of Heidelberg (p), by means of the formula (8.) already employed. By substituting the following numerical values, $A=73°\ 44'$; $\delta=-17°\ 58'$; $t=0°\ 0'$; $p=49°\ 24'$ in the formula, we have

$$S=0\cdot05101.$$

Direct measurement gave

$r=0\cdot1995$ millim.; $r_1=0\cdot9725$ millim.; $d=2590$ millims.; $d_1=2440$ millims.

Hence

$$G=524\cdot7.$$

The observations were made on November 18th, 1858, at $12^h\ 0^m$, when the sun's zenith-distance was 67° 22′. The brightness of the sun's disk, as measured by the eye, is therefore, at this zenith-distance, 524·7 times as great as that of the burning magnesium wire, whilst at the same zenith-distance the chemical brightness of the sun is only 36·6 times as great.

The steady and equable light evolved by magnesium wire burning in the air, and the immense chemical action thus produced, render this source of light valuable as a simple means of obtaining a given amount of illumination expressed in terms of our measurement of light. According to the above experiment, a piece of magnesium wire [illegible] metre long and 0·297 millim. in diameter, effects at a distance of 2·44 metres an [illegible] units of light, of which 10,000 are equal to 1 degree of light. For every [illegible] wire burnt, there is therefore produced 1·1 unit of light at a distance of 1 metre. These numbers cannot be supposed to be very accurate, as the quantity of magnesium upon which we could operate was but small, and we were therefore unable to eliminate the errors arising from the phenomenon of induction as completely as could be wished.

The combustion of magnesium constitutes so definite and simple a source of light for the purposes of photo-chemical measurement, that the wide distribution of this metal becomes desirable. The application of this metal as a source of light may even become of technical importance. A burning magnesium wire of the above thickness evolves, according to a measurement we have made, as much light as 74 stearine candles*, of which five go to the pound. If this light lasted one minute, 0·987 metre of wire, weighing 0·1204 grm., would be burnt. In order to produce a light equal to 74 candles burning for 10 hours, whereby about 20 lbs. of stearine is consumed, 72·2 grms. of magnesium would be required. The magnesium wire can be easily prepared by forcing out the metal by great pressure from a heated steel press having a fine opening at bottom: this wire might be rolled up in coils on a spindle which could be made to revolve by clock-work, and thus the end of the wire, guided by passing through a groove or between rollers, could be continually pushed forward into a gas- or spirit-lamp flame in which it would burn.

V. CHEMICAL ACTION OF THE CONSTITUENT PARTS OF SOLAR LIGHT.

The chemical action effected by the several portions of the solar spectrum depends not only upon the nature of the refracting body, but also upon the thickness of the column of air through which the light has to pass before decomposition. In the following experiments we have employed prisms and lenses of quartz, cut by Mr. Darker of Lambeth, instead of glass prisms, which, as is well known, absorb a large portion of the chemically active rays. In order to render our experiments as free as possible from the irregularities arising from variation in the atmospheric absorption, the observations were made quickly one after the other, so that the zenith-distance of the sun altered but very slightly.

A perfectly cloudless day was chosen for these observations, and the direct sunlight reflected from the speculum-mirror of a Silbermann's heliostat through a narrow slit into our dark room. The spectrum produced by the rays passing through two quartz prisms and a quartz lens, fell upon a white screen which was covered with a solution of sulphate of quinine, to render the ultra-violet rays and the accompanying dark lines visible. In this screen a narrow slit was made through which the rays from any wished-for portion of the spectrum could be allowed to pass, so as to fall directly upon the insolation-vessel situated at a distance of from 4 to 5 feet. A finely divided millimetre-scale was also placed on the screen, by means of which the distance between the Fraunhofer's lines could be accurately measured, and the portion of light employed thus exactly determined.

In order to recognize with accuracy the various portions of the spectrum, we employed a map of the dark lines prepared by Mr. Stokes, which he most kindly placed at our disposal. The figure (fig. 19, Plate XLVII.) contains a copy of Mr. STOKES's map, with the distances measured by him, and letters given according to his notation. We have divided the space between the letter A in the red to the last ray STOKES observed, W in the lavender rays, into 160 equal parts, and we represent the position and breadth of the bundle of rays which effected a given action upon the insolation-vessel as follows:— If a bundle of rays lying between the abscissæ 20·5 and 34 in fig. 15 had to be represented, we should call the edge of the bundle towards A, ½ DE, and that towards W, ¼ FG, whilst the middle of the bundle, or the portion of the spectrum which produces the action, we call "½ DE to ¼ FG." The breadth of this bundle of rays, in which the insolation-vessel was completely bathed, was $\frac{3}{100}$ of the total length of the spectrum.

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* This number must only be regarded as an approximation to the truth, as the diaphragm of the photometer had to be illuminated with light passing a light-blue glass.

The following Table gives the direct results of a series of observations made by perfectly cloudless sky at Heidelberg, on the 14th of August, 1857, under a barometric pressure of 0^{m}·7404. The first column gives the numbers of the observations in the order in which they were made; column II. the times of observation in true solar time; column III. the portion of spectrum under examination; and column IV. the action corresponding to this portion.

I.	II.	III.	IV.
	h m		
1	10 54 A.M.	From ⅜ GH to I	48·80
2	10 58 A.M.	From ½ DE to E	1·27
3	11 4 A.M.	From C to ½ DE	0·47
4	11 8 A.M.	From N_1 to ¼ QR.........	18·28
5	11 13 A.M.	From ½ RS to ⅜ ST......	2·03
6	11 41 A.M.	From ¾ ST to ⅘ UV......	1·27
7	11 47 A.M.	From ½ N_4Q to ½ RS ...	11·73
8	11 50 A.M.	From ½ ST to ¾ UV......	1·02
9	11 54 A.M.	From ⅙ IM_1 to N_1	37·87
10	11 57 A.M.	From H_1 to ½ IM_1	57·42
11	0 1 P.M.	From H_1 to ¼ IM_1	52·30
12	0 4 P.M.	From ⅛ GH to H.........	61·38
13	0 7 P.M.	From ½ FG to G	27·64
14	0 16 P.M.	From ⅓ FG to G	28·74
15	0 20 P.M.	From ¼ DE to F	1·39
16	0 25 P.M.	From ¼ N_4Q to ⅛ RS ...	13·19
17	0 32 P.M.	From ½ N_4Q to ⅓ RS ...	12·41
18	0 40 P.M.	From G to ⅙ GH.........	53·78
19	0 42 P.M.	From ¼ GH to H.........	58·74
20	0 45 P.M.	From ⅝ GH to I	53·9

If the fraction of the unit amount of incident light which is reflected from the mirror of the heliostat at the commencement and at the end of the series of experiments be calculated, we get the numbers 0·644 and 0·642, which differ so slightly, that the variations brought about by the reflexion may be neglected without overstepping the observational errors. At the times of observation on the 14th of August, 1857, the sun's zenith-distance was as follows:—

	h m	° ′
At	10 54 A.M.	37 35
At	0 0 A.M.	35 13
At	0 45 P.M.	36 16

The chemical intensity of the sun's rays at these various periods may be calculated by formula (14.). They are in the proportion of the numbers 1·002, 1·000 and 1·016. Although the differences between these numbers are but small, we have reduced all the observations to that chemical action which would have been observed if they had all been made at $12^h\ 0^m$ A.M. upon the day in question. The following Table contains the numbers thus reduced, the mean value having been taken of those observations which occur more than once:—

No.	True solar time.	Position in the spectrum.	Relative chemical action.
	h m		
1	10 54 A.M.	From $\frac{3}{5}$ GH to I	52·7
2	10 58 A.M.	From $\frac{1}{5}$ DE to E	1·3
3	11 4 A.M.	From C to $\frac{1}{2}$ DE	0·5
4	11 8 A.M.	From N_1 to $\frac{3}{4}$ QR...........	18·9
5	11 13 A.M.	From $\frac{1}{2}$ RS to $\frac{2}{3}$ ST	2·1
6	11 41 A.M.	From $\frac{3}{4}$ ST to $\frac{4}{5}$ UV........	1·2
7	11 47 A.M.	From $\frac{1}{2}$ N_4Q to $\frac{1}{3}$ RS	12·5
8	11 54 A.M.	From $\frac{4}{5}$ IM_1 to N_4...........	38·6
9	0 1 P.M.	From H_1 to $\frac{3}{4}$ IM_1	55·1
10	0 4 P.M.	From $\frac{1}{5}$ GH to H...........	60·5
11	0 16 P.M.	From $\frac{1}{5}$ FG to G	28·4
12	0 20 P.M.	From $\frac{3}{4}$ DE to F	1·4
13	0 40 P.M.	From G to $\frac{4}{5}$ GH	54·5

The lines *a a a a* (fig. 19, Plate XLVII.) give a representation of the relative chemical action which the various parts of the spectrum, the rays of which have only passed through air and quartz, effect on the sensitive mixture of chlorine and hydrogen. It is seen that this action attains many maxima, of which the largest lies by $\frac{1}{5}$ GH to H, and the next at I, and also that the action diminishes much more regularly and rapidly towards the red than towards the violet end of the spectrum.

The sun, when it was employed for these experiments, was 35° 18′ removed from the zenith. If the atmosphere were throughout of the density corresponding to $0^m{\cdot}76$ and 0° C., the perpendicular height which, during our experiment, it would have possessed, is

$$\frac{0{\cdot}7494}{0{\cdot}000095084}=7881 \text{ metres.}$$

The depth of atmosphere through which the rays had to pass in this experiment was, however,

$$\frac{7881}{\cos 35^\circ 13'}=9647 \text{ metres.}$$

We have stated in one of our previous communications*, that the solar rays which at different hours of the day pass through the same column of chlorine, are altered in a very different manner. This shows that rays of different chemical activity are absorbed in very different ways by the air. The above results are therefore only applicable for sunlight which has passed through a column of air, measured at $0^m{\cdot}76$ and 0° C., of 9647 metres in thickness. For rays which have to pass through a column of air of a different length from this, the chemical action of the various constituents of the spectrum must be different. The order and degree in which the chemical rays are absorbed, may be obtained by repeating the observations according to the above method from hour to hour during a whole day. Such a series of experiments we have unfortunately as yet been unable to execute, owing to the variability of the weather in our latitudes. One very imperfect series of observations we can, however, quote, and they suffice to show that the relation between the chemical action of the spectral colours is perceptibly altered when the thickness of air through which the rays pass changes from 9647 to 10,735 metres.

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* Philosophical Transactions, 1857, p. 617, &c.

These experiments were likewise made on August the 14th, 1857, in the short space of time from $9^h\ 44^m$ to $10^h\ 19^m$ A.M., and gave the following numbers reduced to the zenith-distance (42° 46′), corresponding to $10^h\ 0^m$ A.M. They were, however, made with a bundle of rays of a different thickness from the former experiments, and therefore cannot be compared with those.

No.	Time.	Portion of spectrum.	Relative chemical action
	h m		
1	9 44 A.M.	From $\frac{2}{3}$ GH to I	14·5
2	9 48 A.M.	From N_2 to R_2	10·1
3	9 54 A.M.	From $\frac{1}{10}$ R_2S to $\frac{1}{2}$ ST......	2·4
4	9 59 A.M.	From $\frac{1}{2}$ ST to U	0·0
5	10 4 A.M.	From G to $\frac{1}{2}$ GH	13·0
6	10 8 A.M.	From F to $\frac{1}{2}$ FG	7·1
7	10 11 A.M.	From *b* to $\frac{1}{2}$ FG	3·2
8	10 15 A.M.	From $\frac{1}{2}$ DE to $\frac{3}{4}$ EF	0·4

These observations are represented by the lines *b b b* (fig. 10, Plate XLVII.) in which, it must be remembered, a unit different from that of the other experiments is employed. From this it is seen that the relation of the chemical action of the spectrum from the line E to the line H undergoes a considerable alteration when the rays have to pass through a column of air 10,735 metres in height instead of 9647 metres.

An extended series of measurements of the chemical action of the several portions of the solar spectrum under various conditions of atmospheric extinction, may prove of great interest, if, as we can now scarcely doubt, the solar spots appear at regular intervals, and our sun belongs to the class of fixed stars of variable illuminating power. It is possible that such observations, made during the presence and during the absence of the solar spots, may give rise to some unlooked-for relations concerning the singular phenomena occurring on the sun's surface. Whether, however, the atmospheric extinction can ever be determined with sufficient accuracy to render visible the alteration in the light which probably occurs with the spots, is a question which can only be decided by a series of experimental investigations which must extend far beyond the scope of any single observer.

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FIG 1

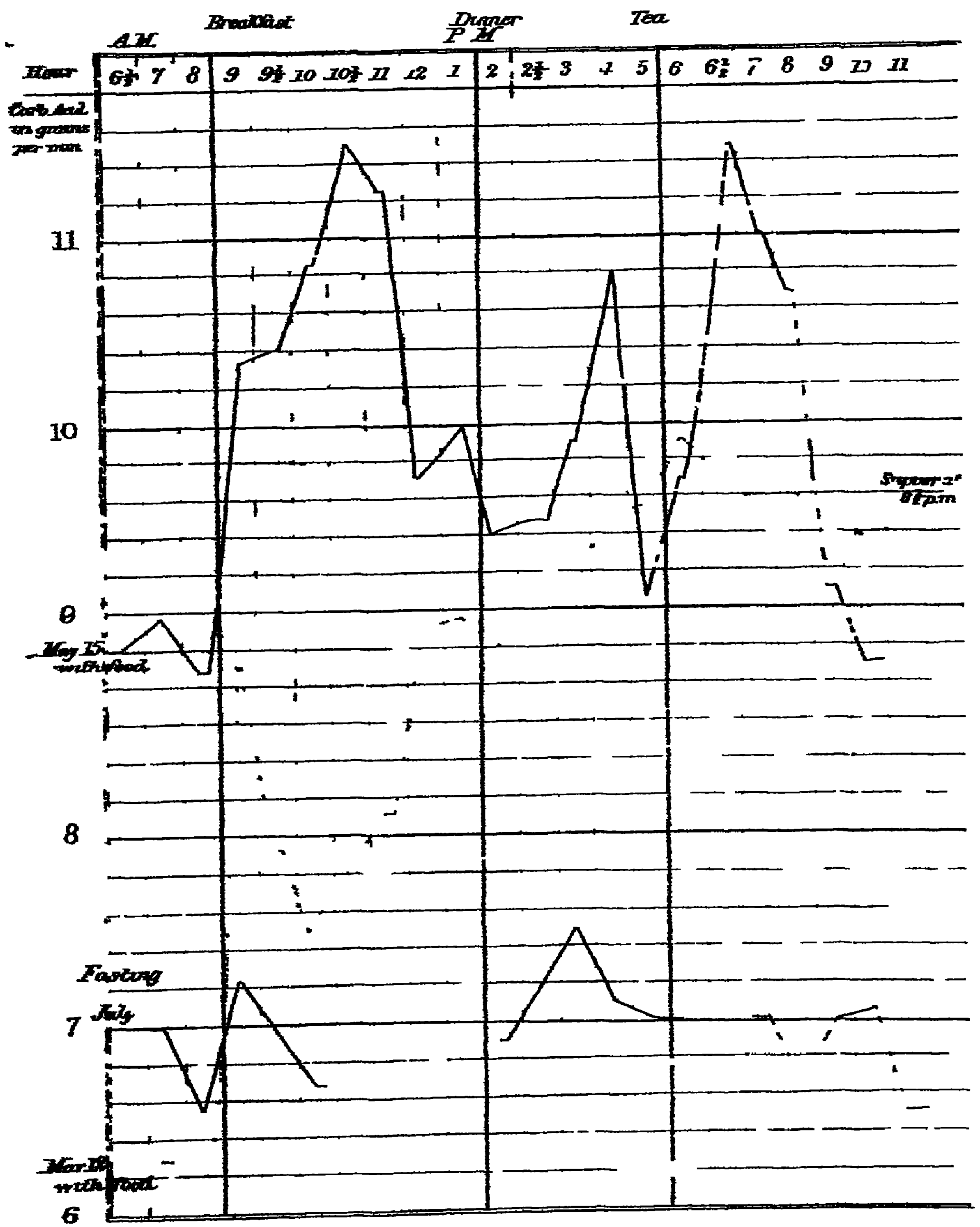

Diagram showing the quantity of Carbonic Acid evolved during the day with food on two occasions and without food on one occasion.

FIG 2

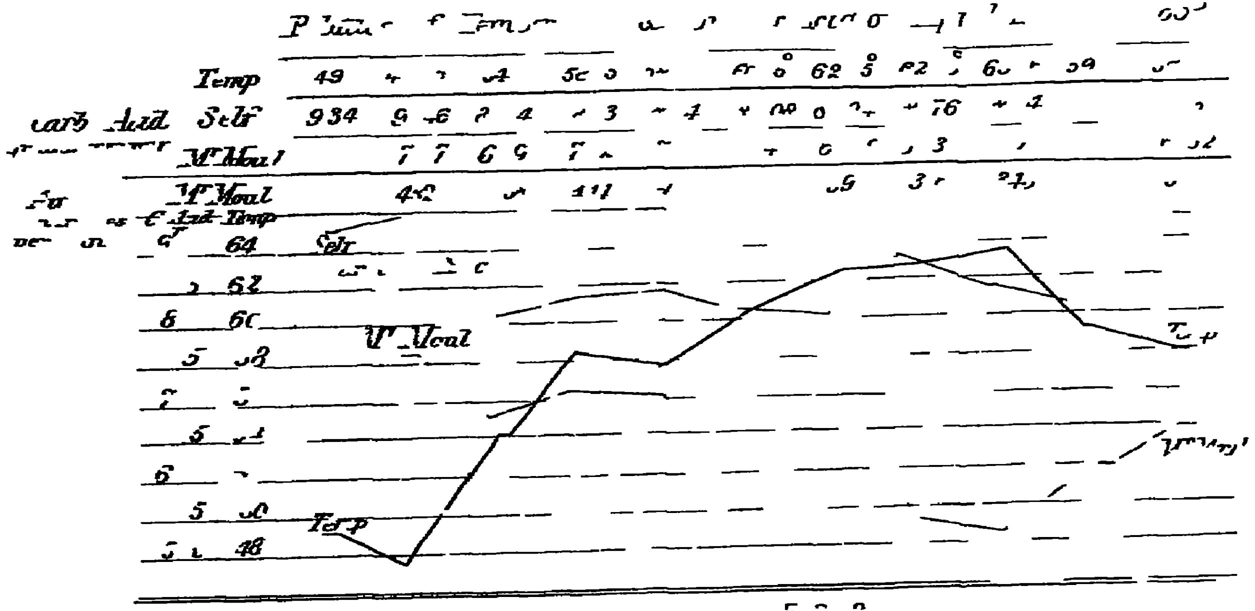

FIG 3

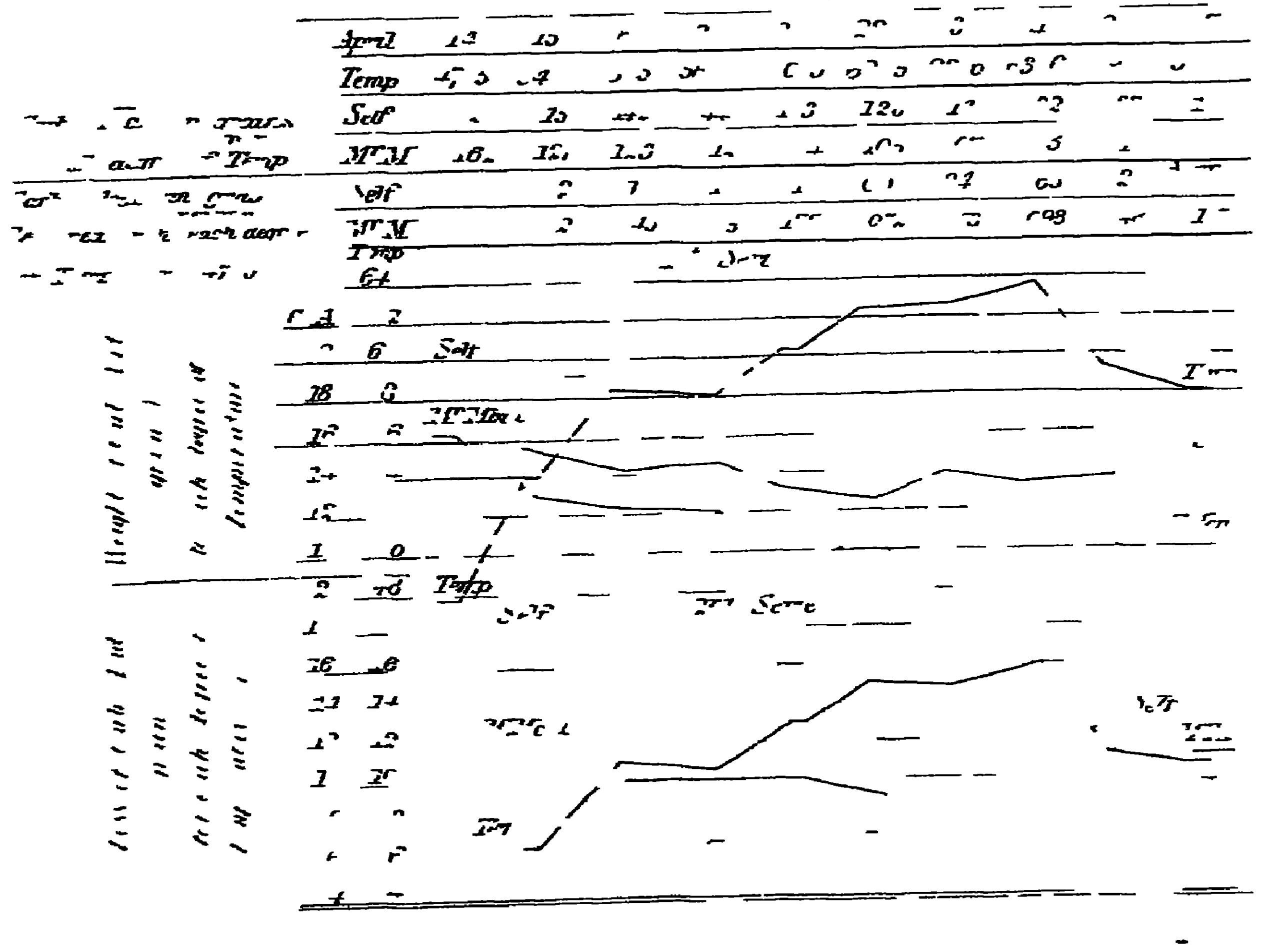

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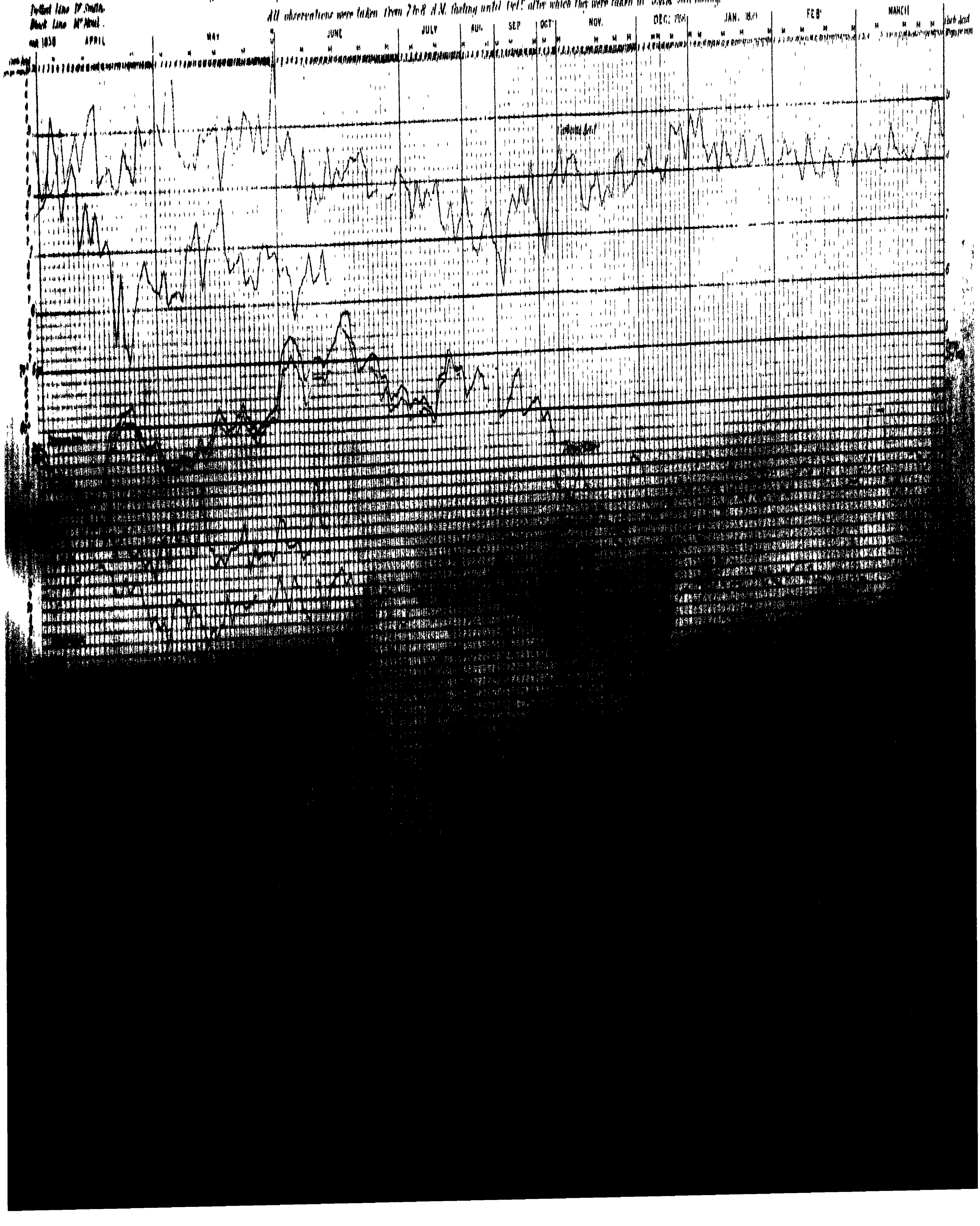
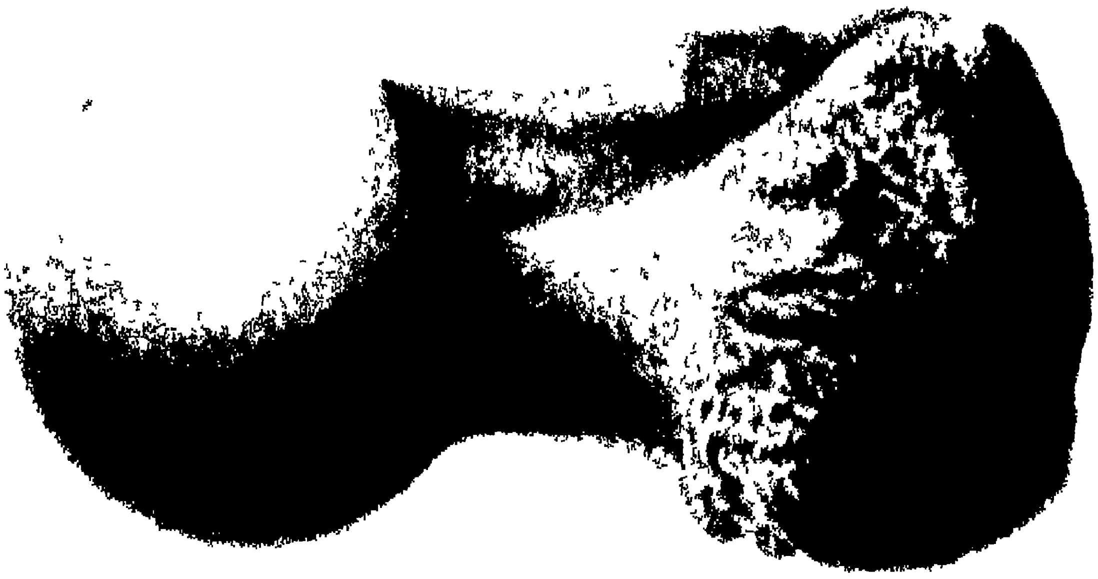
Diagram Shewing the effect of SEASON on the Respiration & Pulsation from a Series of Observations made throughout the Year.

All observations were taken from 7 to 8 A.M. fasting until Oct. after which they were taken at 9 A.M. still fasting.





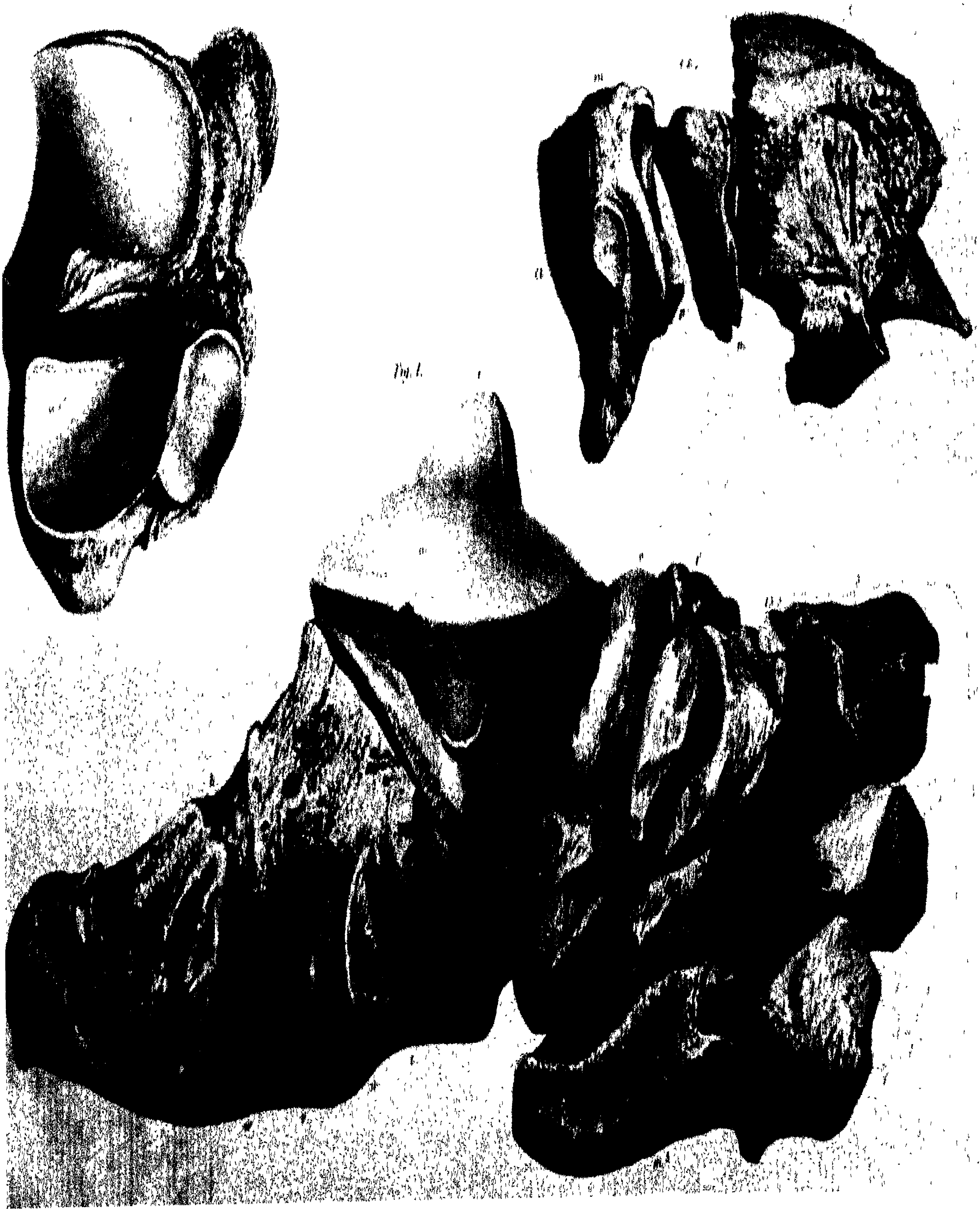
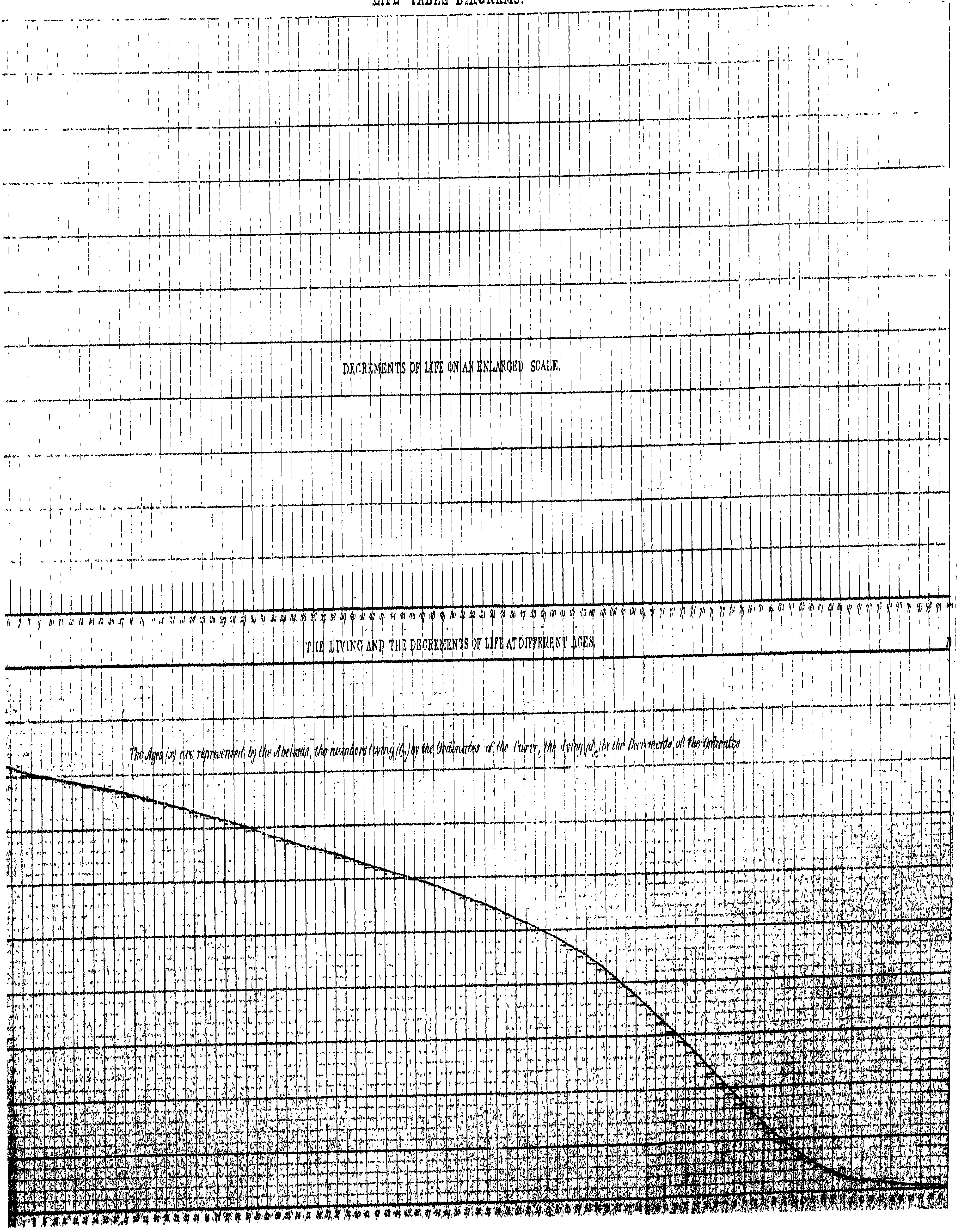


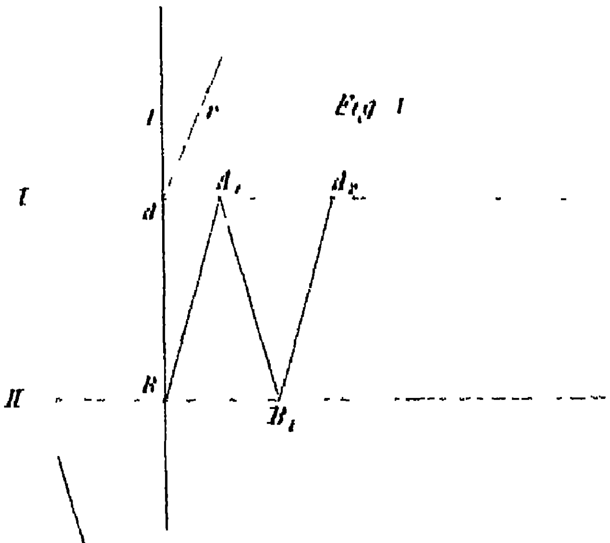
Fig. 1.

HEALTHY DISTRICTS.

LIFE TABLE DIAGRAMS.

Phil. Trans. MDCCCLIV. Plate XIII.

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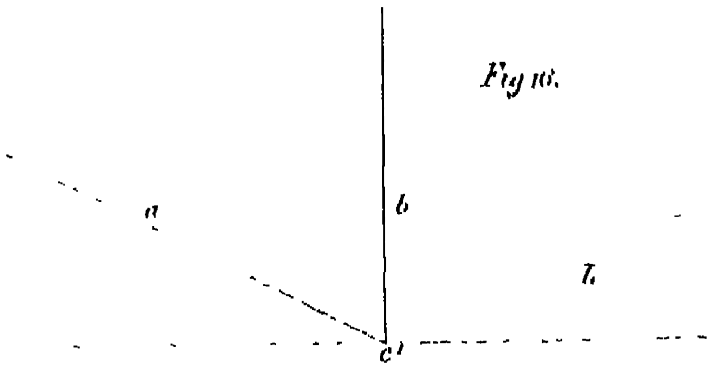
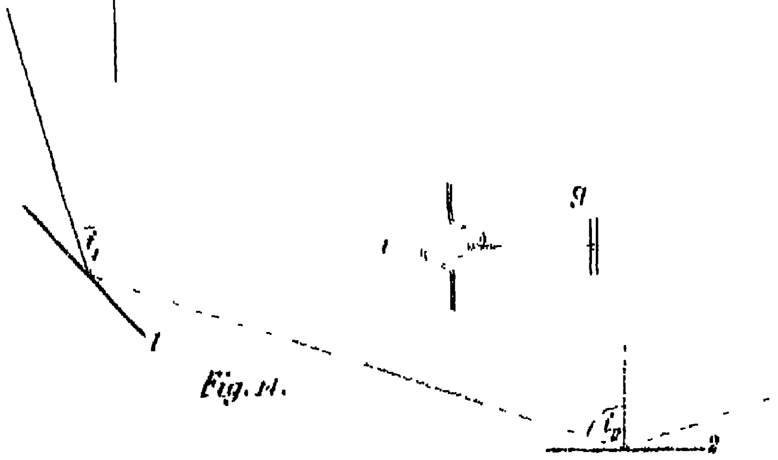


Fig. 10.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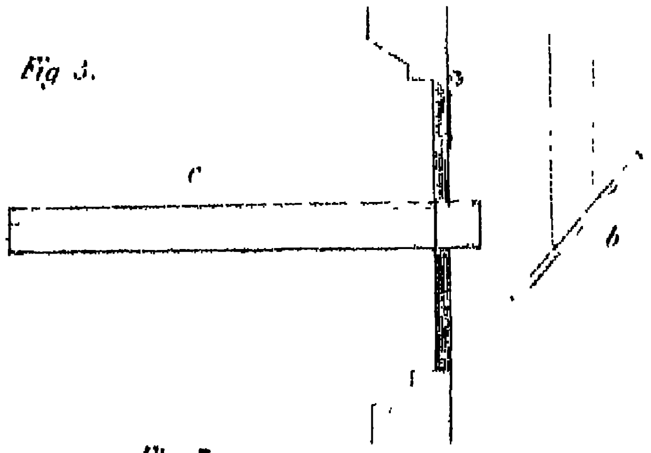
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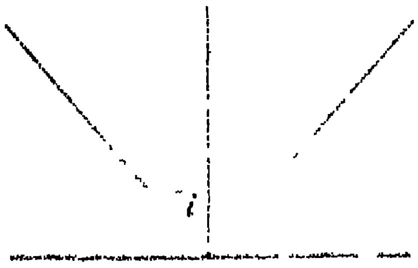
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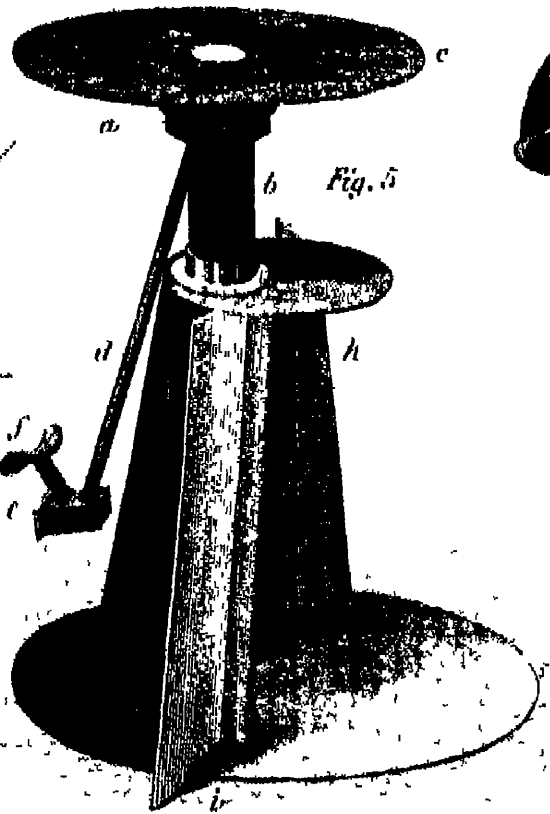
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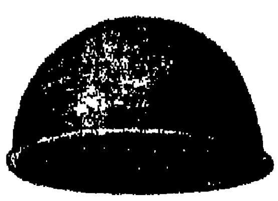
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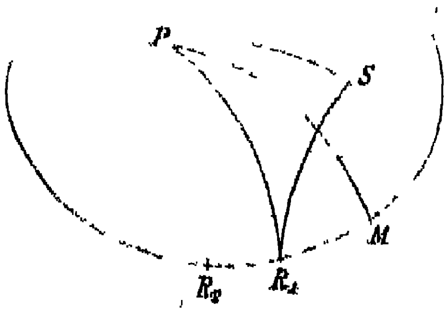
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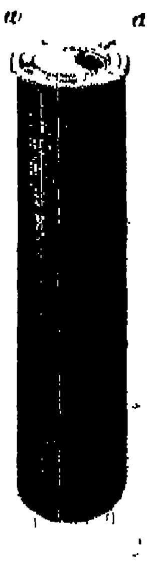
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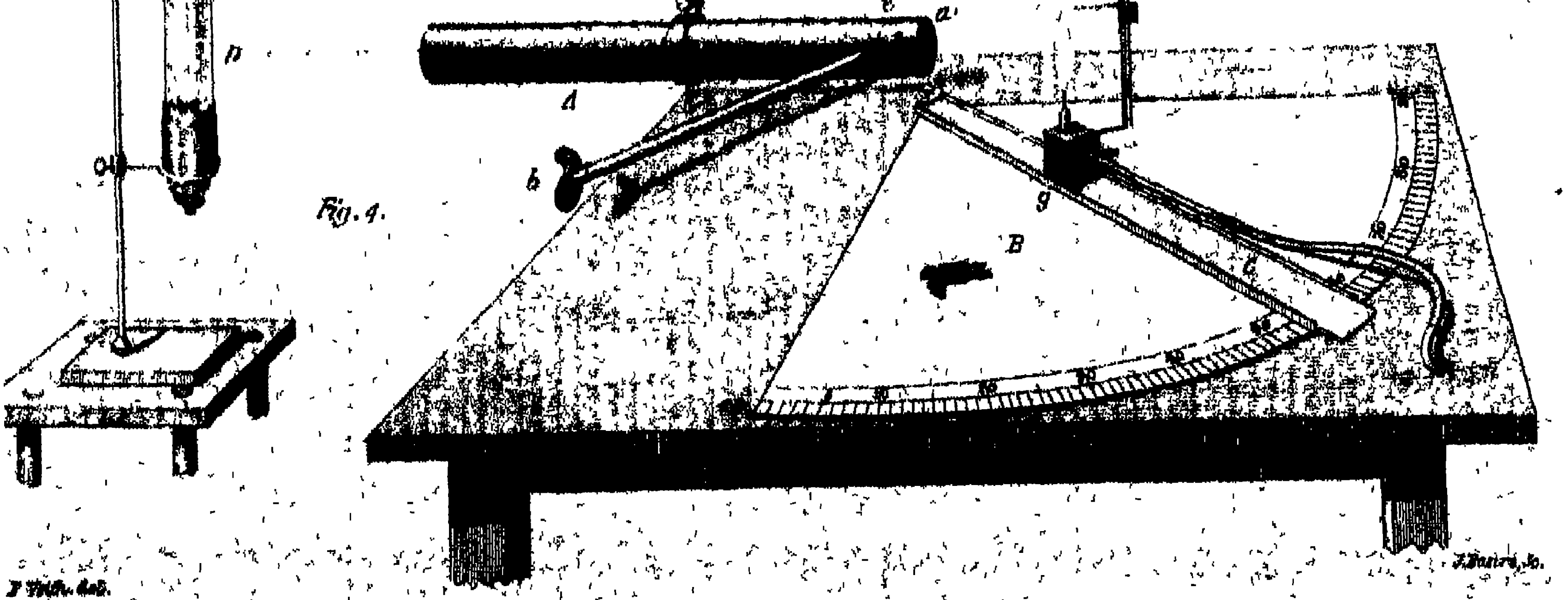
Fig. 4.

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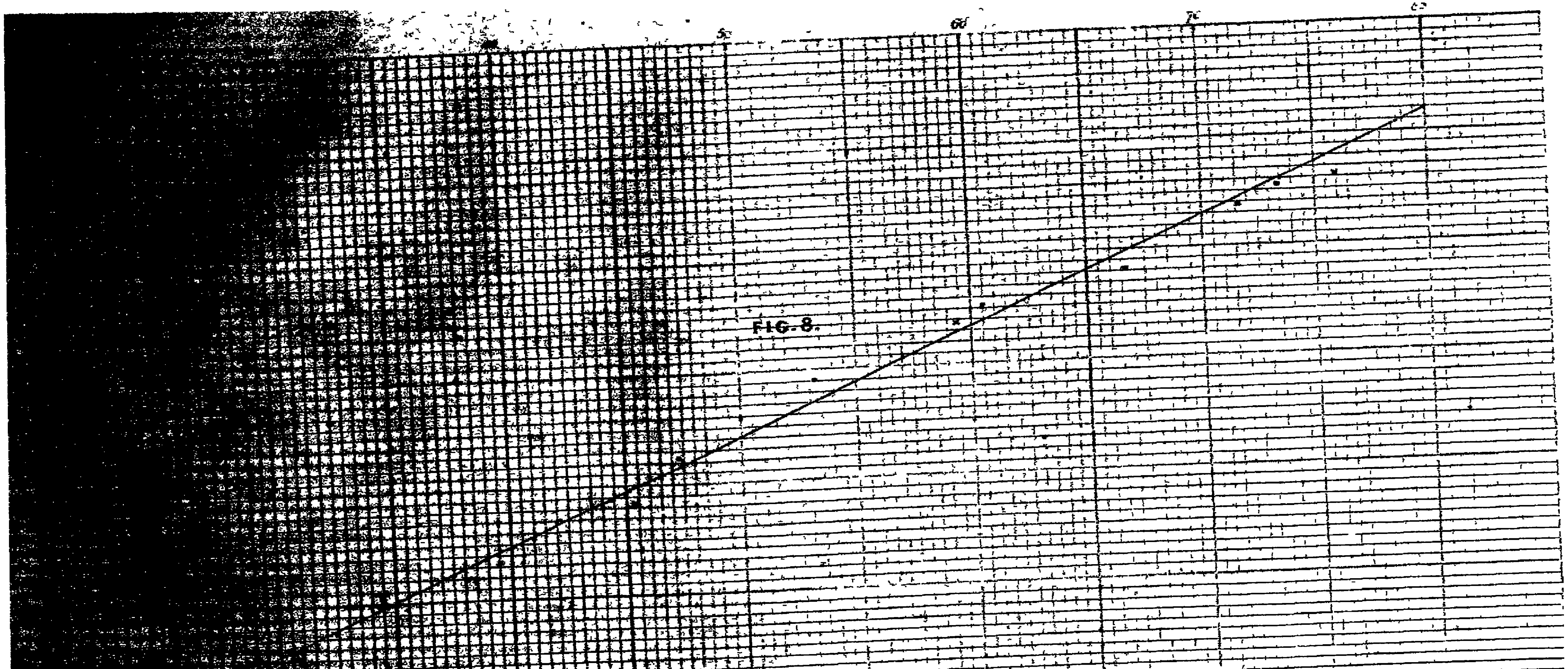

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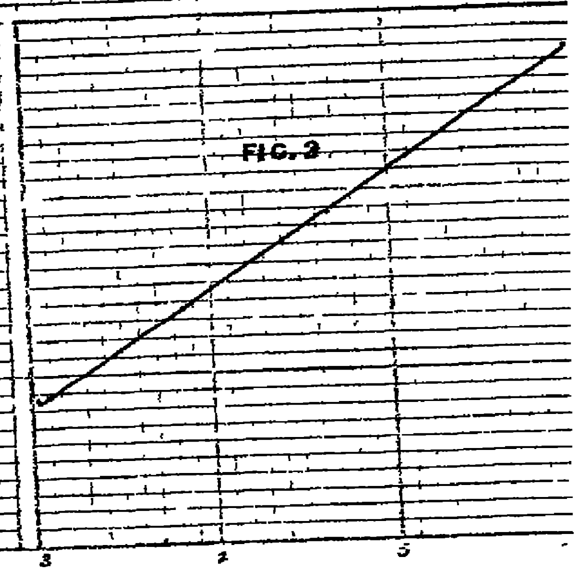
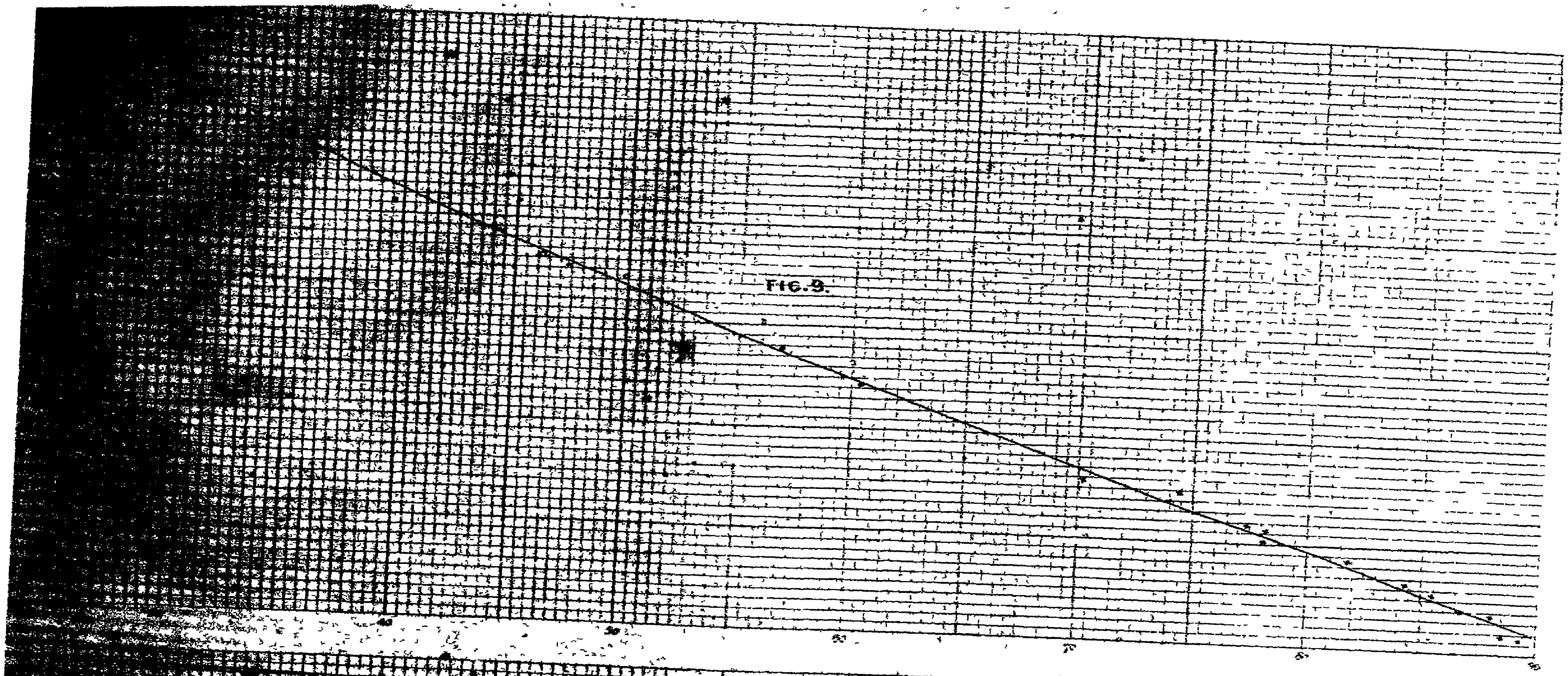


FIG. 3



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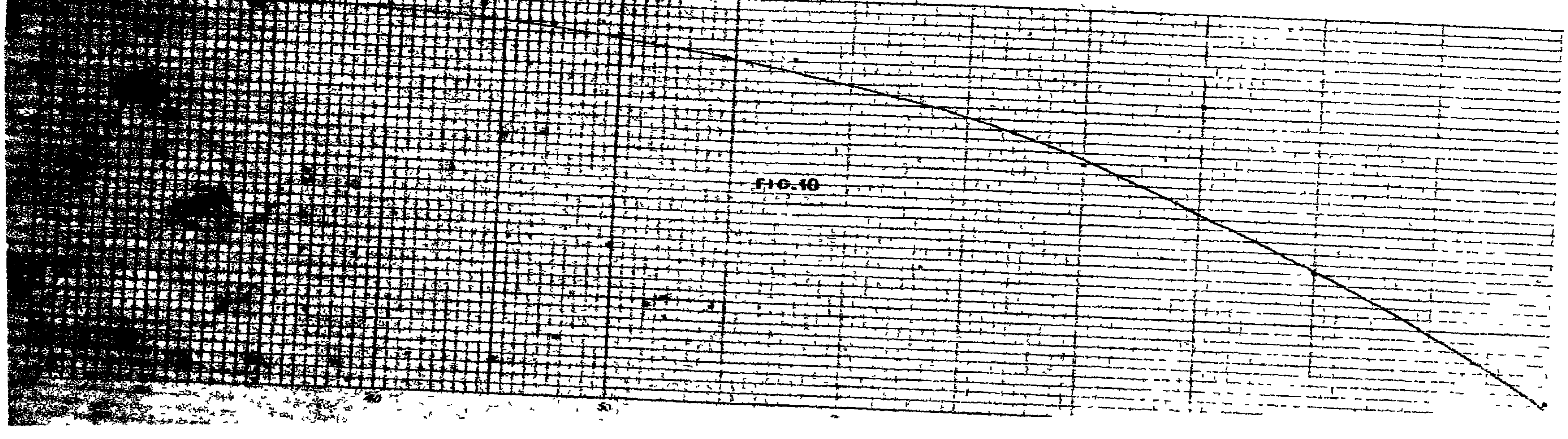
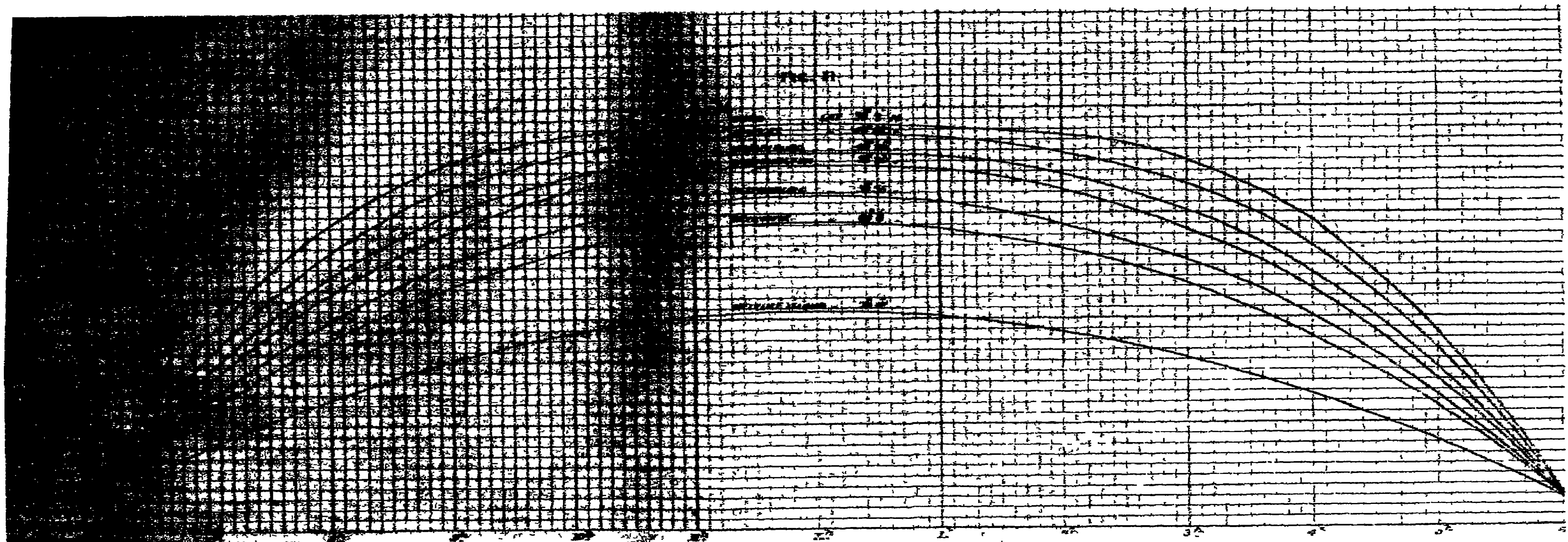


FIG. 10



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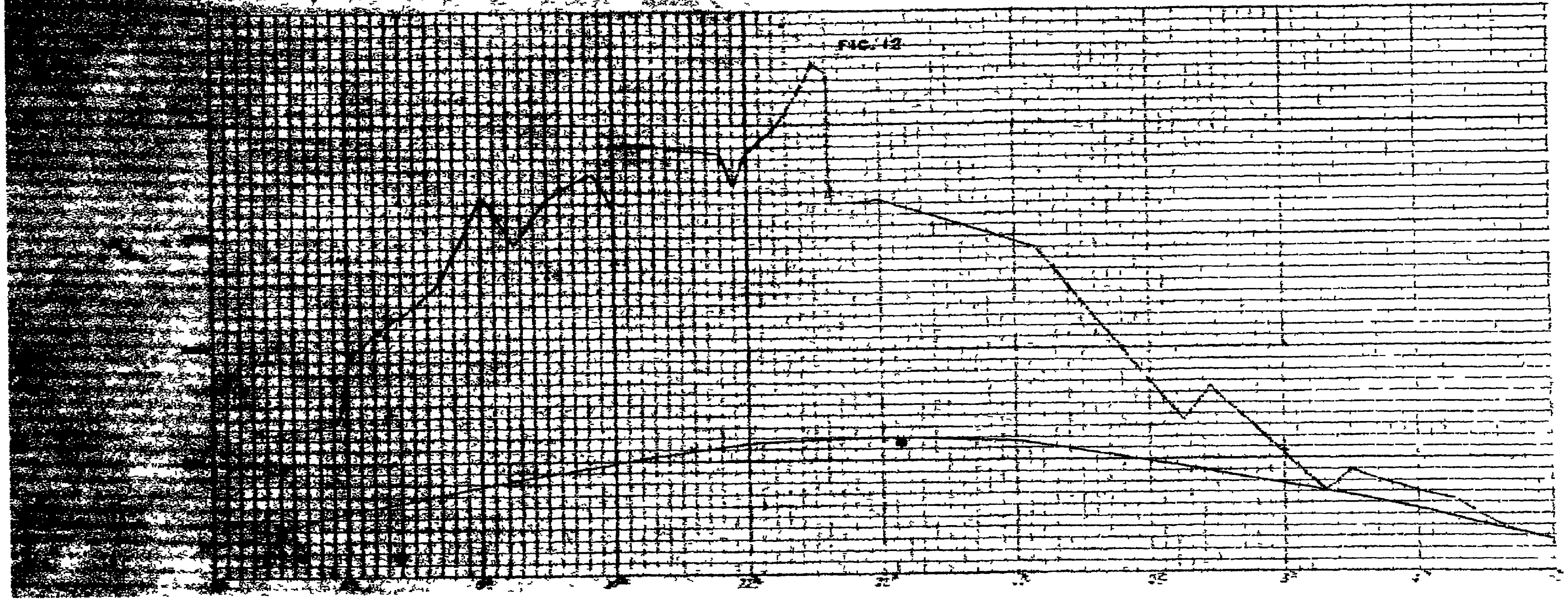
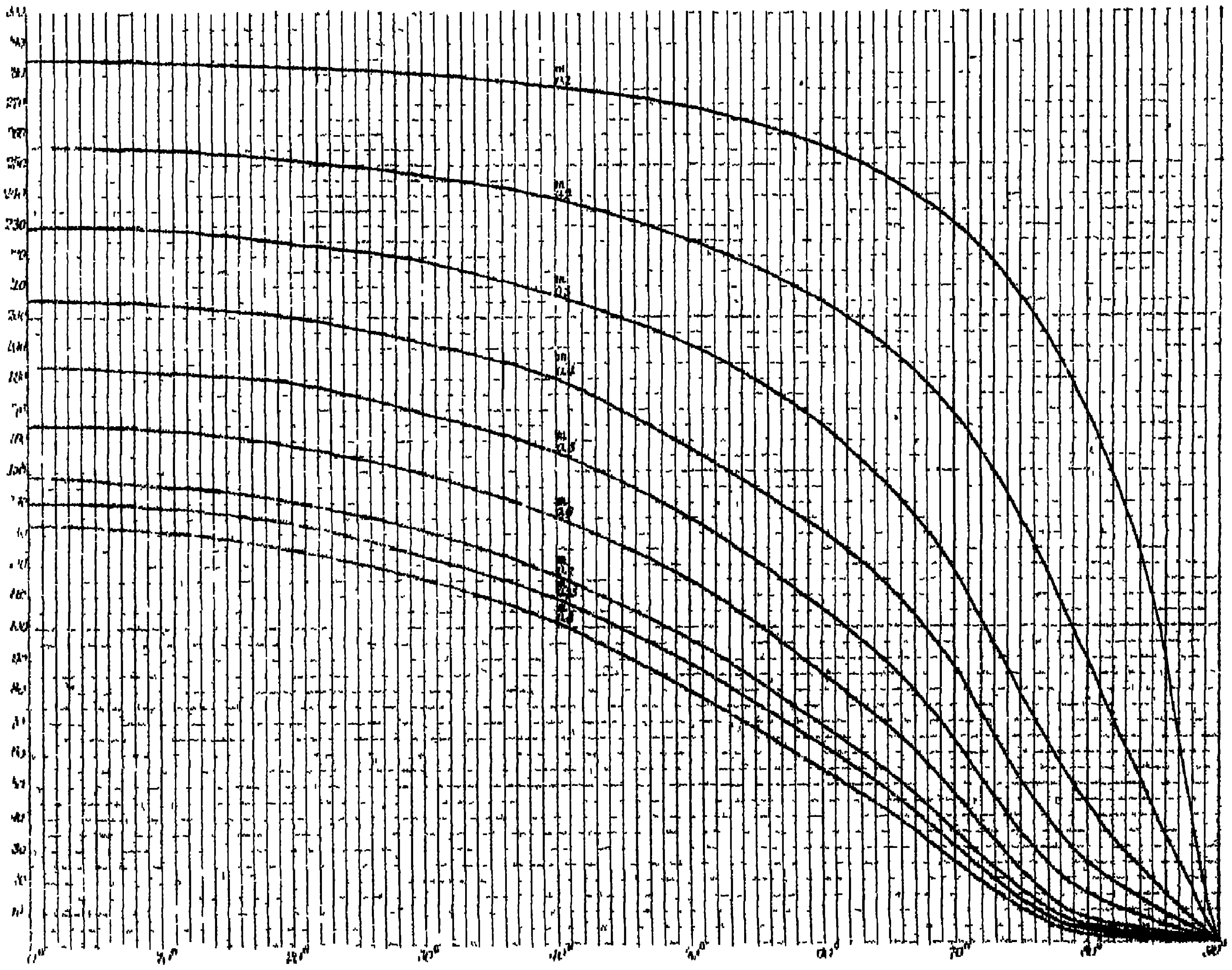


FIG. 12

FIG 17.



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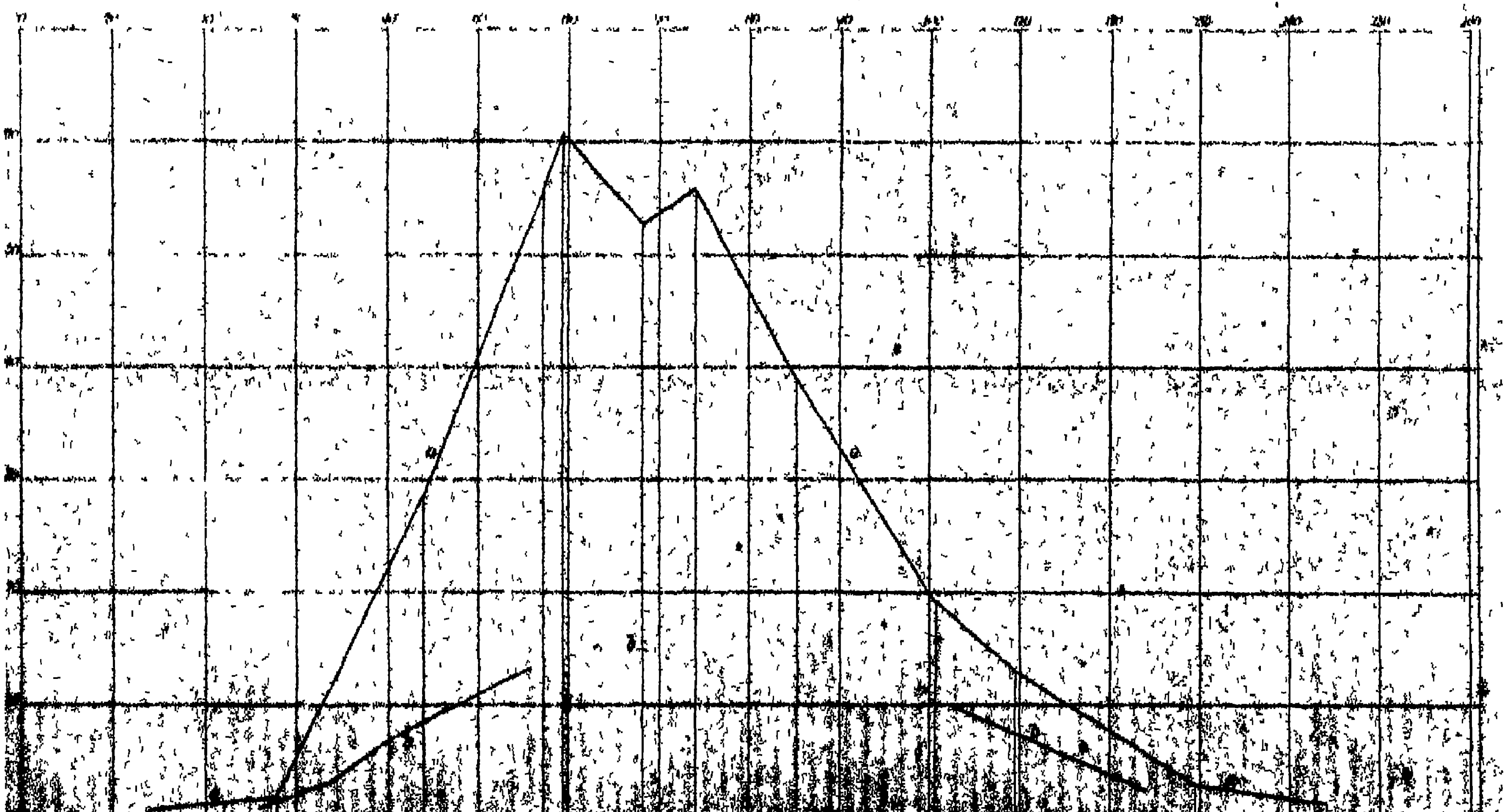
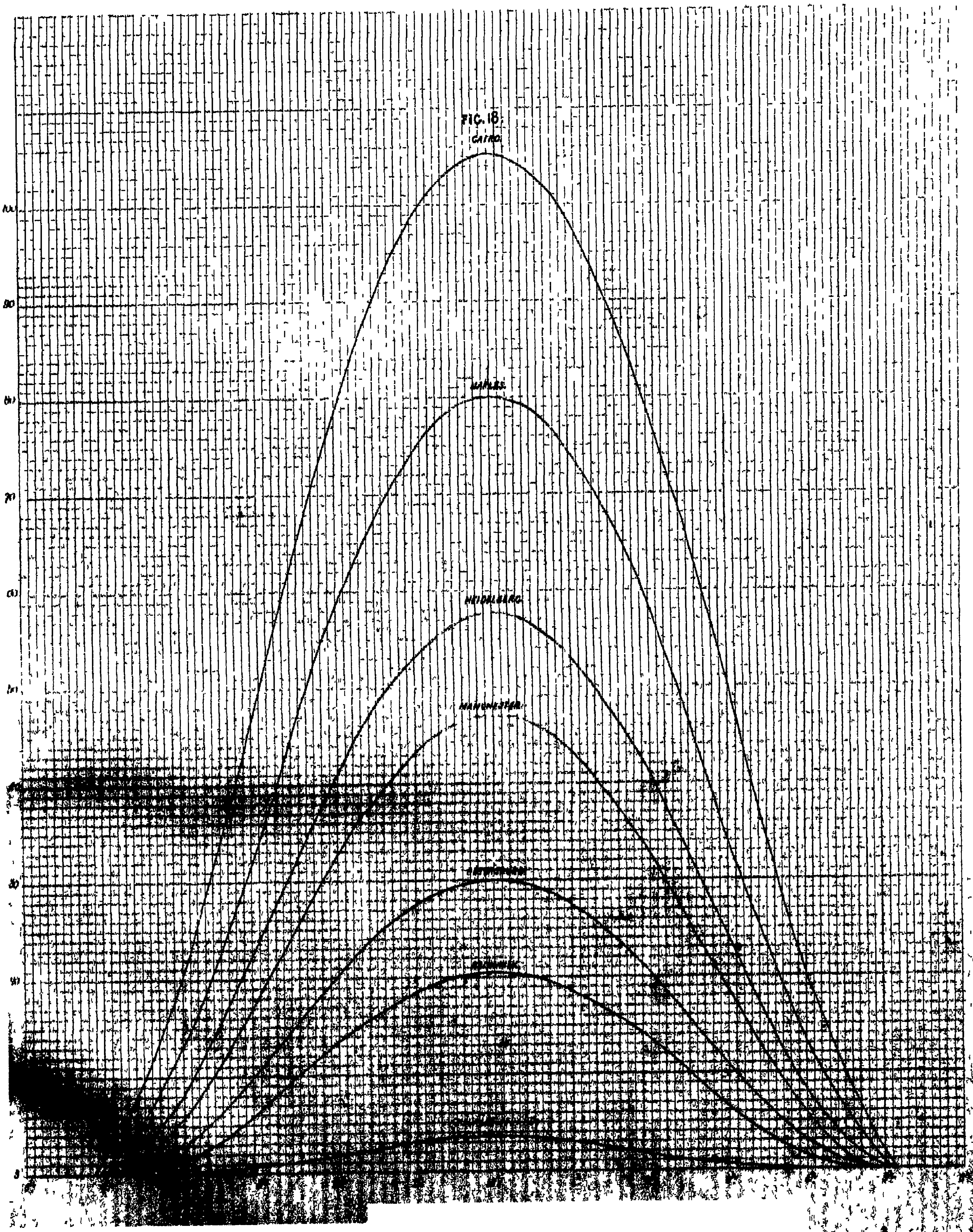


FIG. 18.



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